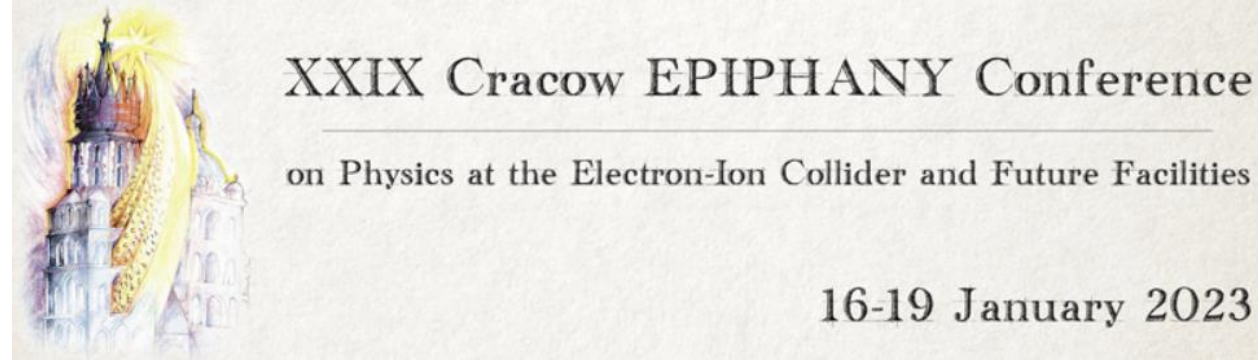


Hunting for gluon **O**rbital **A**ngular **M**omentum at the EIC



Shohini Bhattacharya

RIKEN BNL/BNL

18 January 2023

In Collaboration with:

Renaud Boussarie (CPHT, CNRS)

Yoshitaka Hatta (RIKEN BNL/BNL)

Based on:

PRL 128, 182002 (arXiv: 2201.08709)

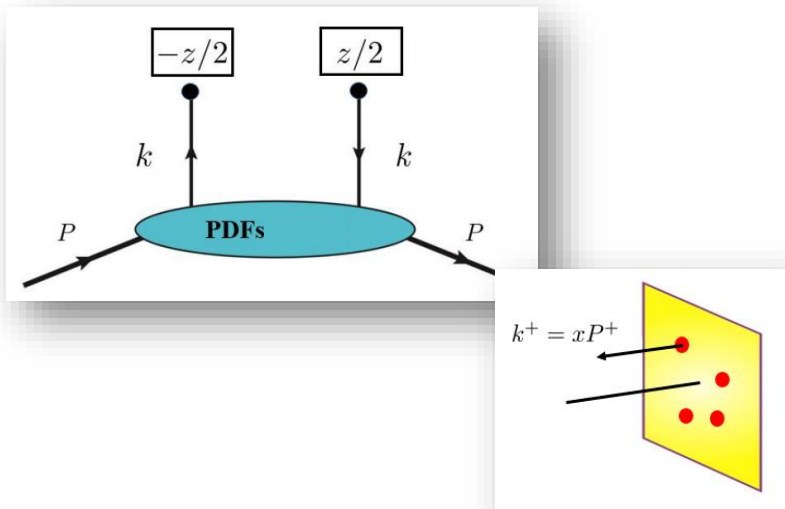


Outline

- **Generalized TMDs (GTMDs) & gluon OAM**
- **Exclusive dijet production as a probe of gluon OAM**
- **Summary**



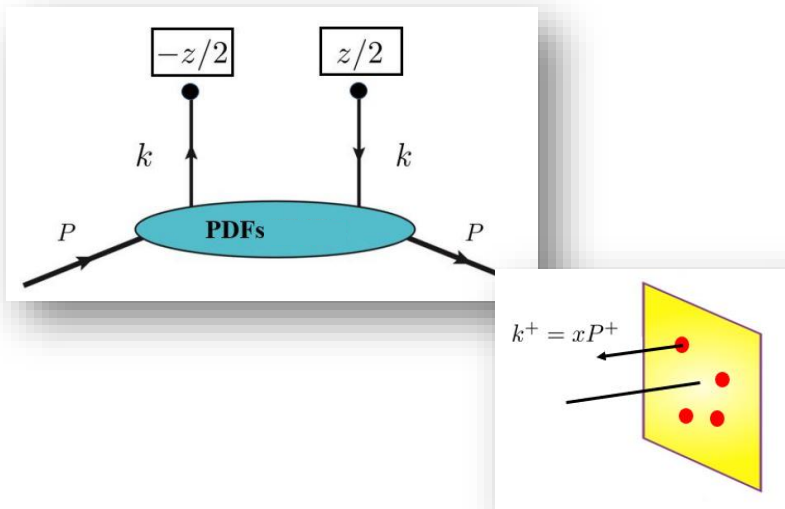
Non-perturbative functions



PDFs (x)

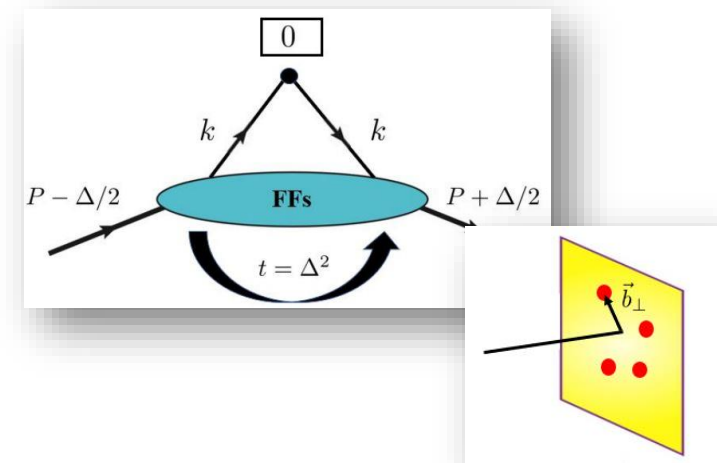


Non-perturbative functions



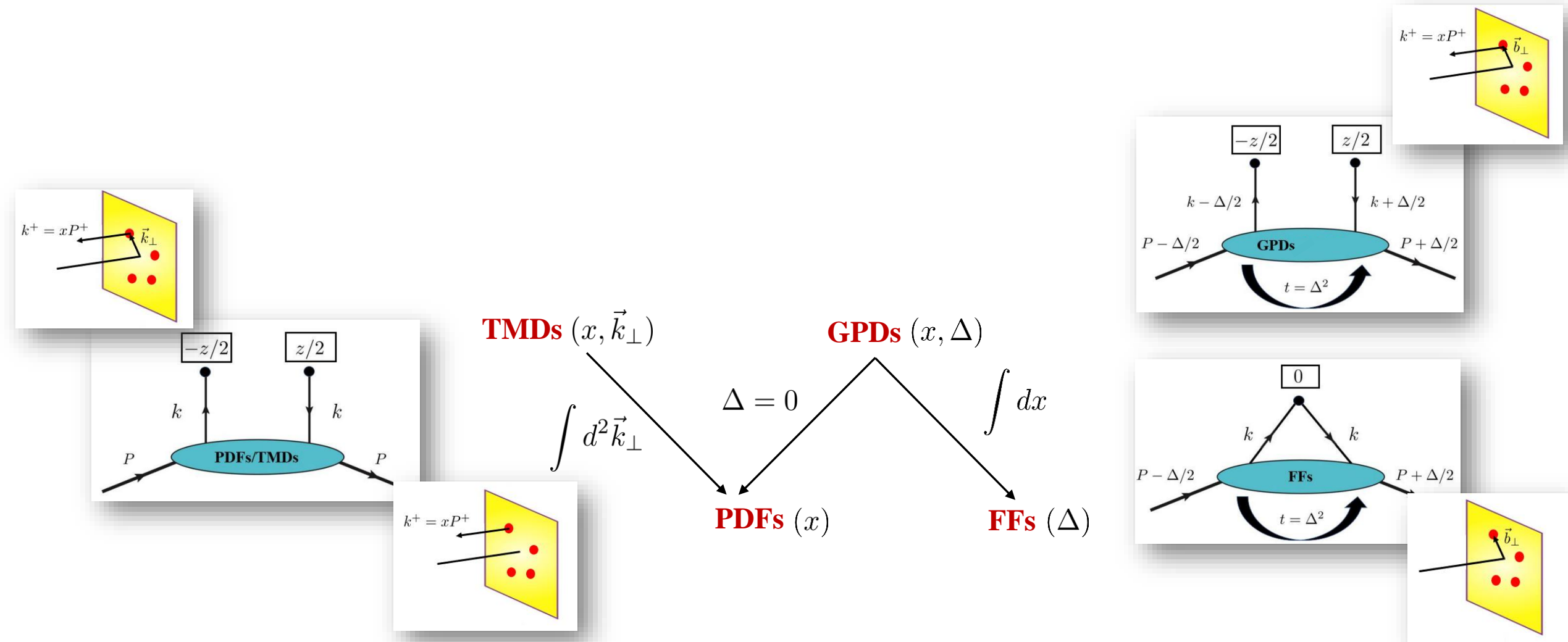
PDFs (x)

FFs (Δ)





Non-perturbative functions

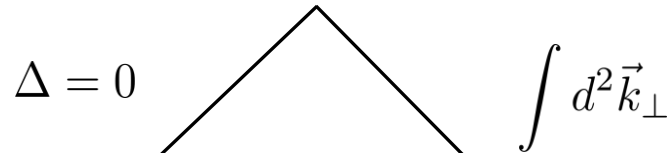




Non-perturbative functions

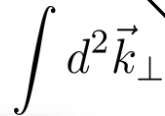
Generalized Transverse Momentum-dependent Distributions

(Meissner, Metz, Schlegel, 2009) **GTMDs** $(x, \vec{k}_\perp, \Delta)$



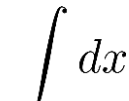
TMDs (x, \vec{k}_\perp)

GPDs (x, Δ)

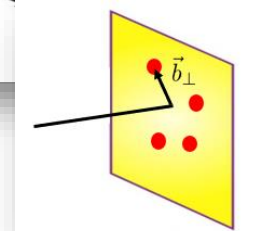
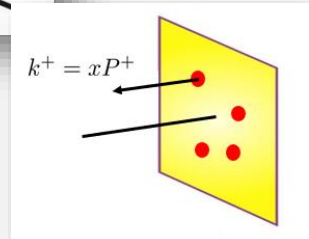
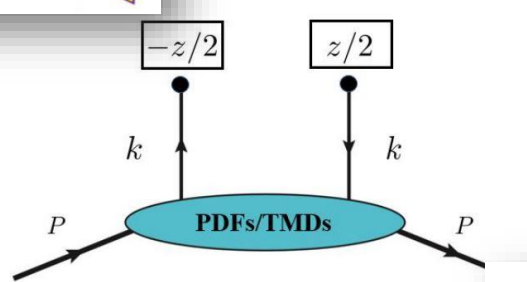
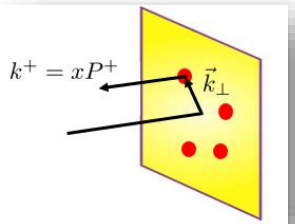
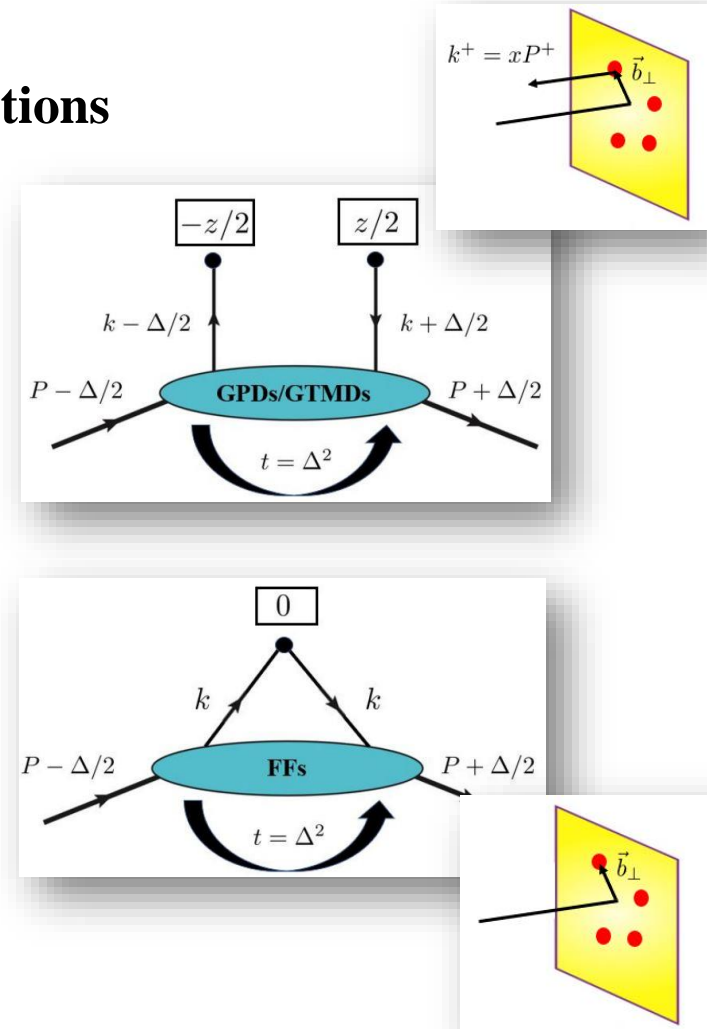


