Vector Boson Scattering: Status and Prospects for the Large Hadron Collider and Beyond

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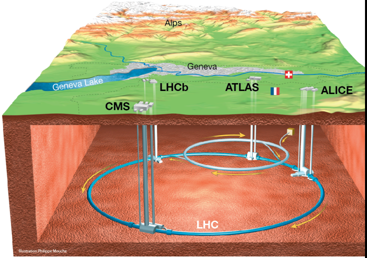
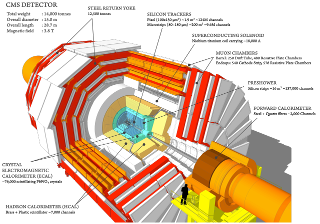
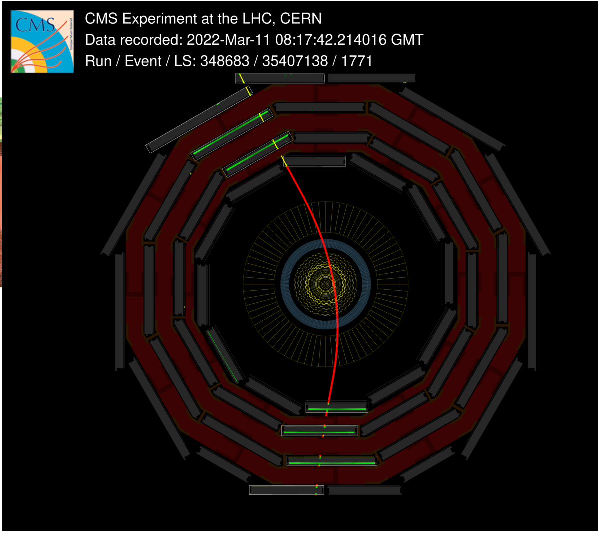
[1] Many, many works. For a review, see Buarque-Franzosi, Gallinaro, RR, et al [2106.01393]

Thank you for the invitation!

A real **cosmic muon** (μ) passing through the **CMS** detector at the **LHC**







Since $|\vec{B}| = 4$ T and radius $\neq 0, \infty$ $\Longrightarrow$ μ **is massive and charged!**

Particle Physics: Then and Now

Since the late 20th, a chief goal of particle physics has been to establish the **spectrum of particles**, their **structures**, and their **properties**

possible with many tools, e.g., production at colliders, tabletop measurements of fundamental symm., and rare decays

Particle Physics: Then and Now

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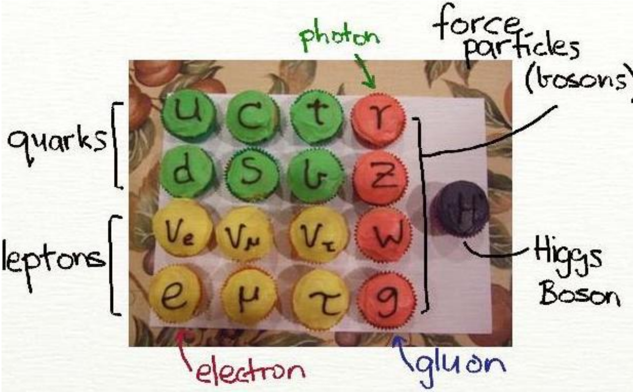
The Standard Model (SM) of particle physics

Position indicates quantum numbers/ charges

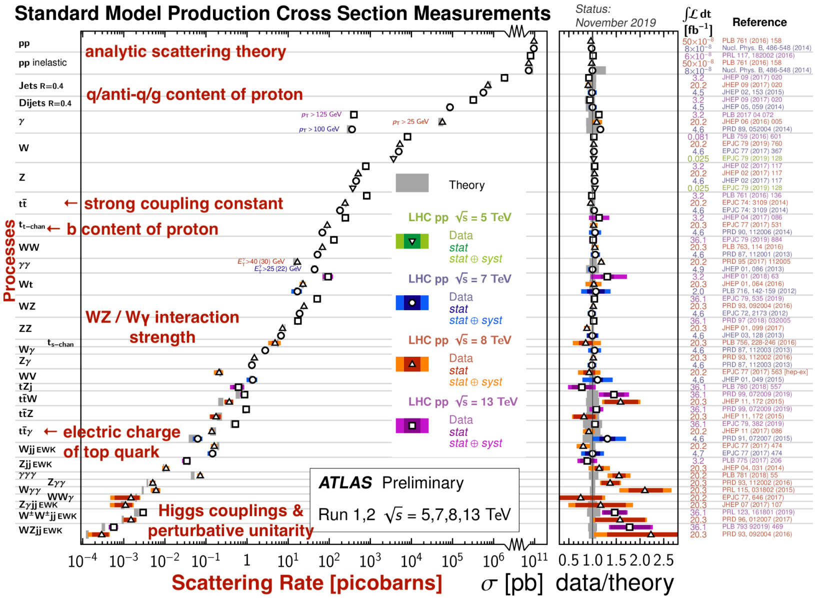
(just like in chemistry!)

E.g., spin, weak isospin, color, electromagnetic, weak hyper charge



Today's goals include understanding the origin of the SM itself

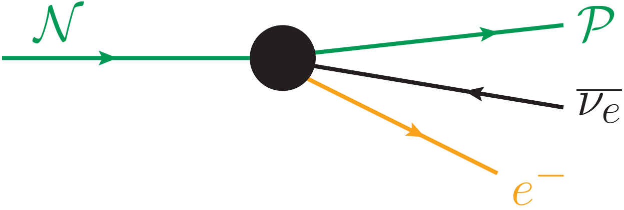
Undoubtedly, the SM is incredibly successful...



... **but not perfect** (we will return to this point!)

first a few ingredients

Nuclear β decay[2]



$$\mathcal{L}_{\rm Fermi} = G_F \left[\overline{\mathcal{N}}\gamma^\mu P_L \mathcal{P}\right] \cdot \left[\overline{\nu_e}\gamma_\mu P_L e\right]$$

Fermi('31)

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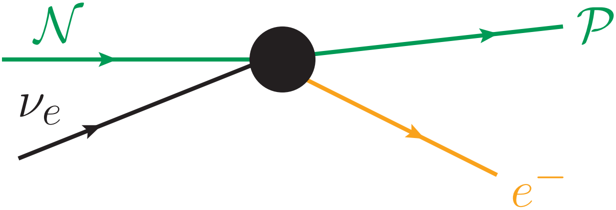
[2] For non-experts: Action $= \mathcal{S} = \int dt\ L = \int d^4x\ \mathcal{L}$. ← HEP uses Lagrangian density with four-vectors x^μ, k^μ

Inverting diagram $\Longrightarrow$ inverse β decay (ν deep-inelastic scattering!)



$$-i\mathcal{M}(\nu_e\mathcal{N} \to e^-\mathcal{P}) \sim G_F\ [\overline{u}(k_{\mathcal{P}})\gamma^\mu P_L u(k_{\mathcal{N}})] \cdot [\overline{u}(k_e)\gamma_\mu P_L u(k_{\nu_e})] \sim G_F\ E^2$$

$$\Longrightarrow\ \sigma(\nu_e\mathcal{N} \to e^-\mathcal{P}) \sim \cancel{f}_{\text{dof}}\ (\text{phase space}) \times |\mathcal{M}|^2 \sim G_F^2 \frac{E^4}{\pi E^2}$$

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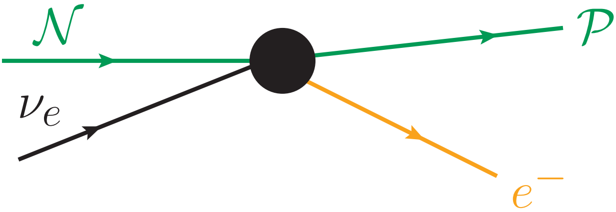

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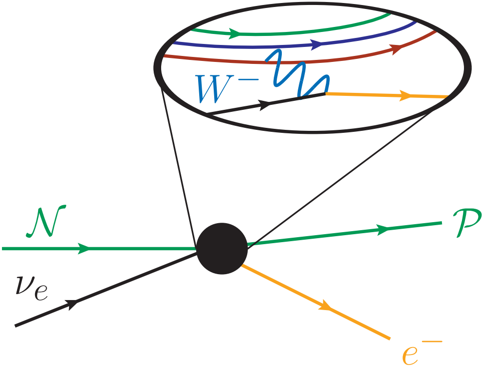
$\Longrightarrow$ **scattering rate (σ) grows with scattering energy!**

$\Longrightarrow$ **violation of unitarity in scattering theory, i.e., $\sum$(prob)≤ 1**

Inverse β decay is a charged-current interaction!



