DIS dijet production beyond eikonal accuracy

Arantxa Tymowska

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arXiv:2212.10484 Tolga Altinoluk, Guillaume Beuf and Alina Czajka

Color Glass Condensate

CGC formalism used for dilute-dense scattering so we apply Semi-classical approximation

- Dense target: classical background field $A^{\mu}_{a}(x)$
- Dilute projectile: virtual photon treated in perturbation theory

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We also adopt Eikonal approximation which amount to taking the high energy limit $s \to \infty$. Beyond eikonal limit give corrections of order $1/s$. We can obtain this limit boosting the target in following way:

$$A_a^\mu(x) \to \begin{cases} \gamma_t A_a^-\left(\gamma_t x^+, \frac{1}{\gamma_t}x^-, x_\perp\right) \\ \frac{1}{\gamma_t} A_a^+\left(\gamma_t x^+, \frac{1}{\gamma_t}x^-, x_\perp\right) \\ A_a^i\left(\gamma_t x^+, \frac{1}{\gamma_t}x^-, x_\perp\right) \end{cases}$$

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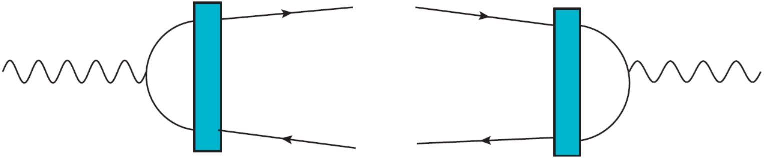
- Taking into account x^- -dependence
  T. Altinoluk, G. Beuf (2021) [arXiv:2109.01620] and T. Altinoluk, G. Beuf. A.Czajka, A.Tymowska (2022) [arXiv:2212.10484]

Inclusive DIS dijet production

We want to calculate DIS dijet production at next-to eikonal order for the inclusive case

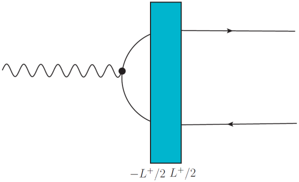


The process will be one of the focus for future EIC experiments.

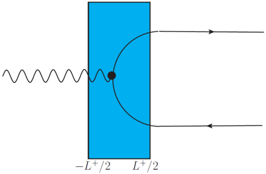
Lower energies at EIC compared to LHC $\rightarrow$ NEik corrections

Diagrams for DIS dijet production at NEik

We need contributions from photon splitting into quark-antiquark pair
First diagrams contributes at both eikonal and next-to eikonal order



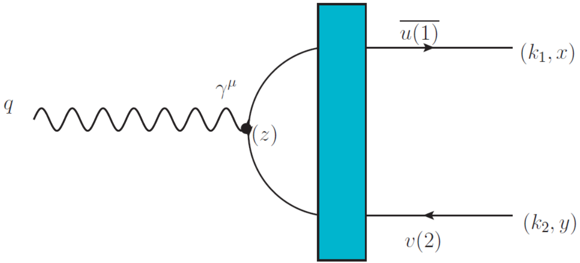
Second diagram contributes only at next-to eikonal order and vanishes when taking the longitudinal polarization of the photon.



LSZ reduction formula

S-matrix element for the virtual photon splitting into a quark-antiquark pair is

$$S_{q_1\bar{q}_2\leftarrow\gamma*} = -iee_f\,\epsilon^{\lambda}_{\mu}(q)\lim_{x^+,y^+\to\infty}\int d^2\mathrm{x}\int dx^-\int d^2\mathrm{y}\int dy^-\int d^4z$$
$$\times\, e^{-iq\cdot z}e^{i\check{k}_1\cdot x}e^{i\check{k}_2\cdot y}\;\bar{u}(1)\gamma^+S_F(x,z)_{\beta\alpha}\gamma^\mu(-S_F(z,y)_{\alpha\delta})\gamma^+v(2)$$



Two polarizations

We have two cases for the vector polarization that we will take into account and for which we calculate the cross sections.

