# Generalized Parton Distributions from Lattice QCD

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OPUS grant No. 2021/43/B/ST2/00497 (2022-2026)

**Outline:**

Introduction

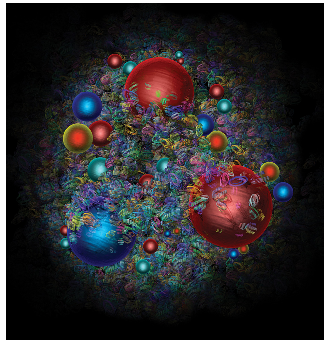
GPDs from lattice:
- how to access
- twist-2 GPDs
- twist-3 GPDs

Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, S. Bhattacharya, M. Constantinou, J. Dodson, X. Gao, K. Hadjiyiannakou, K. Jansen, A. Metz, J. Miller, S. Mukherjee, A. Scapellato, F. Steffens, Y. Zhao

# Generalized parton distributions (GPDs)



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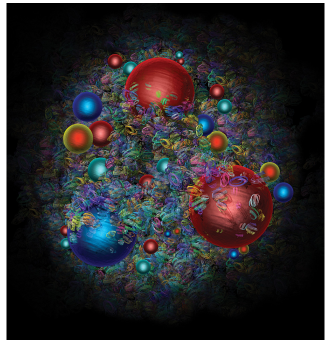

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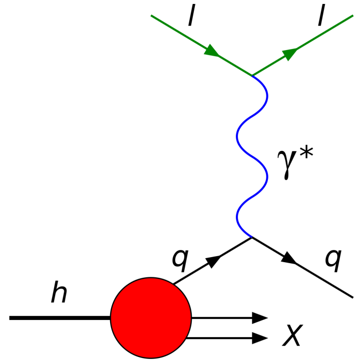
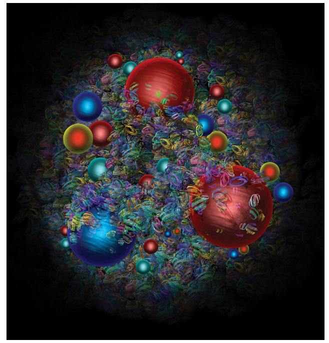
- related to matrix elements with same incoming/outgoing hadron state,
- probed in deep inelastic scattering (DIS) – $ep \longrightarrow eX$.





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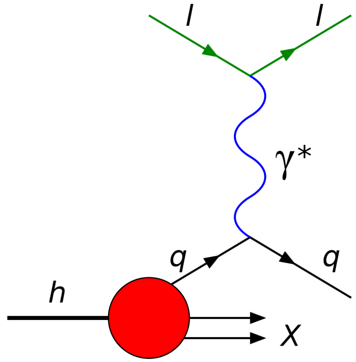


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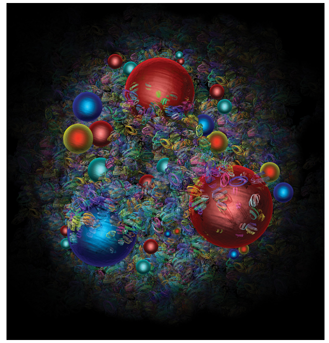

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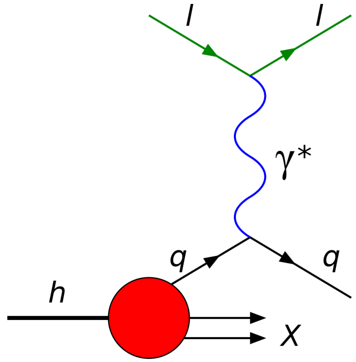


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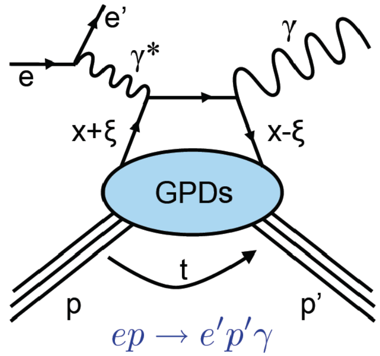
Adding momentum transfer is a natural generalization, leading to **generalized parton distributions** (GPDs):

- experimentally, require exclusive processes like deeply virtual Compton scattering (DVCS) – $ep \longrightarrow e'p'\gamma$,
- reflect spatial distribution of partons in the transverse plane,
- contain information on mechanical properties of hadrons,
- wealth of information on the hadron spin,
- reduce to PDFs in the forward limit, e.g. $H(x,0,0) = q(x)$,
- moments of GPDs are form factors, e.g. $\int dx H(x,\xi,t) = F_1(t)$.



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     typical lattice parameters:
     $L/a = [32, 96]$, $a \in [0.04, 0.15]$ fm, $m_\pi \in [135, 500]$ MeV
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     Monte Carlo simulations to evaluate the discretized path integral
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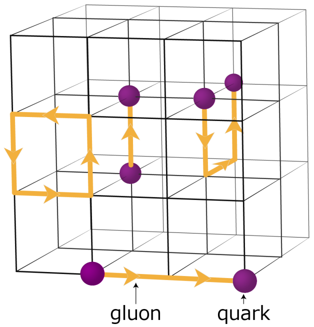

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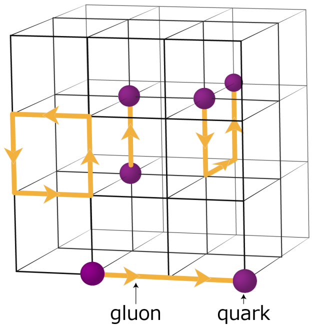


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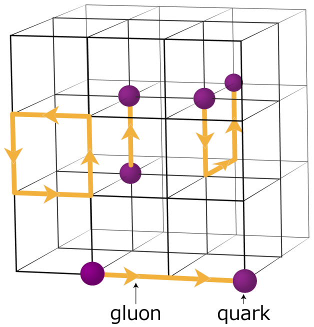


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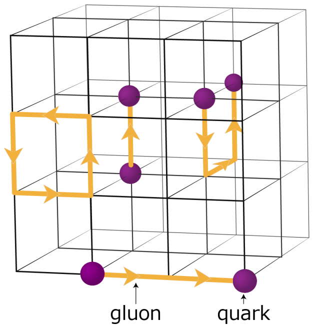


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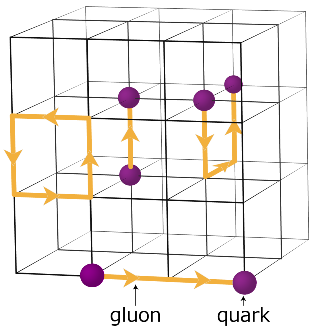


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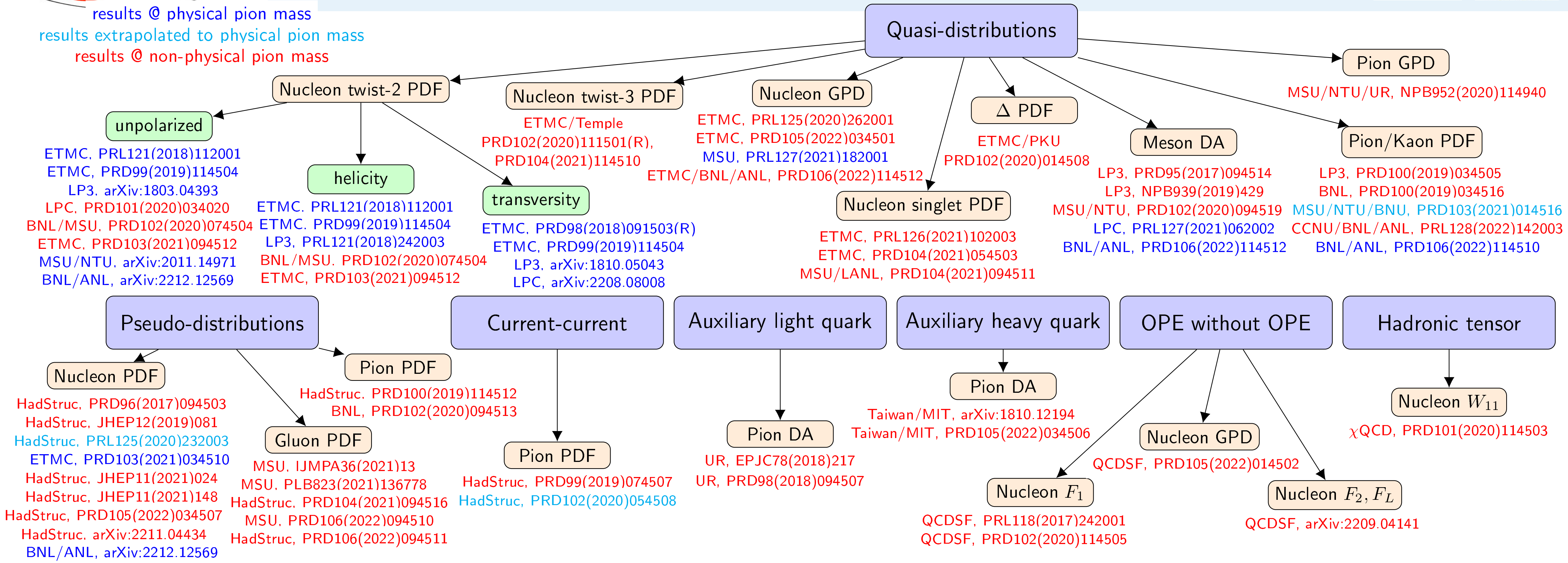
  4. A lot of computing time!
  5. Ingenious analysis techniques, with inputs from perturbation theory.

# Lattice PDFs/GPDs: dynamical progress



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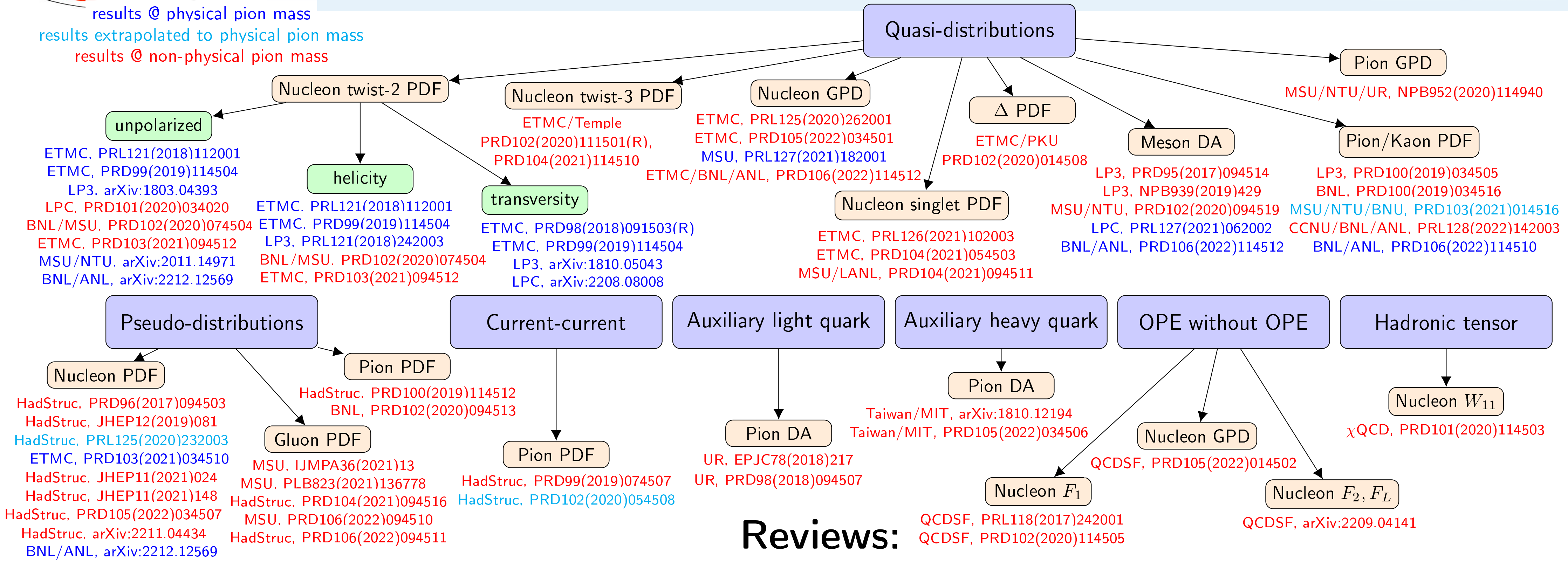


## Reviews:


K. Cichy, *Progress in $x$-dependent partonic distributions from lattice QCD*, plenary talk LATTICE 2021, 2110.07440

K. Cichy, *Overview of lattice calculations of the $x$-dependence of PDFs, GPDs and TMDs*, plenary talk of Virtual Tribute to Quark Confinement 2021, 2111.04552

K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, invited review for a special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD* (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

X. Ji et al., *Large-Momentum Effective Theory*, Rev. Mod. Phys. 93 (2021) 035005

M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, PPNP 121 (2021) 103908

# Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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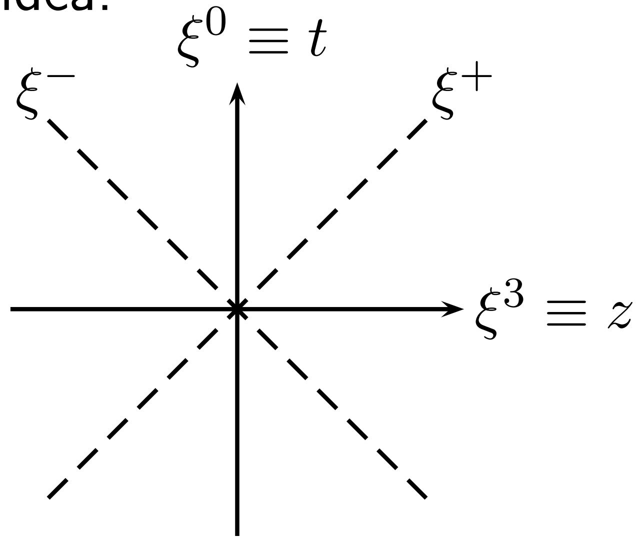

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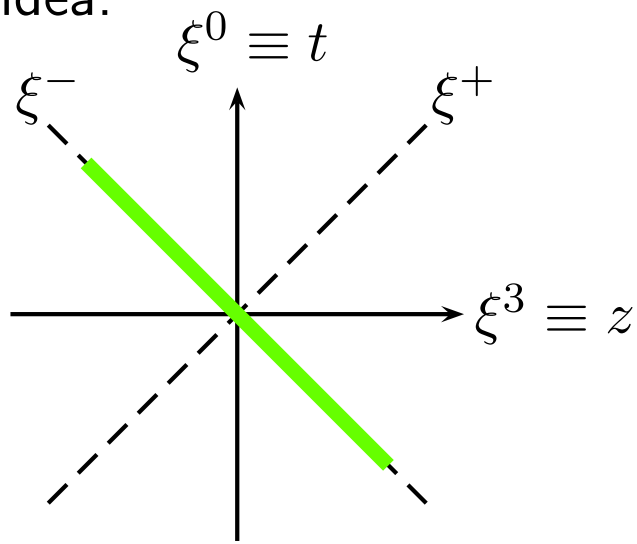


Correlation along the $\xi^-$-direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

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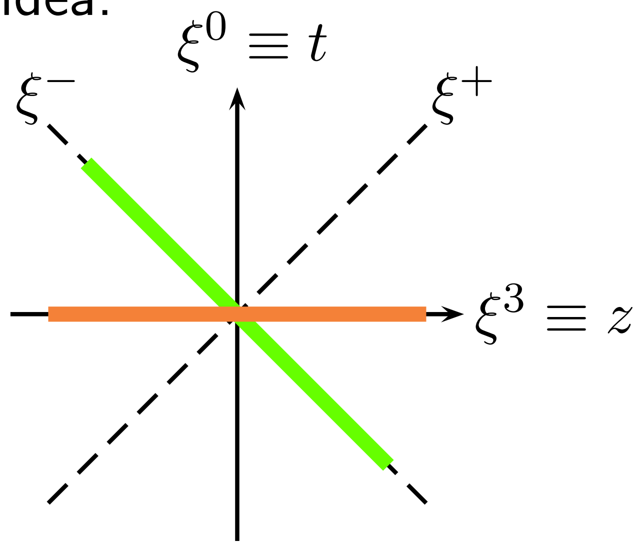


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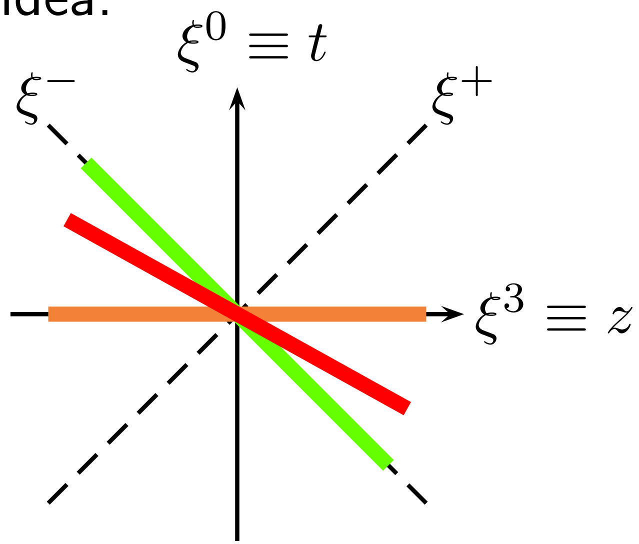


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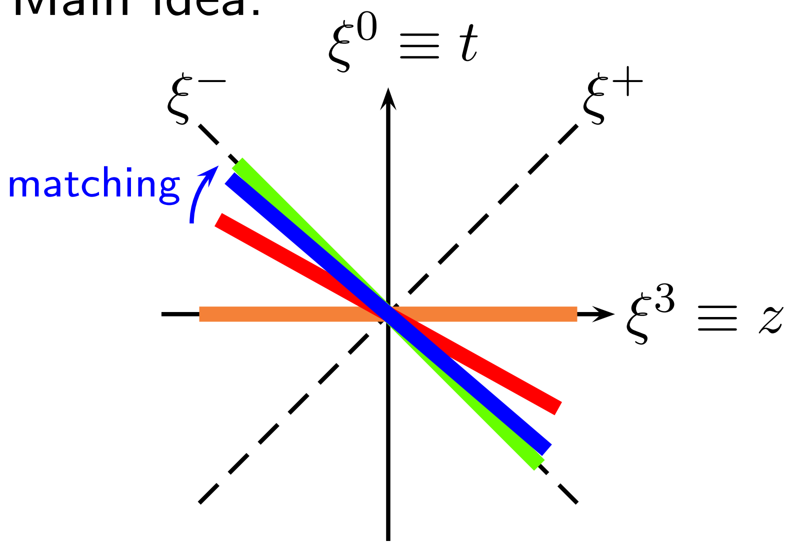


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Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1}\frac{dy}{|y|}\,C\left(\frac{x}{y},\frac{\mu}{P_3}\right)q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF — pert.kernel — PDF — higher-twist effects

# Quasi-GPDs lattice procedure



**spatial correlation in a boosted nucleon**

$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$

$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer

lattice computation of bare ME

↓

renormalization
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intermediate RI scheme

↓

reconstruction of $x$-dependence
$z$-space→$x$-space
Backus-Gilbert

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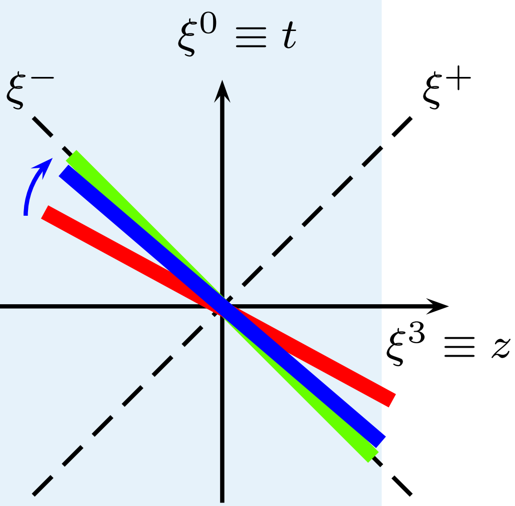

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non-trivial aspect: reconstruction of a continuous distribution from a finite set of ME ("inverse problem")

↓

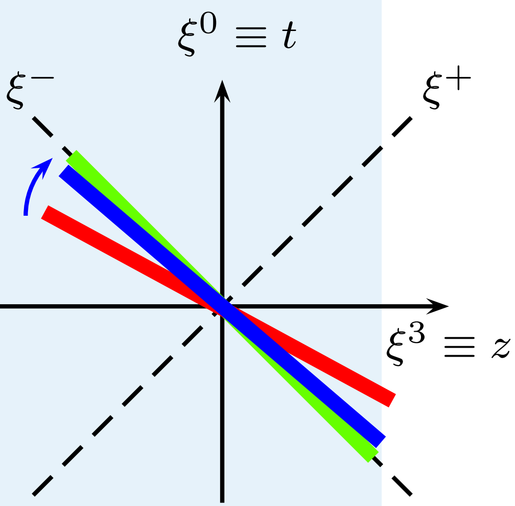
matching to light cone
RI$\rightarrow$ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

needs a sufficiently large momentum
valid up to higher-twist effects

↓

**light-cone GPD**

**the final desired object!**



## Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L \approx 3$ fm,
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Twist-2 unpolarized+helicity GPDs ETMC, Phys. Rev. Lett. 125 (2020) 262001
Twist-2 transversity GPDs ETMC, Phys. Rev. D105 (2022) 034501
Twist-3 axial GPDs S. Bhattacharya et al. (ETMC/Temple), 2112.05538 + in preparation
Twist-2 unpolarized GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) Phys. Rev. D106 (2022) 114512 + in preparation

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## Kinematics:

- three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV,
- momentum transfers: $-t \leq 2.76$ GeV$^2$, most data: $-t = 0.69$ GeV$^2$,
- skewness: $\xi = 0, 1/3$.

