Neutrino Physics

Selected topics

Lecture 2

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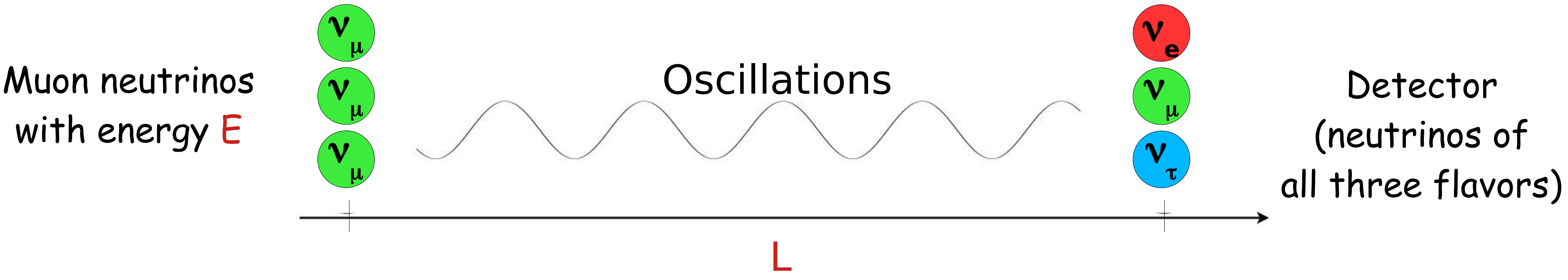
Outline

- Neutrino oscillations
- Reactor neutrinos
- Accelerator neutrinos

Neutrino mixing - basic concept

- Reminder: neutrinos oscillate – change their flavor with time. Experimentally confirmed by a number of experiments: Super Kamiokande, SNO...



Muon neutrinos with energy E

Oscillations

Detector (neutrinos of all three flavors)

- Neutrinos are produced and detected via weak interactions (flavor eigenstates) but propagate in space as the linear superpositions of the mass eigenstates.

Transition coefficients (Unitary 3x3 matrix – 'mixing matrix')

Flavor eigenstates

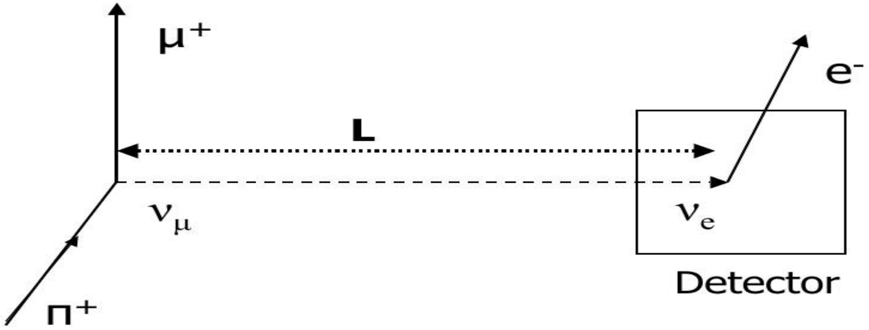
$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Mass eigenstates

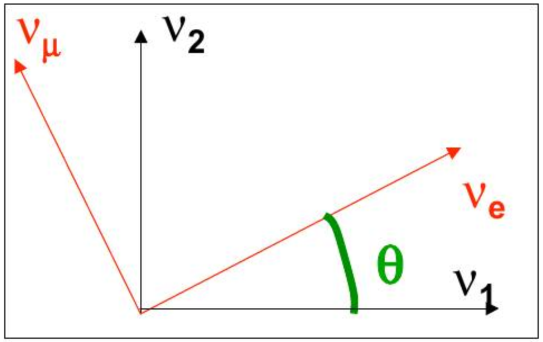
$$U = \begin{pmatrix} U_{e1} & U_{\mu 1} & U_{\tau 1}\\ U_{e2} & U_{\mu 2} & U_{\tau 2}\\ U_{e3} & U_{\mu 3} & U_{\tau 3}\end{pmatrix}$$

