Review of light exotic mesons below 2.6 GeV

Francesco Giacosa
UJK Kielce (Poland) **& Goethe U Frankfurt** (Germany)
in collaboration with:
S. Jafarzade, A. Vereijken, E. Trotti, A. Koenigstein,
M. Piotrowska, V. Shastry
A. Pilloni, R. Pisarski, C. Fischer

XXIX The Cracow Epiphany Conference
16.–19. Jan. 2023
Henryk Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences

Outline

Symmetries of QCD

Conventional mesons: brief recall and open questions

Unconventional mesons:

- Four-quark states
- Glueballs
- Hybrids (briefly -> talk by V. C. Shastry)

Conclusions

A simple introduction to the topic

Vol. 47 (2016) ACTA PHYSICA POLONICA B No 1

MESONS BEYOND THE QUARK–ANTIQUARK PICTURE*

FRANCESCO GIACOSA

Contribution to:
55th Cracow School of Theoretical Physics, 7
e-Print: 1511.04605 [hep-ph]

Symmetries of QCD

Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

Some historical facts

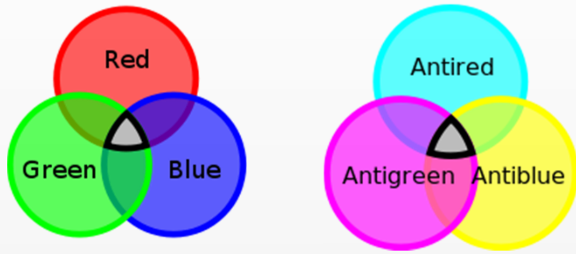
- Nel 1793 un decreto della Convenzione ordinava l'arresto degli stranieri nati in Paesi in guerra con la Repubblica, tra i quali figurava il Regno di Sardegna. La comunità scientifica si schierò in difesa di Lagrange, che fu posto in requisizione con l'incarico di studiare problemi di balistica e venne mantenuto libero in Francia.

- Nel 1799 fu tra i primi membri del Senato della Repubblica francese, previsto dalla nuova Costituzione dell'anno VIII che seguì al colpo di Stato del 18 brumaio (9 novembre) del generale Napoleone Bonaparte. Porta la sua firma il senatoconsulto dell'11 settembre 1802 con cui il Piemonte fu annesso alla Francia.

- Source: https://www.treccani.it/enciclopedia/

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B

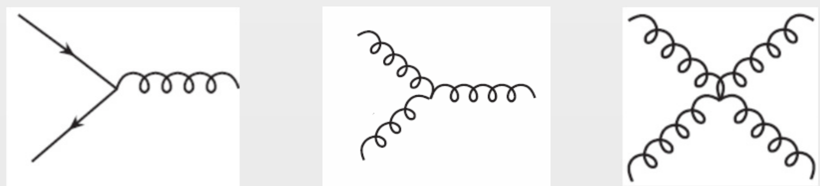


$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix}; \quad i = u,d,s, \ldots$$

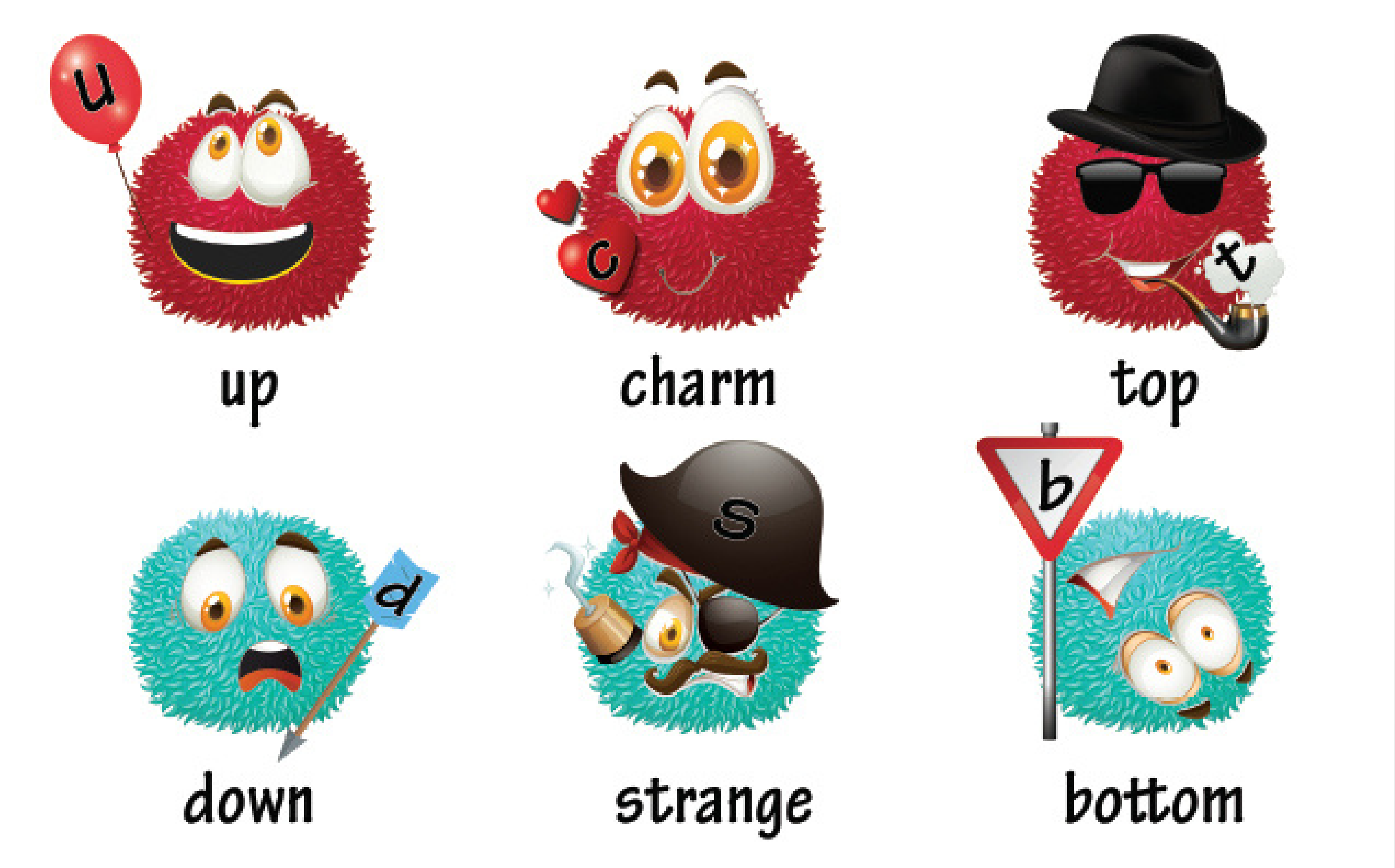
8 type of gluons ($R\overline{G}$, $B\overline{G}$, …)

$$A_\mu^a \; ; \quad a = 1, \ldots, 8$$

$$\boxed{\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}}$$



Confinement: quarks never ‘seen’ directly. How they might look like ☺



Picture by Pawel Piotrowski

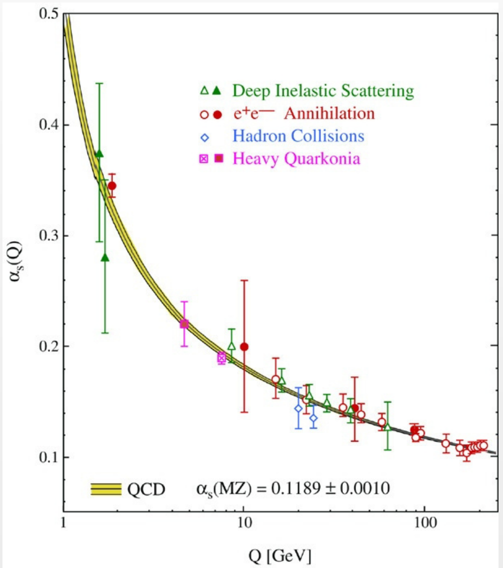
Trace anomaly: the emergence of a dimension

Chiral limit: $m = 0$

$x^{\mu} \rightarrow x'^{\mu} = \lambda^{-1} x^{\mu}$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250\ \mathrm{MeV}$

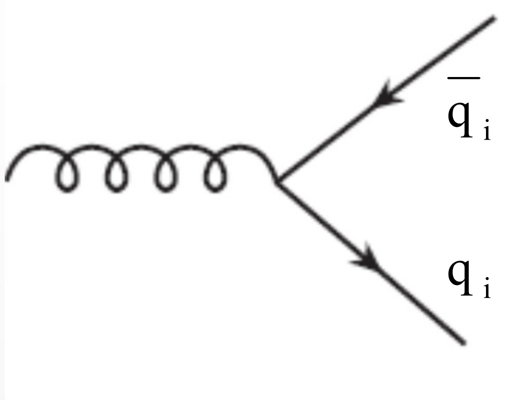
$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \rightarrow m^{*}_{gluon} \approx 500 - 800\ \mathrm{MeV}$

Gluon condensate: $\left\langle G^{a}_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Flavor symmetry



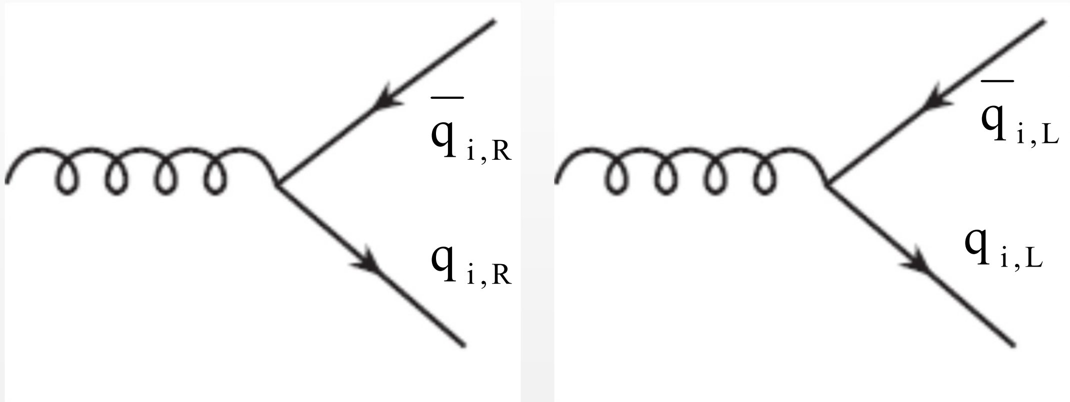
Gluon-quark-antiquark vertex

It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$U \in U(3)_V \rightarrow U^+ U = 1$$

Chiral symmetry



$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \to U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Chiral (or axial) anomaly: explicitely broken by quantum fluctuations

$$\boxed{\partial^\mu(\bar{q}^i\gamma_\mu\gamma_5 q^i) = \frac{3g^2}{16\pi^2}\,\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma})}$$

Spontaneous breaking of chiral symmetry

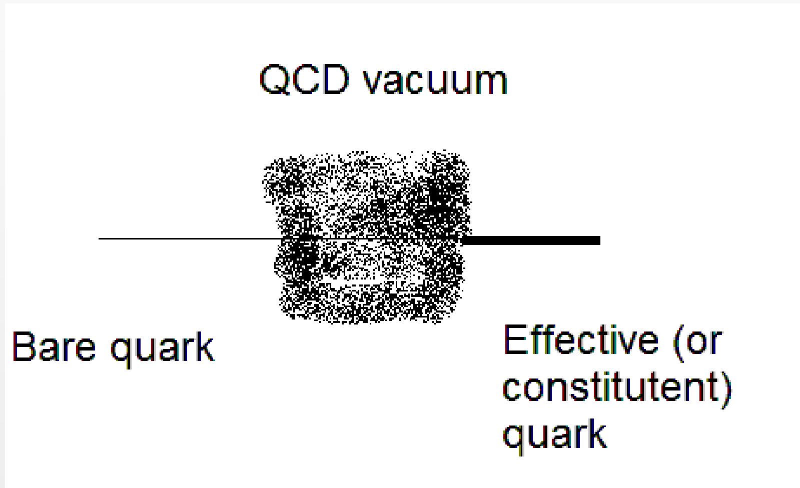
$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

$$\mathrm{SSB:}\ \mathrm{SU(3)_R \times SU(3)_L \rightarrow SU(3)_{V=R+L}}$$

Chiral symmetry → Flavor symmetry

$$\left\langle \bar{q}_i q_i \right\rangle = \left\langle \bar{q}_{i,R} q_{i,L} + \bar{q}_{i,L} q_{i,R} \right\rangle \neq 0$$

$$\mathrm{m \approx m_u \approx m_d \approx 5\ MeV \rightarrow m^* \approx 300\ MeV}$$



$$\mathrm{m_{\rho-meson} \approx 2m^*}$$

$$\mathrm{m_{proton} \approx 3m^*}$$

Symmetries of QCD and breakings

$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**scale anomaly**) and by quark masses.

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (**chiral anomaly**)

Meson

Definition(s):

1) A meson is a strongly interacting particle with integer spin.

2) A meson is a strongly interacting particle with zero baryon number.

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Conventional mesons: quark-antiquark states

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are “white” and are called hadrons.

Hadrons can be:

Mesons: bosonic hadrons

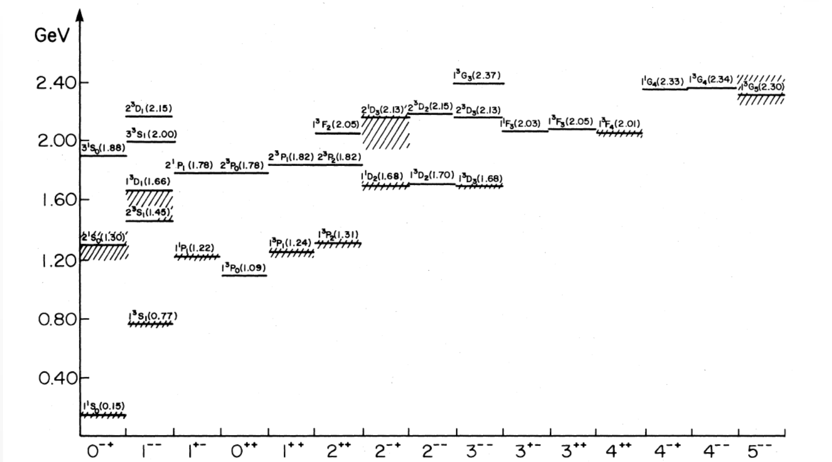
Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Quark model(s) and their QFT extensions

Uniwersytet
Jana Kochanowskiego w Kielcach

Mesons in a Relativized Quark Model
with Chromodynamics
S. Godfrey, N. Isgur
Phys.Rev. D32 (1985) 189-231



QCD phenomenology based
on a chiral effective Lagrangian
Tetsuo Hatsuda, Teiji Kunihiro
Phys.Rept. 247 (1994) 221-367

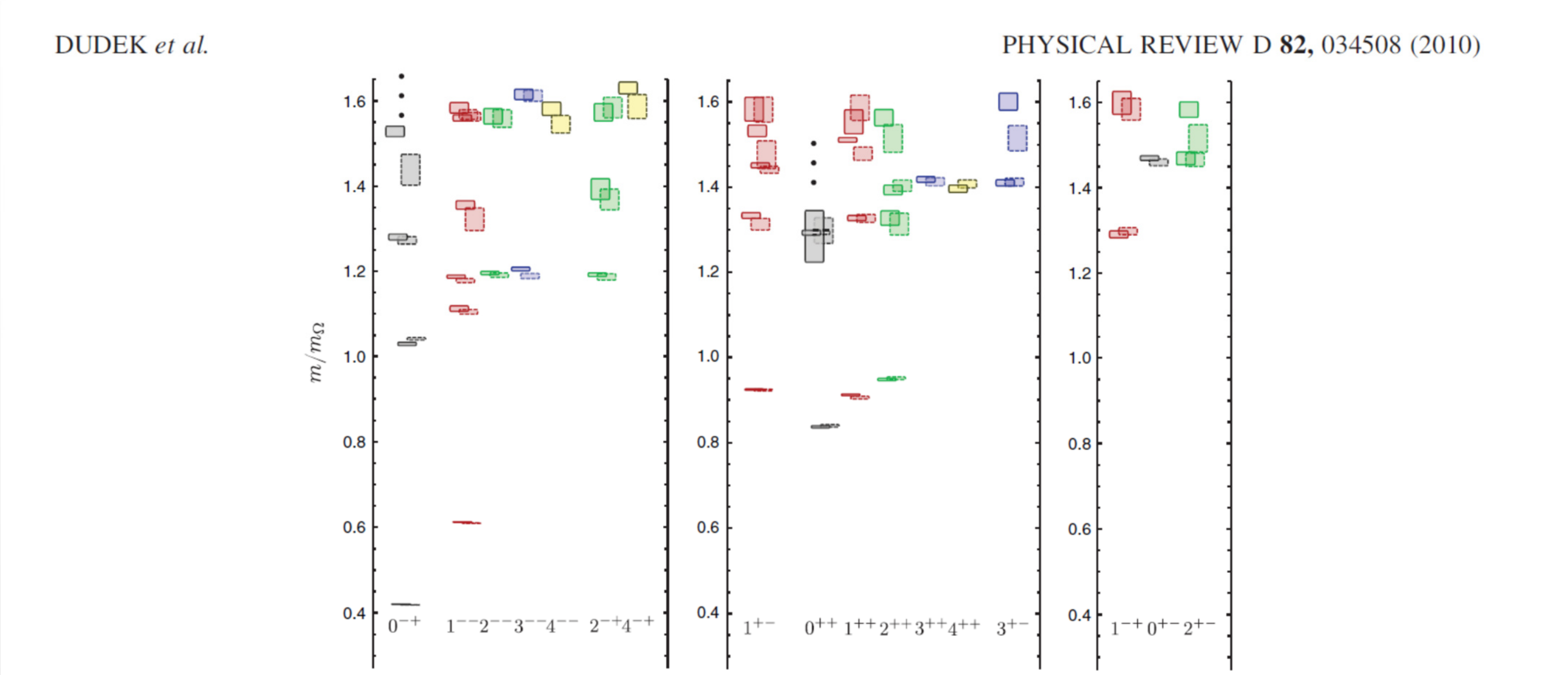
NJL: quark-based model with
chiral symmetry and SSB
chiral condensate
Effective quark mass
Mesons as quarkonia (pion: ok)

The Infrared behavior of QCD Green's functions:
Confinement dynamical symmetry breaking, and hadrons
as relativistic bound states
R. Alkofer, L. von Smekal
Phys.Rept. 353 (2001) 281

DS:
quarks and gluons propagators
from QCD
Condensates
Effective quark and gluon masses
Spectra of mesons as quarkonia
(pion: ok) and baryons as qqq states

G. Eichmann et al,
Baryons as relativistic three-quark bound states,
Prog. Part. Nucl. Phys. 91 (2016), 1-100

Lattice QCD



e.g.

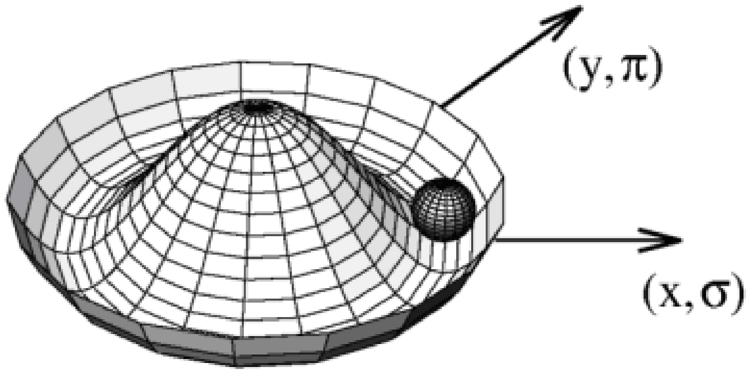
J. J. Dudek et al.

Toward the excited meson spectrum of dynamical QCD,

Phys. Rev. D 82 (2010), 034508 doi:10.1103/PhysRevD.82.034508

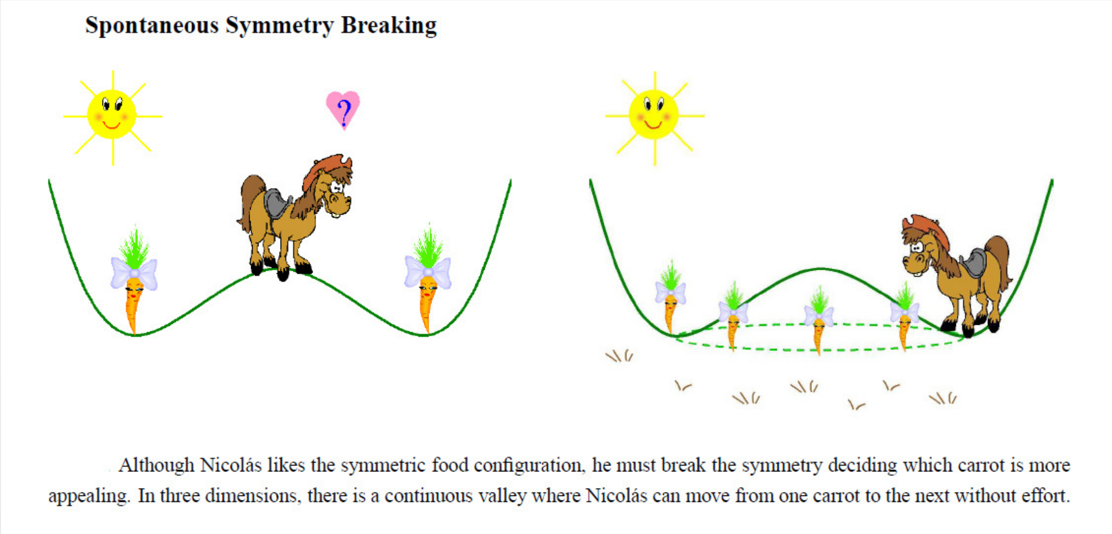
[arXiv:1004.4930 [hep-ph]].

SSB and the donkey of Buridan: hadronic approaches



$$\boxed{\sigma_N \to \sigma_N + \phi}$$

Jean Buridan (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)



- Chiral pertrubation theory (nonlinear realization of CS)

e.g.
S. Scherer and M. R. Schindler,
Chiral perturbation theory for mesons,
Lect. Notes Phys. 830 (2012), 65-144

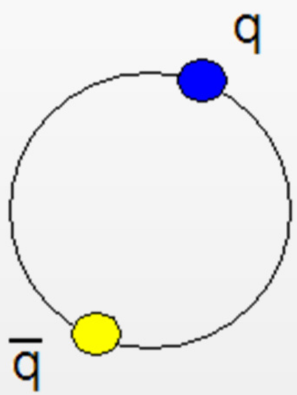
- Effective chiral hadronic models (LSM, eLSM,…linear real.)

e.g.
D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke,
Meson vacuum phenomenology in a three-flavor linear sigma model
with (axial-)vector mesons,
Phys. Rev. D 87 (2013) no.1,
[arXiv:1208.0585 [hep-ph]].