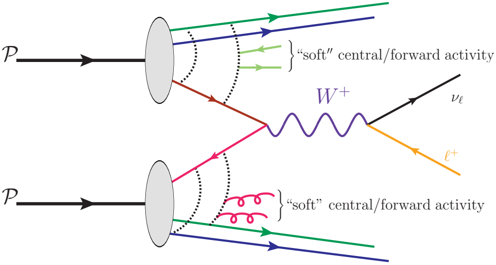
Fermi thry is the low-energy manifestation of the electroweak thry

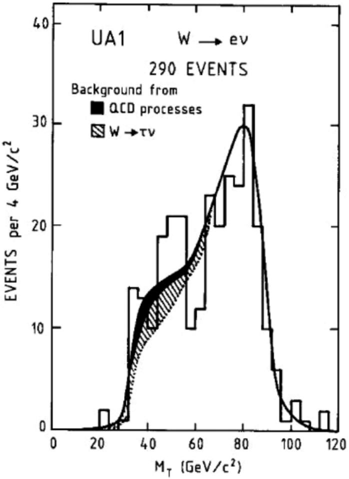
$$\left(\frac{g_W}{\sqrt{2}}\right)^2 \times \left(\frac{g_{\mu\nu}-\frac{q_\mu q_\nu}{M_W^2}}{q^2-M_W^2+i\Gamma_W M_W}\right) \xrightarrow{q^2 \ll M_W^2} \frac{-g_W^2}{2M_W^2} = -2\sqrt{2}G_F$$

$$\implies \sigma(\nu_e \mathcal{N} \to e^- \mathcal{P}) \sim \frac{g_W^4}{\pi}\frac{E^2}{(E^2-M_W^2)^2}$$ ← high-E behavior is regulated (finite)

Rotating diagram $\Longrightarrow$ $W^\pm$ boson production

predicted by Glashow, Weinberg, Salam ('68); + Nobel ('79); discovered by UA1,UA2('83); Nobel ('84)





Transverse mass distribution for all W → $e\nu$ events recorded by UA1

Electroweak sector of Standard Model is powerful:

– explains β decay

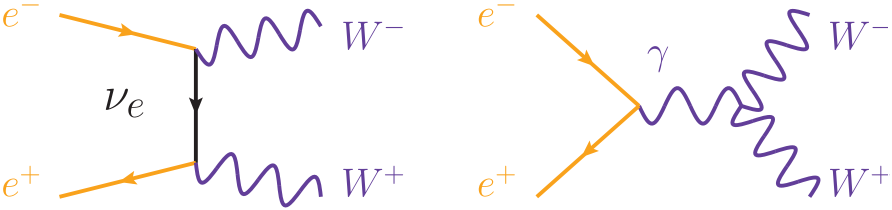
– explains inverse β decay

– predicts $W^\pm$ production in pp collisions

– some inputs needed, e.g., G_F, M_W

A little surgery with diagrams $\Longrightarrow$ W^+W^- pair production

(why make one $W^\pm$ when you can make W^+W^- pairs?)



$$-i\mathcal{M}(e^-e^+ \xrightarrow{\nu} W^+W^-) \sim g_W^2 \times E \times \left(\frac{-E}{E^2}\right) \times \left(\frac{E}{M_W}\right)^2 \sim -g_W^2 \frac{E^4}{E^2 M_W^2}$$

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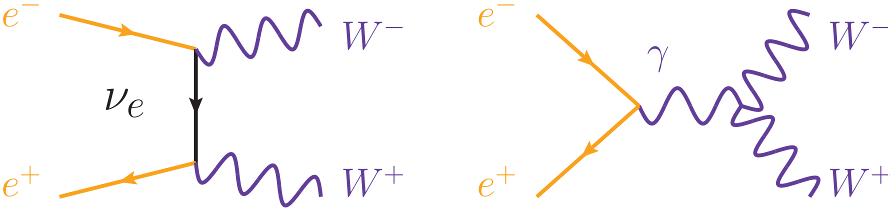

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$\implies$ **scattering amplitude $(\mathcal{M})$ grows with scattering energy!**

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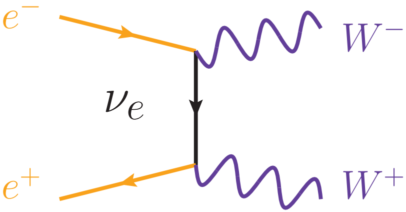

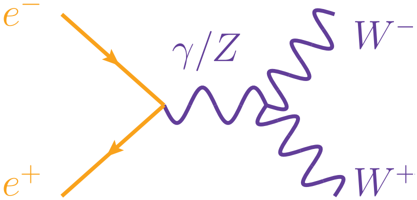

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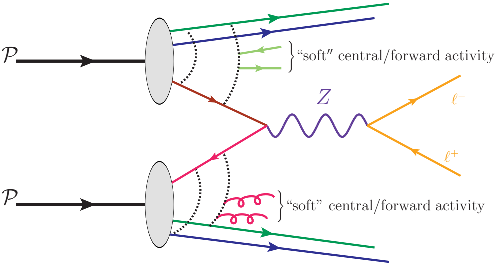
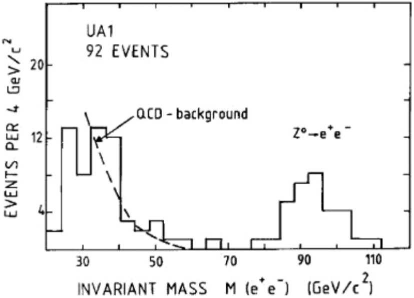
$$-i\mathcal{M}(e^-e^+ \xrightarrow{Z} W^+W^-) \sim \left(\frac{g_W}{\cos\theta_W}\right)(g_W \cos\theta_W) \times (+E) \times \cdots \sim +g_W^2 \frac{E^4}{E^2 M_W^2}$$

Delicate (structural) cancellations when all particles are included!

Diagram fun $\Longrightarrow$ Z boson production

predicted by Glashow, Weinberg, Salam ('68); + Nobel ('79); discovered by UA1,UA2('83); Nobel ('84)





Invariant mass distribution of all e^+e^- pairs recorded by UA1

Electroweak sector of Standard Model is powerful:

– explains β decay

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– some inputs needed, eg, G_F, M_W, M_Z

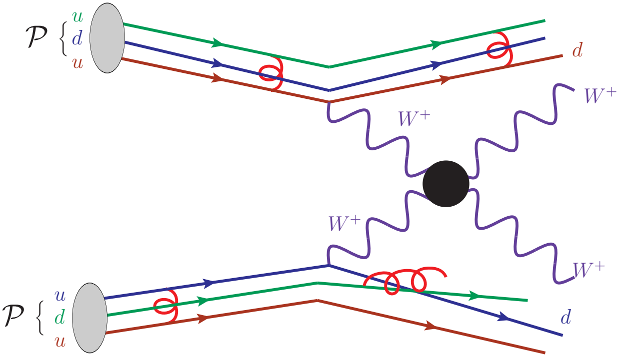
The Standard Model toolbox

- $W^{\pm}$, Z, γ all exist!
- **effective field theories break down at high energies ☺**
- **unitarity violation = bad ☹**
- **breakdown of theory $\Longrightarrow$ unitarity violation ☹**
- **missing contributions $\Longrightarrow$ unitarity violation ☹**
- **small mis-cancellations from new contributions $\Longrightarrow$ E-enhanced scattering rates ☺**

vector boson scattering (VBS) / fusion (VBF)

Cut, rotate, glue, etc. sub-graphs $\Longrightarrow$ $W^+W^+ \to W^+W^+$ scattering

(why make W^+W^- pairs when you can *scatter* them?)