Neutrino mixing – 2 neutrinos example

- Two neutrino example:



- Notice the flavor eigenstates differ from mass eigenstates. Therefore the source produces the linear superposition of the mass eigenstates. In case of two neutrinos:



$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

$$\nu_\alpha = \sum_i^N U_{\alpha i} \nu_i$$

Greek indices for flavor
Latin for mass eigenstates

Neutrino oscillations principles

- Flavor state written as a superposition of the mass states:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle\,,$$

- Mass state propagation (plane wave solutions):

$$|\,\nu_j(t)\,\rangle = e^{-i\left(E_j t - \vec{p}_j\cdot\vec{x}\right)}\,|\,\nu_j(0)\,\rangle\ ,$$

- Ultrarelativistic limit energy approximation. This limit applies to all practical (currently observed) neutrinos, since their masses are less than 1 eV and their energies are at least 1 MeV, so the Lorentz factor, γ, is greater than 10^6 in all cases

$$|\vec{p}_j| = p_j \gg m_j\ ,$$

$$E_j = \sqrt{p_j^2 + m_j^2} \simeq p_j + \frac{m_j^2}{2\,p_j} \approx E + \frac{m_j^2}{2\,E}\ ,$$

- Use above and put t=L plus drop phase factors:

$$|\,\nu_j(L)\,\rangle = e^{-i\left(\frac{m_j^2\,L}{2\,E}\right)}\,|\,\nu_j(0)\,\rangle\ .$$

Neutrino oscillations principles

- Eigenstates with different masses propagate with different frequencies.
- The heavier ones oscillate faster compared to the lighter ones.
- Since the mass eigenstates are combinations of flavor eigenstates, this difference in frequencies causes interference between the corresponding flavor components of each mass eigenstate.
- Constructive interference causes it to be possible to observe a neutrino created with a given flavor to change its flavor during its propagation.

$$P_{\alpha\to\beta} = \left| \langle \nu_\beta | \nu_\alpha(L) \rangle \right|^2 = \left| \sum_j U^*_{\alpha j} U_{\beta j} e^{-i\frac{m_j^2 L}{2E}} \right|^2 .$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \Re(J_{ij}^{\alpha\beta}) \sin^2\Phi_{ij} + 2\sum_{i<j} \Im(J_{ij}^{\alpha\beta}) \sin 2\Phi_{ij}$$

$$\Phi_{ij} = \Delta m_{ij}^2 \frac{L}{4E} = 1.27 \cdot \Delta m_{ij}^2 [eV^2] \cdot \frac{L[km]}{E[GeV]}$$

Term responsible for oscillation

$$J_{ij}^{\alpha\beta} = U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}$$

Mixing matrix elements (Jarlskog invariant)

Neutrino oscillations (3 flavors)

Pontercorvo-Maki-Nakagawa-Sakata matrix (U_{PNMS})

$$U = U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$

$s_{ij} = \sin\theta_{ij}$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \Re(J_{ij}^{\alpha\beta}) \sin^2\Phi_{ij} + 2\sum_{i<j} \Im(J_{ij}^{\alpha\beta}) \sin 2\Phi_{ij}$$