Conventional mesons

Quark: u,d,s,… R,G,B

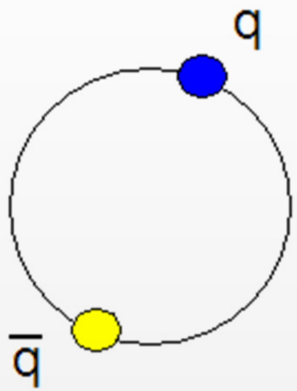
Quark-antiquark bound states: conventional mesons



$$|color\rangle = \sqrt{1/3}\,(\overline{R}R + \overline{B}B + \overline{G}G)$$

Conventional mesons/2

Surely, with quark-antiquark states we can understand a lot of QCD, but definitely not everything.



$$\vec{L}\,,\,\vec{S} \Longrightarrow P = -(-1)^{L} \qquad C = (-1)^{L+S}$$

$$\vec{L}\,,\,\vec{S} \Longrightarrow \vec{J} = \vec{L} + \vec{S} \qquad J^{PC}$$

Exotic quantum numbers

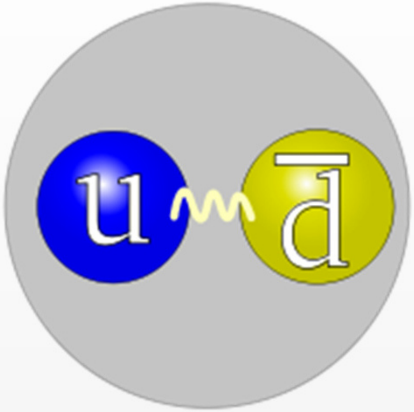
Not all quantum numbers are permitted for a quark-antiquark states.

$$J^{PC} = 0^{+-}\ ,\ 1^{-+}\ ,\ 2^{+-}, \ldots$$

are exotic quantum numbers.

$$P = -(-1)^{L} \qquad C = (-1)^{L+S} \qquad \vec{J} = \vec{L} + \vec{S}$$

Example of conventional quark-antiquark states: the ρ and the π mesons



$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c}\left(|\pi^+\pi^0\rangle + ...\right)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

Rho-meson $m_{\rho^+} = 775 \;\; \text{MeV}$

Pion $m_{\pi^+} = 139 \;\; \text{MeV}$

$$m_u + m_d \approx 7 \;\text{MeV}$$

Mass generation in QCD is a nonpert. penomenon based on SSB
(mentioned previusly).

Quark-antiquark states: the large-Nc limit

As IG have shown, the quark model works.
Theoretical justification relies on the large-Nc expansion.

For a comprehensive review on Nc:

Baryons in the 1/n Expansion
Edward Witten
Nucl.Phys. B160 (1979) 57

$$\left|\rho^+\right\rangle \propto \left|u\bar{d}\right\rangle + \frac{1}{N_c}\left(\left|\pi^+\pi^0\right\rangle + ...\right)$$

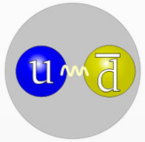
where

$$\left|u\bar{d}\right\rangle = \left|\text{valence } u + \text{valence } \bar{d} + \text{gluons}\right\rangle$$

Mesons beyond q-qbar: the first term in the first expansion is of non-quarkonium type

$L = S = 0 \rightarrow J^{PC} = 0^{-+}$ pseudoscalar mesons

$$|\pi^+\rangle = |u\bar{d}\rangle |space: L=0\rangle |spin: S=0\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$



$$|\pi^-\rangle = |d\bar{u}\rangle |space: L=0\rangle |spin: S=0\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

$$|\pi^0\rangle = |u\bar{u} - d\bar{d}\rangle |space: L=0\rangle |spin: S=0\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

Flavor symmetry: the 3 pions have the same mass.

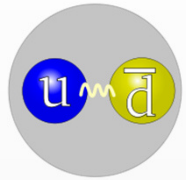
$$|K^+\rangle = |u\bar{s}\rangle |space: L=0\rangle |spin: S=0\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

...

$$|D^0\rangle = |u\bar{c}\rangle |space: L=0\rangle |spin: S=0\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

...

$$L=0\ ,\ S=1 \rightarrow J^{PC}=1^{--}$$ vector mesons



$$\left|\rho^{+}\right\rangle = \left|u\bar{d}\right\rangle \left|space: L=0\right\rangle \left|spin: S=1\right\rangle \left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|K^{*}(892)^{+}\right\rangle = \left|u\bar{s}\right\rangle \left|space: L=0\right\rangle \left|spin: S=1\right\rangle \left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|D^{*0}\right\rangle = \left|u\bar{c}\right\rangle \left|space: L=0\right\rangle \left|spin: S=1\right\rangle \left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|j/\Psi\right\rangle = \left|c\bar{c}\right\rangle \left|space: L=0\right\rangle \left|spin: S=1\right\rangle \left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

$$L=S=1\rightarrow J^{PC}=0^{++}$$ scalar mesons

$$|\sigma\rangle=\left|u\bar{u}+d\bar{d}\right\rangle|space:L=1\rangle|spin:S=1\rangle\left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

corresponds to the resonance $f_0(1370)$.

...

...

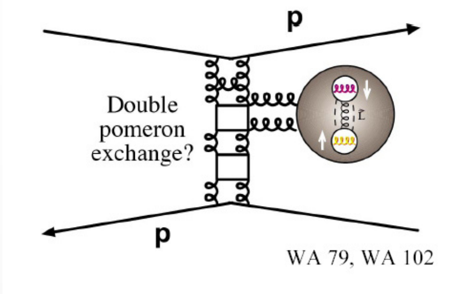
$$\left|\chi_{c0}(1S)\right\rangle=\left|c\bar{c}\right\rangle|space:L=1\rangle|spin:S=1\rangle\left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

How do we see things: experiments

Hadronic experiments

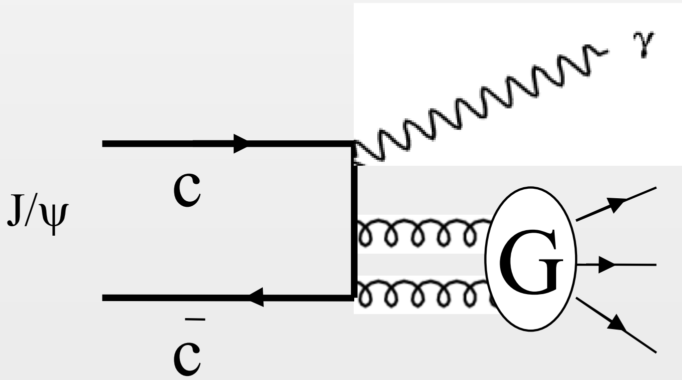
Proton-proton

(WA79,WA102,LHC,RHIC)



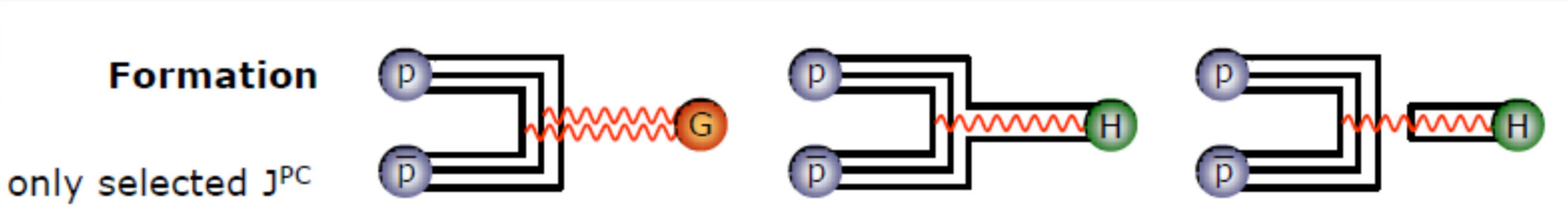
Electron-positron (with strange-antistrange, charm-anticharm or bottom-antibottom formation)

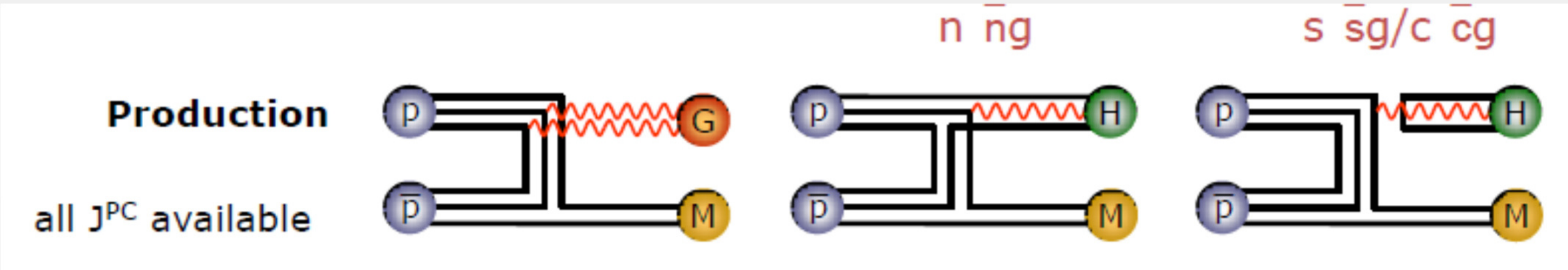
(Belle, Babar, BES, KLOE,…)



Proton-antiproton

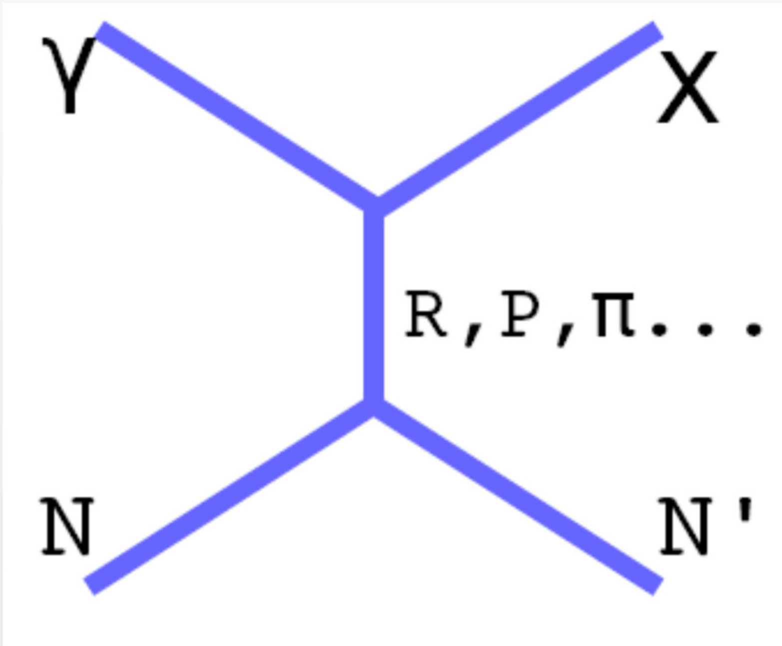
(Lear,Fermilab, and in the future: PANDA)



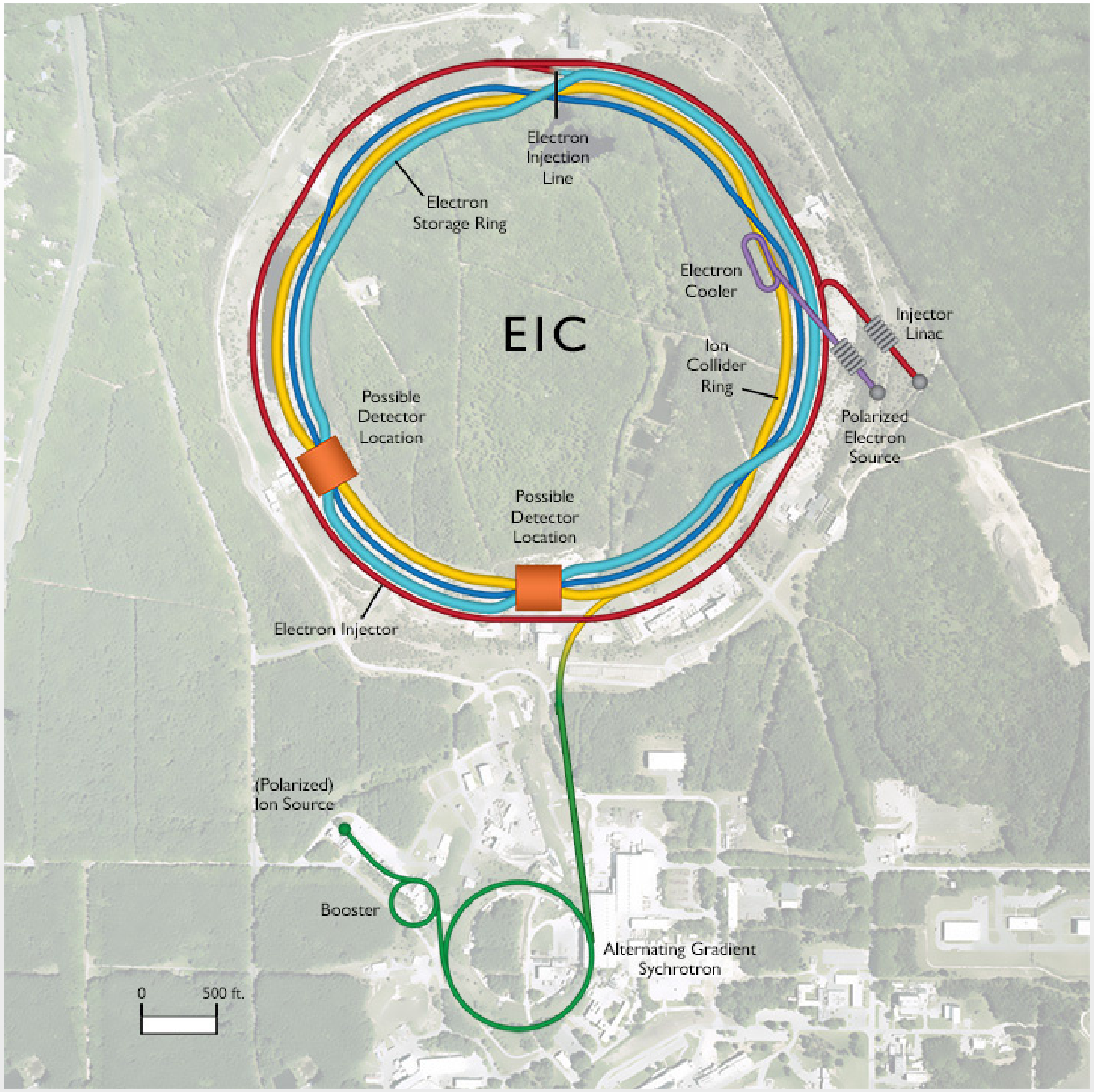


Photoproduction:

Compass at CERN (also with pion instead of the photons)
GlueX AND CLAS12 at Jlab
EIC (planned)



EIC plan


Electron
Injection
Line
Electron
Storage Ring
Electron
Cooler
EIC
Injector
Linac
Ion
Collider
Ring
Possible
Detector
Location
Polarized
Electron
Source
Possible
Detector
Location
Electron Injector
(Polarized)
Ion Source
Booster
Alternating Gradient
Sychrotron
0
500 ft.

B. Touschek et al.

Anectode

È entrato nella storia un ulteriore aneddoto. Oltre alla fisica, Bruno aveva una grande passione per le motociclette. Quando era a Roma aveva una Puch che aveva affettuosamente rinominato Josephine. Sosteneva che quando usciva fino a tardi, e quando alzava un po' il gomito, Josephine sapeva sempre come riportarlo a casa sano e salvo. Una sera, arrivando a Frascati (esattamente a Vermicino), ebbe un incidente in cui andò a scontrarsi con la sua moto contro un camion e fu trasportato immediatamente alla Clinica Neurologica. Bruno era in un evidente stato confusionale e gli venne chiesto di ricostruire l'incidente:
"Io ero fermo- iniziò Touschek- e un camion a marcia indietro mi ha investito"
"Ma come- rispose il medico- ma è stato Lei ad andargli addosso!"
"Ma se Lei fa un'operazione di inversione temporale si renderà conto che è come dico io!".
A questo punto la reazione fu immediata: "È pazzo! Basta, Elettroshock!"

Fortunatamente stava passando di lì Valentino Braitenberg, che qualche tempo dopo diventò un grande neurobiologo, che intervenne dicendo che forse non si trattava di un pazzo ma di un fisico. Telefonò quindi all'Istituto di Fisica e chiese ad Amaldi se "avevano perso un fisico". Amaldi rispose dicendo che, effettivamente, erano un paio di giorni che Bruno Touschek non si faceva vedere.

Manuscript accepted for publication in *The European Physical Journal H*
Historical Perspectives on Contemporary Physics

The Charm of Theoretical Physics (1958-1993)*

Oral History Interview

Luciano Maiani[1,a] and Luisa Bonolis[2,b]

[1] Dipartimento di Fisica and INFN, Piazzale A. Moro 5, 00185 Rome, Italy
[2] Max Planck Institute for the History of Science, Boltzmannstraße 22, 14195 Berlin, Germany

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	−11.3	−24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^{\dagger}$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^{\dagger}$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^{\dagger}$	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		
$3\,^1S_0$	0^{-+}	$\pi(1800)$			$\eta(1760)$		

Quark-antiquark mesons (PDG 2018)

Uniwersytet Jana Kochanowskiego w Kielcach

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	-11.3	-24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		
$3\,^1S_0$	0^{-+}	$\pi(1800)$			$\eta(1760)$		

Some selected nonets

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

J^{PC}, ${}^{2S+1}L_J$	$I = 1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}})$ $I = 1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$ $I = 0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{**}$	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times \times U(1)_{\mathrm{A}}$
0^{-+}, ${}^1S_0$	π K $\eta, \eta'(958)$	$P^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\gamma^5 q^i$	$\Phi = S + \mathrm{i}P$ $(\Phi^{ij} = \bar{q}^j_{\mathrm{R}} q^i_{\mathrm{L}})$	$\Phi \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi U^\dagger_{\mathrm{R}}$
0^{++}, ${}^3P_0$	$a_0(1450)$ $K^*_0(1430)$ $f_0(1370), f_0(1710)^*$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		
1^{--}, ${}^1S_1$	$\rho(770)$ $K^*(892)$ $\omega(782), \phi(1020)$	$V^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L^{ij}_\mu = \bar{q}^j_{\mathrm{L}} \gamma_\mu q^i_{\mathrm{L}})$	$L_\mu \to U_{\mathrm{L}} L_\mu U^\dagger_{\mathrm{L}}$
1^{++}, ${}^3P_1$	$a_1(1260)$ $K_{1,A}$ $f_1(1285), f_1(1420)$	$A^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R^{ij}_\mu = \bar{q}^j_{\mathrm{R}} \gamma_\mu q^i_{\mathrm{R}})$	$R_\mu \to U_{\mathrm{R}} R_\mu U^\dagger_{\mathrm{R}}$
1^{+-}, ${}^1P_1$	$b_1(1235)$ $K_{1,B}$ $h_1(1170), h_1(1380)$	$P^{ij}_\mu = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi^{ij}_\mu = \bar{q}^j_{\mathrm{R}} \mathrm{i}\overleftrightarrow{D}_\mu q^i_{\mathrm{L}})$	$\Phi_\mu \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_\mu U^\dagger_{\mathrm{R}}$
1^{--}, ${}^3D_1$	$\rho(1700)$ $K^*(1680)$ $\omega(1650), \phi(?)$	$S^{ij}_\mu = \frac{1}{2}\bar{q}^j \mathrm{i}\overleftrightarrow{D}_\mu q^i$		
2^{++}, ${}^3P_2$	$a_2(1320)$ $K^*_2(1430)$ $f_2(1270), f'_2(1525)$	$V^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j (\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\mu + \cdots) q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L^{ij}_{\mu\nu} = \bar{q}^j_{\mathrm{L}} (\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots) q^i_{\mathrm{L}})$	$L_{\mu\nu} \to U_{\mathrm{L}} L_{\mu\nu} U^\dagger_{\mathrm{L}}$
2^{--}, ${}^3D_2$	$\rho_2(?)$ $K_2(1820)$ $\omega_2(?), \phi_2(?)$	$A^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j (\gamma^5 \gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots) q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R^{ij}_{\mu\nu} = \bar{q}^j_{\mathrm{R}} (\gamma_\mu \overleftrightarrow{D}_\nu + \cdots) q^i_{\mathrm{R}})$	$R_{\mu\nu} \to U_{\mathrm{R}} R_{\mu\nu} U^\dagger_{\mathrm{R}}$
2^{-+}, ${}^1D_2$	$\pi_2(1670)$ $K_2(1770)$ $\eta_2(1645), \eta_2(1870)$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j (\mathrm{i}\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + \mathrm{i}P_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar{q}^j_{\mathrm{R}} (\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i_{\mathrm{L}})$	$\Phi_{\mu\nu} \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_{\mu\nu} U^\dagger_{\mathrm{R}}$
2^{++}, ${}^3F_2$	$a_2(?)$ $K^*_2(?)$ $f_2(?), f'_2(?)$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j (\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i$		
3^{--}, ${}^3D_3$	$\rho_3(1690)$ $K^*_3(1780)$ $\omega_3(1670), \phi_3(1850)$	$\vdots$	$\vdots$	$\vdots$

Table from:

F.G., R. Pisarski,
A. Koenigstein
Phys.Rev.D 97 (2018) 9, 091901
e-Print: 1709.07454

Three open questions for conventional mesons

Open questions 1: where are the axial-tensor mesons?

