Gluon Propagator in Background Field at Next-to-Eikonal Order

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Outline

- Eikonal approximation and relaxing it
- gluon propagator in background field
  - Gluon propagator at eikonal approximation
  - Next-to-eikonal corrections for gluon propagator
  - Full result
- derivation of s-matrix
- Summary and conclusion

Introduction

- Generally in high energy scattering, in case of projectile target collision:

  Projectile: dilute proton

  Target: dense nucleus(described by CGC)

- In this case for simplification

  Projectile: Gluon parton

  Target: gluon background field $A_\mu(x)$

There is hierarchy between components of $A_\mu(x)$ with respect to Lorentz boost factor γ of the target:

$$A^- = O(\gamma) \gg A_j = O(1) \gg A^+ = O\left(\frac{1}{\gamma}\right)$$

Eikonal Approximation: leading order energy term(leading term in γ)

Eikonal and beyond it

Zero Width

1.Highly boosted background field(target) is localised in the longitudinal direction $x^+ = 0$(zero width).

Leading Component

2.Only leading component of target (- component) is considered and subleading components are neglected (suppressed by Lorentz boost factor).

x^- independence

3.Dynamics of the target are neglected(x^- dependence of target neglected).

Background field of target is : $A^\mu\left(x^-, x^+, \boldsymbol{x}\right) \approx \delta^{\mu-}\delta\left(x^+\right) A^-\left(\boldsymbol{x}\right)$

Beyond Eikonal:

Eikonal and beyond it

Zero Width

Highly boosted background field(target) is localised in the longitudinal direction $x^+ = 0$(zero width).

Leading Component

Only leading component of target (- component) is considered and subleading components are neglected (suppressed by Lorentz boost factor).

x^- independence

Dynamics of the target are neglected(x^- dependence of target neglected).

Background field of target is : $A^{\mu}\left(x^{-}, x^{+}, \boldsymbol{x}\right) \approx \delta^{\mu-} \delta\left(x^{+}\right) A^{-}(\boldsymbol{x})$

Beyond Eikonal:

- In very high energy accelerators like **LHC**(γ~1000 order)**,** NEik order terms **are negligible** while calculating observables.
- But to analyze the data from **RHIC** and future electron ion collider (**EIC**)(γ~ 10-100 order), NEik order terms might be **sizable**!

Eikonal and beyond it

Zero Width

1.Highly boosted background field(target) is localised in the longitudinal direction $x^+ = 0$(zero width).

Leading Component

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x^- independence

3.Dynamics of the target are neglected(x^- dependence of target neglected).

Background field of target is : $A^{\mu}\left(x^{-}, x^{+}, \boldsymbol{x}\right) \approx \delta^{\mu -} \delta\left(x^{+}\right) A^{-}(\boldsymbol{x})$

To go beyond Eikonal:

Finite Width

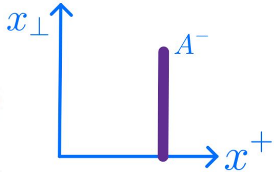
1.Instead of infinite thin shockwave as a target, we consider finite width of a target.

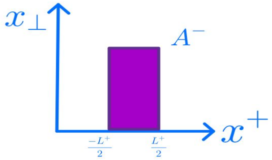
Transverse Component

2.Instead of neglecting sub-leading components, we include transverse component of background field.

x^- dependence

3.We take into account corrections coming due to the x^- dependence of a target(consider background field is x^- dependent).

