Mechanical properties of quantum bound states: EMT form factor for hydrogen atom

Andrzej Czarnecki , University of Alberta
Yizhuang Liu， Jagiellonian University

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The talk is based on the recent works with Xiangdong-Ji:

1. *Momentum-Current Gravitational Multipoles of Hadrons.*
Phys.Rev.D 106 (2022) 3, 034028

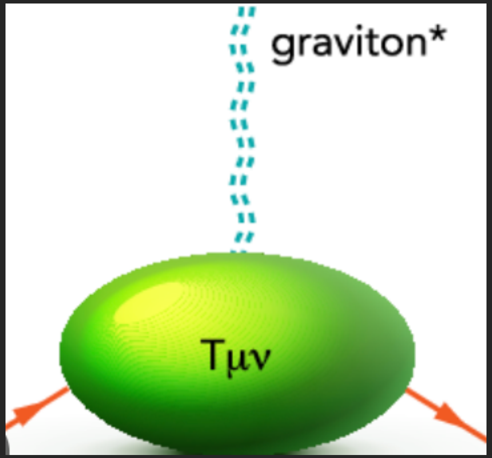
2. *Gravitational Tensor-Monopole Moment of Hydrogen Atom To Order O(α).*
Arxiv: 2208.05029.

Outline

- Energy Momentum Tensor and mass distribution of bound-states.
- D-term for hydrogen atom.
- Results and discussion.

Energy-momentum flow in a bound system

Thought experiment: scattering of a graviton,



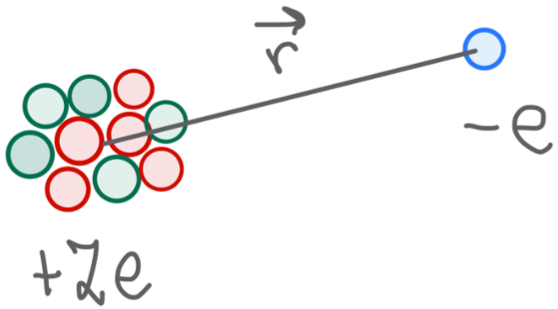
from 2104.02031

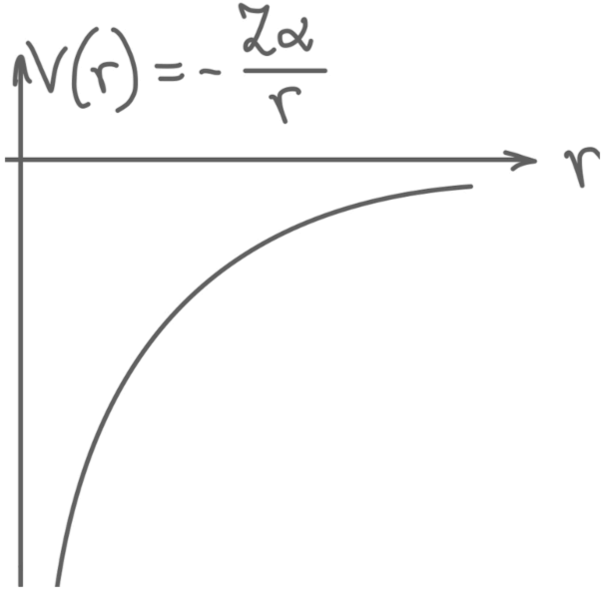
$$\left\langle P-\frac{q}{2}\right| T_{\mu\nu} \left|P+\frac{q}{2}\right\rangle = 2\left[1+\mathcal{O}(q^2)\right] P_\mu P_\nu + \frac{1}{2}\left[1+\mathcal{O}\left(\alpha \ln \alpha\right)\right]\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)$$

We want to better understand the structure of form factors, particularly the so-called D-term.

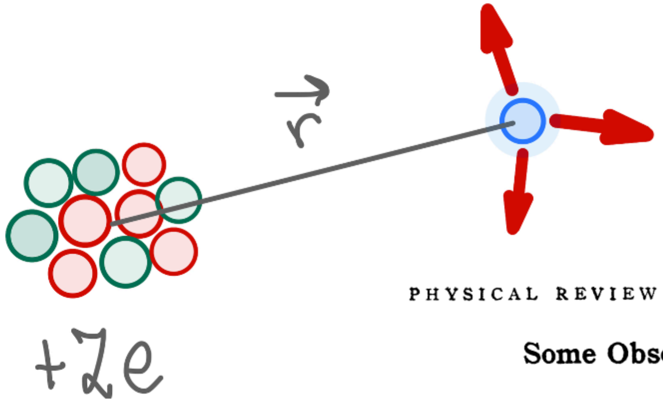
(as discussed in Wojtek Broniowski's talk in the case of a pion)

Interpretation of radiative corrections; Lamb shift as an example





Effect of vacuum fluctuations:



Electron moved in the potential, binding energy changed.

