Small-x Quark and Gluon Helicity Contributions to the Proton Spin Puzzle

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Outline

- Introduction
- Helicity Operators and Observables at small x
- Evolution Equations
- Closed Evolution Equations
  - Large-N_c limit
  - Large-N_c&N_f limit
- Phenomenology

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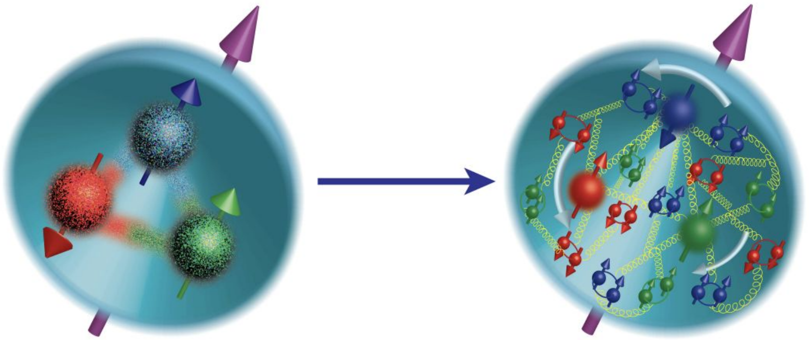
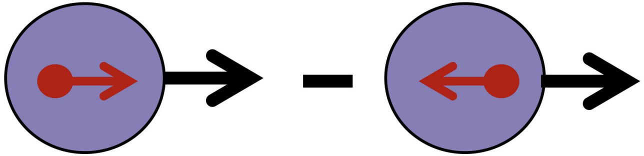
Proton Spin



- In the past, proton spin was thought to be the sum of constituent quarks spins.
- Now, we believe it to be the sum of spins of valence quarks, sea quarks and gluons, together with their orbital angular momenta (OAM).

Helicity PDF



- Helicity-dependent generalization of PDFs
- For each parton f,

$$\Delta f(x, Q^2) \equiv f^+(x, Q^2) - f^-(x, Q^2)$$

- For quarks, we often consider the “flavor-singlet” quark hPDF:

$$\Delta\Sigma(x, Q^2) = \sum_{f=u,d,s} \left[\Delta f(x, Q^2) + \Delta \bar{f}(x, Q^2)\right]$$

- Gluon hPDF: $\Delta G(x, Q^2)$

Proton Helicity Sum Rule

- Jaffe-Manohar sum rule: $$\frac{1}{2} = S_q + S_G + L_q + L_G$$

  where the helicity of quarks (S_q) and gluons (S_G) are

  $$S_q(Q^2) = \frac{1}{2} \int_0^1 dx \, \Delta\Sigma(x, Q^2) \quad \text{and} \quad S_G(Q^2) = \int_0^1 dx \, \Delta G(x, Q^2)$$

- In the late 1980's, EMC measurement implied that $S_q \approx 0.05$, much lower than what would have been (1/2) had all the proton spin been carried by the constituent quarks.

$$\frac{1}{2} = S_q + S_G + L_q + L_G$$

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

$$S_G(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

Current Knowledge of Proton Helicity

- More recently, the proton spin carried by quarks and gluon are estimated to be

$$S_q(Q^2 = 10\ \mathrm{GeV}^2) \approx \frac{1}{2}\int_{0.001}^{1} dx\, \Delta\Sigma(x, 10\ \mathrm{GeV}^2) \in [0.15, 0.20]$$

$$S_G(Q^2 = 10\ \mathrm{GeV}^2) \approx \int_{0.01}^{1} dx\, \Delta G(x, 10\ \mathrm{GeV}^2) \in [0.13, 0.26]$$

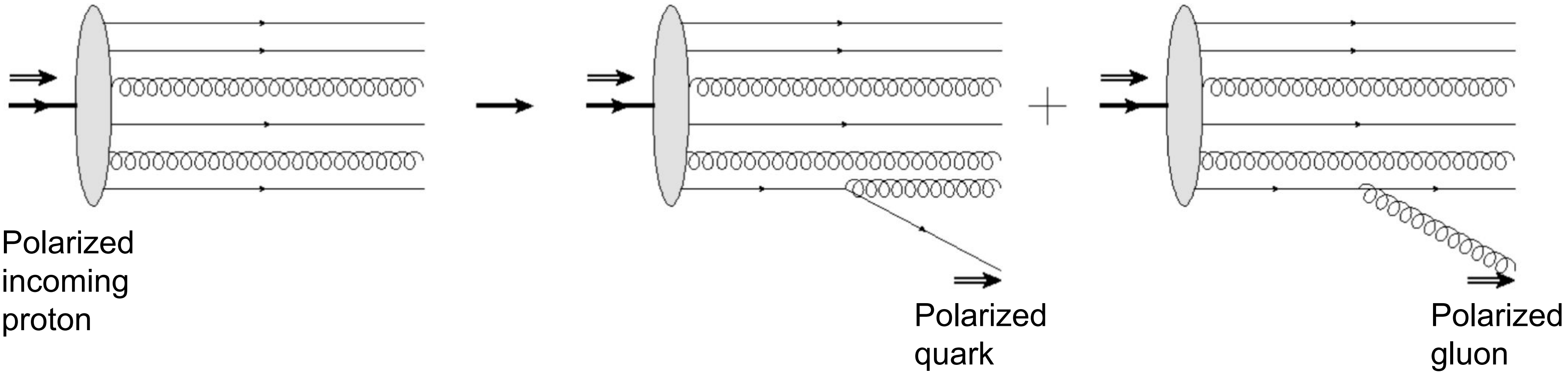
- They do not add to 1/2. The missing spin can come from:
  - Orbital angular momenta, L_q and L_G.
  - Small-x region of $\Delta\Sigma$ and ΔG. Scattering experiments can only access finitely small x. The limit will improve with EIC.

Small-x Evolution for Helicity

- Scattering experiments can only access $\Delta\Sigma$ and ΔG down to finitely small x.
- We attempt to fill the gap by finding small-x asymptotics for $\Delta\Sigma$ and ΔG through evolution in x.
  - Evolution constructed by Y. Kovchegov, D. Pitonyak and M. Sievert (KPS) in 2015-18 [1505.01176, 1511.06737, 1610.06197, 1808.09010]
  - Important additional contribution recently calculated by F. Cougoulic, Y. Kovchegov, A. Tarasov and Y. Tawabutr (KPS-CTT) in 2022 [2204.11898]
  - Employing similar approach to BK/JIMWLK evolution.

Small-x Evolution for Helicity

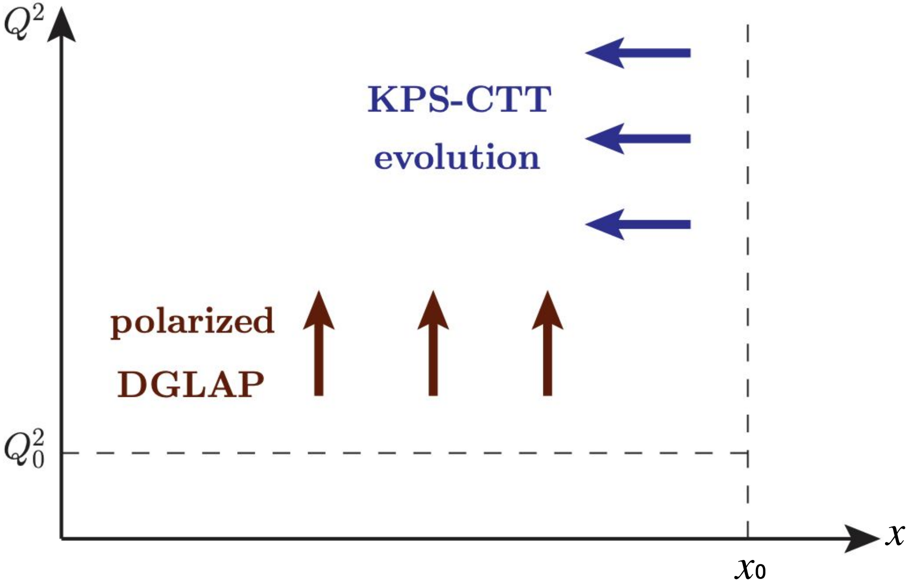
- We attempt to fill the gap by finding small-x asymptotics for $\Delta\Sigma$ and ΔG through evolution in x, employing similar approach to BK/JIMWLK evolution.