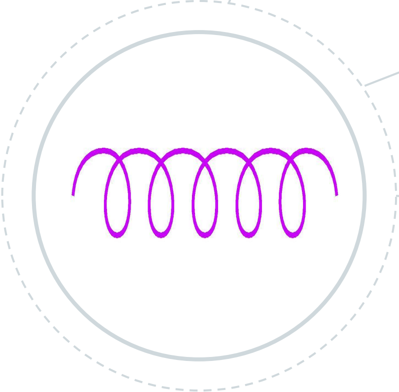




Gluon Propagator

In presence of medium

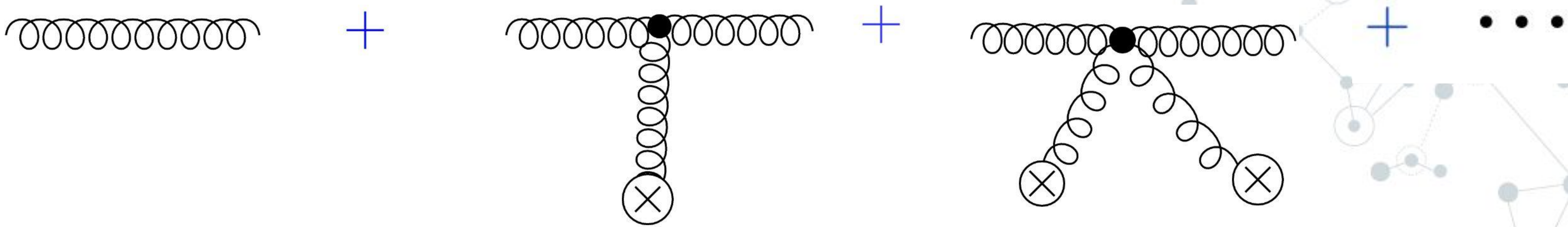
Gluon Propagator



In vacuum Gluon propagator in momentum space is give as:

$$G^{\mu\nu}_{0,F}(p) = \frac{i}{p^2 + i\varepsilon}\left[-g^{\mu\nu} + \frac{p^\mu \eta^\nu + \eta^\mu p^\nu}{p \cdot \eta}\right]$$

This is in Light-cone gauge, $A^+ = 0$ and $\eta^2=0$
Where, $\eta^\mu = g^{\mu +}$



We are computing Gluon propagator in a classical background gluon field $A_\mu(x)$ at next-to-eikonal(NEik) accuracy. Without medium it will reduce to vacuum propagator!!

- We can write gluon propagator as

$$G_F^{\mu\nu}(x,y) = \underbrace{G_{0,F}^{\mu\nu}(x,y)}_{\text{Vacuum contribution}} + \underbrace{\delta G_F^{\mu\nu}(x,y)}_{\text{Medium correction}}$$

- We can write total gluon propagator as

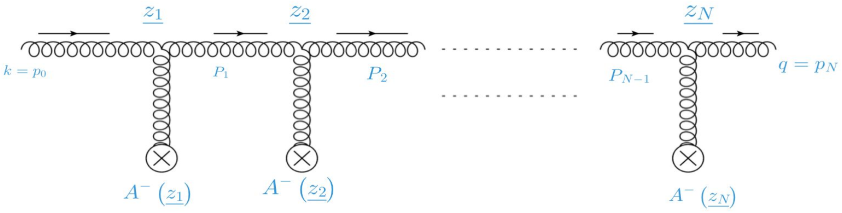
$$G_F^{\mu\nu}(x,y) = G_{0,F}^{\mu\nu}(x,y) + \delta G_F^{\mu\nu}(x,y)$$

Vacuum contribution Medium correction

- We compute gluon propagator in Eikonal approximation then include NEik corrections.
- We will get NEik correction due to three different effects as mentioned before:
  - Due to insertion of transverse components of background field
  - Due to x^- component of background field
  - Due to finite width of target

Method to calculate Eikonal order Gluon Propagator

- To calculate gluon propagator at eikonal order, we re-sum multiple interaction diagrams of Gluon background field as shown in figure below.
- And consider only “-” component (leading) of classical gluon background field.
- Then we take eikonal approximation to obtain following expression.



