Pragmatic extensions of the Standard Model

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3-Higgs Doublet Model (3HDM)

The SM

♠ Gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{L} \supset -\frac{1}{4}\underbrace{F_a^{\mu\nu}F_{a\,\mu\nu}}_{SU(3)_C} -\frac{1}{4}\underbrace{W_i^{\mu\nu}W_{i\,\mu\nu}}_{SU(2)_L} -\frac{1}{4}\underbrace{B^{\mu\nu}B_{\mu\nu}}_{U(1)_Y}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$G_a^\mu\big|_{a=1,\ldots 8} \qquad\qquad W_\mu^\pm, Z_\mu, A_\mu$$

♠ The Higgs sector:

- The minimal choice $\phi = \begin{pmatrix} G^+ \\ (h + iG^0)/\sqrt{2} \end{pmatrix}$ necessary for

$$SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

$$\mathcal{L} \supset (D_\mu \phi)^\dagger D^\mu \phi - V(\phi)$$

for $D_\mu \equiv \partial_\mu + igW^i_\mu T^i + ig' \frac{1}{2} Y B_\mu$ and

$$V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4 \qquad \text{with} \qquad Y_\phi = \frac{1}{2}$$

- If $\mu^2 < 0$ then $\langle 0||\phi|^2|0\rangle = -\frac{1}{2}\frac{\mu^2}{\lambda} \equiv \frac{v^2}{2}$ (spontaneous symmetry breaking, the origin of mass)
- Boson masses: $m_h = \sqrt{2\lambda} v$, $m_{W^\pm} = \frac{1}{2} g v$ and $m_Z = m_W / c_W$, for $c_W \equiv \cos\theta_W = g/(g^2 + g'^{\,2})^{1/2}$

♠ Fermions

fermion	T	T_3	$\frac{1}{2}Y$	Q
$\nu_{i\,L}$	$\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0
$l_{i\,L}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
$u_{i\,L}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
$d_{i\,L}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
$l_{i\,R}$	0	0	-1	-1
$u_{i\,R}$	0	0	$\frac{2}{3}$	$\frac{2}{3}$
$d_{i\,R}$	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$\nu_{i\,R}$	0	0	0	0

$i = 1, \ldots, N = 3$, $\psi_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)\psi$ (parity violation), $Q = T_3 + \frac{1}{2}Y$

Neutrino masses:

- Dirac mass: $f_{ij}\ \bar{L}_{i\,L}\nu_{j\,R}\ \tilde{\phi}$ + H.c. for $\tilde{\phi} \equiv i\tau_2\phi^\star$
- Majorana mass: $\frac{1}{2}M_{ij}\ \overline{\nu_{i\,R}{}^C}\nu_{j\,R}$ + H.c.

Yukawa interactions:

$$\mathcal{L} \supset -\sum_{i,j=1}^{3}\left(\tilde{\Gamma}_{ij}\bar{u}_{i\,R}\tilde{\phi}^{\dagger}Q_{j\,L} + \Gamma_{ij}\bar{d}_{i\,R}\phi^{\dagger}Q_{j\,L} + \text{H.c.}\right)$$

$$\Downarrow$$

if $\langle\phi\rangle \neq 0$ then $m_q \neq 0$

$$\mathcal{L}_{\text{q mass}} = -\sum_{i,j=1}^{3}\left(\bar{u}_{i\,R}\mathcal{M}^{u}_{ij}u_{j\,L} + \bar{d}_{i\,R}\mathcal{M}^{d}_{ij}d_{j\,L} + \text{H.c.}\right)$$

for

$$\mathcal{M}^{u}_{ij} = \frac{v}{\sqrt{2}}\tilde{\Gamma}_{ij} \qquad \mathcal{M}^{d}_{ij} = \frac{v}{\sqrt{2}}\Gamma_{ij}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}$$

$$U_R^\dagger \mathcal{M}^u U_L = \mathrm{diag}(m_u, m_c, m_t) \qquad D_R^\dagger \mathcal{M}^d D_L = \mathrm{diag}(m_d, m_s, m_b)$$

$$\Downarrow$$

$\tilde{\Gamma}, \Gamma$ diagonal $\left(g_f = \sqrt{2}\frac{m_f}{v}\right)$ $\Rightarrow$ no FCNC