- Helicity evolution must keep track of both quark and gluon helicity, in contrast to unpolarized small-x evolution.

Small-x Evolution for Helicity

- The KPS-CTT evolution in x is complementary to the existing polarized DGLAP evolution.



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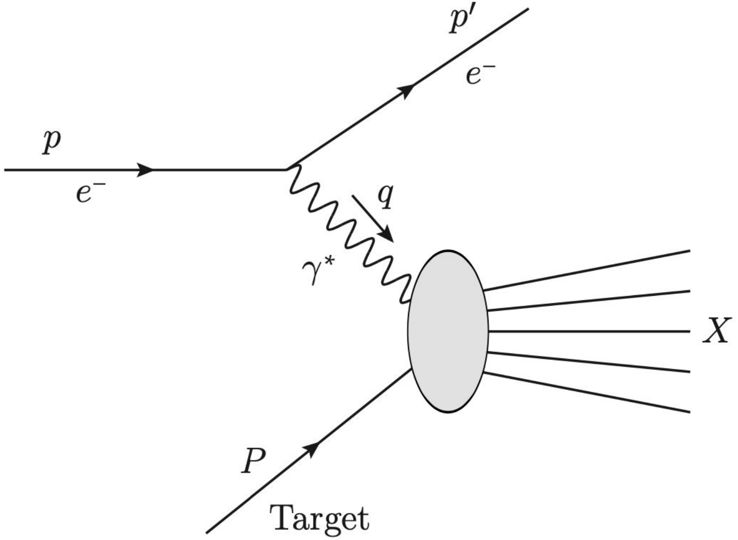
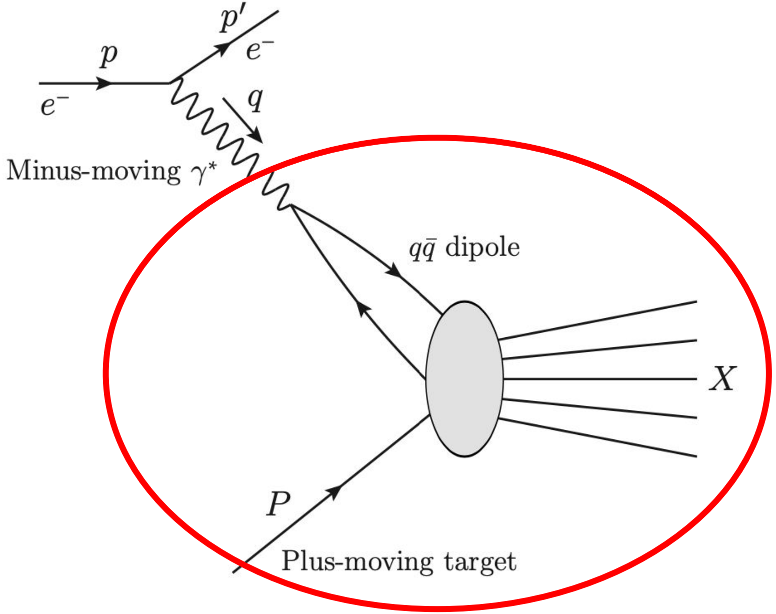
DIS at Small x: The Dipole Picture

- At small-x, DIS is dominated by the contribution where the virtual photon splits into a quark-antiquark dipole, which goes on to interact with the target.



$\Rightarrow$

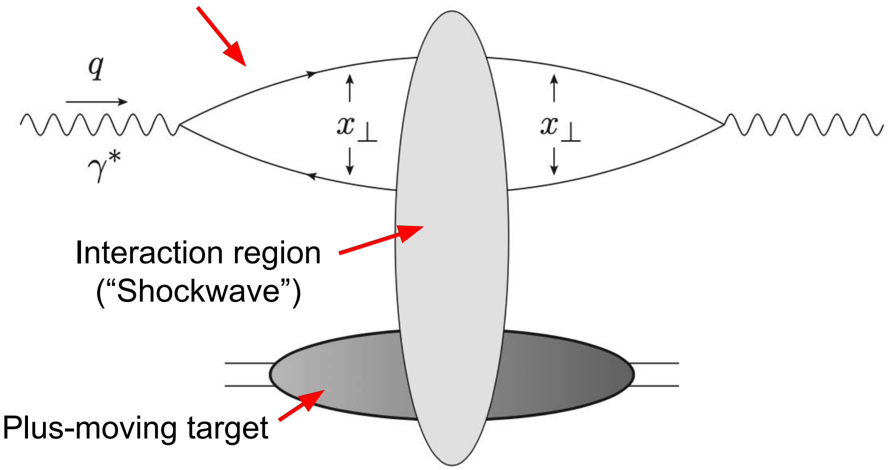


DIS at Small x: The Dipole Picture

- Unpolarized PDF and structure functions, $F_1(x, Q^2)$ and $F_2(x, Q^2)$, relate to the **s-matrix** of dipole-target scattering:

Brackets: Averaging over target's state, including spin

$$S(\underline{x}_1, \underline{x}_0, s) \equiv S_{10}(s) = \frac{1}{N_c} \left\langle \mathrm{tr}\left[V_{\underline{1}} V_{\underline{0}}^\dagger\right] \right\rangle (s)$$



where

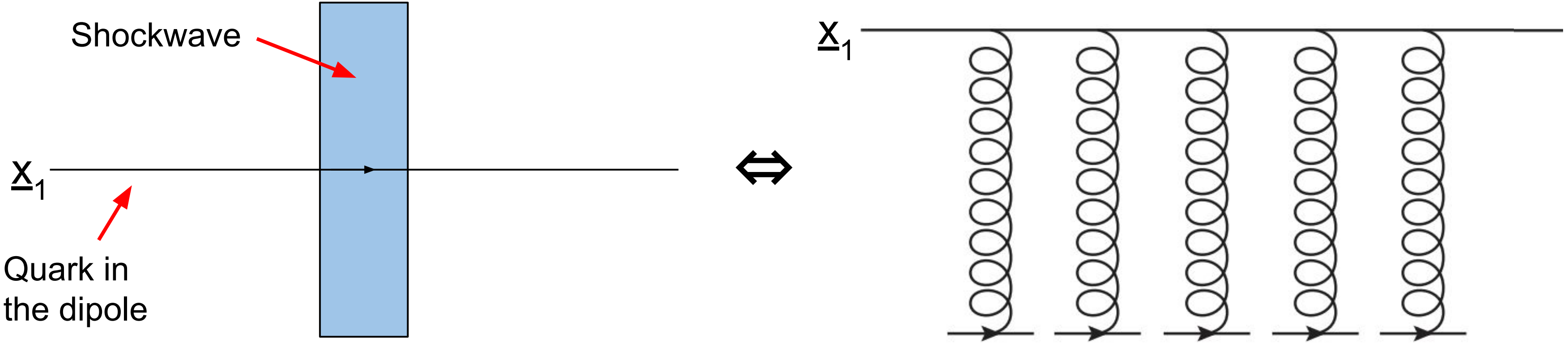
$$V_{\underline{1}}[x_f^-, x_i^-] \equiv V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp\left[ig \int\limits_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1)\right]$$

