Off-shell GPDs and form factors of the pion

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Research with **Vanamali Shastry** and **Enrique Ruiz Arrriola**, see [arXiv:2211.11067]

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[see also talks by Wagner, Kunne, Martinez-Fernandez, Passek-Kumericki, Cichy, Fazio]

GPD of the pion

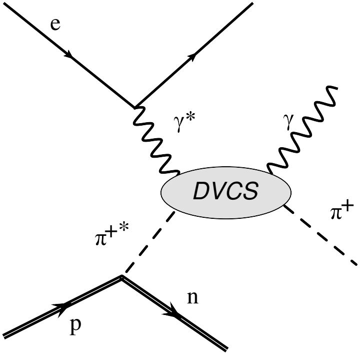
Why the pion?

- Simplest and most fundamental hadron – pseudo-Goldstone boson of the spontaneously broken chiral symmetry
- Simple theoretically – there are approaches working in the non-perturbative regime
- Easier than p on the lattice
- (Indirect) experimental prospects

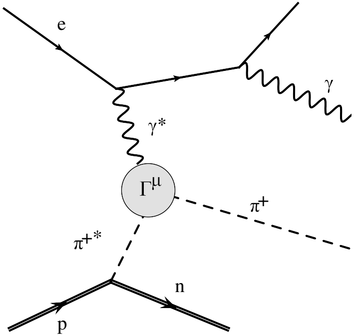
Sullivan process $e\,p \to e\,n\,\pi^+\gamma$

[Sullivan 1972, Shakin+Sun 1994, Aguilar et al. 2019, Chávez et al. 2021, Morgado et. al 2022]

DVCS

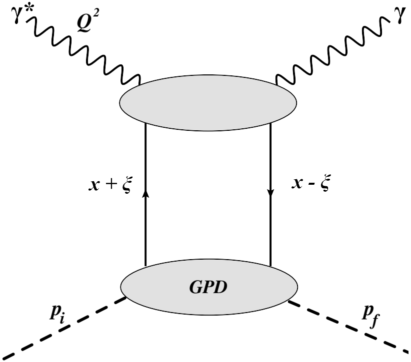


Bethe-Heitler



Intermediate pion is off-shell, interference of DVCS with BH, EIC

DVCS and GPD



Two amplitudes:
$I = 1$ (isovector, symmetric in x, valence, non-singlet) and
$I = 0$ (isoscalar, antisymmetric, valence+sea, singlet)

$$\delta_{ab}\delta_{\alpha\beta}H^0(x,\xi,t,p_i^2,p_f^2) + i\epsilon^{abc}\tau^c_{\alpha\beta}H^1(x,\xi,t,p_i^2,p_f^2) = \int \frac{dz^-}{4\pi} e^{ix\,P^+z^-} \langle \pi^b(p_f)|\overline{\psi}_\alpha(-\tfrac{z}{2})\gamma^+\psi_\beta(\tfrac{z}{2})|\pi^a(p_i)\rangle\Big|_{\substack{z^+=0\\ z^\perp=0}}$$

(in the light-cone gauge)

... gluons ...

Dependence on p_i^2 and p_f^2, off-shellness if either is not m_π^2

Off-shellness

- Up to now no firm assessment of the off-shellness effects in the Sullivan process
- Ignored or assumed negligible at $|p_f^2 - p_i^2| < 0.6 \text{ GeV}^2$ based on a specific model (rainbow diagram resummation [Qin et al. 2017])
- We argue here that the off-shell effects in pion GPDs are larger, of the order of

$$\sim |p_f^2 - p_i^2|/m_\rho^2$$

Formal features of off-shell GPD

Off-shell GPDs and polynomiality

$P^\mu = \frac{1}{2}(p_f^\mu + p_i^\mu), \quad q^\mu = p_f^\mu - p_i^\mu, \quad \xi = -\frac{q^+}{2P^+} \text{ (skewness)}, \quad t = q^2$