Twist-2 unpolarized+helicity GPDs ETMC, Phys. Rev. Lett. 125 (2020) 262001
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# Bare matrix elements

Lattice matrix elements need to be computed with 2 different projections (unpolarized/polarized).
Below for the unpolarized Dirac insertion (for unpolarized GPDs)

**spatial correlation in a boosted nucleon**
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
of bare ME
intermediate RI scheme

↓

reconstruction of $x$-dependence
$z$-space$\to$$x$-space
Backus-Gilbert

↓

matching to light cone
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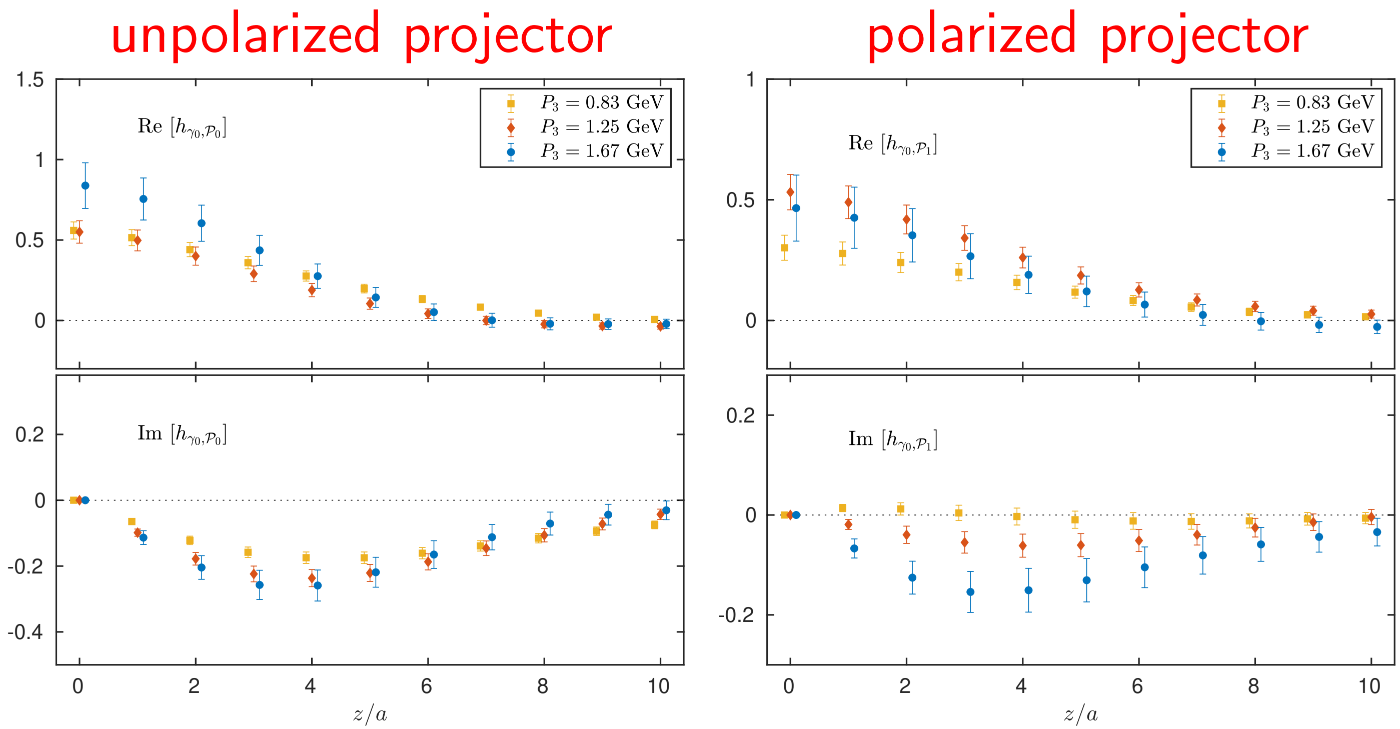


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Zero skewness: $\xi = 0$

ETMC, Phys. Rev. Lett. 125 (2020) 262001

# Disentangled renormalized matrix elements

Removal of divergences and disentangling of $H$- and $E$-GPDs.
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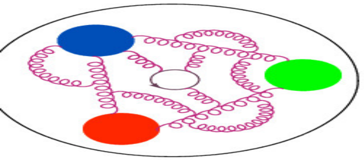

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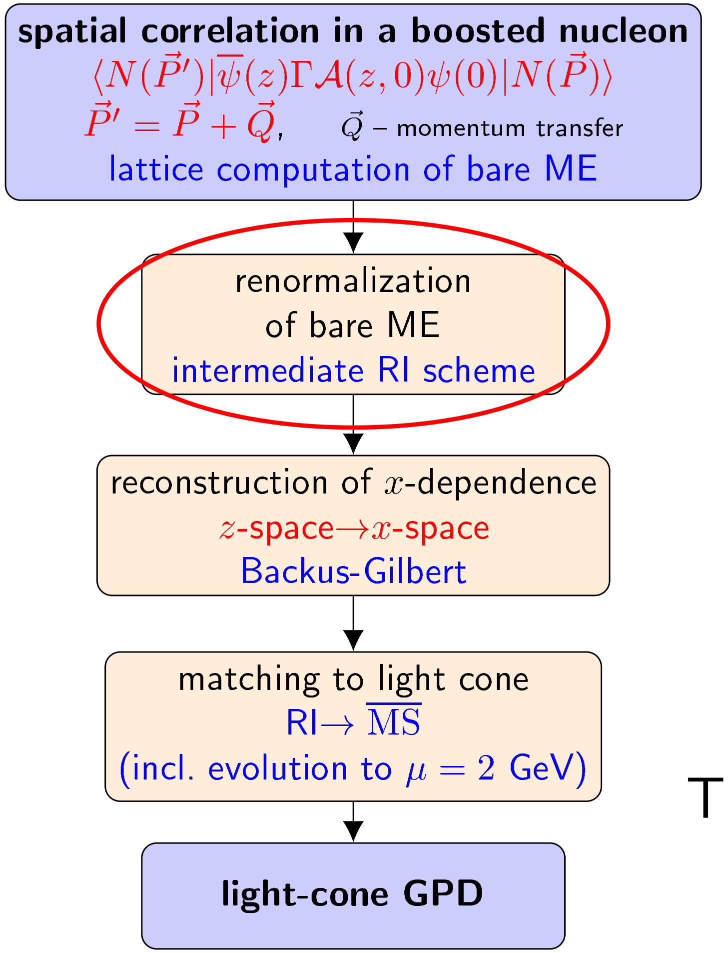
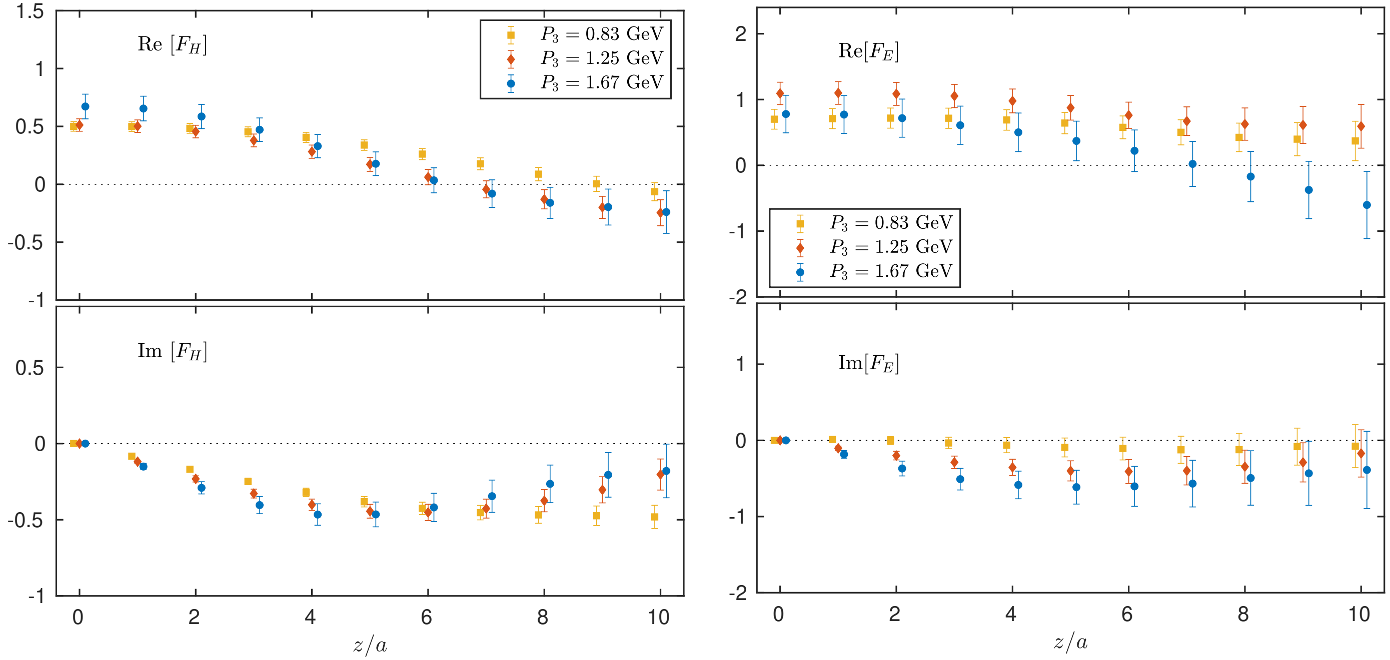



ME of $H$-function　　ME of $E$-function

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ETMC, Phys. Rev. Lett. 125 (2020) 262001

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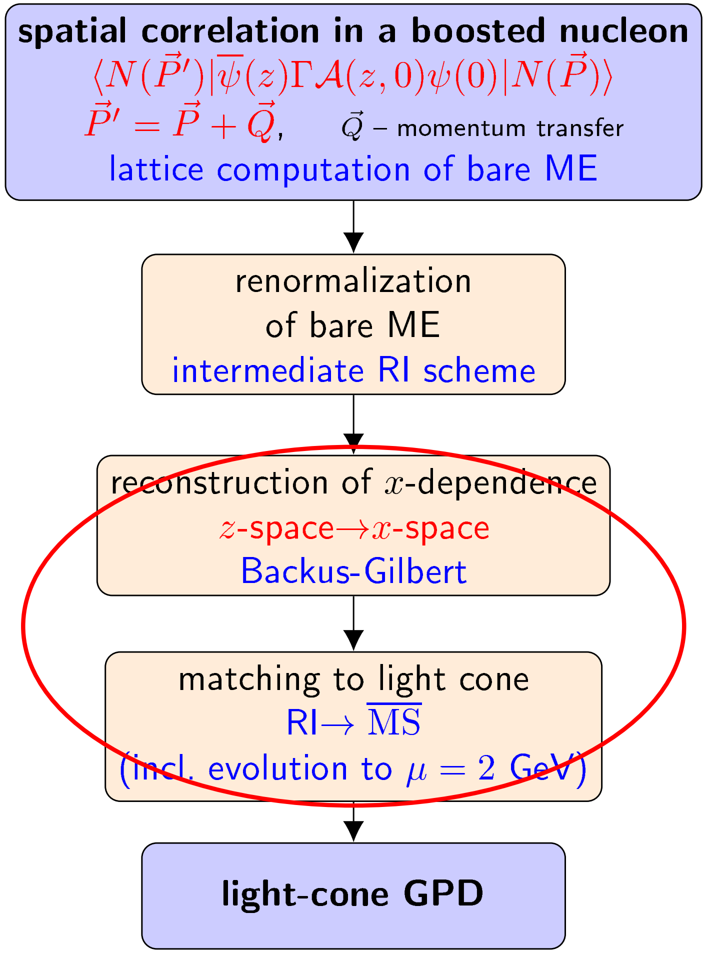

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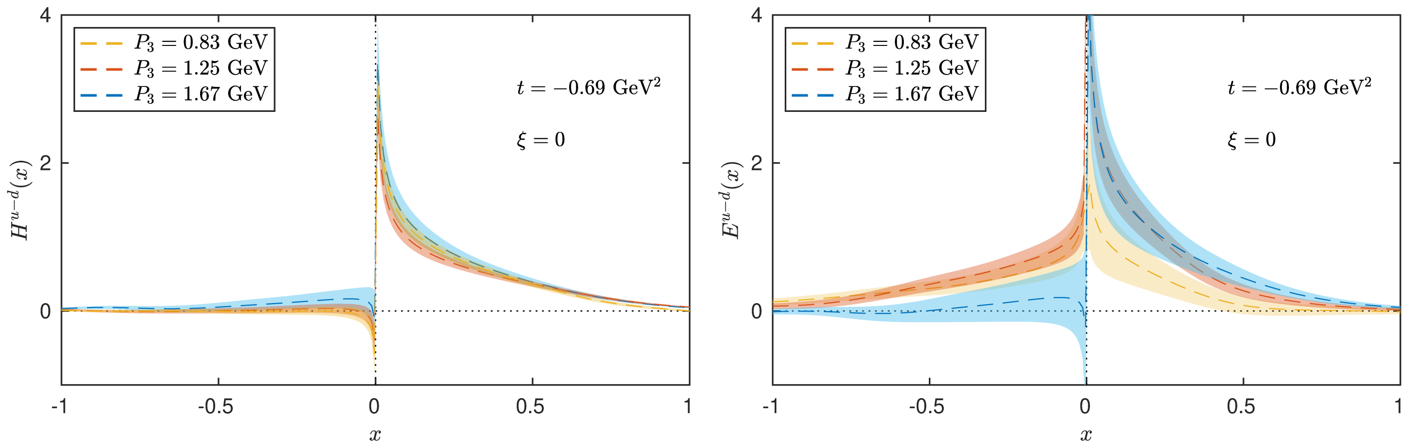
$H$-GPD | $E$-GPD

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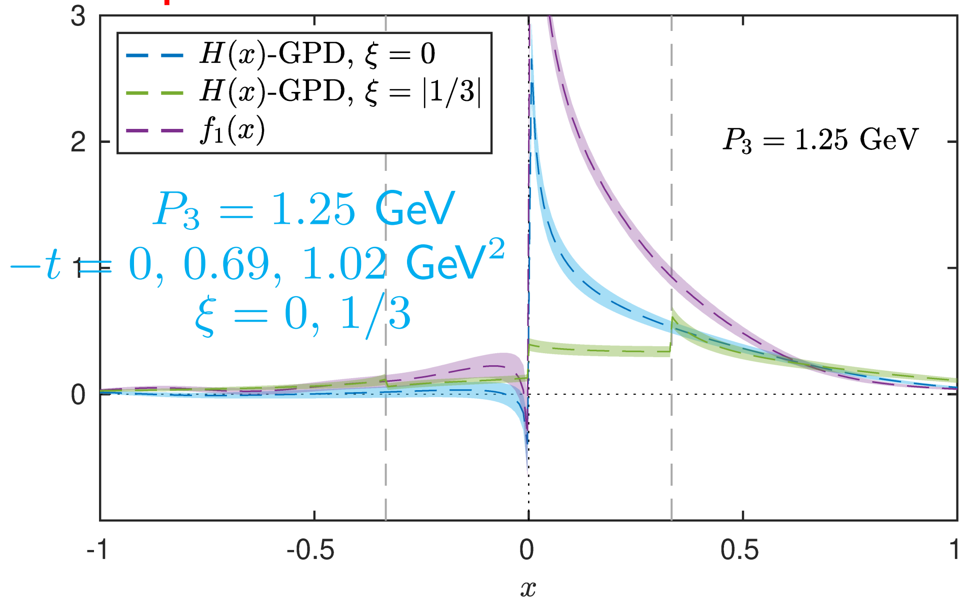
# Comparison of PDFs and $H$-GPDs

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001



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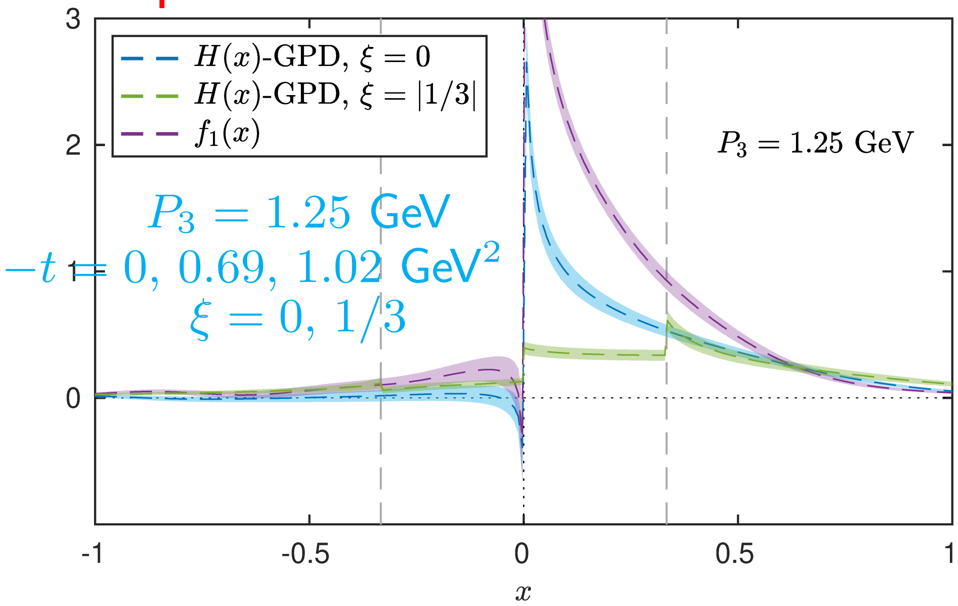


Important insights from models:

S. Bhattacharya, C. Cocuzza, A. Metz
Phys. Lett. B788 (2019) 453
Phys. Rev. D102 (2020) 054201

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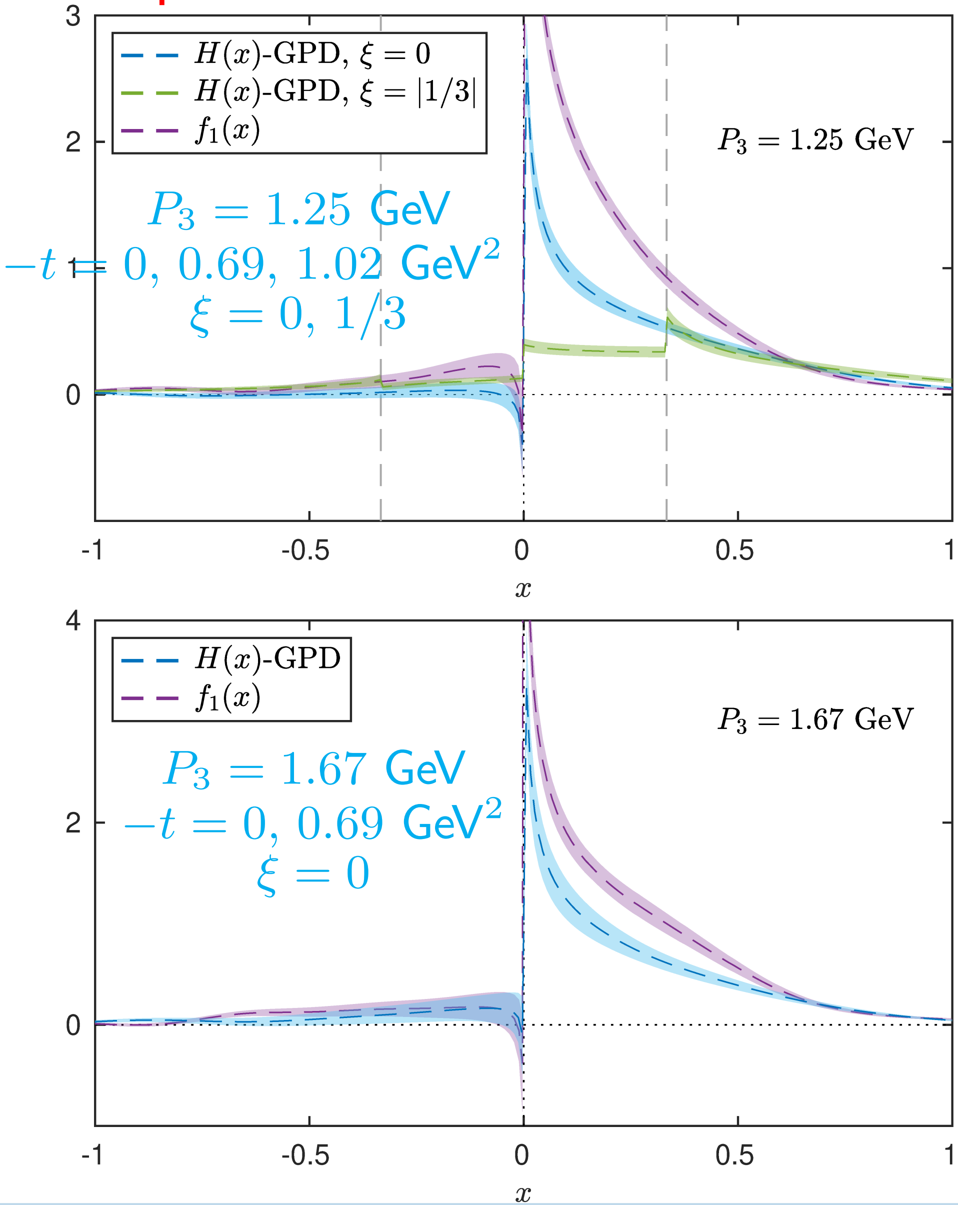

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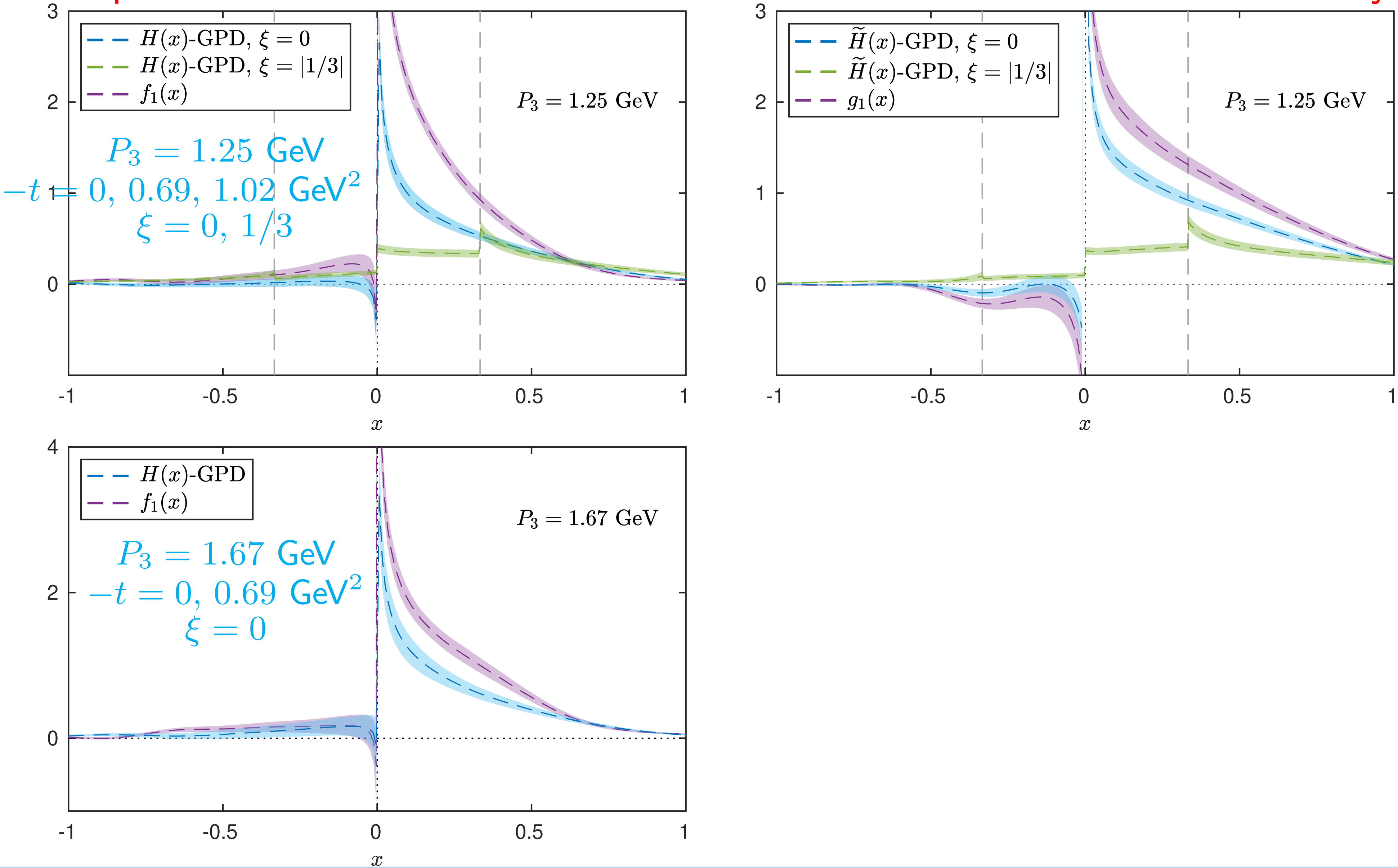
ETMC, Phys. Rev. Lett. 125 (2020) 262001

helicity



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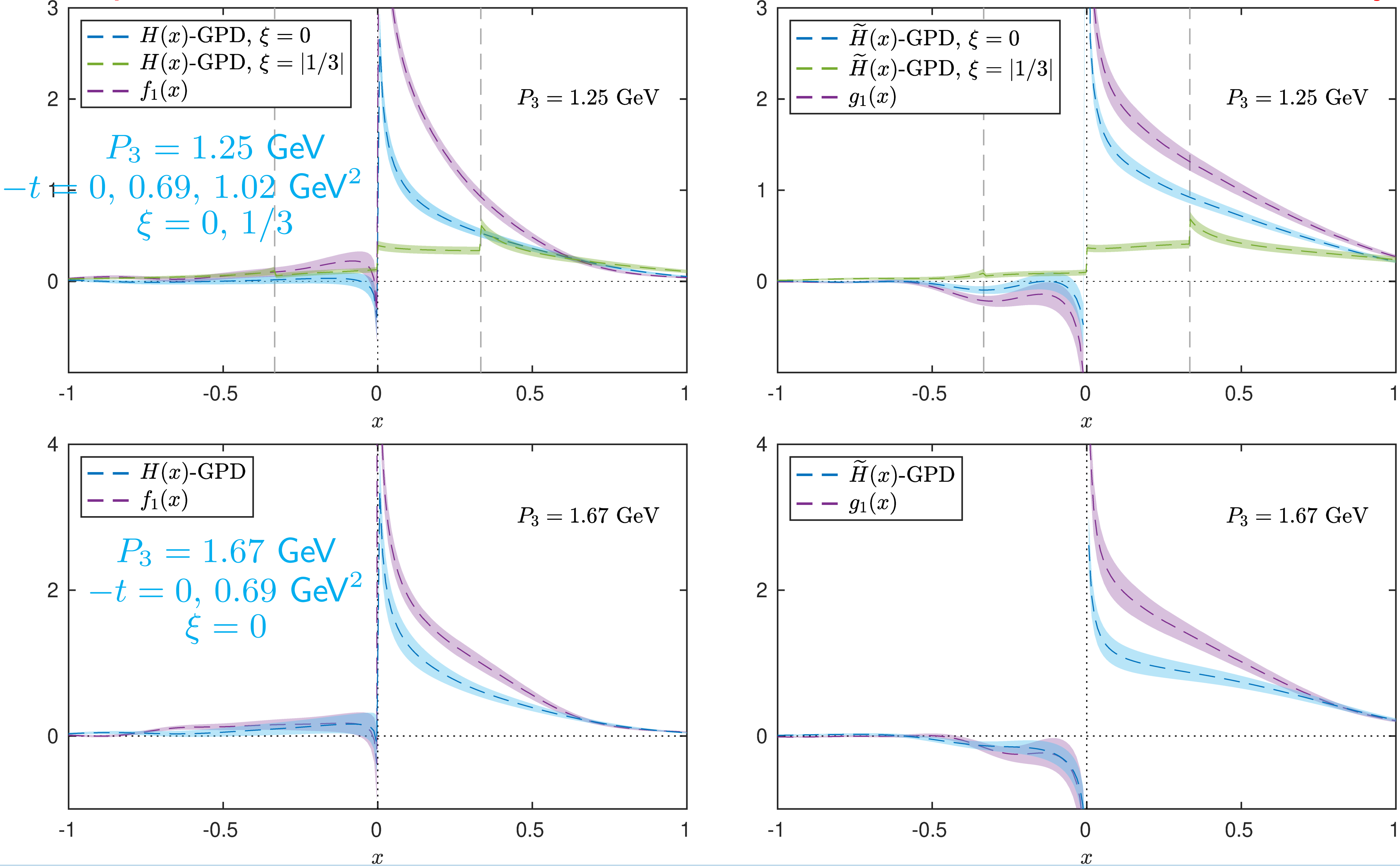
unpolarized ETMC, Phys. Rev. Lett. 125 (2020) 262001 helicity



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Main theoretical tool: S. Bhattacharya et al., PRD106(2022)114512
Lorentz-covariant parametrization of matrix elements (e.g. vector case):

$$F^{\mu}(z,P,\Delta)=\bar{u}(p',\lambda')\left[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+\frac{z^{\mu}i\sigma^{z\Delta}}{m}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p,\lambda),$$

(inspired by: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056).