$$V_{\underline{1}} \equiv V_{\underline{1}}\left[\infty, -\infty\right]$$

Lightcone (unpolarized) Wilson line

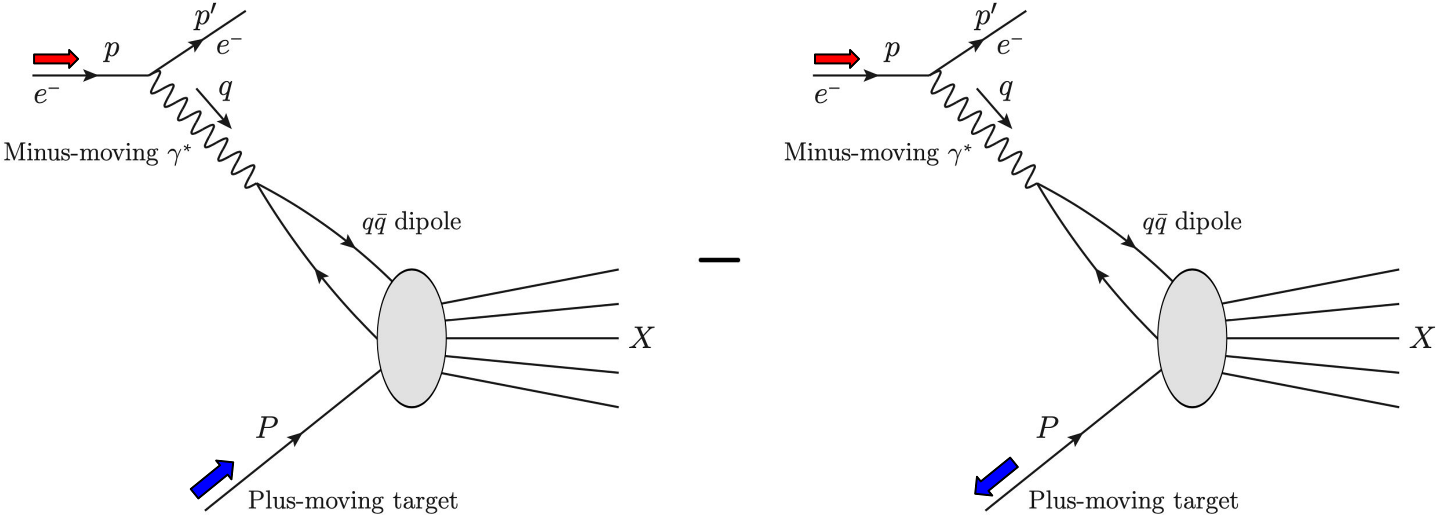
Unpolarized Dipole Amplitude

- Parton **unpolarized PDF,** $\Sigma(x, Q^2)$ and $G(x, Q^2)$, relate to **unpolarized dipole amplitude,** $S_{10}(s) = \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_{\underline{1}} V_{\underline{0}}^{\dagger}\right]\right\rangle(s)$, which obeys BFKL/BK/JIMWLK evolution.
- Quark going through the shockwave at $\underline{x}_1$: unpolarized Wilson line, .
- Multiple parton exchanges at **eikonal** level (leading order in x).



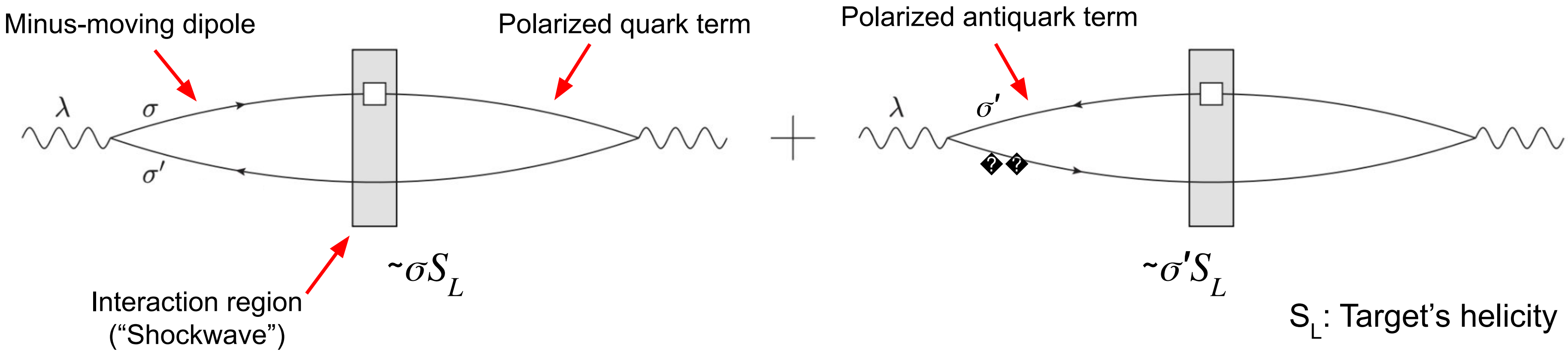
Polarized Dipole-Target Interaction

- Polarized parton PDF and structure function, $g_1(x, Q^2)$, relate to the helicity-dependent part of dipole-target scattering, which is sub-eikonal (suppressed by 1/s) compared to the leading unpolarized term.



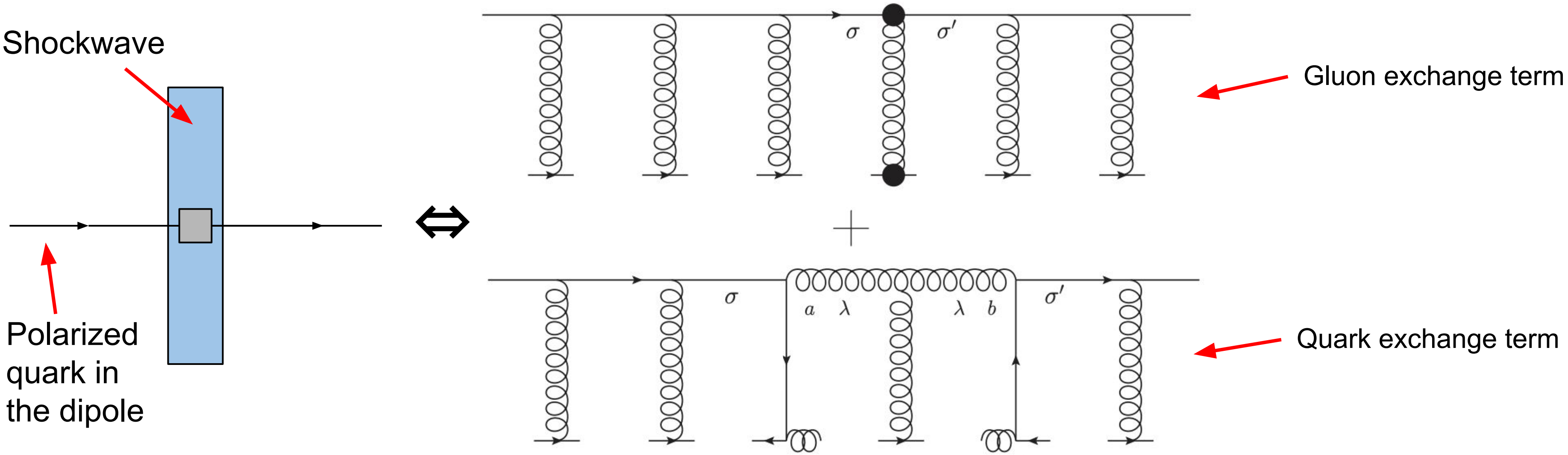
Polarized Dipole-Target Interaction

- Polarized parton PDF and structure function, $g_1(x, Q^2)$, relate to the helicity-dependent part of dipole-target scattering, which is sub-eikonal (suppressed by 1/s) compared to the leading unpolarized term.
- This leads to the generalize dipole picture for a polarized DIS.



Polarized Dipole-Target Interaction

- Helicity-dependent quark line going through the shockwave corresponds to multiple eikonal parton exchanges, except for <u>**one**</u> helicity-dependent exchange, which is **sub-eikonal** (suppressed by an extra factor of x).