For $p_i^2 = p_f^2$, crossing (time-reversal) makes $H^{0,1,g}$ even functions of ξ. This no longer holds if $p_i^2 \neq p_f^2$, i.e., with a virtual pion

$\rightarrow$ x-moments of the GPDs involve also odd powers of ξ and

... polynomiality takes the form

$$\int_{-1}^{1} dx\, x^j H^s(x,\xi,t,p_i^2,p_f^2) = \sum_{k=0}^{j+1} A_{jk}^s(t,p_i^2,p_f^2)\xi^i, \quad s = 0,1,g$$

$A_{jk}^s(t,p_i^2,p_f^2)$ – (off-shell generalized) form factors

Off-shell form factors

- Vector (EM) ff:

$$\int_{-1}^{1} dx\, H^1 = 2(F - G\xi)$$

- Gravitational ff:

$$\int_{-1}^{1} dx\, x[H^0 + H^g] = \theta_2 - \theta_3\xi - \theta_1\xi^2$$

The above ff are functions of (t, p_i^2, p_f^2) and are independent of the factorization scale μ, as they correspond to conserved currents

- Higher rank (generalized) ff:

... (depend on μ)

Ward-Takahashi identities and ff identities

EM vertex

$$\Gamma^\mu(p_i,p_f) \equiv \langle \pi^+(p_f)|J^\mu(0)|\pi^+(p_i)\rangle = 2P^\mu F(t,p_i^2,p_f^2) + q^\mu G(t,p_i^2,p_f^2)$$

WTI: $q_\mu \Gamma^\mu(p_i,p_f) = \Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2)$, where $\Delta(p^2)$ – (full) pion propagator

[Nishijima+Singh 1967, Naus+Koch 1989, Rudy+Fearing+Scherer 1994, Choi et al. 2019]

$$(p_f^2 - p_i^2)F(t,p_i^2,p_f^2) + tG(t,p_i^2,p_f^2) = \Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2)$$

G expressible via F!

$$G(t,p_i^2,p_f^2) = \frac{(p_f^2 - p_i^2)}{t}\left[F(0,p_i^2,p_f^2) - F(t,p_i^2,p_f^2)\right]$$

$G(0,p_i^2,p_f^2) = (p_i^2 - p_f^2)dF(t,p_i^2,p_f^2)/dt|_{t=0}$

Also $F(0,m_\pi^2,p^2) = F(0,p^2,m_\pi^2) = \frac{\Delta^{-1}(p^2)}{(p^2 - m_\pi^2)}$, and $F(0,m_\pi^2,m_\pi^2) = 1$ – charge normalization

Energy-momentum tensor vertex

$$\Gamma^{\mu\nu}(p_i,p_f) \equiv \langle \pi^+(p_f)|\Theta^{\mu\nu}(0)|\pi^+(p_i)\rangle = \tfrac{1}{2}[(q^2 g^{\mu\nu} - q^\mu q^\nu)\theta_1 + 4P^\mu P^\nu \theta_2 + 2(q^\mu P^\nu + q^\nu P_\mu)\theta_3 - g^{\mu\nu}\,\theta_4]$$

WTI: $q_\mu \Gamma^{\mu\nu}(p_i,p_f) = p_i^\nu \Delta^{-1}(p_f^2) - p_f^\nu \Delta^{-1}(p_i^2)$

[Brout+Englert 1966, K. Raman 1971]

→ (new) relations

$$(p_f^2 - p_i^2)\theta_2 + t\theta_3 = \Delta^{-1}(p_f^2) - \Delta^{-1}(p_i^2), \qquad (p_f^2 - p_i^2)\theta_3 - \theta_4 = -[\Delta^{-1}(p_f^2) + \Delta^{-1}(p_i^2)]$$

$$\theta_3(t,p_i^2,p_f^2) = \frac{(p_f^2 - p_i^2)}{t}\left[\theta_2(0,p_i^2,p_f^2) - \theta_2(t,p_i^2,p_f^2)\right]$$
$$\theta_4(t,p_i^2,p_f^2) = (p_f^2 - p_i^2)\theta_3(t,p_i^2,p_f^2) + \Delta^{-1}(p_f^2) + \Delta^{-1}(p_i^2)$$