PDFs (x)

$\Delta = 0$



FFs (Δ)



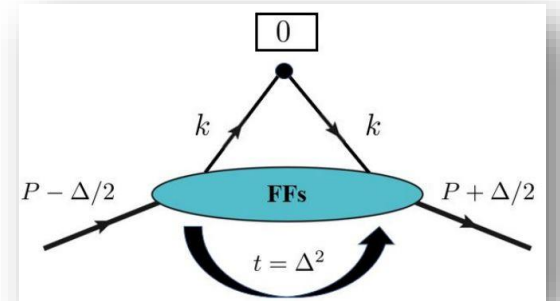
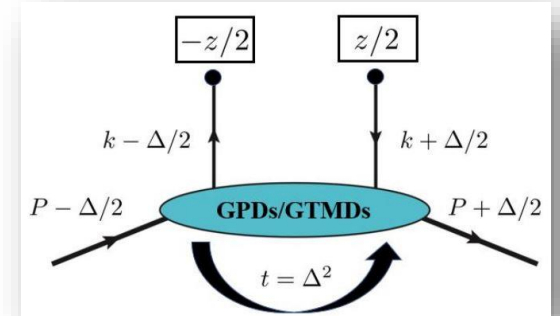
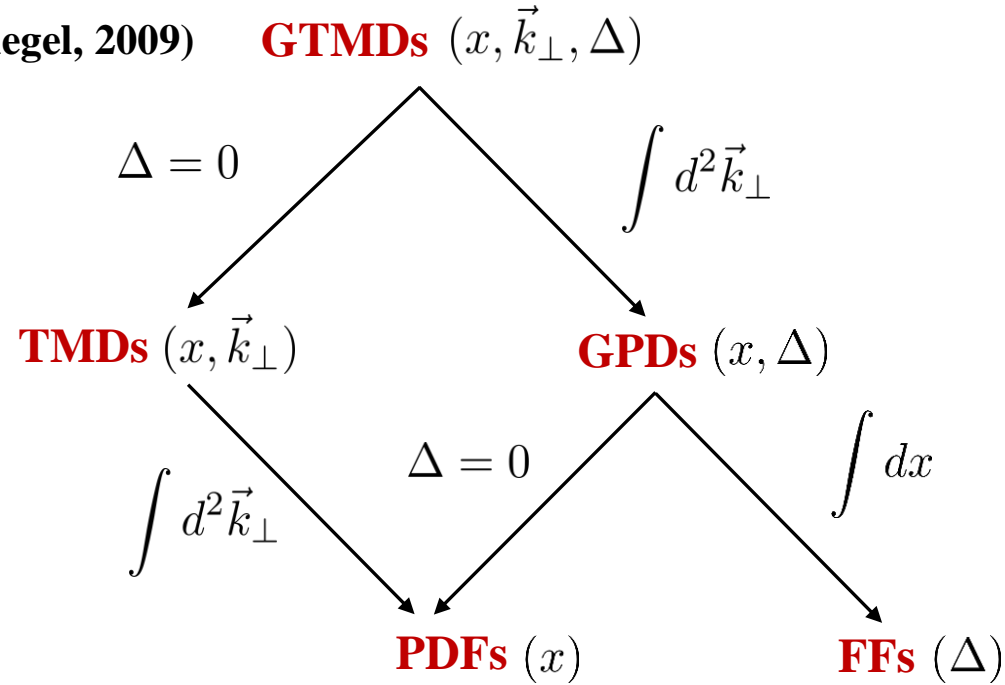
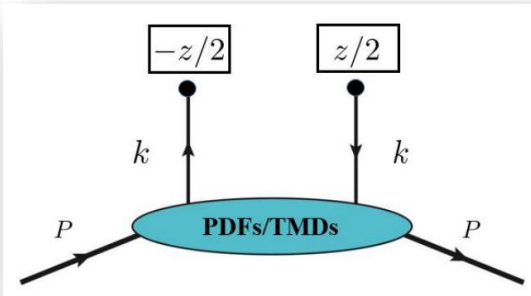


Why are GTMDs interesting?

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1) GTMDs are the “Mother Functions”

(Meissner, Metz, Schlegel, 2009) **GTMDs** $(x, \vec{k}_\perp, \Delta)$

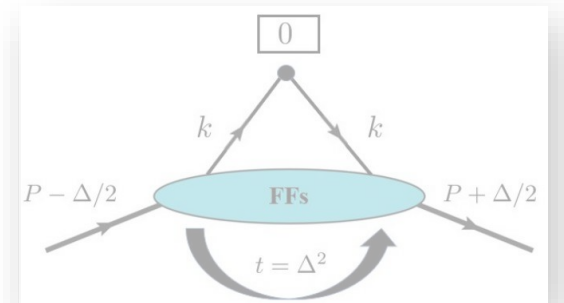
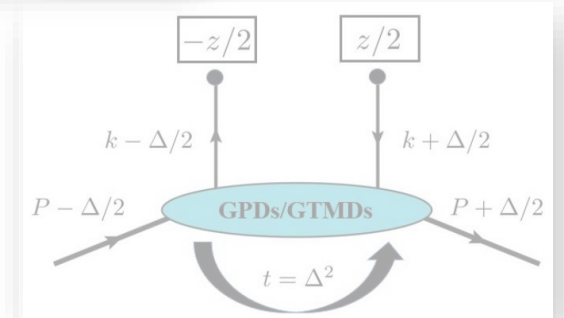
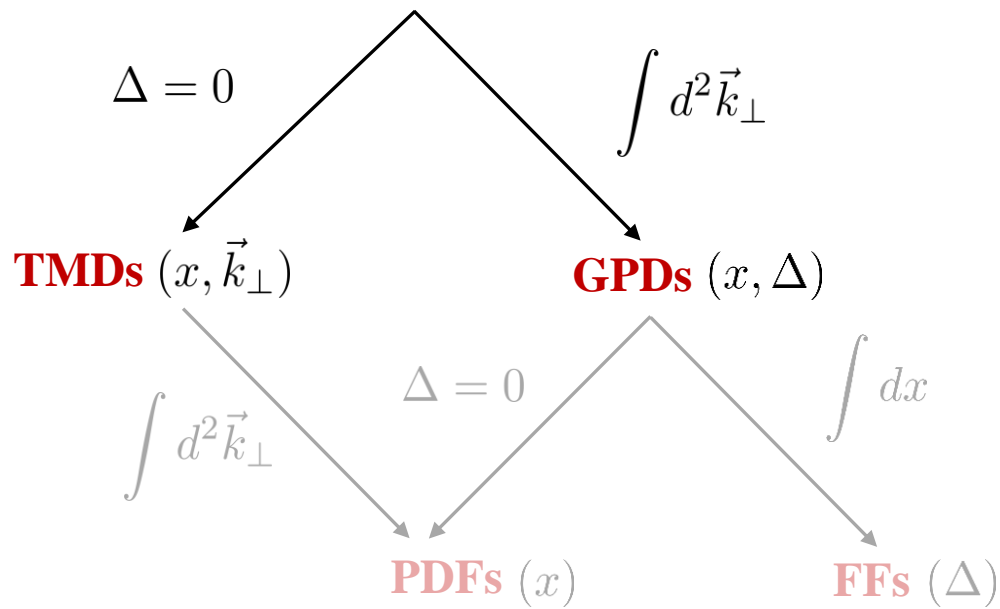
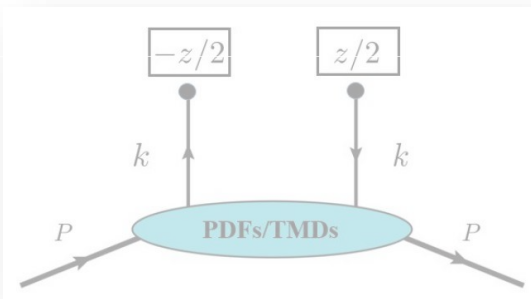


Why are GTMDs interesting?

1) **GTMDs are the “Mother Functions”**

2) **GTMDs contain physics beyond TMDs & GPDs**

(Meissner, Metz, Schlegel, 2009) **GTMDs** $(x, \vec{k}_\perp, \Delta)$





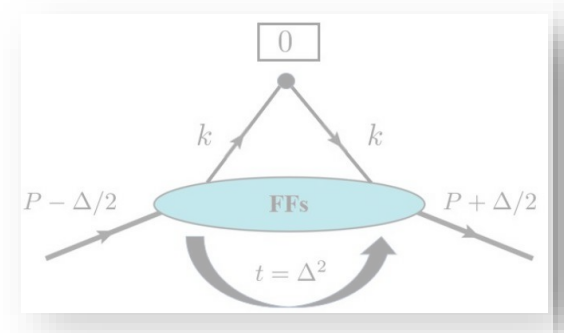
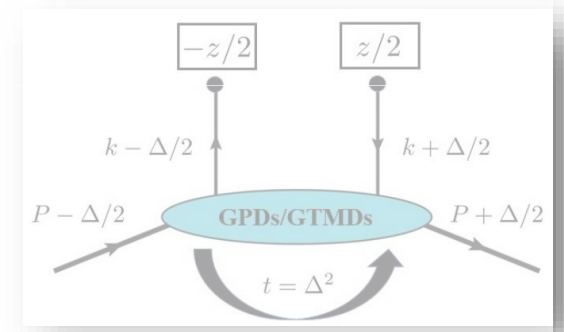
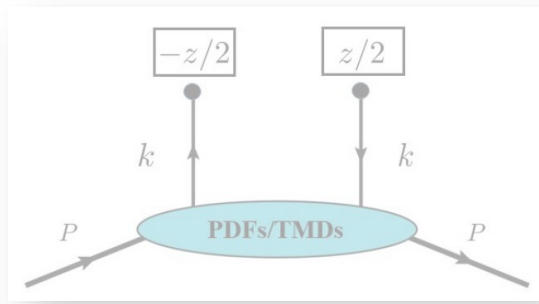
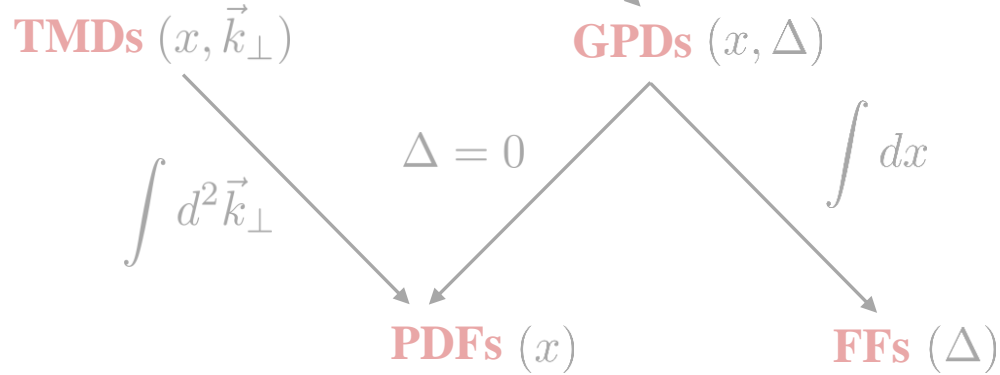
Why are GTMDs interesting?

3) Connection to Wigner functions **Wigner Distribution** $(x, \vec{k}_\perp, \vec{b}_\perp)$ (Belitsky, Ji, Yuan, 2003)

2-D Fourier Transform
 $(\vec{\Delta}_\perp)$ \uparrow $\xi = 0$

(Meissner, Metz, Schlegel, 2009) **GTMDs** $(x, \vec{k}_\perp, \Delta)$

$\Delta = 0$ $\int d^2 \vec{k}_\perp$

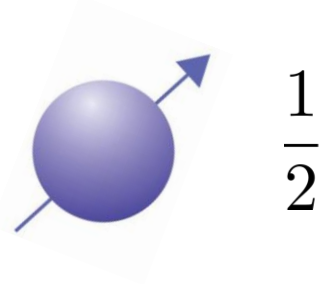


Applications of Wigner distributions



Jaffe-Manohar spin decomposition

- An incomplete story:

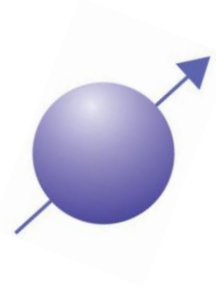




Applications of Wigner distributions

Jaffe-Manohar spin decomposition

- An incomplete story:



A blue sphere representing a quark with a blue arrow pointing upwards and to the right, indicating its spin and momentum.

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma$$

Best known

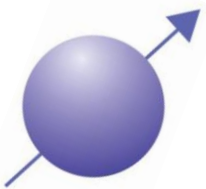
Quark helicity $\sim 30\%$



Applications of Wigner distributions

Jaffe-Manohar spin decomposition

- An incomplete story:


$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G$$

Best known → **Quark helicity** $\sim 30\%$

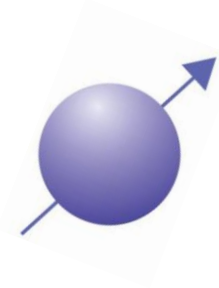
How well do we know? → **Gluon helicity** $\sim 40\%$



Applications of Wigner distributions

Jaffe-Manohar spin decomposition

- An incomplete story:



$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L^q + L^g$$

Best known

How well do we know?

??