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

From well-known tensor mesons to yet unknown axial-tensor mesons

Shahriyar Jafarzade,[1] Arthur Vereijken,[1] Milena Piotrowska,[1] and Francesco Giacosa[1,2]
[1]*Institute of Physics, Jan Kochanowski University, ulica Uniwersytecka 7, 25-406 Kielce, Poland*
[2]*Institute for Theoretical Physics, J. W. Goethe University, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*

(Received 4 April 2022; accepted 25 July 2022; published 10 August 2022)

$$2^{++} \longrightarrow 0^{-+} + 0^{-+}\ ;$$
$$2^{--} \longrightarrow 0^{-+} + 1^{--}\ .$$

$2^{++},\ {}^3P_2$	$\begin{cases} a_2(1320) \\ K_2^*(1430) \\ f_2(1270), f_2'(1525) \end{cases}$	$V_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\mu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}_L^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q_L^i)$	$L_{\mu\nu} \to U_L L_{\mu\nu} U_L^\dagger$
$2^{--},\ {}^3D_2$	$\begin{cases} \rho_2(?) \\ K_2(1820) \\ \omega_2(?), \phi_2(?) \end{cases}$	$A_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}_R^j(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q_R^i)$	$R_{\mu\nu} \to U_R R_{\mu\nu} U_R^\dagger$

$$\mathcal{L}_{g_2^{\text{ten}}} = \frac{g_2^{\text{ten}}}{2}\Big(\mathrm{Tr}\Big[\mathbf{L}_{\mu\nu}\{L^\mu, L^\nu\}\Big] + \mathrm{Tr}\Big[\mathbf{R}_{\mu\nu}\{R^\mu, R^\nu\}\Big]\Big)$$

Small isoscalar mixing angle

$$\begin{pmatrix} f_2(1270) \\ f_2'(1525) \end{pmatrix} = \begin{pmatrix} \cos\beta_T & \sin\beta_T \\ -\sin\beta_T & \cos\beta_T \end{pmatrix} \begin{pmatrix} f_{2,N} \\ f_{2,S} \end{pmatrix}$$

$$\beta_T = (3.16 \pm 0.81)^\circ$$

Postdictions (left) predictions (right)

Decay process (in model)	eLSM (MeV)	PDG (MeV)
$a_2(1320) \longrightarrow \bar{K}\,K$	4.06 ± 0.14	$7.0^{+2.0}_{-1.5} \leftrightarrow (4.9 \pm 0.8)\%$
$a_2(1320) \longrightarrow \pi\,\eta$	25.37 ± 0.87	$18.5 \pm 3.0 \leftrightarrow (14.5 \pm 1.2)\%$
$a_2(1320) \longrightarrow \pi\,\eta'(958)$	1.01 ± 0.03	$0.58 \pm 0.10 \leftrightarrow (0.55 \pm 0.09)\%$
$K_2^*(1430) \longrightarrow \pi\,\bar{K}$	44.82 ± 1.54	$49.9 \pm 1.9 \leftrightarrow (49.9 \pm 0.6)\%$
$f_2(1270) \longrightarrow \bar{K}\,K$	3.54 ± 0.29	$8.5 \pm 0.8 \leftrightarrow (4.6^{+0.5}_{-0.4})\%$
$f_2(1270) \longrightarrow \pi\,\pi$	168.82 ± 3.89	$157.2^{+4.0}_{-1.1} \leftrightarrow (84.2^{+2.9}_{-0.9})\%$
$f_2(1270) \longrightarrow \eta\,\eta$	0.67 ± 0.03	$0.75 \pm 0.14 \leftrightarrow (0.4 \pm 0.08)\%$
$f_2'(1525) \longrightarrow \bar{K}\,K$	23.72 ± 0.60	$75 \pm 4 \leftrightarrow (87.6 \pm 2.2)\%$
$f_2'(1525) \longrightarrow \pi\,\pi$	0.67 ± 0.14	$0.71 \pm 0.14 \leftrightarrow (0.83 \pm 0.16)\%$
$f_2'(1525) \longrightarrow \eta\,\eta$	1.81 ± 0.05	$9.9 \pm 1.9 \leftrightarrow (11.6 \pm 2.2)\%$

Decay process (in model)	eLSM (MeV)	PDG-2020 (MeV)
$a_2(1320) \longrightarrow \rho(770)\,\pi$	71.0 ± 2.6	$73.61 \pm 3.35 \leftrightarrow (70.1 \pm 2.7)\%$
$K_2^*(1430) \longrightarrow \bar{K}^*(892)\,\pi$	27.9 ± 1.0	$26.92 \pm 2.14 \leftrightarrow (24.7 \pm 1.6)\%$
$K_2^*(1430) \longrightarrow \rho(770)\,K$	10.3 ± 0.4	$9.48 \pm 0.97 \leftrightarrow (8.7 \pm 0.8)\%$
$K_2^*(1430) \longrightarrow \omega(782)\,\bar{K}$	3.5 ± 0.1	$3.16 \pm 0.88 \leftrightarrow (2.9 \pm 0.8)\%$
$f_2'(1525) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	19.89 ± 0.73	

Decay process (in model)	eLSM (MeV)
$\rho_2(?) \longrightarrow \rho(770)\,\eta$	$\approx 99 \pm 50$
$\rho_2(?) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 85 \pm 43$
$\rho_2(?) \longrightarrow \omega(782)\,\pi$	$\approx 419 \pm 210$
$\rho_2(?) \longrightarrow \phi(1020)\,\pi$	≈ 0.8
$K_{2,A} \longrightarrow \rho(770)\,K$	$\approx 195 \pm 98$
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\pi$	$\approx 316 \pm 158$
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\eta$	≈ 0.01
$K_{2,A} \longrightarrow \omega(782)\,\bar{K}$	$\approx 51 \pm 26$
$K_{2,A} \longrightarrow \phi(1020)\,\bar{K}$	$\approx 50 \pm 25$
$\omega_{2,N} \longrightarrow \rho(770)\,\pi$	$\approx 1314 \pm 657$
$\omega_{2,N} \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 85 \pm 43$
$\omega_{2,N} \longrightarrow \omega(782)\,\eta$	$\approx 93 \pm 47$
$\omega_{2,N} \longrightarrow \phi(1020)\,\eta$	≈ 0.06
$\omega_{2,S} \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 510 \pm 255$
$\omega_{2,S} \longrightarrow \omega(782)\,\eta$	$\approx 1.0 \pm 0.5$
$\omega_{2,S} \longrightarrow \omega(782)\,\eta'(958)$	≈ 0.3
$\omega_{2,S} \longrightarrow \phi(1020)\,\eta$	$\approx 101 \pm 51$

Decay process (in model)	eLSM (MeV)
$\rho_2(?) \longrightarrow a_2(1320)\,\pi$	≈ 88
$K_{2,A} \longrightarrow K_2^*(1430)\,\pi$	≈ 49
$K_{2,A} \longrightarrow a_2(1320)\,K$	≈ 84
$K_{2,A} \longrightarrow f_2(1270)\,K$	≈ 4
$\omega_{2,S} \longrightarrow K_2^*(1430)\,K + \text{c.c.}$	≈ 15

Open questions 2: mixing in the pseudotesnor meson sector

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Eur. Phys. J. A (2016) **52**: 356
DOI 10.1140/epja/i2016-16356-x

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Phenomenology of pseudotensor mesons and the pseudotensor glueball

Adrian Koenigstein[1,2,a] and Francesco Giacosa[3,1]

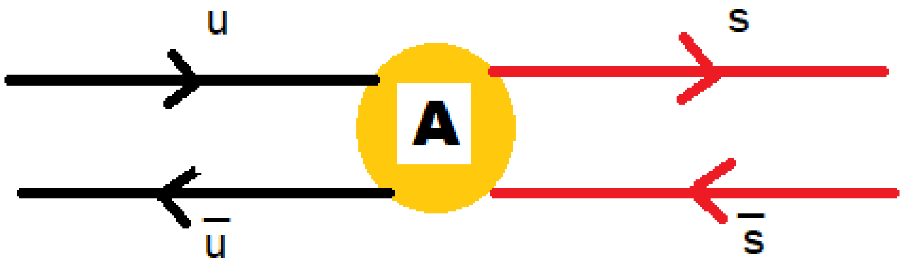
[1] Institute for Theoretical Physics, Johann Wolfgang Goethe University, Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany
[2] Frankfurt Institute for Advanced Studies, Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany
[3] Institute of Physics, Jan Kochanowski University, ul. Swietokrzyska 15, 25-406 Kielce, Poland

Received: 6 September 2016 / Revised: 11 November 2016
Published online: 9 December 2016 – © Società Italiana di Fisica / Springer-Verlag 2016
Communicated by R. Alkofer

Abstract. We study the decays of the pseudotensor mesons ($\pi_2(1670), K_2(1770), \eta_2(1645), \eta_2(1870)$) interpreted as the ground-state nonet of 1^1D_2 $\bar{q}q$ states using interaction Lagrangians which couple them to pseudoscalar, vector, and tensor mesons. While the decays of $\pi_2(1670)$ and $K_2(1770)$ can be well described, the decays of the isoscalar states $\eta_2(1645)$ and $\eta_2(1870)$ can be brought in agreement with the present experimental data only if the mixing angle between nonstrange and strange states is surprisingly large (about $-42°$, similar to the mixing in the pseudoscalar sector, in which the chiral anomaly is active). Such a large mixing angle is however at odd with all other conventional quark-antiquark nonets: if confirmed, a deeper study of its origin will be needed in the future. Moreover, the $\bar{q}q$ assignment of pseudotensor states predicts that the ratio $[\eta_2(1870) \to a_2(1320)\,\pi]/[\eta_2(1870) \to f_2(1270)\,\eta]$ is about 23.5. This value is in agreement with Barberis *et al.*, (20.4 ± 6.6), but disagrees with the recent reanalysis of Anisovich *et al.*, (1.7 ± 0.4). Future experimental studies are necessary to understand this puzzle. If Anisovich's value is confirmed, a simple nonet of pseudoscalar mesons cannot be able to describe data (different assignments and/or additional states, such as an hybrid state, will be needed). In the end, we also evaluate the decays of a pseudoscalar glueball into the aforementioned conventional $\bar{q}q$ states: a sizable decay into $K_2^*(1430)\,K$ and $a_2(1230)\,\pi$ together with a vanishing decay into pseudoscalar-vector pairs (such as $\rho(770)\,\pi$ and $K^*(892)\,K$) are expected. This information can be helpful in future studies of glueballs at the ongoing BESIII and at the future PANDA experiments.

ArXiv: 1608.08777

Large mixing angle: where does it come from?



PHYSICAL REVIEW D **97**, 091901(R) (2018)

Rapid Communications

How the axial anomaly controls flavor mixing among mesons

Francesco Giacosa,[1,2,*] Adrian Koenigstein,[2,†] and Robert D. Pisarski[3,‡]

[1]*Institute of Physics, Jan Kochanowski University, ulica Swietokrzyska 15, 25-406 Kielce, Poland*
[2]*Institute for Theoretical Physics, Johann Wolfgang Goethe University, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*
[3]*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA*

$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos\beta_{pt} & \sin\beta_{pt} \\ -\sin\beta_{pt} & \cos\beta_{pt} \end{pmatrix} \begin{pmatrix} \eta_{2,N} \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} \equiv \bar{s}s \end{pmatrix}$$

$$\underline{\beta_{pt} = -42^\circ}$$

$$\begin{pmatrix} \eta \equiv \eta(547) \\ \eta' \equiv \eta(958) \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_P \simeq -42^\circ$$

For a recent re-analysis with decay widhts partial-wave :
V. Shastry, E. Trotti, F.G., Phys. Rev.D 105 (2022) 5, 054022 • e-Print: 2107.13501

Open question 3: where is the (excited) vector meson phi?

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Citation: R.L. Workman *et al.* (Particle Data Group), to be published (2022)

$\phi(2170)$

$I^G(J^{PC}) = 0^-(1^{--})$

Uniwersytet Jana Kochanowskiego w Kielcach

See the review on "Spectroscopy of Light Meson Resonances."

$\phi(2170)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
2162 ± 7 OUR AVERAGE	Error includes scale factor of 1.1.			

$\phi(2170)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*C*
100^{+31}_{-23} OUR AVERAGE	Error includes scale factor of 2.5.			

Prediction for ϕ(1930)

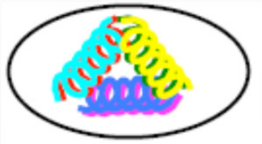
Can one find this state?

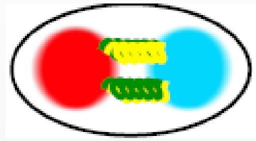
TABLE XII. Summary table for the putative state $\phi(1930)$.

Meson $\phi(1930)$	
Quark composition	$\approx s\bar{s}$
Old spectroscopy notation	(Predom.) $n^{2S+1}L_J = 1^3D_1$
n	(Predom.) 1
S	(Predom.) $1\uparrow\uparrow$
L	(Predom.) 2
J^{PC}	1^{--}
Mass	$\approx 1930 \pm 40$ MeV
Decays	
Decay channel	Decay width (MeV)
$\phi(1930) \rightarrow \bar{K}K$	104 ± 28
$\phi(1930) \rightarrow K\bar{K}^*$	260 ± 109
$\phi(1930) \rightarrow \Phi(1020)\eta$	67 ± 28
$\phi(1930) \rightarrow \Phi(1020)\eta'$	≈ 0
$\phi(1930) \rightarrow \gamma\eta$	0.19 ± 0.12
$\phi(1930) \rightarrow \gamma\eta'$	0.13 ± 0.08

arXiv: 1708.02593; **it does not fit with ϕ(2170)**

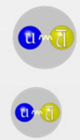
Non-conventional mesons: beyond quark-antiquark

1) Glueballs 

2) Hybrids 

3) Four-quark states

Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

Companion poles (another type of dynamical generation)

Four-quark candidates: light sector

The light scalar mesons

$a_0(980)$ $K_0^*(700)$ $f_0(980)$ $f_0(500)$

$J^{PC} = 0^{++}$

They (most probably!) are not quark-antiquark states!!!

The light scalars can be interpeted as tetraquark state

A tetraquark is the bound state of two diquarks

An example of „good diquark" is:

$$\left|qq\right\rangle = \left|Space: L=0\right\rangle \left|Spin: (\uparrow\downarrow - \downarrow\uparrow)\right\rangle \left|f:(ud-du)\right\rangle \left|c:(RB-BR)\right\rangle$$

Example: $a_0^+(980) = -[\bar{\mathrm{d}},\bar{\mathrm{s}}][\mathrm{u},\mathrm{s}]$ (and not $\mathrm{u}\bar{\mathrm{d}}$)

Uniwersytet Jana Kochanowskiego w Kielcach

$J^{PC}=0^{++}$	M < 1 GeV	Tetraquark interpretation
$I=1$	$a_0(980)$	$\boxed{[u,s][\bar{d},\bar{s}],\ [\bar{u},\bar{s}][d,s],\ ([u,s][\bar{u},\bar{s}]-[d,s][\bar{d},\bar{s}])}$
$I=\frac{1}{2}$	$K_0^*(700)$	$[u,d][\bar{d},\bar{s}],\ [\bar{u},\bar{d}][d,s],$ $[u,d][\bar{u},\bar{s}],\ [\bar{u},\bar{d}][u,s]$
$I=0$	$f_0(500)$	$\approx[\bar{u},\bar{d}][u,d]$
	$f_0(980)$	$\boxed{\approx([u,s][\bar{u},\bar{s}]+[d,s][\bar{d},\bar{s}])}$

Jaffe 1976, see also Ebert, Lucha,…

$J^{PC} = 0^{++}$	M < 1 GeV	Molecular interpretation
$I = 1$	$a_0(980)$	KK bound-state/enhancement
$I = \frac{1}{2}$	$K_0^*(700)$	πK enhanchement
$I = 0$	$f_0(500)$	ππ enhancement
	$f_0(980)$	KK bound-state/enhancement

Oset, Llanes-Strada, Baru, Meissner,…

Existence and pole position of f0(500)

Uniwersytet Jana Kochanowskiego w Kielcach

Complicated PDG history. Existence through the position of the pole.
Now: established.

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015 update

$f_0(500)$ or σ [g]
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

Mass m = (400–550) MeV
Full width Γ = (400–700) MeV

$f_0(500)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	dominant	–
$\gamma\gamma$	seen	–

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015 update

$f_0(500)$ or σ
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

A REVIEW GOES HERE – Check our WWW List of Reviews

$f_0(500)$ T-MATRIX POLE $\sqrt{s}$

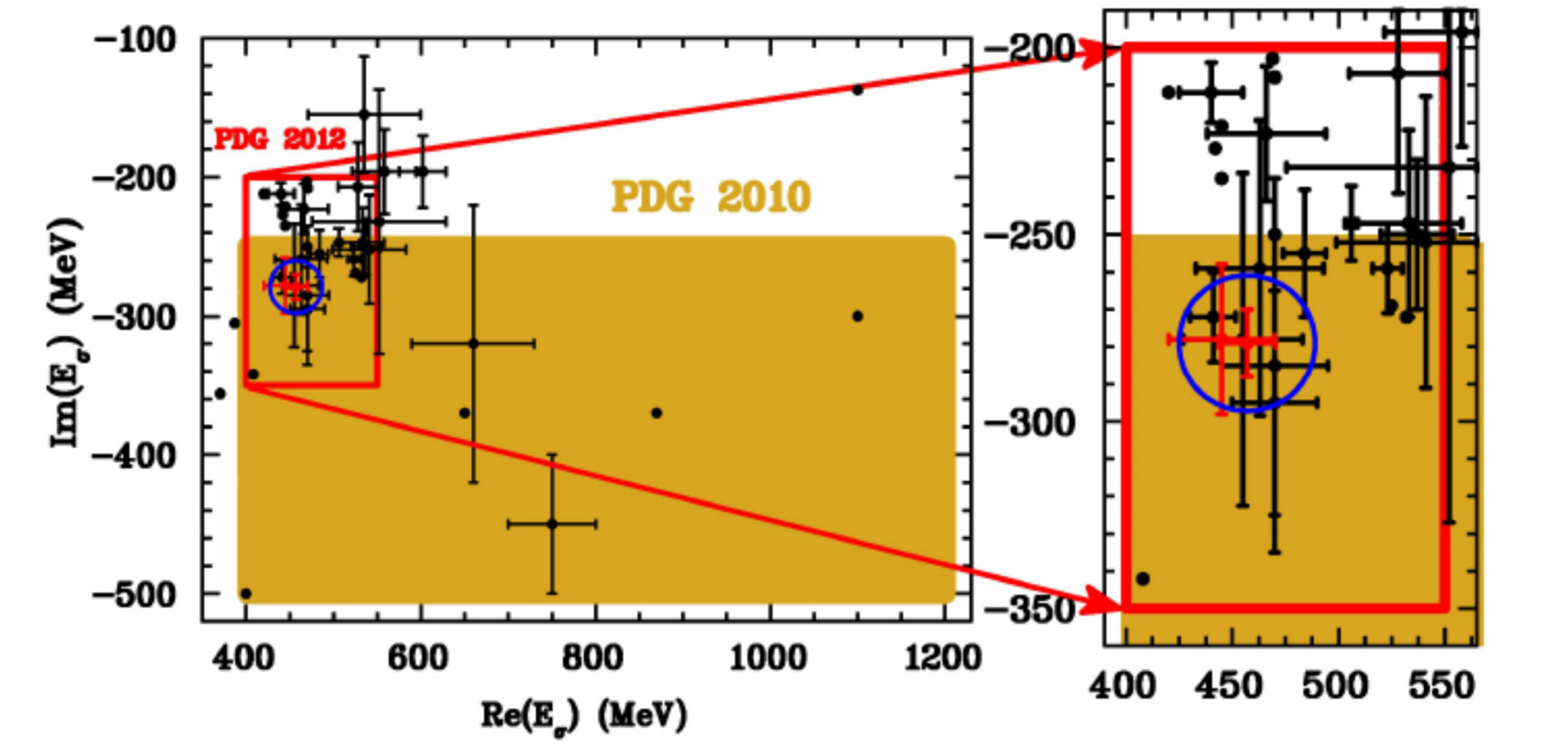
Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\rm pole}})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(400–550)−*i*(200–350) OUR ESTIMATE			

$$\sqrt{s_{pole}} = M - i\frac{\Gamma}{2}$$

Existence and pole position of f0(500)

From 2010 to 2012: update…



J. R. Pelaez,
From controversy to precision on the sigma meson:
a review on the status of the non-ordinary f0(500) resonance,
Phys. Rept. 658 (2016),
[arXiv:1510.00653 [hep-ph]].

Tornqvist and Roos,
Resurrection of the sigma meson,
PRL 76 (1996) 1575
[arXiv:hep-ph/9511210 [hep-ph]].

Madrid-Krakow and Bern results for the poles

$$g^2 = -16\pi \lim_{s \to s_{pole}} (s - s_{pole})\, t_\ell(s)\, (2\ell+1)/(2p)^{2\ell}$$

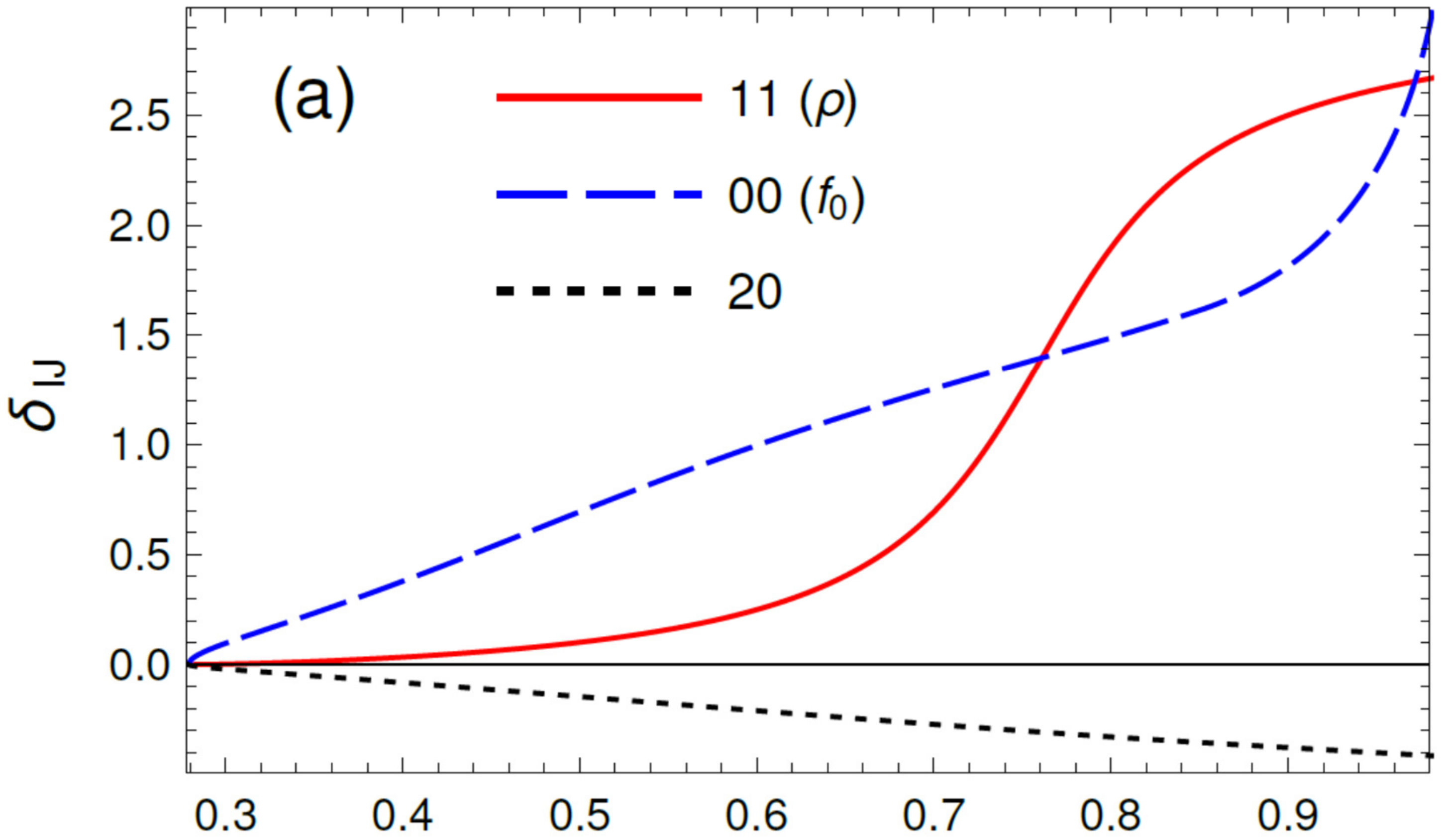
	$\sqrt{s_{\rm pole}}$ (MeV)	$\lvert g \rvert$
$f_0(500)^{\rm GKPY}$	$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	$3.59^{+0.11}_{-0.13}$ GeV
$f_0(500)^{\rm Roy}$	$(445 \pm 25) - i(278^{+22}_{-18})$	3.4 ± 0.5 GeV
$f_0(980)^{\rm GKPY}$	$(996 \pm 7) - i(25^{+10}_{-6})$	2.3 ± 0.2 GeV
$f_0(980)^{\rm Roy}$	$(1003^{+5}_{-27}) - i(21^{+10}_{-8})$	$2.5^{+0.2}_{-0.6}$ GeV
$\rho(770)^{\rm GKPY}$	$(763.7^{+1.7}_{-1.5}) - i(73.2^{+1.0}_{-1.1})$	$6.01^{+0.04}_{-0.07}$
$\rho(770)^{\rm Roy}$	$(761^{+4}_{-3}) - i(71.7^{+1.9}_{-2.3})$	$5.95^{+0.12}_{-0.08}$

*S*0 scattering length

- ChPT + Roy eqs (Bern group): $0.220 \pm 0.005\ m_\pi^{-1}$
- GKPY: $0.220 \pm 0.008\ m_\pi^{-1}$

From R. Kaminski, EEF70, Coimbra (2014).