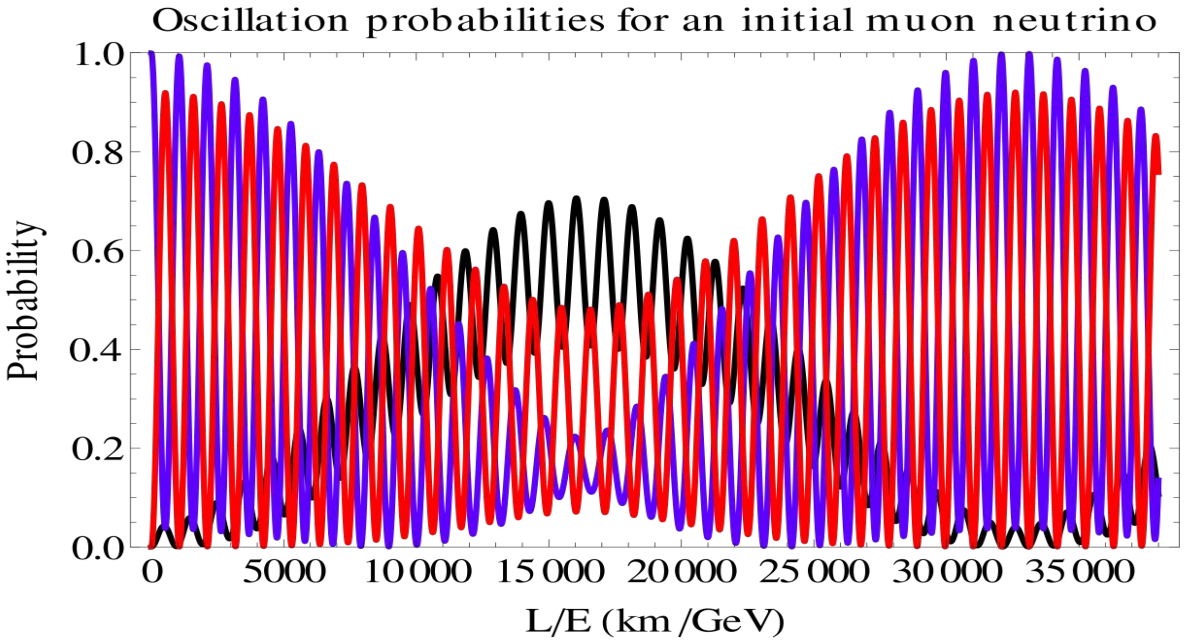
$$\Phi_{ij} = \Delta m_{ij}^2 \frac{L}{4E} = 1.27 \cdot \Delta m_{ij}^2 [eV^2] \cdot \frac{L[km]}{E[GeV]}$$

$$J_{ij}^{\alpha\beta} = U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*$$

- Transition probability $P(\nu_\alpha \rightarrow \nu_\beta)$ in the case of 3 neutrinos depends on:
  - 3 mixing angles: θ_{23}, θ_{13}, θ_{12}
  - 1 complex phase: δ_{CP}
  - 2 independent mass splittings: Δm^2_{32} , Δm^2_{12}
  - Detector-source distance (L), neutrino energy (E) – adjusted experimentally

PMNS model parameters

Neutrino oscillations (3 flavors)



$$P(\nu_\mu \rightarrow \nu_\mu)$$

$$P(\nu_\mu \rightarrow \nu_e)$$

$$P(\nu_\mu \rightarrow \nu_\tau)$$

- Two oscillation frequencies:
  - Slow (solar)
  - Fast (atmospheric)

Experiment		L (m)	E (MeV)	$\lvert\Delta m^2\rvert$ (eV^2)
Solar		10^{10}	1	10^{-10}
Atmospheric		$10^4 - 10^7$	10^2–10^5	$10^{-1} - 10^{-4}$
Reactor	VSBL–SBL–MBL	$10 - 10^3$	1	$1 - 10^{-3}$
	LBL	$10^4 - 10^5$		$10^{-4} - 10^{-5}$
Accelerator	SBL	10^2	10^3–10^4	> 0.1
	LBL	$10^5 - 10^6$	$10^3 - 10^4$	$10^{-2} - 10^{-3}$