Quark helicity $\sim 30\%$

Gluon helicity $\sim 40\%$

OAM of quarks & gluons



Applications of Wigner distributions

Jaffe-Manohar spin decomposition

- An incomplete story:

An intuitive definition

NRQM: $\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x, k) W(x, k)$

$$\frac{1}{2} \Delta \Sigma + \Delta G + L^q + L^g$$

- **OAM as a moment of Wigner distribution** :



Applications of Wigner distributions

Jaffe-Manohar spin decomposition

- An incomplete story:

An intuitive definition

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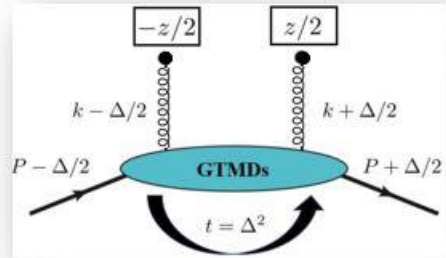
$$\frac{1}{2} \Delta \Sigma + \Delta G + L^q + L^g$$

- **OAM as a moment of Wigner distribution** : (Lorcé, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)

$$L_z^{q,g} = \int dx \int d^2 k_{\perp} d^2 b_{\perp} (\vec{b}_{\perp} \times \vec{k}_{\perp})_z W^{q,g}(x, \vec{b}_{\perp}, \vec{k}_{\perp})$$

Applications of Wigner distributions

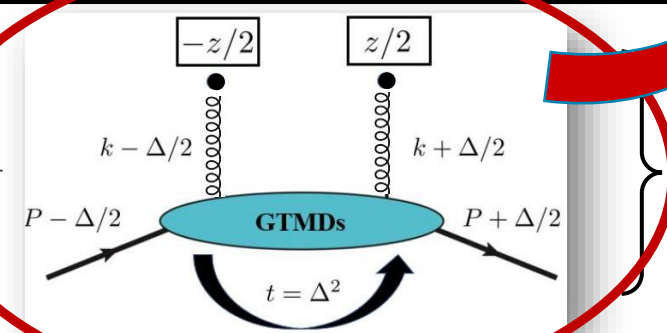
Parameterization of a GTMD correlator (unpolarized gluons):



$$= \frac{1}{2M} \bar{u}(p', \lambda') \left[F_{1,1}^g + \frac{i\sigma^i + k_\perp^i}{P^+} F_{1,2}^g + \frac{i\sigma^i + \Delta_\perp^i}{P^+} F_{1,3}^g + \frac{i\sigma^{ij} k_\perp^i \Delta_\perp^j}{M^2} F_{1,4}^g \right] u(p, \lambda)$$

SB, Metz, Ojha, Tsai, Zhou, 1802.10550

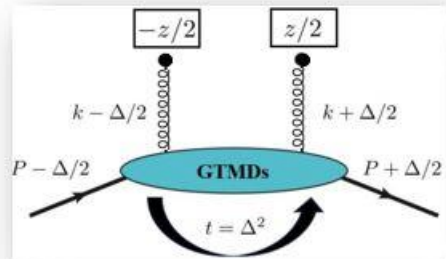
- OAM as a moment of Wigner distribution/GTMD: (Lorcé, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)

$$L_z^{q,g} = \int dx \int d^2 k_\perp d^2 b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \left\{ \int e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \right. \left. \text{Diagram of GTMD correlator} \right\}$$




Applications of Wigner distributions

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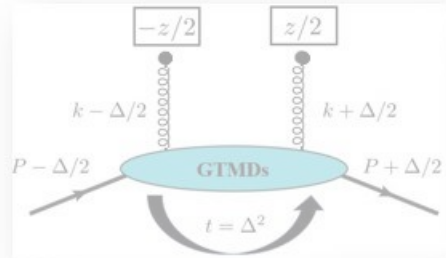
$$L_z^{q,g} = - \int dx \int d^2 \vec{k}_{\perp} \frac{\vec{k}_{\perp}^2}{M^2} F_{1,4}^{q,g}(x, \vec{k}_{\perp}^2)$$

Relation between GTMD $F_{1,4}^{q,g}$ & OAM



Applications of Wigner distributions

Parameterization of a GTMD correlator (unpolarized gluons):



$$= \frac{1}{2M} \bar{u}(p', \lambda') \left[F_{1,1}^g + \frac{i\sigma^{i+} k_{\perp}^i}{P^+} F_{1,2}^g + \frac{i\sigma^{i+} \Delta_{\perp}^i}{P^+} F_{1,3}^g + \frac{i\sigma^{ij} k_{\perp}^i \Delta_{\perp}^j}{M^2} F_{1,4}^g \right] u(p, \lambda)$$

SB, Metz, Ojha,

Big question: Is this measurable?

• OAM as a n

g, Yuan, 2012)

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Relation between GTMD $F_{1,4}^{q,g}$ & OAM

Developments



arXiv: 1601.01585 (2016)

Probing the Small- x Gluon Tomography in Correlated Hard Diffractive Dijet Production in DIS

Yoshitaka Hatta,¹ Bo-Wen Xiao,² and Feng Yuan³

Developments



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**Hunting the Gluon Orbital Angular Momentum at the
Electron-Ion Collider**

Xiangdong Ji,^{1,2} Feng Yuan,³ and Yong Zhao^{1,3}

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Gluon orbital angular momentum at sr arXiv: 1802.10550 (2018)

Yoshitaka

We took a fresh look at this 2016 paper

gluons

in Zhou²

arXiv: 1807.08697 (2018)

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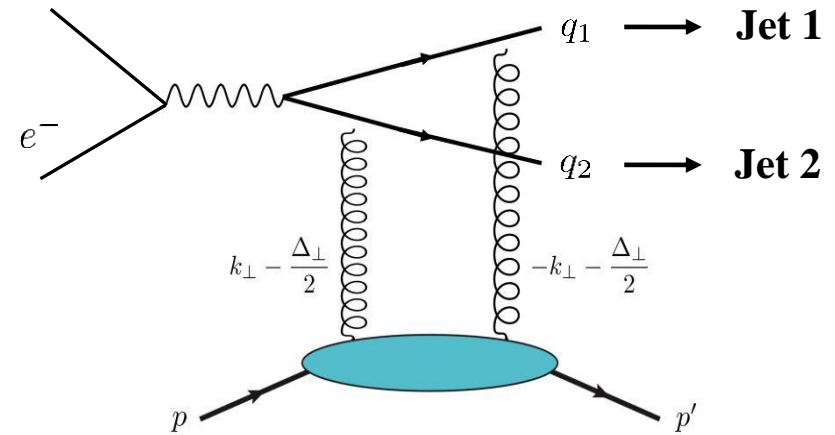


Summary of the 2016 paper

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Hunting the Gluon Orbital Angular Momentum at the
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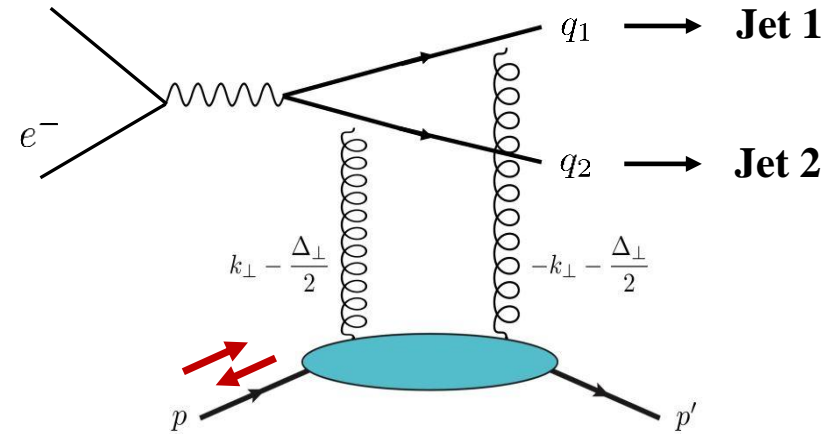
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Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

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Longitudinal single spin asymmetry (SSA):

$$\frac{d\Delta\sigma}{dydQ^2d\Omega} = \sigma_0 h_p \frac{2(\bar{z} - z)(q_\perp \times \Delta_\perp)}{q_\perp^2 + \mu^2} \left[16\beta(1 - y)\Im[F_g^* + 4\xi^2\bar{\beta}F_g'^*][\mathcal{L}_g + 8\xi^2\bar{\beta}\mathcal{L}'_g] \right. \\ \left. + (1 + (1 - y)^2)\Im[F_g^* + 2\xi^2(1 - 2\beta)F_g'^*][\mathcal{L}_g + 2\bar{\beta}(1/z\bar{z} - 2)(\mathcal{L}_g + 4\xi^2(1 - 2\beta)\mathcal{L}'_g)] \right]$$



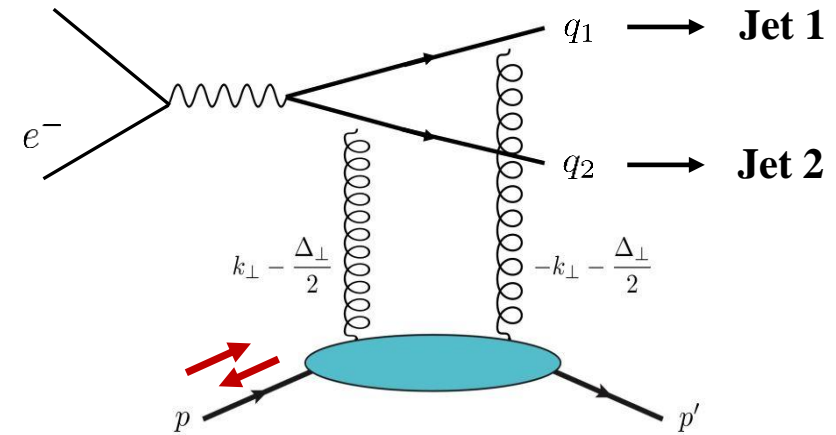
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Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,^{1,2} Feng Yuan,³ and Yong Zhao^{1,3}



Schematic structure of SSA (oversimplified):

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp}) (\bar{z} - z) \left[\Im(F_g^*(\xi) \mathcal{L}_g(\xi)) \right]$$



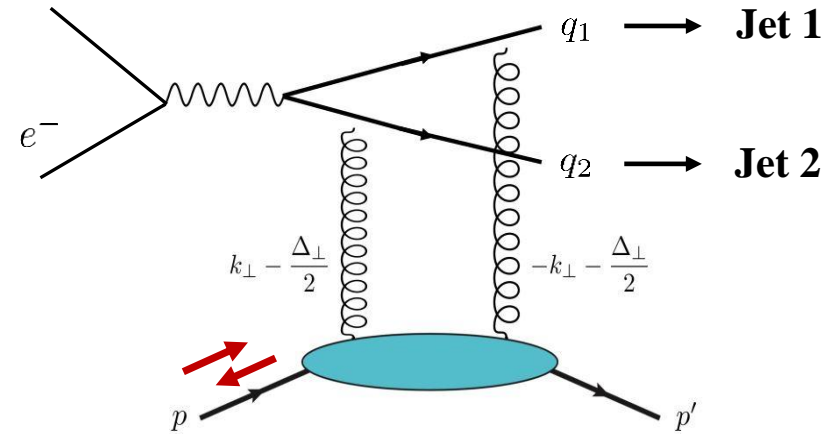
Probing gluon OAM through exclusive dijet production

Summary of the 2016 paper

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,^{1,2} Feng Yuan,³ and Yong Zhao^{1,3}



Signature of OAM is sinusoidal angular modulation

$$q_{\perp} = q_{1\perp} - q_{2\perp}$$

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_{\perp}} - \phi_{\Delta_{\perp}}) (\bar{z} - z) \left[\Im \left(F_g^*(\xi) \mathcal{L}_g(\xi) \right) \right]$$

Moment of GPD

Moment of OAM



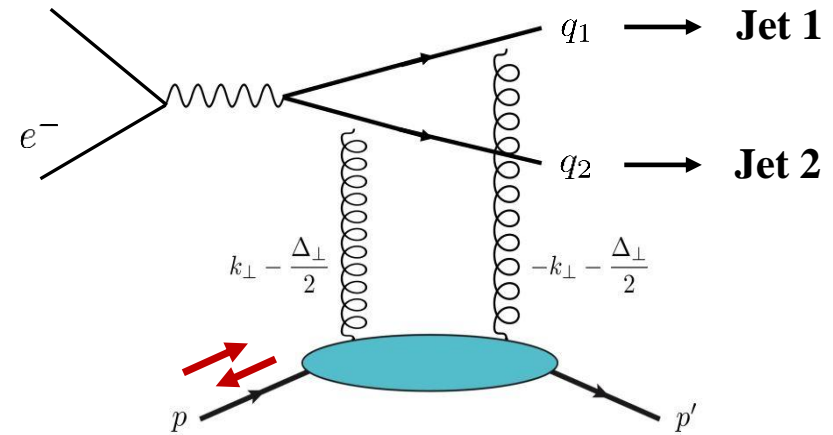
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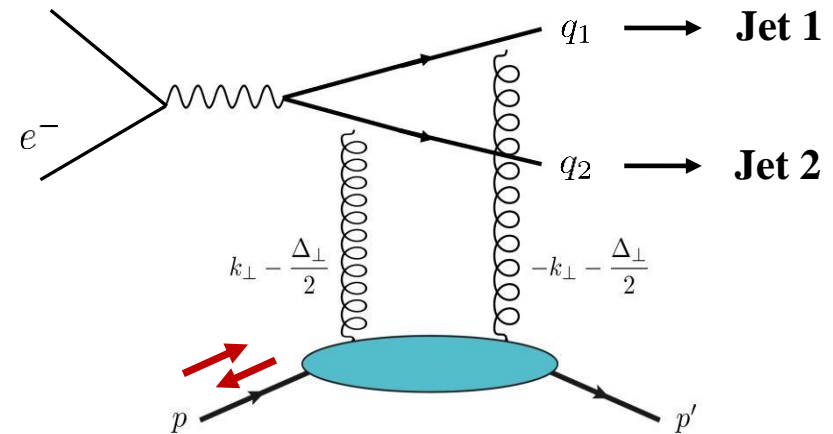
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SSA vanishes for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$