Just one of many examples:

– W^+W^-, $W^\pm Z$, $W^\pm\gamma$, $\gamma\gamma$, ZZ, $Z\gamma$ scattering are all possible

– $W^+W^- \to ZZ$, $W^\pm\gamma \to W^\pm Z$, etc, are also possible

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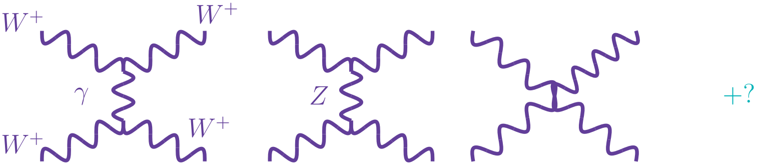

$$-i\mathcal{M}(W^+W^+ \to W^+W^+) \sim \left(\frac{E}{M_W}\right)^4 \times \left(\frac{-M_W^2}{E^2}\right) \times g_W^2(s_\theta^2 + c_\theta^2) \sim \frac{-g_W^2 E^2}{M_W^2}$$

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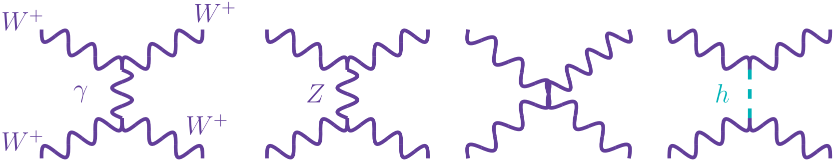


Higgs ('13)

$$-i\mathcal{M}(W^+W^+ \to W^+W^+) \sim \left(\frac{E}{M_W}\right)^4 \times \left(\frac{-M_W^2}{E^2}\right) \times g_W^2(s_\theta^2 + c_\theta^2) \sim \frac{-g_W^2 E^2}{M_W^2}$$

$$-i\mathcal{M}(W^+W^+ \xrightarrow{h} W^+W^+) \sim \left(\frac{E}{M_W}\right)^4 \times \left(\frac{1}{E^2}\right) \times (g_W M_W)^2 \sim \frac{+g_W^2 E^2}{M_W^2}$$

Delicate (structural) cancellations when all particles are included!

Lee, Quigg, and Thacker ('77x2); Chanowitz and Gaillard ('84,'85)

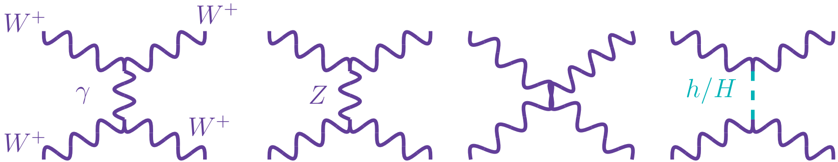
$\Longrightarrow$ **modified $h-V-V$ couplings can partially disrupt cancellations**

Too many contributions?

It is possible that Higgs with $m_h = 125$ GeV is one of several in nature

add'l scalars appears in Two Higgs Doublet Models, Supersymmetry, scalar-singlet dark matter, composite Higgs

$$\underbrace{|h_{\rm SM}\rangle}_{\rm interaction\ eigenstate} = \underbrace{\cos\psi\ |h_{\rm 125\ GeV}\rangle}_{\rm mass\ eigenstate} + \underbrace{\sin\psi\ |H_{\rm several\ TeV}\rangle}_{\rm mass\ eigenstate}$$

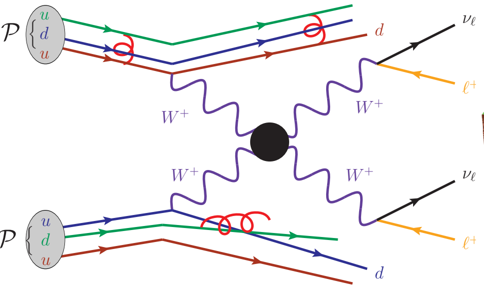


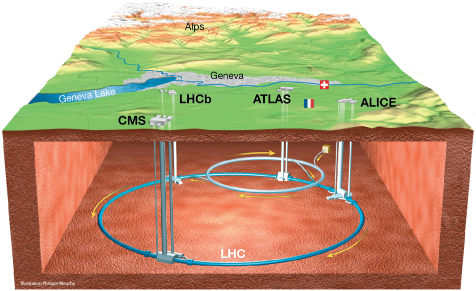
$$-i\mathcal{M}(W^+W^+ \xrightarrow{h/H} W^+W^+) \sim \frac{g_W^2E^2}{M_W^2}\underbrace{\cos^2\psi}_{\mathcal{O}(1)} + \frac{g_W^2E^4}{M_W^2m_H^2}\underbrace{\sin^2\psi}_{\ll 1}$$

$\Longrightarrow$ **$\mathcal{M}$ grows with scattering energy for $E_{(\sim 1\ \rm TeV)} \ll\ m_{H(\rm several\ TeV)}$!**

big idea: studying VBS = studying Higgs sector

The LHC is the **largest, etc.** hadron collider $_{(pp,\, pA,\, AA)}$ at $\sqrt{s} = 13.6$ TeV, with a **broad particle and nuclear physics program**



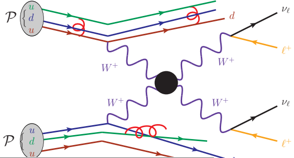


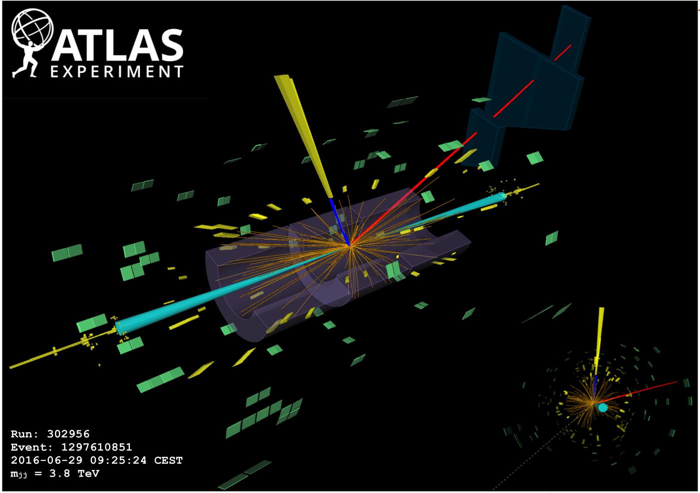
The **ATLAS** and **CMS** detectors at the **LHC** were designed to study VBS

Using VBS to measure SM physics with high precision and search for new phenomena is part of the **LHC's** long-term plan

Buarque (ed.), Gallinaro (ed.), RR (ed.), et al, Rev. Physics ('22) [arXiv:2106.01393]

ATLAS $W^+W^+ \to W^+W^+$ candidate event $(pp \to e^+\nu_e\mu^+\nu_\mu jj)$





[PRL('19)]

ATLAS $W^+W^+ \to W^+W^+$ candidate event
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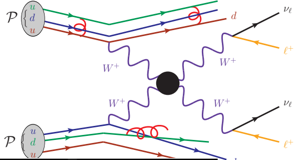
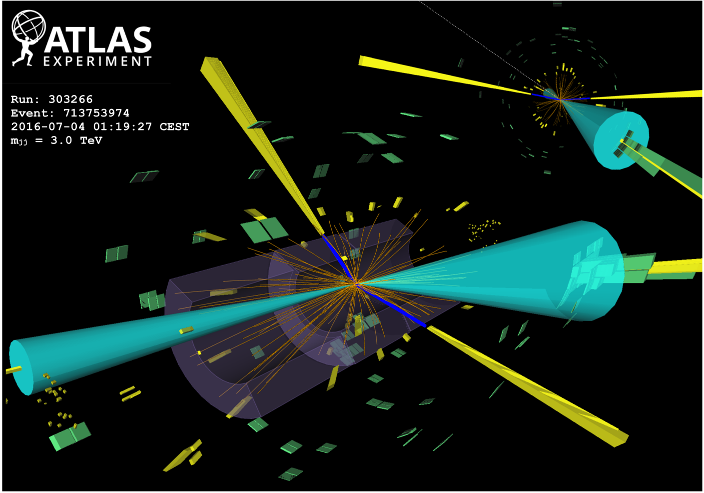
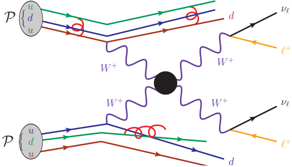
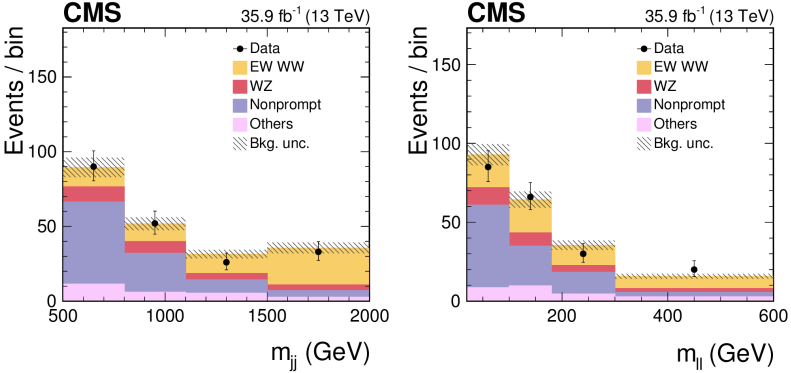