Polarized Dipole-Target Interaction

- Polarized quark line also corresponds to **polarized Wilson line.**

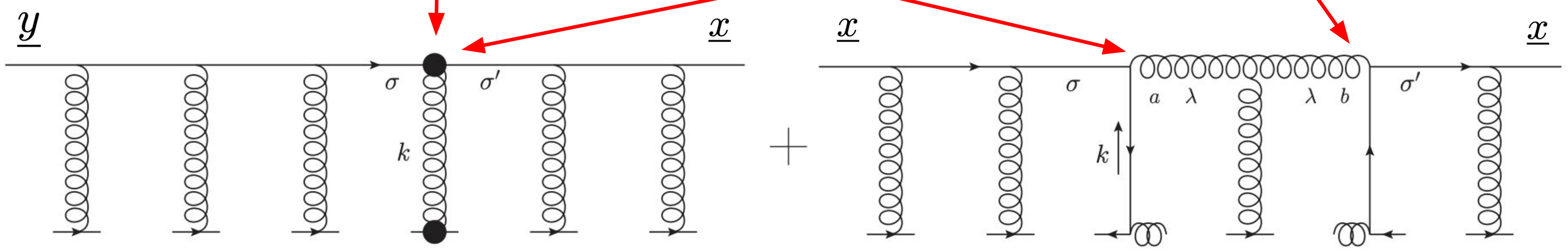
$$V_{\underline{x},\underline{y};\sigma',\sigma}\Big|_{\text{sub-eikonal}} = \sigma\delta_{\sigma,\sigma'}V_{\underline{x}}^{\text{pol}[1]}\delta^2(\underline{x}-\underline{y}) + \delta_{\sigma,\sigma'}V_{\underline{x},\underline{y}}^{\text{pol}[2]}$$

Type-1 polarized Wilson line:

$$V_{\underline{x}}^{\text{pol}[1]} = V_{\underline{x}}^{\text{G}[1]} + V_{\underline{x}}^{\text{q}[1]}$$

Type-2 polarized Wilson line:

$$V_{\underline{x},\underline{y}}^{\text{pol}[2]} = V_{\underline{x},\underline{y}}^{\text{G}[2]} + V_{\underline{x}}^{\text{q}[2]}\delta^2(\underline{x}-\underline{y})$$



Type-1 Polarized Wilson Line

- Found and studied previously by [KPS, 1610.06188, 1706.04236, 1808.09010; Chirilli, 1807.11435, 2101.12744; Altinoluk et al, 2012.03886].
- Comes with $\sigma\delta_{\sigma,\sigma'}$, and thought to be the only relevant object in helicity.
- Defines the **type-1 polarized dipole amplitude:**

$$Q(x_{10}^2, zs) = \frac{zs}{2N_c} \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) \mathrm{Re} \left\langle \mathrm{T\,tr} \left[V_{\underline{0}} V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger} \right] + \mathrm{T\,tr} \left[V_{\underline{1}}^{\mathrm{pol}[1]} V_{\underline{0}}^{\dagger} \right] \right\rangle$$

$$\left\langle \ldots \right\rangle \equiv \frac{1}{2} \sum_{S_L} S_L \, \frac{1}{2P^+V^-} \, \langle P, S_L | \ldots | P, S_L \rangle$$

$$V_{\underline{x}}^{\mathrm{pol}[1]} = V_{\underline{x}}^{\mathrm{G}[1]} + V_{\underline{x}}^{\mathrm{q}[1]}$$

Bracket: average over target's state,
but antisymmetrize over target's helicity

Type-1 Polarized Wilson Line

$$Q(x_{10}^2, zs) = \frac{zs}{2N_c} \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) \mathrm{Re} \left\langle \mathrm{T\,tr} \left[V_{\underline{0}} V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger} \right] + \mathrm{T\,tr} \left[V_{\underline{1}}^{\mathrm{pol}[1]} V_{\underline{0}}^{\dagger} \right] \right\rangle$$

$$\Big\langle \dots \Big\rangle \equiv \frac{1}{2} \sum_{S_L} S_L \, \frac{1}{2P^+V^-} \, \langle P, S_L | \dots | P, S_L \rangle$$

$$V_{\underline{x}}^{\mathrm{pol}[1]} = V_{\underline{x}}^{\mathrm{G}[1]} + V_{\underline{x}}^{\mathrm{q}[1]}$$

Target's longitudinal spin

$$V_{\underline{x}}^{\mathrm{G}[1]} = \frac{i\, g\, P^+}{s} \int\limits_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-] \, F^{12}(x^-, \underline{x}) \; V_{\underline{x}}[x^-, -\infty]$$

$\vec{\mu} \cdot \vec{B}$

$$V_{\underline{x}}^{\mathrm{q}[1]} = \frac{g^2 P^+}{2\, s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \; \left[\gamma^+ \gamma^5\right]_{\alpha\beta} \; \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty]$$

Axial current

Type-2 Polarized Wilson Line

- First studied in [Altinoluk et al, 2012.03886; Kovchegov, Santiago, 2108.03667] and introduced to helicity evolution in [2204.11898].
- Comes with $\delta_{\sigma,\sigma'}$, and previously thought not to contribute to helicity.
- The gluon exchange term, $V_{\underline{x},\underline{y}}^{\mathrm{G}[2]}$, contributes, but not the quark exchange term.
- Defines the **type-2 polarized dipole amplitude:**

$$G_2(x_{10}^2, zs) = \frac{\epsilon^{ij}(x_{10})_\perp^j}{x_{10}^2} \int d^2\left(\frac{x_0+x_1}{2}\right) \frac{zs}{2N_c} \left\langle \mathrm{tr}\left[V_{\underline{0}}^\dagger V_{\underline{1}}^{i\,\mathrm{G}[2]} + \left(V_{\underline{1}}^{i\,\mathrm{G}[2]}\right)^\dagger V_{\underline{0}}\right]\right\rangle$$

$$\left\langle \dots \right\rangle \equiv \frac{1}{2}\sum_{S_L} S_L \, \frac{1}{2P^+V^-} \, \langle P, S_L | \dots | P, S_L \rangle$$

Can be written in term of $V_{\underline{x},\underline{y}}^{\mathrm{G}[2]}$, which is the gluon exchange term in type-2 polarized Wilson line

Target's longitudinal spin

Type-2 Polarized Wilson Line

$$G_2(x_{10}^2, zs) = \frac{\epsilon^{ij} (x_{10})_\perp^j}{x_{10}^2} \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) \frac{zs}{2N_c} \left\langle \mathrm{tr} \left[V_{\underline{0}}^\dagger V_{\underline{1}}^{i\,\mathrm{G}[2]} + \left(V_{\underline{1}}^{i\,\mathrm{G}[2]} \right)^\dagger V_{\underline{0}} \right] \right\rangle$$

$$\left\langle \dots \right\rangle \equiv \frac{1}{2} \sum_{S_L} S_L \, \frac{1}{2P^+V^-} \, \langle P, S_L | \dots | P, S_L \rangle$$

Target's longitudinal spin

Can be written in term of $V_{\underline{x},\underline{y}}^{\mathrm{G}[2]}$, which is the gluon exchange term in type-2 polarized Wilson line

$$V_{\underline{z}}^{i\,\mathrm{G}[2]} = \frac{P^+}{2s} \int_{-\infty}^{\infty} dz^- \, V_{\underline{z}}[\infty, z^-] \left[D^i(z^-, \underline{z}) - \overleftarrow{D}^i(z^-, \underline{z}) \right] V_{\underline{z}}[z^-, -\infty]$$

$$V_{\underline{x},\underline{y}}^{\mathrm{G}[2]} = -\frac{i\,P^+}{s} \int_{-\infty}^{\infty} dz^- d^2 z \; V_{\underline{x}}[\infty, z^-] \, \delta^2(\underline{x} - \underline{z}) \, \overleftarrow{D}^i(z^-, \underline{z}) \, D^i(z^-, \underline{z}) \, V_{\underline{y}}[z^-, -\infty] \, \delta^2(\underline{y} - \underline{z})$$