$\theta_4 = 0$ if both the initial and final pions are on mass shell. Does not contribute to the x-moment upon the light-cone projection, as $n_\mu g^{\mu\nu} n_\nu = n^2 = 0$, with $n^\mu = (1,0,0,-1)/P^+$

Energy-momentum tensor vertex (2)

In addition:

Vector-gravitational relation at $t=0$

$\theta_2(0,p_i^2,p_f^2) = F(0,p_i^2,p_f^2)$

$\theta_2(0,m_\pi^2,p^2) = \theta_2(0,p^2,m_\pi^2) = \frac{\Delta^{-1}(p^2)}{(p^2-m_\pi^2)}$
$\theta_2(0,m_\pi^2,m_\pi^2) = 1$ (momentum sum rule)

θ_1 (D-term/pressure), corresponding to a transverse tensor, does not enter any constraints from current conservation

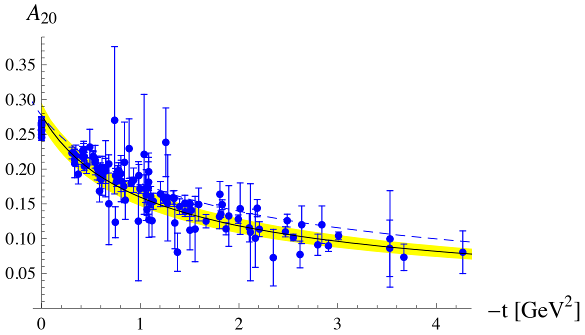
In the chiral limit and on-shell, $m_\pi^2 = 0$, one has the low-energy theorem $\theta_1(0,0,0) = \theta_2(0,0,0)$ [Donoghue+Leutwyler 1991]

Intermediate summary

(up to now completely general)

- GPDs are off-shell if connected to a virtual hadrons (here pion)
- Current conservation enforces nontrivial constraints on off-shell GPDs via WTI

BTW, there exists info on the on-shell gravitational ff of the pion from the lattice $\theta_2(t) \equiv A_{20}(t)$ – quark part → data: [Brommel 2007]



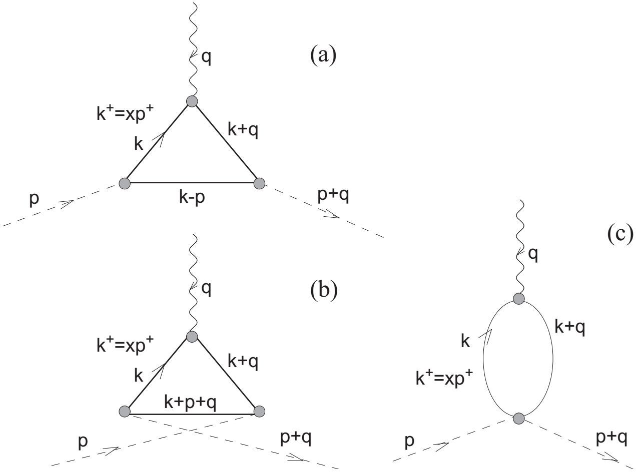
Distribution of mass is more compact than for charge, with $\langle r^2 \rangle_M \simeq \frac{1}{2}\langle r^2 \rangle_Q$ [WB+ERA 2008]

(θ_1 is noisy)

A non-perturbative model illustration

Non-perturbative approaches: models based on chiral symmetry breaking (Nambu–Jona-Lasinio, instanton liquid), Dyson-Schwinger rainbow diagram resummation, ...