[PRL('19)]

Plotted: in $pp \to \ell_1^\pm \ell_2^\pm \nu\nu jj$, invariant mass of (L) (jj)-system, (R) ($\ell_1\ell_2$)-system





[PRL('18)]

VBS observed for first time during LHC's Run II [CMS('18),ATLAS('19)]

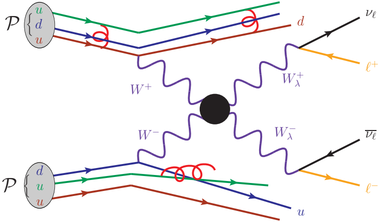
- VBS at the LHC probes multi-TeV energy scales
- First measurements of VBS within 20% of SM predictions

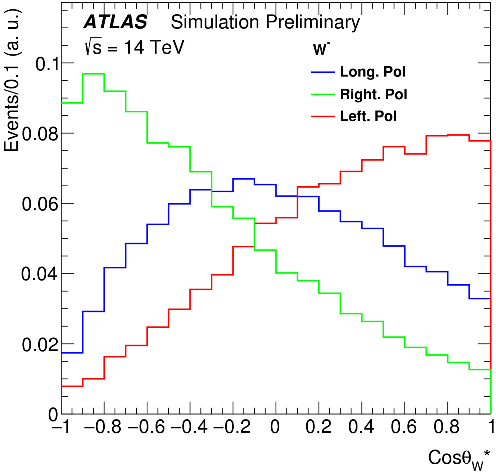
polarization

The $W^{\pm}_{\lambda}, Z_{\lambda}$ bosons are massive, spin-1 objects

Plotted: angle of outgoing W^- in $pp \to W^+ W^-_{\lambda} jj$ via VBS

- 2 transverse polarizations (L,R)
- 1 longitudinal polarization (0)





polarizations of vector bosons imprint on kinematics!

ATLAS [ATL-PHYS-PUB-2018-023]

The $W^\pm_\lambda, Z_\lambda$ bosons are massive, spin-1 objects

Plotted: angle of outgoing ℓ^- in $pp \to W^+W^-_\lambda jj \to W^+\ell^-\overline{\nu_\ell}jj$ via VBS

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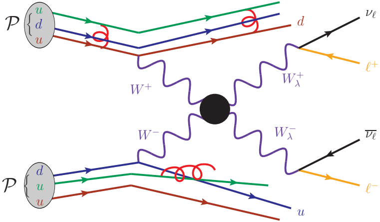

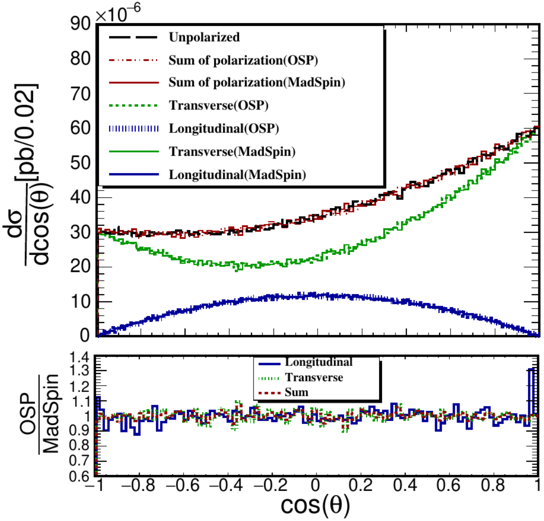

polarizations also imprint on kinematics of decay products!

Buarque Franzosi, RR, et al [(JHEP'20)]

First measurement of polarization in $W^\pm W^\pm$ scattering

CMS (PLB'20)

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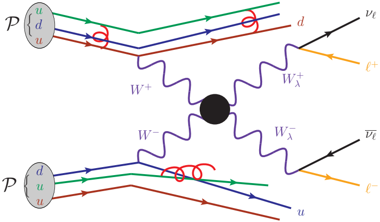
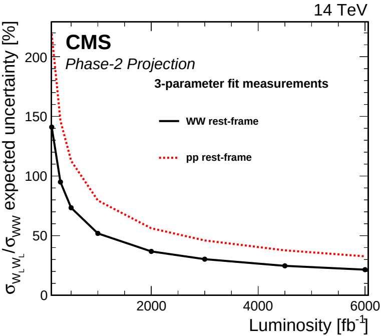


polarizations also imprint on kinematics of decay products!

Process	$\sigma\,\mathcal{B}$ (fb)	Theoretical prediction (fb)
$W^\pm_L W^\pm_L$	$0.32^{+0.42}_{-0.40}$	0.44 ± 0.05
$W^\pm_X W^\pm_T$	$3.06^{+0.51}_{-0.48}$	3.13 ± 0.35
$W^\pm_L W^\pm_X$	$1.20^{+0.56}_{-0.53}$	1.63 ± 0.18
$W^\pm_T W^\pm_T$	$2.11^{+0.49}_{-0.47}$	1.94 ± 0.21

uncertainties sizable but will improve with time



CMS [CMS-PAS-FTR-21-001]

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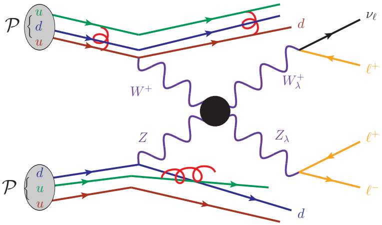
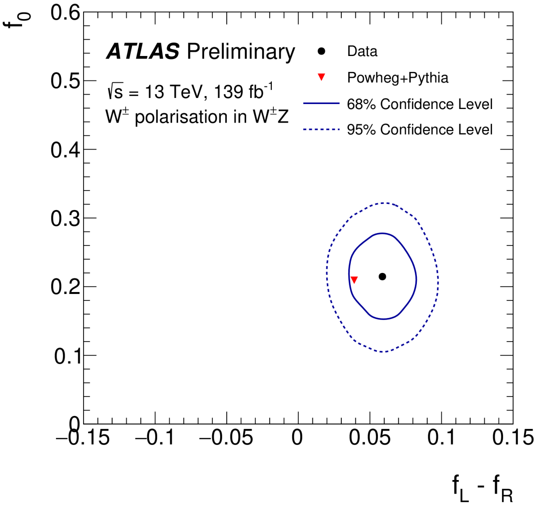


polarization also imprints on kinematics of decay products!