Relations with Helicity PDFs and g_1 Structure Function

- Through an expansion in x, **helicity PDFs,** $\Delta\Sigma(x, Q^2)$ and $\Delta G(x, Q^2)$, relate to **polarized dipole amplitudes,** $Q(x_{10}^2, zs)$ (type 1) and $G_2(x_{10}^2, zs)$ (type 2) by

$$\Delta\Sigma(x,Q^2) = -\frac{N_c\,N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2},\frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\,G_2(x_{10}^2, zs)\right]$$

$$\Delta G(x,Q^2) = \frac{2N_c}{\alpha_s \pi^2} \left[\left(1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2}\right) G_2\left(x_{10}^2, zs = \frac{Q^2}{x}\right)\right]_{x_{10}^2 = \frac{1}{Q^2}}$$

- Similarly, g_1 structure function relates to both polarized dipole amplitudes by

$$g_1(x,Q^2) = -\sum_f \frac{N_c\,Z_f^2}{4\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2},\frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\,G_2(x_{10}^2, zs)\right]$$

Recap

$$\frac{1}{2} = S_q + S_G + L_q + L_G$$

$$S_q(Q^2) = \frac{1}{2} \int\limits_0^1 dx\, \Delta\Sigma(x, Q^2)$$

$$S_G(Q^2) = \int\limits_0^1 dx\, \Delta G(x, Q^2)$$

$$\Delta\Sigma(x, Q^2) = -\frac{N_c\, N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2}, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\, G_2(x_{10}^2, zs)\right]$$

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s \pi^2} \left[\left(1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2}\right) G_2\left(x_{10}^2, zs = \frac{Q^2}{x}\right)\right]_{x_{10}^2 = \frac{1}{Q^2}}$$

$$Q(x_{10}^2, zs) = \frac{zs}{2N_c} \int d^2\left(\frac{\underline{x}_0 + \underline{x}_1}{2}\right) \mathrm{Re} \left\langle \mathrm{T\,tr}\left[V_{\underline{0}} V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{T\,tr}\left[V_{\underline{1}}^{\mathrm{pol}[1]} V_{\underline{0}}^{\dagger}\right]\right\rangle$$

$$G_2(x_{10}^2, zs) = \frac{\epsilon^{ij}\, (x_{10})_\perp^j}{x_{10}^2} \int d^2\left(\frac{\underline{x}_0 + \underline{x}_1}{2}\right) \frac{zs}{2N_c} \left\langle \mathrm{tr}\left[V_{\underline{0}}^{\dagger} V_{\underline{1}}^{i\,\mathrm{G}[2]} + \left(V_{\underline{1}}^{i\,\mathrm{G}[2]}\right)^{\dagger} V_{\underline{0}}\right]\right\rangle$$

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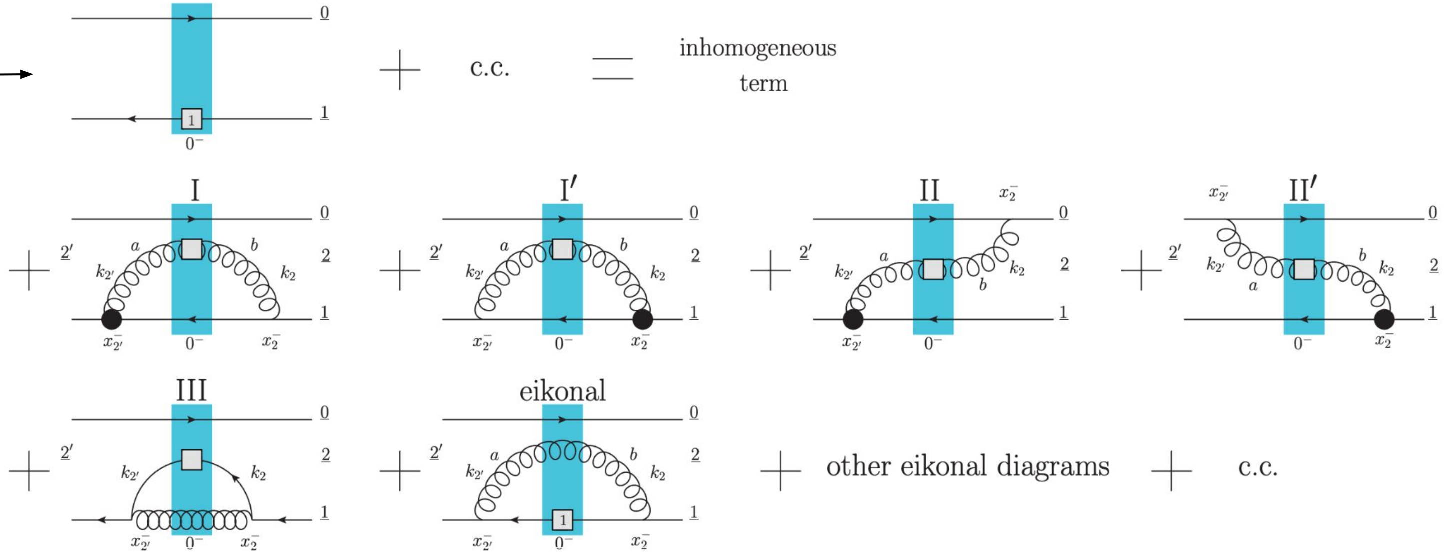
Evolution Equations

- We employ a blend of Brodsky & Lepage's LCPT and background field method inspired operator treatment. We refer to the latter as the **light-cone operator treatment (LCOT).**
- The largest contributions contain logarithmic integrals in both transverse size (UV & IR) and longitudinal momentum fraction of the daughter dipole.
- As a result, the leading contribution is at **double-logarithmic approximation (DLA),** resumming powers of $\alpha_s \ln^2(1/x)$.
- The complete single-logarithmic corrections are in progress.

Evolution Equations

- Quark (fundamental) dipole of type 1:

$Q(x_{10}^2, zs) \longrightarrow$



Evolution Equations

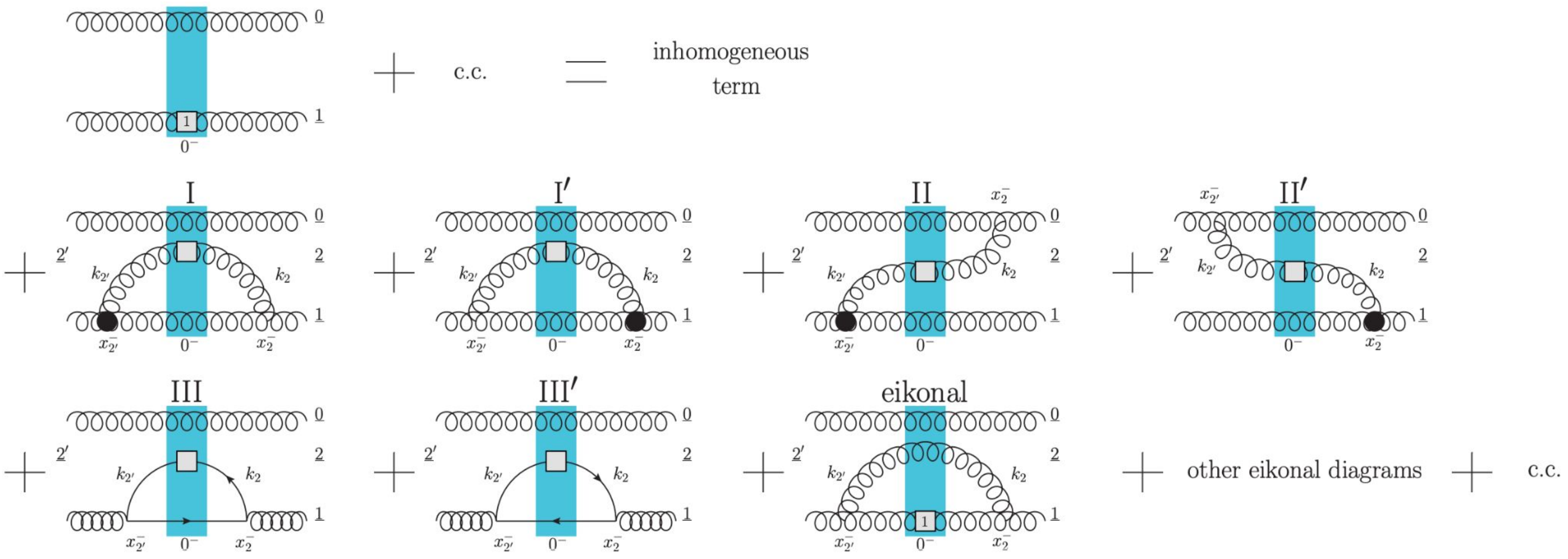
- Quark (fundamental) dipole of type 1:

$$
\begin{aligned}
&\frac{1}{2N_c}\Big\langle\!\Big\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{c.c.}\Big\rangle\!\Big\rangle(zs) = \frac{1}{2N_c}\Big\langle\!\Big\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{c.c.}\Big\rangle\!\Big\rangle_0(zs) \\
&+ \frac{\alpha_s\, N_c}{2\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2\Bigg\{\left[\frac{1}{x_{21}^2} - \frac{\underline{x}_{21}}{x_{21}^2}\cdot\frac{\underline{x}_{20}}{x_{20}^2}\right]\frac{1}{N_c^2}\Big\langle\!\Big\langle \mathrm{tr}\left[t^b V_{\underline{0}} t^a V_{\underline{1}}^\dagger\right]\left(U_{\underline{2}}^{\mathrm{pol}[1]}\right)^{ba} + \mathrm{c.c.}\Big\rangle\!\Big\rangle(z's) \\
&+ \left[2\frac{\epsilon^{ij}\, x_{21}^j}{x_{21}^4} - \frac{\epsilon^{ij}\,(x_{20}^j + x_{21}^j)}{x_{20}^2\, x_{21}^2} - \frac{2\, \underline{x}_{20}\times\underline{x}_{21}}{x_{20}^2\, x_{21}^2}\left(\frac{x_{21}^i}{x_{21}^2} - \frac{x_{20}^i}{x_{20}^2}\right)\right]\frac{1}{N_c^2}\Big\langle\!\Big\langle \mathrm{tr}\left[t^b V_{\underline{0}} t^a V_{\underline{1}}^\dagger\right]\left(U_{\underline{2}}^{i\,\mathrm{G}[2]}\right)^{ba} + \mathrm{c.c.}\Big\rangle\!\Big\rangle(z's)\Bigg\} \\
&+ \frac{\alpha_s N_c}{4\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int\frac{d^2x_2}{x_{21}^2}\Bigg\{\frac{1}{N_c^2}\Big\langle\!\Big\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{2}}^{\mathrm{pol}[1]\,\dagger}\right] U_{\underline{1}}^{ba}\Big\rangle\!\Big\rangle(z's) + 2\,\frac{\epsilon^{ij}\, x_{21}^j}{x_{21}^2}\,\frac{1}{N_c^2}\Big\langle\!\Big\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{2}}^{i\,\mathrm{G}[2]\,\dagger}\right] U_{\underline{1}}^{ba}\Big\rangle\!\Big\rangle(z's) + \mathrm{c.c.}\Bigg\} \\
&+ \frac{\alpha_s\, N_c}{2\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2\,\frac{x_{10}^2}{x_{21}^2\, x_{20}^2}\Bigg\{\frac{1}{N_c^2}\Big\langle\!\Big\langle \mathrm{tr}\left[t^b\, V_{\underline{0}}\, t^a\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right] U_{\underline{2}}^{ba}\Big\rangle\!\Big\rangle(z's) - \frac{C_F}{N_c^2}\Big\langle\!\Big\langle \mathrm{tr}\left[V_{\underline{0}}\, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger}\right]\Big\rangle\!\Big\rangle(z's) + \mathrm{c.c.}\Bigg\}
\end{aligned}
$$

Evolution equations do not close.

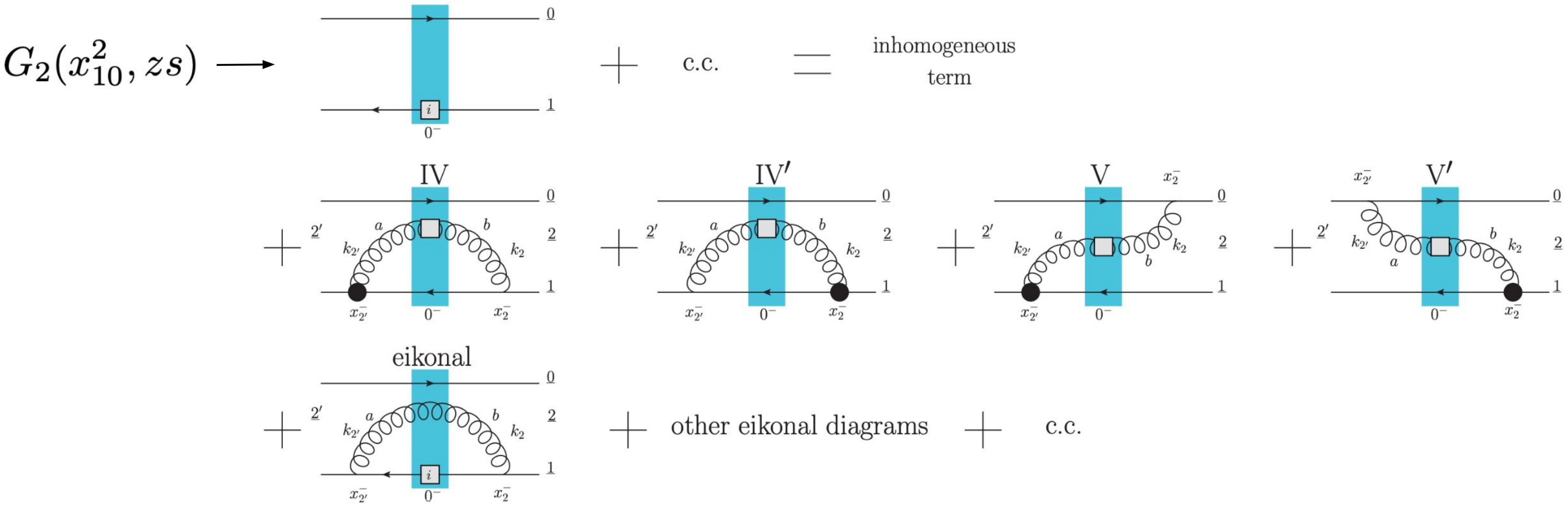
Evolution Equations

- Gluon (adjoint) dipole of type 1:



Evolution Equations

- Quark (fundamental) dipole of type 2:



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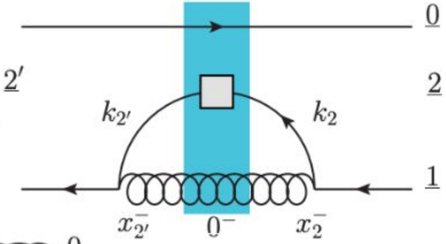
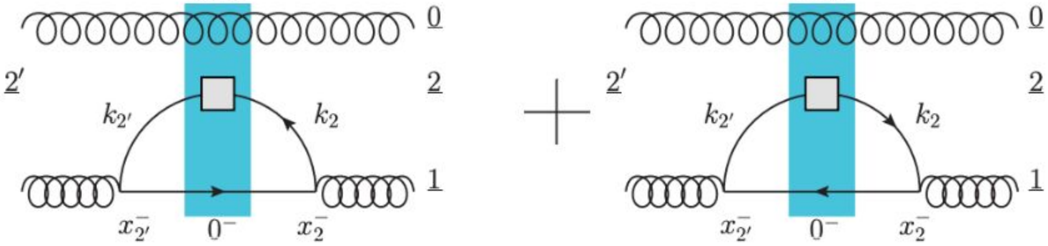
Large-N_c Limit

- For all types of dipoles, the equations do not close in general.
- Similar to BK, we obtain a closed system of equations in the **large-N_c limit:**
  - $N_c \gg 1$
  - $\alpha_s N_c \ll 1$, a fixed constant
  - Discard sub-eikonal quark-exchange terms.