PHYSICAL REVIEW VOLUME 74, NUMBER 9 NOVEMBER 1, 1948

Some Observable Effects of the Quantum-Mechanical Fluctuations of the Electromagnetic Field

THEODORE A. WELTON*

Interpretation of the Lamb shift (Welton)

Vacuum energy in one mode $E_k \exp ikr$

$$\frac{\hbar\omega}{2} = 2 \cdot V \cdot \frac{\epsilon_0}{2} E_k^2$$

$$\delta U = \langle U_c(\boldsymbol{r} + \boldsymbol{q}) - U_c(\boldsymbol{r}) \rangle \simeq \frac{1}{6} \overline{\delta^2} \cdot \nabla^2 U_c$$

Total position disturbance, summed over modes $\overline{\delta^2} = \frac{4\alpha}{\pi m^2} \ln \frac{1}{\alpha}$

$$\langle \delta U \rangle_{2S} = \frac{1}{6\pi^2 \hbar} \alpha^5 \ln \frac{1}{\alpha} \cdot mc^2 \sim 1000 \text{ MHz.}$$

D-term versus Lamb shift

Vacuum fluctuations smear electron's position, $\overline{\delta^2} = \frac{4\alpha}{\pi m^2} \ln \frac{1}{\alpha}$

Lamb

$$E = E^{(0)} \left(1 - \frac{16\alpha^3}{3\pi} \ln \frac{1}{\alpha}\right)$$

$$\frac{\Delta E}{E^{(0)}} \sim \frac{\overline{\delta^2}}{a_B^2}$$

Only S-states are affected

D-term

$$D = D^{(0)} \left(1 - \frac{4\alpha}{3\pi} \ln \frac{1}{\alpha}\right)$$

$$\frac{\Delta D}{D^{(0)}} \sim \frac{\overline{\delta^2}}{\lambda_C^2}$$

Universal log-correction: all states

Energy Momentum Tensor (EMT)

- Energy-momentum tensor (EMT) is not an unfamiliar object .

1. $T^{\mu\nu}(x)$ describes the distribution of mass and momentum flow in many systems. Such as:

- Classical continuum matter. σ^{ij} called stress tensor. Often decomposed further into pressure ($\propto p\delta^{ij}$) and shear-force (traceless).
- Classical electromagnetic dynamics. T^{0i} is the famous Poynting vector. T^{ij} is the famous Maxwell stress tensor.

EMT in QFT.

- In QFT, $T^{\mu\nu}(x)$ is more subtle.

1. Formally as Noether's currents of space-time translation symmetry.
2. $T^{00}(x)$: the energy density. $T^{0i}(x)$ the momentum density. $T^{ij}(x)$: momentum current density. $\partial_\mu T^{\mu\nu} = 0$.
3. Special attention required to handle UV problem.

Hadronic EMT form factor.

- The EMT form factor of spin-$\frac{1}{2}$ particle ($q = P' - P, \bar{P} = \frac{P+P'}{2}$)
- $\langle P'|T^{\mu\nu}|P\rangle =$

$$\bar{u}(P')\left(A(q)\gamma^{(\mu}\bar{P}^{\nu)} + B(q)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M} + C(q)(q^\mu q^\nu - g^{\mu\nu}q^2) \right)u(P)$$

1. The Mass, Spin form factors. $A(0) = B(0) = 1$. Mass and spin sum-rule.
2. What about $C(0)$? This is called the D-term and remains unknown.
3. Is $C(0)$ always negative due to stability?

Hadronic EMT form factor and GPDs

hep-ph/9609381, Ji
hep-ph/9807358, Ji
hep-ph/0307382, Diehl

- Where can the EMT form factor be probed ?