First measurement of polarization fractions (f_λ) in $W^\pm Z$ scattering

ATLAS ('22) [ATLAS-CONF-2022-053]



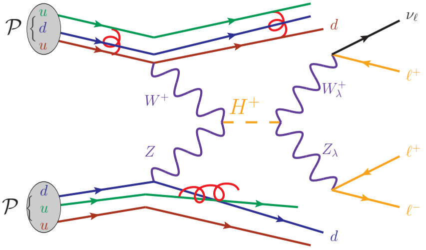
singly and doubly charged scalars

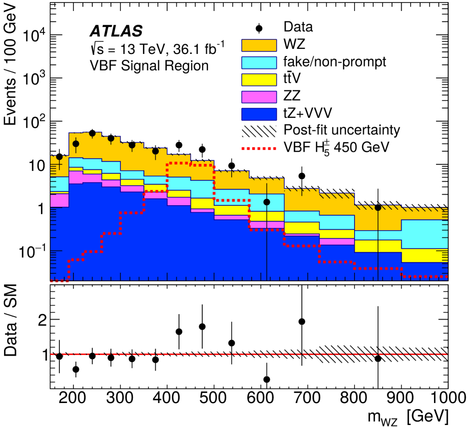
Singly ($H^\pm$) and doubly ($H^{\pm\pm}$) charged scalars are predicted in several popular models

Two Higgs Doublet Models, Supersymmetry, Type II Seesaw, Georgi-Machacek model

Plotted: invariant mass of (WZ)-system in $pp \to W^\pm(\to jj)Z(\to \ell^+\ell^-)jj$

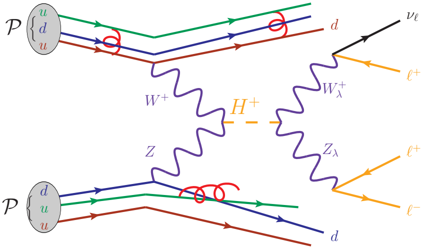
ATLAS [PRL('15)]





Searches for $H^\pm$ in $W^\pm Z$ scattering with early Run II data gave *suggestive hints of something new* ☺!

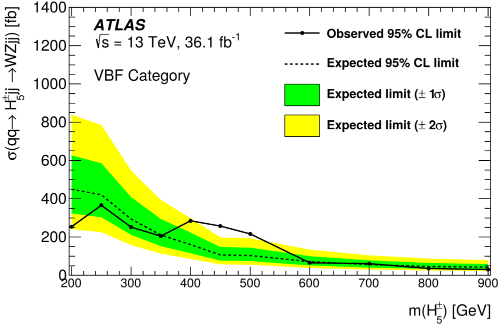
Singly $(H^{\pm})$ and doubly $(H^{\pm\pm})$ charged scalars are predicted in several popular models Two Higgs Doublet Models, Supersymmetry, Type II Seesaw, Georgi-Machacek model



Searches for $H^{\pm}$ in $W^{\pm}Z$ scattering with early Run II data gave *suggestive hints of something new* ☺!

Plotted: excluded upperlimit on scattering rate of $pp \to W^{\pm}Zjj$ via $H^{\pm}$ as a function of $m_H^{\pm}$

ATLAS [PRL('15)]



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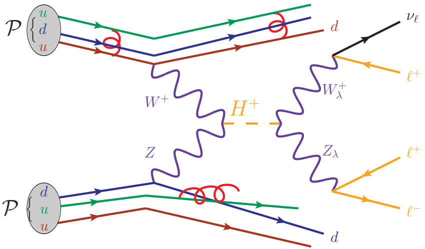
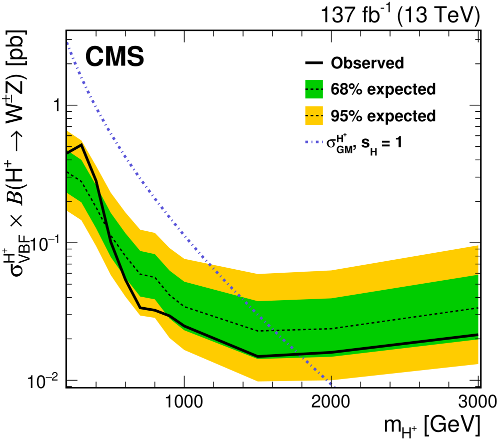
Two Higgs Doublet Models, Supersymmetry, Type II Seesaw, Georgi-Machacek model



Searches for $H^{\pm}$ in $W^{\pm}Z$ scattering with all Run II data shows "bump"

just a statistical fluctuation ☹

Plotted: excluded upperlimit on scattering rate of $pp \rightarrow W^{\pm}Zjj$ via $H^{\pm}$ as a function of $m_H^{\pm}$ CMS [EPJC('21)]



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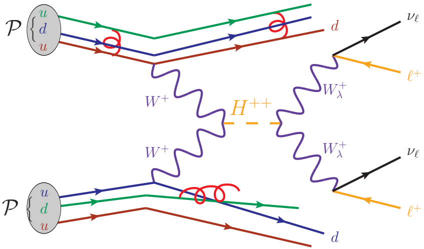
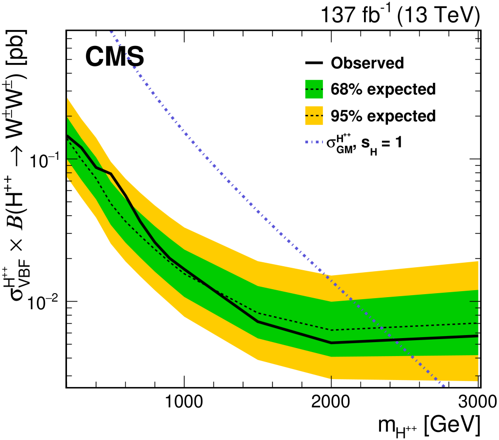




Searches for $H^{\pm\pm}$ in $W^{\pm}W^{\pm}$ scattering with all Run II data explores *new mass and coupling scales*☺

effective field theories[3]

---

[3] too long to get into many details!

Effective field theories are power frameworks to parameterize the impact of new phenomena (and our ignorance!)

$$\mathcal{L} \;=\; \mathcal{L}_{\rm SM} \;+\; \frac{C_5}{\Lambda}\mathcal{O}^{(5)} \;+\; \sum_k \frac{C_{6,k}}{\Lambda^2}\mathcal{O}_k^{(6)} \;+\; \dots$$

Example: the origin of tiny, sub-eV neutrino masses (in the SM, $m_\nu = 0$)

$$\mathcal{L}_5 = \frac{C_5^{\ell\ell'}}{\Lambda}\left[\Phi\cdot\overline{L}_\ell^c\right]\left[L_{\ell'}\cdot\Phi\right] \quad \xrightarrow{\text{low energies (EWBS)}} \quad \frac{1}{2}\underbrace{\frac{C_5^{\ell\ell'}}{\Lambda}\langle\Phi\rangle^2}_{=m_\nu^{\ell\ell'}} \times \overline{\nu_{L\ell}^c}\nu_{L\ell'}$$

With strong but reasonable assumptions, m_ν can be parametrized

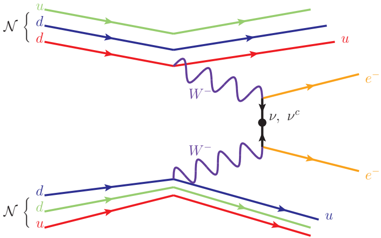
$\mathcal{O}^{(5)}$ is the so-called "dimension-five Weinberg operator," Weinberg ('79)

The Weinberg op. has long-predicted:

– neutrinos are their own antiparticle (Majorana!)

– $0\nu\beta\beta$ decay of heavy isotopes

absence $\implies$ limits on size of C_5^{ee}/Λ.



What about the other $C_5^{\ell\ell'}$?

Effective field theories are power frameworks to parameterize the impact of new phenomena (and our ignorance!)

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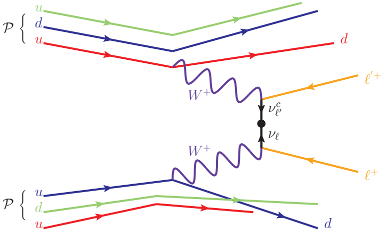
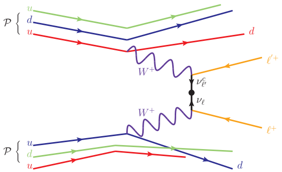


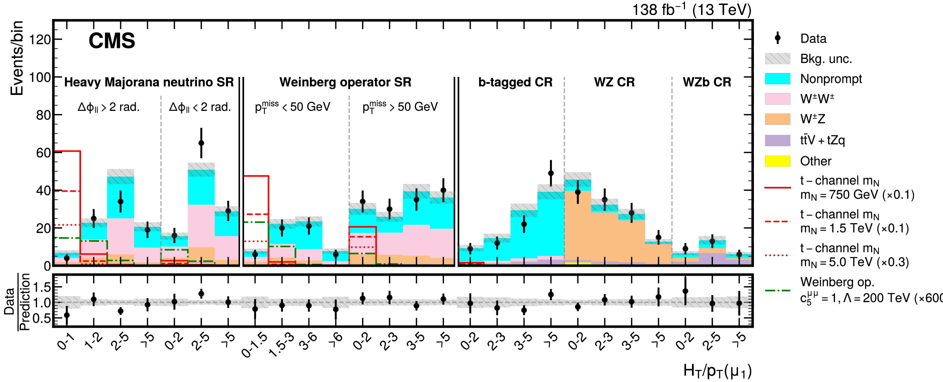
absence $\Longrightarrow$ limits on size of C_5^{ee}/Λ.