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

S. Bhattacharya et al., PRD106(2022)114512

The relation between lattice-calculated matrix elements and the Lorentz-invariant amplitudes $A_i$ is different in the symmetric and the non-symmetric frame.

# Example

S. Bhattacharya et al., PRD106(2022)114512

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For example: ($\gamma_0$ insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0) = C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

asymmetric frame:

$$\begin{aligned}\Pi_0^a(\Gamma_0) &= C\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1 - \frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3 + \frac{(E_i-E_f)P_3 z}{4m}A_4 \\ &+ \frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5 + \frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6 + \frac{E_f P_3(E_f-E_i)^2 z}{2m^3}A_8\Bigg).\end{aligned}$$

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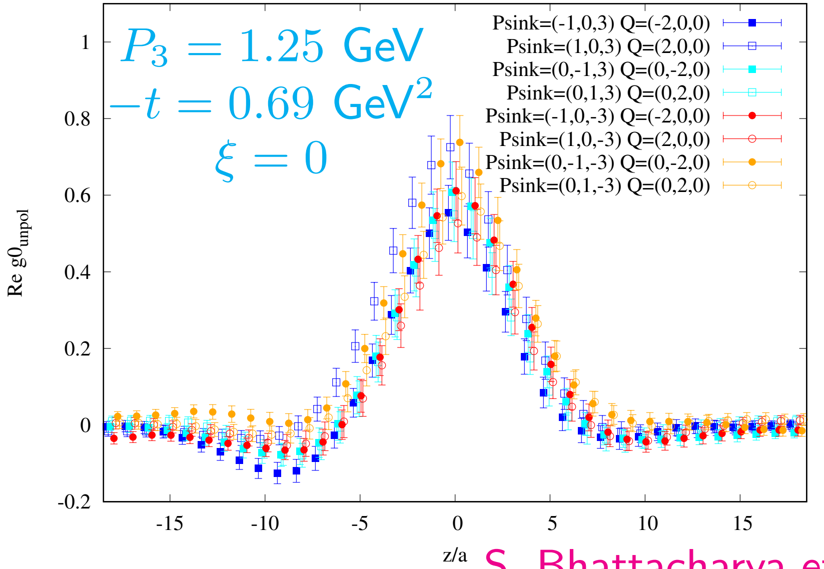
Thus,

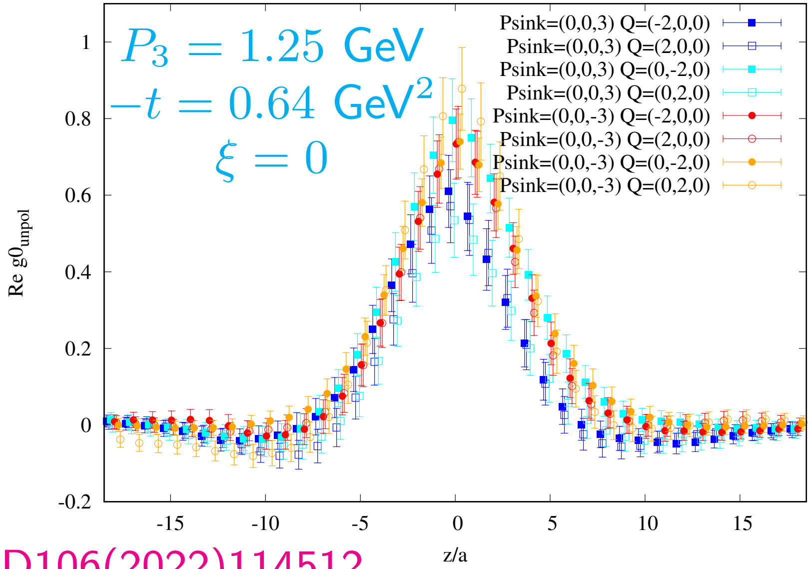
- matrix elements $\Pi_\mu(\Gamma_\nu)$ are frame-dependent,
- but the amplitudes $A_i$ are frame-invariant.

# Bare matrix elements of $\Pi_0(\Gamma_0)$

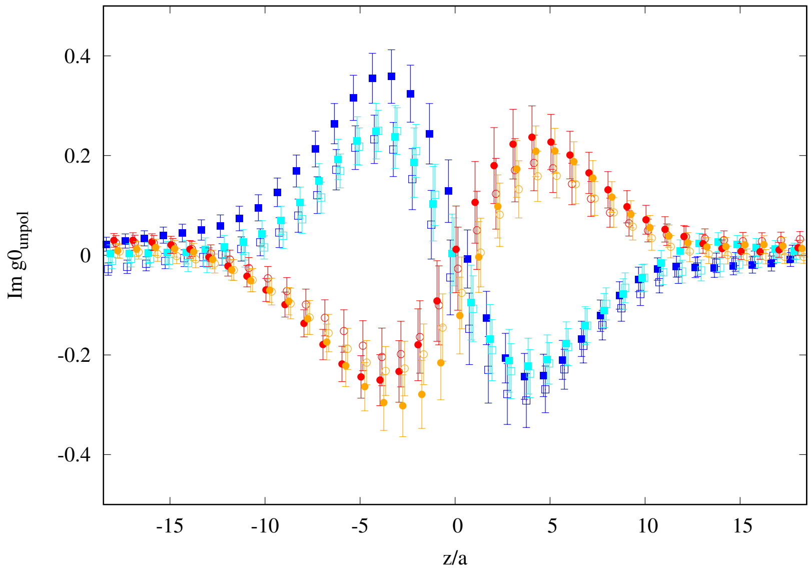
## symmetric frame | non-symmetric frame




Re




S. Bhattacharya et al., PRD106(2022)114512

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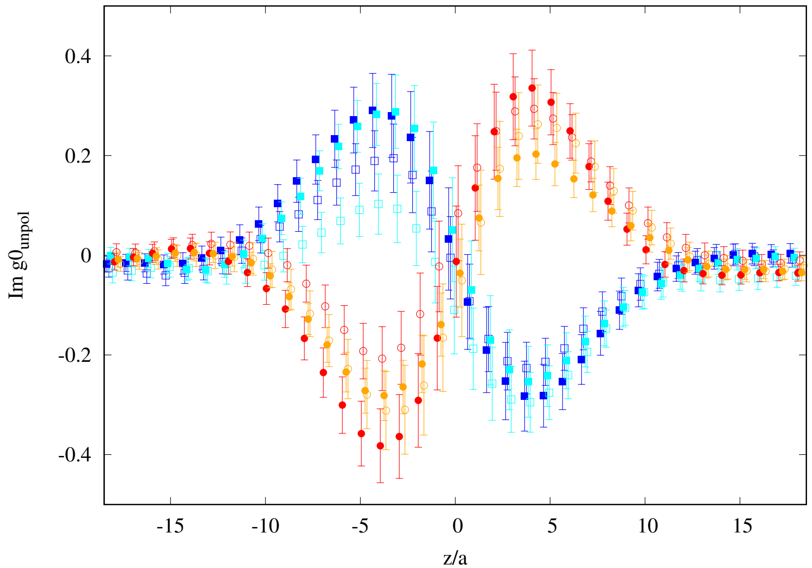
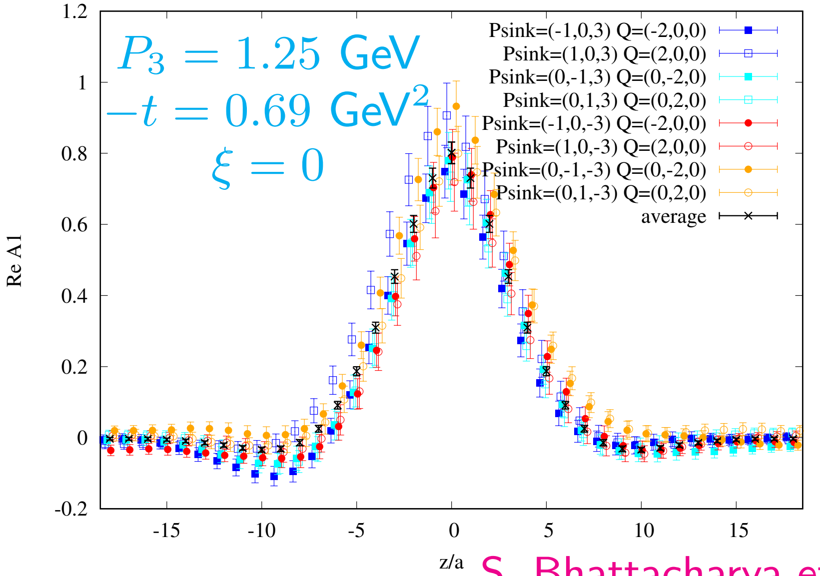
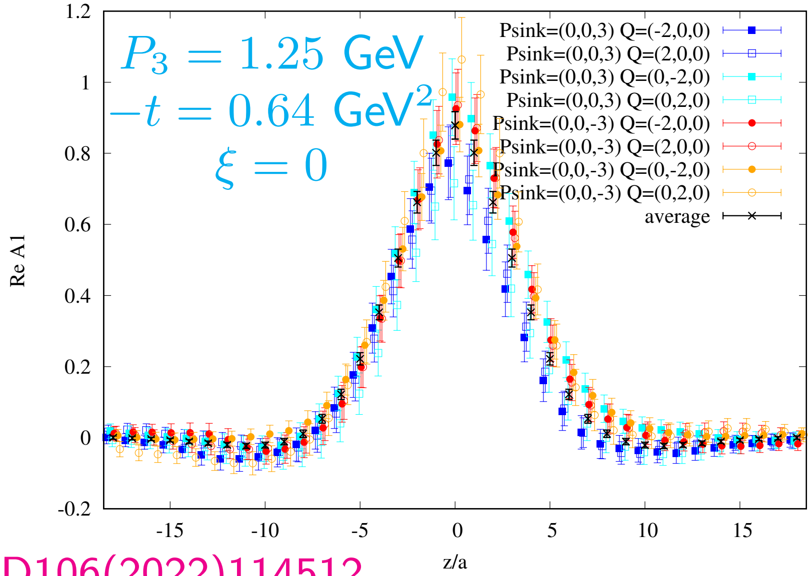


# Example amplitude $A_1$

symmetric frame — non-symmetric frame

Re

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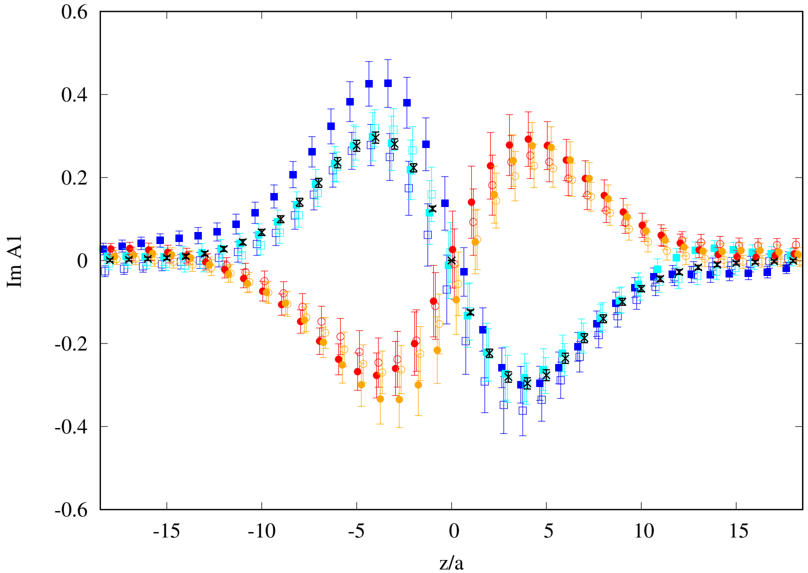
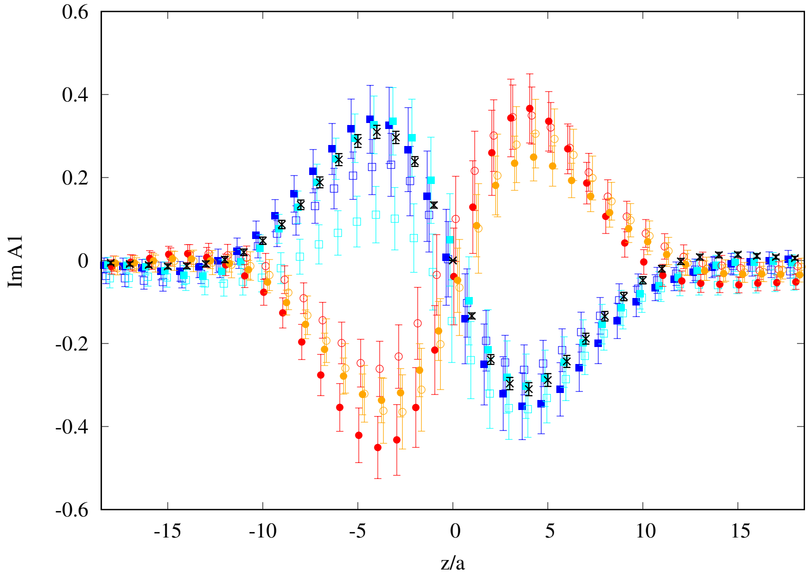
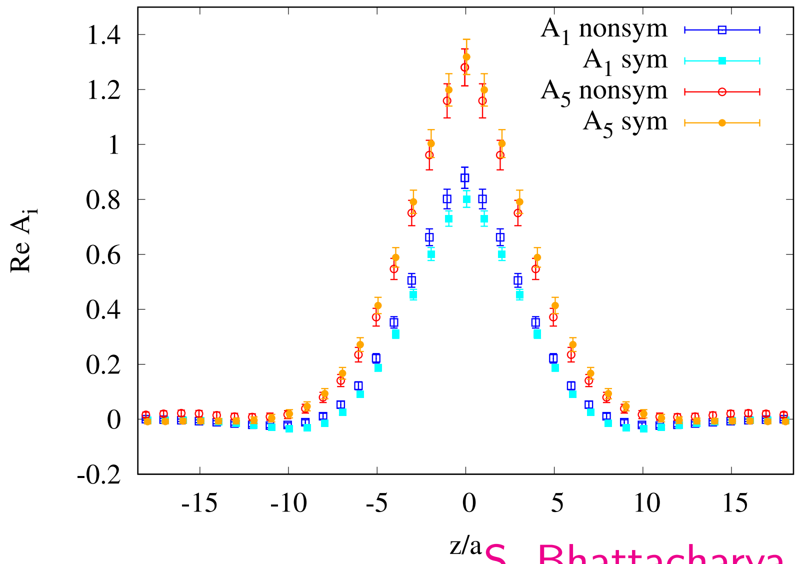


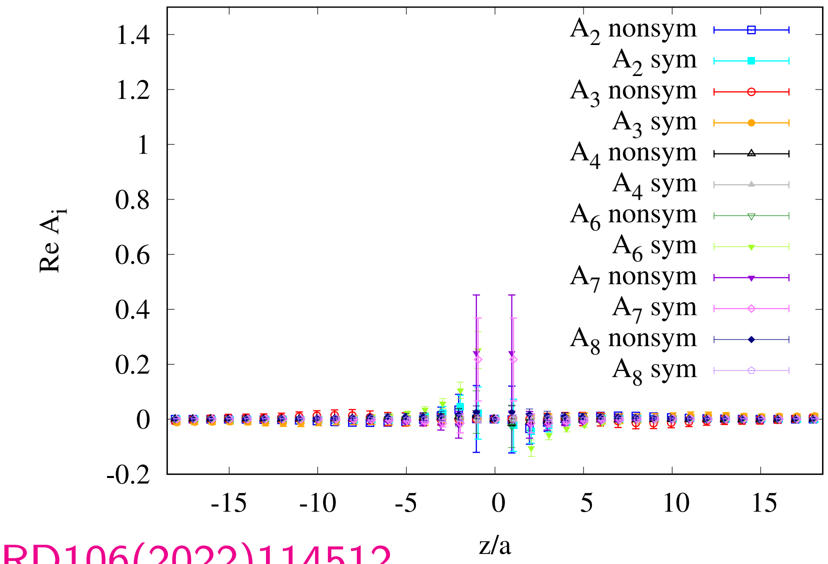



# Comparison of amplitudes between frames

$A_1$, $A_5$ (leading ones) $A_2$, $A_3$, $A_4$, $A_6$, $A_7$, $A_8$ (subleading ones)

Re

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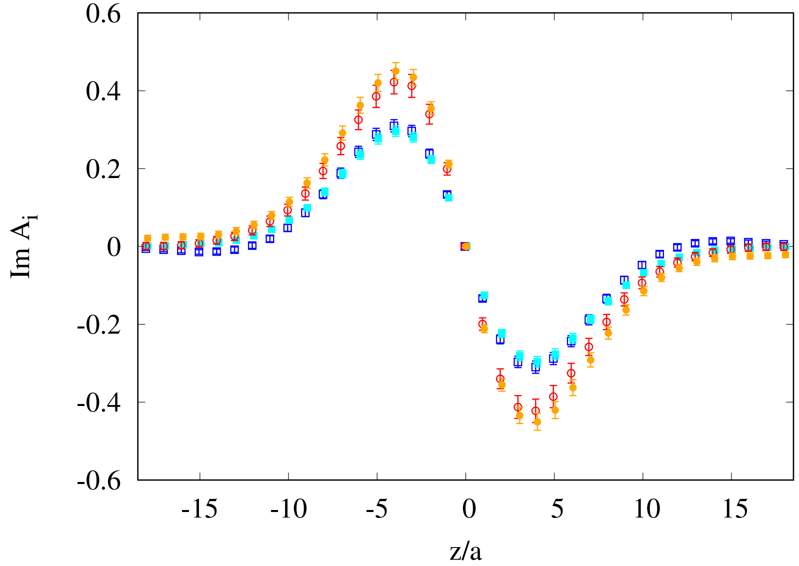


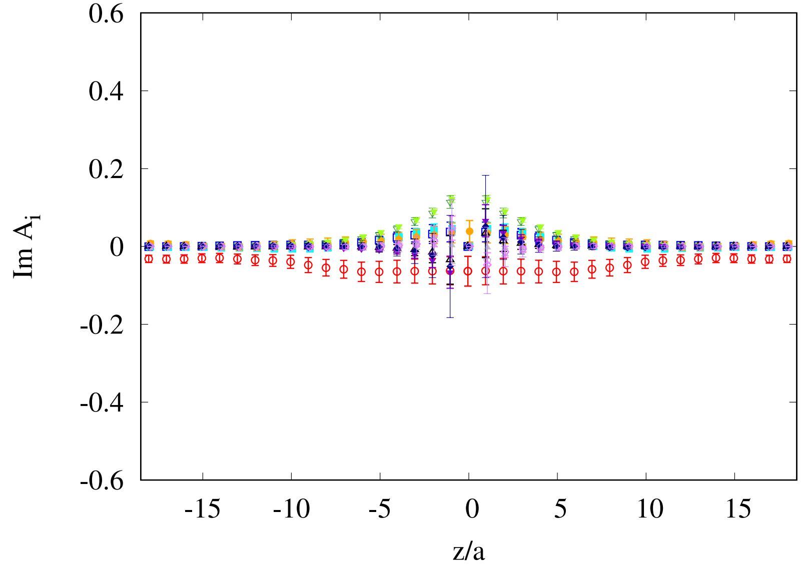

# $H$ and $E$ GPDs – standard definition

S. Bhattacharya et al., PRD106(2022)114512

The standard definition of $H$ and $E$ GPDs:

$$F^0(z, P, \Delta) = \bar{u}(p', \lambda')\left[\gamma^0 F_{H^{(0)}}(z, P, \Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m} F_{E^{(0)}}(z, P, \Delta)\right]u(p, \lambda)\,.$$

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Thus-defined GPDs are obviously frame-dependent! In terms of $A_i$'s ($\xi = 0$ case):

symmetric frame:

$$F_{H^{(0)}} = A_1 + \frac{z(\Delta_1^2+\Delta_2^2)}{2P_3} A_6\,,$$

$$F_{E^{(0)}} = -A_1 - \frac{m^2 z}{P_3} A_4 + 2A_5 - \frac{z\left(4E^2+\Delta_1^2+\Delta_2^2\right)}{2P_3} A_6\,.$$

asymmetric frame:

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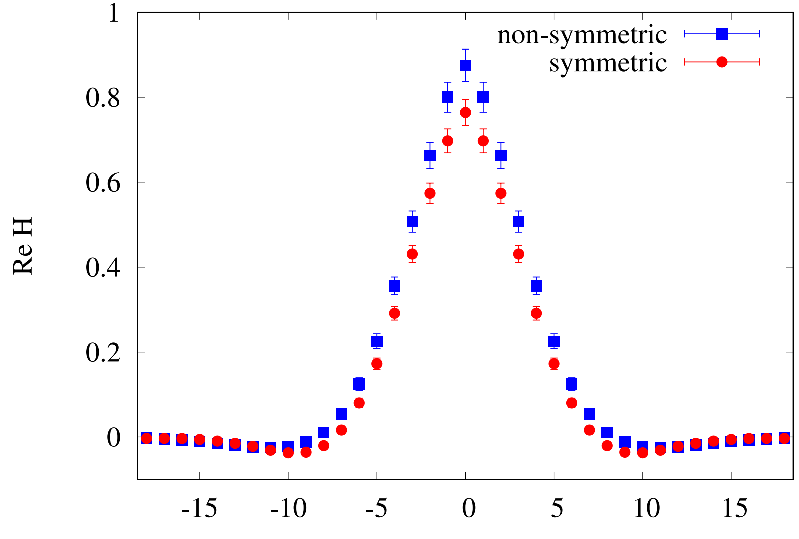
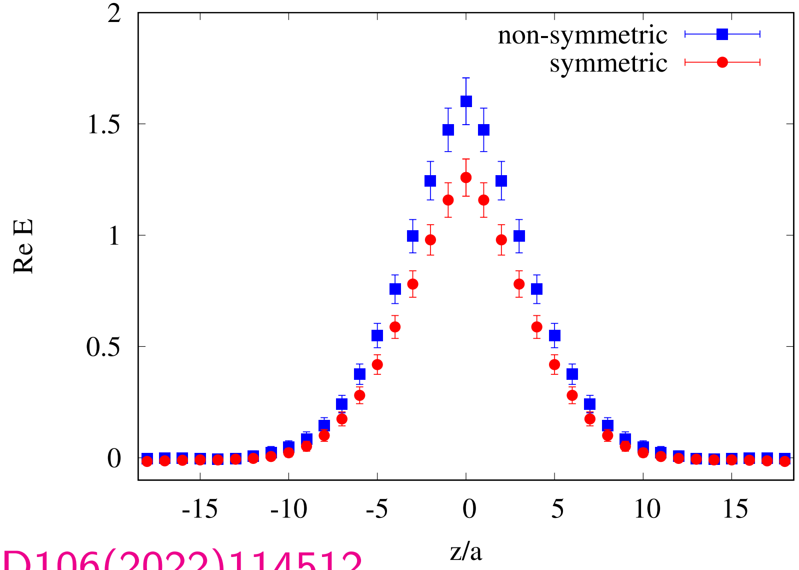
Note: the standard definition is frame-dependent, but still valid in the sense of approaching the correct GPDs in the light-cone limit.

# $H$ and $E$ GPDs – standard definition

## $H$-GPD | $E$-GPD

**Re**

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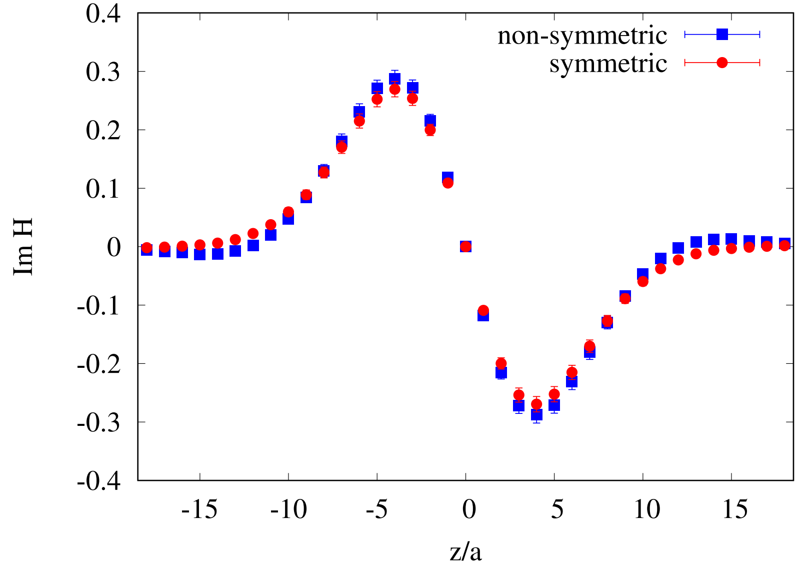


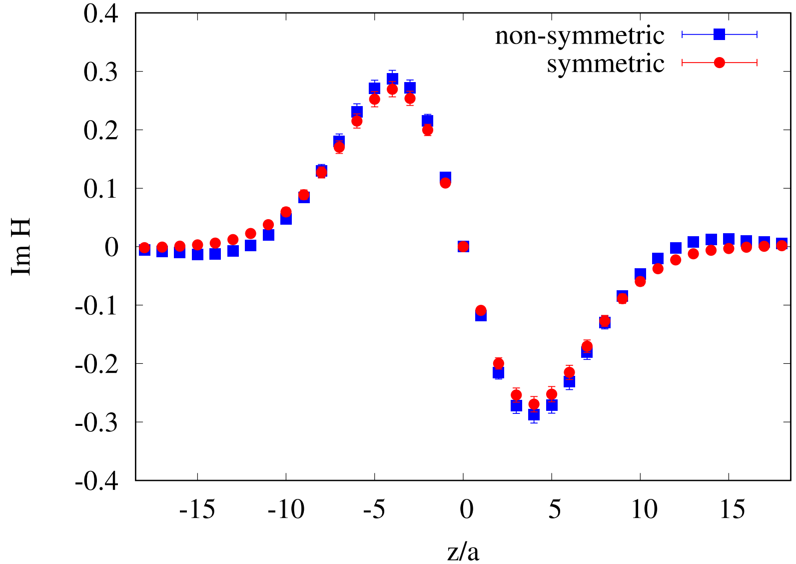

# $H$ and $E$ GPDs – Lorentz-invariant definition

The definition of $H$ and $E$ GPDs can be made Lorentz-invariant in the following way:

S. Bhattacharya et al., PRD106(2022)114512

$$F_H = A_1 + \frac{\Delta \cdot z}{P \cdot z} A_3 \,,$$

$$F_E = -A_1 - \frac{\Delta \cdot z}{P \cdot z} A_3 + 2A_5 + 2P \cdot z A_6 + 2\Delta \cdot z A_8 \,.$$

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At zero-skewness:

$$F_H = A_1 \,,$$

$$F_E = -A_1 + 2A_5 + 2zP_3 A_6 \,.$$

With respect to the standard definition, removed/reduced contribution from $A_3$, $A_4$, $A_6$, $A_8$.