Pion-pion phase-shift



W.Broniowski, F.G., V. Begun, 1506.01260, PRC:
cancellation of the sigma meson in thermal models

$K_0^*(700)$ from PDG

$K_0^*(800)$

or κ

$I(J^P) = \frac{1}{2}(0^+)$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$K_0^*(800)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
682 ±29 OUR AVERAGE		Error includes scale factor of 2.4. See the ideogram below.		

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022)

$K_0^*(700)$

$I(J^P) = \frac{1}{2}(0^+)$

also known as κ; was $K_0^*(800)$

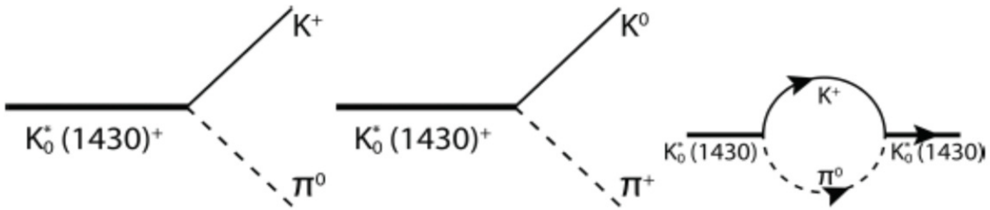
See the related review(s):

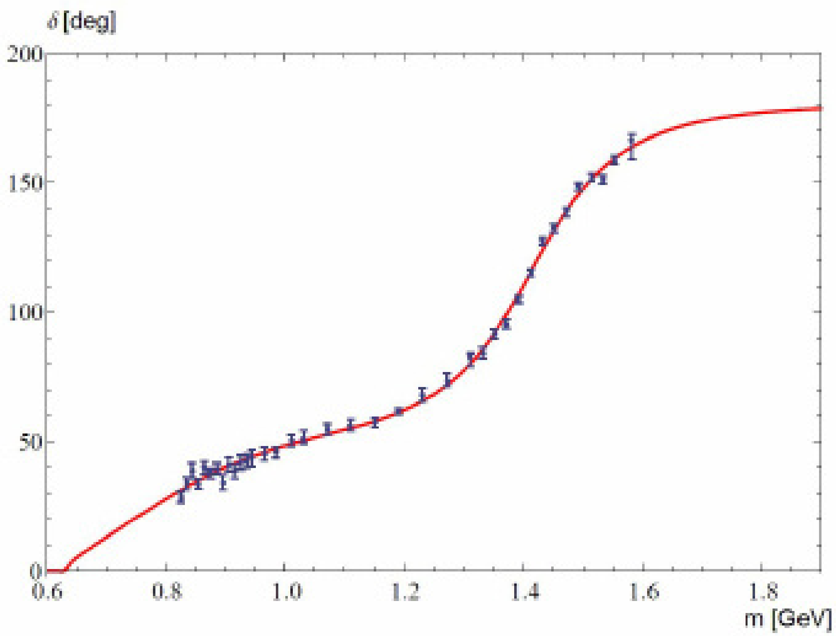
Scalar Mesons below 1 GeV

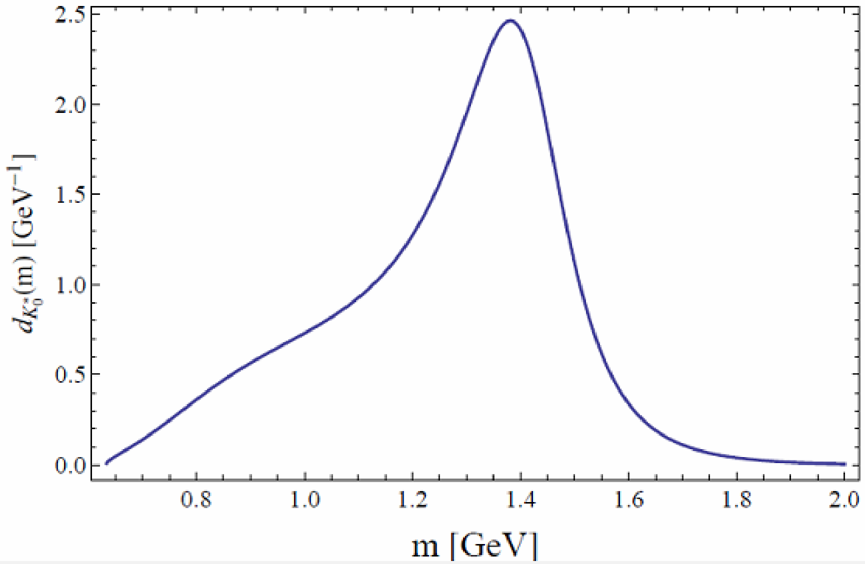
$K_0^*(700)$ T-Matrix Pole $\sqrt{s}$

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
(630–730) − *i* (260–340) OUR ESTIMATE	(see Fig. 64.1 in the review)			
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$(648 \pm 7) - i\,(280 \pm 16)$	[1] PELAEZ	20	RVUE	$\pi K \to \pi K$
$(670 \pm 18) - i\,(295 \pm 28)$	[2] PELAEZ	17	RVUE	$\pi K \to \pi K$
$(764 \pm 63^{+71}_{-54}) - i\,(306 \pm 149^{+143}_{-85})$	[3] ABLIKIM	11B	BES2	1.3k $J/\psi \to K^0_S K^0_S \pi^+ \pi^-$

K0*(1430) and K0*(700) together







Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022)

$K_0^*(1430)$ $\quad I(J^P) = \frac{1}{2}(0^+)$

$K_0^*(1430)$ T-MATRIX POLE $\sqrt{s}$

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(1431 \pm 6) - i\,(110 \pm 19)$	[1] PELAEZ 17	RVUE	$\pi K \to \pi K$

[1] Reanalysis of ESTABROOKS 78 and ASTON 88 satisfying Forward Dispersion Relations and using sequences of Pade approximants.

$K_0^*(1430)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1425 ±50 OUR ESTIMATE				

K0*(700) as a companion pole of K0*(1430)

based on M. Soltysiak, T. Wolkanowski and F. G., K0*(800) as a companion pole of K0*(1430),' Nucl. Phys. B 909 (2016) 418 [arXiv:1512.01071 [hep-ph]].

Glueballs:
where are You????

Masses of glueballs

Glueball masses: bag models

A. Chodos, et al., Phys. Rev. D 9 (1974) 3471.

R.L. Jaffe, K. Johnson, Phys. Lett. B60 (1976) 201

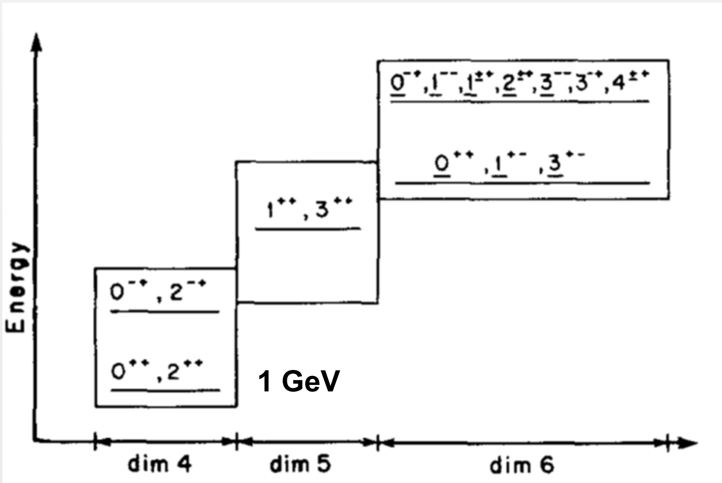
ANNALS OF PHYSICS **168**, 344–367 (1986)

Qualitative Features of the Glueball Spectrum*

R. L. JAFFE, K. JOHNSON, AND Z. RYZAK†

Center for Theoretical Physics, Laboratory for Nuclear Science, and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Received September 13, 1985



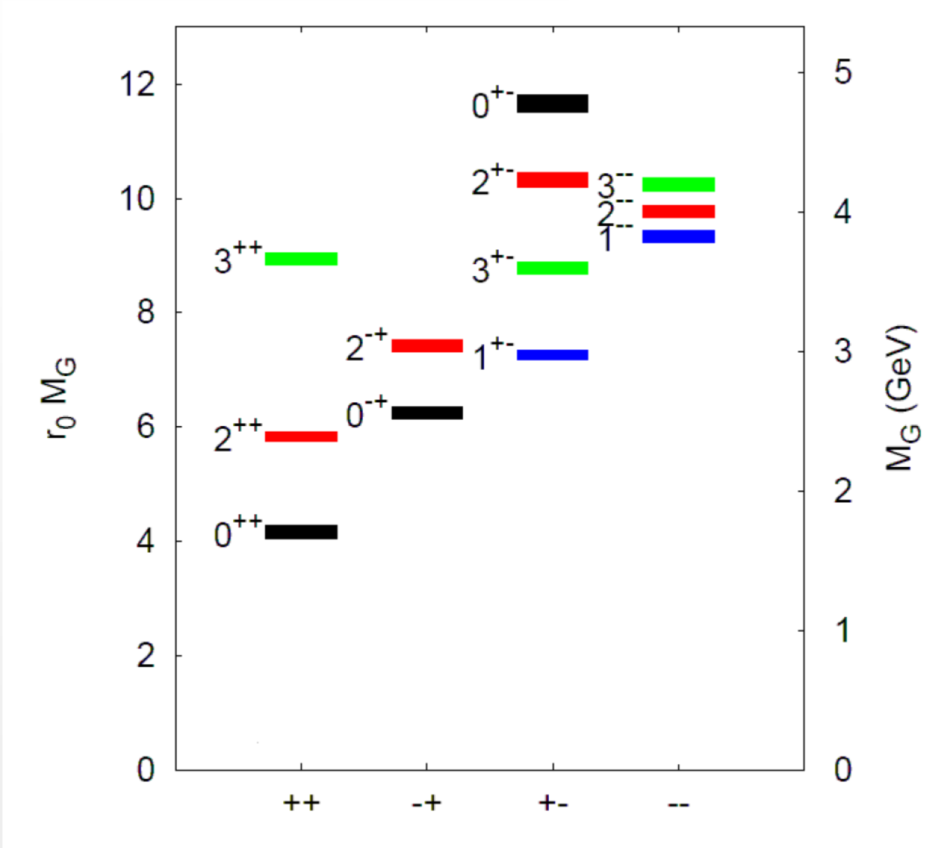
Lattice

PHYSICAL REVIEW D **73,** 014516 (2006)

Glueball spectrum and matrix elements on anisotropic lattices

Y. Chen,[1,2] A. Alexandru,[2] S. J. Dong,[2] T. Draper,[2] I. Horváth,[2] F. X. Lee,[3,4] K. F. Liu,[2] N. Mathur,[2,4] C. Mornin
M. Peardon,[6] S. Tamhankar,[2] B. L. Young,[7] and J. B. Zhang[8]
[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[2]*Department of Physics & Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA*
[3]*Center for Nuclear Studies, Department of Physics, George Washington University, Washington, D.C. 20052 US*
[4]*Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[5]*Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*
[6]*School of Mathematics, Trinity College, Dublin, Dublin 2, Ireland*
[7]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*
[8]*CSSM and Department of Physics, University of Adelaide, Adelaide, SA 5005, Australia*
(Received 13 October 2005; published 26 January 2006)

The glueball-to-vacuum matrix elements of local gluonic operators in scalar, tensor, and pseudoscalar channels are investigated numerically on several anisotropic lattices with the spatial lattice spacing ranging from 0.1–0.2 fm. These matrix elements are needed to predict the glueball branching ratios in J/ψ radiative decays which will help identify the glueball states in experiments. Two types of improved local gluonic operators are constructed for a self-consistent check and the finite-volume effects are studied. We find that lattice spacing dependence of our results is very weak and the continuum limits are reliably extrapolated, as a result of improvement of the lattice gauge action and local operators. We also give updated glueball masses with various quantum numbers.



Quoted by the PDG in the 'Quark Model' review.

See also: Gregory et al, **JHEP 1210 (2012) 170**

Towards the glueball spectrum from unquenched lattice QCD

Conclusions and future prospects The most conservative interpretation of our results is that the masses in terms of lattice representations are broadly consistent with results from quenched QCD. We do not see any evidence for large unquenching effects, however a definitive calculation requires a continuum extrapolation, and the inclusion of fermionic operators.

Other approaches/1

QCD Sum rules

H. G. Dosch and S. Narison, Nucl. Phys. Proc. Suppl. 121 (2003) 114 [arXiv:hep-ph/0208271].
H. Forkel, Phys. Rev. D 71, 054008 (2005) [arXiv:hep-ph/0312049].

$$M_{0++} \simeq 1.3 \text{ GeV}$$
$$M_{2++} \simeq 2.0 \text{ GeV}$$
$$M_{0-+} \simeq 2.2 \text{ GeV}$$

Hamiltonian QCD

A.P. Szczepaniak, E.S. Swanson, Phys. Lett. B577 (2003) 61. hep-ph/0308268.

$$M_{0++} \simeq 1.9 \text{ GeV}$$
$$M_{2++} \simeq 2.4 \text{ GeV}$$
$$M_{0-+} \simeq 2.2 \text{ GeV}$$

Other approaches/2

- Flux-tube models

N. Isgur, J.E. Paton,
Phys. Lett. B124 (1983) 247

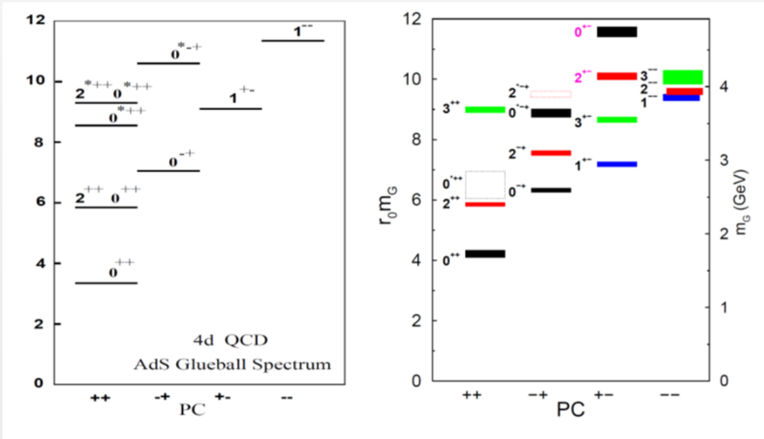
$$M_{0^{++}} \simeq 1.5 \text{ GeV}$$
$$M_{2^{++}} \simeq 2.8 \text{ GeV}$$
$$M_{0^{-+}} \simeq 2.8 \text{ GeV}$$

- Bethe-Salpeter approach

Sanchis-Alepuz et al,
Phys. Rev. D 92, 034001 (2015)

$$M_{0^{++}} \simeq 1.6 \text{ GeV}$$
$$M_{0^{-+}} \simeq 4.5 \text{ GeV}$$

- ADS/QCD



R. C. Brower, S. D. Mathur and C. I. Tan, *Nucl. Phys. B* **587** (2000) 249 [arXiv:hep-th/0003115]

Info from finite temperature YM theory?

Arxiv: 2212.03272

LQCD papers	Number of glueballs	Lattice Parameter	T_c (using Eqs. (2.5)-(2.6))
Chen et.al [51]	12	$r_0^{-1} = 410(20)$ MeV	317 ± 23 MeV
Meyer [52]	22	$\sqrt{\sigma} = 440(20)$ MeV	277 ± 13 MeV
Athenodorou and Teper [53]	20	$r_0^{-1} = 418(5)$ MeV	323 ± 18 MeV

Table 1. Parameters entering lattice simulations of the glueball mass spectrum.

$n\,J^{PC}$	M[MeV]			$n\,J^{PC}$	M[MeV]		
	Chen et.al. [51]	Meyer [52]	A & T [53]		Chen et.al. [51]	Meyer [52]	A & T [53]
1 0^{++}	1710(50)(80)	1475(30)(65)	1653(26)	**1 1^{--}**	3830(40)(190)	3240(330)(150)	4030(70)
2 0^{++}		2755(30)(120)	2842(40)	**1 2^{--}**	4010(45)(200)	3660(130)(170)	3920(90)
3 0^{++}		3370(100)(150)		**2 2^{--}**		3740(200)(170)	
4 0^{++}		3990(210)(180)		**1 3^{--}**	4200(45)(200)	4330(260)(200)	
1 2^{++}	2390(30)(120)	2150(30)(100)	2376(32)	**1 0^{+-}**	4780(60)(230)		
2 2^{++}		2880(100)(130)	3300(50)	**1 1^{+-}**	2980(30)(140)	2670(65)(120)	2944(42)
1 3^{++}	3670(50)(180)	3385(90)(150)	3740(70)	**2 1^{+-}**			3800(60)
1 4^{++}		3640(90)(160)	3690(80)	**1 2^{+-}**	4230(50)(200)		4240(80)
1 6^{++}		4360(260)(200)		**1 3^{+-}**	3600(40)(170)	3270(90)(150)	3530(80)
1 0^{-+}	2560(35)(120)	2250(60)(100)	2561(40)	**2 3^{+-}**		3630(140)(160)	
2 0^{-+}		3370(150)(150)	3540(80)	**1 4^{+-}**			4380(80)
1 2^{-+}	3040(40)(150)	2780(50)(130)	3070(60)	**1 5^{+-}**		4110(170)(190)	
2 2^{-+}		3480(140)(160)	3970(70)				
1 5^{-+}		3942(160)(180)					
1 1^{-+}			4120(80)				
2 1^{-+}			4160(80)				
3 1^{-+}			4200(90)				

Pressure of a gas of glueballs

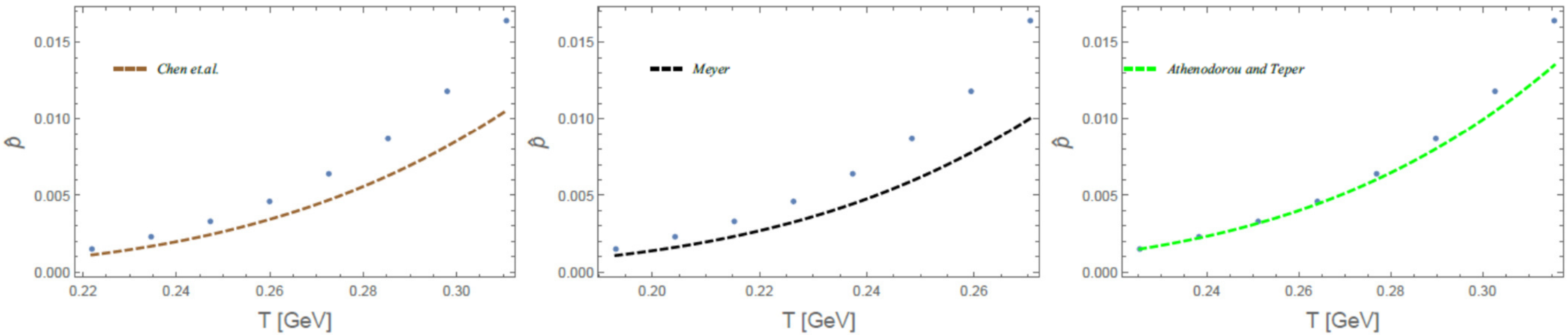


Figure 1. Normalized pressure of the GRG as function of the temperature for three different sets of lattice masses compared with the pressure evaluated in Ref. [28]. Left: GRG with masses from [51]; center: GRG with masses from [52]; right: GRG with masses from [53].

Masses: summary

Agreement of different methods.

Lightest state: a scalar glueball between 1- 2 GeV

Next: tensor and pseudoscalar glueballs.

Glueballs with exotic quantum numbers (oddballs) are also predicted to exist.

Decays of glueballs: simple features

Glueball decays: blindness

Flavour blindness (widely used for the scalar glueball):

$$\left|\frac{A_{G\to\pi\pi}}{A_{G\to KK}}\right|^2 = \frac{3}{4} \qquad \left|\frac{A_{G\to\eta\eta}}{A_{G\to KK}}\right|^2 = \frac{3}{4} \qquad A_{G\to\eta\eta'} = 0$$

Even more: chiral blindness (could be relevant for heavier glueballs):

$$\left|\frac{A_{G\to\rho\rho}}{A_{G\to a_1(1230)a_1(1230)}}\right|^2 = 1$$

Glueball decay: Large-Nc

Glueball masses are Nc-indipentent for large-Nc
(just a convetional quark-antiquark mesons)

$$M_G \propto N_c^0$$

Decay amplitude of a glueball into (conventional) mesons scales as:

$$A_{G\to M_1 M_2} \propto N_c^{-1} \qquad \Gamma_{G\to M_1 M_2} \propto N_c^{-2}$$

Recall that for a conventional quark-antiquark state:

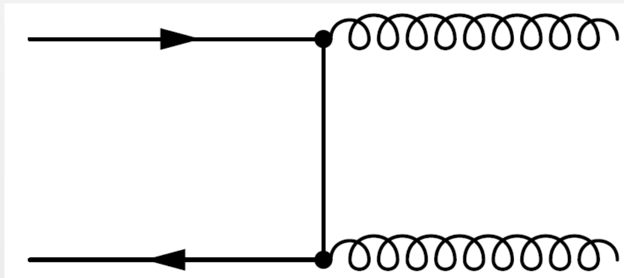
$$A_{M\to M_1 M_2} \propto N_c^{-1/2} \qquad \Gamma_{\mathrm{M}\to M_1 M_2} \propto N_c^{-1}$$

Glueball-quarkonium: mixing and large-Nc

The picture is complicated by mixing.