Gluon propagator at Eikonal approximation

$$G_F^{\mu\nu}(x,y)|_{Eik} = G_{0,F}^{\mu\nu}(x,y) + \delta G_F^{\mu\nu}(x,y)|_{Eik}$$

$$\begin{aligned} G_F^{\mu\nu}(x,y)|_{Eik} &= i\delta^2(x_\perp - y_\perp)\ \delta(x^+ - y^+)\eta^\mu\eta^\nu \left[\int \frac{dk^+}{2\pi} \frac{e^{-i(x^- - y^-)k^+}}{k^+ k^+} \right] \\ &+ \Bigg\{ \int \frac{d^3 q}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3} \frac{e^{-ix\cdot\check{q}}\ e^{iy\cdot\check{k}}}{2k^+} \left[2\pi\ \delta(k^+ - q^+) \right] \\ &\left[-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+ k^+}(\check{q}\cdot\check{k}) \right] \left[\int d^2 z_\perp\ e^{-i(q_\perp - k_\perp)z_\perp} \right] \Bigg\} \\ &\left[\theta(x^+ - y^+)\ \theta(k^+)\ U_A(x^+, y^+; z_\perp) - \theta(y^+ - x^+)\ \theta(-k^+)\ U_A^\dagger(x^+, y^+; z_\perp) \right] \end{aligned}$$

Gluon propagator at Eikonal approximation

$$G_F^{\mu\nu}(x,y)|_{Eik} = G_{0,F}^{\mu\nu}(x,y) + \delta G_F^{\mu\nu}(x,y)|_{Eik}$$

$$G_F^{\mu\nu}(x,y)|_{Eik} = i\delta^2(x_\perp - y_\perp)\;\delta(x^+ - y^+)\eta^\mu\eta^\nu\Bigg[\int\frac{dk^+}{2\pi}\,e^{-i(x^-\cdots}$$

$$+\Bigg\{\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\,\frac{e^{-ix\cdot\check{q}}\;e^{iy\cdot\check{k}}}{2k^+}\Big[2\pi\;\delta(\cdots$$

$$\left[-g^{\mu\nu}+\frac{\check{k}^\mu\eta^\nu}{k^+}+\frac{\eta^\mu\check{q}^\nu}{q^+}-\frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right]\left[\int d^2z_\perp\;e^{-i(q_\perp-k_\perp)z_\perp}\right]\Bigg\}$$

$$\left[\theta(x^+-y^+)\;\theta(k^+)\;U_A(x^+,y^+;z_\perp) - \theta(y^+-x^+)\;\theta(-k^+)\;U_A^\dagger(x^+,y^+;z_\perp)\right]$$

This is **Wilson line**, which contains summation of all the A^- insertions!

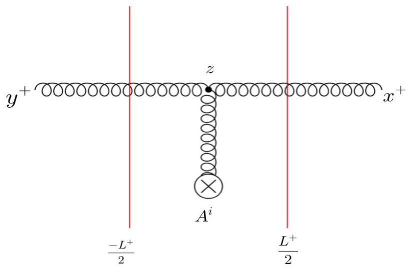
Where, $$U_A(x^+,y^+;z_\perp) = 1 + \sum_{N=1}^{\infty}\frac{1}{N!}\mathcal{P}_+\left[-ig\int_{y^+}^{x^+}dz^+A^-(z^+,z_\perp)\cdot T\right]^N$$

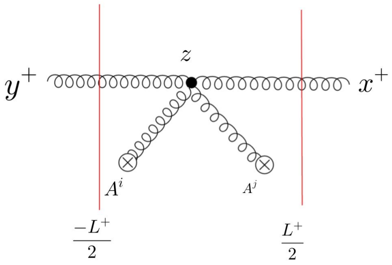
NEik Corrections: Due to Insertion of transverse component

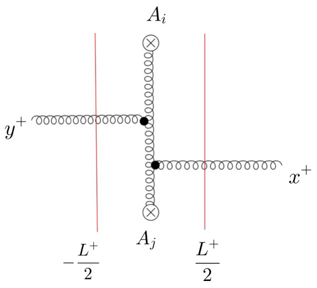
Single three gluon vertex

Four gluon vertex

Double three gluon vertex







NEik correction: due to including transverse component of A_μ

We will calculate NEik corrections to Gluon propagator by evaluating following kind of equations:

$$\delta G_F^{\mu\nu}(x,y) = \int d^4z \; G_F^{\mu\mu'}(x,z)\Big|_{Eik} X_{\mu'\nu'}(\underline{z}) \; G_F^{\nu'\nu}(z,y)\Big|_{Eik}$$