Oscillations - experimental aspects

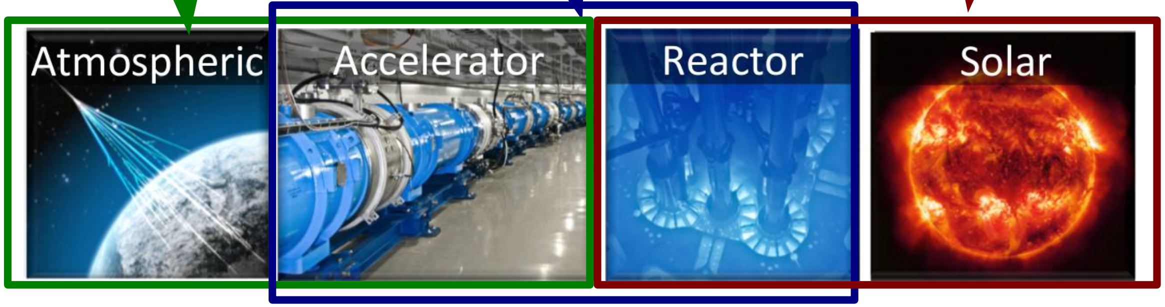
- Neutrino oscillation experiments measure:
  - Appearance probability: $P(\nu_\alpha \rightarrow \nu_\beta) = ?$
  - Disappearance (survival) probability: $P(\nu_\alpha \rightarrow \nu_\alpha) = ?$
- Number of observed (detected) neutrino interactions is proportional to:
  - neutrino-target (eg. neutrino-nucleus) cross section σ,
  - neutrino flux Φ
  - number of interacting targets (eg. number of target nuclei) T

$$N_{Obs} \approx \sigma * \Phi * T$$

Neutrino oscillations - experiments

$$U_{PMNS}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\cdot\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{CP}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13}\end{pmatrix}\cdot\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$

ν_μ disappearance

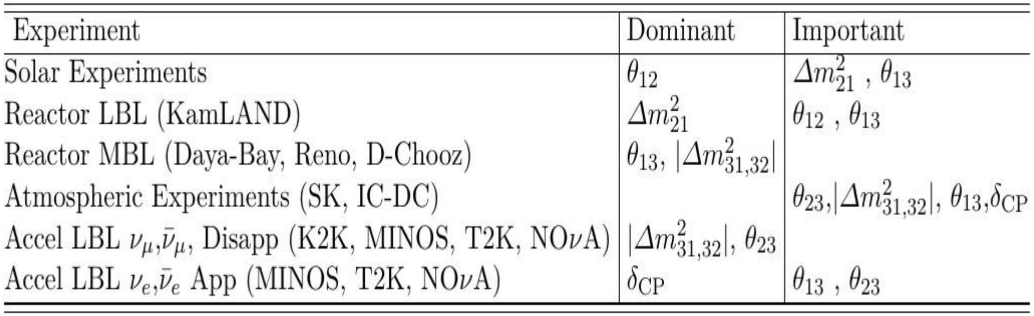


ν_e disappearance,

$\overline{\nu}_e$ disappearance

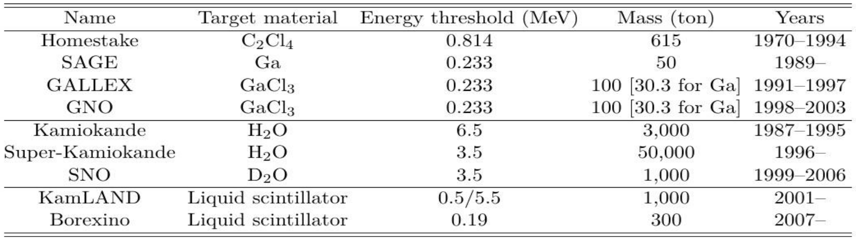
ν_e appearance) , $\overline{\nu}_e$ disappearance

Table 14.6: Experiments contributing to the present determination of the oscillation parameters.