- charged currents: $\sum \bar{u}_{i\,L} \gamma^\mu d_{i\,L} = (\bar{u}, \bar{c}, \bar{t})_L \underbrace{U_L^\dagger D_L}_{U^{CKM}} \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$
- neutral currents: $\sum \bar{u}_{i\,L} \gamma^\mu u_{i\,L}$, $\sum \bar{d}_{i\,L} \gamma^\mu d_{i\,L}$ remain unchanged upon $U_{L,R}$, $D_{L,R}$ transformations

U^{CKM}:

- unitary complex $N \times N$ matrix, $q_{i\,L} \to e^{i\alpha_i} q_{i\,L} \;\Rightarrow\; \frac{1}{2}(N-1)(N-2)$ phases in U^{CKM}
- $N \geq 3 \qquad \Rightarrow \qquad$ CP violation in charged currents

♠ Masses in the SM: $\quad m_V \propto g v \qquad m_h \propto \lambda^{1/2} v \qquad m_f \propto g_f v$

Leptons:

$$m_{\nu_e} \lesssim 3 \text{ eV} \qquad m_{\nu_\mu} \lesssim 0.2 \text{ MeV} \qquad m_{\nu_\tau} \lesssim 18 \text{ MeV}$$
$$m_e = 0.5 \text{ MeV} \qquad m_\mu = 105.5 \text{ MeV} \qquad m_\tau = 1.78 \text{ GeV}$$

Quarks:

$$m_u \simeq 2 \text{ MeV} \qquad m_c \simeq 1.2 \text{ GeV} \qquad m_t \simeq 174 \text{ GeV}$$
$$m_d = 5 \text{ MeV} \qquad m_s = 0.1 \text{ GeV} \qquad m_b = 4.3 \text{ GeV}$$

Bosons:

$$m_{W^\pm} = 80.4 \text{ GeV} \qquad m_Z = 91.2 \text{ GeV} \qquad m_\gamma = 0 \qquad m_h = 125.3 \text{ GeV}$$

$$\Downarrow$$

$$\frac{m_{\nu_e}}{m_t} \lesssim 1.72 \cdot 10^{-11} \qquad \Rightarrow \qquad \frac{g_{\nu_e}}{g_t} \lesssim 1.72 \cdot 10^{-11}$$

Difficulties of the SM

- Lack of the DM candidate within the SM
  - Strong experimental evidence for DM:
    - Galaxy rotation curves
    - Gravitational lensing
    - Cosmic microwave background
    - Structure formation
  - Modified Newtonian Dynamics (MOND/TeVeS) not an attractive possibility, it is not sufficient to explain data (DM is still needed)
- Unexplained baryon asymmetry
  The Sakhrov conditions:
  - B-violation
  - C- and CP-violation
  - Thermal inequilibrium

SM: $CPV \propto J$ (Jarlskog invariant)

$$\propto Im\left[{U^{CKM}}_{us}{U^{CKM}}_{cb}{U^{CKM\star}}_{ub}{U^{CKM\star}}_{cs}\right] = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta \sim 3{\cdot}10^{-5}$$

- The strong CP problem
  - symmetries of the SM allow for

$$\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(F_{\mu\nu}F_{\alpha\beta}\right) \xrightarrow{P} -\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

  - odd under CP

$$\mathcal{L}_\theta = \theta\frac{g_s^2}{32\pi^2}F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu} \quad \Rightarrow \quad \text{neutron-EDM} \quad D_n \simeq 2.7\cdot 10^{-16}\theta \text{ e cm}$$

$$\Downarrow$$

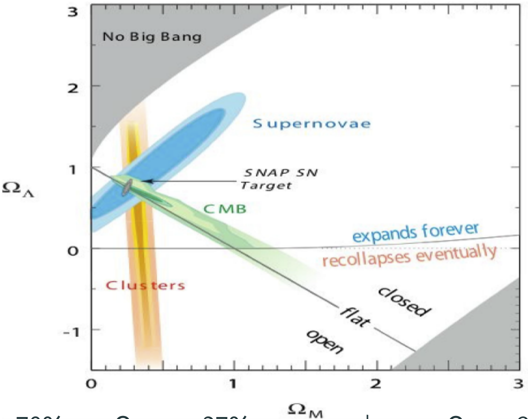
$$D_n \lesssim 1.1\cdot 10^{-25}\ \mathrm{e\ cm} \quad \Rightarrow \quad \theta \lesssim 3\cdot 10^{-10}$$

The strong CP problem: why is θ so small?