# $H$ and $E$ GPDs – Lorentz-invariant definition

The definition of $H$ and $E$ GPDs can be made Lorentz-invariant in the following way:

S. Bhattacharya et al., PRD106(2022)114512

$$F_H = A_1 + \frac{\Delta \cdot z}{P \cdot z} A_3 \,,$$

$$F_E = -A_1 - \frac{\Delta \cdot z}{P \cdot z} A_3 + 2A_5 + 2P \cdot z A_6 + 2\Delta \cdot z A_8 \,.$$

At zero-skewness:

$$F_H = A_1 \,,$$

$$F_E = -A_1 + 2A_5 + 2zP_3 A_6 \,.$$

With respect to the standard definition, removed/reduced contribution from $A_3$, $A_4$, $A_6$, $A_8$.

In terms of matrix elements:

- standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,
- Lorentz-invariant definition – additionally:
  - ⋆ symmetric: $\Pi_{1/2}(\Gamma_3)$,
  - ⋆ non-symmetric: $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$.

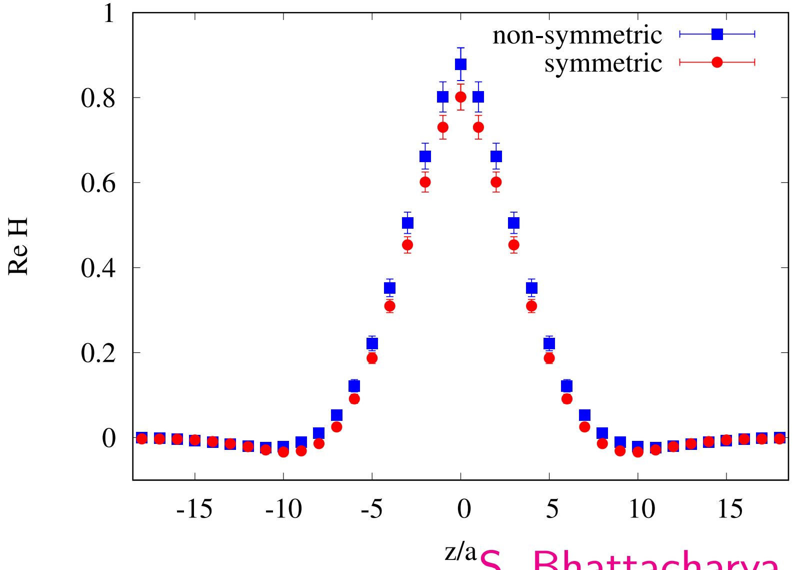
Thus, adding info from additional MEs potentially improves convergence (to be investigated).

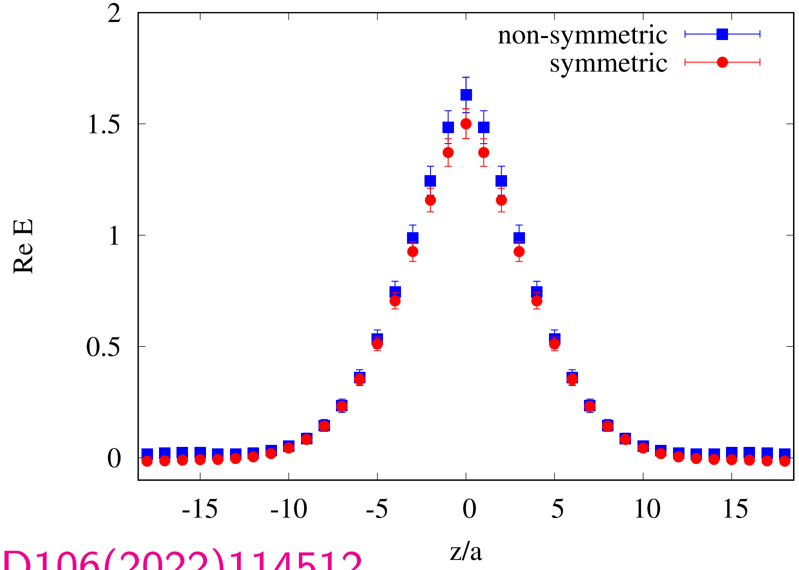
# $H$ and $E$ GPDs – Lorentz-invariant definition

## $H$-GPD

## $E$-GPD

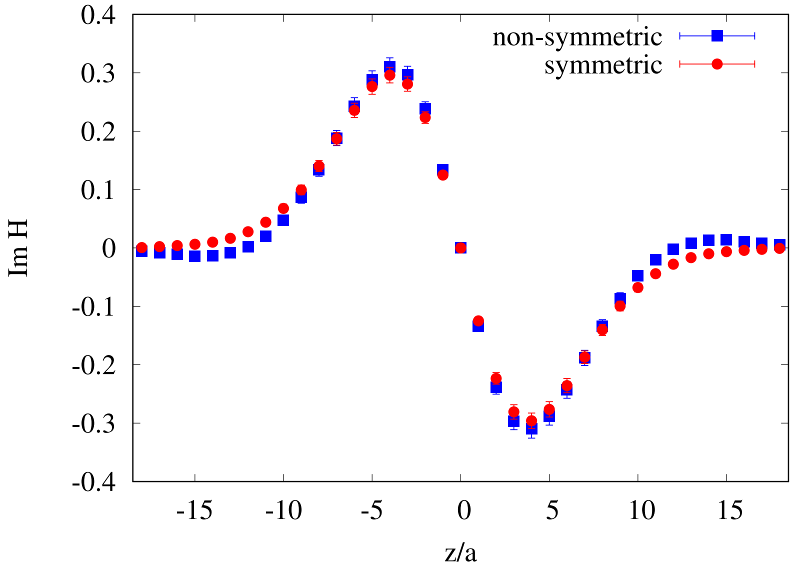
Re

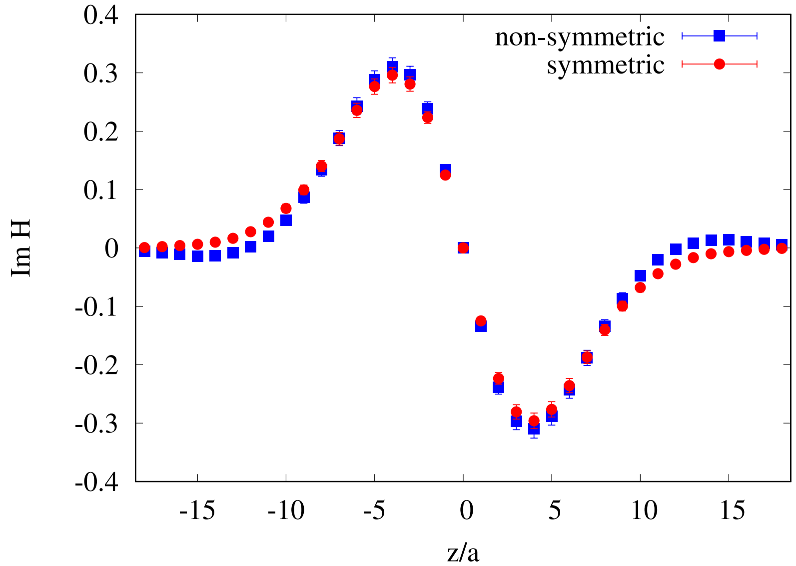


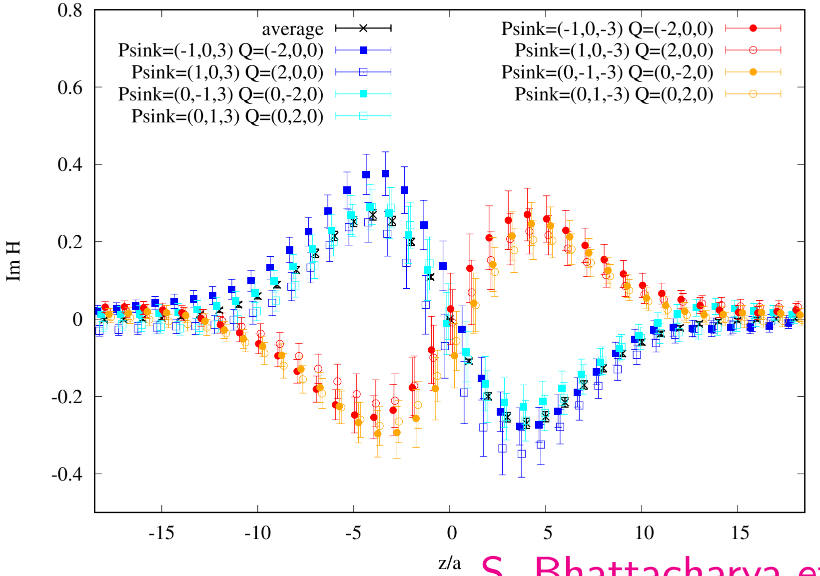

S. Bhattacharya et al., PRD106(2022)114512

Im



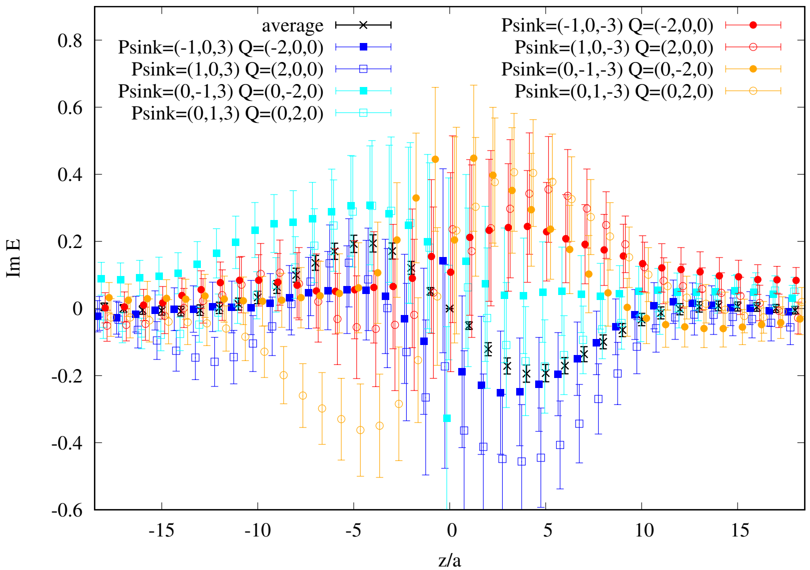



standard




Im $H$

S. Bhattacharya et al., PRD106(2022)114512

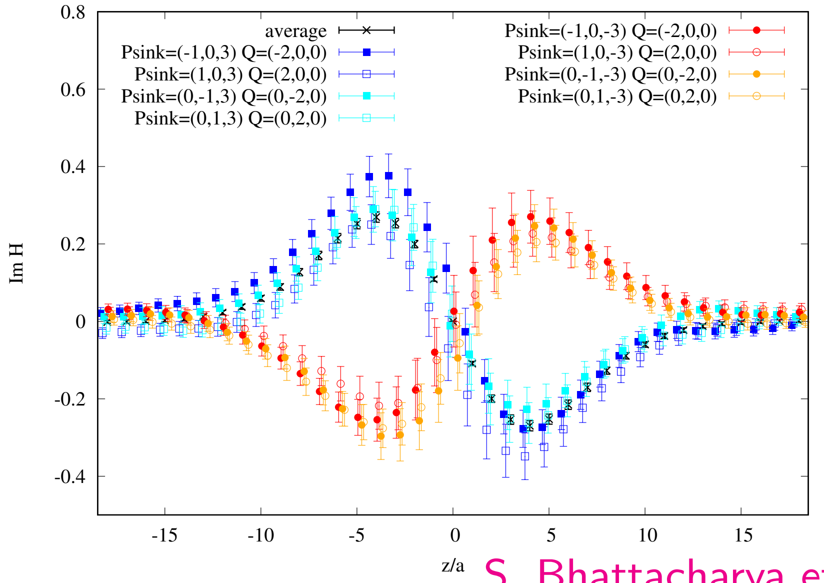



Im $E$

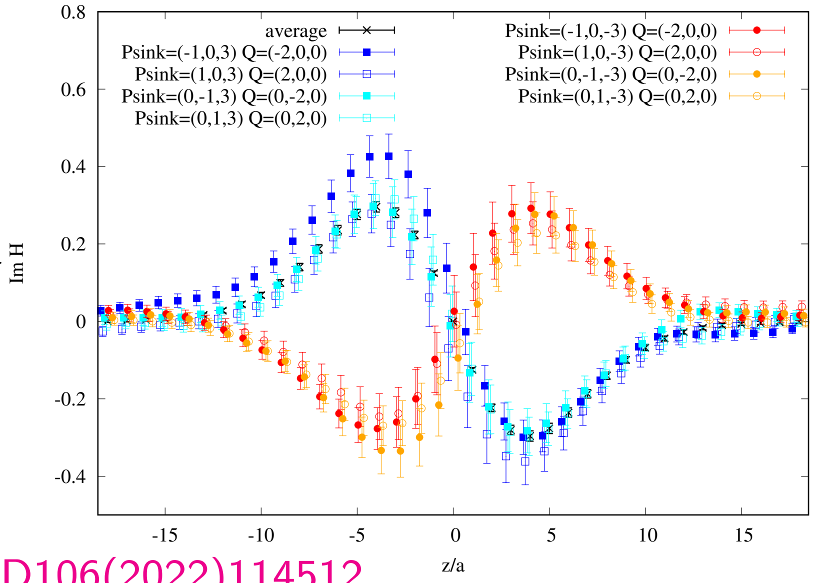
# $H$ and $E$ GPDs – signal improvement

standard

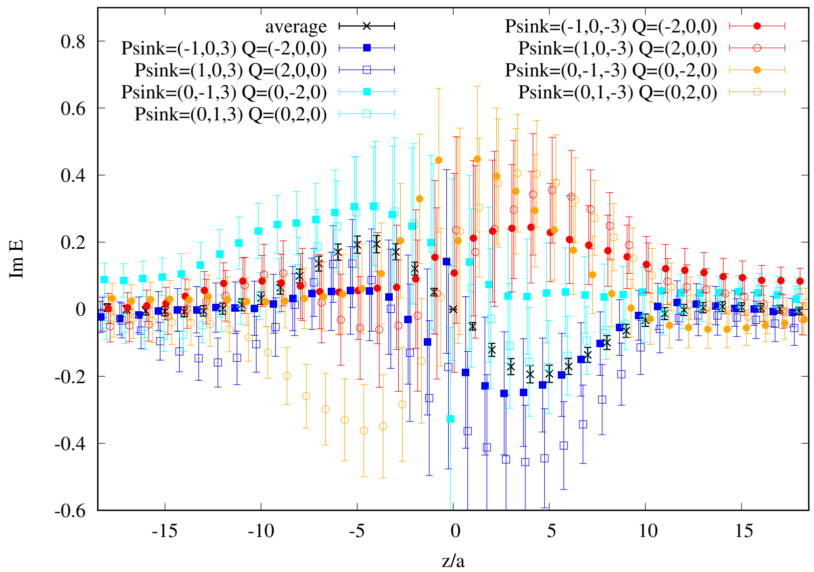
Lorentz-invariant



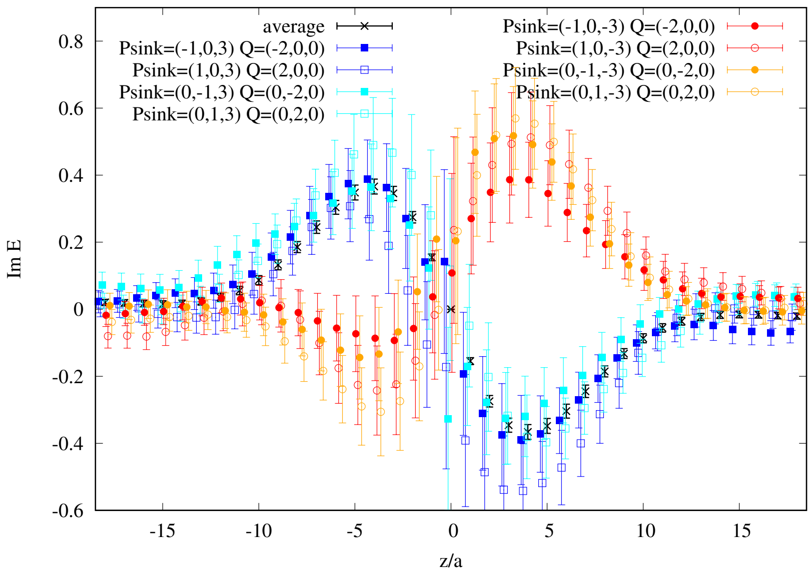

Im $H$



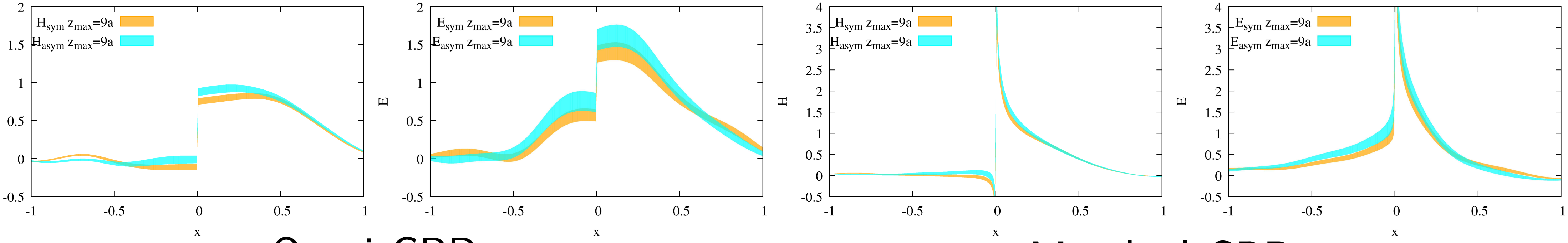

S. Bhattacharya et al., PRD106(2022)114512




Im $E$



STANDARD DEFINITION

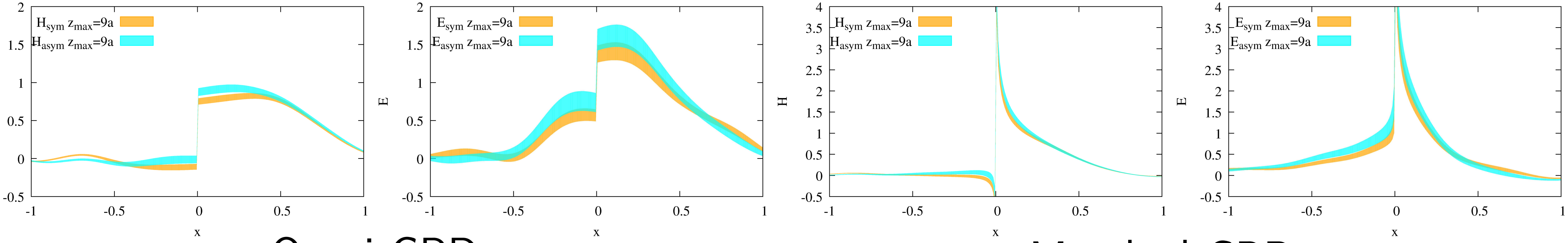



Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

$H$-GPD $E$-GPD $H$-GPD $E$-GPD

# Quasi- and matched $H$ and $E$ GPDs

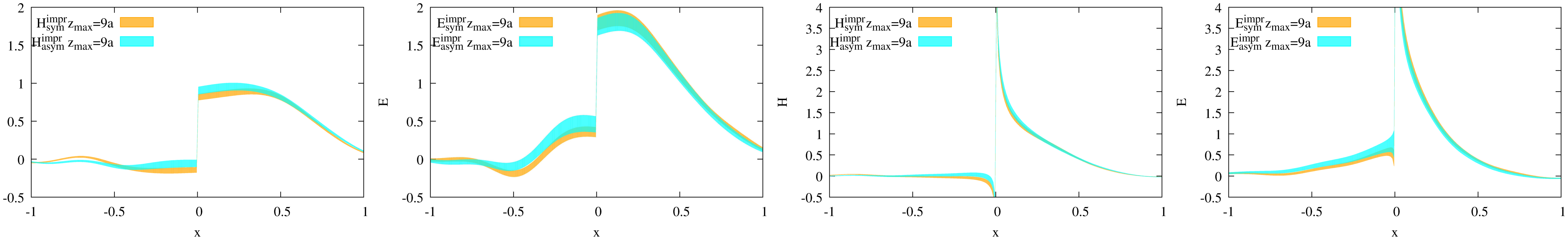
STANDARD DEFINITION




Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

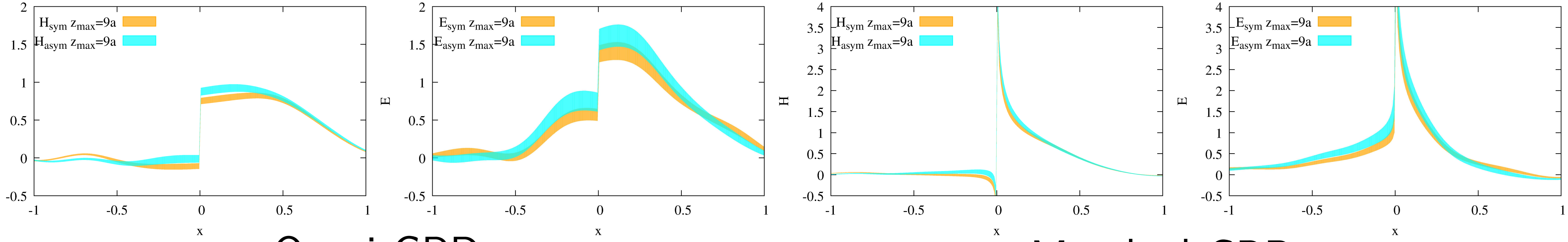
$H$-GPD $E$-GPD $H$-GPD $E$-GPD

LORENTZ-INVARIANT DEFINITION



# Quasi- and matched $H$ and $E$ GPDs

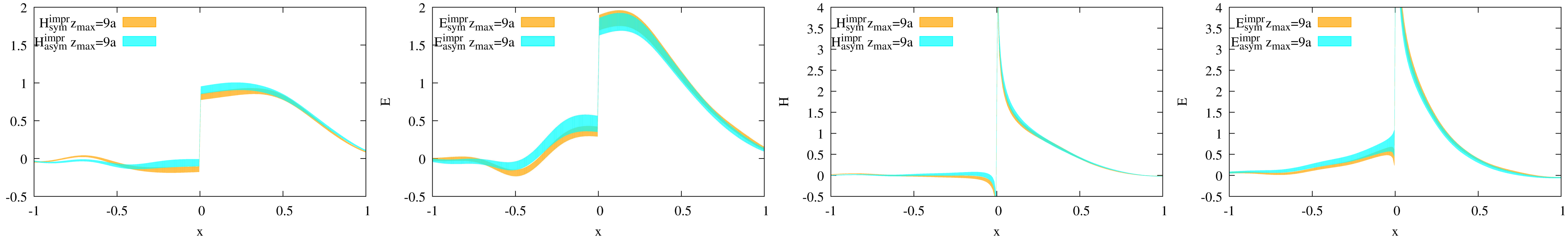
## STANDARD DEFINITION




Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

$H$-GPD $E$-GPD $H$-GPD $E$-GPD

## LORENTZ-INVARIANT DEFINITION




Main conclusions:

– GPDs can be computed in non-symmetric frames, reducing the computational cost

– GPDs can be made frame-independent (Lorentz-invariant definition) – potentially better convergence

**Overall, it gives much better perspectives for lattice GPDs!**

# Helicity GPDs – work in progress

Lorentz-covariant parametrization of matrix elements (axial vector case):

$$F^{[\gamma^\mu\gamma_5]} = \bar{u}(p',\lambda')\Big[\frac{i\epsilon^{\mu P z\Delta}}{m}A_1+\gamma^\mu\gamma_5 A_2+\gamma_5\Big(\frac{P^\mu}{m}A_3+mz^\mu A_4+\frac{\Delta^\mu}{m}A_5\Big)+m\not{z}\gamma_5\Big(\frac{P^\mu}{m}A_6+mz^\mu A_7+\frac{\Delta^\mu}{m}A_8\Big)\Big]u(p,\lambda)$$

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Two definitions of $\widetilde{H}$ :

standard: $F_{\tilde{H}} = A_2 + zP_3 A_6 - m^2 z^2 A_7\,,$

Lorentz-invariant: $F_{\tilde{H}} = A_2 + zP_3 A_6\,.$

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standard: $F_{\tilde{H}} = A_2 + zP_3A_6 - m^2z^2A_7\,,$