Longitudinal polarization

$$\epsilon_\mu^\lambda(q) \rightarrow \epsilon_\mu^L(q) \equiv \frac{Q}{q^+} g_\mu^+ \ .$$

Transverse polarization

$$\begin{aligned} \epsilon_\lambda^+(q) &= 0 \\ \epsilon_\lambda^i(q) &= \varepsilon_\lambda^i \\ \epsilon_\lambda^-(q) &= \frac{\mathrm{q}^i \varepsilon_\lambda^i}{q^+} \end{aligned}$$

Quark propagator from before to after the medium at NEik order

$$S_F(x,y) = \int \frac{d^3\underline{p}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, \theta(p^+)\theta(k^+) e^{-ix\cdot\check{p}}\, e^{iy\cdot\check{k}} \int dz^- e^{iz^-(p^+-k^+)} \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{p}-\mathbf{k})}$$
$$\times \frac{(\check{\not{p}}+m)}{2p^+}\gamma^+ \Bigg\{ \mathcal{U}_F(\mathbf{z},z^-)$$
$$- \frac{(\mathbf{p}^j+\mathbf{k}^j)}{2(p^++k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}, z^-\right) \overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\, \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}, z^-\right)\right]$$
$$- \frac{i}{(p^++k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}, z^-\right) \overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\, \overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\, \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}, z^-\right)\right]$$
$$+ \frac{[\gamma^i,\gamma^j]}{4(p^++k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\, \mathcal{U}_F\left(\frac{L^+}{2}, z^+; \mathbf{z}, z^-\right) gt\cdot\mathcal{F}_{ij}(z)\, \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; \mathbf{z}, z^-\right) \Bigg\} \frac{(\check{\not{k}}+m)}{2k^+} + O(\mathrm{NNEik}),$$

$$\overrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\partial_{z^\mu}} + igt\cdot\mathcal{A}_\mu(z)$$
$$\overleftarrow{\mathcal{D}_{z^\mu}} \equiv \overleftarrow{\partial_{z^\mu}} - igt\cdot\mathcal{A}_\mu(z)$$
$$\overleftrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}} - \overleftarrow{\mathcal{D}_{z^\mu}} = \overleftrightarrow{\partial_{z^\mu}} + 2igt\cdot\mathcal{A}_\mu(z)$$
$$\mathcal{F}^a_{\mu\nu}(z) \equiv \partial_{z^\mu}\mathcal{A}^a_\nu(z) - \partial_{z^\nu}\mathcal{A}^a_\mu(z) - gf^{abc}\,\mathcal{A}^b_\mu(z)\,\mathcal{A}^c_\nu(z)$$

Quark propagator from inside to after the medium at NEik order

For the case of the quark ($x^+ > L^+/2$ and $-L^+/2 < y^+ < L^+/2$)

$$S_F(x,y)\Big|^{\text{IA},\, q}_{\text{Eik.}} = \int \frac{d^3\underline{p}}{(2\pi)^3}\,\frac{\theta(p^+)}{2p^+}\,e^{-ix\cdot\check{p}}\,(\check{\not{p}}+m)\,\mathcal{U}_F(x^+,y^+;\mathbf{y})\left[1-\frac{\gamma^+\gamma^i}{2p^+}\,i\overleftarrow{\mathcal{D}_{\mathbf{y}^i}}\right]\,e^{iy^-p^+}\,e^{-i\mathbf{y}\cdot\mathbf{p}}$$