Where, $X_{\mu'\nu'}(\underline{z})$ is insertion factor.(depends upon insertions of background field)

Calculating corrections

We will calculate NEik corrections to Gluon propagator by evaluating following kind of equations:

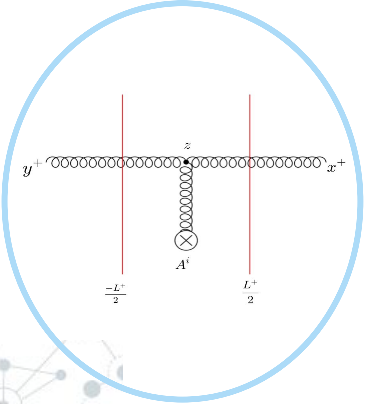
$$\delta G_F^{\mu\nu}(x,y) = \int d^4z \; G_F^{\mu\mu'}(x,z)\Big|_{Eik} X_{\mu'\nu'}(\underline{z}) \;\; G_F^{\nu'\nu}(z,y)\Big|_{Eik}$$

Where, $X_{\mu'\nu'}(\underline{z})$ is insertion factor.(depends upon insertions of background field)

After many of filled blackboards and simplifications of integration, we get……

NEik result

Correction due to single insertion of $A_\perp$ (Due to three gluon vertex):

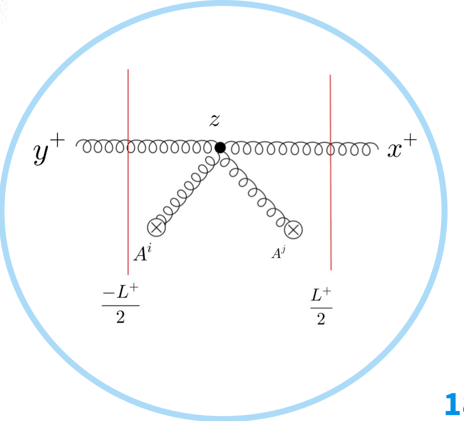


$$
\begin{aligned}
\delta G_F^{\mu\nu}{}_{ab}(x,y)\big|_{\text{single}A_\perp} = g &\int d^3z \int \frac{d^3q}{(2\pi)^3}\,\frac{e^{-ix\cdot\check{q}}}{2q^+}\,\theta(x^+-z^+)\theta(q^+) \int \frac{d^3k}{(2\pi)^3}\,\frac{e^{iy\cdot\check{k}}}{2k^+}\theta(k^+)\left(2\pi\delta(q^+-k^+)\right) \\
&\Big[U_A(x^+,z^+;z_\perp)\Big]_{aa'}\; e^{-iq_\perp z_\perp}\Bigg\{2\Bigg[\left(g^{\mu j}g^{\nu i} - \frac{\eta^\mu g^{\nu i}q^j}{q^+} - \frac{g^{\mu j}k^i\eta^\nu}{q^+} + \frac{\eta^\mu\eta^\nu k^i q^j}{q^+q^+}\right) \\
&-\left(g^{\mu i}g^{j\nu} - \frac{\eta^\mu q^i g^{j\nu}}{q^+} - \frac{g^{\mu i}k^j\eta^\nu}{q^+} + \frac{\eta^\mu\eta^\nu q^i k^j}{q^+q^+}\right)\Bigg]\left[\frac{\overleftarrow{d}}{dz^i}\left(T\cdot A^j(z)\right) + \left(T\cdot A^j(z)\right)\frac{\overrightarrow{d}}{dz^i}\right] \\
&+\left[g^{\mu\nu} - \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\check{k}^\mu\eta^\nu}{q^+} + \frac{\eta^\mu\eta^\nu}{q^+q^+}(\check{q}\cdot\check{k})\right]\left[\frac{\overleftarrow{d}}{dz^j}\left(T\cdot A^j(z)\right) - \left(T\cdot A^j(z)\right)\frac{\overrightarrow{d}}{dz^j}\right] \\
&\Bigg\}\; e^{ik_\perp z_\perp}\;\Big[U_A(z^+,y^+;z_\perp)\Big]_{b'b}\;\theta(z^+-y^+)
\end{aligned}
$$