Experiment	Dominant	Important
Solar Experiments	θ_{12}	Δm^2_{21} , θ_{13}
Reactor LBL (KamLAND)	Δm^2_{21}	θ_{12} , θ_{13}
Reactor MBL (Daya-Bay, Reno, D-Chooz)	θ_{13}, $\lvert\Delta m^2_{31,32}\rvert$	
Atmospheric Experiments (SK, IC-DC)		θ_{23},$\lvert\Delta m^2_{31,32}\rvert$, θ_{13},$\delta_{\rm CP}$
Accel LBL ν_μ,$\bar\nu_\mu$, Disapp (K2K, MINOS, T2K, NOνA)	$\lvert\Delta m^2_{31,32}\rvert$, θ_{23}	
Accel LBL ν_e,$\bar\nu_e$ App (MINOS, T2K, NOνA)	$\delta_{\rm CP}$	θ_{13} , θ_{23}

Table 14.2: List of solar neutrino experiments



Name	Target material	Energy threshold (MeV)	Mass (ton)	Years
Homestake	C_2Cl_4	0.814	615	1970–1994
SAGE	Ga	0.233	50	1989–
GALLEX	$GaCl_3$	0.233	100 [30.3 for Ga]	1991–1997
GNO	$GaCl_3$	0.233	100 [30.3 for Ga]	1998–2003
Kamiokande	H_2O	6.5	3,000	1987–1995
Super-Kamiokande	H_2O	3.5	50,000	1996–
SNO	D_2O	3.5	1,000	1999–2006
KamLAND	Liquid scintillator	0.5/5.5	1,000	2001–
Borexino	Liquid scintillator	0.19	300	2007–

Neutrino oscillations - what has been measured?

$$\sin^2(\theta_{12}) = 0.307 \pm 0.013$$

$$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2$$

$$\sin^2(\theta_{23}) = 0.547 \pm 0.021 \quad \text{(Inverted order)}$$

$$\sin^2(\theta_{23}) = 0.545 \pm 0.021 \quad \text{(Normal order)}$$

$$\Delta m^2_{32} = (-2.546^{+0.034}_{-0.040}) \times 10^{-3}\ \mathrm{eV}^2 \quad \text{(Inverted order)}$$

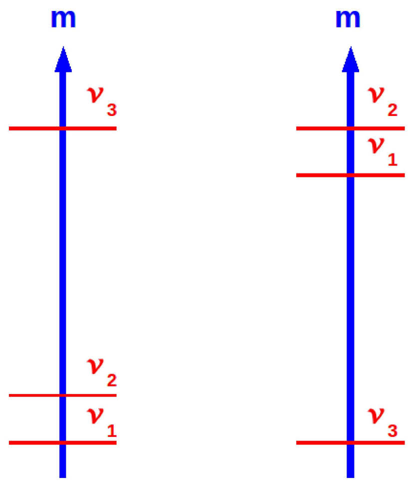
$$\Delta m^2_{32} = (2.453 \pm 0.034) \times 10^{-3}\ \mathrm{eV}^2 \quad \text{(Normal order)}$$

$$\sin^2(\theta_{13}) = (2.18 \pm 0.07) \times 10^{-2}$$

$$\delta,\ CP \text{ violating phase} = 1.36 \pm 0.17\ \pi \text{ rad}$$

- Most important open questions:
  - What is the value of δ_{CP}? CP symmetry violation in neutrino sector?
  - What is the neutrino mass ordering? Normal: $\mathbf{m_3 > m_2} > m_1$ (NO) or inverted: $m_2 > \mathbf{m_1 > m_3}$ (IO)?
  - What is the value of θ_{23}? If not 45 degrees, then in which octant it lies?