for the antiquark ($y^+ > L^+/2$ and $-L^+/2 < x^+ < L^+/2$) we have

$$S_F(x,y)\Big|^{\text{IA},\, \bar{q}}_{\text{Eik.}} = \int \frac{d^3\underline{k}}{(2\pi)^3}\,(-1)\,\frac{\theta(-k^+)}{2k^+}\,e^{iy\cdot\check{k}}\,e^{-ix^-k^+}\,e^{i\mathbf{x}\cdot\mathbf{k}}\left[1-\frac{\gamma^+\gamma^i}{2k^+}\,i\overrightarrow{\mathcal{D}_{\mathbf{x}^i}}\right]\,\mathcal{U}_F^\dagger(y^+,x^+;\mathbf{x})\,(\check{\not{k}}+m)$$

the S-matrix element for the case of splitting of the photon inside of the medium is:

$$S^{\text{in}}_{q_1\bar{q}_2\leftarrow\gamma_T^*} = ee_f\,\varepsilon^i_\lambda\,2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{q^+}{4k_1^+k_2^+}\,\bar{u}(1)\gamma^+\left(\frac{(k_2^+ - k_1^+)}{q^+}\,\delta^{ij} + \frac{1}{2}\,[\gamma^i,\gamma^j]\right)v(2)$$
$$\times\int d^2\mathbf{z}\,e^{-i(\mathbf{k}_1+\mathbf{k}_2-\mathbf{q})\cdot\mathbf{z}}\int_{-L^+/2}^{L^+/2} dz^+\left[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\mathcal{U}_F^\dagger\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\right]$$

NEik DIS dijet production cross section via longitudinal photon I

$$S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*} = S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*}\Big|_{\text{Gen. Eik}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*}\Big|_{\text{dec. on } q} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*}\Big|_{\text{dec. on } \bar{q}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*}\Big|_{L^+ \text{ phase}} + S^{\text{bef}}_{q_1\bar{q}_2\leftarrow\gamma^*}\Big|_{\text{dyn. target}} + O(NNEik)$$

Cross section for the longitudinal case:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|_{\text{Gen. Eik}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|_{\text{NEik corr.}} + O(\text{NNEik})$$

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Big|_{\text{Gen. Eik}} = N_c\,\frac{\alpha_{\text{em}}}{\pi}\,e_f^2\,Q^2\,\theta\Big(q^+ + k_1^+ - k_2^+\Big)\theta\Big(q^+ - k_1^+ + k_2^+\Big)\frac{k_1^+k_2^+}{(q^+)^5}\,(q^+ + k_1^+ - k_2^+)^2(q^+ - k_1^+ + k_2^+)^2$$
$$\times \int_{\mathrm{v},\mathrm{v}',\mathrm{w},\mathrm{w}'} e^{i\mathrm{k}_1\cdot(\mathrm{v}'-\mathrm{v})}\,e^{i\mathrm{k}_2\cdot(\mathrm{w}'-\mathrm{w})}K_0(\hat{Q}|\mathrm{w}'-\mathrm{v}'|)K_0(\hat{Q}|\mathrm{w}-\mathrm{v}|)$$
$$\times \int d(\Delta b^-)e^{i\Delta b^-(k_1^+ + k_2^+ - q^+)}\Big\{Q\Big(\mathrm{w}',\mathrm{v}',\mathrm{v},\mathrm{w},\frac{\Delta b^-}{2}\Big) - d\big(\mathrm{w}',\mathrm{v}'\big) - d\big(\mathrm{v},\mathrm{w}\big) + 1\Big\}$$

$$\hat{Q} = \sqrt{m^2 + \frac{(q^+ + k_1^+ - k_2^+)(q^+ - k_1^+ + k_2^+)}{4(q^+)^2}\,Q^2} \qquad \bar{Q} \equiv \sqrt{m^2 + Q^2\frac{k_1^+k_2^+}{(q^+)^2}}$$