What about the other $C_5^{\ell\ell'}$?

Fuks, Ruiz, et al (PRD'21×2)

Plotted: “hadronic energy / lepton energy”
for different signal categories
in $pp \to \mu^\pm \mu^\pm jj$ via $W^\pm W^\pm \to \mu^\pm \mu^\pm$





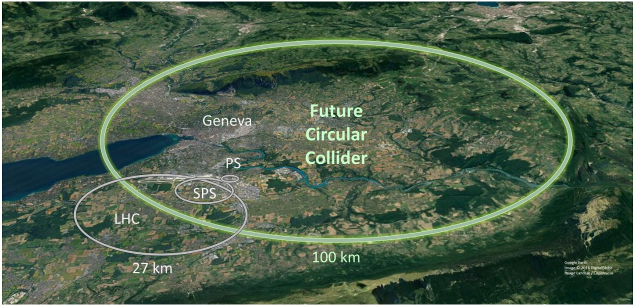
For the first time collider searches for Weinberg operator constrains

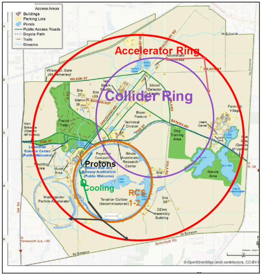
$$\Lambda / C_5^{\mu\mu} \gtrsim 5 \text{ TeV}$$

CMS [PRL’22]

a future beyond the LHC

Many physics and technical discussions are taking place over the successor of the **LHC** (beyond '30s-'40s)

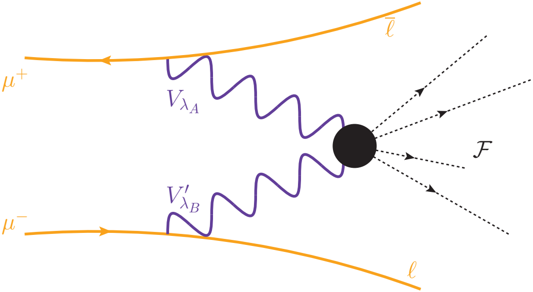




Multi-stage 100 TeV *pp* collider at CERN (FCC program) and 14-30 TeV $\mu^+\mu^-$ at CERN or Fermilab are most supported

European Strategy for Particle Physics [1910.11775,CERN-ESU-013]; Black (ed.), Jindariani (ed.), Li (ed.), F. Maltoni (ed.), et al, [2209.01318]

Why?[4] Situation where scattering formalism is **theoretically interesting**



Partonic collisions at $Q \sim \mathcal{O}(10)$ TeV explore when **electroweak (EW)** symmetry is nearly restored, i.e., $(M^2_{W/Z/H}/Q^2) \to 0$

See C. Bauer, et al ('16,'17,'18); T. Han, et al ('16,'20,'21); A. Manohar, et al ('14,'18) + others

When momentum transfers reach $Q \sim \mathcal{O}(10)$ TeV, vector boson scattering (**VBS/VBF**) **acts a bit... funny**

w/ A. Costantini, et al [2005.10289]

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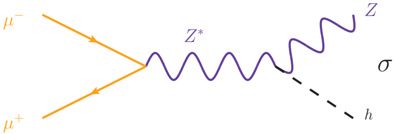
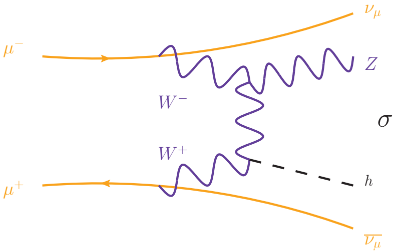
[4] Many motivations, e.g., Al Ali, et al. [2103.14043]; R&D progress as reported in the European Strategy Update (Delahaye, et al) [1901.06150], muoncollider.web.cern.ch; Snowmass (on-going this week)

some examples of VBS at higher energies

Quick interlude: *s*-channel annihilation vs VBF/S



$$\sigma^{s-channel} \sim \frac{g^4}{s}$$

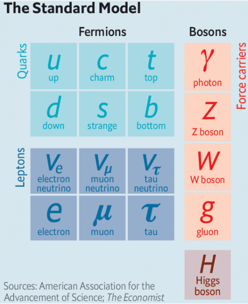


$$\sigma^{VBF} \sim \frac{g^8}{M_{WW}^2} \log^2\left(\frac{M_{WW}^2}{M_W^2}\right)$$

More legs $\implies$ more propagators $\implies$ $\int dk^2/(k^2 - M_W^2) \sim \log(\Lambda^2/M_W^2)$

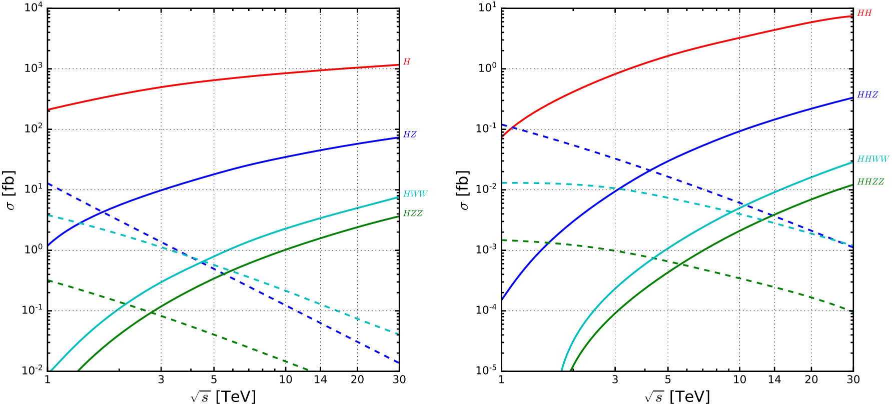
Larger s $\implies$ larger (M_{WW}^2/M_W^2) $\implies$ collinear V compensate for g

Higgs production



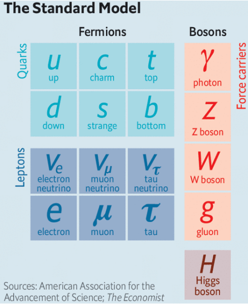
Sources: American Association for the Advancement of Science; *The Economist*

cross sections (σ) vs $\sqrt{s}$ for s-channel annihilation (dash) vs VBF (solid)

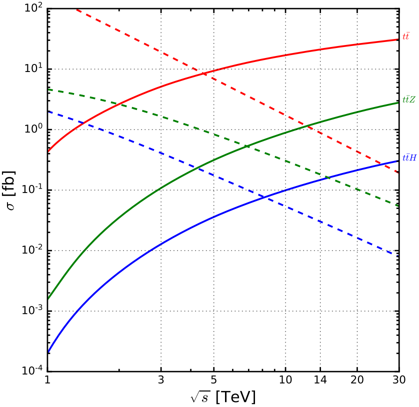


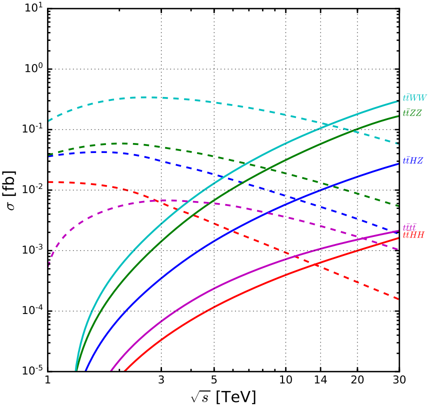
- Eventually, $\sigma^{VBF} > \sigma^{s-channel}$ since
  - $\sigma^{s-channel} \sim 1/s$
  - $\sigma^{VBF} \sim \log^2(M_{VV}^2/M_V^2)/M_{VV}^2$ due to forward emission of $V = W/Z$