NEik result

Correction due to four gluon vertex:

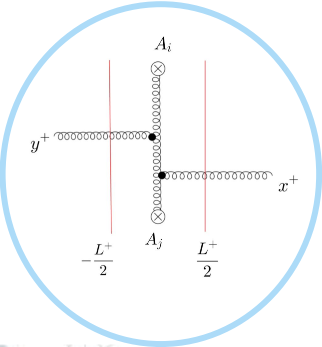
$$
\begin{aligned}
\delta G^{\mu\nu}_{F\ ab}(x,y)\big|_{\text{double}A_\perp} = {} & \frac{1}{2}\int d^3z \int \frac{d^3q}{(2\pi)^3}\,\frac{e^{-ix\cdot\check{q}}}{2q^+}\,\theta(x^+-z^+)\theta(q^+)\int\frac{d^3k}{(2\pi)^3}\,\frac{e^{iy\cdot\check{k}}}{2k^+}\,\theta(k^+)\,\Big[U_A(x^+,z^+;z_\perp)\Big]_{aa'} \\
& \Big[2\pi\delta(q^+-k^+)\Big]\, e^{-i(q_\perp-k_\perp)z_\perp}\,\Big(2ig^2\Big)\,\Big[T\cdot A^i(z)\Big]_{a'e}\Big[T\cdot A^j(z)\Big]_{eb'} \\
& \Bigg[\bigg(-g^{\mu\nu}g_{ij}+\frac{\check{k}^\mu\eta^\nu g_{ij}}{q^+}+\frac{\eta^\mu\check{q}^\nu g_{ij}}{q^+}-\frac{\eta^\mu\eta^\nu g_{ij}}{q^+q^+}(\check{k}\cdot\check{q})\bigg) \\
& +\bigg(g^\nu_i g^\mu_j-\frac{k_i g^\mu_j\eta^\nu}{q^+}-\frac{\eta^\mu g^\nu_i q_j}{q^+}+\frac{\eta^\mu\eta^\nu k_i q_j}{q^+q^+}\bigg)\Bigg] \\
& \Big[U_A(z^+,y^+;z_\perp)\Big]_{b'b}\,\theta(z^+-y^+)
\end{aligned}
$$



NEik result

Correction due to two three-gluon-vertex:

$$\delta G^{\mu\nu}_{ab}(x,y)|_{inst.A_iA_j} = g^2 \int d^3z \int \frac{d^3q}{(2\pi)^3}\frac{e^{-ix\cdot\check{q}}}{2q^+}\,\theta(x^+-z^+)\,\theta(q^+)\int\frac{d^3k}{(2\pi)^3}\,\frac{e^{iy\cdot\check{k}}}{2k^+}\,U_A(x^+,z^+;z_\perp)\,e^{-iq_\perp z_\perp}$$
$$\left[2\pi\,\delta(q^+-k^+)\right](-i)\left[T\cdot A^i(z)\right]\,\left[T\cdot A^j(z)\right]$$
$$\left[g^{\mu i}g^{j\nu}-\frac{g^{\mu i}k^j\eta^\nu}{k^+}-\frac{\eta^\mu q^i g^{j\nu}}{q^+}+\frac{\eta^\mu\eta^\nu q^ik^j}{q^+k^+}\right]\,e^{ik_\perp z_\perp}\,\theta(z^+-y^+)\,\theta(k^+)\,U_A(z^+,y^+;z_\perp)$$



Correction due to pure A⁻ component (finite width)