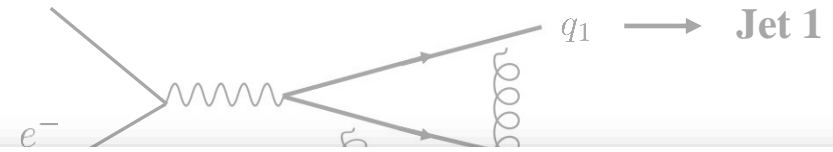
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Electron-Ion Collider

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“Compton Form Factor”:

$$\mathcal{L}_g(\xi) = \int dx \frac{x^2 \xi L_g(x, \xi)}{(x^2 - \xi^2 + i\xi\epsilon)^3}$$

Third pole at $x = \pm\xi \longrightarrow$ potentially dangerous for collinear factorization
(See Cui, Hu, Ma, 1804.05293)

Issues with SSA:

$$q_{\perp} = q_{1\perp} - q_{2\perp}$$

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_{\perp}} - \phi_{\Delta_{\perp}}) (\bar{z} - z) \left[\Im(F_g^*(\xi) \mathcal{L}_g(\xi)) \right]$$

SSA vanishes for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$

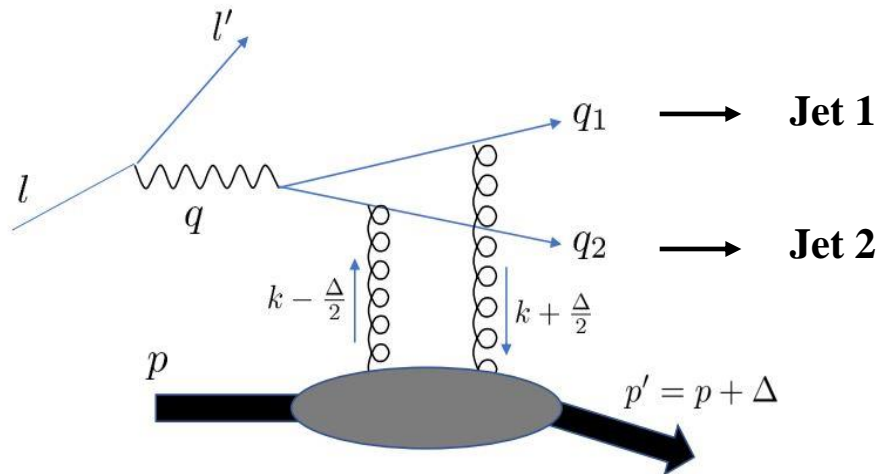
Probing gluon OAM through exclusive dijet production



Our work

Signature of the gluon orbital angular momentum

Shohini Bhattacharya,^{1,*} Renaud Boussarie,^{2,†} and Yoshitaka Hatta^{1,3,‡}





Probing gluon OAM through exclusive dijet production

Our work

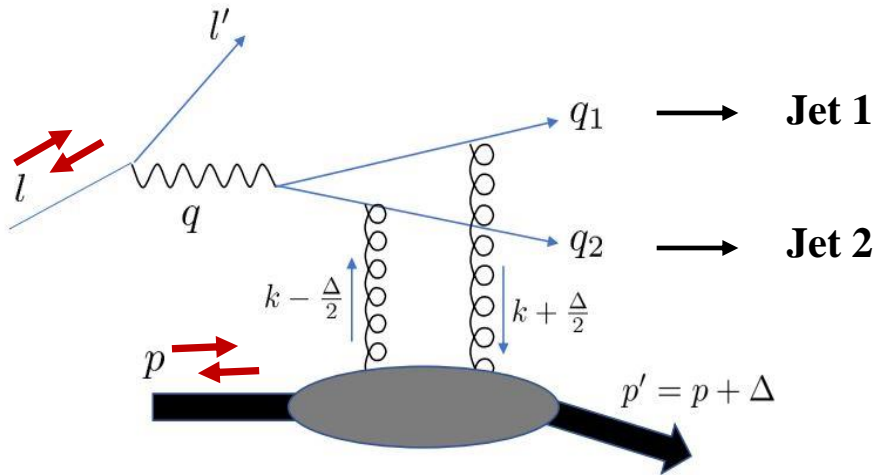
Signature of the gluon orbital angular momentum

Shohini Bhattacharya,^{1,*} Renaud Boussarie,^{2,†} and Yoshitaka Hatta^{1,3,‡}

Distinct feature in our work

Double spin asymmetry (DSA):-

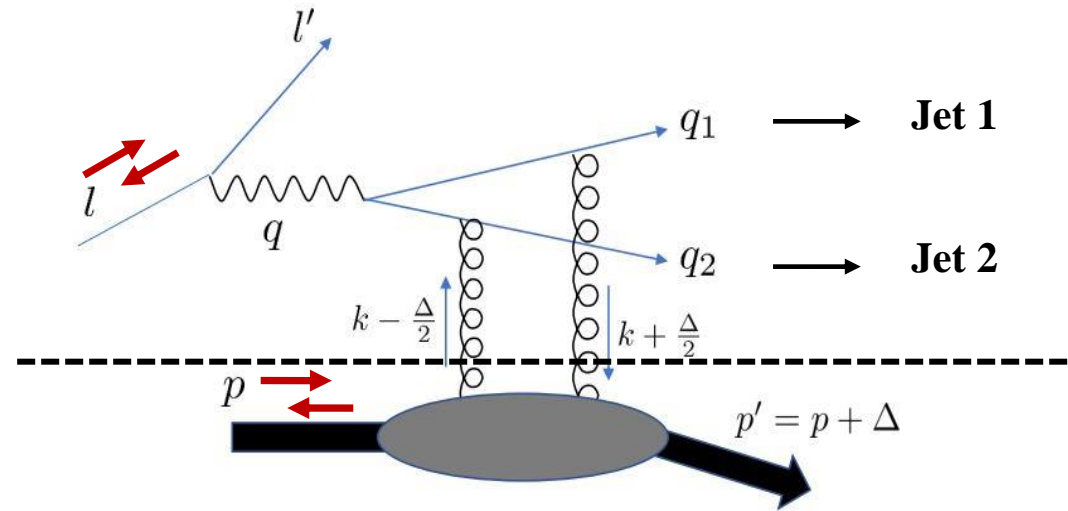
Both electron & incoming proton are longitudinally polarized





Probing gluon OAM through exclusive dijet production

Scattering amplitude



- 6 leading-order Feynman diagrams
- Scattering amplitude:

$$A \propto \int dx \int d^2 k_{\perp} \mathcal{H}(x, \xi, q_{\perp}, k_{\perp}, \Delta_{\perp}) x f_g(x, \xi, k_{\perp}, \Delta_{\perp})$$

Hard part

Soft part

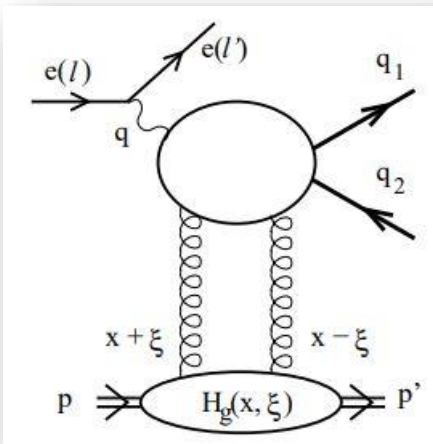


Probing gluon OAM through exclusive dijet production

Scattering amplitude

Twist expansion:

- **Twist-2 amplitude:** Proportional to gluon GPD



Braun, Ivanov, 0505263

$$A_T^2 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{1}{q_\perp^2 + \mu^2} (\bar{u}(q_1) \not{\epsilon}_\perp v(q_2)) \int dx \frac{1}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \times \left(1 + \frac{2\xi^2(1 - 2\beta)}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \right) \int d^2 k_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$A_L^2 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{1}{(q_\perp^2 + \mu^2)^2} 4\xi z \bar{z} QW (\bar{u}(q_1) \gamma^- v(q_2)) \int dx \frac{1}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \times \left(1 + \frac{4\xi^2 \bar{\beta}}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \right) \int d^2 k_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$



High exclusive dijet production

g amplitude

$$L_z^{q,g} = - \int dx \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, \vec{k}_\perp^2)$$

Relation between GTMD $F_{1,4}^{q,g}$ & OAM

Twist expansion:

- **Twist-3 amplitude:** Proportional to gluon OAM

$$A_T^3 = - \frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$- \frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \int d^2 k_\perp \epsilon_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

Probing gluon OAM through exclusive dijet production



Scattering amplitude

Twist expansion:

- **Twist-3 amplitude:** Proportional to gluon OAM

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$-\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \int d^2k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

Factorization-breaking third poles at $x = \pm\xi$

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$



Probing gluon OAM through exclusive dijet production

Twist expansion:

- **Twist-3 amplitude:** Proportion

Note: Gluon GPDs may contain $\sim \theta(\xi - |x|)(x^2 - \xi^2)^2$
(See Radyushkin, 9805342)

Hence, integrals containing third poles are divergent

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$-\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

Factorization-breaking third poles at $x = \pm\xi$

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Probing gluon OAM through exclusive dijet production



Scattering amplitude

Twist expansion:

Switch off the factorization-breaking third poles by setting $z = \bar{z} = \frac{1}{2}$

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_1^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$-\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_1^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \int d^2k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

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Probing gluon OAM through exclusive dijet production

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$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_1^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

Recall: Not possible in SSA

Factorization-breaking third poles at $x = \pm\xi$

Issues with SSA:

$$A_L^3 \Big|_{q_\perp = q_{1\perp} - q_{2\perp}} \frac{d\sigma}{dy dQ^2 d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_\Delta) (\bar{z} - z) \left[\text{Im}(F_g^*(\xi) \mathcal{L}_g(\xi)) \right] \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

SSA vanishes for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$



Probing gluon OAM through exclusive dijet production

Scattering amplitude

Twist expansion:

Switch off the factorization-breaking third poles by setting $z = \bar{z} = \frac{1}{2}$

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(\cancel{\frac{(2\xi)^3(1-2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)}} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$-\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \int d^2 k_\perp \epsilon_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

DSA is sensitive to OAM through an interference between twist-2 amplitude A^2 & twist-3 amplitude A_T^3 (No third pole)

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left(\cancel{\frac{8\xi^2(1-\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)}} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

Probing gluon OAM through exclusive dijet production



Scattering amplitude

Main result ($z = 1/2$):

DSA's OAM part:

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \Re \left[\left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$



Probing gluon OAM through exclusive dijet production

Scattering amplitude

Main result ($z = 1/2$): **DSA does not vanish for symmetric jet configurations** $z = \bar{z} = \frac{1}{2}$

DSA's OAM part:

Consequence:

Elimination of factorization-breaking third poles at $x = \pm\xi$

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[\left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

“Compton Form Factors”:

$$\mathcal{H}_g^{(1)}(\xi) = \int_{-1}^1 dx \frac{H_g(x, \xi)}{(x - \xi + i\epsilon)(x + \xi - i\epsilon)}$$

$$\mathcal{H}_g^{(2)}(\xi) = \int_{-1}^1 dx \frac{\xi^2 H_g(x, \xi)}{(x - \xi + i\epsilon)^2 (x + \xi - i\epsilon)^2}$$

$$\mathcal{L}_g(\xi) = \int_{-1}^1 dx \frac{x^2 L_g(x, \xi)}{(x - \xi + i\epsilon)^2 (x + \xi - i\epsilon)^2}$$



Probing gluon OAM through exclusive dijet production

Scattering amplitude

Main result ($z = 1/2$):

DSA's OAM part:

Scattered lepton angle

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \Re \left[\left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

Signature of gluon OAM is cosine angular modulation

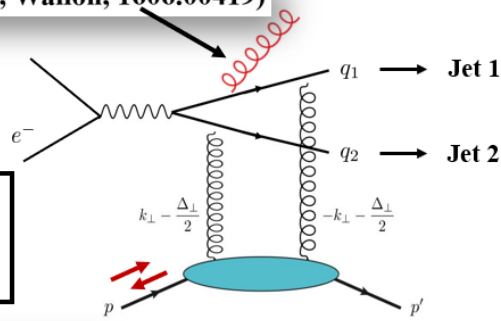
Exclusive dijet production

(Boussarie, Grabovsky, Szymanowski, Wallon, 1606.00419)

Schematic structure of SSA (oversimplified):

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp}) (\bar{z} - z) \left[\Im(F_g^*(\xi) \mathcal{L}_g(\xi)) \right]$$