One-quark loop

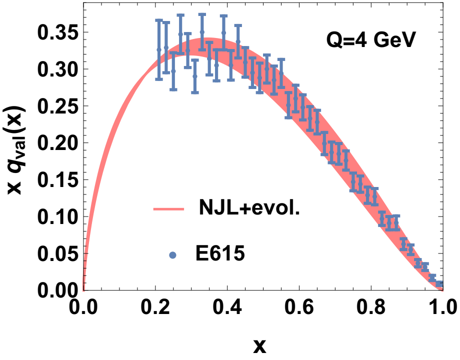


- Non-perturbative modeling
- Leading N_c
- Massive quarks ($M \sim 300$ MeV) due to the chiral symmetry breaking
- Obtained at a low quark-model scale
- All formal constraints satisfied (support, polynomiality, (EM) gauge, positivity...)
- Amended with the QCD evolution provides surprisingly good phenomenology (PDF, DA), existing predictions for GPD, TDA, quasi ...)

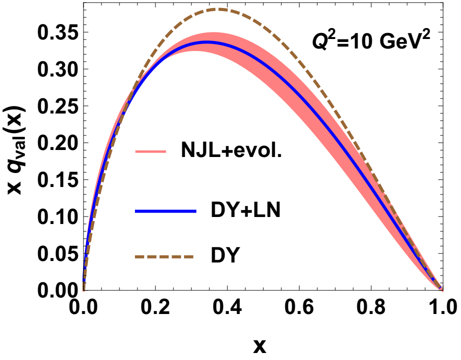
Quark PDF of the pion in the NJL-like models

[Davidson+Arriola 1995, ...]

low-energy quark model initial condition at the scale ~ 320 MeV + DGLAP evolution



E615 [Conway et al. 1989]



E615+NA10+HERA [Barry et al. 2018]

Half-off-shell form factors in the spectral quark model

SQM [ERA+WB] - a way of putting in the vector meson dominance into the quark model. All analytic, but half-off-shell form factors in the chiral limit are manageably simple:

$$F(t,p^2,0) = \frac{M_V^4}{(M_V^2-p^2)(M_V^2-t)}, \qquad G(t,p^2,0) = \frac{p^2 M_V^2}{(M_V^2-p^2)(M_V^2-t)}$$

$$\theta_1(t,p^2,0) = \frac{M_V^2\left[\frac{p^2(t-p^2)}{M_V^2-p^2} + (t-2p^2)L\right]}{(t-p^2)^2}, \qquad \theta_2(t,p^2,0) = \frac{M_V^2\left[\frac{p^2(p^2-t)}{M_V^2-p^2} + tL\right]}{(t-p^2)^2}$$

$$\theta_3(t,p^2,0) = \ldots \qquad \theta_4(t,p^2,0) = \ldots$$

with $L = \log\frac{M_V^2-p^2}{M_V^2-t}$ and M_V being the ρ meson mass

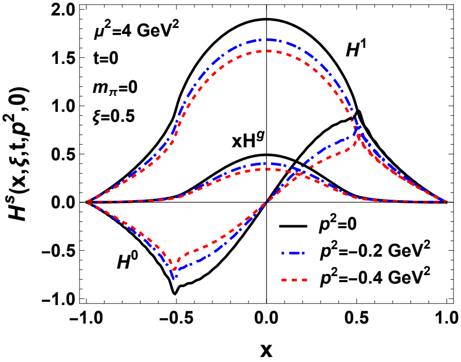
All ff relations matched! Generally, no factorization of t and off-shellness p^2

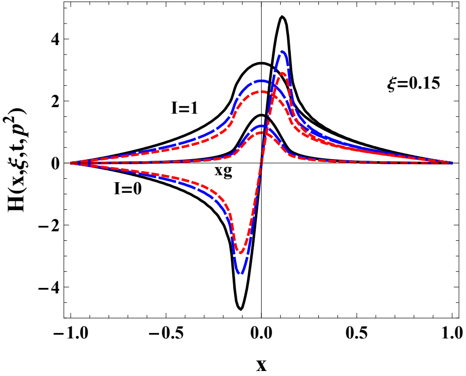
At low t off-shellness effects $\sim p^2/M_V^2$