Medium correction due to pure A⁻ component of background field contains gluon propagator at eikonal order plus NEik correction, given as,

$$\delta G_F^{\mu\nu}(x,y)\,|_{pureA^-} = G_F^{\mu\nu}(x,y)\,|_{Eik} + \delta G_F^{\mu\nu}(x,y)\,|_{pureA^-}^{NEik}$$

$$\delta G_F^{\mu\nu}(x,y)|_{\mathrm{pure}A^-} = \int\frac{d^3\underline{q}}{(2\pi)^3}\int\frac{d^3\underline{k}}{(2\pi)^3}\,2\pi\delta(q^+-k^+)e^{-ix\cdot\check{q}}e^{iy\cdot\check{k}}\left[\frac{1}{2k^+}\left(-g^{\mu\nu}+\frac{\check{k}^\mu\eta^\nu}{k^+}+\frac{\eta^\mu\check{q}^\nu}{q^+}-\frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right)\right]$$
$$\theta(k^+)\int d^2z_\perp\;e^{-iz_\perp(q_\perp-k_\perp)}\Bigg\{\left[U_A\left(\frac{L^+}{2},-\frac{L^+}{2};z_\perp\right)-1\right]$$
$$-\frac{q^j+k^j}{4k^+}\int_{\frac{-L^+}{2}}^{\frac{L^+}{2}}dz^+\left[U_A\left(\frac{L^+}{2},z^+;z_\perp\right)\overrightarrow{\frac{d}{dz^j}}-\overleftarrow{\frac{d}{dz^j}}U_A\left(z^+,-\frac{L^+}{2};z_\perp\right)\right]$$
$$-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+U_A\left(\frac{L^+}{2},z^+;z_\perp\right)\overleftarrow{\frac{d}{dz^j}}\,\overrightarrow{\frac{d}{dz^j}}\,U_A\left(z^+,-\frac{L^+}{2};z_\perp\right)\Bigg\}+\mathrm{NNEik}$$

NEIK correction due to transverse brownian motion and drift!

NEik Gluon Propagator : Compact form

We add all the mentioned corrections and simplify the total expression to write it in compact form. (For the case $x^+ > L^+/2$ and $y^+ < -L^+/2$)

To write in compact form, we split the gluon propagator in two part, such as :

$$\delta G_F^{\mu\nu}(x,y)\big|_{\text{pure}A^-,\text{perp}} = \delta G_{1,F}^{\mu\nu}(x,y) + \delta G_{2,F}^{\mu\nu}(x,y)$$

Where, one part is

$$\delta G_{1F}^{\mu\nu}(x,y) = \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+) \frac{e^{-ix\cdot\check{q}}\ e^{iy\cdot\check{k}}}{2k^+\ 2k^+} \Big(-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \Big)$$
$$\theta(k^+) \int d^2 z_\perp\ e^{-iz_\perp(q_\perp - k_\perp)} \Big\{ -\frac{q^j + k^j}{2} \int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[U_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big) \Big(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\Big)$$
$$U_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\Big] - i \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[U_A\Big(\frac{L^+}{2}, z^+; z_\perp\Big) \Big(\overleftarrow{D}_{z^j} \overrightarrow{D}_{z^j}\Big)\ U_A\Big(z^+, -\frac{L^+}{2}; z_\perp\Big)\Big]$$
$$\Big\} + \text{NNEik}$$

Where, $D_i = \partial_i + igT \cdot A_i$ is covariant derivative.

NEik Gluon Propagator: Compact form

And other one is in terms of field strength given as:

$$\delta G^{\mu\nu}_{2F\,ab}(x,y) = \int dz^+ \int \frac{d^3q}{(2\pi)^3}\,\frac{e^{-ix\cdot\check{q}}}{2q^+} \int \frac{d^3k}{(2\pi)^3}\,\frac{e^{iy\cdot\check{k}}}{2k^+}\theta(k^+)\;2\pi\delta(q^+-k^+)\bigg(g^{\mu j}g^{\nu i} - \frac{\eta^\mu g^{\nu i}q^j}{q^+} - \frac{g^{\mu j}k^i\eta^\nu}{q^+} + \frac{\eta^\mu\eta^\nu k^i q^j}{q^+q^+}\bigg)\int d^2z_\perp e^{-i(-q_\perp-k_\perp)z_\perp}U_A\bigg(\frac{L^+}{2},z^+;z_\perp\bigg)\;gT\cdot F_{ij}\;U_A\bigg(z^+,-\frac{L^+}{2};z_\perp\bigg)$$