Jet angle affected by gluon emissions



Scattered lepton angle

$$\int d\phi_{q_\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[\left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

Signature of gluon OAM is cosine angular modulation

Probing gluon OAM through exclusive dijet production



Scattering amplitude

Main result ($z = 1/2$):

DSA's OAM part:

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

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“Compton Form Factors”:

$$O(x, \xi) \equiv \int d^2 \tilde{k}_\perp \frac{\tilde{k}_\perp^2}{M^2} F_{1,2}(x, \xi, \tilde{\Delta}_\perp = 0)$$

$$O(\xi) = \int_{-1}^1 dx \frac{x O(x, \xi)}{(x - \xi + i\epsilon)^2 (x + \xi - i\epsilon)^2}$$

Probing gluon OAM through exclusive dijet production



Scattering amplitude

Not the end of the story:



Probing gluon OAM through exclusive dijet production

Scattering amplitude

Not the end of the story:

- **Interference between unpolarized & helicity GPD** ($z = 1/2$):

Helicity GPD



$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \Re \left[\left(\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left(\tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$

Probing gluon OAM through exclusive dijet production



Scattering amplitude

Not the end of the story:

- **Interference between unpolarized & helicity GPD** ($z = 1/2$):

Helicity GPD

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10} \pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1 - \xi^2) \xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \Re \left[\left(\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1 - \xi^2} \mathcal{E}_g^{(1)*} \right) \left(\tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1 - \xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$

Helicity contributes to the same angular modulation as that of OAM

DSA is a simultaneous probe of gluon OAM & it's helicity

Probing gluon OAM through exclusive dijet production



Numerical estimate of cross section

Probing gluon OAM through exclusive dijet production



Numerical estimate of cross section

Ingredients for non-perturbative functions

OAM

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[\left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

Helicity

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[\left(\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left(\tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$

Probing gluon OAM through exclusive dijet production



Numerical estimate of cross section

Ingredients for non-perturbative functions

- Neglect contributions from $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$ Very simple formula

OAM

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[\left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

Helicity

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[\left(\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left(\tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$



Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Ingredients for non-perturbative functions

- Neglect contributions from $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$ Very simple formula
- Model (H_g, \tilde{H}_g) according to the Double distribution approach (see for instance Radyushkin, 9805342)

$$\begin{pmatrix} H_g(x, \xi) \\ \tilde{H}_g(x, \xi) \end{pmatrix} = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) \times \frac{15}{16} \frac{[(1 - |\beta|)^2 - \alpha^2]^2}{(1 - |\beta|)^5} \times \begin{cases} \beta G(\beta) \\ \beta \Delta G(\beta) \end{cases}$$

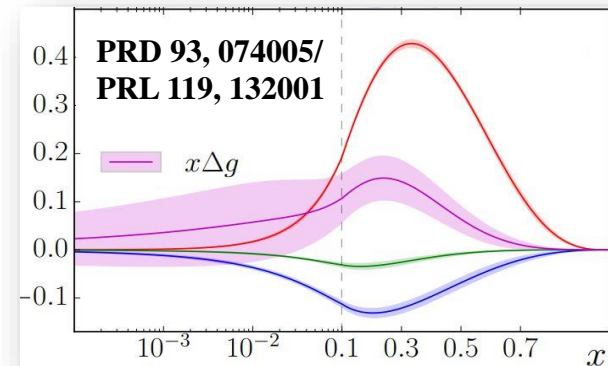
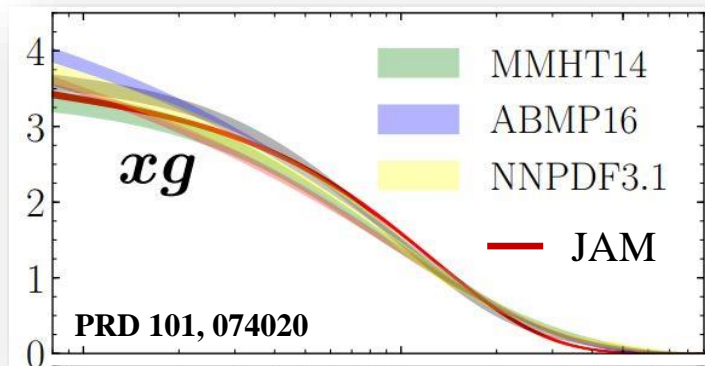
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Numerical estimate of cross section

Ingredients for non-perturbative functions

- Neglect contributions from $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$ Very simple formula
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Numerical estimate of cross section

Ingredients for non-perturbative functions

- Neglect contributions from $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$ Very simple formula
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- Model for OAM:

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Numerical estimate of cross section

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- Neglect contributions from $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$ Very simple formula
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- Model for OAM:
 1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^g(\boldsymbol{x}) = x \int_x^1 \frac{dx'}{x'^2} (H_g(x') + E_g(x')) - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x') + \text{genuine twist-three}$$



Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Ingredients for non-perturbative functions

- Neglect contributions from $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$ Very simple formula
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- Model for OAM:

1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^g(\boldsymbol{x}) \overset{\text{WW approx}}{\approx} x \int_x^1 \frac{dx'}{x'^2} (H_g(x') + E_g(x')) - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x') + \text{genuine twist-three}$$

$$H_g(x') = x'G(x')$$

Neglect E_g



Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Ingredients for non-perturbative functions

- Neglect contributions from $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$ Very simple formula
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2. Use the Double distribution approach to construct $xL_g(x, \boldsymbol{\xi})$ from $xL_g(x)$ (GPD-like approach)

Probing gluon OAM through exclusive dijet production



Numerical estimate of cross section

Realistic EIC kinematics

\sqrt{s} [GeV]	Q^2 [GeV ²]	y	ξ
120	2.7	0.7	$\lesssim 10^{-3}$
	4.8		
	10.0		

Focus on:
 $z = \bar{z} = \frac{1}{2}$



Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

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120	2.7	0.7	$\lesssim 10^{-3}$
	4.8		
	10.0		

Focus on:
 $z = \bar{z} = \frac{1}{2}$

Cross section:

$$\frac{d\sigma}{dydQ^2d\phi_{l\perp}dzdq_{\perp}^2d^2\Delta_{\perp}} = \frac{\alpha_{em}y}{2^{11}\pi^7Q^4} \frac{\int d\phi_{q\perp} L^{\mu\nu} A_{\mu}^* A_{\nu}}{(W^2 + Q^2)(W^2 - M_J^2)z\bar{z}}$$



Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

Realistic EIC kinematics

\sqrt{s} [GeV]	Q^2 [GeV ²]	y	ξ
120	2.7	0.7	$\lesssim 10^{-3}$
	4.8		
	10.0		

Focus on:
 $z = \bar{z} = \frac{1}{2}$

Study cross section as differential in the skewness variable

Cross section:

$$\frac{d\sigma}{dy dQ^2 d\phi_{\perp} dz dq_{\perp}^2 d\Delta_{\perp}} = \frac{\alpha_{em} y}{2^{11} \pi^7 Q^4} \frac{d\sigma}{dy dQ^2 dz d\xi d\delta \phi}$$

Relation between skewness & jet momenta:

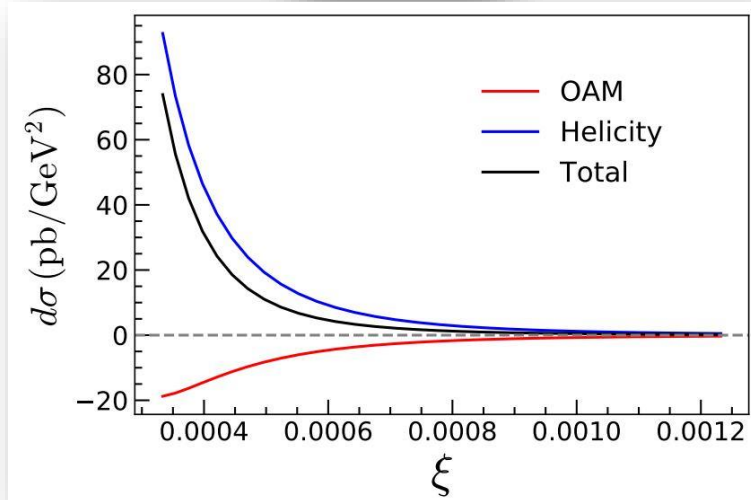
$$\xi = \frac{q_{\perp}^2 + z\bar{z}Q^2}{-q_{\perp}^2 + z\bar{z}(Q^2 + 2W^2)}$$

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Numerical estimate of cross section

$$Q^2 = 2.7$$

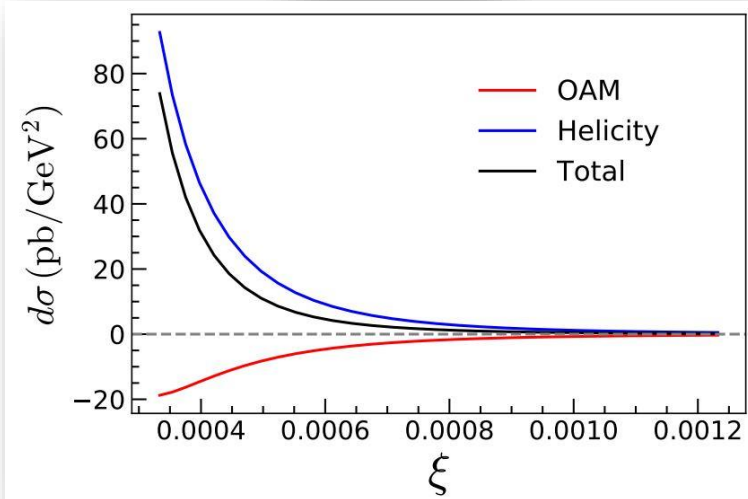




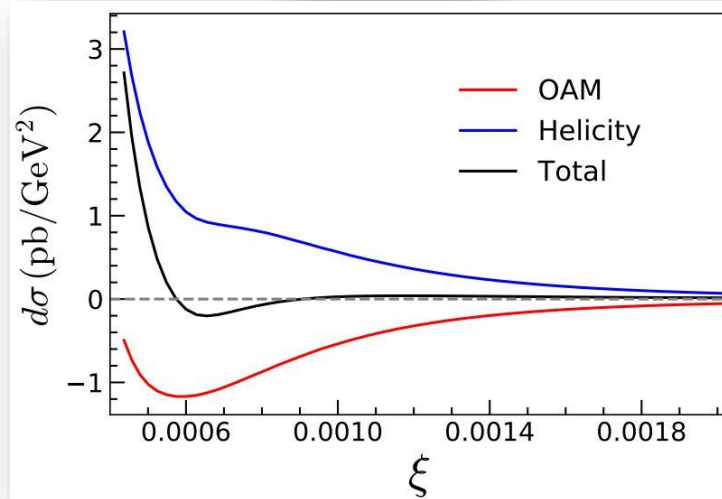
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Numerical estimate of cross section

$Q^2 = 2.7$



$Q^2 = 4.8$

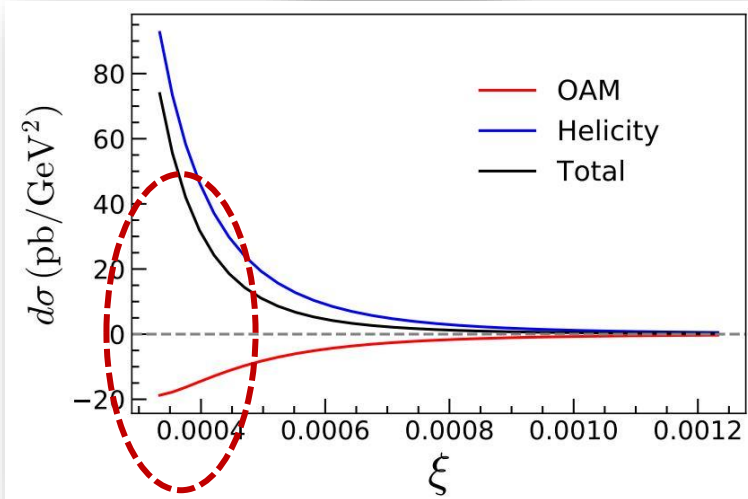


Probing gluon OAM through exclusive dijet production

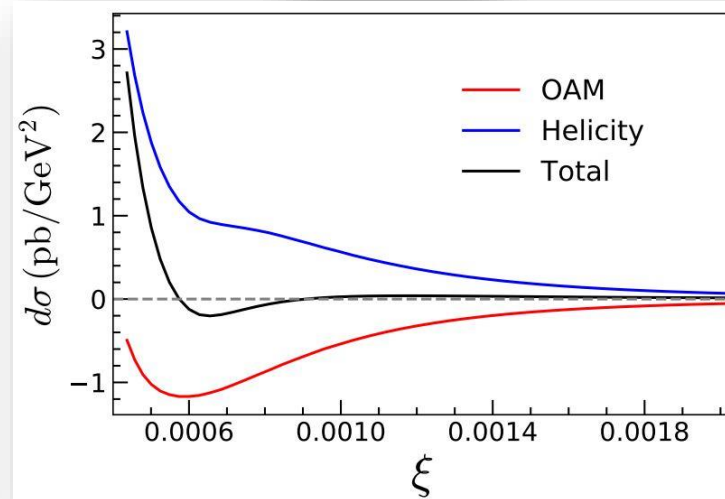


Numerical estimate of cross section

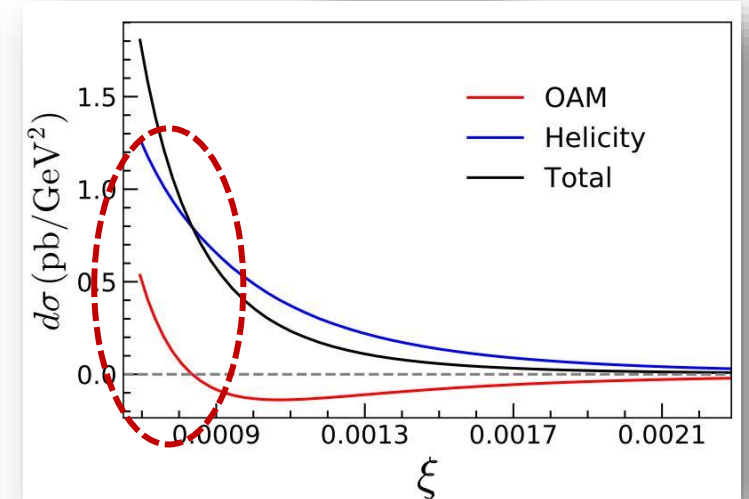
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$