1. Twist-two parts. First moment of the Generalized parton distributions (GPD). $F(x,\xi,q) \sim H + E$.
2. $\int_{-1}^{1} x\, H_g(x,\xi,q) = A_g(q) + (2\xi)^2 C_g(q)$,
   $\int_{-1}^{1} x\, E_g(x,\xi,q) = A_g(q) - (2\xi)^2 C_g(q)$. And similar for quark.
3. Can be probed in DVCS, where $\xi = -\frac{q^+}{2\,\bar{P}^+}$ is called the skewness.
4. Near threshold J/ψ production: Clue to $C(0)$?

PhysRevD.103.096010, Guo, Ji, Liu
PhysRevD.106.086004,Mamo&Zahed

DVCS and vector meson production

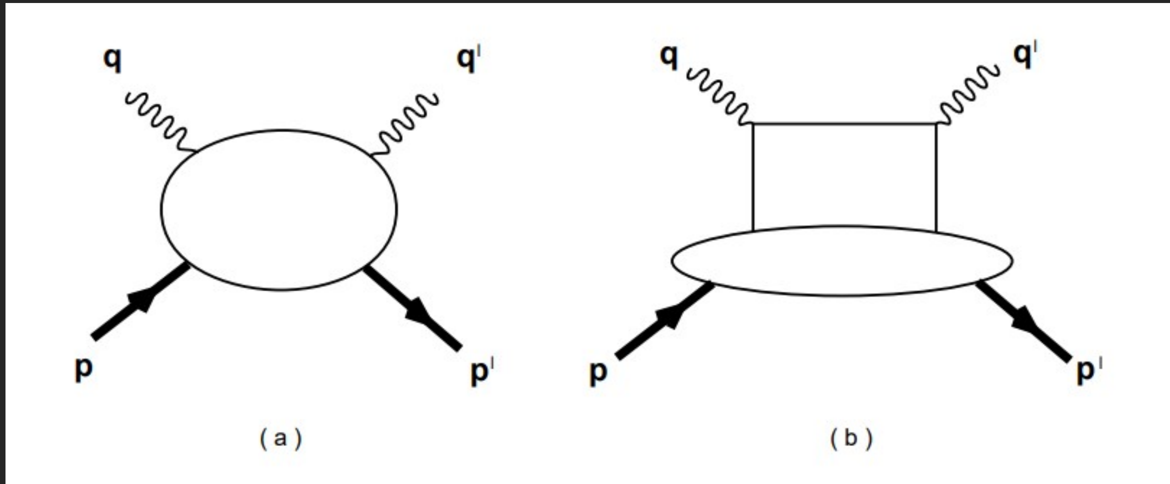


Figure 2. (*a*) Compton scattering; (*b*) leading Feynman diagrams for DVCS.

In the Bjorken limit, the DVCS amplitudes factorizes into GPDs and Hard kernels.