Back to GPDs

Half-of-shell GPDs from SQM + evolution

Expressions at the quark model scale are analytic, no factorization in x, ξ, t, or p^2!
Evolved to $4\ \mathrm{GeV}^2$ with DGLAP-ERBL [code from Golec-Biernat+Martin 1998]



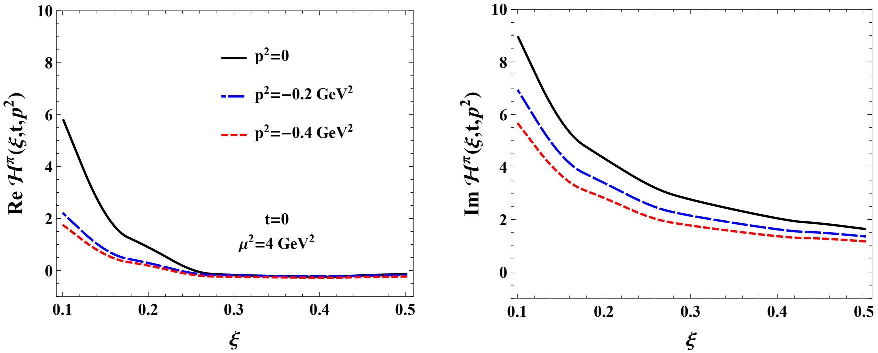


Quantitative assessment of p^2 effects

For on-shell pion GPD predictions, see [WB, ERA, Golec-Biernat 2007]

Half-off-shell Compton form factor

$$\mathcal{H}_{\pi^+}(\xi,t,p^2) = \sum_{q=u,\bar{d}} e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q(x,\xi,t,p^2)$$

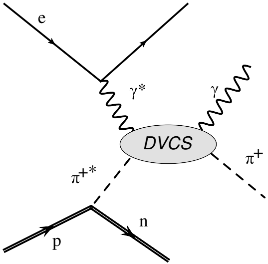


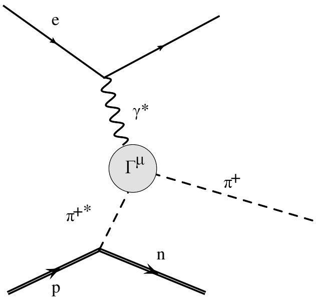
NLO effects relevant [Moutarde+Pire+Sabatie+Szymanowski+Wagner 2013], gluons dominant [Morgado et al. 2022]

Off-shell pion propagator

Off-shellness in the pion propagator

Effects must be treated together:





In SQM for $(m_\pi = 0)$, the correction to the pole term in the propagator is

$$\Delta(p^2) = \frac{M_V^2 - p^2}{M_V^2} \frac{1}{p^2} = \frac{1}{p^2} - \frac{1}{M_V^2}$$

BTW, there are attempts to extract the G ff from TJLAB data in [Choi et al. 2019], with $-p^2$ up to $0.36\ \mathrm{GeV}^2$

Conclusions

Conclusions

- The virtual pion is off shell (amplitudes, propagator) – care needed!
- GPDs are not even functions of the skewness $\xi \to$ new set of form factors
- Off-shell effects show in GPDs, form factors, propagators $\to$ contribute to more uncertainty in extraction of (on-shell) GPDs from the data, in addition to excited states or $G_{\pi NN}$
- Lattice QCD? (ambiguity of the inerpolating current, full process on the lattice distant)

Message to take home

Off-shellness effects $\sim |p_f^2 - p_i^2|/\Lambda^2 \sim |p_f^2 - p_i^2|/(0.5 \text{ GeV}^2)$

Thank you!