A glueball with conventional quantum numbers mix with nearby quark-antiquark states.

$$|\bar{q}q\rangle \longleftrightarrow |gg\rangle$$



Mixing suppressed in large-Nc but phenom. relevant

$$A_{G\text{-}M} \propto N_c^{-1/2}$$

For comparison:

$$A_{\sqrt{\frac{1}{2}}(\bar{\mathbf{u}}\mathbf{u}+\bar{\mathbf{d}}\mathbf{d})\text{-}\bar{\mathbf{s}}\mathbf{s}} \propto N_c^{-1}$$

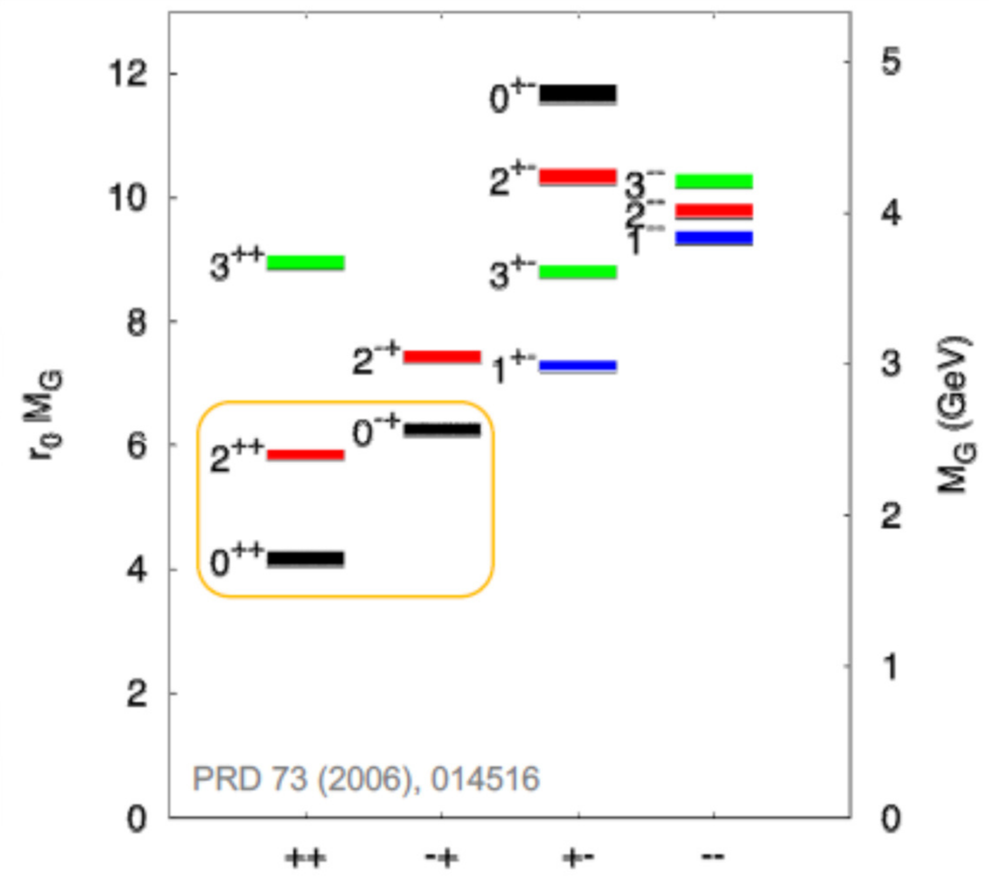
Glueball production and decays: gluon-rich processes

Glueballs should be find in gluon-rich processes
 (such as J/ψ decays, proton-antiproton fusion, …)

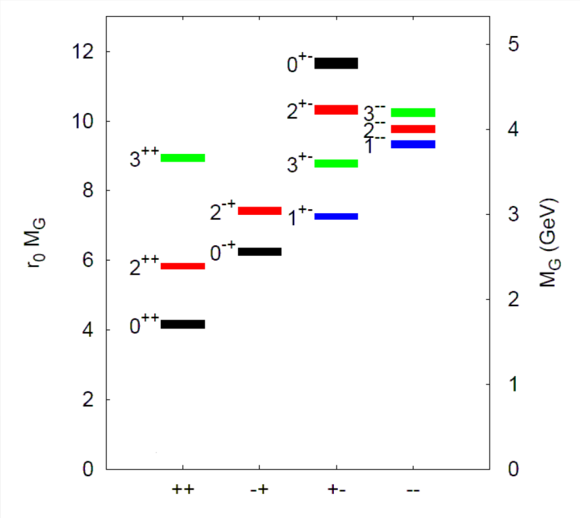
Glueball should have suppressed decay into flavor breaking channels (eg eta-etaprime)

Moreover, glueballs should have a suppressed (but nonzero!) decay into photons.

Light glueballs



The scalar glueball



Ellis-Lanik warning (1984)

IS SCALAR GLUONIUM OBSERVABLE?

John ELLIS
CERN, Geneva, Switzerland

and

Jozef LÁNIK
JINR, Dubna, USSR

Received 26 October 1984

We discuss couplings of scalar gluonium σ on the basis of the low energy theorems of broken chiral symmetry and the anomalous trace of the energy–momentum tensor, implemented using a phenomenological lagrangian. Taking the ITEP value of the gluon consensate as input, we find $\Gamma(\sigma \to \pi\pi) \simeq 0.6$ GeV $\times (m_\sigma/1\ \mathrm{GeV})^5$ and $\Gamma(\sigma \to \gamma\gamma) \simeq 90$ eV $\times (m_\sigma/1\ \mathrm{GeV})^5$, while m_σ is undetermined. These results suggest that if the scalar gluonium mass is above 1 GeV, it is probably unobservably wide, while production in $\gamma\gamma$ collisions is probably too small to be detectable if $m_\sigma < 1.5$ GeV. We comment on the observability of $J/\psi \to \sigma + \gamma$ and on the relevance of our results to other gluonia.

Physics Letters 150 B, 1984

Dilaton Lagrangian which mimics the trace anomaly: very large glueball is found.
Decay into pion reads:

$$\Gamma = 0.6(M_G/1\,\mathrm{GeV})^5\ \mathrm{GeV}$$

For a glueball of about 1.5 GeV in mass, one gets a width of about 4.5 GeV!

Disagreement with the large-Nc expectation

M. Migdal and Shifman, Phys. Lett. 114B, 445 (1982)

J. Schechter et al, Phys. Rev. D 24, 2545 (1981)

Mixing pattern

Above 1 GeV one has two quark-antiquark states and a bare glueball.

$$\sqrt{\tfrac{1}{2}}(\bar{u}u + \bar{d}d)$$

$$\bar{s}s$$

$$\text{Glueball: } gg$$

They mix to form the 3 resonances on the right.

Note:
$a_0(980)$ $k(800)$ $f_0(980)$ $f_0(500)$
are regarded as non-quarkonium objects

$f_0(1370)$ $I^G(J^{PC}) = 0^+(0^{++})$

See also the mini-reviews on scalar mesons under $f_0(500)$ (see the index for the page number) and on non-$q\bar{q}$ candidates in PDG 06, Journal of Physics **G33** 1 (2006).

$f_0(1370)$ T-MATRIX POLE POSITION

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
(1200–1500)−i(150–250) OUR ESTIMATE			

$f_0(1500)$ $I^G(J^{PC}) = 0^+(0^{++})$

See also the mini-reviews on scalar mesons under $f_0(500)$ (see the index for the page number) and on non-$q\bar{q}$ candidates in PDG 06, Journal of Physics **G33** 1 (2006).

$f_0(1500)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1504± 6 OUR AVERAGE		Error includes scale factor of 1.3. See the ideogram below.		

$f_0(1500)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
109± 7 OUR AVERAGE				

$f_0(1710)$ $I^G(J^{PC}) = 0^+(0^{++})$

See our mini-review in the 2004 edition of this *Review*, Physics Letters **B592** 1 (2004). See also the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$f_0(1710)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1723^{+6}_{-5} OUR AVERAGE		Error includes scale factor of 1.6. See the ideogram below.		

$f_0(1710)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
139 ± 8 OUR AVERAGE		Error includes scale factor of 1.1.		

Close-Amsler (1995 and 1996)

C. Amsler and F.E. Close Phys. Lett. **B353** (1995) 385

C. Amsler and F.E. Close Phys. Rev. **D53**, 295 (1996)

Is $f_0(1500)$ a scalar glueball?

Claude Amsler*
Physik-Institut, Universität Zürich, CH-8057 Zürich, Switzerland

Frank E. Close†
Particle Theory, Rutherford-Appleton Laboratory, Chilton, Didcot OX11 0QX, United Kingdom

(Received 24 July 1995)

Following the discovery of two new scalar mesons $f_0(1370)$ and $f_0(1500)$ at the Low Energy Antiproton Ring at CERN, we argue that the observed properties of this pair are incompatible with them both being $Q\bar{Q}$ mesons. We show instead that $f_0(1500)$ is compatible with the ground state glueball expected around 1500 MeV mixed with the nearby states of the $0^{++}Q\bar{Q}$ nonet. Tests of this hypothesis include the prediction of a further scalar state f_0' (1500–1800) which couples strongly to $K\bar{K}$, $\eta\eta$, and $\eta\eta'$. Signatures for a possible tensor glueball at ~ 2 GeV are also considered.

Decuplet: nonet+glueball.
3P0 model for decay.
Flavour symmetry.
Fit to data on decays.

$$\begin{pmatrix} \mathrm{f}_0(1370) \\ \mathrm{f}_0(1500) \\ \mathrm{f}_0(1710) \end{pmatrix} = \begin{pmatrix} -0.91 & 0.07 & 0.40 \\ -0.41 & 0.35 & -0.84 \\ 0.09 & 0.93 & 0.36 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{\mathrm{u}}\mathrm{u}+\bar{\mathrm{d}}\mathrm{d}) \\ \bar{\mathrm{s}}\mathrm{s} \\ \text{Glueball: } \mathrm{gg} \end{pmatrix}$$

Lee and Weingarten (1999)

PHYSICAL REVIEW D, VOLUME 61, 014015

Scalar quarkonium masses and mixing with the lightest scalar glueball

W. Lee* and D. Weingarten
IBM Research, P.O. Box 218, Yorktown Heights, New York 10598
(Received 18 August 1999; published 10 December 1999)

We evaluate the continuum limit of the valence (quenched) approximation to the mass of the lightest scalar quarkonium state, for a range of different quark masses, and to the mixing energy between these states and the lightest scalar glueball. Our results support the interpretation of $f_0(1710)$ as composed mainly of the lightest scalar glueball.

One of the first lattice studies.
Coupling of the glueball to pions and kaons (via mixing)

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.82 & 0.29 & -0.49 \\ -0.40 & 0.91 & -0.13 \\ 0.41 & 0.30 & 0.85 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ \text{Glueball: } gg \end{pmatrix}$$

My PhD time (2005)

PHYSICAL REVIEW D **72,** 094006 (2005)

Scalar nonet quarkonia and the scalar glueball: Mixing and decays in an effective chiral approach

F. Giacosa, Th. Gutsche, V. E. Lyubovitskij, and Amand Faessler

Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14,D-72076 Tübingen, Germany

(Received 22 September 2005; published 9 November 2005)

We study the strong and electromagnetic decay properties of scalar mesons above 1 GeV within a chiral approach. The scalar-isoscalar states are treated as mixed states of quarkonia and glueball configurations. A fit to the experimental mass and decay rates listed by the Particle Data Group is performed to extract phenomenological constraints on the nature of the scalar resonances and to the issue of the glueball decays. A comparison to other experimental results and to other theoretical approaches in the scalar meson sector is discussed.

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi_+\rangle + \frac{1}{2}\langle D_\mu S D^\mu S - M_S^2 S^2 \\ &+ \frac{1}{2}(\partial_\mu G \partial^\mu G - M_G^2 G^2) + c_d^s\langle S u_\mu u^\mu\rangle \\ &+ c_m^s\langle S\chi_+\rangle + \frac{c_d^g}{\sqrt{3}} G\langle u_\mu u^\mu\rangle + \frac{c_m^g}{\sqrt{3}} G\langle\chi_+\rangle \\ &+ c_e^s\langle S F_{\mu\nu}^+ F^{+\mu\nu}\rangle + \frac{c_e^g}{\sqrt{3}} G\langle F_{\mu\nu}^+ F^{+\mu\nu}\rangle + \mathcal{L}_{\rm mix}^P\end{aligned}$$

Starting point: Lagrangian (with both derivative and non-derivative terms is in agreement with ChPt; nonlinear real. of chiral symmetry)

Flavour symmetry is crucial.

Fit to all available PDG data.

$$\begin{pmatrix} \mathrm{f}_0(1370) \\ \mathrm{f}_0(1500) \\ \mathrm{f}_0(1710) \end{pmatrix} = \begin{pmatrix} 0.86 & 0.24 & 0.45 \\ -0.45 & -0.06 & 0.89 \\ -0.24 & 0.97 & -0.06 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ \text{Glueball: } gg \end{pmatrix}$$

My PhD time (2005)/2

But we found also an alternative solution:

$$\begin{pmatrix} \mathbf{f_0(1370)} \\ \mathbf{f_0(1500)} \\ \mathbf{f_0(1710)} \end{pmatrix} = \begin{pmatrix} \mathbf{0.81} & \mathbf{0.19} & \mathbf{0.54} \\ \mathbf{-0.49} & \mathbf{0.72} & \mathbf{0.49} \\ \mathbf{-0.30} & \mathbf{0.67} & \mathbf{-0.68} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\mathbf{\bar{u}u} + \mathbf{\bar{d}d}) \\ \mathbf{\bar{s}s} \\ \mathbf{Glueball:\ gg} \end{pmatrix}$$

Various works have followed

Uniwersytet
Jana Kochanowskiego w Kielcach

IOP PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS
J. Phys. G: Nucl. Part. Phys. **40** (2013) 043001 (68pp) doi:10.1088/0954-3899/40/4/043001

TOPICAL REVIEW

The status of glueballs

Wolfgang Ochs

Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, Föhringer Ring 6, D-80805 München, Germany

Progress in Particle and Nuclear Physics 63 (2009) 74–116

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

The experimental status of glueballs

V. Crede [a], C.A. Meyer [b,*]

[a] Florida State University, Tallahassee, FL 32306, USA
[b] Carnegie Mellon University, Pittsburgh, PA 15213, USA

Int.J.Mod.Phys. E18 (2009) 1-49

Vincent Mathieu
Groupe de Physique Nucléaire Théorique, Université de Mons-Hainaut, Académie universitaire Wallonie-Bruxelles, Place du Parc 20, BE-7000 Mons, Belgium.
vincent.mathieu@umh.ac.be

Nikolai Kochelev
Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region, 141980 Russia.
kochelev@theor.jinr.ru

Vicente Vento
Departament de Física Teòrica and Institut de Física Corpuscular, Universitat de València-CSIC, E-46100 Burjassot (Valencia), Spain.
vicente.vento@uv.es

Glueballs are particles whose valence degrees of freedom are gluons and therefore in their description the gauge field plays a dominant role. We review recent results in the physics of glueballs with the aim set on phenomenology and discuss the possibility of finding them in conventional hadronic experiments and in the Quark Gluon Plasma. In order to describe their properties we resort to a variety of theoretical treatments which include, lattice QCD, constituent models, AdS/QCD methods, and QCD sum rules. The review is supposed to be an informed guide to the literature. Therefore, we do not discuss in detail technical developments but refer the reader to the appropriate references.

F.E. Close, Rep. Progress Phys. 51 (1988) 833.
C. Amsler, Rev. Modern Phys. 70 (1998) 1293. hep-ex/9708025.
S. Godfrey, J. Napolitano, Rev. Modern Phys. 71 (1999) 1411. hep-ph/9811410.
C. Amsler, N.A. Tornqvist, Phys. Rep. 389 (2004) 61.
A. Masoni, C. Cicalo, G.L. Usai, J. Phys. G32 (2006) R293.

Janowski et al (2014)

PHYSICAL REVIEW D **90,** 114005 (2014)

Is $f_0(1710)$ a glueball?

Stanislaus Janowski,[1] Francesco Giacosa,[1,2] and Dirk H. Rischke[1]

[1]*Institute for Theoretical Physics, Goethe University, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*

[2]*Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland*

(Received 26 August 2014; published 2 December 2014)

We study the three-flavor chirally and dilatation invariant extended linear sigma model with (pseudo)scalar and (axial-)vector mesons as well as a scalar dilaton field whose excitations are interpreted as a glueball. The model successfully describes masses and decay widths of quark-antiquark mesons in the low-energy region up to 1.6 GeV. Here we study in detail the vacuum properties of the scalar-isoscalar $J^{PC} = 0^{++}$ channel and find that (i) a narrow glueball is only possible if the vacuum expectation value of the dilaton field is (at tree level) quite large (i.e. larger than what lattice QCD and QCD sum rules suggest) and (ii) only solutions in which $f_0(1710)$ is predominantly a glueball are found. Moreover, the resonance $f_0(1370)$ turns out to be mainly $(\bar{u}u + \bar{d}d)/\sqrt{2}$ and thus corresponds to the chiral partner of the pion, while the resonance $f_0(1500)$ is mainly $\bar{s}s$.

DOI: 10.1103/PhysRevD.90.114005 PACS numbers: 12.39.Fe, 12.39.Mk, 12.40.Yx, 13.25.Jx

Based on the
extended Linear Sigma Model:
Chiral model with (pseudo)scalar and (axial-)vector mesons.
Dilaton built in.
Valid up to 1.7 GeV.
We did not study the scalar sector alone but as part of a larger picture.

Unique assignment is found:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.91 & -0.24 & 0.33 \\ 0.30 & 0.94 & -0.17 \\ -0.27 & 0.26 & 0.93 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \bar{s}s \\ \text{Glueball: } gg \end{pmatrix}$$

Details of the model:
D. Parganlija, P. Kovacs, G. Wolf , F. G., D. H. Rischke, **Phys.Rev. D87 (2013) 014011 arXiv:1208.0585**

Lattice result on J/Psi decay into glueball

PRL **110**, 021601 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2

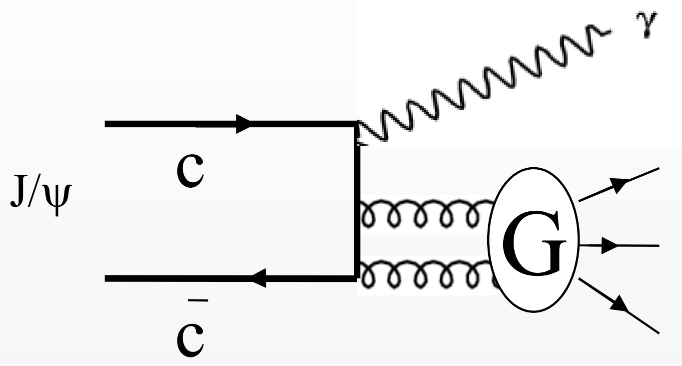
Scalar Glueball in Radiative J/ψ Decay on the Lattice

Long-Cheng Gui,[1,2] Ying Chen,[1,2,*] Gang Li,[3] Chuan Liu,[4] Yu-Bin Liu,[5] Jian-Ping Ma,[6]
Yi-Bo Yang,[1,2] and Jian-Bo Zhang[7]

(CLQCD Collaboration)

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[2]*Theoretical Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[3]*Department of Physics, Qufu Normal University, Qufu 273165, People's Republic of China*
[4]*School of Physics and Center for High Energy Physics, Peking University, Beijing 100871, People's Republic of China*
[5]*School of Physics, Nankai University, Tianjin 300071, People's Republic of China*
[6]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*
[7]*Department of Physics, Zhejiang University, Zhejiang 310027, People's Republic of China*
(Received 5 June 2012; published 10 January 2013)

The form factors in the radiative decay of J/ψ to a scalar glueball are studied within quenched lattice QCD on anisotropic lattices. The continuum extrapolation is carried out by using two different lattice spacings. With the results of these form factors, the partial width of J/ψ radiatively decaying into the pure gauge scalar glueball is predicted to be 0.35(8) keV, which corresponds to a branching ratio of $3.8(9) \times 10^{-3}$. By comparing with experiments, out results indicate that $f_0(1710)$ has a larger overlap with the pure gauge glueball than other related scalar mesons.



From the PDG (decay of the j/ψ): the radiative decays into $f_0(1710)$ are larger than into $f_0(1500)$.

$\gamma f_0(1710) \to \gamma K\overline{K}$	$(8.5\ ^{+1.2}_{-0.9}) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma\pi\pi$	$(4.0 \pm 1.0) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma\omega\omega$	$(3.1 \pm 1.0) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma\eta\eta$	$(2.4\ ^{+1.2}_{-0.7}) \times 10^{-4}$

$\gamma f_0(1500) \to \gamma\pi\pi$	$(1.01 \pm 0.32) \times 10^{-4}$
$\gamma f_0(1500) \to \gamma\eta\eta$	$(1.7\ ^{+0.6}_{-1.4}) \times 10^{-5}$

Decays of the scalar glueball in holography

PHYSICAL REVIEW D **91,** 106002 (2015)

Glueball decay rates in the Witten-Sakai-Sugimoto model

Frederic Brünner, Denis Parganlija, and Anton Rebhan
Institut für Theoretische Physik, Technische Universität Wien,
Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria
(Received 13 March 2015; published 14 May 2015)

We revisit and extend previous calculations of glueball decay rates in the Sakai-Sugimoto model, a holographic top-down approach for QCD with chiral quarks based on D8-$\overline{\text{D8}}$ probe branes in Witten's holographic model of nonsupersymmetric Yang-Mills theory. The rates for decays into two pions, two vector mesons, four pions, and the strongly suppressed decay into four π^0 are worked out quantitatively, using a range of the 't Hooft coupling which closely reproduces the decay rate of ρ and ω mesons and also leads to a gluon condensate consistent with QCD sum rule calculations. The lowest holographic glueball, which arises from a rather exotic polarization of gravitons in the supergravity background, turns out to have a significantly lower mass and larger width than the two widely discussed glueball candidates $f_0(1500)$ and $f_0(1710)$. The lowest nonexotic and predominantly dilatonic scalar mode, which has a mass of 1487 MeV in the Witten-Sakai-Sugimoto model, instead provides a narrow glueball state, and we conjecture that only this nonexotic mode should be identified with a scalar glueball component of $f_0(1500)$ or $f_0(1710)$. Moreover the decay pattern of the tensor glueball is determined, which is found to have a comparatively broad total width when its mass is adjusted to around or above 2 GeV.