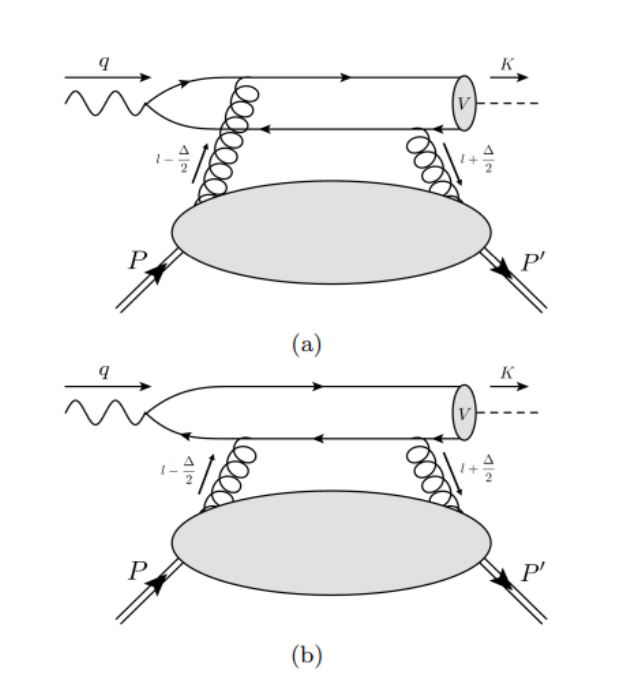
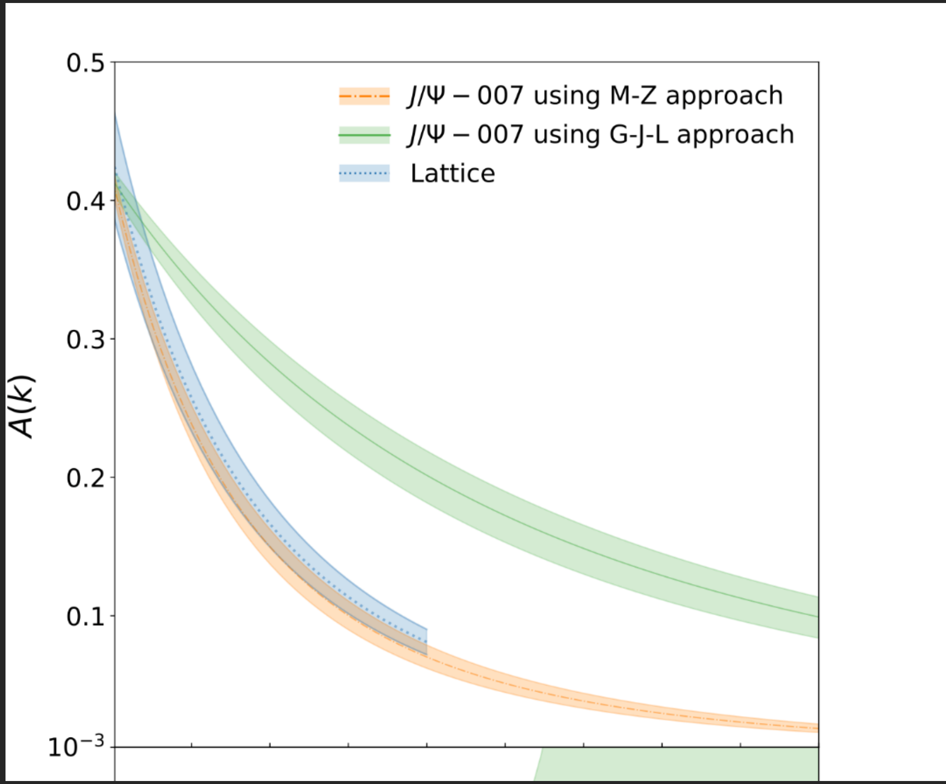


FIG. 4: Examples of leading Feynman diagrams that contribute to heavy vector meson photoproduction.

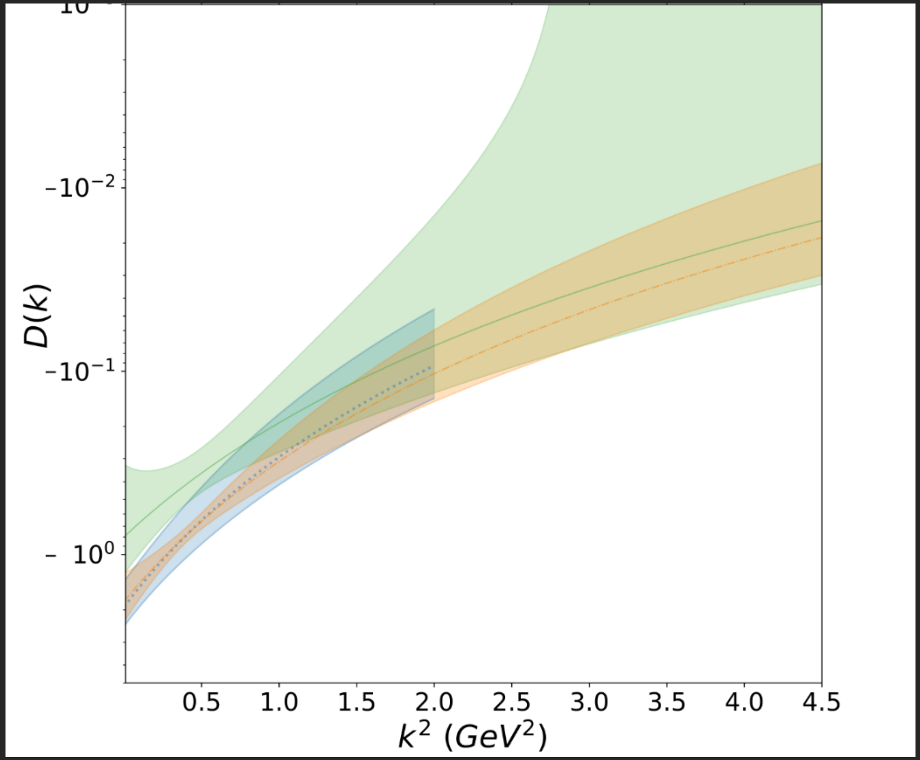
The vector meson production can also be used to probe GPDs. Near threshold may probe $\xi \sim 1$ region.