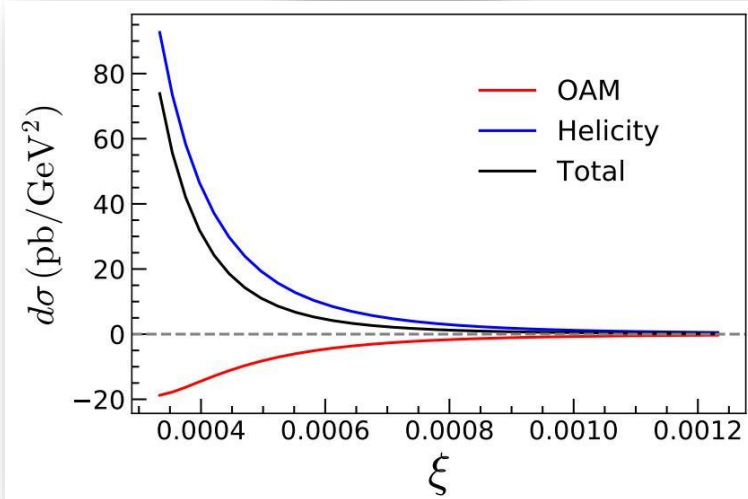




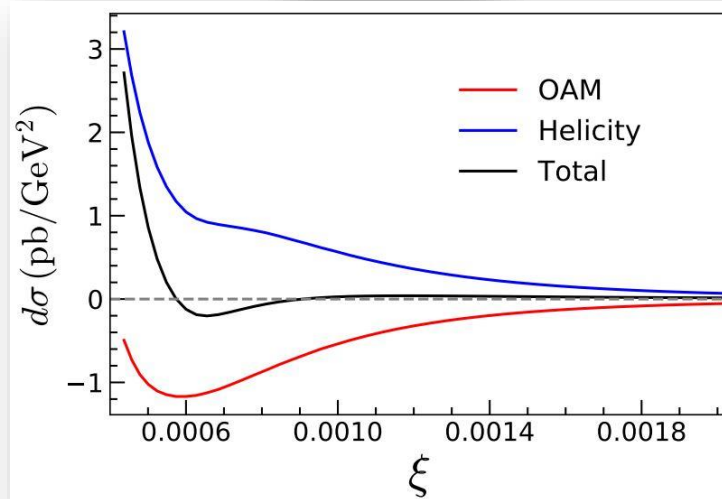
Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

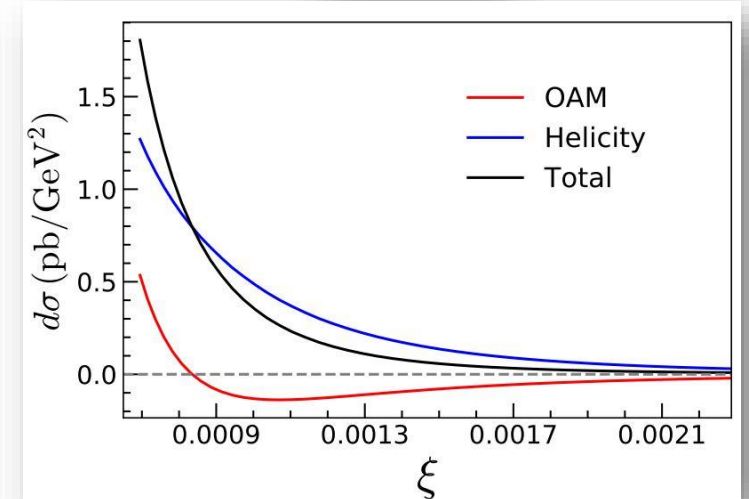
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$



DSA:
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \Big|_{\delta\phi=0} \sim \Re \left[\mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] - \Re \left[\left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + Q^2/4} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right]$$



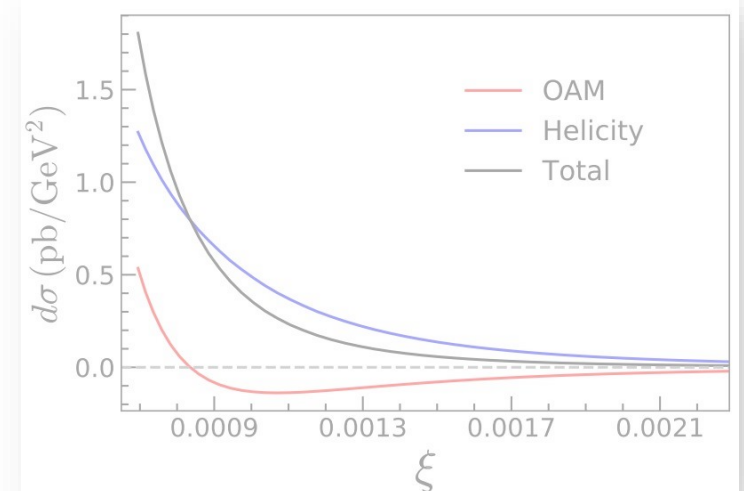
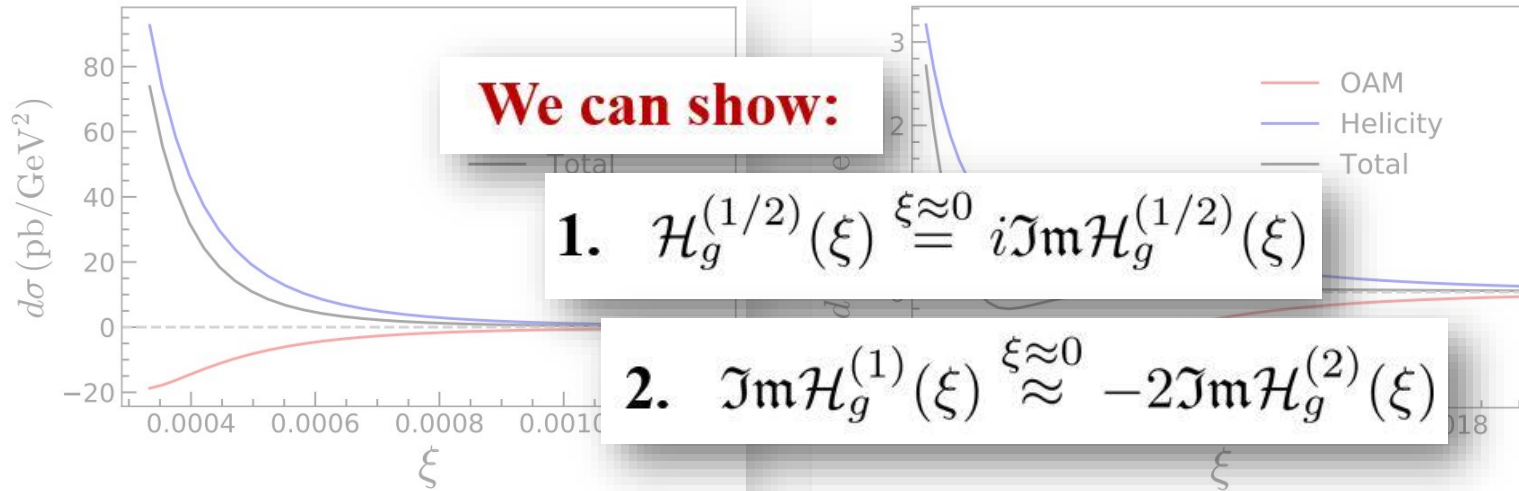
Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

$Q^2 = 2.7$

$Q^2 = 4.8$

$Q^2 = 10$



DSA:
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \Big|_{\delta\phi=0} \sim \Re \left[\mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] - \Re \left[\left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + Q^2/4} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right]$$



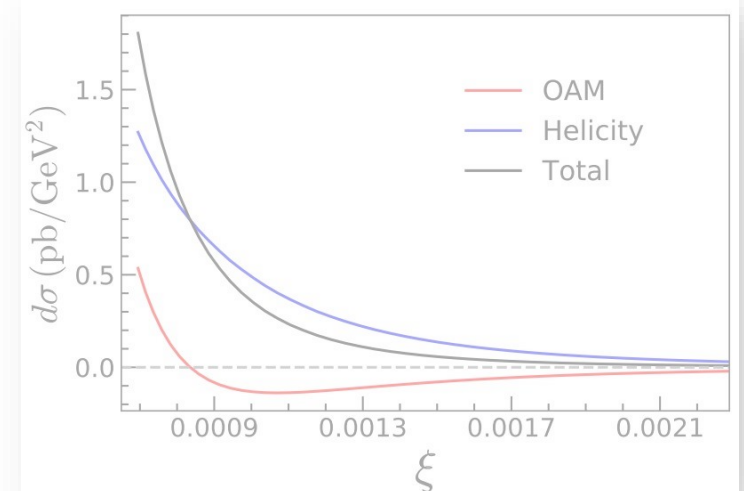
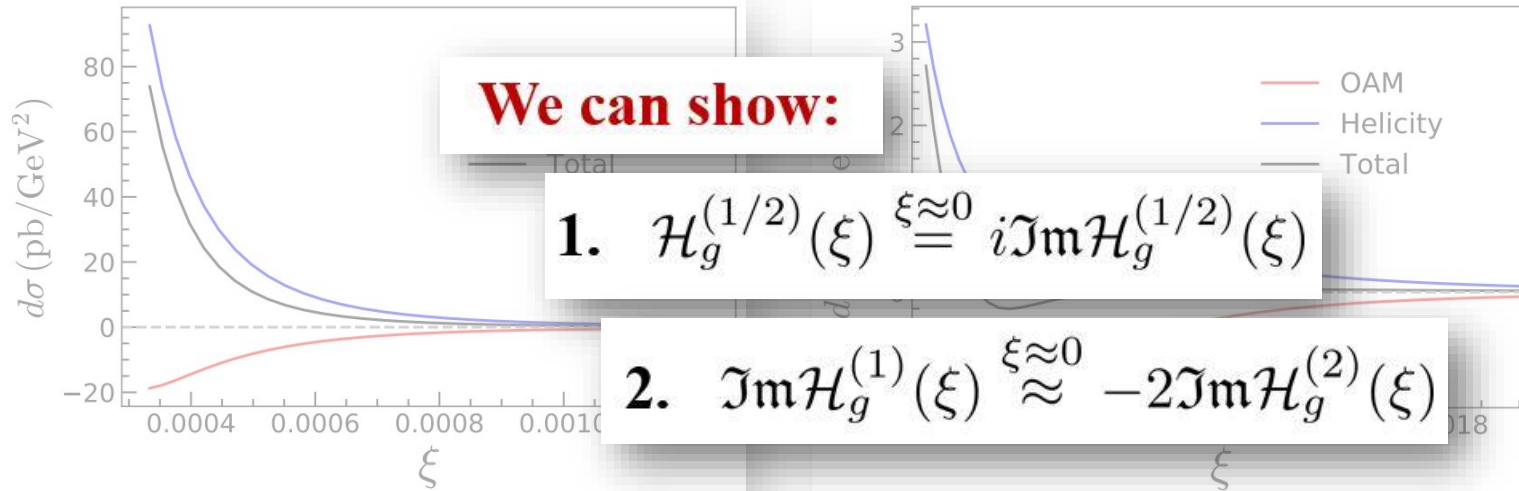
Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

$Q^2 = 2.7$

$Q^2 = 4.8$

$Q^2 = 10$



We can show:

$$1. \mathcal{H}_g^{(1/2)}(\xi) \stackrel{\xi \approx 0}{\approx} i \mathcal{I}m \mathcal{H}_g^{(1/2)}(\xi)$$

$$2. \mathcal{I}m \mathcal{H}_g^{(1)}(\xi) \stackrel{\xi \approx 0}{\approx} -2 \mathcal{I}m \mathcal{H}_g^{(2)}(\xi)$$

