The latent meaning of forcing in quantum mechanics

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1. Motivations and mathematical background
2. Forcing and quantum mechanics
   - Some historical remarks
   - Micro to macroscale shift
3. Relations with topology
   - The Sierpiński-Erdös duality theorem
   - Cosmological models on exotic smooth $\mathbb{R}^4$
4. Exemplary application
   - The cosmological constant problem
5. Overview of the perspectives

- QM-GR INCOMPATIBILITY
- COMMON USE OF THE REAL LINE $\mathbb{R}$ IN PHYSICS

ZF(C) = ZERMELO-FRAENKEL SET THEORY (WITH THE AXIOM OF CHOICE)

MATHEMATICAL BACKGROUND

- $\mathbb{R}$ WITHIN FRAMEWORK OF ZFC
- MODEL OF AN AXIOMATIC THEORY
- NON-ISOMORPHIC MODELS OF ZFC

Forcing as a formal tool:

- changing the model
- "adding reals"

.... For <u>some</u> models of ZFC ....

$\mathcal{M}_1, \mathcal{M}_2$ - non-isomorphic models of ZFC

$$\mathbb{R}_{\mathcal{M}_1}, \mathbb{R}_{\mathcal{M}_2} \subset \mathbb{R} \qquad \mathbb{R}_{\mathcal{M}_1} \neq \mathbb{R}_{\mathcal{M}_2}$$

Different models of ZFC $\Leftrightarrow$ **Different sets of reals**

Paul A. Benioff (1976)

QM formalism not in
a single model of ZFC

Particular position of QM
formalism in mathematics

William Boos (1996) ⋄ Robert A. van Wesep (2006)

If there are "semiclassical states" realizing LHV program, then they are generic ultrafilters (objects specific to forcing constructions).

Jerzy Król (2004)

"Dynamical network of models"
(adding result of a measurement by forcing)

Suppose that the real numbers, which parametrize space, come from the quantum realm via continous measurement ($\sim$ position observable).

There is always a nontrivial forcing on the MEASURE ALGEBRA on $\mathbb{R}^3$ when passing from the quantum to classical (GR) regime

(Jerzy Król, P. K. - 2015)

THE DUALITY THEOREM*

$$f\colon \mathbb{R} \xrightarrow{1-1} \mathbb{R}$$
$$f = f^{-1}$$
$$f(x) \in \mathcal{O} \Leftrightarrow x \in \mathcal{M}$$
$$f(x) \in \mathcal{M} \Leftrightarrow x \in \mathcal{O}$$

*Under Continuum Hypothesis

$$\mathcal{O} = \{x \subset \mathbb{R} | \mu(x) = 0\}$$
$$\mathcal{M} = \{x \subset \mathbb{R} | x \text{ is } \textit{meager}\}$$

μ - Lebesgue measure

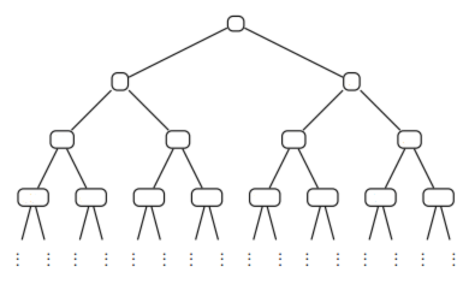
MEASURE ALGEBRA $\mathfrak{Bor}(\mathbb{R})/\mathcal{O}$ $\leftrightarrow$ **COHEN ALGEBRA** $\mathfrak{Bor}(\mathbb{R})/\mathcal{M}$

QM→GR SHIFT

ANY PHYSICAL MEANING?

→ Unique atomless complete Boolean algebra with countable dense subset

Infinite binary tree



It represents Cantor set 2^ω and Casson handles. Nodes correspond to the countable dense subset.

Casson Handles

The description of smooth geometries, especially on $\mathbb{R}^4$

⇒ Measure algebra has no such countable dense subset

Infinite binary tree $\rightarrow$ exotic smooth structures
they exist only in dimension 4!

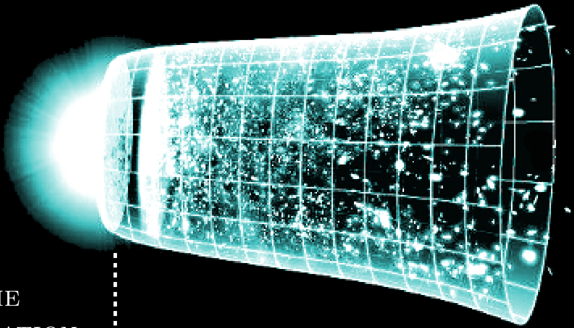
Torsten Asselmeyer-Maluga, Jerzy Król (2014)

We can use exotic smooth $\mathbb{R}^4$ (instead of the standard one) while building cosmological models, obtaining realistic parameters:

- the Cosmological Constant value
- the Shape of the Primordial Inflation Potential
- the Expansion Rate of the Universe

[T. Asselmeyer-Maluga, J. Król, Adv. High Energy Phys., 867460 (2014)]

Exemplary application



Spacetime parametrization via changing $\mathbb{R}_{\mathcal{M}}$'s

$\star$ The full real line $\mathbb{R}$ parametrization

$\star$ The growth of inherent density of reals $\leftrightarrow$ spacetime inflation

The zero-point energy of quantum field corresponding to a particle of mass m:

$$\frac{E}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{\mathbf{k}^2+m^2}}{2}$$

$\Rightarrow$ CONTRIBUTIONS TO Ω_Λ FAR TOO BIG

ZERO MODES INTERNAL IN ZFC MODELS $\rightarrow$ INTEGRATING OVER A MEAGER $\mathbb{R}^3_{\mathcal{M}} \subset \mathbb{R}^3$

MATHEMATICAL FACT

ALL MEASURABLE SUBSETS OF $\mathbb{R}^3_{\mathcal{M}}$ HAVE MEASURE 0!

$\Rightarrow$ CONTRIBUTIONS VANISH (COMPLETELY)

Current Research:

- ⋆ Attempt to find the roots of primordial inflation on a formal level
- ⋆ Further exploration of the presented microscale $\leftrightarrow$ macroscale formal connection via Cohen algebra

Plans for future work:

1. Attempt to find the impact of meager ($\sim$ rarefied) real line on the various distortions of CMB
2. Description of quantum entanglement in the forcing language

$\sim$ Thank You $\sim$

for Your Attention