Threshould J/Ψ production and EMT form factor

Experiment performed in J-lab. 2207.05212



Extraction of the proton A form factor based on various approaches vs Lattice.



Extraction of the proton $D = 4C$ form factor.

PhysRevD.103.096010, Guo, Ji, Liu
PhysRevD.106.086004,Mamo&Zahed

EMT form factor and Sign of $C(0)$: clue from QED?

- Despite progress, uncertainty still large for $C(0)$ and its physical meaning. Is $C(0)$ always negative due to stability? hep-ph/9902451, Polyakov & Weiss
- Insights from QED will be helpful.
- The C-form factor for single electron has been used as an example.

Phys.Lett.B 820 (2021) 136501, Metz&Pasquini&Rodini
Phys. Rev. D 15 (1977) 538, Milton,

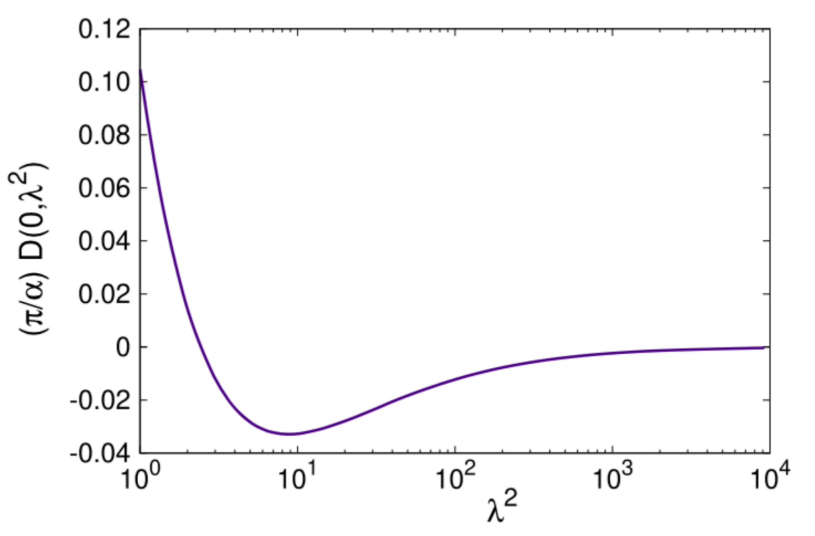


Fig. 2. The total D-term of the electron as a function of λ^2.

The D-term of single electron is negative for small photon mass.

Figure from
Metz, Pasquini, & Rodini

However, single electron is not bound state.

But hydrogen atom is.

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Bound states in QED: Scale separation and NRQED

- The most famous bound state in QFT: the hydrogen atom.

1. Hard to calculate in the underlying microscopic theory.
2. Great simplification due to the scale separation,
   $(Z\alpha)^2 m_e \ll (Z\alpha) m_e \ll m_e$:
   Power & log expansion organized by the NRQED.

- Very similar to power-expansion/OPE in high-energy physics.

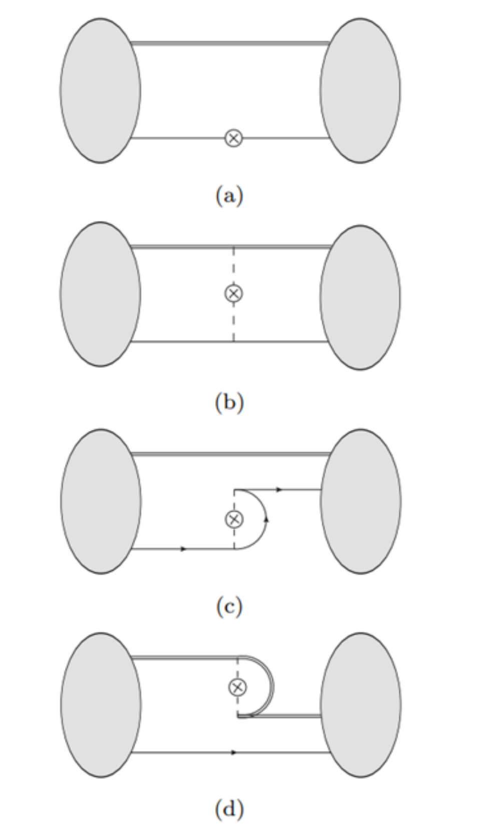
Caswell/Lepage, Phys. Lett. B 167, 437.
Labelle, Phys. Rev. D 58, 093013.
Pineda and Soto, Nucl. Phys. B Proc. Suppl. 64,428