- Cosmology

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}$$

$$\rho_c = \frac{3H_0^2}{8\pi G_N}$$



$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2} \simeq 70\%, \quad \Omega_{DM} \simeq 27\% \quad \text{and} \quad \Omega_B \simeq 3\%$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \simeq 0.7 \quad \Rightarrow \quad \rho_\Lambda \simeq 10^{-46}\ \text{GeV}^4$$

The gravitational prediction: $\rho_\Lambda \sim M_{\rm Pl}^4 \sim 10^{76}\ \text{GeV}^4$

- Parameters of the SM

$$
\begin{array}{cccccc}
m_e & m_\mu & m_\tau & m_u & m_c & m_t \\
m_{\nu_e} & m_{\nu_\mu} & m_{\nu_\tau} & m_d & m_s & m_b
\end{array}
$$

$$
\underbrace{g \quad , \quad g'}_{(\alpha_{QED}, \sin\theta_W)}, \underbrace{g_s}_{(\alpha_{QCD})}, \underbrace{m_h, \ \lambda}_{(\mu,\lambda)}, \underbrace{U_{CKM}}_{\theta_{1,2,3},\delta_{CP}}
$$

21 parameters !

- Why is there only one Higgs boson?
  - The Higgs field was introduced just to make the model renormalizable (unitary)
  - There exist many fermions and vector bosons, so why only one scalar? Why, for instance, not a dedicated scalar for each fermion?

Interpretation of the LHC Higgs data

SM as an effective field theory:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c}{\Lambda}\mathcal{O}_5 + \sum_i \frac{c_i}{\Lambda^4}\mathcal{O}_i$$

The 125-GeV Higgs boson is SM-like

-

$$H_{125} \simeq H_{SM}$$

$$\Downarrow$$

$$\Lambda \gg v = 246 \text{ GeV}$$

No new physics in the TeV energy/mass range!

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n-Higgs doublet model (nHDM) in the alignment limit: $\Lambda \sim 300$ GeV

Fundamental (renormalizable) extensions of the SM

♠ Extra gauge symmetries

- GUTs, e.g. $SU(5)$: unification of gauge couplings, ...
- $L - R$ symmetry, $SU(2)_L \times SU(2)_R \times U(1)$: spontaneous parity violation
- $SU(2)_L \times U(1) \times U(1)'$: just extra Z'

♠ Extra fermions

- vector-like quarks

♠ Extra Higgs bosons

- SM-like Higgs-boson discovery by ATLAS and CMS at the LHC announced on 4 July 2012

$$m_h = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\ \text{GeV}$$

- SM single Higgs doublet is rather unnatural, why only one?
  - Higgs-boson representation:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W}, \qquad \text{SM} \qquad \Rightarrow \qquad \rho = 1 + \mathcal{O}(\alpha)$$

for general Higgs multiplets:

$$\rho = \frac{\sum_i \left[T_i(T_i+1) - T_{i\,3}^2\right] v_i^2}{\sum_i 2T_{i\,3}^2 v_i^2}$$

data: $\rho = 1.0002 \begin{cases} +0.0024 \\ -0.0009 \end{cases} \qquad \Rightarrow \qquad T = \frac{1}{2}$

Doublets (nHDM) and

♠ Extra Higgs bosons

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Doublets (nHDM) and extra singlet (real or complex) are favored.

- Scalar $SU(2)$ singlets:
    - real $\Rightarrow$ DM
    - complex $\Rightarrow$ pseudo-Goldstone DM with suppressed DM-nucleon coupling
- Scalar $SU(2)$ doublets: extra sources of CPV from the scalar potential and from Yukawas (for baryogenesis)

Real singlet scalar $S \bigoplus SM$

$$V = -\mu_\phi^2|\phi|^2 + \lambda_\phi|\phi|^4 - \mu_S^2 S^2 + \lambda_S S^4 + \kappa S^2|\phi|^2$$

- ϕ is the SM Higgs doublet
- Z_2 symmetry $S \to -S$, S is DM candidate

B.G. and J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics," Phys. Rev. Lett. **103**, 091802 (2009)

A. Drozd, B.G. and J. Wudka, "Multi-Scalar-Singlet Extension of the Standard Model - the Case for Dark Matter and an Invisible Higgs Boson," JHEP **04**, 006 (2012)

Complex singlet (DM) $\bigoplus$ SM (CPC)

$$V = -\mu_\phi^2|\phi|^2+\lambda_\phi|\phi|^4-\mu_S^2|S|^2+\lambda_S|S|^4+\kappa|S|^2|\phi|^2+\mu^2(S^2+S^{*\,2})$$