Where field strength(longitudinal chromomagnetic field of target) : $F_{ij} = \left[D_i, D_j\right]$

Correction due to x- dependence of background field

$$\delta G_F^{\mu\nu}(x,y)\big|_{Eik,z^-} = i\delta^2(x_\perp - y_\perp)\;\delta(x^+ - y^+)\eta^\mu\eta^\nu\left[\int\frac{dk^+}{2\pi}\;\frac{e^{-i(x^- - y^-)k^+}}{k^+k^+}\right]$$
$$+\left\{\int\frac{d^3q}{(2\pi)^3}\;e^{-ix\cdot\check{q}}\;\theta(q^+)\;\int\frac{d^3k}{(2\pi)^3}\;e^{iy\cdot\check{k}}\;\theta(k^+)\;\frac{1}{q^+ + k^+}\right.$$
$$\times\left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right]\;\int d^2z_\perp e^{-i(q_\perp - k_\perp)z_\perp}$$
$$\left.\times\int dz^-\;e^{i(q^+ - k^+)z^-)}\;u_A(x^+,y^+,z_\perp,z^-)\right\}$$

Due to dynamics of target

Correction due to x^- dependence of background field

$$
\begin{aligned}
\delta G_F^{\mu\nu}(x,y)\big|_{Eik,z^-} = {} & i\delta^2(x_\perp - y_\perp)\ \delta(x^+ - y^+)\eta^\mu\eta^\nu\left[\int \frac{dk^+}{2\pi}\ \frac{e^{-i(x^- - y^-)k^+}}{k^+k^+}\right] \\
& + \left\{ \int \frac{d^3q}{(2\pi)^3}\ e^{-ix\cdot\check{q}}\ \theta(q^+) \int \frac{d^3k}{(2\pi)^3}\ e^{iy\cdot\check{k}}\ \theta(k^+)\ \frac{1}{q^+ + k^+} \right. \\
& \times \left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \right] \int d^2z_\perp e^{-i(q_\perp - k_\perp)z_\perp} \\
& \left. \times \int dz^-\ e^{i(q^+ - k^+)z^-}\ u_A(x^+, y^+, z_\perp, z^-) \right\}
\end{aligned}
$$

Wilson line is z^- dependent

Due to dynamics of target

Total gluon propagator

Total gluon propagator upto NEik order travelling through the entire medium(dynamic gluon background field) for the case $x^+ > L^+/2$ and $y^+ < -L^+/2$ with $x^+ > y^+$ is:

$$
\begin{aligned}
G_F^{\mu\nu}(x,y) &= \int \frac{d^3q}{(2\pi)^3}\; e^{-ix\cdot\check{q}}\;\theta(q^+)\;\int \frac{d^3k}{(2\pi)^3}\; e^{iy\cdot\check{k}}\;\theta(k^+)\;\frac{1}{q^+ + k^+} \\
&\times \left[-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \right] \int d^2z_\perp e^{-i(q_\perp - k_\perp)z_\perp} \\
&\times \int dz^-\; e^{i(q^+ - k^+)z^-}\; U_A(\frac{L^+}{2}, \frac{-L^+}{2}, z_\perp, z^-) \\
&+ \int \frac{d^3\underline{q}}{(2\pi)^3}\frac{e^{-ix\cdot\check{q}}}{2q^+} \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{e^{iy\cdot\check{k}}}{2k^+}\;\theta(k^+)\;2\pi\delta(q^+ - k^+) \\
&\times \int d^2z_\perp\; e^{-iz_\perp(q_\perp - k_\perp)} \Bigg\{ \Bigg(\left(-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k}) \right) \\
&\times \left(-\frac{q^j + k^j}{2} \int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[U_A\left(\frac{L^+}{2}, z^+; z_\perp\right)\left(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\right) \right.\right. \\
&\left.\left. \times U_A\left(z^+, -\frac{L^+}{2}; z_\perp\right)\right] - i\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[U_A\left(\frac{L^+}{2}, z^+; z_\perp\right)\left(\overleftarrow{D}_{z^j}\overrightarrow{D}_{z^j}\right) U_A\left(z^+, -\frac{L^+}{2}; z_\perp\right)\right]\right) \\
&+ \left(g^{\mu j} g^{\nu i} - \frac{\eta^\mu g^{\nu i} q^j}{q^+} - \frac{g^{\mu j} k^i \eta^\nu}{q^+} + \frac{\eta^\mu \eta^\nu k^i q^j}{q^+ q^+} \right) \\
&\times \left(\left(\int dz^+\; U_A\left(\frac{L^+}{2}, z^+; z_\perp\right)\; gT\cdot F_{ij}\; U_A\left(z^+, -\frac{L^+}{2}; z_\perp\right) \right)\right) \Bigg\}
\end{aligned}
$$