The leading order results

- $\langle 0|T^{ij}(q)|0\rangle_H = (q^i q^j - \delta^{ij} q^2)\frac{C_H(q)}{m_e}$

$$\frac{C_{\mathrm{H}}(q)}{m_e} = \frac{1}{2m_e(\frac{q^2}{\alpha^2 m_e^2}+4)} - \frac{\alpha}{4|q|}\left(\frac{\pi}{2} - \mathrm{Arctan}\,\frac{q}{2\alpha m_e}\right) + \frac{\alpha\pi}{|q|}\frac{1}{(\frac{q^2}{\alpha^2 m_e^2}+4)^2} + \frac{\alpha\pi}{|q|}\frac{1}{(\frac{q^2}{\alpha^2 M^2}+4)^2}.$$

- $\frac{C_H(0)}{m_e} = \frac{1}{4m_e}$ is positive.



The leading order diagrams.

The NLO contribution for bound-state.

Figure (b) vanishes.

- For the bound-state, the pure-radiative diagram is activated in the ultra-soft region.
- Dipoles expansion performed in ultra-soft region as usual.
- Tadpole diagram remains the same as free-NRQED.
- Cancellation of longitudinal parts between (a), (c) and (d).

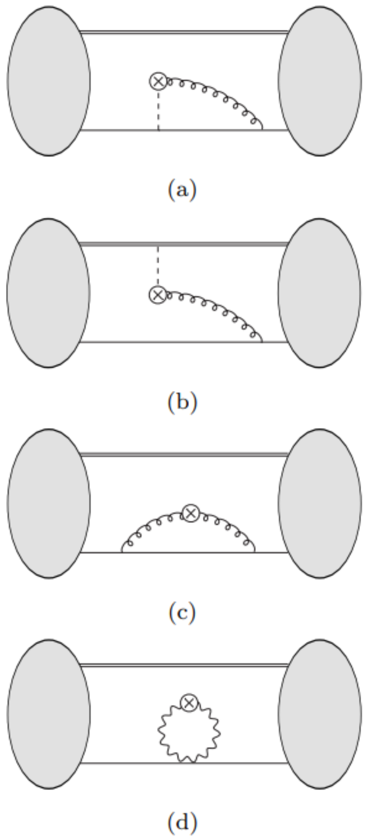


FIG. 7: The order-$\mathcal{O}(\alpha)$ contributions to $T^{ij}_{\gamma} + T^{ij}_{\gamma p}$ for a bound state. Dashed lines represent Coulomb photons and crossed circles denote the operator insertions.

The NLO contribution: The total results

- The D-term for hydrogen-atom

1. $\langle 0|T^{ij}(q)|0\rangle_H = (q^i q^j - \delta^{ij} q^2)\frac{c_H(q)}{m_e}$

2. $\frac{c_H(0)}{m_e} = \tau_H = \frac{1}{4m_e} + \frac{\alpha}{6\pi}\sum_{M\neq 0}\frac{2\vec{v}_{0M}\cdot\vec{v}_{M0}}{D(E_M - E_0)}\left(\ln\frac{4(E_M - E_0)^2}{m_e^2} - \frac{1}{4}\right)$

- The logarithm couples the UV and IR. Similar to the Bethe logarithm.
- Both continuum and discrete spectrum contribute.

The NLO contribution: The total results

- Numerically, one has

$$\frac{\tau_H}{\tau_0} - 1 = \frac{4\alpha}{3\pi}(\ln\alpha - 0.028) = -1.54\times10^{-2}.$$

- The NLO contribution is small and negative.

Discussion and Summary

- To summarize, the EMT form factor in QFT are important quantities to understand the mass-structure of bound-states.
- We use the hydrogen-like atom in QED as a non-trivial example to show that the D-term is not necessarily negative.
- Simple understanding of the origin of the logarithm, like the Lamb's shift?
- Full $T^{ij}(q)$? Mass radius ?