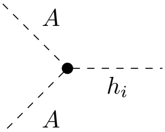
Symmetries:

- $S = \frac{1}{\sqrt{2}}(v_S + \phi_S + iA)$, with $\langle S\rangle = \frac{v_S}{\sqrt{2}}$,
- $U(1)$ softly broken by $\mu^2(S^2+S^{*\,2})$, $\Rightarrow$ pseudo-Goldstone boson,
- $U(1)$ softly broken by $\mu^2(S^2+S^{*\,2})$, $\Rightarrow$ residual symmetry: $S \overset{Z_2}{\to} -S$ (only even powers of S).

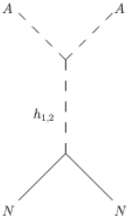
D. Azevedo, M. Duch, B.G., D. Huang, M. Iglicki and R. Santos, "Testing scalar versus vector dark matter," Phys. Rev. D **99**, no.1, 015017 (2019), "One-loop contribution to dark-matter-nucleon scattering in the pseudo-scalar dark matter model," JHEP **01**, 138 (2019)

The DM direct detection signals are naturally suppressed in the pGDM model.

$$V \supset \frac{A^2}{2v_S}(\sin\alpha\, m_1^2 h_1 + \cos\alpha\, m_2^2 h_2)\,,$$



$$= -iR_{2i}\frac{m_i^2}{v_S}$$



$$i\mathcal{M} = -i\frac{\sin 2\alpha f_N m_N}{2vv_S}\left(\frac{m_1^2}{q^2-m_1^2}-\frac{m_2^2}{q^2-m_2^2}\right)\bar{u}_N(p_4)u_N(p_2) \to 0$$

Properties of nHDM

PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

A Theory of Spontaneous T Violation*

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(Received 11 April 1973)

A theory of spontaneous T violation is presented. The total Lagrangian is assumed to be invariant under the time reversal T and a gauge transformation (e.g., the hypercharge gauge), but the physical solutions are not. In addition to the spin-1 gauge field and the known matter fields, in its simplest form the theory consists of two complex spin-0 fields. Through the spontaneous symmetry-breaking mechanism of Goldstone and Higgs, the vacuum expectation values of these two spin-0 fields can be characterized by the shape of a triangle and their quantum fluctuations by its vibrational modes, just like a triangular molecule. T violations can be produced among the known particles through virtual excitations of the vibrational modes of the triangle which has a built-in T-violating phase angle. Examples of both Abelian and non-Abelian gauge groups are discussed. For renormalizable theories, all spontaneously T-violating effects are finite. It is found that at low energy, below the threshold of producing these vibrational quanta, T violation is always quite small.

I. INTRODUCTION

In this paper we discuss a theory of spontaneous T violation. To illustrate the theory, we shall first discuss a simple model in which the weak-interaction Lagrangian, as well as the strong- and electromagnetic-interaction Lagrangians, is assumed to be invariant under (1) the time reversal T and (2) a gauge transformation, e.g., that of the hypercharge Y. Yet the physical solutions are required to exhibit both T violation and Y nonconservation. In its construction, the model is similar to those gauge-group spontaneous symmetry-violating theories[1-4] that have been extensively discussed in the literature. The only difference is that one now has, in addition, the spontaneous violation of a discrete symmetry.[5] As we shall see

$$\phi_k \to e^{i\Lambda}\phi_k$$

and (1)

$$B_\mu \to B_\mu + f^{-1}\frac{\partial\Lambda}{\partial x_\mu},$$

where f is the hypercharge coupling constant and the subscript $k=1$ and 2. As usual, T is assumed to commute with Y,

$$TYT^{-1} = Y. \tag{2}$$

This gives then a well-defined difference between T and either CT or CPT. Since T is an antiunitary operator, we can always choose the phase of ϕ_k such that

$$T\phi_k T^{-1} = \phi_k. \tag{3}$$

$$
\begin{aligned}
&V(\Phi_1,\Phi_2)\\
&= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}\right]\right\}\\
&+\frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\
&+\left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2) + \text{H.c.}\right\}
\end{aligned}
$$

In a general basis, the vacuum may be complex:

$$
\Phi_j = e^{i\xi_j}\begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2,
$$

Spontaneous or explicit violation of CP is possible

B.G., H. E. Haber, O. M. Ogreid and P. Osland, "Heavy Higgs boson decays in the alignment limit of the 2HDM," JHEP **12**, 056 (2018)