DOI: 10.1103/PhysRevD.91.106002 PACS numbers: 11.25.Tq, 13.25.Jx, 14.40.Be, 14.40.Rt

f_0(1710) fits well into the picture.

Total width calculated to be about 100 MeV.

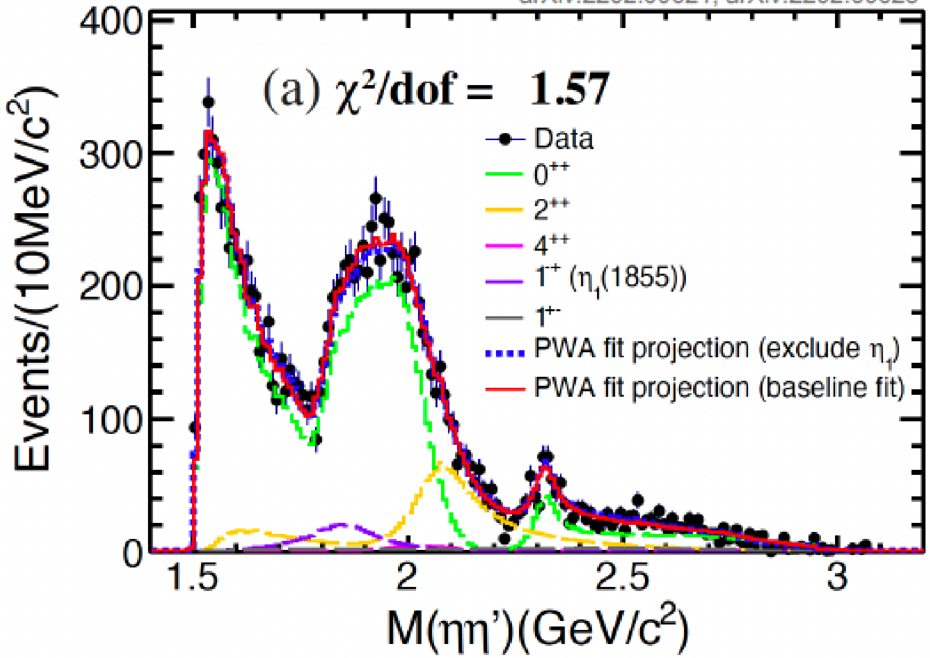
Alternative direction to study glueballs.

Recent BES results

Radiative J/ψ decays

- scalar glueball decays to $\eta\eta'$ expected to be suppressed $\frac{B(G\to\eta\eta')}{B(G\to\pi\pi)} < 0.04$
  PRD 92, 121902 (2015)
- significant $f_0(1500)$ contribution, but no $f_0(1710)$ (there is a small $f_0(1810)$ in the fit)
- $\frac{B(f_0(1500)\to\eta\eta')}{B(f_0(1500)\to\pi\pi)} = (8.96^{+2.95}_{-2.87})\times 10^{-2}$,
- $\frac{B(f_0(1710)\to\eta\eta')}{B(f_0(1710)\to\pi\pi)} < 1.61\times 10^{-3}$ (90% CL)
- $\frac{B(f_0(1810)\to\eta\eta')}{B(f_0(1710)\to\pi\pi)} = (1.39^{+0.62}_{-0.52})\times 10^{-2}$

arXiv:2202.00621, arXiv:2202.00623



Nils Hüsken
on behalf of the BESIII collaboration

Workshop: Recent results and perspectives in hadron physics
Orsay, October 17th, 2022

The splitted scalar glueball scenario

Klempt et al.,
Phys.Lett.B 826 (2022) 136906 • e-Print: 2112.04348 [hep-ph]

Phys.Lett.B 816 (2021) 136227 • e-Print: 2103.09680 [hep-ph]

$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(1770)$	$f_0(2020)$	$f_0(2100)$
$(5{\pm}4)\%$	$< 5\%$	$(12{\pm}6)\%$	$(25{\pm}10)\%$	$(16{\pm}9)\%$	$(17{\pm}8)\%$

glueball-glueball scattering: a new state?

Eur. Phys. J. C (2022) 82:487
https://doi.org/10.1140/epjc/s10052-022-10403-z

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

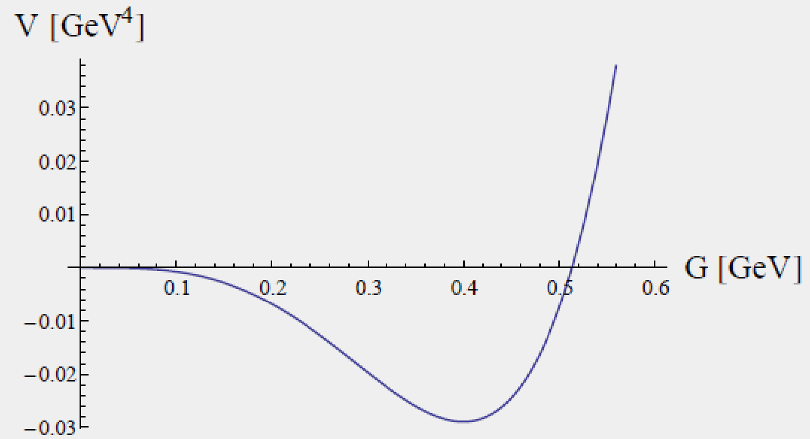
Glueball–glueball scattering and the glueballonium

Francesco Giacosa[1,2], Alessandro Pilloni[3,4], Enrico Trotti[1,a]

$$\mathcal{L}_{\text{dil}} = \frac{1}{2}(\partial_\mu G)^2 - V(G),$$

with

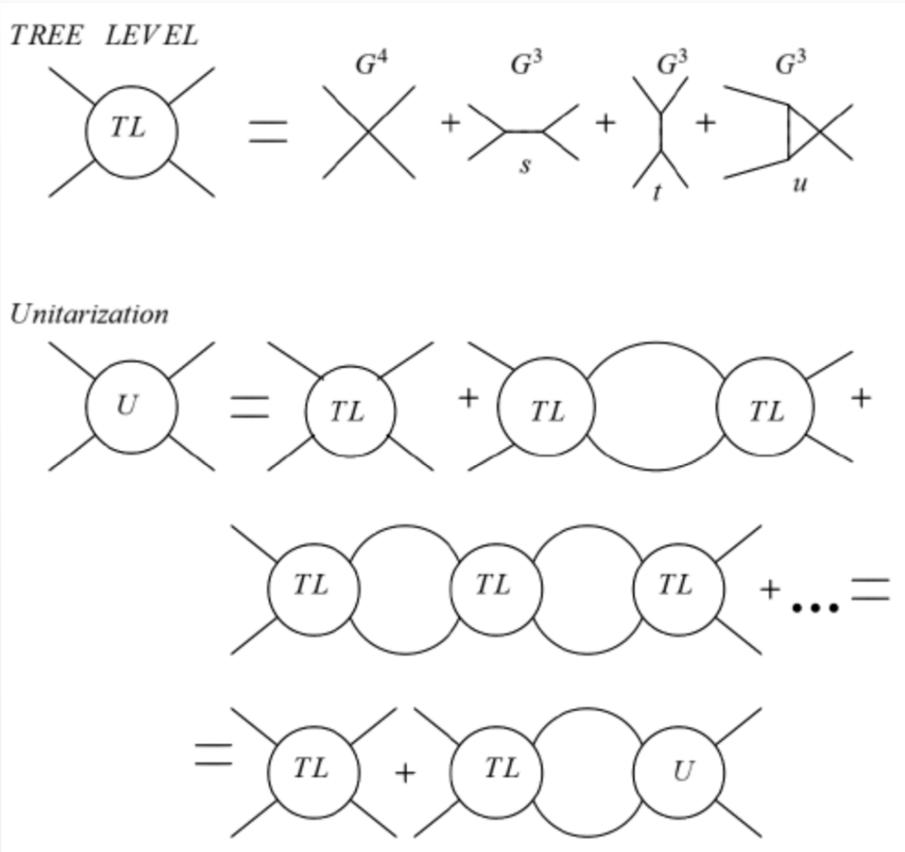
$$V(G) = \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right).$$



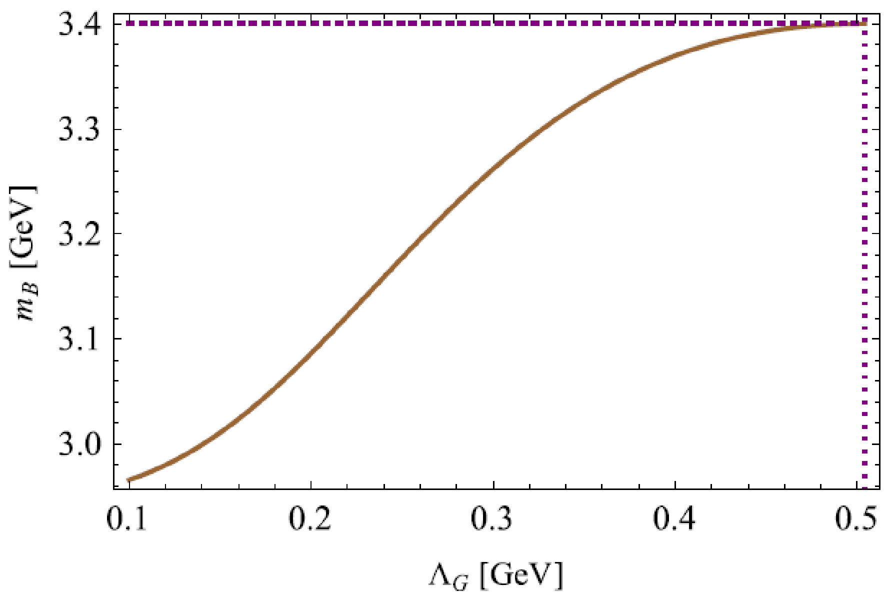
A. A. Migdal and M. A. Shifman, "Dilaton Effective Lagrangian In Gluodynamics," Phys. Lett.B114, 445 (1982)

Glueballonium mass

$$V(G) = V(\Lambda_G) + \frac{1}{2} m_G^2 G^2 + \frac{1}{3!}\left(5\frac{m_G^2}{\Lambda_G}\right) G^3 + \frac{1}{4!}\left(11\frac{m_G^2}{\Lambda_G^2}\right) G^4 + \frac{1}{5!}\left(6\frac{m_G^2}{\Lambda_G^3}\right) G^5 + \dots$$



$U = TL + (TL)\Sigma U$



Can one see that? In YM-lattice, probably yes.
In experiment? Hard, but…

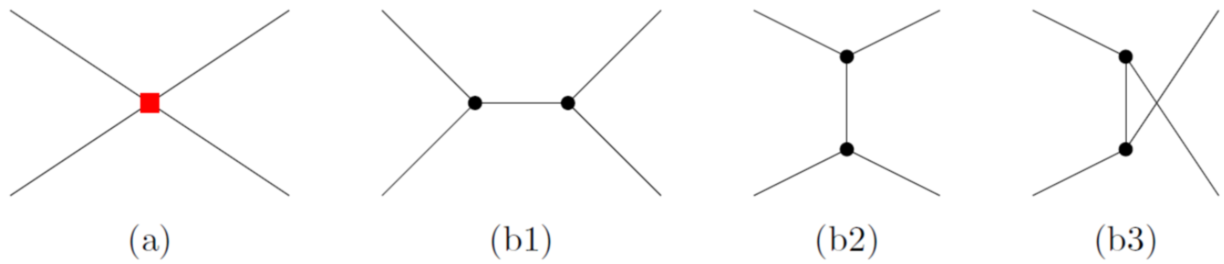
Higgsonium?

2212.01272 [hep-ph]

$$V(H) = V(v) + \frac{m_H^2}{2!}(H-v)^2 + \frac{g}{3!}(H-v)^3 + \frac{\lambda}{4!}(H-v)^4 + \frac{g_{5H}}{5!}(H-v)^5 + \ldots$$
$$= V(v) + \frac{m_H^2}{2!}h^2 + \frac{g}{3!}h^3 + \frac{\lambda}{4!}h^4 + \frac{g_{5H}}{5!}h^5 + \ldots$$

$$g = d_3\frac{3m_H^2}{v} \qquad \lambda = d_4\frac{3m_H^2}{v^2}$$

$$i\mathcal{M}_a = -i\lambda$$
$$i\mathcal{M}_{b1} = -ig^2\frac{1}{s - m_H^2 + i\epsilon}$$
$$i\mathcal{M}_{b2} = -ig^2\frac{1}{t - m_H^2 + i\epsilon}$$
$$i\mathcal{M}_{b3} = -ig^2\frac{1}{u - m_H^2 + i\epsilon}$$

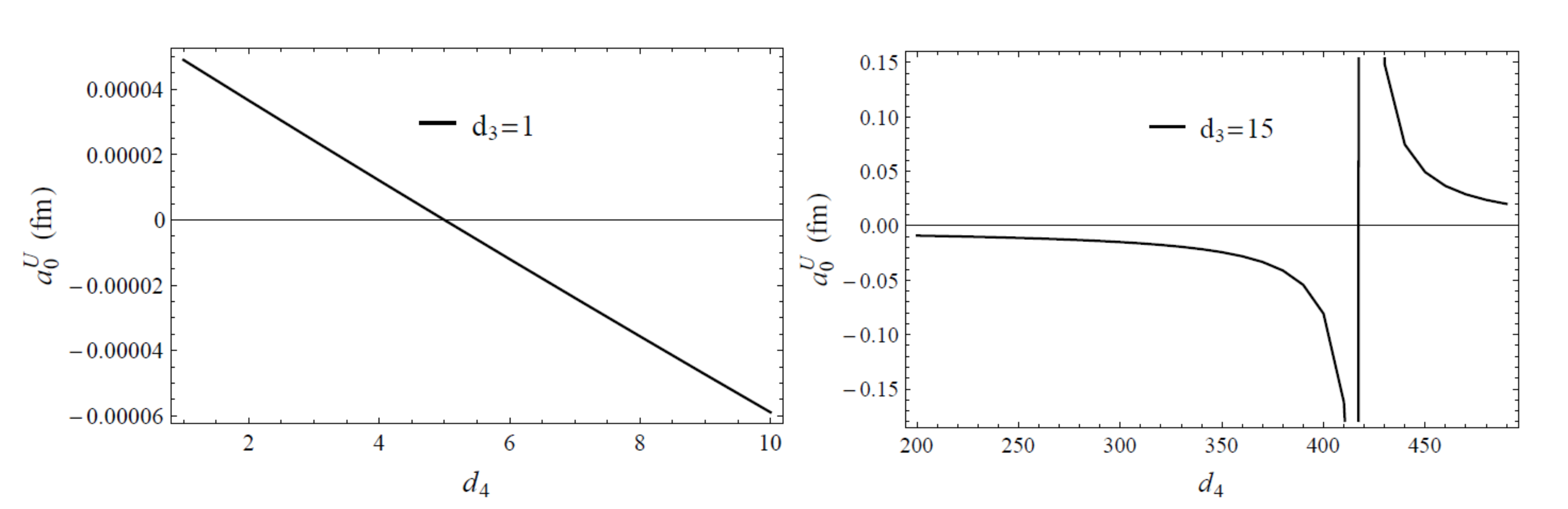


Higgsonium/2

$$a_0^{TL} = \frac{1}{32\pi m_H} A_0(s = 4m_H^2) = \frac{-\lambda + \frac{5g^2}{3m_H^2}}{32\pi m_H} = (4.86 \pm 0.01) \times 10^{-5} \text{ fm.}$$

$$a_2^{TL} = \frac{g^2}{30\pi m_H^7} = \frac{3d_3^2}{10\pi v^2 m_H^3} \stackrel{\text{SM}}{=} 2.4 \times 10^{-16} \text{ fm }^5,$$

$$a_4^{TL} = \frac{8g^2}{315\pi m_H^{11}} = \frac{8d_3^2}{35\pi v^2 m_H^7} \stackrel{\text{SM}}{=} 1.1 \times 10^{-27} \text{ fm}^9 .$$



Tensor glueball

Candidate: $f_J(2220)$
It is not on the Regge trajectories

V. V. Anisovich, JETP Lett. 80, 715 (2004) [Pisma Zh. Eksp. Teor. Fiz. 80, 845 (2004)] [arXiv:hep-ph/0412093];
V. V. Anisovich, M. A. Matveev, J. Nyiri and A. V. Sarantsev, arXiv:hep-ph/0506133.

No decay into photon-photon
Two-pion/two-kaon ratio in agreement with flavor blindness.

Details and further refs. in:
C. Amsler and N. A. Tornqvist, Phys. Rept. 389, 61 (2004).

F. G, T.Gutsche, V. E. Lyubovitskij and A.~Faessler,
``*Decays of tensor mesons and the tensor glueball in an effective field approach*,''
Phys. Rev. D 72 (2005) 114021 [hep-ph/0511171].

$$\mathcal{L}_{\rm eff}^{G} = c_{GPP}\, G_{\mu\nu} \, \langle \Theta_P^{\mu\nu} \rangle + c_{GVV}\, G_{\mu\nu} \, \langle \mathcal{V}^\mu \mathcal{V}^\nu \rangle$$

$$\pi\pi : \overline{K}K : \eta\eta : \eta\eta' : \eta'\eta' = 1 : 0.79 : 0.17 : 0 : 0.001$$

$$\rho\rho : \overline{K}^* K^* : \omega\omega : \omega\phi : \phi\phi = 1 : 0.84 : 0.32 : 0 : 0.11$$

Tensor glueball

$f_J(2220)$ $\quad I^G(J^{PC}) = 0^+(2^{++} \text{ or } 4^{++})$

OMITTED FROM SUMMARY TABLE
Needs confirmation. See our mini-review in the 2004 edition of this *Review*, PDG 04.

$f_J(2220)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*

$f_J(2220)$ WIDTH

VALUE (MeV)	*CL%*	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
$\mathbf{23^{+\ 8}_{-\ 7}}$ **OUR AVERAGE**					

PDG 2014

$f_J(2220)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	seen
Γ_2	$\pi^+\pi^-$	seen
Γ_3	$K\overline{K}$	seen
Γ_4	$p\overline{p}$	
Γ_5	$\gamma\gamma$	not seen
Γ_6	$\eta\eta'(958)$	seen
Γ_7	$\phi\phi$	not seen
Γ_8	$\eta\eta$	not seen

PDG 2022

$f_J(2220)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	not seen
Γ_2	$\pi^+\pi^-$	not seen
Γ_3	$K\overline{K}$	not seen
Γ_4	$p\overline{p}$	not seen
Γ_5	$\gamma\gamma$	not seen
Γ_6	$\eta\eta'(958)$	seen
Γ_7	$\phi\phi$	not seen
Γ_8	$\eta\eta$	not seen

$\Gamma(K\overline{K})/\Gamma_{total}$ $\qquad \Gamma_3/\Gamma$

VALUE	*DOCUMENT ID*		*COMMENT*
not seen	[1] DOBBS	15	$J/\psi \to \gamma K\overline{K}$
not seen	[1] DOBBS	15	$\psi(2S) \to \gamma K\overline{K}$

[1] Using CLEO-c data but not authored by the CLEO Collaboration.

$\Gamma(\pi\pi)/\Gamma(K\overline{K})$ $\qquad \Gamma_1/\Gamma_3$

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
1.0±0.5	BAI	96B	BES	$e^+e^- \to J/\psi \to \gamma 2\pi, K\overline{K}$

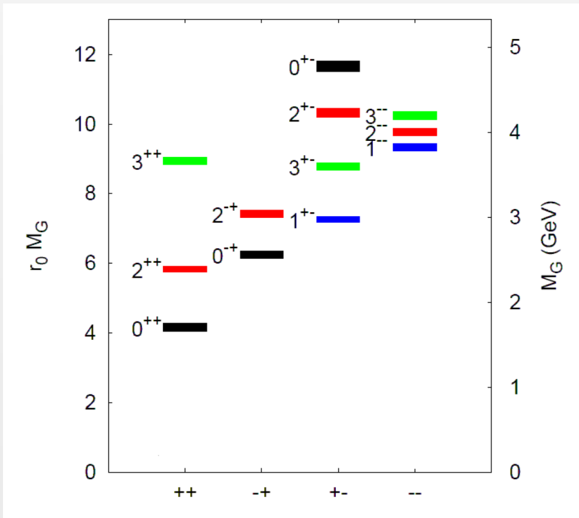
Present status in PDG

Resonances	Masses (MeV)	Decay Widths (MeV)	Decay Channels
$f_2(1910)$	1900 ± 9	167 ± 21	$\pi\pi,\ KK,\ \eta\eta,\ \omega\omega,\ \eta\eta',\ \eta'\eta',\ \rho\rho$
$f_2(1950)$	1936 ± 12	464 ± 24	$\pi\pi,\ K^*K^*,\ KK,\ \eta\eta$
$f_2(2010)$	2011^{+60}_{-80}	202 ± 60	$\phi\phi,\ KK$
$f_2(2150)$	2157 ± 12	152 ± 30	$\pi\pi,\ \eta\eta,\ KK$
$f_J(2220)$	2231.1 ± 3.5	23^{+8}_{-7}	$\eta\eta'$
$f_2(2300)$	2297 ± 28	149 ± 41	$\phi\phi,\ KK$
$f_2(2340)$	2345^{+50}_{-40}	322^{+70}_{-60}	$\phi\phi,\ \eta\eta$

Recent work:
Klempt et al., Phys.Lett.B 830 (2022) 137171 • e-Print: 2205.07239 [hep-ph]

Our work: ongoing

Pseudoscalar glueball



Francesco Giacosa

Pseudoscalar glueball

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **32** (2006) R293–R335 doi:10.1088/0954-3899/32/9/R01

TOPICAL REVIEW

The case of the pseudoscalar glueball

A Masoni[1,2], C Cicalò[1,2] and G L Usai[1,2]

[1] INFN, Sezione di Cagliari, Cagliari, Italy
[2] Dipartimento di Fisica, Università di Cagliari, Cagliari, Italy

Received 20 January 2006
Published 10 August 2006
Online at stacks.iop.org/JPhysG/32/R293

Abstract
Glueballs represent a key requirement of quantum chromodynamics as a non-Abelian field theory. Their search provides one of the strongest motivations for meson spectroscopy. The first glueball candidate was identified in 1980 in the J/Ψ radiative decay. Its discovery actually dates back to 1963 and for four decades about 30 experiments, using six different production mechanisms, were dedicated to studying the pseudoscalar states lying in the 1.4–1.5 GeV/c^2 mass region. Today, the presence of two pseudoscalar states and an axial vector can be considered as established (see 2004 edition of the *Review of Particle Properties*). Assuming that $\eta(1295)$ is established and the nonet filled, the lower mass pseudoscalar, $\eta(1405)$, becomes a supernumerary and shows the properties of a non-$\overline{q}q$ state. Here, we review the experimental effort related to this long search, which can be considered a sort of paradigm for light-quark spectroscopy.