Application: Calculating s-matrix

For the process of gluon scattering on a background field, using LSZ- type reduction formula,

$$S_{g_2 \leftarrow g_1} = \lim_{x^+ \to +\infty} (-1)(2p_2^+) \int d^2x \int dx^- \, e^{+ix\cdot\check{p}_2} \, \epsilon_\mu^{\lambda_2}(\underline{p_2})^* \times \lim_{y^+ \to -\infty} (-1)(2p_1^+) \int d^2y \int dy^- \, e^{-iy\cdot\check{p}_1} \, \epsilon_\nu^{\lambda_1}(\underline{p_1}) \, G_F^{\mu\nu}(x,y)_{a_2\, a_1}$$

Application: Calculating s-matrix

Simplest observable: For the process of gluon scattering on a background field, we get s-matrix as,

$$
\begin{aligned}
S =&(2p_2^+)\,(2p_1^+)\int d^2z_\perp e^{-i(p_{2\perp}-p_{1\perp})z_\perp}\Bigg\{\epsilon^{i\,*}_{\lambda_2}\,\epsilon^i_{\lambda_1}\,\Bigg[\frac{1}{p_2^+ + p_1^+}\int dz^-\; e^{i(p_2^+-p_1^+)z^-}\;U_A\Big(\frac{L^+}{2},\frac{-L^+}{2},z_\perp,z^-\Big)\\
&+\frac{2\pi\delta(p_2^+-p_1^+)}{2p_2^+2p_1^+}\Bigg(-\frac{p_2^j+p_1^j}{2}\int_{\frac{-L^+}{2}}^{\frac{L^+}{2}}dz^+\Big[U_A\Big(\frac{L^+}{2},z^+;z_\perp\Big)\Big(\overrightarrow{D}_{z^j}-\overleftarrow{D}_{z^j}\Big)\\
&\times U_A\Big(z^+,-\frac{L^+}{2};z_\perp\Big)\Big]-i\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\Big[U_A\Big(\frac{L^+}{2},z^+;z_\perp\Big)\Big(\overleftarrow{D}_{z^j}\overrightarrow{D}_{z^j}\Big)\;U_A\Big(z^+,-\frac{L^+}{2};z_\perp\Big)\Big]\Bigg)\Bigg]\\
&+\frac{2\pi\delta(p_2^+-p_1^+)}{2p_2^+2p_1^+}\epsilon^{j\,*}_{\lambda_2}\,\epsilon^i_{\lambda_1}\int dz^+\,U_A\Big(\frac{L^+}{2},z^+;z_\perp\Big)\;gT\cdot F_{ij}\;U_A\Big(z^+,-\frac{L^+}{2};z_\perp\Big)\Bigg\}
\end{aligned}
$$

Currently, we are calculating cross section for single inclusive gluon production in forward pA collisions.

Summary and Conclusion

- Gluon propagator at next-to-eikonal correction in dynamic gluon field is calculated.
  - Correction due to finite width, transverse component of target and x^- dependence of target field included.
- It is used to derive s-matrix for gluon scattering on background field.
- NEik corrections might be sizable in future EIC.
- The obtained expression for gluon propagator is of general form therefore of general use; it can be used for different scattering processes.

Thank you!