The model contains three neutral scalars, which are linear combinations of the η_i,

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix},$$

with the 3×3 orthogonal rotation matrix R satisfying

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2),$$

and with $M_1 \leq M_2 \leq M_3$. The rotation matrix R can conveniently be parametrized as

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix}$$

Physical/observable input parameter set:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

$$\begin{aligned} e_i &\propto H_i W^+ W^-, H_i ZZ \\ q_i &\propto H_i H^+ H^- \end{aligned}$$

$$e_1^2 + e_2^2 + e_3^2 = v^2 \equiv v_1^2 + v_2^2$$

Weak-bases transformation:

$$\begin{pmatrix} \bar{\Phi}_1 \\ \bar{\Phi}_2 \end{pmatrix} = \underbrace{e^{i\psi} \begin{pmatrix} \cos\theta & e^{-i\tilde{\xi}} \sin\theta \\ -e^{i\chi} \sin\theta & e^{i(\chi-\tilde{\xi})} \cos\theta \end{pmatrix}}_{U(2)} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

Observables are weak-basis independent

Flavor-Changing Neutral Currents in nHDM

$$\mathcal{L} \supset -\sum_{\alpha=1}^{n} \sum_{i,j=1}^{3} \left(\tilde{\Gamma}_{ij}^{\alpha} \bar{u}_{i\,R} \tilde{\phi}^{\alpha\,\dagger} Q_{j\,L} + \Gamma_{ij}^{\alpha} \bar{d}_{i\,R} \phi^{\alpha\,\dagger} Q_{j\,L} + \text{H.c.} \right)$$

$$\mathcal{M}_{ij}^{u} = \sum_{\alpha=1}^{n} \tilde{\Gamma}_{ij}^{\alpha} \frac{v_\alpha}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^{d} = \sum_{\alpha=1}^{n} \Gamma_{ij}^{\alpha} \frac{v_\alpha}{\sqrt{2}}$$

$n = 2$

$$\mathcal{M}_{ij}^{u} = \tilde{\Gamma}_{ij}^{1} \frac{v_1}{\sqrt{2}} + \tilde{\Gamma}_{ij}^{2} \frac{v_2}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^{d} = \Gamma_{ij}^{1} \frac{v_1}{\sqrt{2}} + \Gamma_{ij}^{2} \frac{v_2}{\sqrt{2}}$$

$$Z_2 : \phi_2 \to -\phi_2, u_{i\,R} \to -u_{i\,R}$$

$$\mathcal{M}_{ij}^{u} = \cancel{\tilde{\Gamma}_{ij}^{1} \frac{v_1}{\sqrt{2}}} + \tilde{\Gamma}_{ij}^{2} \frac{v_2}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^{d} = \Gamma_{ij}^{1} \frac{v_1}{\sqrt{2}} + \cancel{\Gamma_{ij}^{2} \frac{v_2}{\sqrt{2}}}$$

no: FCNC

The Inert Higgs Doublet model (IDM)

$Z_2 : \phi_2 \to -\phi_2$ unbroken, i.e. $v_2 = 0$:

$$V(\Phi_1, \Phi_2) =$$
$$-\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \cancel{\left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}\right]}\right\}$$
$$+\frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \cancel{[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2)} + \text{H.c.}\right\}$$

$$\Phi_2 = \cancel{e^{i\xi_2}}\begin{pmatrix} \varphi_2^+ \\ (\cancel{v_2} + \eta_2 + i\chi_2)/\sqrt{2} \end{pmatrix},$$

$$\Downarrow$$

DM candidates: η_2, χ_2

The Higgs alignment

$$H_{125} \simeq H_{SM}$$

$$\Downarrow$$

$$e_1 \simeq v$$

$$\Downarrow$$

$$2HDM: \; e_1 = v, e_2 = e_3 = 0 \qquad (e_1^2 + e_2^2 + e_3^2 = v^2)$$

$$\Downarrow$$

$$e_1 = v\cos(\alpha_2)\cos(\alpha_1 - \beta) = v,$$

where $\tan\beta = v_2/v_1$.