Large-N_c Limit

- Define $G(x_{10}^2, zs)$ as the counterpart of $Q(x_{10}^2, zs)$, with the quark exchange term neglected.
- The equation for $G_2(x_{10}^2, zs)$ remains the same because type-2 polarized Wilson line only has gluon exchange.
- Dipole amplitudes, G and G_2, form a system of integral equations with the auxiliary **neighbor dipole amplitudes,** Γ and Γ_2.

$$\begin{pmatrix} G \\ \Gamma \\ G_2 \\ \Gamma_2 \end{pmatrix} = \begin{pmatrix} G \\ \Gamma \\ G_2 \\ \Gamma_2 \end{pmatrix}_0 + \mathcal{K} \otimes \begin{pmatrix} G \\ \Gamma \\ G_2 \\ \Gamma_2 \end{pmatrix}$$

Large-N_c Limit

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z's) + 3\, G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z's) \right]$$

Type-1 polarized dipole amplitude
(without quark exchange term)

$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z''s) + 3\, G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) + 2\, \Gamma_2(x_{10}^2, x_{32}^2, z''s) \right]$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \left[G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) \right]$$

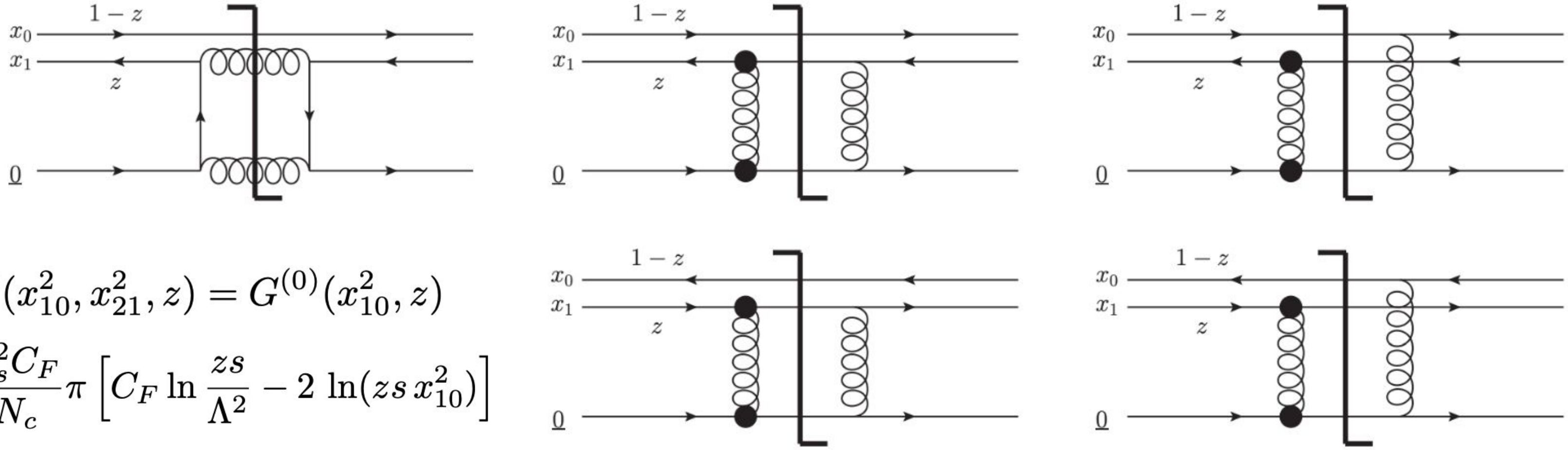
Type-2 polarized dipole amplitude

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max\left[x_{10}^2, \frac{1}{z''s}\right]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) \right]$$

Initial condition: Born-level calculation

Initial Conditions

- Given by Born-level diagrams. For type-1 dipole amplitude, we have



$$\Gamma^{(0)}(x_{10}^2, x_{21}^2, z) = G^{(0)}(x_{10}^2, z)$$
$$= \frac{\alpha_s^2 C_F}{N_c}\pi \left[C_F \ln\frac{zs}{\Lambda^2} - 2\ln(zs\, x_{10}^2)\right]$$

- Similar Born-level calculation is done for type-2 dipole amplitude, G_2 and Γ_2.

Asymptotics at Large N_c

- Generally,

$$\Delta\Sigma(x,Q^2) \sim \Delta G(x,Q^2) \sim g_1(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h}$$

- In [2204.11898], α_h is numerically computed to be approx $3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}$, which agrees with the previous work [9603204] by Bartels, Ermolaev and Ryskin (BER).
  - This is a much better agreement than what was obtained from KPS equations, which yielded α_h of $2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}$ for $\Delta\Sigma$ and g_1 and $1.88\sqrt{\frac{\alpha_s N_c}{2\pi}}$ for ΔG.

Asymptotics at Large N_c

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- More recently, the analytic expression was calculated for KPS-CTT:
  - KPS-CTT: $\alpha_h = \frac{4}{3^{1/3}}\sqrt{\mathrm{Re}\left[(-9+i\sqrt{111})^{1/3}\right]}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 3.661\sqrt{\frac{\alpha_s N_c}{2\pi}}$
  - BER (calculated by KPS in 2016): $\alpha_h = \sqrt{\frac{17+\sqrt{97}}{2}}\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 3.664\sqrt{\frac{\alpha_s N_c}{2\pi}}$

This implies that there is still a small disagreement between our result and BER.

Outline

- Introduction
- Helicity Operators and Observables at small x
- Evolution Equations
- **Closed Evolution Equations**
  - Large-N_c limit
  - **Large-N_c&N_f limit**
- Phenomenology

Large-N_c&N_f Limit

- A more realistic extension to the large-N_c limit (since the quark is included)
  - $N_c, N_f \gg 1$
  - $\alpha_s N_c \sim \alpha_s N_f \ll 1$, fixed constants
  - $\frac{N_f}{N_c} =$ constant of order 1
- The evolution equation forms a closed system of equations with 6 amplitudes.
- This equations can be solved numerically using a similar technique.

Large-N_c&N_f Limit

- The evolution equation forms a closed system of equations with 6 amplitudes.
- This equations can be solved numerically using a similar technique.

$$Q(x_{10}^2, zs) = Q^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int_{1/z's}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[2\,\widetilde{G}(x_{21}^2, z's) + 2\,\widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) + Q(x_{21}^2, z's) - \overline{\Gamma}(x_{10}^2, x_{21}^2, z's) + 2\,\Gamma_2(x_{10}^2, x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \Big] + \frac{\alpha_s N_c}{4\pi} \int_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int_{1/z's}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2} \left[Q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],$$

$$\overline{\Gamma}(x_{10}^2, x_{21}^2, z's) = Q^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int_{1/z''s}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \Big[2\,\widetilde{G}(x_{32}^2, z''s) + 2\,\widetilde{\Gamma}(x_{10}^2, x_{32}^2, z''s) + Q(x_{32}^2, z''s) - \overline{\Gamma}(x_{10}^2, x_{32}^2, z''s) + 2\,\Gamma_2(x_{10}^2, x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \Big] + \frac{\alpha_s N_c}{4\pi} \int_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int_{1/z''s}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2} \left[Q(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right],$$

$$\widetilde{G}(x_{10}^2, zs) = \widetilde{G}^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z} \frac{dz'}{z'} \int_{1/z's}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[3\,\widetilde{G}(x_{21}^2, z's) + \widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) + \left(2 - \frac{N_f}{2N_c}\right) \Gamma_2(x_{10}^2, x_{21}^2, z's) - \frac{N_f}{4N_c} \overline{\Gamma}(x_{10}^2, x_{21}^2, z's) \Big] - \frac{\alpha_s N_f}{8\pi} \int_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int_{\max\{x_{10}^2,\, 1/z's\}}^{x_{10}^2 z/z'} \frac{dx_{21}^2}{x_{21}^2} \left[Q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],$$