Lorentz-invariant: $F_{\tilde{H}} = A_2 + zP_3A_6\,.$

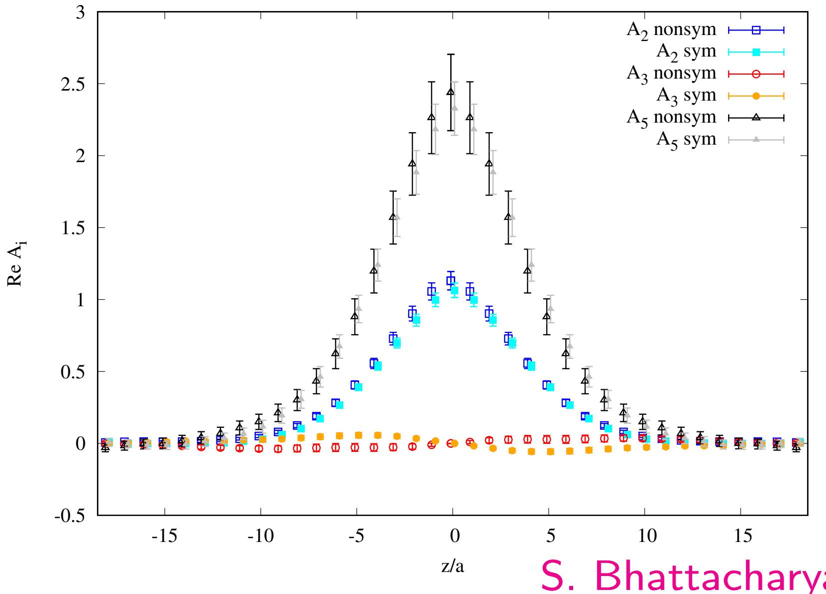
$\widetilde{E}$ seems impossible to extract at $\xi = 0$:

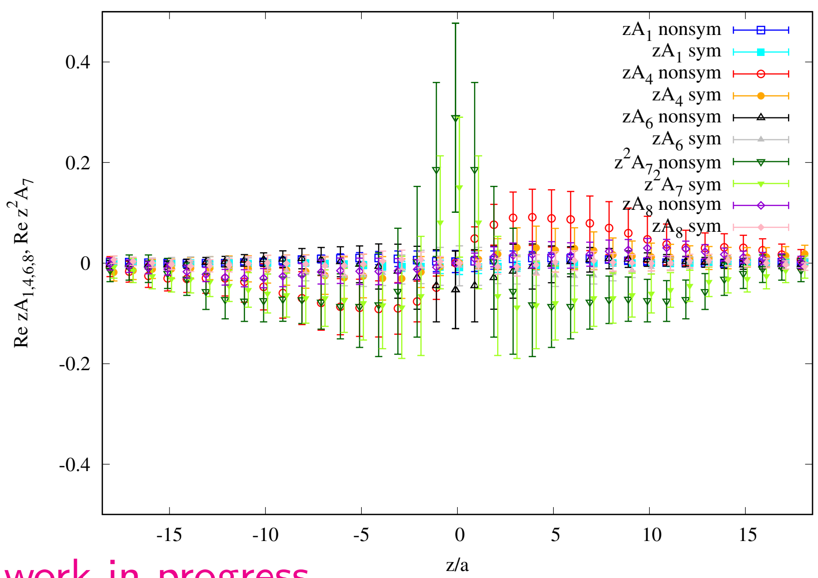
$$F_{\tilde{E}} = 2\,\frac{P\cdot z}{\Delta\cdot z}A_3 + 2A_5\,.$$

# Helicity GPDs – work in progress

## $A_2$, $A_3$, $A_5$ $\quad$ $zA_1$, $zA_4$, $zA_6$, $z^2A_7$, $zA_8$

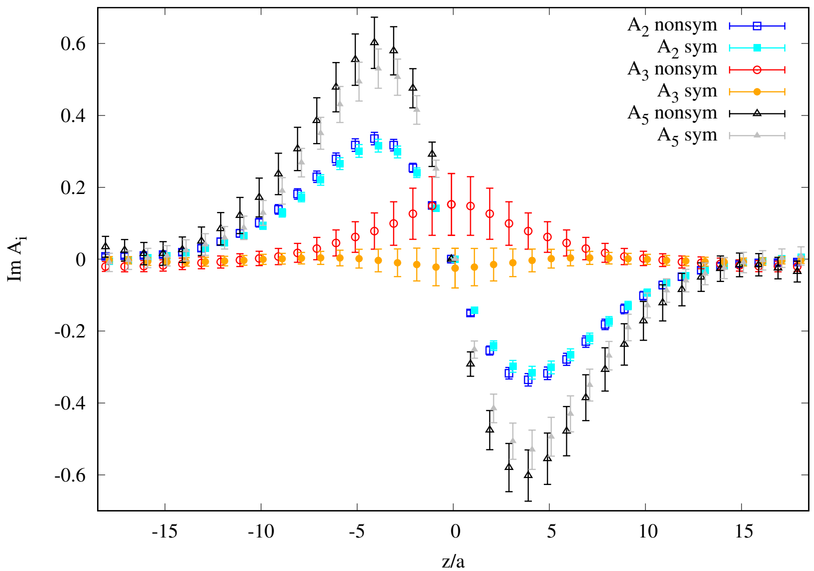
**Re**

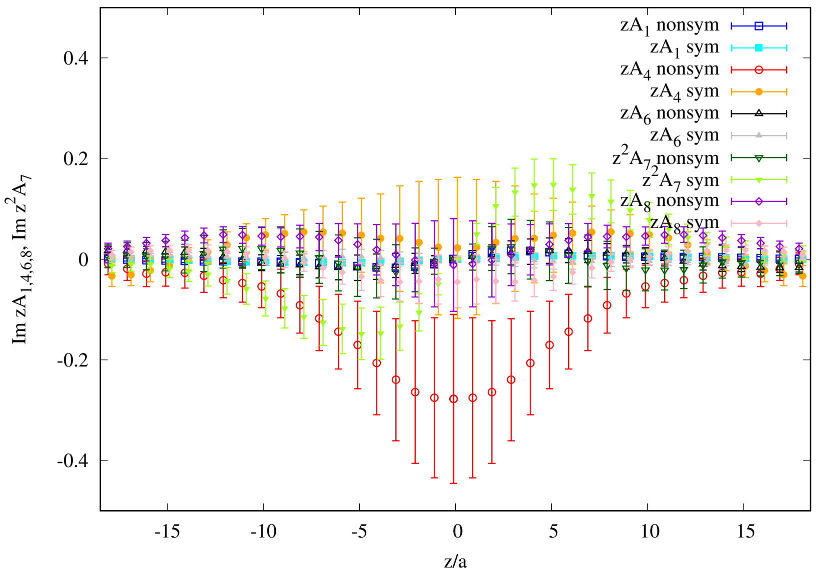



S. Bhattacharya et al., work in progress

**Im**

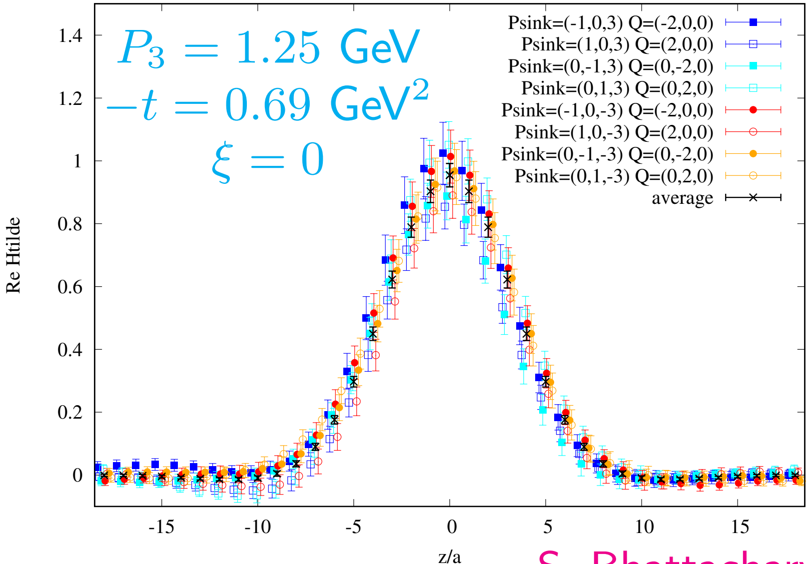


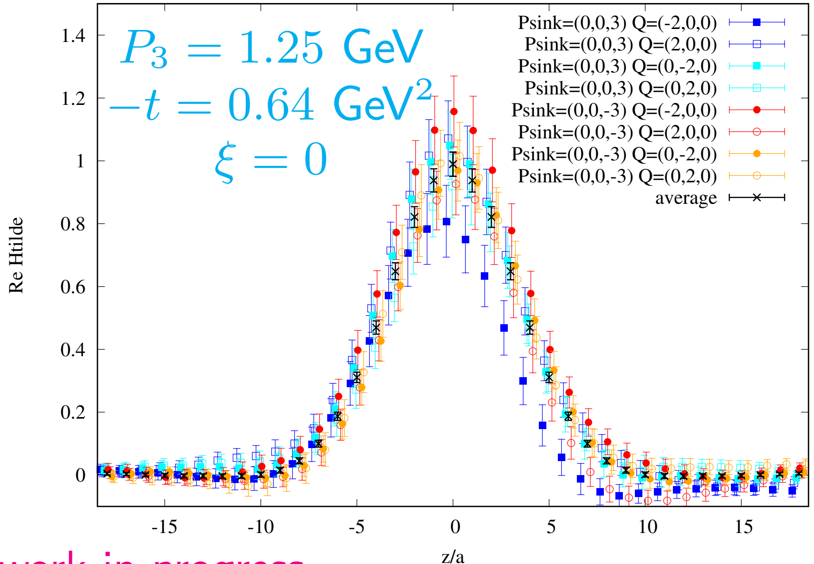



# Helicity GPD $\widetilde{H}$ (std. def.) – work in progress

symmetric frame | non-symmetric frame

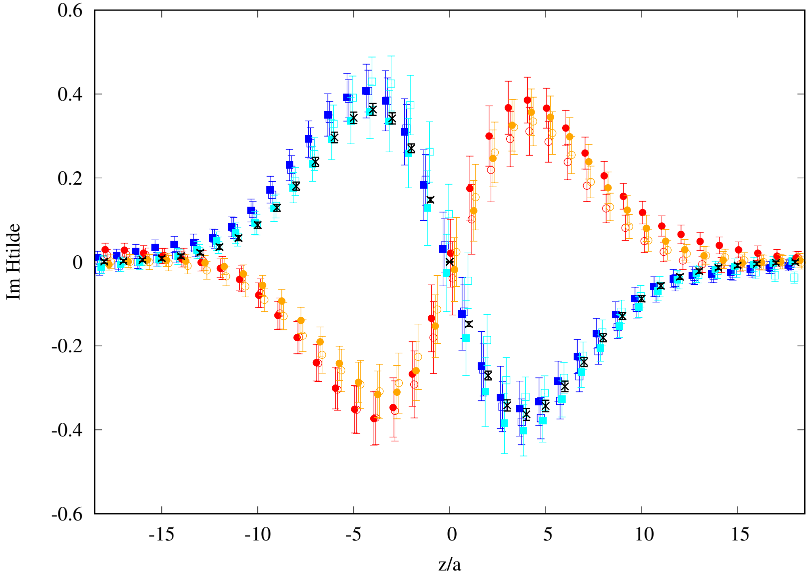
Re

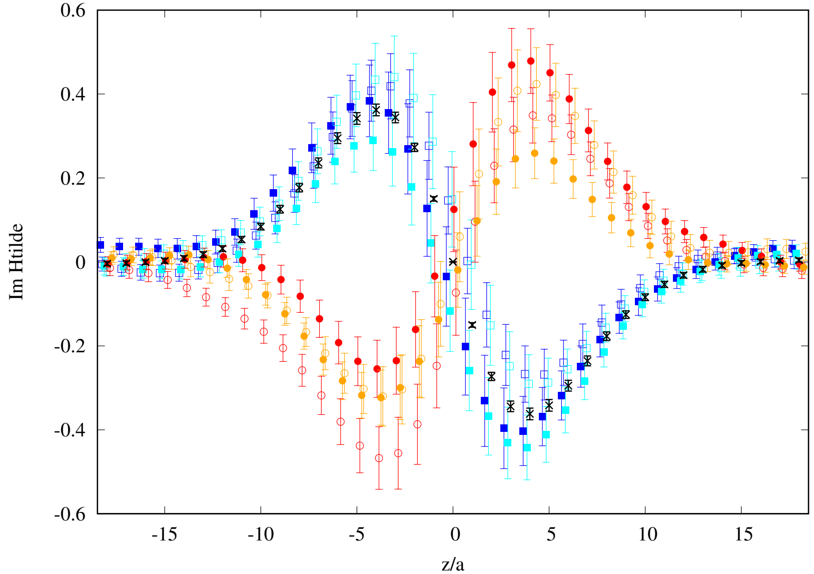


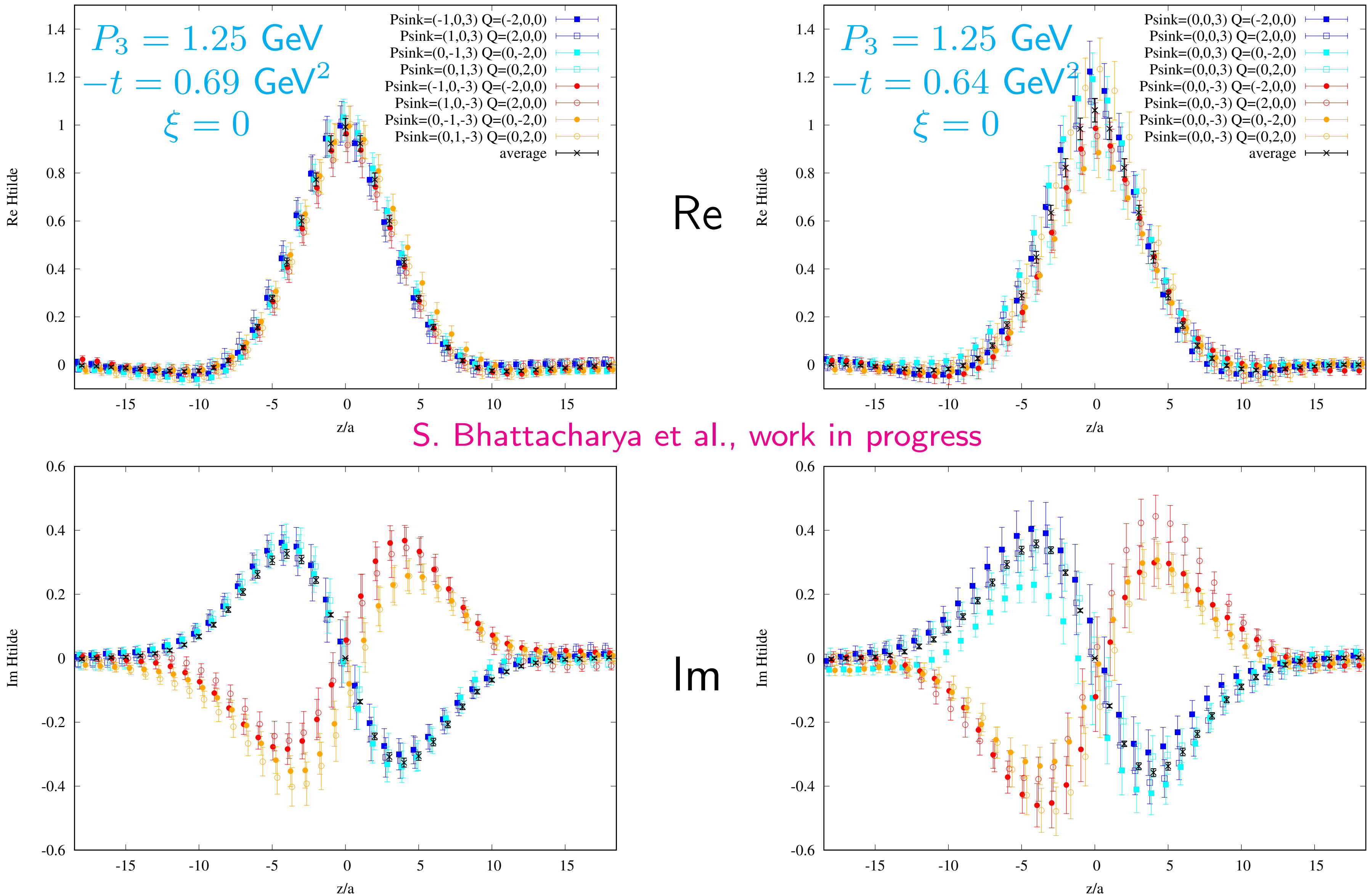

S. Bhattacharya et al., work in progress

Im



symmetric frame | non-symmetric frame




S. Bhattacharya et al., work in progress

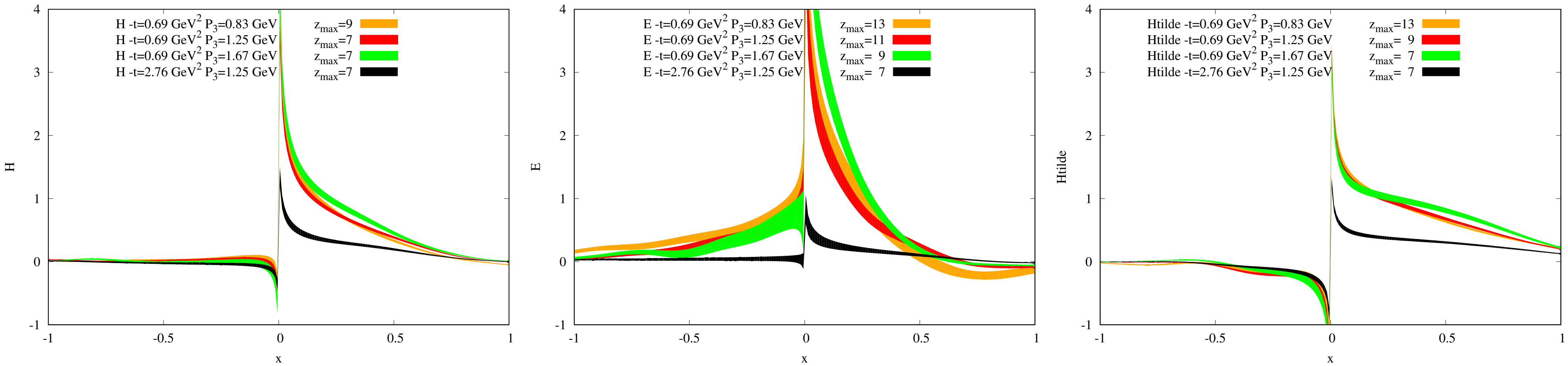
# Further work in progress

Current extensions include also:

- investigation of convergence towards the light cone: $P_3 = 0.83,\ 1.25,\ 1.67$ GeV for $-t = 0.69$ GeV$^2$ ($Q = (2,0,0)$),
- additional momentum transfers:
  - ⋆ symmetric: $-t = 0.69,\ 1.38,\ 2.76$ GeV$^2$ ($Q = (2,0,0),\ (2,2,0),\ (4,0,0)$),
  - ⋆ non-symmetric: $-t = 0.17,\ 0.33,\ 0.64,\ 0.80,\ 1.37,\ 1.50,\ 2.26$ GeV$^2$ ($Q = (1,0,0),\ (1,1,0),\ (2,0,0),\ (2,1,0),\ (3,0,0),\ (3,1,0),\ (4,0,0)$),
- transversity GPDs,
- twist-3 GPDs
- pion and kaon GPDs.