NEik DIS dijet production cross section via longitudinal photon II

The corrections are:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|^{\text{dec. on } q}_{\text{NEik corr.}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, 8N_c\, \frac{\alpha_{\text{em}}}{\pi}\, e_f^2\, Q^2\, \frac{(k_1^+)^2(k_2^+)^3}{(q^+)^5}$$
$$\times\, 2\mathrm{Re}\int_{\mathrm{v},\mathrm{v}',\mathrm{w},\mathrm{w}'} e^{i\mathrm{k}_1\cdot(\mathrm{v}'-\mathrm{v})}\, e^{i\mathrm{k}_2\cdot(\mathrm{w}'-\mathrm{w})}\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)$$
$$\times\left\{\left[\frac{(\mathrm{k}_2^j-\mathrm{k}_1^j)}{2}+\frac{i}{2}\,\partial_{\mathrm{w}^j}\right]\left[Q_j^{(1)}(\mathrm{w}',\mathrm{v}',\mathrm{v}_*,\mathrm{w}) - d_j^{(1)}(\mathrm{v}_*,\mathrm{w})\right] - i\left[Q^{(2)}(\mathrm{w}',\mathrm{v}',\mathrm{v}_*,\mathrm{w}) - d^{(2)}(\mathrm{v}_*,\mathrm{w})\right]\right\}$$

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S}}\Bigg|^{\text{dec. on } \bar{q}}_{\text{NEik corr.}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, 8N_c\, \frac{\alpha_{\text{em}}}{\pi}\, e_f^2\, Q^2\, \frac{(k_1^+)^3(k_2^+)^2}{(q^+)^5}$$
$$\times\, 2\mathrm{Re}\int_{\mathrm{v},\mathrm{v}',\mathrm{w},\mathrm{w}'} e^{i\mathrm{k}_1\cdot(\mathrm{v}'-\mathrm{v})}\, e^{i\mathrm{k}_2\cdot(\mathrm{w}'-\mathrm{w})}\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)$$
$$\times\left\{\left[-\frac{(\mathrm{k}_2^j-\mathrm{k}_1^j)}{2}+\frac{i}{2}\,\partial_{\mathrm{v}^j}\right]\left[Q_j^{(1)}(\mathrm{v}',\mathrm{w}',\mathrm{w}_*,\mathrm{v})^\dagger - d_j^{(1)}(\mathrm{w}_*,\mathrm{v})^\dagger\right] - i\left[Q^{(2)}(\mathrm{v}',\mathrm{w}',\mathrm{w}_*,\mathrm{v})^\dagger - d^{(2)}(\mathrm{w}_*,\mathrm{v})^\dagger\right]\right\}$$

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|^{\text{dyn. target}}_{\text{NEik corr.}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, 8N_c\, \frac{\alpha_{\text{em}}}{\pi}\, e_f^2\, Q^2\, \frac{(k_1^+)^2(k_2^+)^2(k_1^+-k_2^+)}{(q^+)^5}\, 2\mathrm{Re}\,(-i)\int_{\mathrm{v},\mathrm{v}',\mathrm{w},\mathrm{w}'} e^{i\mathrm{k}_1\cdot(\mathrm{v}'-\mathrm{v})}\, e^{i\mathrm{k}_2\cdot(\mathrm{w}'-\mathrm{w})}$$
$$\times\left[\tilde{Q}(\mathrm{w}',\mathrm{v}',\mathrm{v}_*,\mathrm{w}_*) - \tilde{d}(\mathrm{v}_*,\mathrm{w}_*)\right]\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\left[\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right) - \frac{(\bar{Q}^2-m^2)}{2\bar{Q}}\,|\mathrm{w}-\mathrm{v}|\,\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)\right]$$