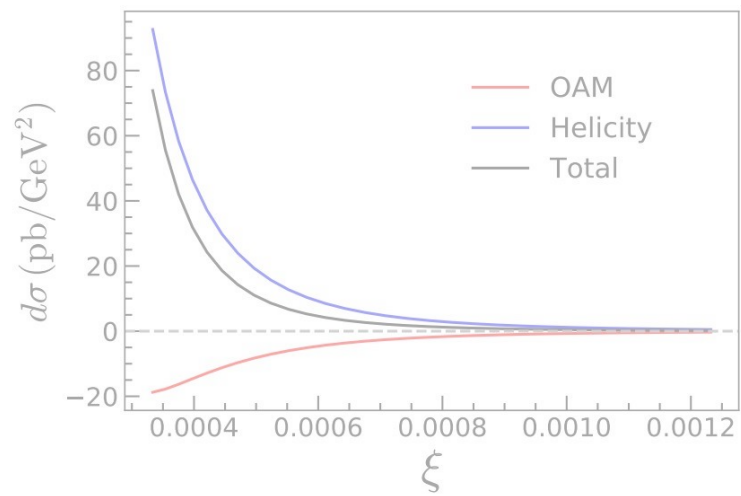
$$\text{DSA: } \int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \Big|_{\delta\phi=0} \sim \mathcal{H}_g^{(1)*}(\xi) \left(\tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4} \mathcal{L}_g(\xi) \right)$$



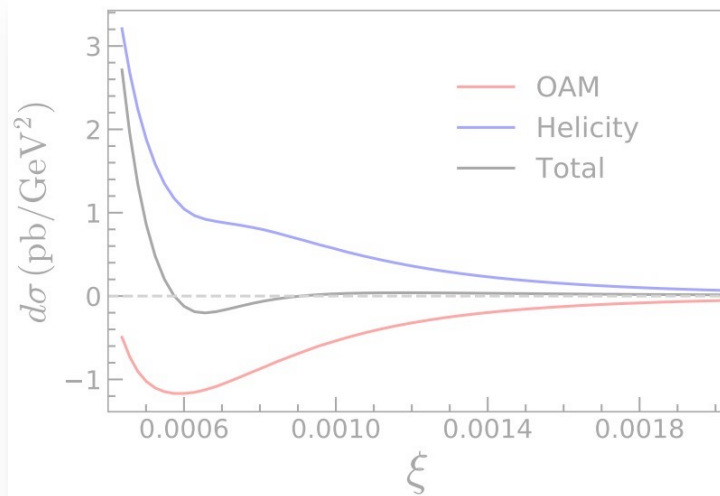
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Numerical estimate of cross section

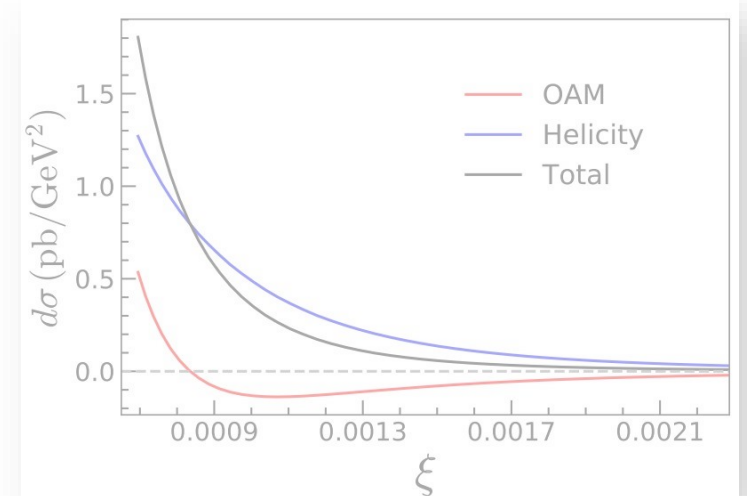
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$



DSA:
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \Big|_{\delta\phi=0} \sim \mathcal{H}_g^{(1)*}(\xi) \left(\tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4} \mathcal{L}_g(\xi) \right)$$

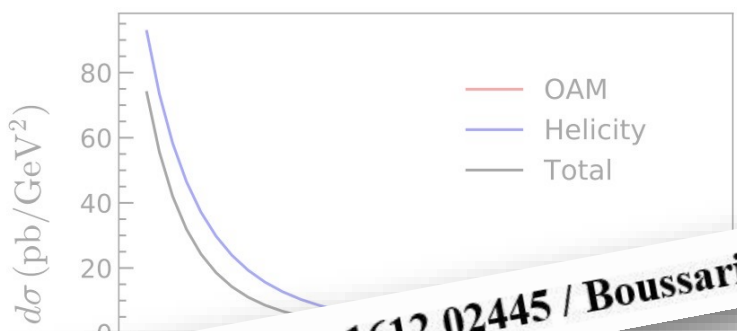
$\tilde{\mathcal{H}}_g^{(2)}$ & \mathcal{L}_g interfere positively/negatively depending upon sign of $q_\perp^2 - \frac{Q^2}{4}$



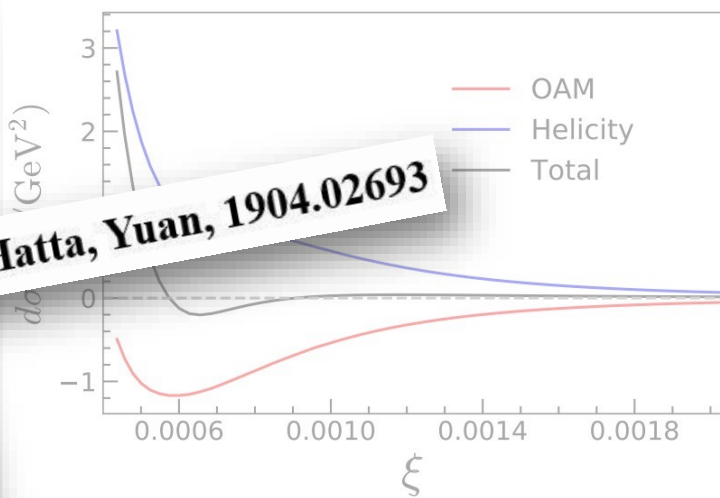
Cancellation expected between Helicity & OAM at small x

$$\Delta G(x) \approx -L_g(x)$$

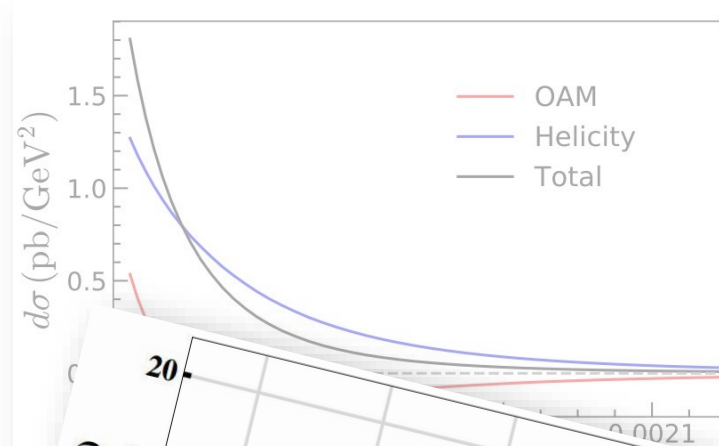
$Q^2 = 2.7$



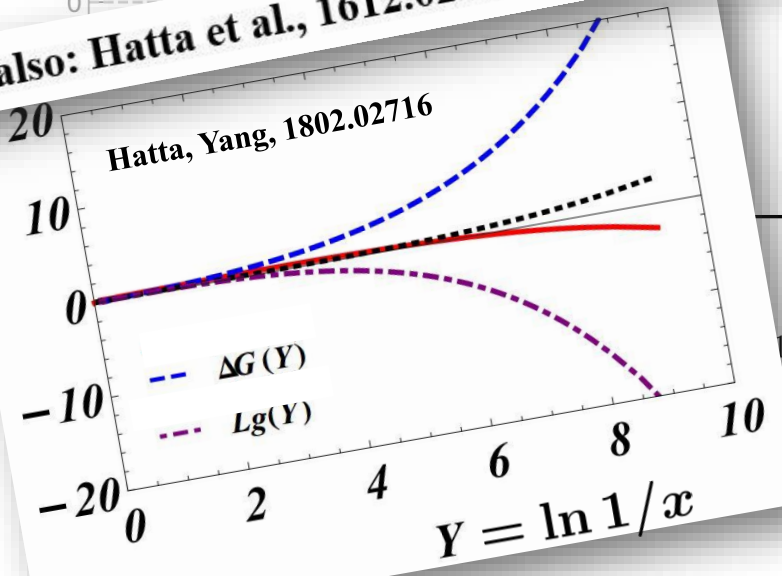
$Q^2 = 4.8$



$Q^2 = 10$

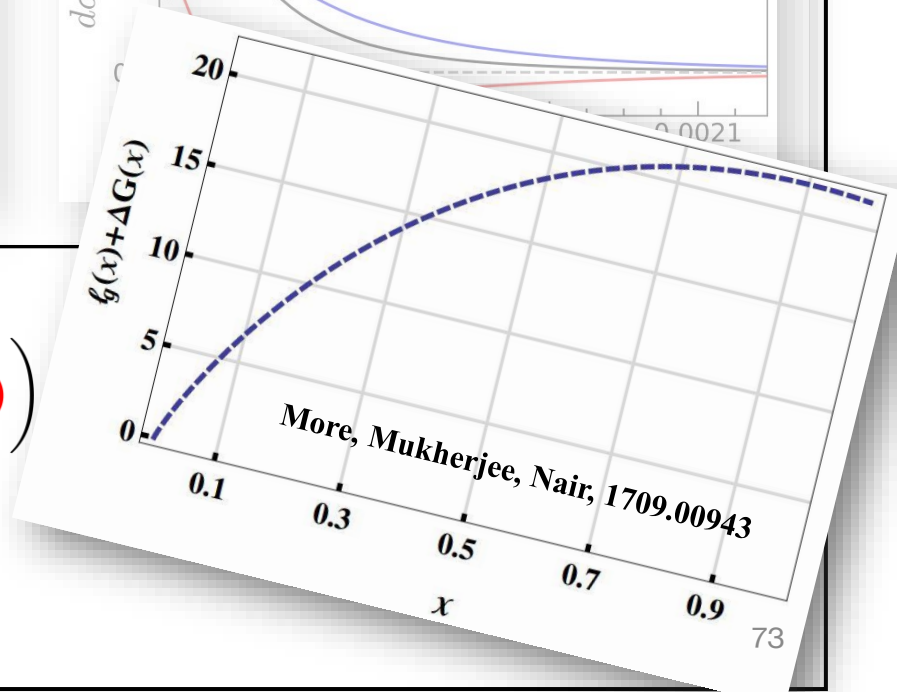


See also: Hatta et al., 1612.02445 / Boussarie, Hatta, Yuan, 1904.02693



$$(\dots)^*(\xi) \left(\tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4} \mathcal{L}_g(\xi) \right)$$

\downarrow \downarrow
 $\Delta G(x)$ $L_g(x)$



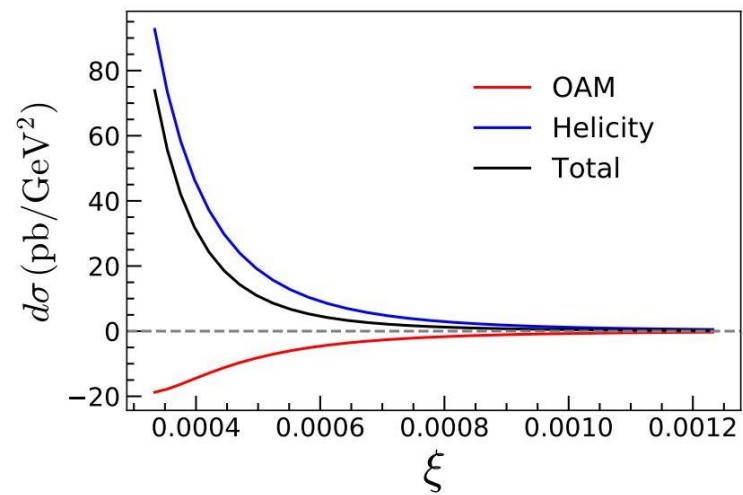
More, Mukherjee, Nair, 1709.00943

Cancellation expected between Helicity & OAM at small x

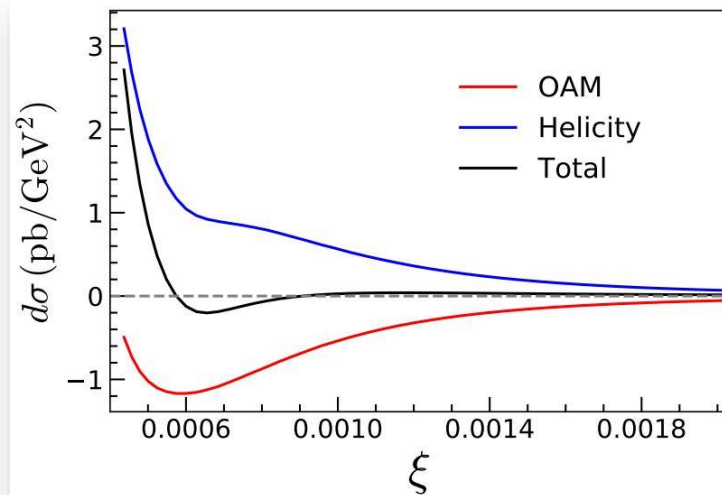


$$\Delta G(x) \approx -L_g(x)$$

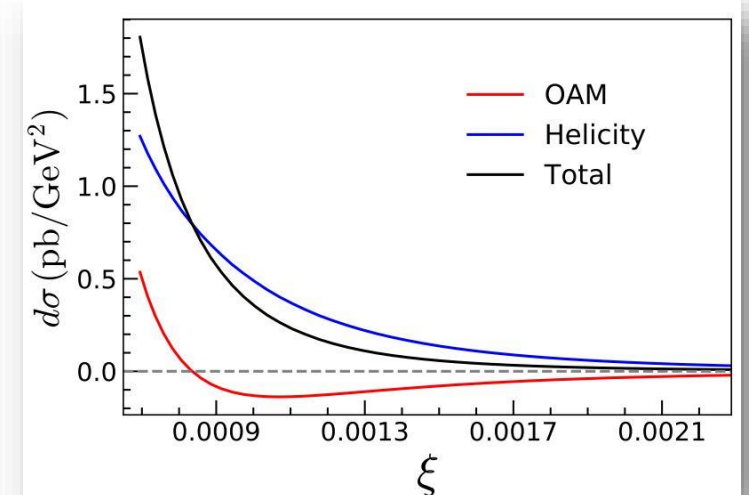
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$