# Convergence of different definitions

STANDARD DEFINITION




UNPOLARIZED

$H$-GPD

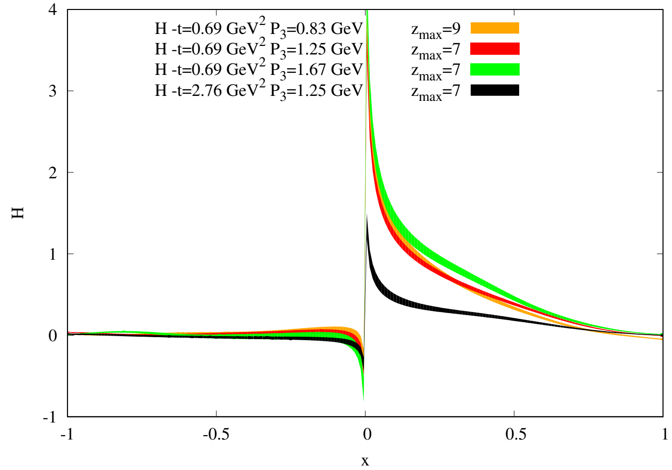
$E$-GPD

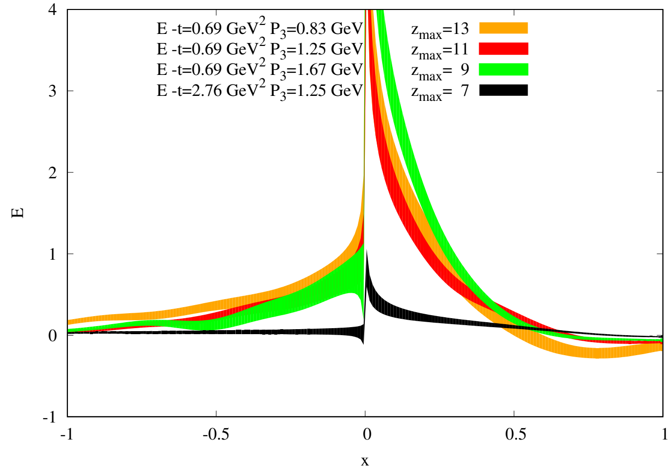
HELICITY

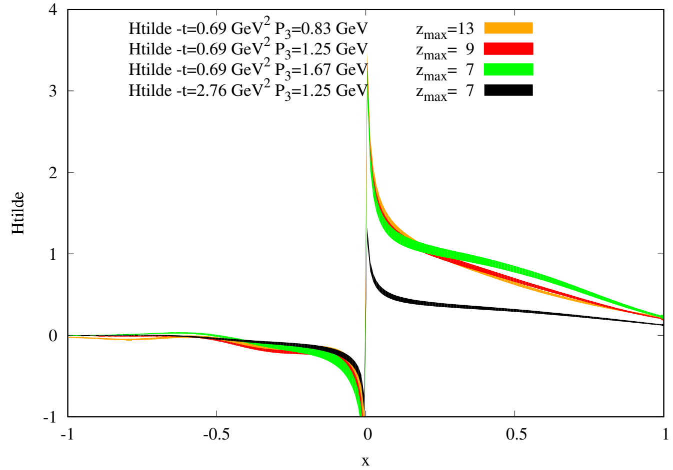
$\widetilde{H}$-GPD

# Convergence of different definitions

STANDARD DEFINITION




UNPOLARIZED

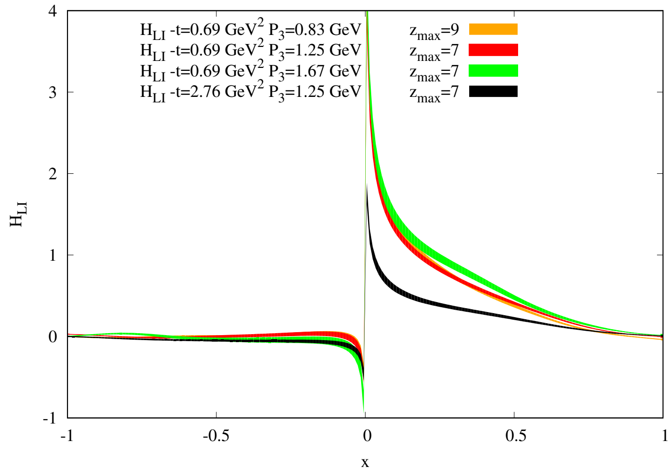
HELICITY

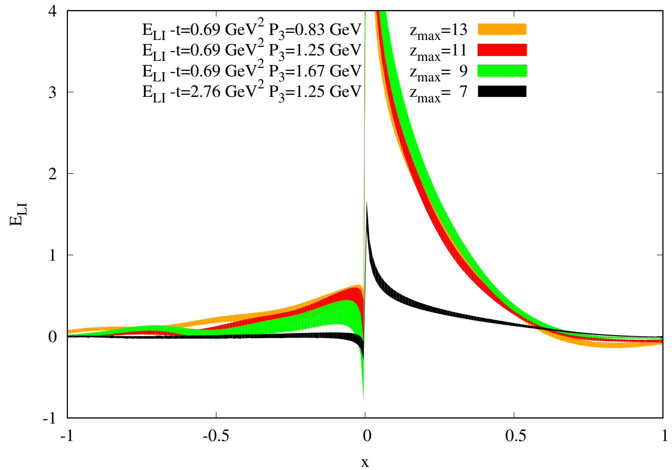
$H$-GPD

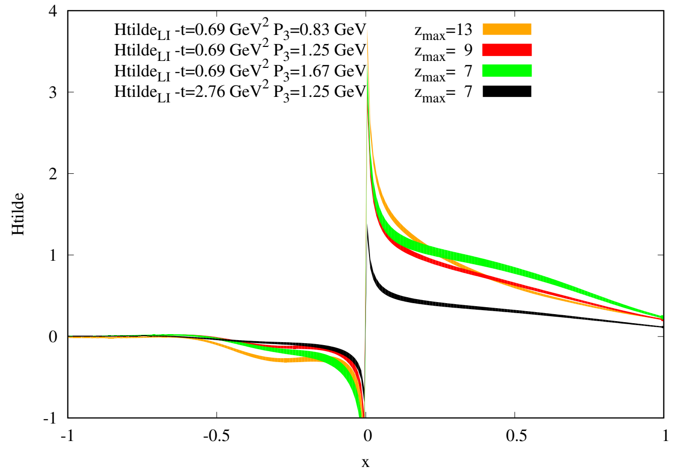
$E$-GPD

$\widetilde{H}$-GPD

LORENTZ-INVARIANT DEFINITION

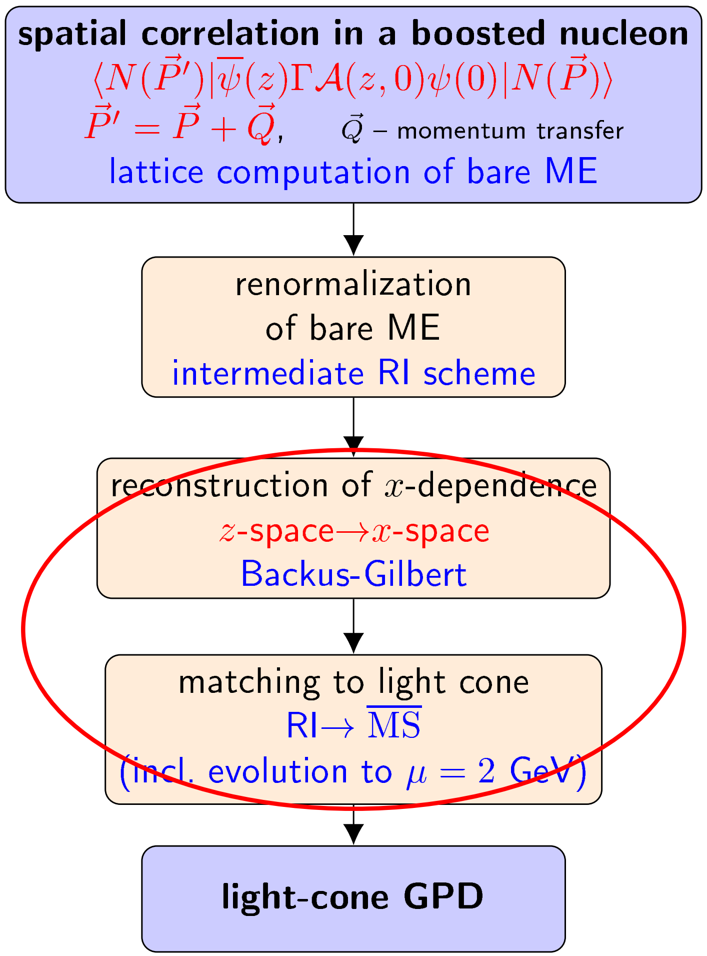


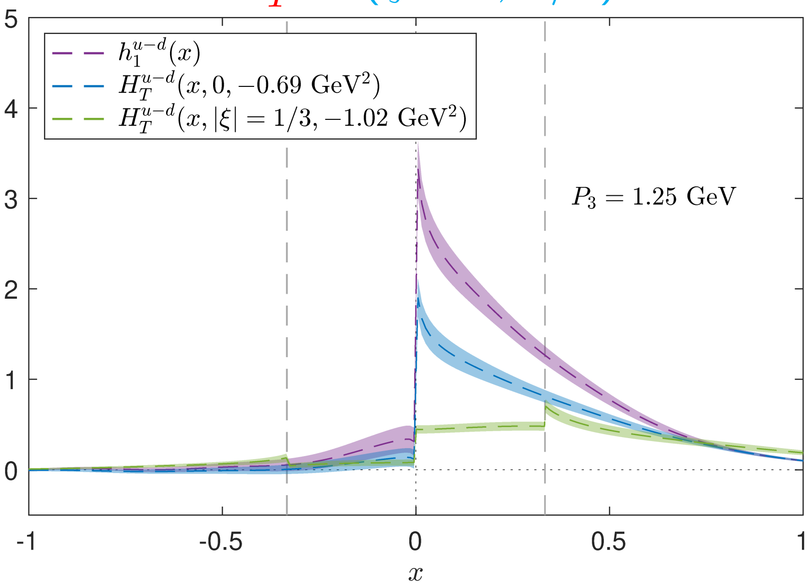



ETMC, Phys. Rev. D105 (2022) 034501

Transversity GPDs:
4 GPDs: $H_T$, $E_T$, $\tilde{H}_T$, $\tilde{E}_T$




$H_T^{u-d}$ $(\xi = 0,\, 1/3)$



# Transversity GPDs

Transversity GPDs:
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**spatial correlation in a boosted nucleon**
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
of bare ME
intermediate RI scheme

↓

reconstruction of $x$-dependence
$z$-space→$x$-space
Backus-Gilbert

↓

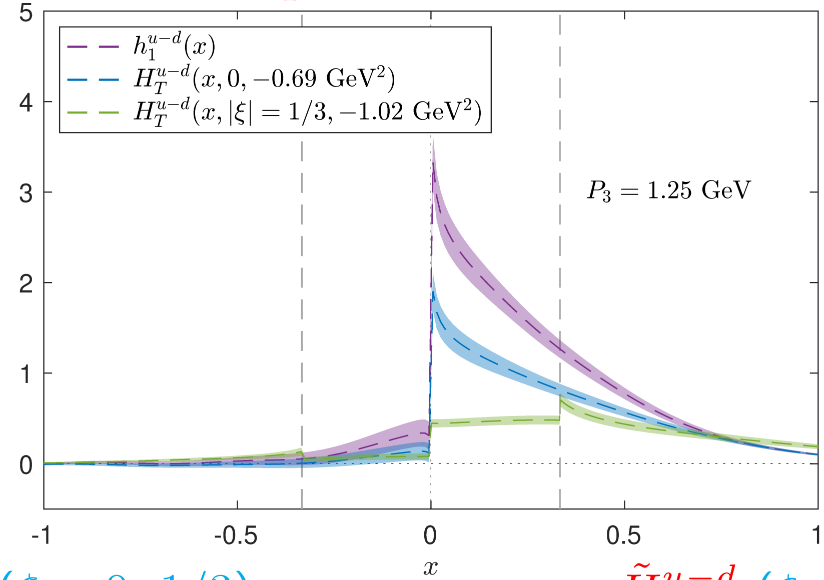
matching to light cone
RI→ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

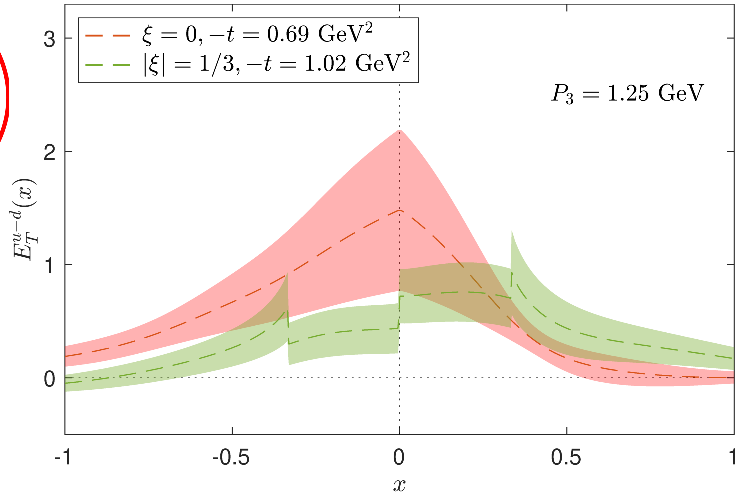
**light-cone GPD**

ETMC, Phys. Rev. D105 (2022) 034501

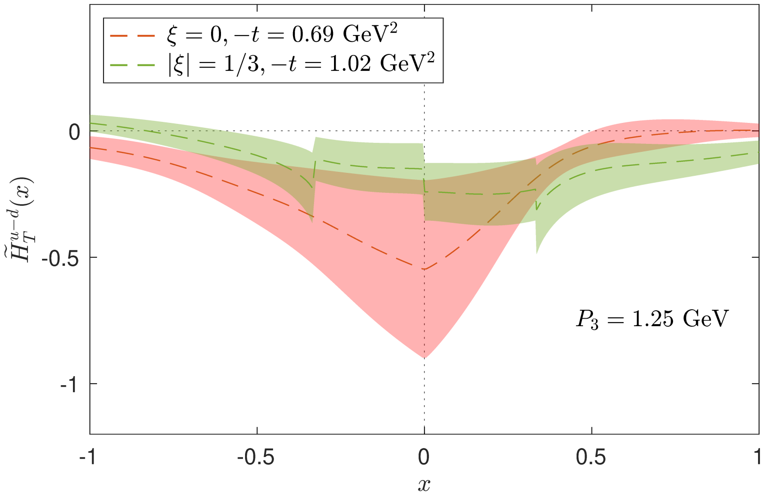
$H_T^{u-d}$ $(\xi = 0,\ 1/3)$




$E_T^{u-d}$ $(\xi = 0,\ 1/3)$




$\tilde{H}_T^{u-d}$ $(\xi = 0,\ 1/3)$



# Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

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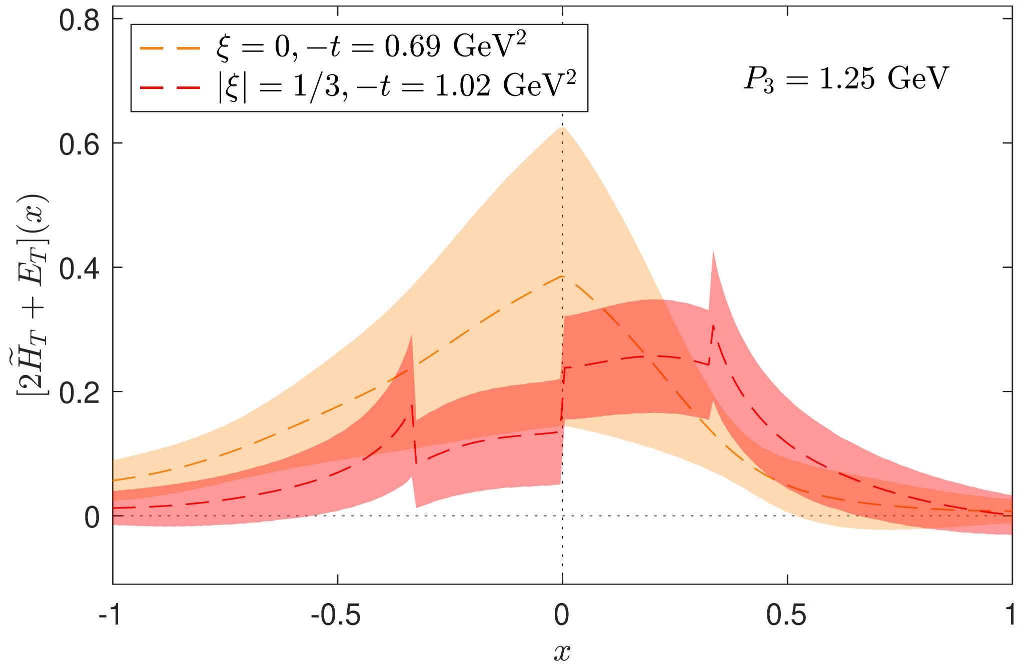
↓

**light-cone GPD**

More fundamental quantity: $E_T + 2\tilde{H}_T$

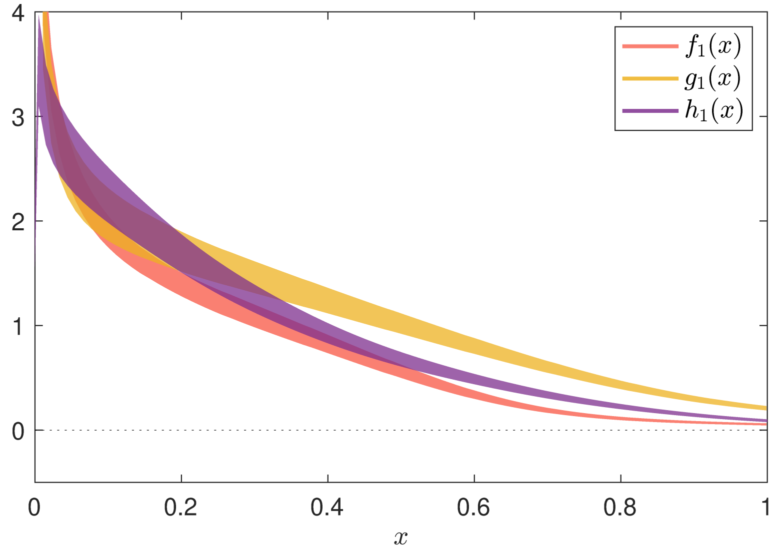
– related to the transverse spin structure of the proton
– physically interpreted as lateral deformation in the distribution of transversely polarized quarks in an unpolarized proton
– lowest Mellin moment in the forward limit: transverse spin-flavor dipole moment in an unpolarized target ($k_T$)
– second moment related to the transverse-spin quark angular momentum in an unpolarized proton



# Comparison of different types of PDFs/GPDs

ETMC, Phys. Rev. Lett. 125 (2020) 262001

ETMC, Phys. Rev. D105 (2022) 034501



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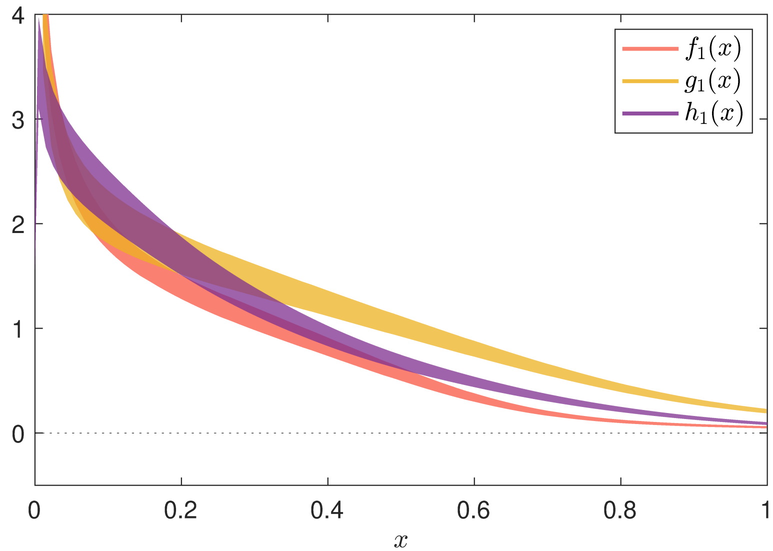
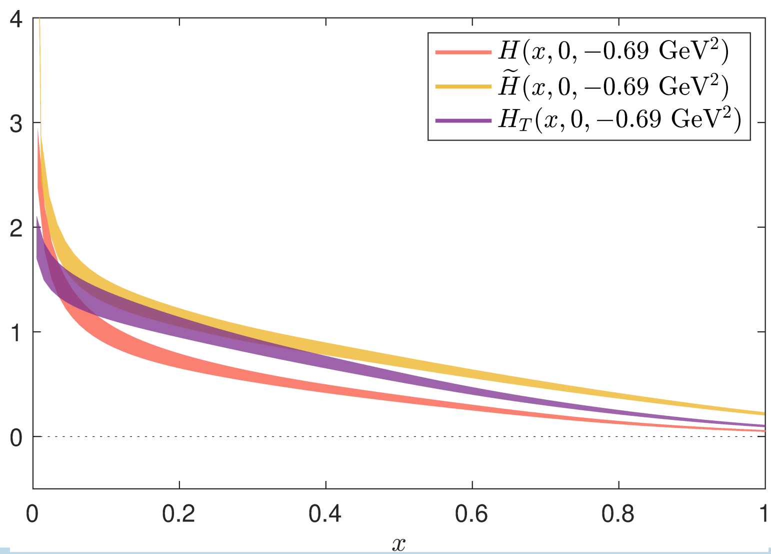
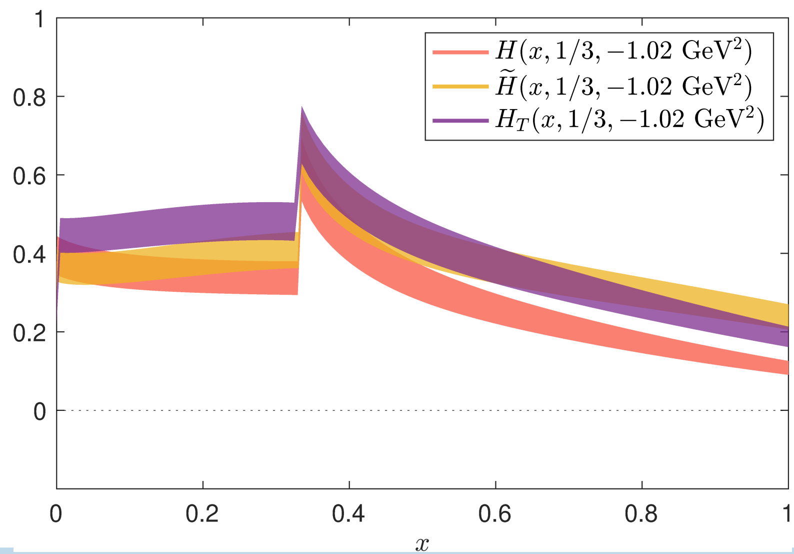
ETMC, Phys. Rev. D105 (2022) 034501



# Twist-3

PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.
LT: **twist-2** – probability densities for finding partons carrying fraction $x$ of the hadron momentum.

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LT: **twist-2** – probability densities for finding partons carrying fraction $x$ of the hadron momentum.

**Twist-3:**

- no density interpretation,
- contain important information about $qgq$ correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements very difficult.

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Exploratory studies:

- matching for twist-3 PDFs: $g_T$, $h_L$, $e$
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025
  BC-type sum rules S. Bhattacharya, A. Metz, 2105.07282
  Note: neglected $qgq$ correlations
  see also: V. Braun, Y. Ji, A. Vladimirov, JHEP 05(2021)086, 11(2021)087

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**Twist-3:**

| QUASI | TMF | $m_\pi = 260$ MeV | $a{=}0.093$ fm |
|---|---|---|---|

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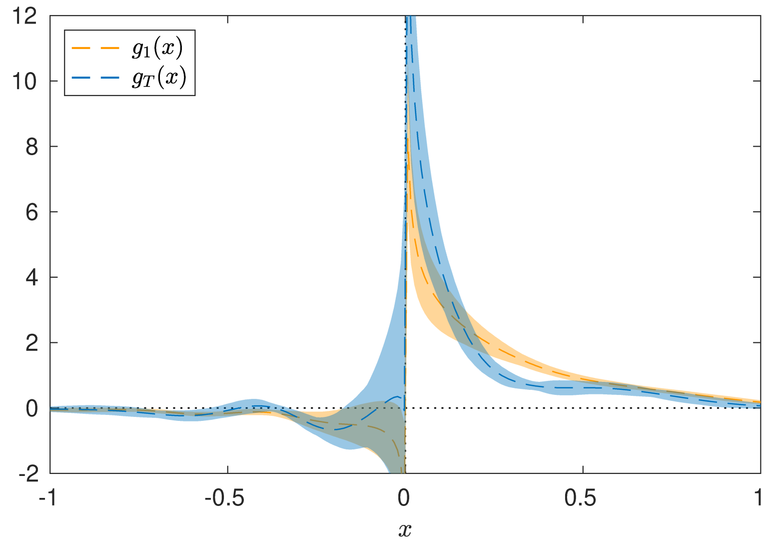


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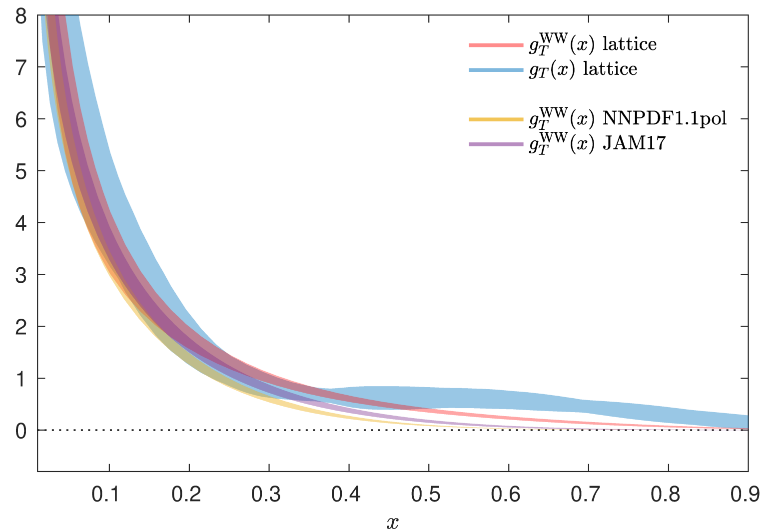

# Twist-3

PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

LT: **twist-2** – probability densities for finding partons carrying fraction $x$ of the hadron momentum.

**Twist-3:**

| QUASI | TMF | $m_\pi = 260$ MeV | $a=0.093$ fm |
|---|---|---|---|

- no density interpretation,
- contain important information about $qgq$ correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
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- important for JLab's 12 GeV program + for EIC,
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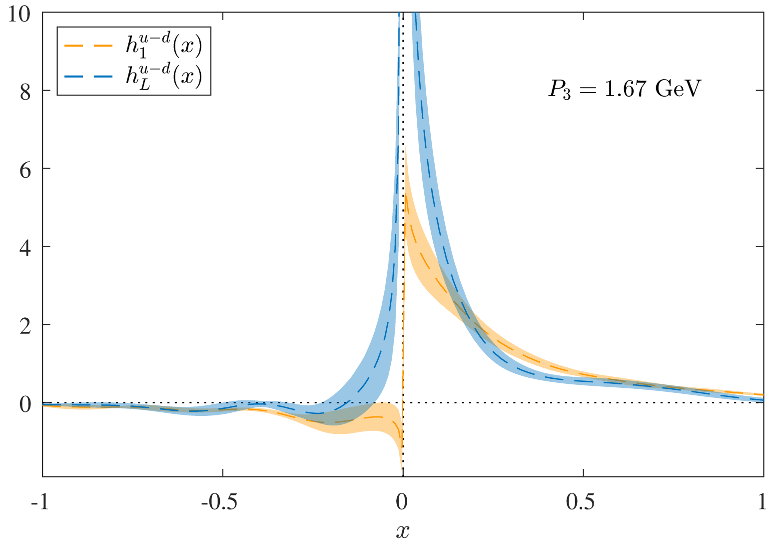


Exploratory studies:

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  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
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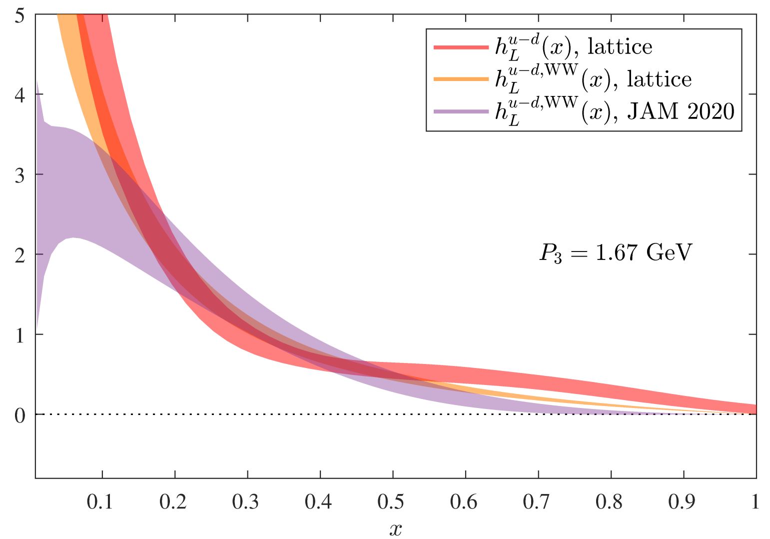

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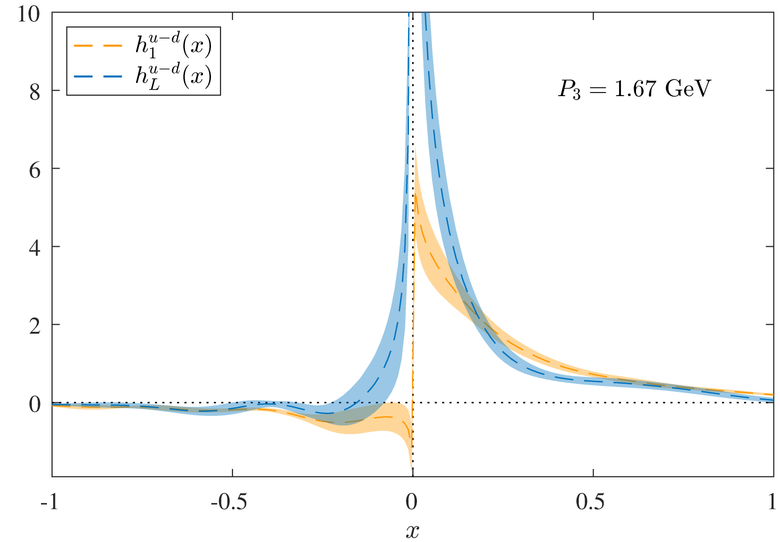