Gluonium content in η'

Eur. Phys. J. C (2010) 68: 619–681
DOI 10.1140/epjc/s10052-010-1351-1

THE EUROPEAN PHYSICAL JOURNAL C

Review

Physics with the KLOE-2 experiment at the upgraded DAΦNE

$|q\bar{q}\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$ $\quad |s\bar{s}\rangle$

$$\boxed{\begin{aligned} |\eta'\rangle &= \cos\psi_G \sin\psi_P\,|q\bar{q}\rangle + \cos\psi_G\cos\psi_P\,|s\bar{s}\rangle \\ &\quad + \sin\psi_G\,|GG\rangle\,, \\ |\eta\rangle &= \cos\psi_P\,|q\bar{q}\rangle - \sin\psi_P\,|s\bar{s}\rangle\,, \end{aligned}}$$

$$Z_G^2 = \sin^2\psi_G$$

Table 12 Fit to the gluonium content in η' assuming 1% error on the η' branching fractions. The $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$ constraint is used (not used) in the left (right) column

	with $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$	without $\eta' \to \gamma\gamma/\pi^0 \to \gamma\gamma$
Z_G^2	0.11 ± 0.03	0.11 ± 0.04

In general, one has a 3-body mixing.
Chiral anomaly important.
However, the mixing should be suppressed by the large glueball mass (2.6 GeV).

Results not conclusive yet...

JHEP

PUBLISHED BY INSTITUTE OF PHYSICS PUBLISHING FOR SISSA

RECEIVED: *March 22, 2007*
REVISED: *April 19, 2007*
ACCEPTED: *April 23, 2007*
PUBLISHED: *May 2, 2007*

On the gluon content of the η and η' mesons

Rafel Escribano
Grup de Física Teórica and IFAE, Universitat Autónoma de Barcelona
E-08193 Bellaterra, Barcelona, Spain
E-mail: Rafel.Escribano@ifae.es

Jordi Nadal
Institut de Física d'Altes Energies, Universitat Autónoma de Barcelona
E-08193 Bellaterra, Barcelona, Spain
E-mail: jnadal@ifae.es

ABSTRACT: A phenomenological analysis of radiative $V \to P\gamma$ and $P \to V\gamma$ decays is performed with the purpose of determining the gluonic content of the η and η' wave functions. Our results show that within our model there is no evidence for a gluonium contribution in the η, $Z_\eta^2 = 0.00 \pm 0.12$, or the η', $Z_{\eta'}^2 = 0.04 \pm 0.09$. In terms of a mixing angle description this corresponds to $\phi_P = (41.4 \pm 1.3)^\circ$ and $|\phi_{\eta' G}| = (12 \pm 13)^\circ$. In addition, the η-η' mixing angle is found to be $\phi_P = (41.5 \pm 1.2)^\circ$ if we don't allow for a gluonium component.

Situation not clarified yet.

According to Lattice,
the pseudoscalar glueball
has a mass of about 2.6 GeV.

Mixing should be suppressed.

Pseudoscalar glueball

The bare states

$$\sqrt{\tfrac{1}{2}}(\bar{u}u + \bar{d}d)$$

$$\bar{s}s$$

$$\text{Glueball: } gg$$

mix and form the 3 resonances.
Usually, in such studies the glueball turns out to sit mostly in the eta(1405) state.

Conflict with Lattice!

$\boxed{\eta(1295)}$ $\quad I^G(J^{PC}) = 0^+(0^{-+})$

See also the mini-review under $\eta(1405)$

$\eta(1295)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1294±4 OUR AVERAGE	Error includes scale factor of 1.6. See the ideogram below.			

$\boxed{\eta(1405)}$ $\quad I^G(J^{PC}) = 0^+(0^{-+})$

A REVIEW GOES HERE – Check our WWW List of Reviews

$\eta(1405)$ MASS

VALUE (MeV)	DOCUMENT ID
1408.8±1.8 OUR AVERAGE	Includes data from the 2 datablocks that follow this one. Error includes scale factor of 2.1. See the ideogram below.

$\boxed{\eta(1475)}$ $\quad I^G(J^{PC}) = 0^+(0^{-+})$

See also the $\eta(1405)$.

$\eta(1475)$ MASS

$K\overline{K}\pi$ MODE ($K^*(892)$ K dominant)

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1476± 4 OUR AVERAGE	Error includes scale factor of 1.3. See the ideogram below.			

Pseudoscalar glueball

Issues with both approaches (one should at least try a 5-mixing) and with the very existence of two separated pseudoscalar resonances between 1.4-1.5 GeV.

Severe conflict with Lattice expectations.

The pseudoscalar glueball: predictions from the eLSM

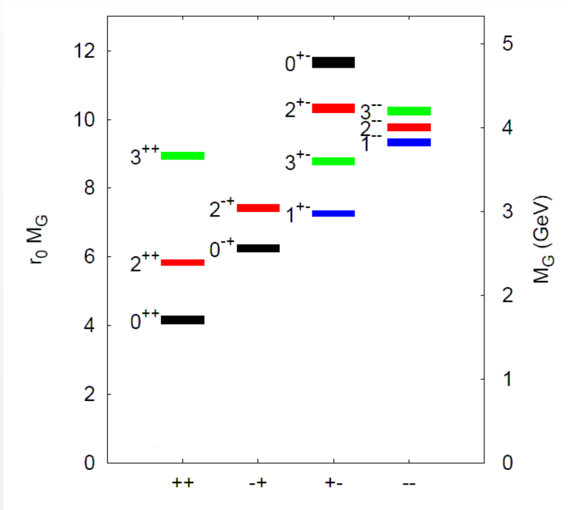
$$\mathcal{L}^{int}_{\tilde{G}\text{-mesons}} = ic_{\tilde{G}\Phi}\tilde{G}\left(\det\Phi - \det\Phi^{\dagger}\right)$$

Quantity	Value
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{tot}_{\tilde{G}}$	0.049
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.019
$\Gamma_{\tilde{G}\to \eta\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.016
$\Gamma_{\tilde{G}\to \eta\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0017
$\Gamma_{\tilde{G}\to \eta\eta'\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.00013
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{tot}_{\tilde{G}}$	0.46
$\Gamma_{\tilde{G}\to \eta\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.16
$\Gamma_{\tilde{G}\to \eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.094

$$\boxed{\Gamma_{\tilde{G}\to\pi\pi\pi} = 0}$$

MG = 2.6 GeV as been used as an input.

Quantity	Value
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}}$	0.059
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}}$	0.083
$\Gamma_{\tilde{G}\to \eta\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.028
$\Gamma_{\tilde{G}\to \eta\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0.012
$\Gamma_{\tilde{G}\to \eta'\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.019



X(2370) found at BESIII is a possible candidate.

Future experimental search, e.g. at BES and PANDA

Details in:

W. Eshraim, S. Janowski, F.G., D. Rischke, **Phys.Rev. D87 (2013) 054036. arxiv: 1208.6474** .

W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., **Acta Phys. Pol. B**, Prc. Suppl. 5/4, **arxiv: 1209.3976**

Pseudoscalar glueball/novel result?

PHYSICAL REVIEW LETTERS **129,** 042001 (2022)

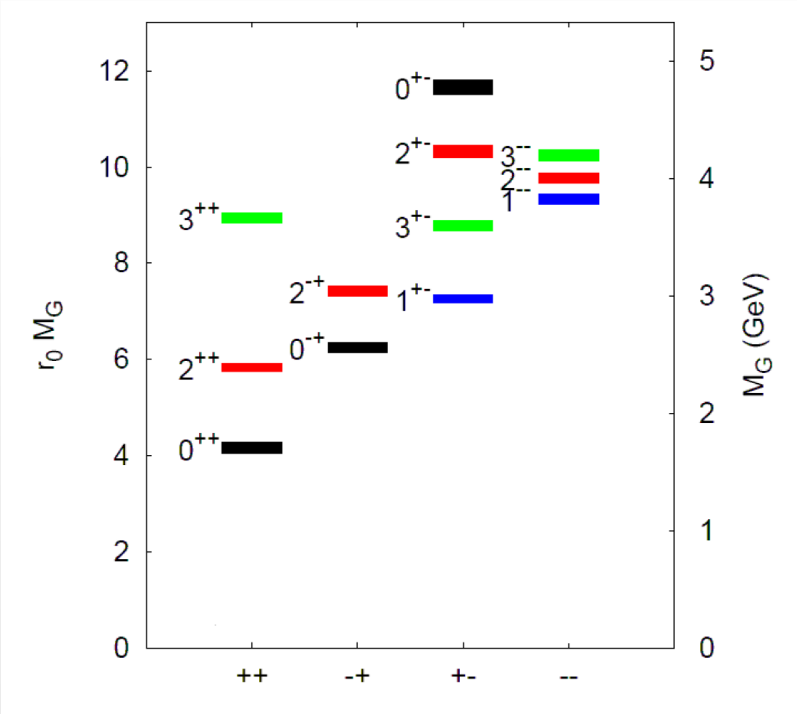
Observation of a State $X(2600)$ in the $\pi^+\pi^-\eta\prime$ System in the Process $J/\psi \to \gamma\pi^+\pi^-\eta\prime$

$\pi^+\pi^-$ invariant mass spectrum. A simultaneous fit on the $\pi^+\pi^-\eta'$ and $\pi^+\pi^-$ invariant mass spectra with the two η' decay modes indicates that the mass and width of the $X(2600)$ state are $2618.3 \pm 2.0^{+16.3}_{-1.4}$ MeV$/c^2$ and $195 \pm 5^{+26}_{-17}$ MeV, where the first uncertainties are statistical, and the second systematic.

Other glueballs: ???

vector, pseudotensor,... glueballs.

Oddballs.



F.G. et al., decays of the vector glueball,
Phys.Rev.D 95 (2017) 11, 114004 • e-Print: 1607.03640 [hep-ph]

Remarks

- We did not find glueballs yet.
- We have some good candidates for the scalar sector and less good candidates for the tensor and pseudoscalar sector.
- Are (at least some) glueballs narrow enough? This is a decisive information. Help from Lattice would be very welcome!
- Ongoing and future experiments:

  BESIII

  COMPASS@CERN - LHCb@CERN

  CLAS12@JLAB, GLUEX@JLAB

  BELLE2

  RHIC/STAR

  PANDA@FAIR is very well suited for the search of glueballs.

Toward a nonet of hybrid state/PDG

$\pi_1(1600)$

$I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Spectroscopy of Light Meson Resonances" and a note in PDG 06, Journal of Physics **G33** 1 (2006).

$\pi_1(1600)$ T-Matrix Pole $\sqrt{s}$

$\pi_1(1600)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1661^{+15}_{-11} OUR AVERAGE		Error includes scale factor of 1.2.		

$\pi_1(1600)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
240± 50 OUR AVERAGE		Error includes scale factor of 1.7. See the ideogram below.		

$\pi_1(1600)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi\pi$	seen
Γ_2	$\rho^0\pi^-$	seen
Γ_3	$f_2(1270)\pi^-$	not seen
Γ_4	$b_1(1235)\pi$	seen
Γ_5	$\eta'(958)\pi^-$	seen
Γ_6	$\eta\pi$	
Γ_7	$f_1(1285)\pi$	seen

$\pi_1(1400)$

$I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1400)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1354 ±25 OUR AVERAGE		Error includes scale factor of 1.8. See the ideogram below.			

$\pi_1(1400)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
330 ±35 OUR AVERAGE					

$\pi_1(1400)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\pi^0$	seen
Γ_2	$\eta\pi^-$	seen

Uniwersytet Jana Kochanowskiego w Kielcach

A unique I=1 hybrid state

PHYSICAL REVIEW LETTERS **122,** 042002 (2019)

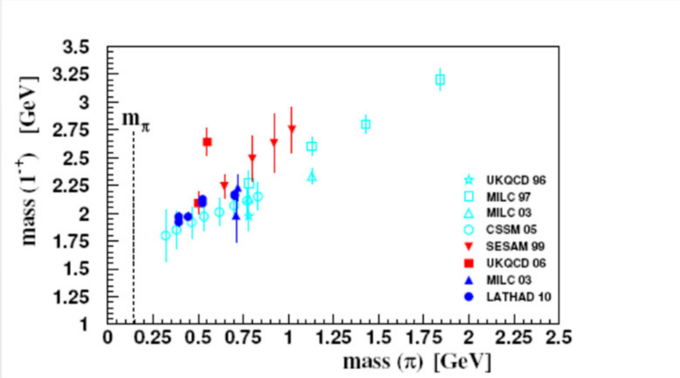
Determination of the Pole Position of the Lightest Hybrid Meson Candidate

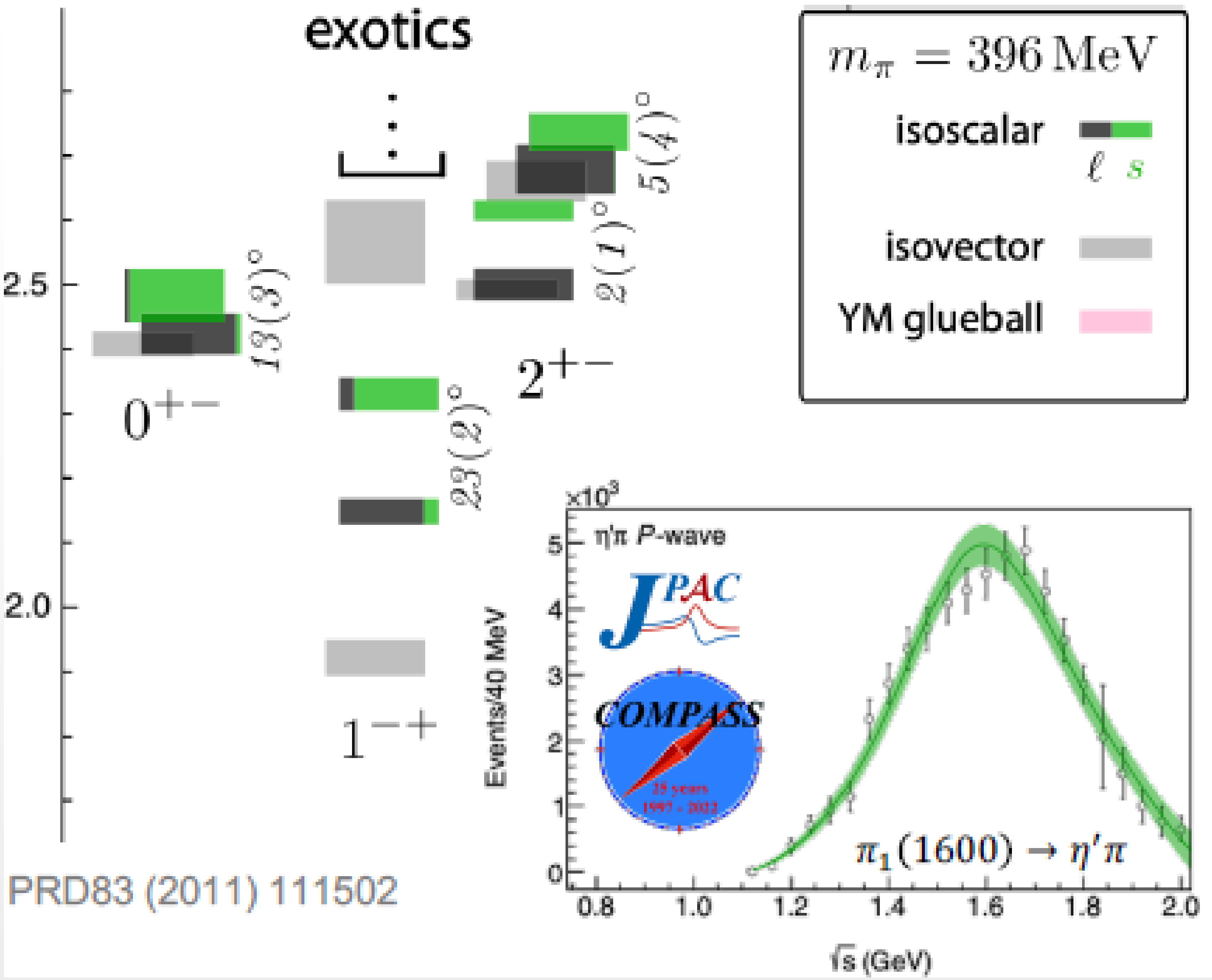
A. Rodas,[1,*] A. Pilloni,[2,3,†] M. Albaladejo,[2,4] C. Fernández-Ramírez,[5] A. Jackura,[6,7] V. Mathieu,[2] M. Mikhasenko,[8] J. Nys,[9] V. Pauk,[10] B. Ketzer,[8] and A. P. Szczepaniak[2,6,7]

Mapping states with explicit gluonic degrees of freedom in the light sector is a challenge, and has led to controversies in the past. In particular, the experiments have reported two different hybrid candidates with spin-exotic signature, $\pi_1(1400)$ and $\pi_1(1600)$, which couple separately to $\eta\pi$ and $\eta'\pi$. This picture is not compatible with recent Lattice QCD estimates for hybrid states, nor with most phenomenological models. We consider the recent partial wave analysis of the $\eta^{(\prime)}\pi$ system by the COMPASS Collaboration. We fit the extracted intensities and phases with a coupled-channel amplitude that enforces the unitarity and analyticity of the S matrix. We provide a robust extraction of a single exotic π_1 resonant pole, with mass and width $1564 \pm 24 \pm 86$ and $492 \pm 54 \pm 102$ MeV, which couples to both $\eta^{(\prime)}\pi$ channels. We find no evidence for a second exotic state. We also provide the resonance parameters of the $a_2(1320)$ and $a_2'(1700)$.

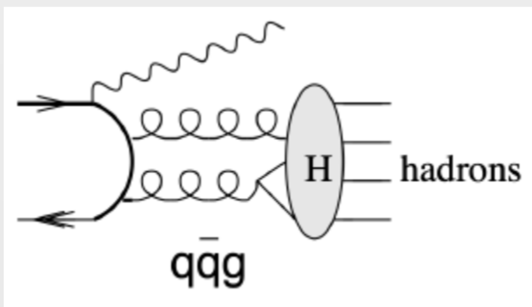
π1(1600) and π1(1400) are the same state
(in agreement with various models and lattice QCD)

C. Meyer and E. Swanson,
Hybrid Mesons,
Prog. Part. Nucl. Phys. 82 (2015) 21
[arXiv:1502.07276 [hep-ph]].





exotics
$m_\pi = 396\,\mathrm{MeV}$
isoscalar
ℓ s
isovector
YM glueball
2.5
2.0
0^{+-}
1^{-+}
2^{+-}
$13(3)^\circ$
$23(2)^\circ$
$2(1)^\circ$
$5(4)^\circ$
×10³
η'π P-wave
JPAC
COMPASS
25 years
1997 - 2022
Events/40 MeV
$\pi_1(1600) \to \eta'\pi$
√s (GeV)
0
1
2
3
4
5
0.8
1.0
1.2
1.4
1.6
1.8
2.0
PRD83 (2011) 111502



H
hadrons
qq̄g

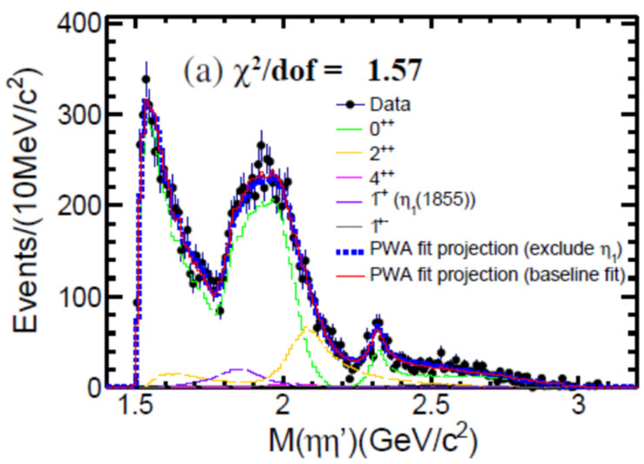
New experimental finding: η1(1855)

Observation of an isoscalar resonance with exotic $J^{PC} = 1^{-+}$ quantum numbers in $J/\psi \to \gamma\eta\eta'$

M. Ablikim[1], M. N. Achasov[10,b], P. Adlarson[68], S. Ahmed[14], M. Albrecht[4], R. Aliberti[28], A. Amoroso[67A,67C], M. R. An[32],

Using a sample of $(10.09\pm0.04)\times10^9$ J/ψ events collected with the BESIII detector operating at the BEPCII storage ring, a partial wave analysis of the decay $J/\psi \to \gamma\eta\eta'$ is performed. The first observation of an isoscalar state with exotic quantum numbers $J^{PC} = 1^{-+}$, denoted as $\eta_1(1855)$, is reported in the process $J/\psi \to \gamma\eta_1(1855)$ with $\eta_1(1855) \to \eta\eta'$. Its mass and width are measured to be $(1855\pm9^{+6}_{-1})$ MeV/c^2 and $(188\pm18^{+3}_{-8})$ MeV, respectively, where the first uncertainties are statistical and the second are systematic, and its statistical significance is estimated to be larger than 19σ.

Phys.Rev.Lett. 129 (2022) 19, 192002 2202.00621 [hep-ex]



A nonet of hybrid states?

Physics Letters B 834 (2022) 137478

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$

Vanamali Shastry [a,*], Christian S. Fischer [b,c], Francesco Giacosa [a,d]

arXiv:2203.04327

Beides π1(1600) and η1(1855), we expect also: K1(1750) and η1(1660). The last two not yet seen.

	M (MeV)	Γ (MeV)
K_1^{hyb}	1761	312 ± 97 170 ± 65
η_1^L	1661	81 ± 15 83 ± 16
η_1^H	1855	259 ± 92 157 ± 68

Table 7
The partial widths and branching ratios of various decay channels and the total width for the hybrid kaon $K_1^{hyb}(1750)$. We have assumed the mass of the state to be 1761 MeV [44].

Channel	Width (MeV)		Channel	Width (MeV)	
	Set-1	Set-2		Set-1	Set-2
$\Gamma_{K_1(1270)\pi}$	125 ± 42	48 ± 25	$\Gamma_{\rho K}$	2.18 ± 0.56	2.19 ± 0.57
$\Gamma_{K_1(1400)\pi}$	103 ± 45	98 ± 43	$\Gamma_{\omega K}$	0.82 ± 0.21	0.82 ± 0.21
$\Gamma_{h_1(1170)K}$	1.53 ± 0.28	1.37 ± 0.24	$\Gamma_{\phi K}$	0.49 ± 0.12	0.49 ± 0.13
$\Gamma_{\eta K}$	0.29 ± 0.07	0.29 ± 0.07	$\Gamma_{K^*\pi}$	0.67 ± 0.17	0.67 ± 0.17
$\Gamma_{\eta' K}$	2.77 ± 0.62	2.81 ± 0.62	$\Gamma_{K^*\eta}$	0.30 ± 0.08	0.30 ± 0.08
$\Gamma_{\rho K^*}$	0.045 ± 0.016	0.047 ± 0.016	$\Gamma_{\omega K^*}$	0.011 ± 0.004	0.012 ± 0.004
$\Gamma_{a_1 K}$	11.0 ± 2.32	11.3 ± 2.35	$\Gamma_{b_1 K}$	64 ± 14	3.11 ± 2.88
			Γ_{tot}	312 ± 97	170 ± 65

Table 6
The partial widths and branching ratios of various decay channels and the total width of the η_1^L (left) and the $\eta_1(1855)$ (right) for $\theta_h = 15°$. This corresponds to the "Scenario-2" discussed in the text.