$$\Downarrow$$

$$\alpha_1 = \beta, \quad \alpha_2 = 0$$

Weak-basis CPV invariants

$$
\begin{aligned}
\Im J_1 &= \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk} M_i^2 e_i e_k q_j \\
&= \frac{1}{v^5}[M_1^2 e_1(e_3 q_2 - e_2 q_3) + M_2^2 e_2(e_1 q_3 - e_3 q_1) + M_3^2 e_3(e_2 q_1 - e_1 q_2)] \\
\Im J_2 &= \frac{2}{v^9}\sum_{i,j,k}\epsilon_{ijk} e_i e_j e_k M_i^4 M_k^2 \\
&= \frac{2 e_1 e_2 e_3}{v^9}(M_1^2 - M_2^2)(M_2^2 - M_3^2)(M_3^2 - M_1^2) \\
\Im J_{30} &= \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk} q_i M_i^2 e_j q_k
\end{aligned}
$$

CP is conserved if and only if $\Im J_1 = \Im J_2 = \Im J_{30} = 0$

B.G., O. M. Ogreid and P. Osland, "Measuring CP violation in Two-Higgs-Doublet models in light of the LHC Higgs data", JHEP **11**, 084 (2014)

Alignment: e_1=v, e_2=e_3=0

$$\Im J_1 = 0,$$
$$\Im J_2 = 0,$$
$$\Im J_{30} = \frac{q_2 q_3}{v^4}(M_3^2 - M_2^2)$$

- $e_1 = v$ implies no CP violation in the couplings to gauge bosons ($\Im J_1 = \Im J_2 = 0$), the only possible CP violation may appear in cubic scalar couplings q_2 and q_3.
- The necessary condition for CP violation is that both $(H_2 H^+ H^-)$ *and* $(H_3 H^+ H^-)$ couplings must exist *together* with a non-zero ZH_2H_3 vertex ($\propto e_1$).
- If $\lambda_6 = \lambda_7 = 0$ (Z_2-symmetric 2HDM), then $\Im J_{30} = 0$, so no CPV!

2HDM conclusions

- Violation of CP in the scalar potential
- Weak-basis invariance of observables
- Flavor-Changing Neutral Currents ($Z_2 : \phi_2 \to -\phi_2$)
- Symmetries of the potential (Z_2 the only possibility with Yukawa invariance)
- Inert Doublet Model (IDM): exactly Z_2-symmetric ($\phi_2 \to -\phi_2$) 2HDM, DM candidates, no CPV
- No CPV in the potential in Z_2-symmetric models in the alignment limit

$$\Downarrow$$

For CPV in the potential the Z_2 must be abandoned, so FCNC appear

Minimal models with CPV and DM

- minimal: real or complex singlet (DM) $\bigoplus$ 2HDM (CPV)

Minimal models with CPV and DM

- minimal: real or complex singlet (DM) $\bigoplus$ 2HDM (CPV)
- next to minimal: inert doulet (DM) $\bigoplus$ 2HDM (CPV)

Real singlet (DM) $\bigoplus$ 2HDM (CPV)

$Z_2 \times Z_2'$ $(\phi_2 \to -\phi_2,\, \varphi \to -\varphi)$ symmetry, Z_2 softly broken

$$\begin{aligned} V(\phi_1,\phi_2,\varphi) \;\; = \;\; & -\frac{1}{2}\left\{m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \mathrm{H.c.}\right]\right\} \\ & +\frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 \\ & +\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \mathrm{H.c.}\right] \\ & +\mu_\varphi^2\varphi^2 + \frac{1}{24}\lambda_\varphi\varphi^4 + \varphi^2(\eta_1\phi_1^\dagger\phi_1 + \eta_2\phi_2^\dagger\phi_2). \end{aligned}$$

φ - DM candidate

B.G. and P. Osland, “Tempered Two-Higgs-Doublet Model”, Phys.Rev.D82, 125026 (2010)

Singlet Complex Scalar $\bigoplus$ General 2HDM

$$S \xrightarrow{\mathrm{U}(1)} e^{i\alpha} S$$

The $U(1)$ softly broken ($Z_2 : S \to -S$):

$$V = -\frac{1}{2}\left[m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 + \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{H.c.}\right)\right] + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4$$
$$+ \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \left[\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_2)|\Phi_1|^2 + \lambda_7(\Phi_1^\dagger\Phi_2)|\Phi_2|^2 +\right.$$
$$- \mu_s^2|S|^2 + \frac{\lambda_s}{2}|S|^4 + (\mu^2 S^2 + \mathrm{H.c.}) + |S|^2\left[\kappa_1|\Phi_1|^2 + \kappa_2|\Phi_2|^2 + \left(\frac{\kappa_3}{2}\Phi_1^\dagger\Phi_2 + \mathrm{H.c.}\right)\right]$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + \eta_1 + i\chi_1}{\sqrt{2}} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + \eta_2 + i\chi_2}{\sqrt{2}} \end{pmatrix}, \qquad S = \frac{v_s + s + iA}{\sqrt{2}}$$

In the general 2HDM with pGDM the tree-level DM-quark amplitude also vanishes at the zero momentum transfer limit.