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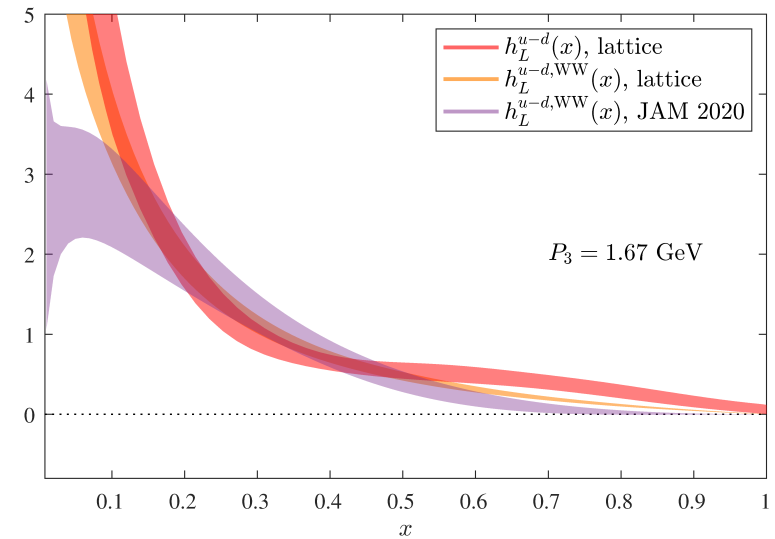
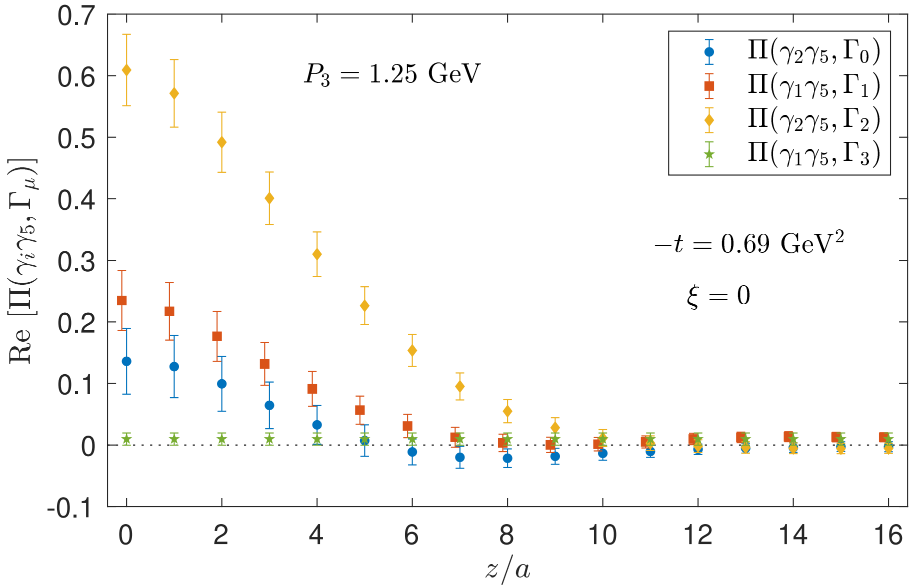


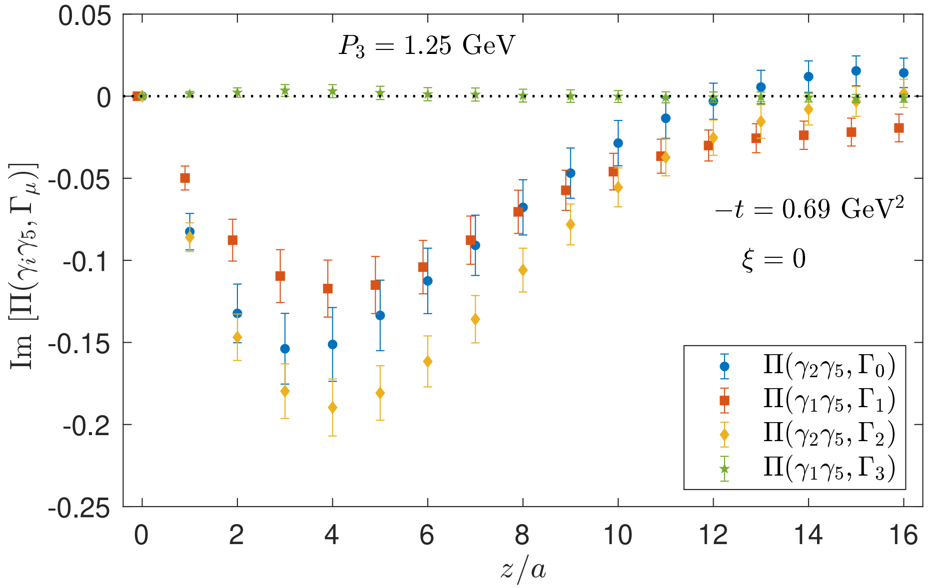
Very recently, we combined our explorations of GPDs and of twist-3 distributions
S. Bhattacharya et al., 2112.05538

Twist-3 axial GPDs: $\widetilde{G}_1, \widetilde{G}_2, \widetilde{G}_3, \widetilde{G}_4$

$$h_{\gamma^j\gamma_5} = \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma_5}{2m}\rangle\rangle [F_{\widetilde{E}} + F_{\widetilde{G}_1}] + \langle\langle g_\perp^{j\rho}\gamma_\rho\gamma_5\rangle\rangle [F_{\widetilde{H}} + F_{\widetilde{G}_2}] + \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma^+\gamma_5}{P^+}\rangle\rangle F_{\widetilde{G}_3} + \langle\langle \frac{i\epsilon_\perp^{j\rho}\Delta_\rho\gamma^+}{P^+}\rangle\rangle F_{\widetilde{G}_4}\,.$$

Bare ME: (same lattice setup)

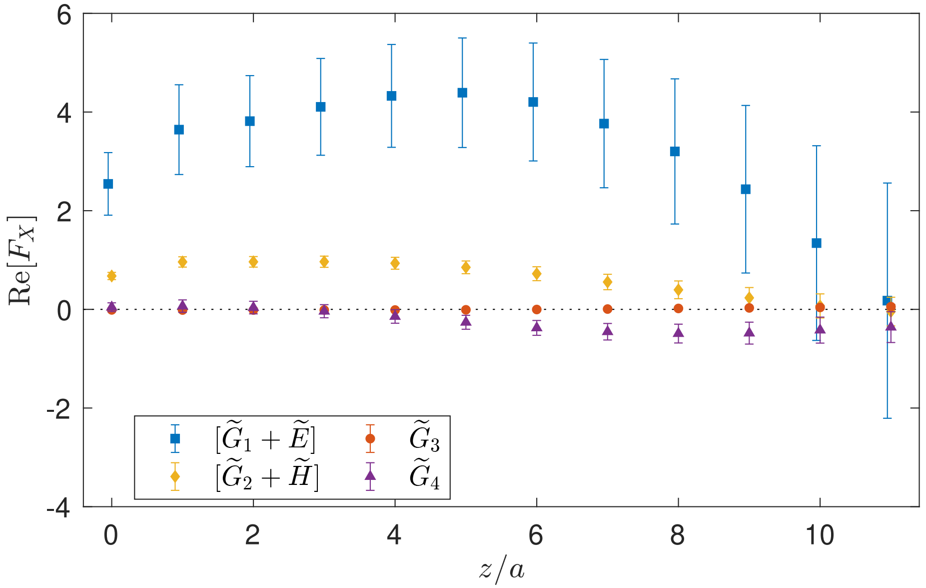


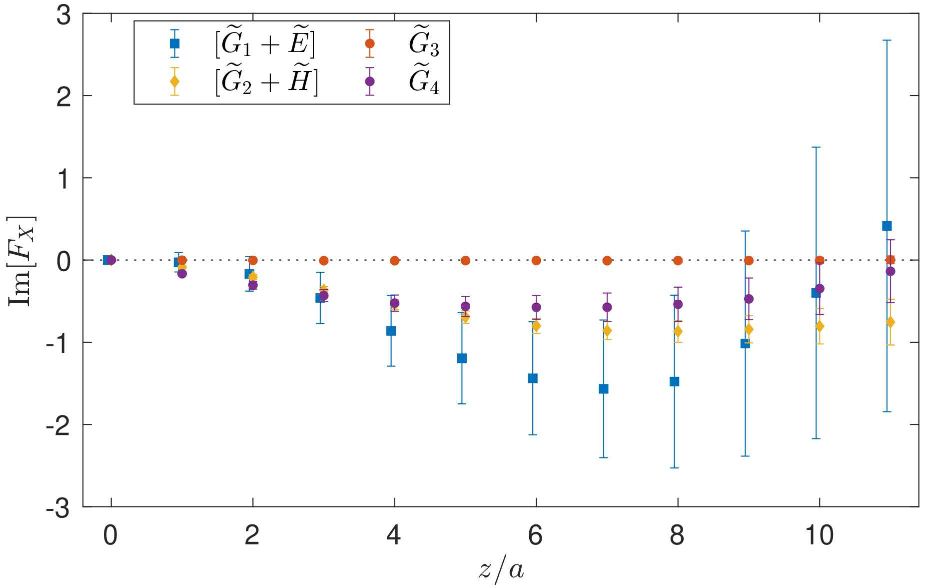



Contributions from different insertions and projectors ($\vec{Q} = (Q_x, 0, 0)$):

$\Pi(\gamma^2\gamma^5, \Gamma_0)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^2\gamma^5, \Gamma_2)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^1\gamma^5, \Gamma_1)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{E} + \widetilde{G}_1$,
$\Pi(\gamma^1\gamma^5, \Gamma_3)$: $\widetilde{G}_3$.

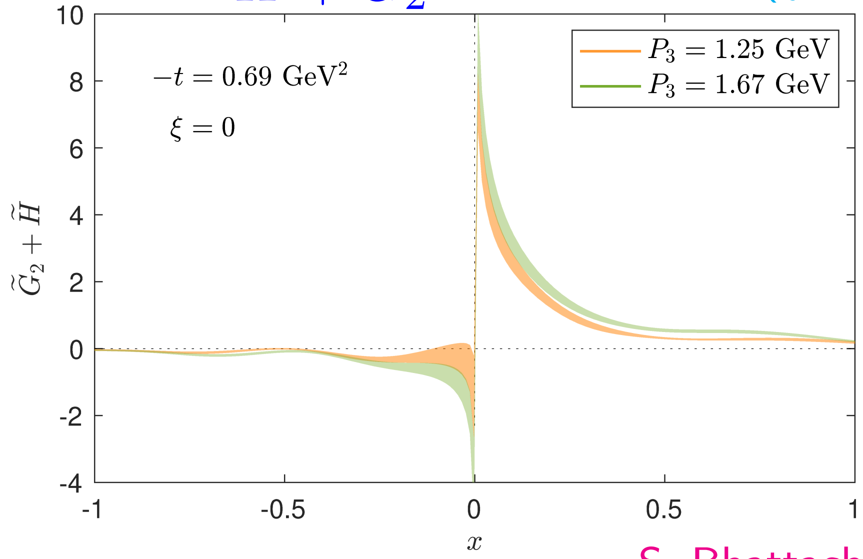
S. Bhattacharya et al., 2112.05538

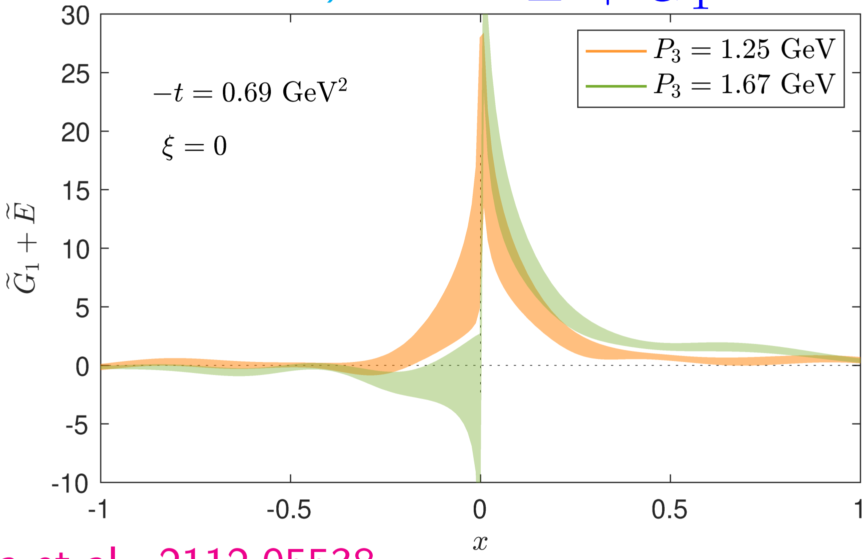


## First exploration of twist-3 GPDs

$\widetilde{H} + \widetilde{G}_2$ $(\xi = 0, -t = 0.69\ \text{GeV}^2)$ $\widetilde{E} + \widetilde{G}_1$

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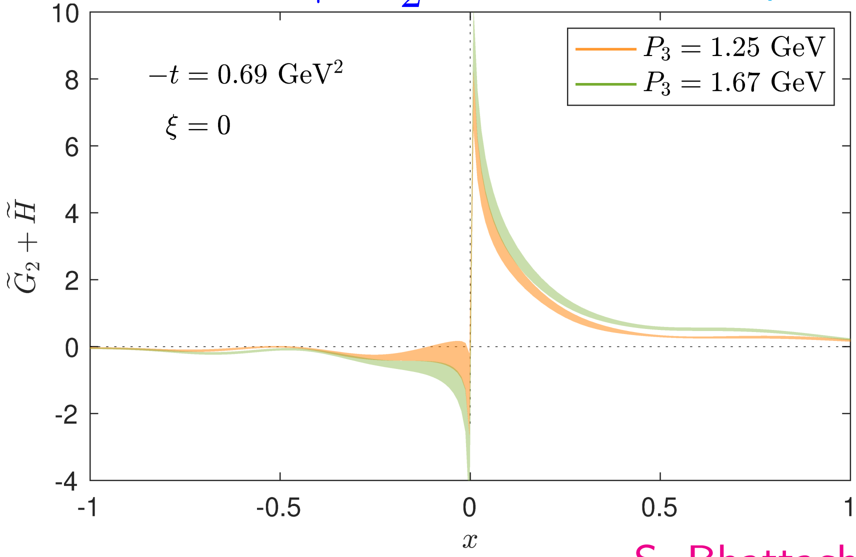
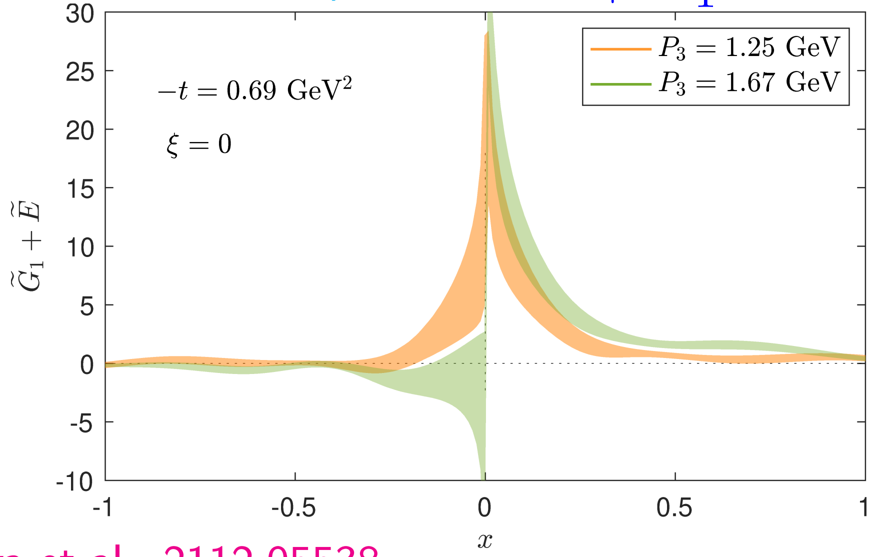
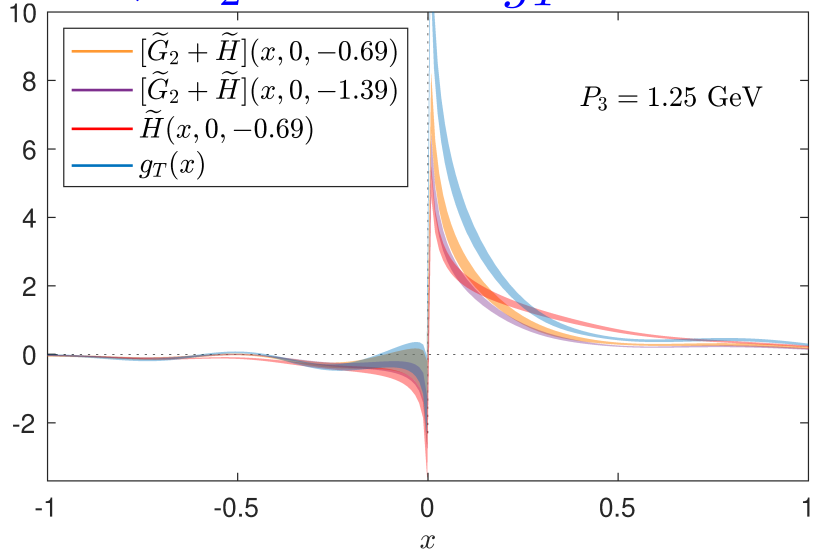
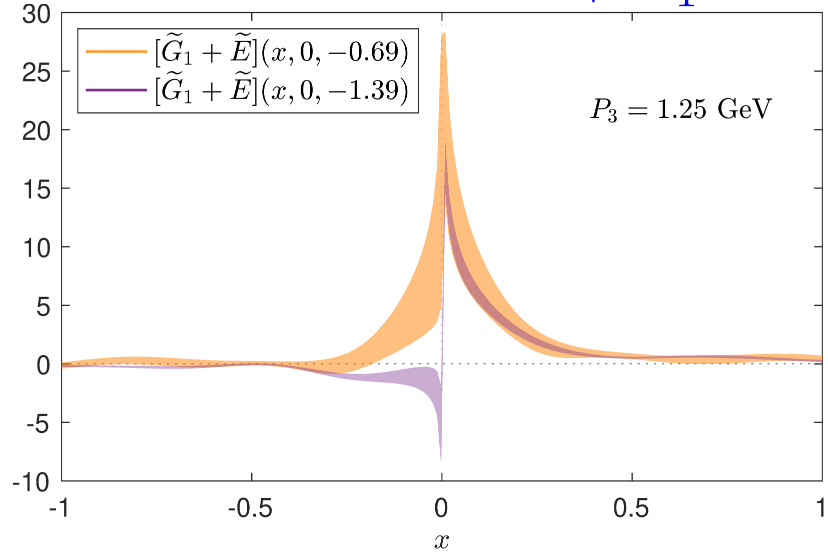



S. Bhattacharya et al., 2112.05538

$\widetilde{H} + \widetilde{G}_2$ vs. $\widetilde{H}$ vs. $g_T$ $(\xi = 0, -t = 0.69, 1.39$ GeV$^2)$ $\widetilde{E} + \widetilde{G}_1$



# Conclusions and prospects

- **Huge progress in lattice calculations of GPDs!**
- Recent breakthrough:
  - ⋆ computationally more efficient calculations in non-symmetric frames,
  - ⋆ alternative definitions of GPDs can provide faster convergence to the light-cone.
- Overall very encouraging results!
- Still several challenges to overcome (control of systematics).
- Obviously, GPDs much more challenging than PDFs.
- Expect slow, but consistent progress and complementary role to pheno.

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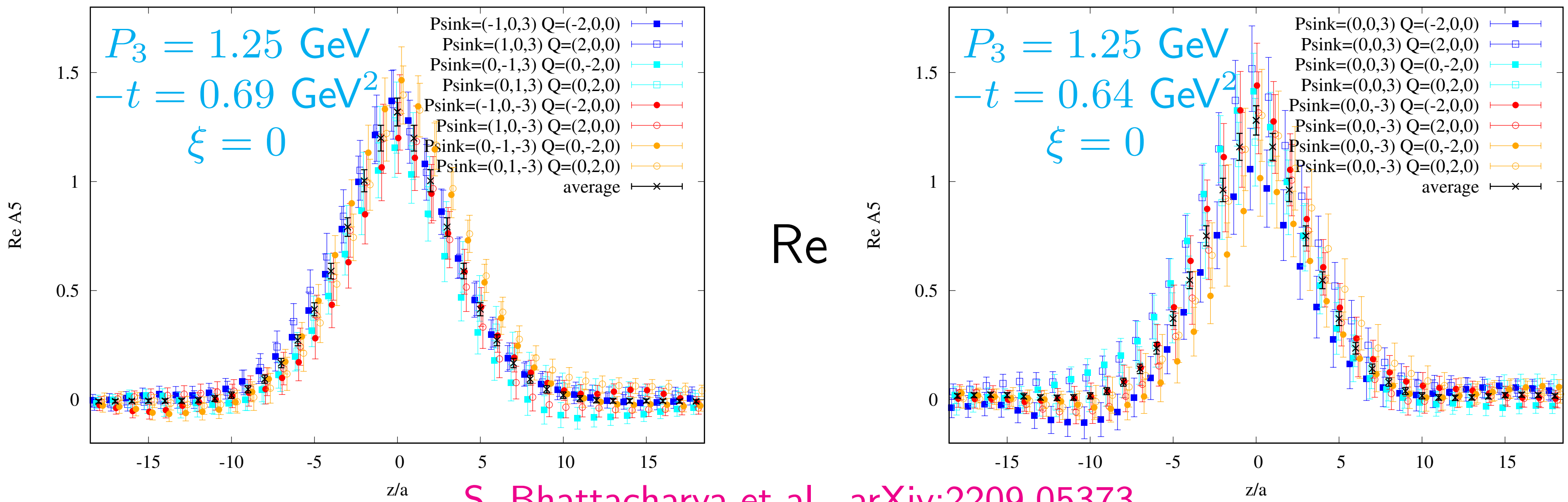
Thank you for your attention!