$$\widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) = \widetilde{G}^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int_{\max\{\Lambda^2, 1/x_{10}^2\}/s}^{z'} \frac{dz''}{z''} \int_{1/z''s}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \Big[3\,\widetilde{G}(x_{32}^2, z''s) + \widetilde{\Gamma}(x_{10}^2, x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) + \left(2 - \frac{N_f}{2N_c}\right) \Gamma_2(x_{10}^2, x_{32}^2, z''s) - \frac{N_f}{4N_c} \overline{\Gamma}(x_{10}^2, x_{32}^2, z''s) \Big] - \frac{\alpha_s N_f}{8\pi} \int_{\Lambda^2/s}^{z' x_{21}^2/x_{10}^2} \frac{dz''}{z''} \int_{\max\{x_{10}^2,\, 1/z''s\}}^{x_{21}^2 z'/z''} \frac{dx_{32}^2}{x_{32}^2} \left[Q(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right],$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\frac{z}{z'} x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\widetilde{G}(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int_{\max\left[x_{10}^2, \frac{1}{z''s}\right]}^{\frac{z'}{z''} x_{21}^2} \frac{dx_{32}^2}{x_{32}^2} \left[\widetilde{G}(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right]$$

Asymptotics at Large N_c & N_f

- For $N_f \leq 5$, we also have

$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim g_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h}$$

- Recently, a preliminary numerical computation shows that α_h decreases with N_f. However, there is a small (second decimal) disagreement with the corresponding BER intercepts with quarks.
- Possible explanations for the remaining disagreement are under investigation.

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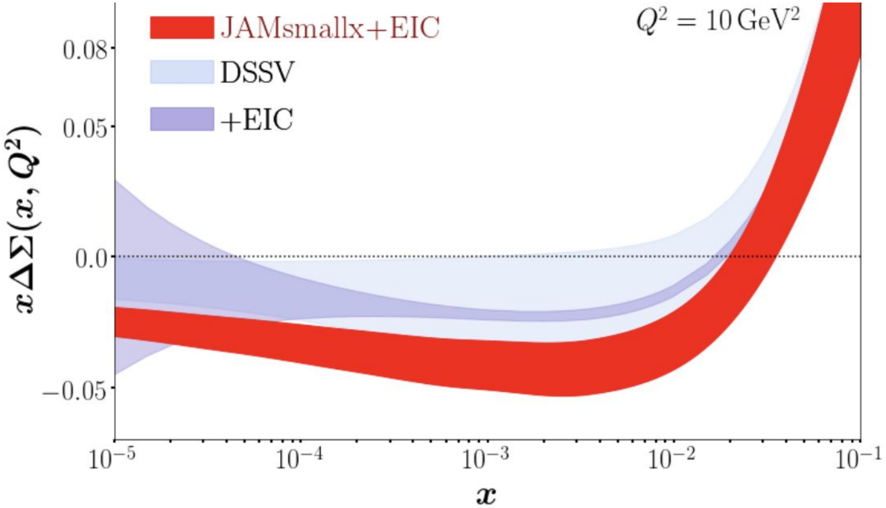
Fits to Polarized DIS Data

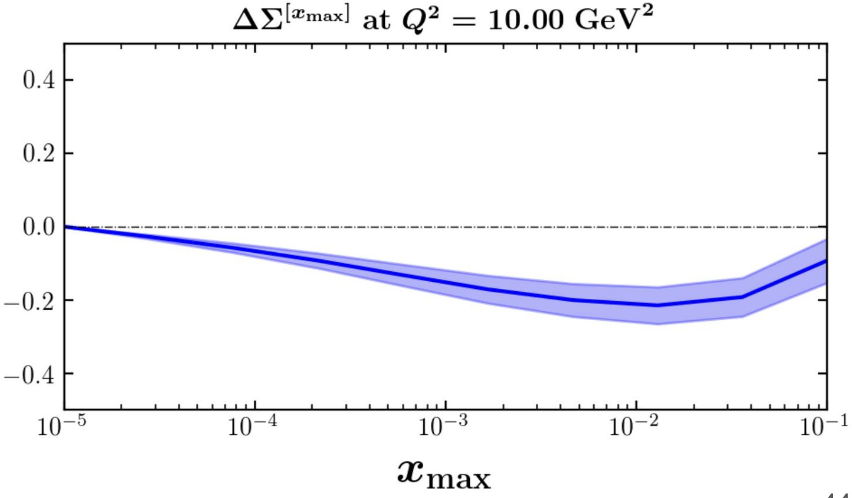
- In [2102.06159], the fit was performed by D. Adamiak, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert and Y. Kovchegov with polarized DIS data using the large-N_c limit of KPS equations (without the recent corrections).
  - It worked well, with $\chi^2/Npts$ = 1.01 ($\chi^2/Npts$ = 1.07 for JAM16)
  - Small-x evolution starts at x_0= 0.1 (Unpolarized BK/JIMWLK evolution starts around x_0= 0.01.)
  - Our approach fails at larger x as expected (x_0= 0.3 gives $\chi^2/Npts$ = 4.75).
- An updated version with the complete KPS-CTT equations at large N_c&N_f is in progress. This work includes both polarized DIS and SIDIS data.

Fits to Polarized DIS Data

- The KPS evolution is able to constraint e.g. the quark spin at small x.
- Potentially negative 10-20% of the proton spin carried by small-x quarks.

Warning: this is preliminary. An updated version with the complete KPS-CTT equations at large N_c&N_f is in progress.

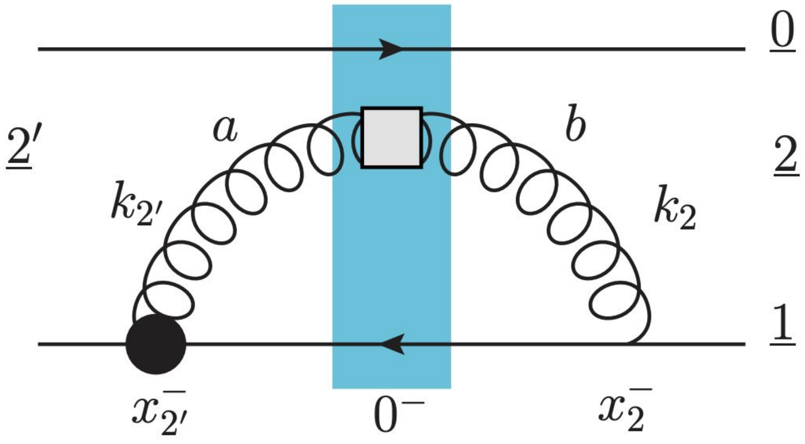




Conclusion

- New evolution equations in x for the quark and gluon helicity distributions have been constructed (and corrected).
- These equations have now been studied at large N_c, yielding the small-x asymptotics of $\Delta\Sigma(x, Q^2)$ and $\Delta G(x, Q^2)$. The study of the large-N_c&N_f equations and OAM distributions are under way.
- First successful fit of polarized world DIS data for x < 0.1 was done using solely the small-x helicity evolution (old KPS version of evolution). There is a clear possibility of a significant amount of proton spin to be found at small x.
- More precise and comprehensive phenomenology to come in the future (helicity+OAM), in preparation for EIC, with the aim of resolving the small-x part of the proton spin puzzle.

Neighbor Polarized Dipole Amplitudes (Γ and Γ_2)



Cougoulic, Kovchegov, Tarasov, Tawabutr, 2204.11898

- At large N_c, this diagram contains a dipole at x_{20} and a dipole at x_{21}, one of which is polarized.
- The DLA limit requires $x_{21} << x_{10} \sim x_{20}$.
- In the contribution where the x_{20}-dipole is polarized, the dipole has transverse separation x_{20}, but its lifetime is $z' x_{20}^2$, which is much smaller than $z' x_{21}^2$.
- Such the dipole must "know about" both x_{20} and x_{21}. Hence, it is different from a normal polarized dipole.