Top production



Sources: American Association for the Advancement of Science; *The Economist*

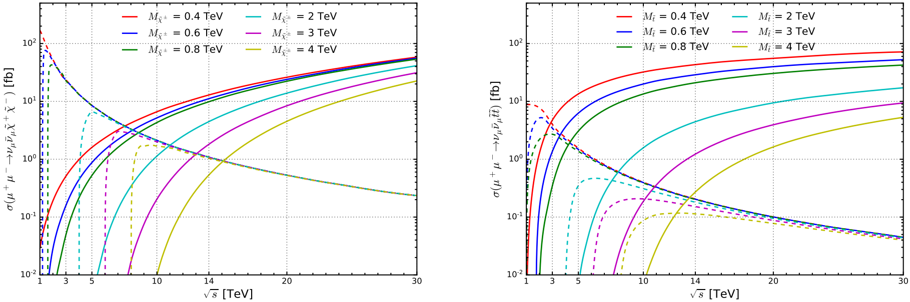




- Do you notice a pattern?

Supersymmetry

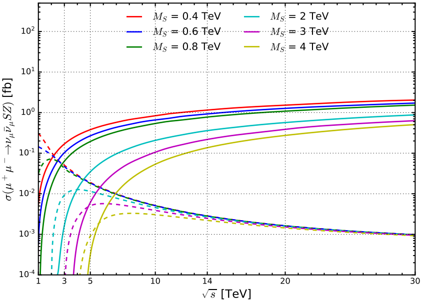
(L) chargino pairs (R) stop pairs



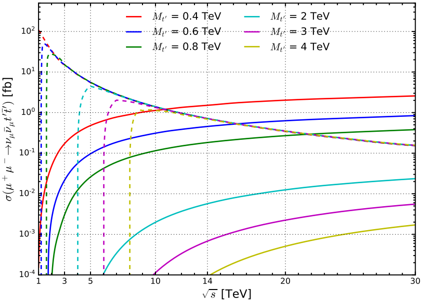
- And now?

Simple Extensions

(L) *S*inglet + *Z* production

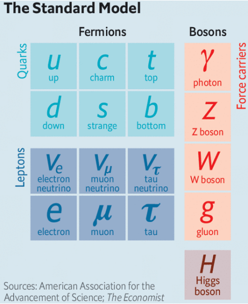


(R) vector-like top pair production



- . . . a little different but a lot of the same

Many-boson production[5]



Sources: American Association for the Advancement of Science; *The Economist*

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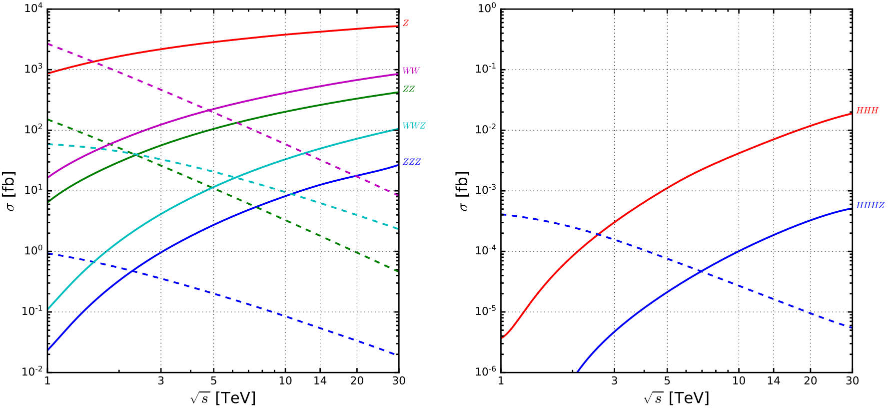
[5] My favorite! I find these processes really neat!



- **VBF is the dominant production vehicle for many processes**

Evidence for universal behavior when the production of X by VBF and annihilation are driven by same physics

Consider a generic s-channel process

$$\sigma^{s-ch.} \sim \frac{(s-M_X^2)}{(s-M_V^2)^2} \sim \frac{(s-M_X^2)}{s^2}$$

Think of $W/Z/\gamma$ as constituents of $\mu^\pm$, to express $\sigma^{\rm VBF}$ in terms of $\sigma^{\rm s-ch.}$

$$\frac{d\sigma^{\rm VBF}}{dz_1 dz_2} \sim f_V(z_1) f_{V'}(z_2) \frac{(z_1 z_2 s - M_X^2)}{(z_1 z_2 s - M_V^2)^2} \sim f_V(z_1) f_{V'}(z_2) \frac{(z_1 z_2 s - M_X^2)\sigma^{s-ch.}}{(z_1 z_2)^2\ (s-M_X^2)}$$

Solve for collider energy $E = \sqrt{s}$ when $\sigma^{\rm VBF} \ > \ \sigma^{\rm s-channel}$

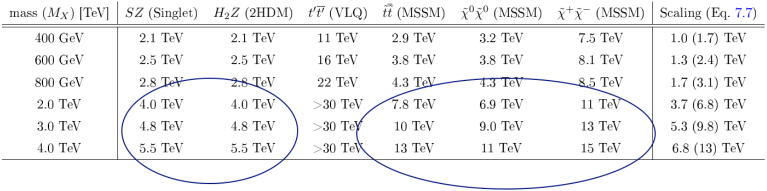
$$\frac{\sigma^{\rm VBF}}{\sigma^{s-ch.}} \sim \mathcal{S} \left(\frac{g_W^2}{4\pi}\right)^2 \left(\frac{s}{M_X^2}\right) \log^2 \frac{s}{M_V^2} \log \frac{s}{M_X^2} > 1$$

Evidence for universal behavior: when the production of X by VBF and annihilation are driven by same physics, VBF dominates for $\sqrt{s}$ given by

w/ A. Costantini, et al [2005.10289]

$$\frac{\sigma^{\rm VBF}}{\sigma^{s-ch.}} \sim \mathcal{S}\left(\frac{g_W^2}{4\pi}\right)^2\left(\frac{s}{M_X^2}\right)\log^2\frac{s}{M_V^2}\log\frac{s}{M_X^2} > 1$$

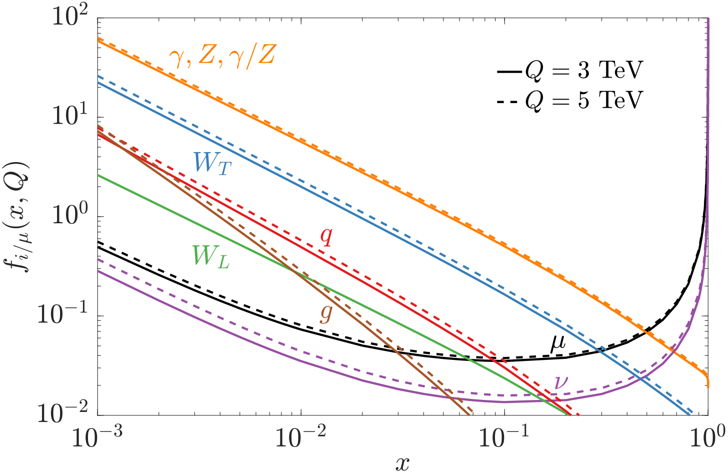
Scaling estimate not so bad if $M_X \gg M_V$. Difference is about $\mathcal{O}(10\%)$



mass (M_X) [TeV]	SZ (Singlet)	H_2Z (2HDM)	$t'\overline{t'}$ (VLQ)	$\tilde{t}\bar{\tilde{t}}$ (MSSM)	$\tilde{\chi}^0\tilde{\chi}^0$ (MSSM)	$\tilde{\chi}^+\tilde{\chi}^-$ (MSSM)	Scaling (Eq. 7.7)
400 GeV	2.1 TeV	2.1 TeV	11 TeV	2.9 TeV	3.2 TeV	7.5 TeV	1.0 (1.7) TeV
600 GeV	2.5 TeV	2.5 TeV	16 TeV	3.8 TeV	3.8 TeV	8.1 TeV	1.3 (2.4) TeV
800 GeV	2.8 TeV	2.8 TeV	22 TeV	4.3 TeV	4.3 TeV	8.5 TeV	1.7 (3.1) TeV
2.0 TeV	4.0 TeV	4.0 TeV	>30 TeV	7.8 TeV	6.9 TeV	11 TeV	3.7 (6.8) TeV
3.0 TeV	4.8 TeV	4.8 TeV	>30 TeV	10 TeV	9.0 TeV	13 TeV	5.3 (9.8) TeV
4.0 TeV	5.5 TeV	5.5 TeV	>30 TeV	13 TeV	11 TeV	15 TeV	6.8 (13) TeV