NEik DIS dijet production cross section via longitudinal photon III

$$\mathcal{U}^{(1)}_{F;j}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\, \mathcal{U}_F\Big(\frac{L^+}{2}, v^+; \mathrm{v}\Big) \overleftrightarrow{\mathcal{D}_{\mathrm{v}^j}} \mathcal{U}_F\Big(v^+, -\frac{L^+}{2}; \mathrm{v}\Big)$$

$$\mathcal{U}^{(2)}_{F}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\, \mathcal{U}_F\Big(\frac{L^+}{2}, v^+; \mathrm{v}\Big) \overleftarrow{\mathcal{D}_{\mathrm{v}^j}}\, \overrightarrow{\mathcal{D}_{\mathrm{v}^j}} \mathcal{U}_F\Big(v^+, -\frac{L^+}{2}; \mathrm{v}\Big)$$

$$\mathcal{U}^{(3)}_{F;ij}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\, \mathcal{U}_F\Big(\frac{L^+}{2}, v^+; \mathrm{v}\Big) gt\cdot \mathcal{F}_{ij}(\underline{v}) \mathcal{U}_F\Big(v^+, -\frac{L^+}{2}; \mathrm{v}\Big)$$

$$\begin{aligned}
d^{(1)}_j(\mathrm{v}_*, \mathrm{w}) &= \Big\langle \frac{1}{N_c} \mathrm{Tr}\Big[\mathcal{U}^{(1)}_{F;j}(\mathrm{v})\mathcal{U}^\dagger_F(\mathrm{w})\Big]\Big\rangle \\
d^{(2)}(\mathrm{v}_*, \mathrm{w}) &= \Big\langle \frac{1}{N_c} \mathrm{Tr}\Big[\mathcal{U}^{(2)}_{F}(\mathrm{v})\mathcal{U}^\dagger_F(\mathrm{w})\Big]\Big\rangle \\
Q^{(1)}_j(\mathrm{w}', \mathrm{v}', \mathrm{v}_*, \mathrm{w}) &= \Big\langle \frac{1}{N_c} \mathrm{Tr}\Big[\mathcal{U}_F(\mathrm{w}')\mathcal{U}^\dagger_F(\mathrm{v}')\mathcal{U}^{(1)}_{F;j}(\mathrm{v})\mathcal{U}^\dagger_F(\mathrm{w})\Big]\Big\rangle \\
Q^{(2)}(\mathrm{w}', \mathrm{v}', \mathrm{v}_*, \mathrm{w}) &= \Big\langle \frac{1}{N_c} \mathrm{Tr}\Big[\mathcal{U}_F(\mathrm{w}')\mathcal{U}^\dagger_F(\mathrm{v}')\mathcal{U}^{(2)}_{F}(\mathrm{v})\mathcal{U}^\dagger_F(\mathrm{w})\Big]\Big\rangle,
\end{aligned}$$

$$\begin{aligned}
\tilde{d}(\mathrm{v}_*, \mathrm{w}_*) &= \Big\langle \frac{1}{N_c} \mathrm{Tr}\Big[\Big(\mathcal{U}_F(\mathrm{v}, b^-)\overleftrightarrow{\partial_-}\mathcal{U}^\dagger_F(\mathrm{w}, b^-)\Big)\Big|_{b^-=0}\Big]\Big\rangle, \\
\tilde{Q}(\mathrm{w}', \mathrm{v}', \mathrm{v}_*, \mathrm{w}_*) &= \Big\langle \frac{1}{N_c} \mathrm{Tr}\Big[\mathcal{U}_F(\mathrm{w}')\mathcal{U}^\dagger_F(\mathrm{v}')\Big(\mathcal{U}_F(\mathrm{v}, b^-)\overleftrightarrow{\partial_-}\mathcal{U}^\dagger_F(\mathrm{w}, b^-)\Big)\Big|_{b^-=0}\Big]\Big\rangle.
\end{aligned}$$