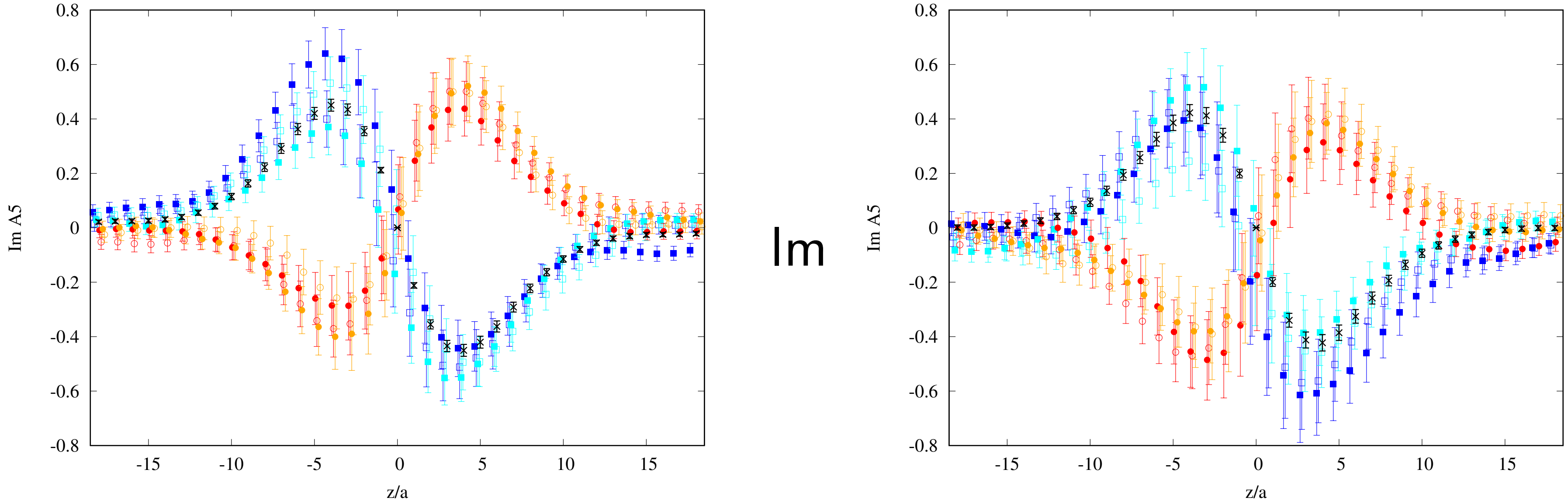
# Backup slides

# Example amplitude $A_5$

symmetric frame non-symmetric frame

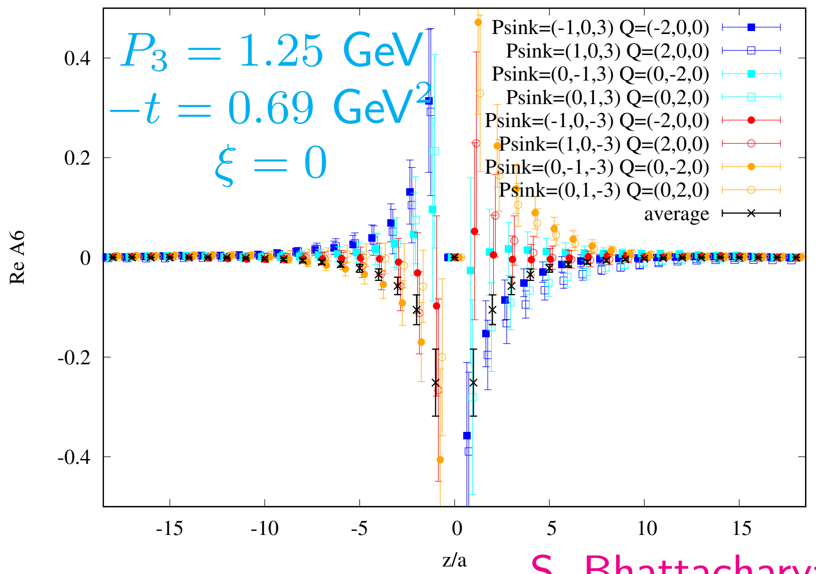


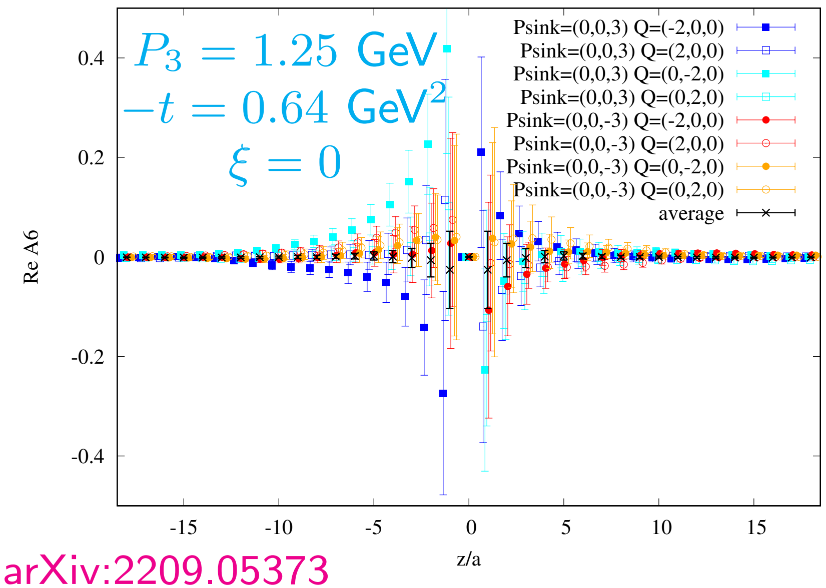

S. Bhattacharya et al., arXiv:2209.05373



# Example amplitude $A_6$

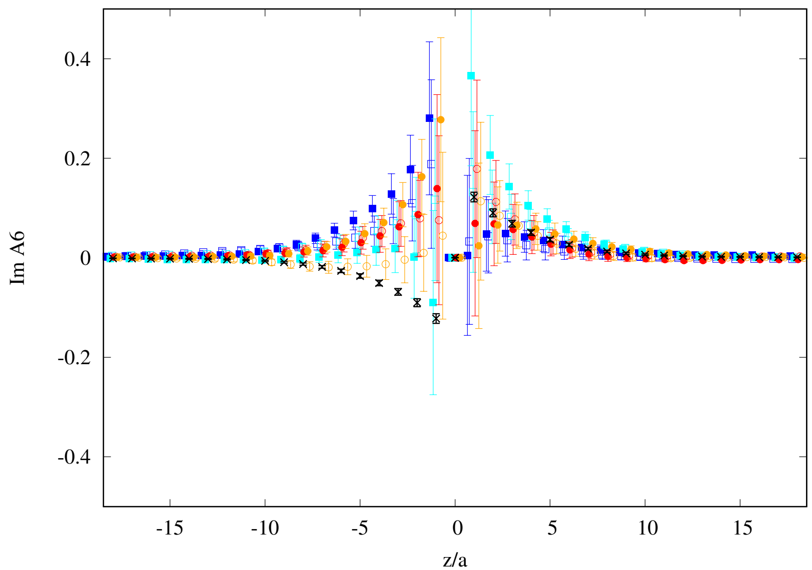
symmetric frame | non-symmetric frame

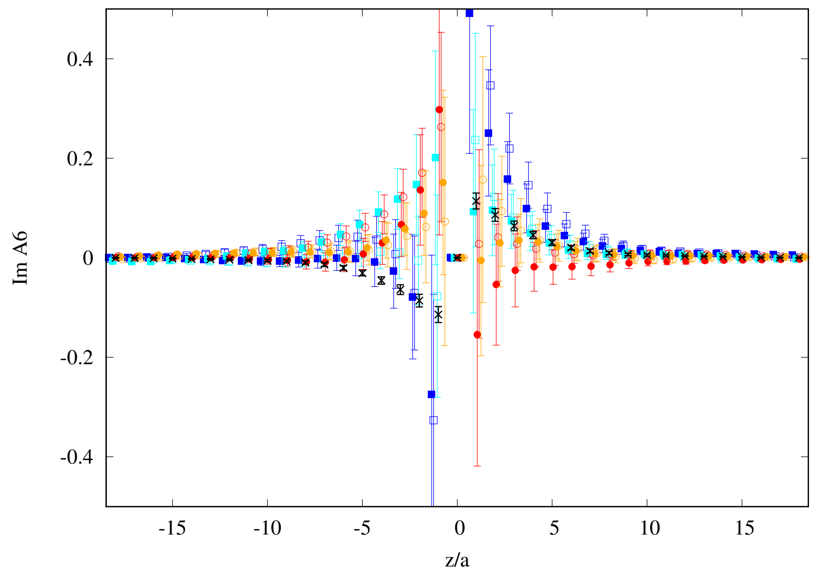



Re

S. Bhattacharya et al., arXiv:2209.05373




Im

Transversity GPDs: ETMC, Phys. Rev. D105 (2022) 034501
4 GPDs: $H_T$, $E_T$, $\tilde{H}_T$, $\tilde{E}_T$

**spatial correlation in a boosted nucleon**
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
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intermediate RI scheme

↓

reconstruction of $x$-dependence
$z$-space→$x$-space
Backus-Gilbert

↓

matching to light cone
RI→ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

**light-cone GPD**

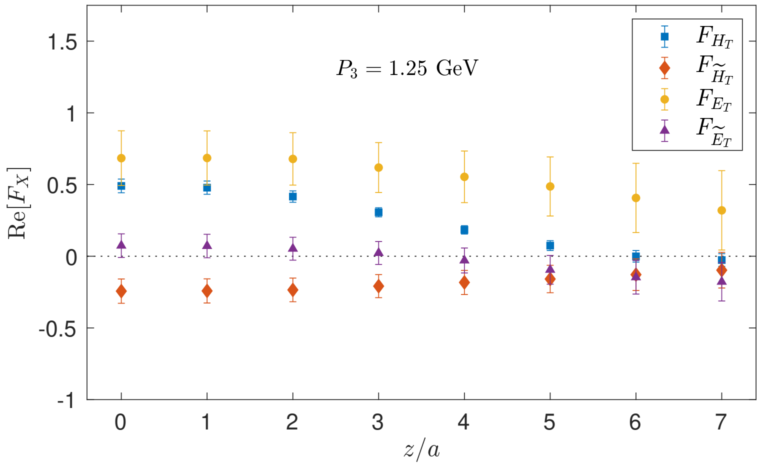
Three nucleon boosts ($\xi = 0$): $P_3 = 0.83, 1.25, 1.67$ GeV
Nucleon boost ($\xi \neq 0$): $P_3 = 1.25$ GeV

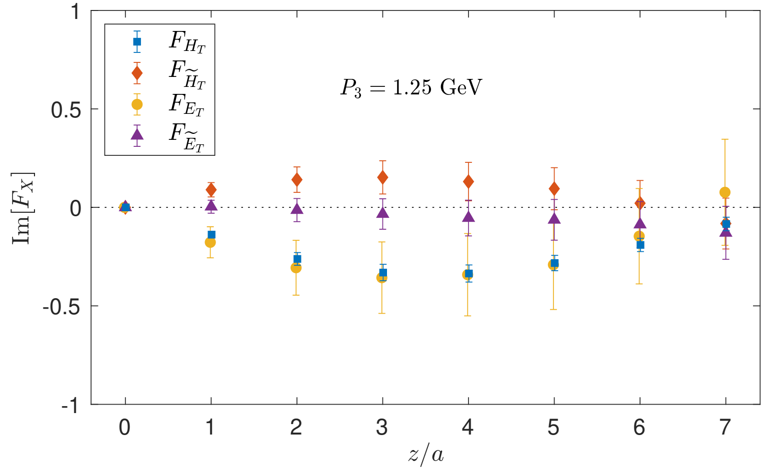
Momentum transfer ($\xi = 0$): $-t = 0.69$ GeV$^2$
Momentum transfer ($\xi \neq 0$): $-t = 1.02$ GeV$^2$

Renormalized ME

Real part | Imaginary part

$\xi = 1/3$



# Transversity GPDs

ETMC, Phys. Rev. D105 (2022) 034501

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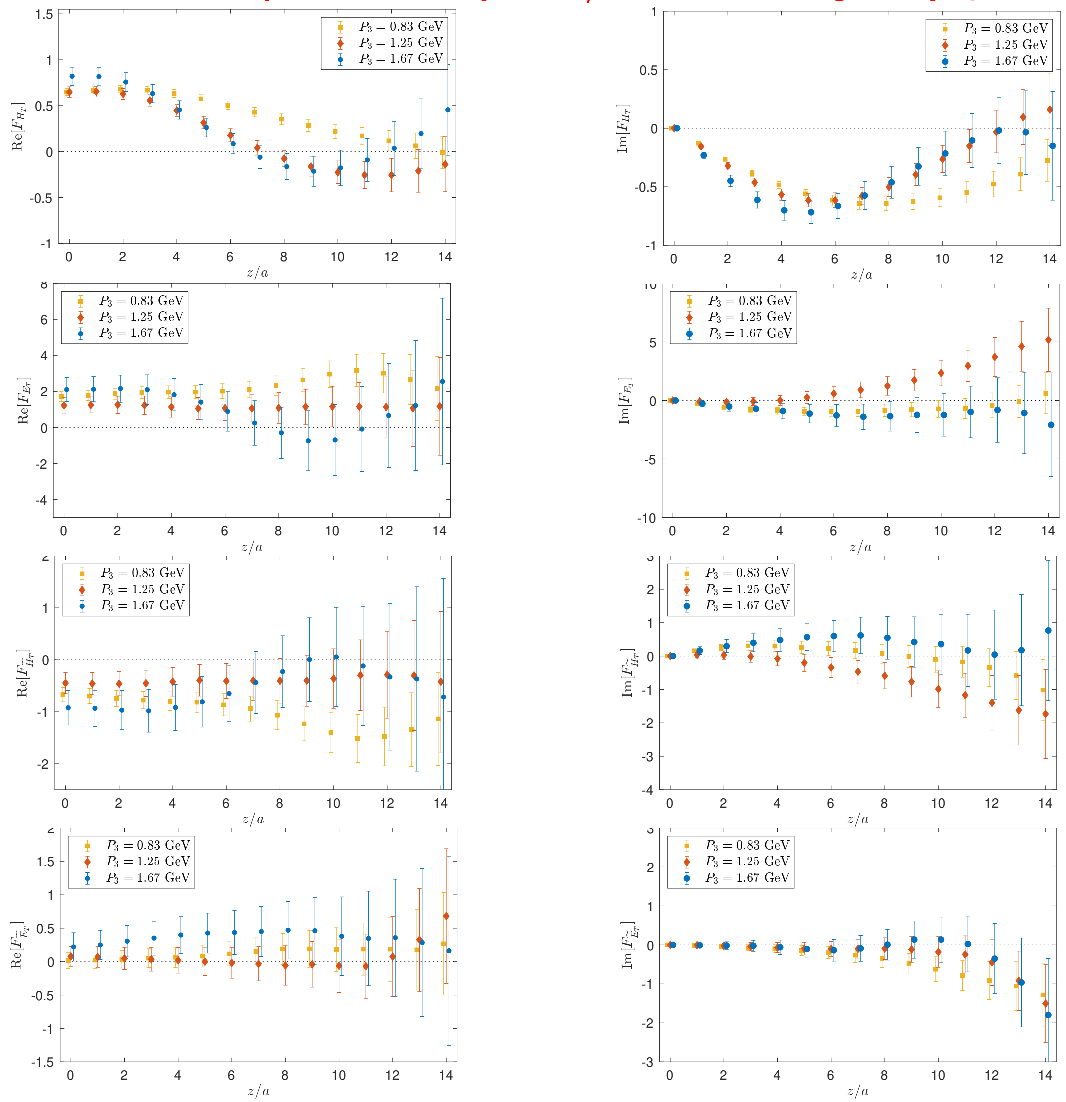

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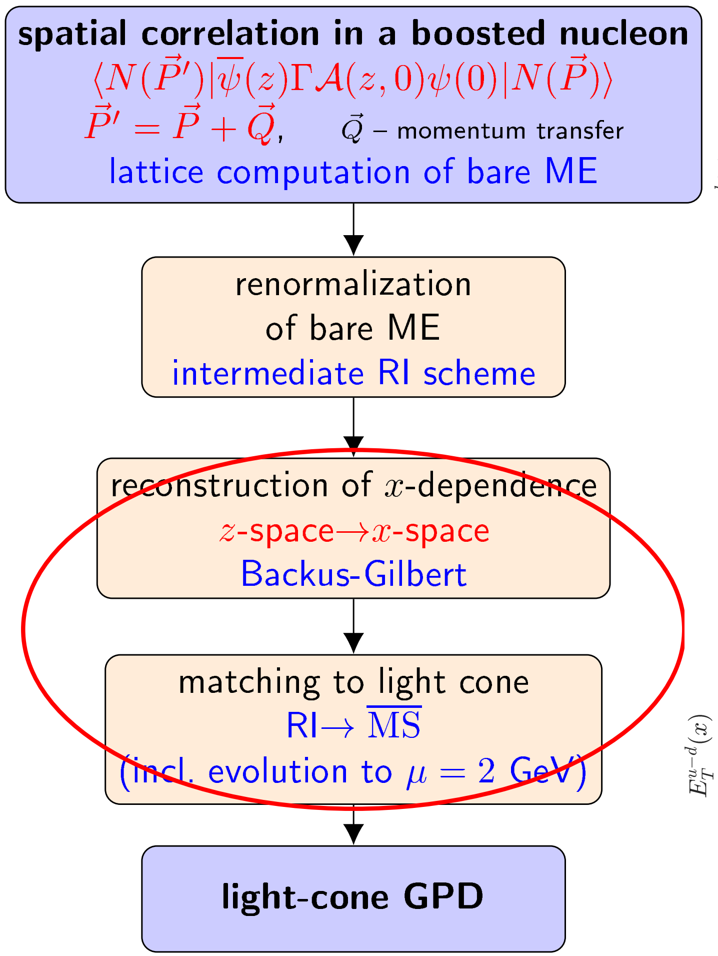


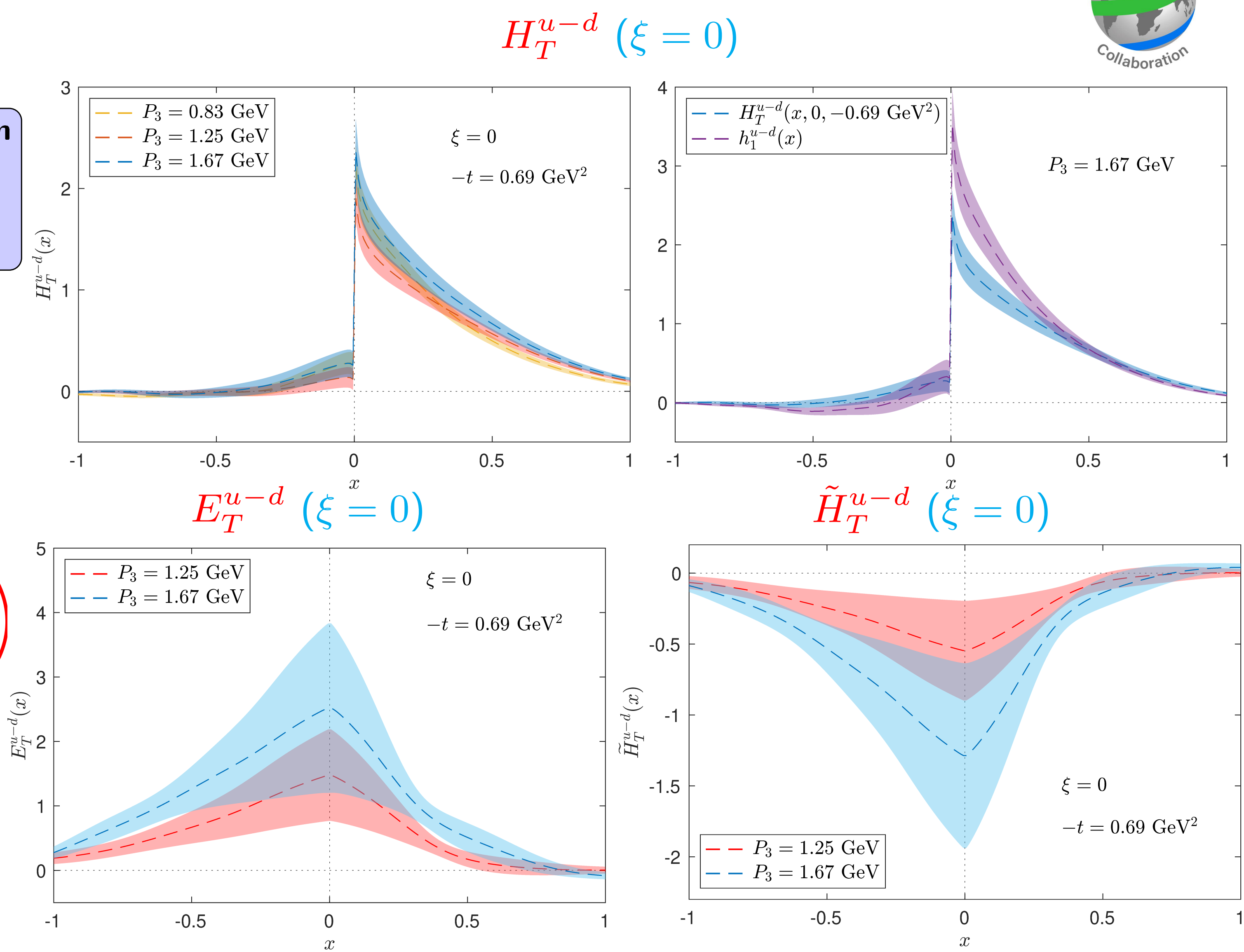

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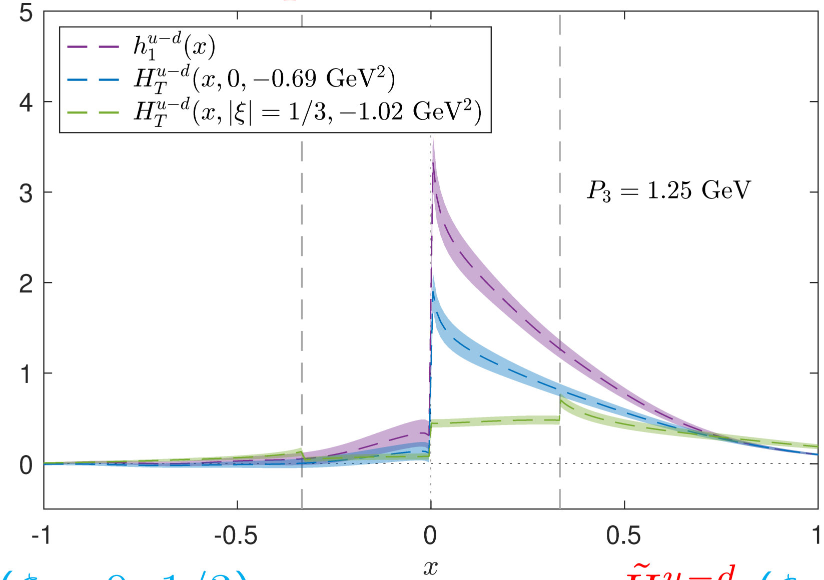
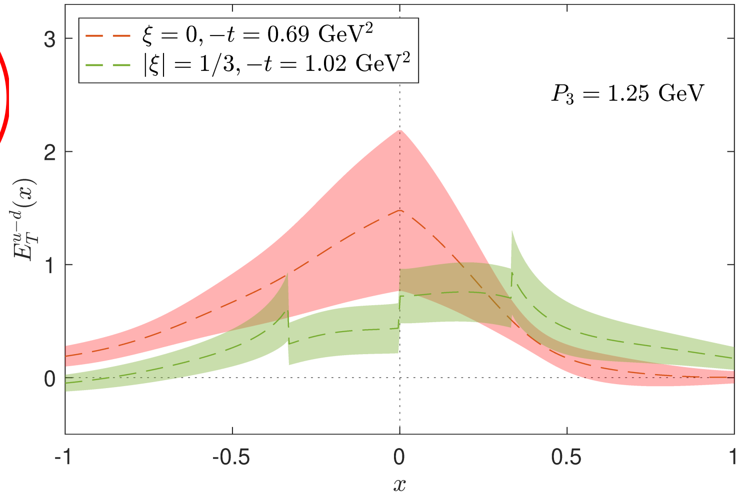
ETMC, Phys. Rev. D105 (2022) 034501

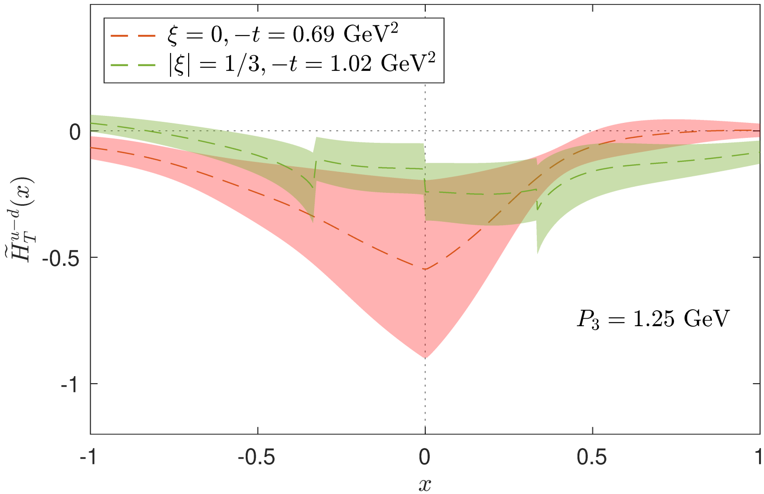
$H_T^{u-d}$ $(\xi = 0,\ 1/3)$




$E_T^{u-d}$ $(\xi = 0,\ 1/3)$




$\tilde{H}_T^{u-d}$ $(\xi = 0,\ 1/3)$



# Moments of transversity GPDs

$n = 0$ Mellin moments:

$$\begin{aligned}
\int_{-1}^{1} dx\, H_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, H_{Tq}(x,\xi,t,P_3) = A_{T10}(t)\,,\\
\int_{-1}^{1} dx\, E_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, E_{Tq}(x,\xi,t,P_3) = B_{T10}(t)\,,\\
\int_{-1}^{1} dx\, \widetilde{H}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{H}_{Tq}(x,\xi,t,P_3) = \widetilde{A}_{T10}(t)\,,\\
\int_{-1}^{1} dx\, \widetilde{E}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{E}_{Tq}(x,\xi,t,P_3) = 0\,,
\end{aligned} \quad (1)$$

- lowest moments of GPDs skewness-independent,
- lowest moments of quasi-GPDs boost-independent.

$n = 1$ Mellin moments (related to GFF of one-derivative tensor operator):

$$\begin{aligned}
\int_{-1}^{1} dx\, x\, H_T(x,\xi,t) &= A_{T20}(t)\,,\\
\int_{-1}^{1} dx\, x\, E_T(x,\xi,t) &= B_{T20}(t)\,,\\
\int_{-1}^{1} dx\, x\, \widetilde{H}_T(x,\xi,t) &= \widetilde{A}_{T20}(t)\,, \quad (3)\\
\int_{-1}^{1} dx\, x\, \widetilde{E}_T(x,\xi,t) &= 2\xi \widetilde{B}_{T21}(t)\,, \quad (2)
\end{aligned}$$

- skewness-dependence only in for $\widetilde{E}_T$ (only $\xi$-odd GPD).

| Moments of | $H_T(x,\xi=0,t=-0.69\,\mathrm{GeV}^2)$ | | | $H_T(x,\xi=1/3,t=-1.02\,\mathrm{GeV}^2)$ |
|---|---|---|---|---|
| | $P_3=0.83$ GeV | $P_3=1.25$ GeV | $P_3=1.67$ GeV | $P_3=1.25$ GeV |
| $H_{Tq}$ | 0.65(4) | 0.64(6) | 0.81(10) | 0.49(5) |
| $H_T$ | 0.69(4) | 0.67(6) | 0.84(10) | 0.45(4) |
| $xH_T$ | 0.20(2) | 0.21(2) | 0.24(3) | 0.15(2) |
| $A_{T10}\ (z=0)$ | 0.65(4) | 0.65(6) | 0.82(10) | 0.49(5) |

**Mellin moments $P_3$-independent, preserved by matching, suppressed with increasing $-t$.**

| Moments of | $E_T(x,\xi=0,t=-0.69\,\mathrm{GeV}^2)$ | | | $H_T(x,\xi=1/3,t=-1.02\,\mathrm{GeV}^2)$ |
|---|---|---|---|---|
| | $P_3=0.83$ GeV | $P_3=1.25$ GeV | $P_3=1.67$ GeV | $P_3=1.25$ GeV |
| $E_{Tq}$ | | 1.20(42) | 2.05(65) | 0.67(19) |
| $E_T$ | | 1.15(43) | 2.10(67) | 0.73(19) |
| $xE_T$ | | 0.06(4) | 0.13(5) | 0.11(11) |
| $B_{T10}\ (z=0)$ | 1.71(28) | 1.22(43) | 2.10(67) | 0.68(19) |

| Moments of | $\widetilde{H}_T(x,\xi=0,t=-0.69\,\mathrm{GeV}^2)$ | | | $\widetilde{H}_T(x,\xi=1/3,t=-1.02\,\mathrm{GeV}^2)$ |
|---|---|---|---|---|
| | $P_3=0.83$ GeV | $P_3=1.25$ GeV | $P_3=1.67$ GeV | $P_3=1.25$ GeV |
| $\widetilde{H}_{Tq}$ | | -0.44(20) | -0.90(32) | -0.26(9) |
| $\widetilde{H}_T$ | | -0.42(21) | -0.92(33) | -0.27(9) |
| $x\widetilde{H}_T$ | | -0.17(8) | -0.30(10) | -0.05(5) |
| $\widetilde{A}_{T10}\ (z=0)$ | -0.67(14) | -0.45(21) | -0.92(33) | -0.24(8) |

**Similar conclusions (but very large errors).**