N. Darvishi and B.G."Pseudo-Goldstone dark matter model with CP violation," JHEP **06**, 092 (2022)

Inert doublet (DM) $\bigoplus$ 2HDM (CPV)

$Z_2 \times Z_2'$ ($\phi_2 \to -\phi_2$, $\eta \to -\eta$) symmetry, Z_2 softly broken:

$$V(\Phi_1, \Phi_2, \eta) = V_{12}(\Phi_1, \Phi_2) + V_3(\eta) + V_{123}(\Phi_1, \Phi_2, \eta)$$

where

$$\begin{aligned}
V_{12}(\Phi_1, \Phi_2) &= -\frac{1}{2}\left\{m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}\right]\right\} \\
&\quad + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
&\quad + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\left[\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}\right], \\
V_3(\eta) &= m_\eta^2 \eta^\dagger \eta + \frac{\lambda_\eta}{2}(\eta^\dagger \eta)^2, \\
V_{123}(\Phi_1, \Phi_2, \eta) &= \lambda_{1133}(\Phi_1^\dagger \Phi_1)(\eta^\dagger \eta) + \lambda_{2233}(\Phi_2^\dagger \Phi_2)(\eta^\dagger \eta) \\
&\quad + \lambda_{1331}(\Phi_1^\dagger \eta)(\eta^\dagger \Phi_1) + \lambda_{2332}(\Phi_2^\dagger \eta)(\eta^\dagger \Phi_2) \\
&\quad + \frac{1}{2}\left[\lambda_{1313}(\Phi_1^\dagger \eta)^2 + \text{H.c.}\right] + \frac{1}{2}\left[\lambda_{2323}(\Phi_2^\dagger \eta)^2 + \text{H.c.}\right]
\end{aligned}$$

B.G., O.M. Ogreid, P. Osland, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys.Rev.D80, 055013 (2009)

Summary and conclusions

- The SM is not perfect (DM, Λ, θ, BA, 21, etc.)
- The SM should be extended: extra scalar doublets and singlets are likely/favored
- $2HDM$, $nHDM$ $\Rightarrow$ CP violation (explicit or spontaneous)
- Extra scalars (real or complex singlets, $SU(2)$-doublets) $\Rightarrow$ DM
- No CPV in Z_2 symmetric 2HDM in the alignment limit ($e_1 = 1$, $e_2 = e_3 = 0$) $\Rightarrow$ generic 2HDM with FCNC in Yukawa interactions
- Complex scalar with soft breaking of a global symmetry (e.g. $U(1)$) $\Rightarrow$ pGDM with suppressed $DM - nucleus$ coupling.
- Minimal "pragmatic" models (CPV & DM):
  - minimal: real or complex singlet (DM) $\bigoplus$ 2HDM (CPV), but FCNC
  - next to minimal: inert doublet (DM) $\bigoplus$ 2HDM (CPV), but FCNC
- $3HDM$ e.g. inert doublet (DM) $\bigoplus$ 2HDM (CPV), $N_f = N_h = 3$

3HDM of Weinberg:

- CPV in $H^{\pm}$ interactions.
- Natural flavour conservation by a Z_2

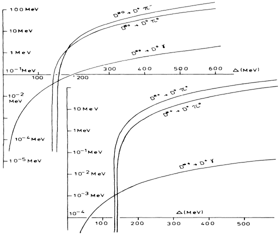

FIG. 1. Decay rates $\Gamma_{D^* \to D\pi}$ and $\Gamma_{D^* \to D\gamma}$ in the charmed quark model: $\Delta \equiv D^{*0} - D^0$.