NEik DIS dijet production cross section via transverse photon I

The generalized eikonal contribution is:

$$\frac{d\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|_{\mathrm{Gen.\,Eik}} = N_c\,\frac{\alpha_{\mathrm{em}}}{\pi}\,e_f^2\,\frac{2k_1^+k_2^+}{q^+}\,\theta(q^++k_1^+-k_2^+)\,\theta(q^++k_2^+-k_1^+)\int_{\mathrm{v},\mathrm{v}',\mathrm{w},\mathrm{w}'} e^{i\mathbf{k}_1\cdot(\mathrm{v}'-\mathrm{v})}\,e^{i\mathbf{k}_2\cdot(\mathrm{w}'-\mathrm{w})}$$
$$\times\left\{2\,m^2\,\mathrm{K}_0\left(\hat{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\,\mathrm{K}_0\left(\hat{Q}\,|\mathrm{w}-\mathrm{v}|\right)+\left[1+\left(\frac{k_2^+-k_1^+}{q^+}\right)^2\right]\hat{Q}^2\,\frac{(\mathrm{w}'-\mathrm{v}')\cdot(\mathrm{w}-\mathrm{v})}{|\mathrm{w}'-\mathrm{v}'||\mathrm{w}-\mathrm{v}|}\,\mathrm{K}_1\left(\hat{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\,\mathrm{K}_1\left(\hat{Q}\,|\mathrm{w}-\mathrm{v}|\right)\right\}$$
$$\times\int d(\Delta b^-)e^{i\Delta b^-(k_1^++k_2^+-q^+)}\left\{Q\left(\mathrm{w}',\mathrm{v}',\mathrm{v},\mathrm{w},\frac{\Delta b^-}{2}\right)-d\left(\mathrm{w}',\mathrm{v}'\right)-d\left(\mathrm{v},\mathrm{w}\right)+1\right\}$$

$$\frac{d\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|^{\mathrm{in}}_{\mathrm{NEik\,corr.}} = 2\pi\delta(k_1^++k_2^+-q^+)\,N_c\,\alpha_{\mathrm{em}}\,e_f^2\left[1+\left(\frac{k_2^+-k_1^+}{q^+}\right)^2\right]2\mathrm{Re}\,(i)\int_{\mathrm{z},\mathrm{v}',\mathrm{w}'} e^{i\mathbf{k}_1\cdot(\mathrm{v}'-\mathrm{z})}\,e^{i\mathbf{k}_2\cdot(\mathrm{w}'-\mathrm{z})}$$
$$\times\frac{(\mathrm{w}'^j-\mathrm{v}'^j)}{|\mathrm{w}'-\mathrm{v}'|}\,\bar{Q}\,\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\int_{-L^+/2}^{L^+/2}dz^+\left\langle\frac{1}{N_c}\mathrm{Tr}\left[\mathcal{U}_F(\mathrm{w}')\mathcal{U}_F^\dagger(\mathrm{v}')-1\right]\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathrm{z}\right)\overleftrightarrow{\mathcal{D}_{\mathrm{z}^j}}\mathcal{U}_F^\dagger\left(\frac{L^+}{2},z^+;\mathrm{z}\right)\right]\right\rangle$$

$$\frac{d\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|^{L^+\ \mathrm{phase}}_{\mathrm{NEik\,corr.}} = 2\pi\delta(k_1^++k_2^+-q^+)\,N_c\,\alpha_{\mathrm{em}}\,e_f^2\left[1+\left(\frac{k_2^+-k_1^+}{q^+}\right)^2\right]2\mathrm{Re}\,(-i)\,\frac{L^+}{2}\int_{\mathrm{z},\mathrm{v}',\mathrm{w}'} e^{i\mathbf{k}_1\cdot(\mathrm{v}'-\mathrm{z})}\,e^{i\mathbf{k}_2\cdot(\mathrm{w}'-\mathrm{z})}$$
$$\times\frac{(\mathrm{w}'^j-\mathrm{v}'^j)}{|\mathrm{w}'-\mathrm{v}'|}\,\bar{Q}\,\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\left\langle\frac{1}{N_c}\mathrm{Tr}\left[\mathcal{U}_F(\mathrm{w}')\mathcal{U}_F^\dagger(\mathrm{v}')-1\right]\left[\mathcal{U}_F(\mathrm{z})\overleftrightarrow{\partial_{\mathrm{z}^j}}\mathcal{U}_F^\dagger(\mathrm{z})\right]\right\rangle$$