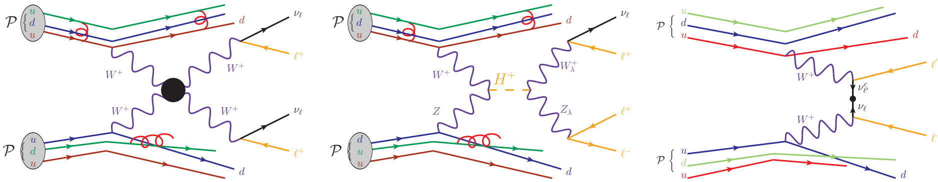
Table 9. For representative processes and inputs, the required muon collider energy $\sqrt{s}$ [TeV] at which the VBF production cross section surpasses the s-channel, annihilation cross section, as shown in figure 17. Also shown are the cross over energies as estimated from the scaling relationship in equation (7.7) assuming a mass scale M_X ($2M_X$).

Idea: $W_\lambda/Z_\lambda/\gamma_\lambda$ content of μ



Han, et al [2007.14300]

summary and outlook



Vector boson scattering is a powerful probe of the Standard Model and new phenomena

Long-predicted but observed first during Run I/II of LHC!

Take-away: With Run II data, first measurements of VBS have established our understanding of a new tool

Take-away: Run III (now-'25) will see VBS used as new probe for the first time in many situations

Take-away: Run IV ('30-'40) will see legacy precision measurements

Thank you!

backup

neutrino masses

For the experts (1 slide)

To generate m_ν via the Higgs mechanism, we need ν_R

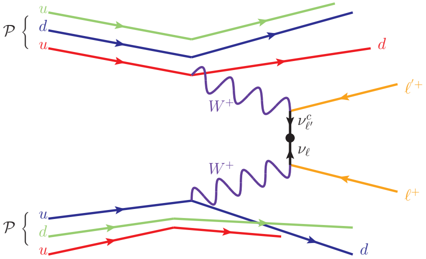
$$\mathcal{L}_{\nu\ \mathrm{Yuk.}} = -y_\nu \overline{L}\tilde{\Phi}\nu_R + \ldots \quad = \quad -y_\nu \left(\overline{\nu_L} \quad \overline{\ell_L}\right) \begin{pmatrix} \langle \Phi \rangle + h \\ 0 \end{pmatrix} \nu_R + \ldots$$

$$= \quad \underbrace{-y_\nu \langle \Phi \rangle}_{\equiv m_\nu} \overline{\nu_L}\nu_R + \ldots$$

ν_R do not exist in the SM, so $m_\nu = 0$!

Dilemma: postulating ν_R requires either new conservation laws or violation of **lepton number** and/or **lepton flavor number** symmetries

(expected but no evidence! suggestive that there is more to the picture)

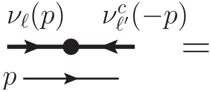


The helicity amplitude for the $0\nu\beta\beta$ process $q\overline{q'} \to \ell_1^+\ell_2^+\overline{f}f'$ is

$$\mathcal{M}_{LNV} = J^{\mu}_{f_1 f_1'} J^{\nu}_{f_2 f_2'} \Delta^W_{\mu\alpha} \Delta^W_{\nu\beta} \underbrace{T^{\alpha\beta}_{LNV}\mathcal{D}(p_\nu)}_{\text{lepton current}}$$

Difficult to simulate since Weinberg op. modifies propagator of ν_ℓ

modern Monte Carlo tools work in mass basis and do not like the idea of modifying $\langle 0|\overline{\nu_{\ell'}}\nu_\ell|0\rangle$



$$= \frac{i\not{p}}{p^2}\,\frac{-iC_5^{\ell\ell'}v^2}{\Lambda}\,\frac{i\not{p}}{p^2} = \frac{im_{\ell\ell'}}{p^2}$$

Solution: Treat vertex as a particle! Invent unphysical Majorana fermion with (small) mass $m_{\ell\ell}$ that couples to all lepton flavors

recovers right behavior!

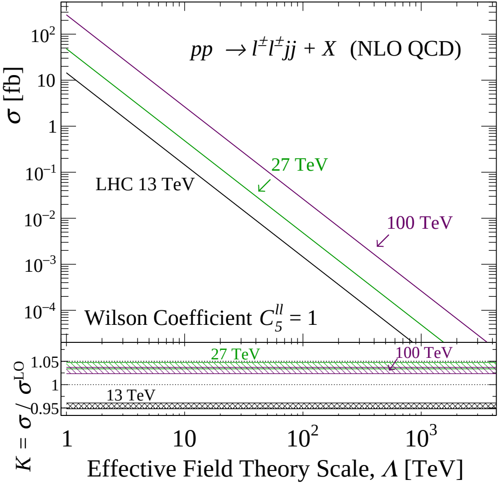
$$T^{\alpha\beta}_{LNV}\mathcal{D}(p_\nu) \propto \gamma^\alpha P_L \frac{i(\not{p}+m_{\ell\ell'})}{p^2-m^2_{\ell\ell'}}\gamma^\beta P_R = \gamma^\alpha P_L \frac{im_{\ell\ell'}}{p^2} P_L \gamma^\beta \times \left[1+\mathcal{O}\left(\left|\frac{m^2_{\ell\ell'}}{p^2}\right|\right)\right]$$

Plotted: Normalized production rate $(C_5 = 1)$ vs scale (Λ)

w/ Fuks, Neundorf, Peters, Saimpert [2012.09882]

Full $2 \to 4$ calculation at NLO(+PS) in QCD is more involved

Used `mg5amc` + NEW `SMWeinberg` UFO libraries



Driven by $W_0^+ W_0^+$ scattering

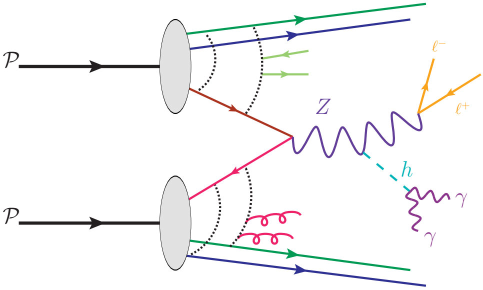
$$\hat{\sigma}(W^+ W^+ \to \ell^+ \ell^+) \sim \frac{|C_5^{\ell\ell}|^2}{18\pi\Lambda^2}$$

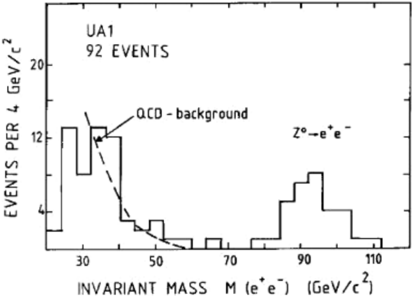
Once σ is obtained for a "high" scale, i.e., $C_5^{\ell\ell'} = 1, \Lambda = 200$ TeV, rescale for other Λ / C_5.

C_5^{ee}/Λ is heavily constrained. **What can the LHC say about $C_5^{\ell\ell'}$?**

Diagram games $\Longrightarrow$ h boson production

predicted by Brout, Engler ('64), Higgs ('64); + Nobel ('14); discovered by ATLAS and CMS ('12)





Invariant mass distribution of all e^+e^- pairs recorded by UA1

Electroweak sector of Standard Model is powerful:

– explains β decay

– explains inverse β decay

– explains masses of $W^\pm, Z$, e, others

– inputs needed, eg, G_F, M_W, M_Z, m_h