Channel	Width (MeV)		Channel	Width (MeV)	
	Set-1	Set-2		Set-1	Set-2
$\Gamma_{a_1\pi}$	80 ± 15	82 ± 16	$\Gamma_{K_1(1270)K}$	253 ± 92	151 ± 67
Γ_{K^*K}	0.29 ± 0.075	0.29 ± 0.075	Γ_{K^*K}	1.45 ± 0.37	1.46 ± 0.38
$\Gamma_{\eta'\eta}$	0.41 ± 0.09	0.41 ± 0.09	$\Gamma_{\eta'\eta}$	2.28 ± 0.51	2.31 ± 0.51
$\Gamma_{K_1(1270)K}$	0	0	$\Gamma_{a_1\pi}$	0	0
$\Gamma_{\rho\rho}$	0.081 ± 0.028	0.082 ± 0.029	$\Gamma_{\rho\rho}$	0	0
$\Gamma_{K^*K^*}$	0	0	$\Gamma_{K^*K^*}$	0.075 ± 0.027	0.077 ± 0.028
$\Gamma_{\omega\phi}$	0	0	$\Gamma_{\omega\phi}$	$\sim 10^{-4}$	$\sim 10^{-4}$
$\Gamma_{f_1\eta}$	0	0	$\Gamma_{f_1\eta}$	2.15 ± 0.56	2.21 ± 0.57
Γ_{tot}	81 ± 15	83 ± 16	Γ_{tot}	259 ± 92	157 ± 68

Conclusions and outlook

Many nonets fit well in the quark-antiquark picture, but…

- axial-tensor mesons basically unknown;
- pseudotensor mesons, is there a large isoscalar mixing?
- vector mesons: which is the orbitally excited ϕ meson?

Unconventional mesons:

- hybrid mesons: a new nonet?
- Glueballonium (possible), Higgsonium (improbable)

Outlook:

- tensor glueball (ongoing)

Thanks

Back-up slides

Phenomenology of $J^{PC} = 3^{--}$ tensor mesons

Shahriyar Jafarzade[*]

Institute of Physics, Jan Kochanowski University, ul. Uniwersytecka 7, P-25-406 Kielce, Poland

Adrian Koenigstein[†]

Institute for Theoretical Physics, Goethe-University, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany

Francesco Giacosa[‡]

Institute of Physics, Jan Kochanowski University, ul. Uniwersytecka 7, P-25-406 Kielce, Poland, and Institute for Theoretical Physics, Goethe-University, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany

(Received 19 January 2021; accepted 15 April 2021; published 28 May 2021)

We study the strong and radiative decays of the antiquark-quark ground state $J^{PC} = 3^{--}$ ($n^{2S+1}L_J = 1^3D_3$) nonet $\{\rho_3(1690), K_3^*(1780), \phi_3(1850), \omega_3(1670)\}$ in the framework of an effective quantum field theory approach, based on the $SU_V(3)$-flavor symmetry. The effective model is fitted to experimental data listed by the Particle Data Group. We predict numerous experimentally unknown decay widths and branching ratios. An overall agreement of theory (fit and predictions) with experimental data confirms the $\bar{q}q$ nature of the states and qualitatively validates the effective approach. Naturally, experimental clarification as well as advanced theoretical description is needed for trustworthy quantitative predictions, which is observed from some of the decay channels. Besides conventional spin-3 mesons, theoretical predictions for ratios of strong and radiative decays of a hypothetical glueball state $G_3(4200)$ with $J^{PC} = 3^{--}$ are also presented.

Decays of J=3-mesons

TABLE III. Effective relativistic interaction terms describing the strong decays of mesons with $J^{PC} = 3^{--}$.

Decay mode	Interaction Lagrangians
$3^{--} \to 0^{-+} + 0^{-+}$	$\mathcal{L}_{w_3pp} = g_{w_3pp}\mathrm{tr}[W_3^{\mu\nu\rho}[P, (\partial_\mu\partial_\nu\partial_\rho P)]_-]$
$3^{--} \to 0^{-+} + 1^{--}$	$\mathcal{L}_{w_3v_1p} = g_{w_3v_1p}\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}[W_{3,\mu\alpha\beta}\{(\partial_\nu V_{1,\rho}), (\partial^\alpha\partial^\beta\partial_\sigma P)\}_+]$
$3^{--} \to 0^{-+} + 2^{++}$	$\mathcal{L}_{w_3a_2p} = g_{w_3a_2p}\varepsilon_{\mu\nu\rho\sigma}\mathrm{tr}[W_3{}^{\mu}{}_{\alpha\beta}[(\partial^\nu A_2^{\rho\alpha}), (\partial^\sigma\partial^\beta P)]_-]$
$3^{--} \to 0^{-+} + 1^{+-}$	$\mathcal{L}_{w_3b_1p} = g_{w_3b_1p}\mathrm{tr}[W_3^{\mu\nu\rho}\{B_{1,\mu}, (\partial_\nu\partial_\rho P)\}_+]$
$3^{--} \to 0^{-+} + 1^{++}$	$\mathcal{L}_{w_3a_1p} = g_{w_3a_1p}\mathrm{tr}[W_3^{\mu\nu\rho}[A_{1,\mu}, (\partial_\nu\partial_\rho P)]_-]$
$3^{--} \to 1^{--} + 1^{--}$	$\mathcal{L}_{w_3v_1v_1} = g_{w_3v_1v_1}\mathrm{tr}[W_3^{\mu\nu\rho}[(\partial_\mu V_{1,\nu}), V_{1,\rho}]_-]$

$$W_3^{\mu\nu\rho} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_{3,N}^{\mu\nu\rho}+\rho_3^{0\mu\nu\rho}}{\sqrt{2}} & \rho_3^{+\mu\nu\rho} & K_3^{+\mu\nu\rho} \\ \rho_3^{-\mu\nu\rho} & \frac{\omega_{3,N}^{\mu\nu\rho}-\rho_3^{0\mu\nu\rho}}{\sqrt{2}} & K_3^{0\mu\nu\rho} \\ K_3^{-\mu\nu\rho} & \bar{K}_3^{0\mu\nu\rho} & \omega_{3,S}^{\mu\nu\rho} \end{pmatrix}$$

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

$$V_1^{\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_{1,N}^{\mu}+\rho_1^{0\mu}}{\sqrt{2}} & \rho_1^{+\mu} & K_1^{*+\mu} \\ \rho_1^{-\mu} & \frac{\omega_{1,N}^{\mu}-\rho_1^{0\mu}}{\sqrt{2}} & K_1^{*0\mu} \\ K_1^{*-\mu} & \bar{K}_1^{*0\mu} & \omega_{1,S}^{\mu} \end{pmatrix}$$

TABLE IV. Decay amplitudes for different decay modes.

Decay mode	$\frac{1}{7}\lvert\mathcal{M}\rvert^2$
$3^{--} \to 0^{-+} + 0^{-+}$	$g_{w_3pp}^2 \frac{2}{35}\lvert\vec{k}_{p^{(1)},p^{(2)}}\rvert^6$
$3^{--} \to 0^{-+} + 1^{--}$	$g_{w_3v_1p}^2 \frac{8}{105}\lvert\vec{k}_{v_1,p}\rvert^6 m_{w_3}^2$
$3^{--} \to 0^{-+} + 2^{++}$	$g_{w_3a_2p}^2 \frac{2}{105}\lvert\vec{k}_{a_2,p}\rvert^4 \frac{m_{w_3}^2}{m_{a_2}^2}(2\lvert\vec{k}_{a_2,p}\rvert^2 + 7m_{a_2}^2)$
$3^{--} \to 0^{-+} + 1^{+-}$	$g_{w_3b_1p}^2 \frac{2}{105}\lvert\vec{k}_{b_1,p}\rvert^4(7 + 3\frac{\lvert\vec{k}_{b_1,p}\rvert^2}{m_{b_1}^2})$
$3^{--} \to 0^{-+} + 1^{++}$	$g_{w_3a_1p}^2 \frac{2}{105}\lvert\vec{k}_{a_1,p}\rvert^4(7 + 3\frac{\lvert\vec{k}_{a_1,p}\rvert^2}{m_{a_1}^2})$
$3^{--} \to 1^{--} + 1^{--}$	$g_{w_3v_1v_1}^2 \frac{1}{105}(m_{v_1^{(1)}}^2 m_{v_1^{(2)}}^2)^{-1}\lvert\vec{k}_{v_1^{(1)},v_2^{(2)}}\rvert^2[6\lvert\vec{k}_{v_1^{(1)},v_1^{(2)}}\rvert^4 + 35m_{v_1^{(1)}}^2 m_{v_1^{(2)}}^2 + 14\lvert\vec{k}_{v_1^{(1)},v_1^{(2)}}\rvert^2(m_{v_1^{(1)}}^2 + m_{v_1^{(2)}}^2)]$

Results (post- and predictions)

TABLE V. Decays of $J^{PC} = 3^{--}$ mesons into two pseudoscalars. Experimental data is taken from Ref. [1].

Decay process	Theory Γ/MeV	Experiment Γ/MeV
$\rho_3(1690) \to \pi\pi$	32.7 ± 2.3	38.0 ± 3.2
$\rho_3(1690) \to \bar{K}K$	4.0 ± 0.3	2.54 ± 0.45
$K_3^*(1780) \to \pi\bar{K}$	18.5 ± 1.3	29.9 ± 4.3
$K_3^*(1780) \to \bar{K}\eta$	7.4 ± 0.5	48 ± 22
$K_3^*(1780) \to \bar{K}\eta'(958)$	0.021 ± 0.001	
$\omega_3(1670) \to \bar{K}K$	3.0 ± 0.2	
$\phi_3(1850) \to \bar{K}K$	18.8 ± 1.3	Seen

TABLE VI. Decays of $J^{PC} = 3^{--}$ mesons into a pseudoscalar-vector pair. Experimental data taken from Ref. [1].

Decay process	Theory Γ/MeV	Experiment Γ/MeV
$\rho_3(1690) \to \rho(770)\eta$	3.8 ± 0.8	Seen
$\rho_3(1690) \to \bar{K}^*(892)K$	3.4 ± 0.7	
$\rho_3(1690) \to \omega(782)\pi$	35.8 ± 7.4	25.8 ± 9.8
$\rho_3(1690) \to \phi(1020)\pi$	0.036 ± 0.007	
$K_3^*(1780) \to \rho(770)K$	16.8 ± 3.5	49.3 ± 15.7
$K_3^*(1780) \to \bar{K}^*(892)\pi$	27.2 ± 5.6	31.8 ± 9.0
$K_3^*(1780) \to \bar{K}^*(892)\eta$	0.09 ± 0.02	
$K_3^*(1780) \to \omega(782)\bar{K}$	4.3 ± 0.9	
$K_3^*(1780) \to \phi(1020)\bar{K}$	1.2 ± 0.3	
$\omega_3(1670) \to \rho(770)\pi$	97 ± 20	Seen
$\omega_3(1670) \to \bar{K}^*(892)K$	2.9 ± 0.6	
$\omega_3(1670) \to \omega(782)\eta$	2.8 ± 0.6	
$\omega_3(1670) \to \phi(1020)\eta$	$(7.6 \pm 1.6) \times 10^{-6}$	
$\phi_3(1850) \to \rho(770)\pi$	1.1 ± 0.2	
$\phi_3(1850) \to \bar{K}^*(892)K$	35.5 ± 7.3	Seen
$\phi_3(1850) \to \omega(782)\eta$	0.015 ± 0.003	
$\phi_3(1850) \to \omega(782)\eta'(958)$	0.003 ± 0.001	
$\phi_3(1850) \to \phi(1020)\eta$	3.8 ± 0.8	

TABLE VII. Theoretical predictions for the radiative decays $W_3 \to \gamma P$.

Decay process	Theory Γ/keV
$\rho_3^{\pm/0}(1690) \to \gamma\pi^{\pm/0}$	69 ± 14
$\rho_3^0(1690) \to \gamma\eta$	157 ± 32
$\rho_3^0(1690) \to \gamma\eta'(958)$	20 ± 4
$K_3^{\pm}(1780) \to \gamma K^{\pm}$	58 ± 12
$K_3^0(1780) \to \gamma K^0$	231 ± 48
$\omega_3(1670) \to \gamma\pi^0$	560 ± 120
$\omega_3(1670) \to \gamma\eta$	19 ± 4
$\omega_3(1670) \to \gamma\eta'(958)$	1.4 ± 0.3
$\phi_3(1850) \to \gamma\pi^0$	4 ± 1
$\phi_3(1850) \to \gamma\eta$	129 ± 26
$\phi_3(1850) \to \gamma\eta'(958)$	35 ± 7

Isoscalar mixing is small

$$\begin{pmatrix} \omega_3(1670) \\ \phi_3(1850) \end{pmatrix} = \begin{pmatrix} \cos\beta_{w_3} & \sin\beta_{w_3} \\ -\sin\beta_{w_3} & \cos\beta_{w_3} \end{pmatrix} \begin{pmatrix} \omega_{3,N} \\ \omega_{3,S} \end{pmatrix}$$

$$\beta_{w_3} = 3.5°$$

The eLSM: a chiral model of QCD

PHYSICAL REVIEW D **87,** 014011 (2013)

Meson vacuum phenomenology in a three-flavor linear sigma model with (axial-)vector mesons

D. Parganlija,[1,2,*] P. Kovács,[2,3,†] Gy. Wolf,[3,‡] F. Giacosa,[2,§] and D. H. Rischke[2,4,||]

[1]*Institute for Theoretical Physics, Vienna University of Technology, Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria*
[2]*Institute for Theoretical Physics, Johann Wolfgang Goethe University, Max-von-Laue-Strasse 1, D-60438 Frankfurt am Main, Germany*
[3]*Institute for Particle and Nuclear Physics, Wigner Research Center for Physics, Hungarian Academy of Sciences, H-1525 Budapest, Hungary*
[4]*Frankfurt Institute for Advanced Studies, Ruth-Moufang-Strasse 1, D-60438 Frankfurt am Main, Germany*
(Received 7 August 2012; published 8 January 2013)

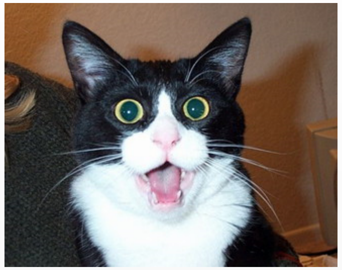
PHYSICAL REVIEW D **90,** 114005 (2014)

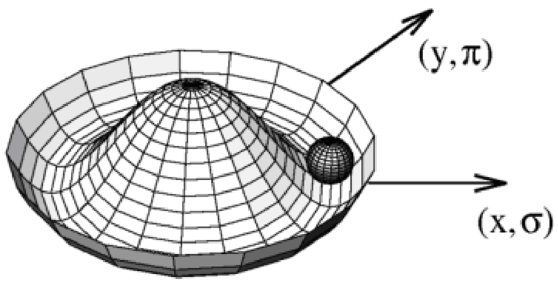
Is $f_0(1710)$ a glueball?

Stanislaus Janowski,[1] Francesco Giacosa,[1,2] and Dirk H. Rischke[1]

[1]*Institute for Theoretical Physics, Goethe University, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*
[2]*Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland*
(Received 26 August 2014; published 2 December 2014)

Model of QCD – eLSM with scalar Glueball





$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right) + \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi)\right] \\
& - m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\Phi^\dagger\Phi\right] - \lambda_1\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 - \lambda_2 \mathrm{Tr}\left[(\Phi^\dagger\Phi)^2\right] \\
& + \left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left((L^\mu)^2 + (R^\mu)^2\right)\right] \\
& - \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \mathrm{Tr}\left[H\left(\Phi^\dagger + \Phi\right)\right] \\
& + c_1[\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\
& + h_2 \mathrm{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \mathrm{Tr}[\Phi R_\mu \Phi^\dagger L^\mu]
\end{aligned}
$$

$$
\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}
$$

$$
L^\mu, R^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{\star +} \pm K_1^+ \\ \rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{\star 0} \pm K_1^0 \\ K^{\star -} \pm K_1^- & \bar{K}^{\star 0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S} \end{pmatrix}
$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011**)
D. Parganlija, P. Kovacs, G. Wolf , F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011**

Results of the eLSM (11 parameters, 21 exp. quantities)

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

Error from PDG or 5% of exp.
Scalar-isoscalar sector not included.

$$\chi^2_{red} = 1.2$$

arXiv:1208.0585

Pseudotensor: Lagrangians and decays

Pseudotensor mesons: {$\pi_2(1670)$, $K_2(1770)$, $\eta_2(1645)$, $\eta_2(1870)$}
Lagrangians based on flavour symmetry

$$\mathcal{L}_{TVP} = c_{TVP}\,\mathrm{Tr}\{\,T_{\mu\nu}\,[V^{\mu},\,(\partial^{\nu}P)]_{-}\}\,,$$

$$P = \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \quad V^{\mu} = \begin{pmatrix} \frac{\omega_N^{\mu}+\rho^{0\mu}}{\sqrt{2}} & \rho^{+\mu} & K^{*+\mu} \\ \rho^{-\mu} & \frac{\omega_N^{\mu}-\rho^{0\mu}}{\sqrt{2}} & K^{*0\mu} \\ K^{*-\mu} & \bar{K}^{*0\mu} & \omega_S^{\mu} \end{pmatrix},$$

$$T^{\mu\nu} = \begin{pmatrix} \frac{\eta_{2,N}^{\mu\nu}+\pi_2^{0\mu\nu}}{\sqrt{2}} & \pi_2^{+\mu\nu} & K_2^{+\mu\nu} \\ \pi_2^{-\mu\nu} & \frac{\eta_{2,N}^{\mu\nu}-\pi_2^{0\mu\nu}}{\sqrt{2}} & K_2^{0\mu\nu} \\ K_2^{-\mu\nu} & \bar{K}_2^{0\mu\nu} & \eta_{2,S}^{\mu\nu} \end{pmatrix}.$$

$$\mathcal{L}_{TXP} = c_{TXP}\,\mathrm{Tr}(\,T_{\mu\nu}\,\{X^{\mu\nu},\,P\}_{+})$$

$$P = \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \quad X^{\mu\nu} = \begin{pmatrix} \frac{f_{2,N}^{\mu\nu}+a_2^{0\mu\nu}}{\sqrt{2}} & a_2^{+\mu\nu} & K_2^{*+\mu\nu} \\ a_2^{-\mu\nu} & \frac{f_{2,N}^{\mu\nu}-a_2^{0\mu\nu}}{\sqrt{2}} & K_2^{*0\mu\nu} \\ K_2^{*-\mu\nu} & \bar{K}_2^{*0\mu\nu} & f_{2,S}^{\mu\nu} \end{pmatrix},$$

$$T^{\mu\nu} = \begin{pmatrix} \frac{\eta_{2,N}^{\mu\nu}+\pi_2^{0\mu\nu}}{\sqrt{2}} & \pi_2^{+\mu\nu} & K_2^{+\mu\nu} \\ \pi_2^{-\mu\nu} & \frac{\eta_{2,N}^{\mu\nu}-\pi_2^{0\mu\nu}}{\sqrt{2}} & K_2^{0\mu\nu} \\ K_2^{-\mu\nu} & \bar{K}_2^{0\mu\nu} & \eta_{2,S}^{\mu\nu} \end{pmatrix}.$$

Tree-level decay widths:

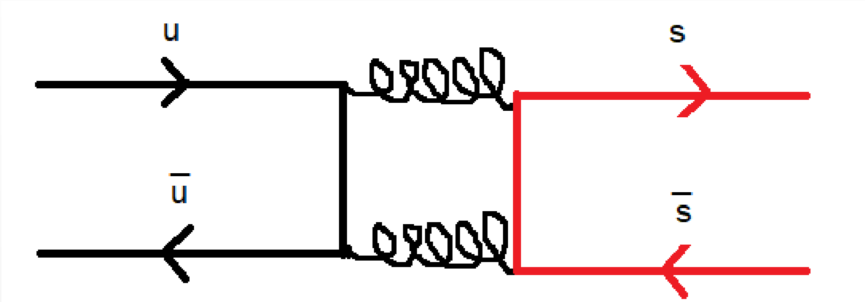
$$\Gamma^{tl}_{T\to VP} = \frac{k_f}{8\pi\,m_T}\,\frac{g^2_{TVP}}{15}\left(2\,\frac{k_f^4}{m_V^2} + 5\,k_f^2\right)\Theta(m_T - m_V - m_P)\,,$$

and

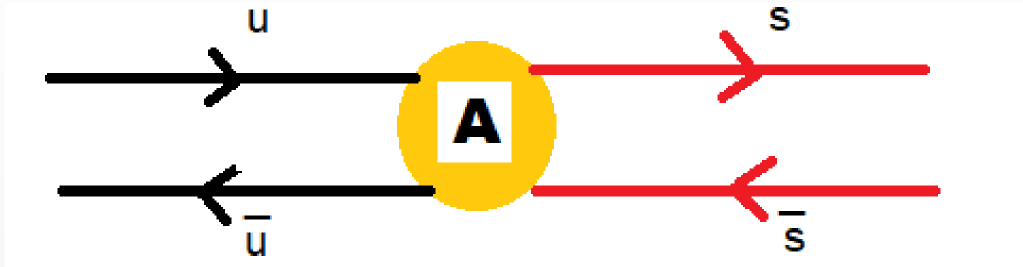
$$\Gamma^{tl}_{T\to XP} = \frac{k_f}{8\pi\,m_T}\,\frac{g^2_{TXP}}{45}\left(4\,\frac{k_f^4}{m_X^4} + 30\,\frac{k_f^2}{m_X^2} + 45\right)\Theta(m_T - m_X - m_P)\,.$$

Large mixing angle: where does it come from?

Such a mixing is suppressed...



But this can be large



- For pseudoscalar mesons: η(547) and η'(958). Θmix = -42° Large mixing caused by the axal anomaly.

- For vector mesons: ω(782) and φ(1020). Θmix = -3° Very small mixing.

- For tensor mesons: f2(1270) and f'2(1525). Θmix = 3° Also very small mixing. Why?

- Pseudotensor mesons: also large, but confirmation is needed.

Details in: **1709.07454**