NEik DIS dijet production cross section via transverse photon II

$$d_{ij}^{(3)}(\mathrm{v}_*, \mathrm{w}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\left[\mathcal{U}_{F;ij}^{(3)}(\mathrm{v})\mathcal{U}_F^{\dagger}(\mathrm{w})\right] \right\rangle$$

$$Q_{ij}^{(3)}(\mathrm{w}', \mathrm{v}', \mathrm{v}_*, \mathrm{w}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\left[\mathcal{U}_F(\mathrm{w}')\mathcal{U}_F^{\dagger}(\mathrm{v}')\mathcal{U}_{F;ij}^{(3)}(\mathrm{v})\mathcal{U}_F^{\dagger}(\mathrm{w})\right] \right\rangle$$

$$\frac{d\sigma_{\gamma_T^* \to q_1 \bar{q}_2}}{d\mathrm{P.S.}}\Bigg|_{\mathrm{NEik\,corr.}}^{\mathrm{dyn.\ target}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, N_c\, \frac{\alpha_{\mathrm{em}}}{\pi}\, e_f^2\, \frac{k_1^+ k_2^+ (k_2^+ - k_1^+)}{(q^+)^3}\, 2\mathrm{Re}\,(-i) \int_{\mathrm{v},\mathrm{v}',\mathrm{w},\mathrm{w}'} e^{i\mathrm{k}_1\cdot(\mathrm{v}'-\mathrm{v})}\, e^{i\mathrm{k}_2\cdot(\mathrm{w}'-\mathrm{w})}$$

$$\times \left[\tilde{Q}(\mathrm{w}', \mathrm{v}', \mathrm{v}_*, \mathrm{w}_*) - \tilde{d}(\mathrm{v}_*, \mathrm{w}_*)\right] \Bigg\{ \frac{1}{2} \left[1 + \left(\frac{k_2^+ - k_1^+}{q^+}\right)^2\right] \frac{(\mathrm{w}'-\mathrm{v}')\cdot(\mathrm{w}-\mathrm{v})}{|\mathrm{w}'-\mathrm{v}'|}\, \bar{Q}\, \mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\, Q^2\, \mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)$$

$$+ m^2\, Q^2\, \mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}'-\mathrm{v}'|\right) \frac{|\mathrm{w}-\mathrm{v}|}{\bar{Q}}\, \mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right) + 2\, \frac{(\mathrm{w}'-\mathrm{v}')\cdot(\mathrm{w}-\mathrm{v})}{|\mathrm{w}'-\mathrm{v}'||\mathrm{w}-\mathrm{v}|}\, \bar{Q}^2\, \mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}'-\mathrm{v}'|\right)\, \mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right) \Bigg\}$$