Unique opportunity to study interplay between

$$\Delta G(x) \text{ \& } L_g(x)$$

which has been so far only studied theoretically!

$$\left(\tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_{\perp}^2 - Q^2/4}{q_{\perp}^2 + Q^2/4} \mathcal{L}_g(\xi) \right)$$

\downarrow
 $\Delta G(x)$

\downarrow
 $L_g(x)$



Summary

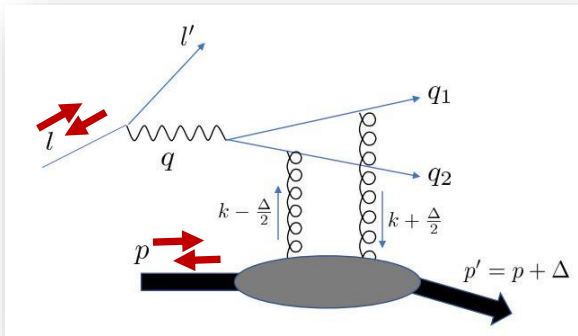
Summary

- **Glucn OAM related to the Wigner distribution**

Summary

Summary

- **Glucan OAM related to the Wigner distribution**
- **DSA in exclusive dijet production is a unique observable to access the glucan OAM @ EIC:**



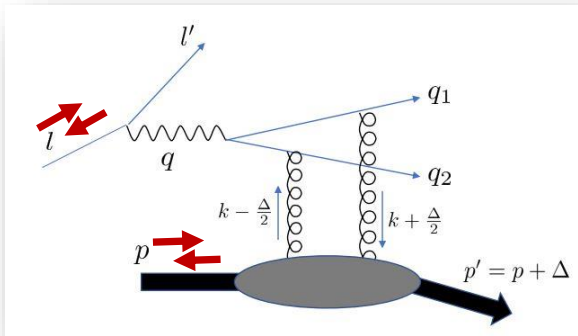
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \sim -\Re \left[\left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

$$+ \Re \left[\mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

Summary

Summary

- Gluon OAM related to the Wigner distribution
- DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:



$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \sim -\Re \left[\left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

$$+ \Re \left[\mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

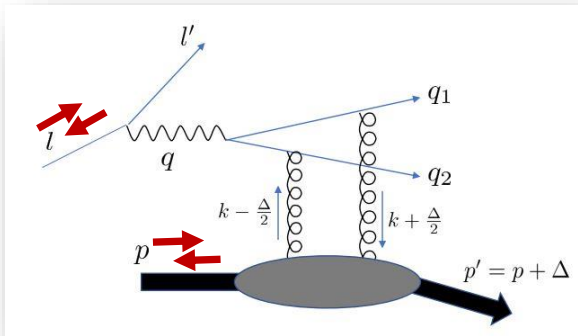
Signature of gluon OAM is cosine angular modulation

Summary

Summary

DSA does not vanish for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$

- DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:



$$\int d\phi_{q_\perp} L^{\mu\nu} A_\mu^* A_\nu \sim -\Re \left[\left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$+ \Re \left[\mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

Signature of gluon OAM is cosine angular modulation

Summary

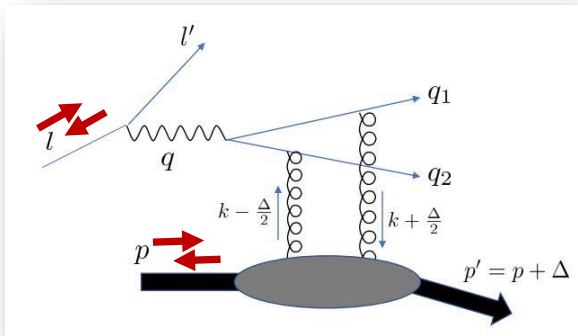
Summary

DSA does not vanish for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$

Consequence:

- DSA in exclusive dijet production is

Elimination of factorization-breaking third poles at $x = \pm\xi$



$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \sim -\Re \left[\left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

$$+ \Re \left[\mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

Signature of gluon OAM is cosine angular modulation



Summary

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DSA does not vanish for symmetric jet configurations $z = \bar{z} = \frac{1}{2}$

Consequence:

Elimination of factorization-breaking third poles at $x = \pm\xi$

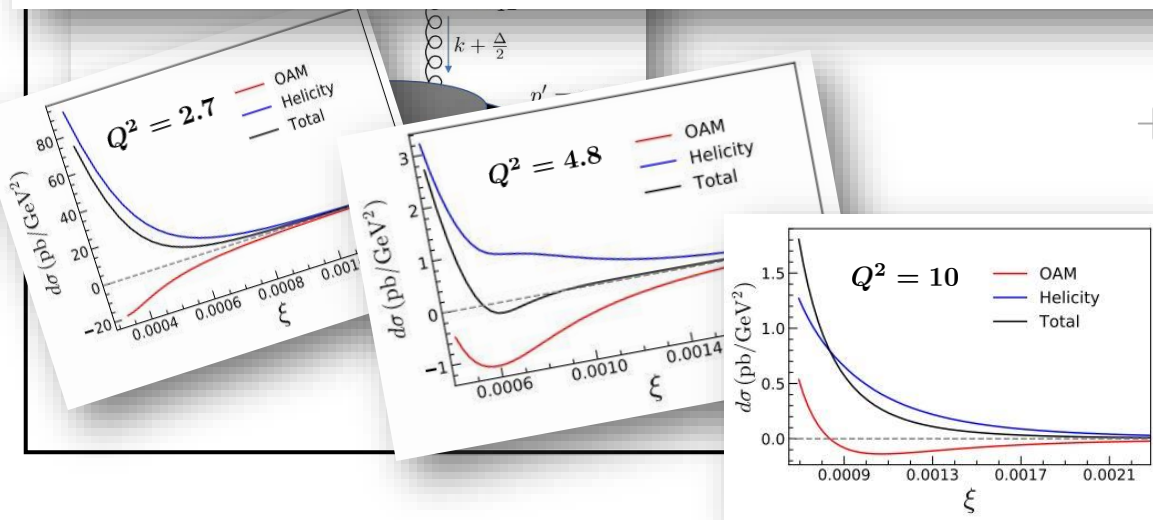
- DSA in exclusive dijet production is

DSA is a unique observable to study interplay between gluon OAM & helicity

$$\left\{ \mathcal{L}_g(\xi) \right\} \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

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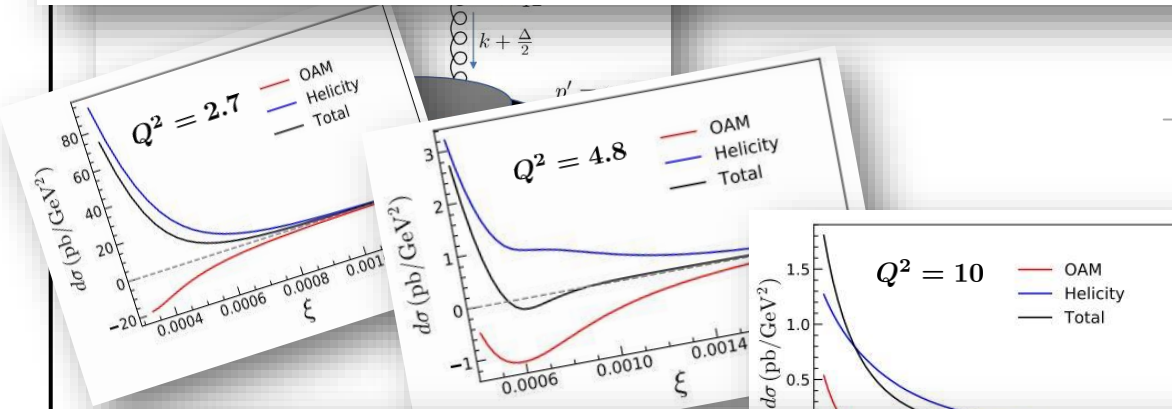
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First realistic numerical calculation of observable sensitive to OAM @ EIC

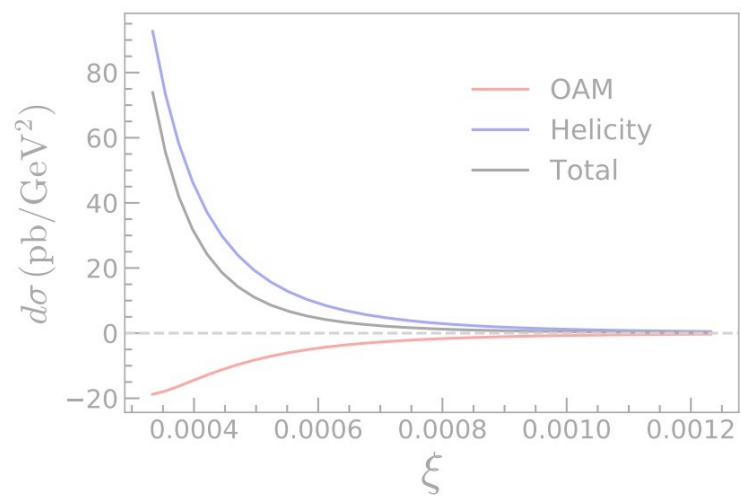
Backup slides



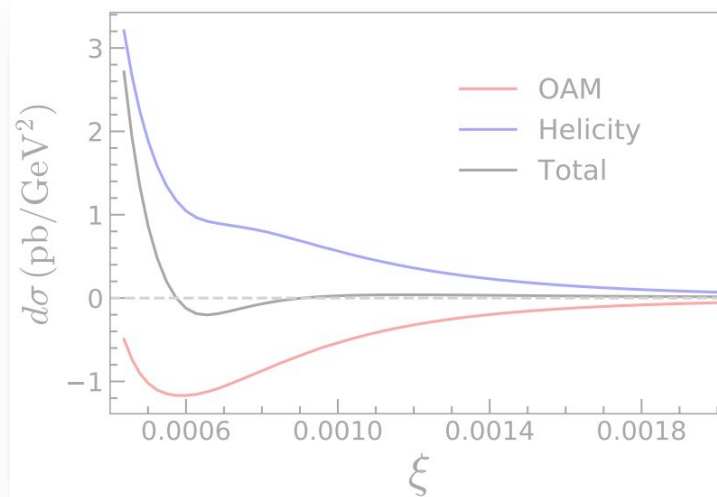
Probing gluon OAM through exclusive dijet production

Numerical estimate of cross section

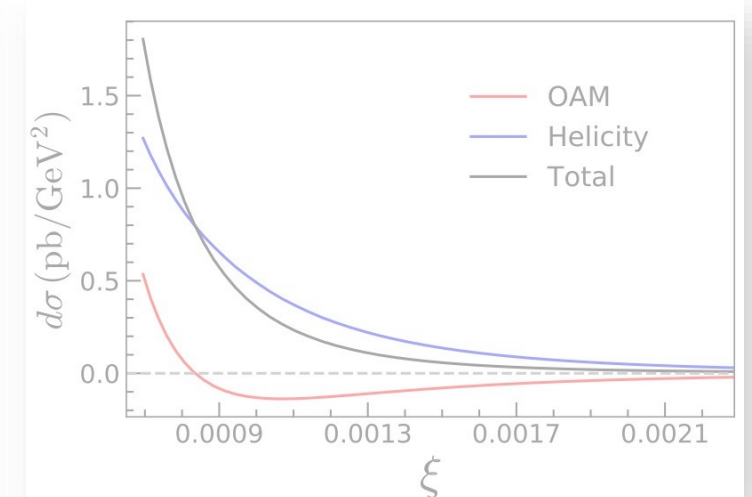
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$



Caveat:

- In practice, measurements are done in a window in z around $z = 1/2$

Corrections of order $\sim (z - 1/2)^2$ should be calculable in k_t -factorization approach

Probing gluon OAM through exclusive dijet production



Cross section

Jet azimuthal angle (ϕ_{q_\perp}) integrated out

$$\frac{d\sigma}{dydQ^2d\phi_{l_\perp}dzdq_\perp^2d^2\Delta_\perp} = \frac{\alpha_{em}y}{2^{11}\pi^7Q^4} \frac{\int d\phi_{q_\perp} L^{\mu\nu} A_\mu^* A_\nu}{(W^2 + Q^2)(W^2 - M_J^2)z\bar{z}}$$

Integrate assuming a Gaussian form factor

$$\sim e^{-b\Delta_\perp^2}$$



Slope = 5

(See Braun, Ivanov, 0505263)