For $\Delta > 200$ MeV one gets

$$\begin{aligned} &\Gamma_{D^{*0} \to D^+ \pi^-} : \Gamma_{D^{*0} \to D^0 \pi^0} : \Gamma_{D^{*0} \to D^0 \gamma} \\ &\quad \simeq 1:0.5:10^{-2}, \\ &\Gamma_{D^{*+} \to D^0 \pi^+} : \Gamma_{D^{*+} \to D^+ \pi^0} : \Gamma_{D^{*+} \to D^+ \gamma} \\ &\quad \simeq 1:0.5:3\times 10^{-3}. \end{aligned} \tag{17}$$

In view of the fact that, experimentally, Δ appears to be about 140 MeV, very nearly a pion mass, and is not measured with high accuracy, one may not be able to calculate the decay rates of $D^* \to D\pi$ very accurately.

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[1] V. Lüth, in Proceedings of the International Neutrino Conference, Aachen, West Germany, 1976 (to be published).
[2] A. De Rújula, in Proceedings of the International Neutrino Conference, Aachen, West Germany, 1976 (to be published).
[3] S. Ono, Nucl. Phys. B107, 522 (1976).
[4] S. Ono, to be published.
[5] D. B. Lichtenberg, Phys. Rev. D 12, 3760 (1975).
[6] A. Le Yaounac, L. Oliver, O. Pène, and J. C. Raynal, Nucl. Phys. B37, 552 (1972).
[7] S. Ono, Phys. Rev. D 9, 2005, 2670 (1974).

Gauge Theory of *CP* Nonconservation*

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It is proposed that *CP* nonconservation arises purely from the exchange of Higgs bosons.

Ever since the discovery[1] that *CP* conservation is not exact, the mystery has been why it is so feebly violated.[2] In many proposed theories,[3] one must arrange to have *CP* to be approximately conserved, by making the appropriate constants in the Lagrangian to have sufficiently small values. However, one would prefer a more natural explanation.

Renormalizable gauge theories[4] of the weak and electromagnetic interactions provide a mechanism which could violate *CP* conservation with about the right strength: the Higgs boson. The coupling strength of a Higgs boson to a quark or lepton of mass m is of order $mG_F^{1/2}$ (where G_F is the Fermi coupling constant), so that the exchange of a Higgs boson of mass m_H produces an effective Fermi interaction with coupling of order $G_F m^2/m_H^2$. For reasonable mass values,[5] this is "milliweak." However, in order for the Higgs exchange to appear as a natural explanation for a feeble *CP* nonconservation, one must understand why *CP* conservation is strongly violated in the Higgs exchange, *and nowhere else*. In this paper, I wish to present a realistic gauge theory, in which *CP* nonconservation automatically arises in just this way.[6]

We assume an $SU(2)\otimes U(1)$ gauge theory,[4] with the usual four quarks[7]: $\mathcal{P}_1$ and $\mathcal{P}_2$ have charge $+\frac{2}{3}$

- explanation of weak mixing angles through horizontal symmetries

$$\mathcal{L} \supset -\sum_{i,j=1}^{3} \sum_{\alpha=1}^{N_H} \left(\tilde{\Gamma}^{\alpha}_{ij} \bar{u}_{i\,R} \tilde{H}^{\alpha\,\dagger} Q_{j\,L} + \Gamma^{\alpha}_{ij} \bar{d}_{i\,R} H^{\alpha\,\dagger} Q_{j\,L} + \text{H.c.} \right)$$

$$H^{\alpha} \to \mathcal{H}^{\alpha}_{\beta} H^{\beta}\,, \qquad u_{i\,R} \to \mathcal{U}^{j}_{i} u_{j\,R}\,, \qquad d_{i\,R} \to \mathcal{D}^{j}_{i} d_{j\,R}\,, \qquad Q_{i\,L} \to \mathcal{Q}^{j}_{i} Q_{j\,L}$$

$$\Downarrow$$

constraints on fermion mass-matrices:

$$\mathcal{M}^{u}_{ij} = \sum_{\alpha=1}^{N_H} \tilde{\Gamma}^{\alpha}_{ij} \frac{v_\alpha}{\sqrt{2}}, \qquad \mathcal{M}^{d}_{ij} = \sum_{\alpha=1}^{N_H} \Gamma^{\alpha}_{ij} \frac{v_\alpha}{\sqrt{2}}$$

$$U_R^{\dagger} \mathcal{M}^{u} U_L = \mathrm{diag}(m_u, m_c, m_t) \qquad D_R^{\dagger} \mathcal{M}^{d} D_L = \mathrm{diag}(m_d, m_s, m_b)$$

If $\mathcal{M}^{u,d}$ constrained, then $U^{CKM} \equiv U_L^{\dagger} D_L = U^{CKM}(m_q/m_{q'})$