NEik DIS dijet production cross section via transverse photon III

$$\frac{d\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|_{\text{NEik corr.}}^{\text{dec. on } q} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, N_c\, \frac{\alpha_{\text{em}}}{\pi}\, e_f^2\, \frac{2k_2^+}{q^+}\, 2\text{Re}\int_{\mathrm{v,v',w,w'}} e^{i\mathrm{k}_1\cdot(\mathrm{v'}-\mathrm{v})}\, e^{i\mathrm{k}_2\cdot(\mathrm{w'}-\mathrm{w})}$$
$$\times\Bigg\{\Bigg[\bigg(\frac{(\mathrm{k}_2^j-\mathrm{k}_1^j)}{2}+\frac{i}{2}\partial_{\mathrm{w}^j}\bigg)\Big(Q_j^{(1)}(\mathrm{w',v',v_*,w})-d_j^{(1)}(\mathrm{v_*,w})\Big) - i\Big(Q^{(2)}(\mathrm{w',v',v_*,w})-d^{(2)}(\mathrm{v_*,w})\Big)\Bigg]$$
$$\times\Bigg[\frac{1}{2}\bigg(1+\Big(\frac{k_2^+-k_1^+}{q^+}\Big)^2\bigg)\frac{(\mathrm{w'}-\mathrm{v'})\cdot(\mathrm{w}-\mathrm{v})}{|\mathrm{w'}-\mathrm{v'}||\mathrm{w}-\mathrm{v}|}\bar{Q}^2\,\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w'}-\mathrm{v'}|\right)\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)+m^2\,\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w'}-\mathrm{v'}|\right)\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)\Bigg]$$
$$+\frac{(k_1^+-k_2^+)}{q^+}\frac{(\mathrm{w'}^i-\mathrm{v'}^i)(\mathrm{w}^j-\mathrm{v}^j)}{|\mathrm{w'}-\mathrm{v'}||\mathrm{w}-\mathrm{v}|}\bar{Q}^2\,\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w'}-\mathrm{v'}|\right)\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)\Big(Q_{ij}^{(3)}(\mathrm{w',v',v_*,w})-d_{ij}^{(3)}(\mathrm{v_*,w})\Big)\Bigg\}$$

$$\frac{d\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|_{\text{NEik corr.}}^{\text{dec. on } \bar{q}} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, N_c\, \frac{\alpha_{\text{em}}}{\pi}\, e_f^2\, \frac{2k_1^+}{q^+}\, 2\text{Re}\int_{\mathrm{v,v',w,w'}} e^{i\mathrm{k}_1\cdot(\mathrm{v'}-\mathrm{v})}\, e^{i\mathrm{k}_2\cdot(\mathrm{w'}-\mathrm{w})}$$
$$\times\Bigg\{\Bigg[\frac{1}{2}\bigg[1+\Big(\frac{k_2^+-k_1^+}{q^+}\Big)^2\bigg]\frac{(\mathrm{w'}-\mathrm{v'})\cdot(\mathrm{w}-\mathrm{v})}{|\mathrm{w'}-\mathrm{v'}||\mathrm{w}-\mathrm{v}|}\bar{Q}^2\,\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w'}-\mathrm{v'}|\right)\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)+m^2\,\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w'}-\mathrm{v'}|\right)\mathrm{K}_0\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)\Bigg]$$
$$\times\bigg\{\Big[-\frac{(\mathrm{k}_2^j-\mathrm{k}_1^j)}{2}+\frac{i}{2}\partial_{\mathrm{v}^j}\Big]\Big[Q_j^{(1)}(\mathrm{v',w',w_*,v})^\dagger-d_j^{(1)}(\mathrm{w_*,v})^\dagger\Big]-i\Big[Q^{(2)}(\mathrm{v',w',w_*,v})^\dagger-d^{(2)}(\mathrm{w_*,v})^\dagger\Big]\bigg\}$$
$$+\frac{(k_1^+-k_2^+)}{q^+}\frac{(\mathrm{w'}^i-\mathrm{v'}^i)(\mathrm{w}^j-\mathrm{v}^j)}{|\mathrm{w'}-\mathrm{v'}||\mathrm{w}-\mathrm{v}|}\bar{Q}^2\,\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w'}-\mathrm{v'}|\right)\mathrm{K}_1\left(\bar{Q}\,|\mathrm{w}-\mathrm{v}|\right)\Big[Q_{ij}^{(3)}(\mathrm{v',w',w_*,v})^\dagger-d_{ij}^{(3)}(\mathrm{w_*,v})^\dagger\Big]\Bigg\}$$

Outlook

- We computed the cross section for the case of photon longitudinal polarization and transverse polarization for DIS dijet production at full NEik order from the gluon background field
- Next-to eikonal corrections include:
  - Relaxing the shockwave approximation $\rightarrow$ transverse motion through the target
  - Including interactions with transverse component of the background field
  - Taking into account z^--dependence $\rightarrow$ effects of longitudinal momentum exchange with the target

Thank you for your attention