Neutrino mass ordering

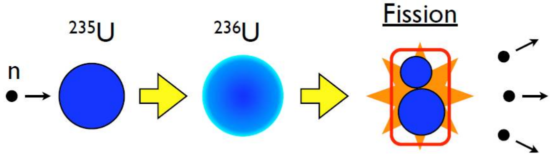


Normal Ordering NO

Inverted Ordering IO

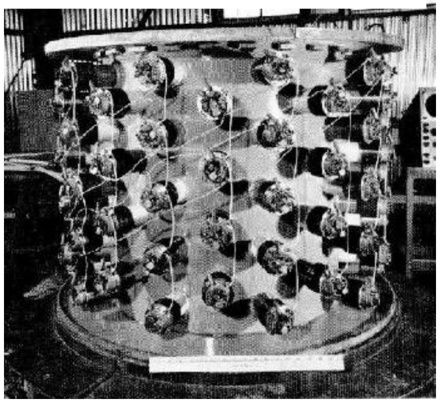
Artificial neutrino sources: reactors

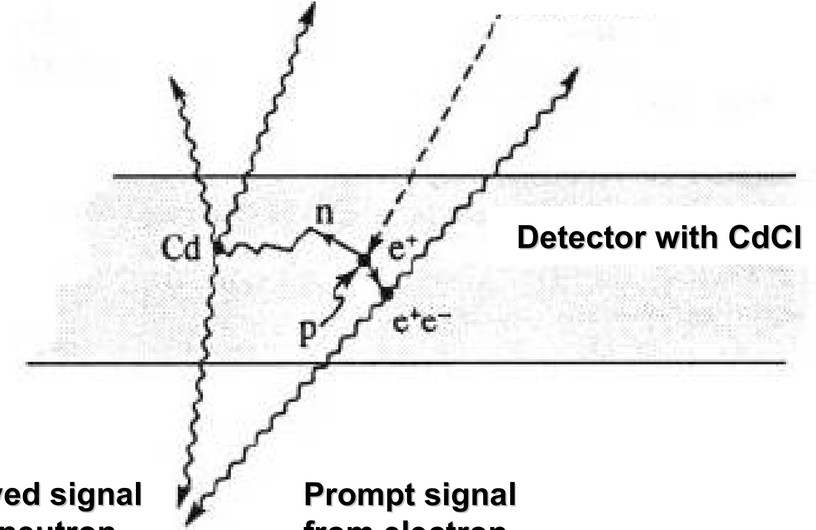
- In the nuclear reactors electron antineutrinos are produced as a result of the nuclear fission of heavy isotopes ^{235}U, ^{238}U, ^{239}Pu, ^{241}Pu



- 6 antineutrinos are produced per one fission reaction
- Typical nuclear powerplant (1 GW reactor) produces 2×10^{20} antineutrinos per second
- Number of detected antineutrinos is proportional to the reactor power
- There’s a long tradition of using reactor antineutrinos to study neutrino properties. It started in 1953 by Reines & Cowan (detector located next to the Savannah River powerplant)

Reines & Cowan detector





Reactor antineutrino

Detector with CdCl

Delayed signal from neutron

Prompt signal from electron

Frederick Reines & Clyde Cowan

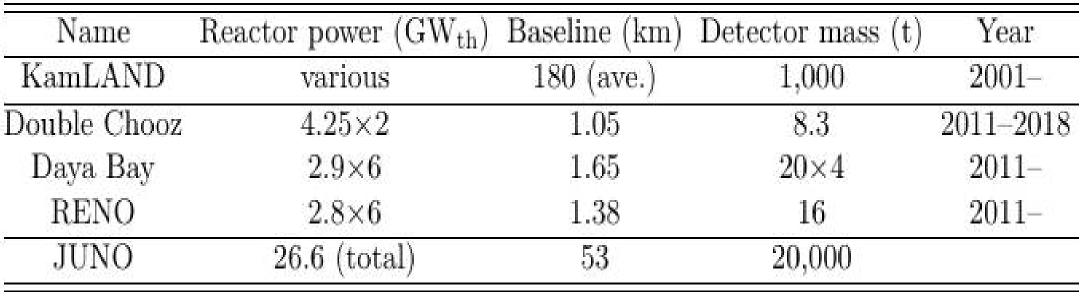
Nobel prize for Reines in 1995

Reactor neutrinos

- Reactor experiments are measuring the probability of disappearance of electron antineutrinos from the reactor.

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = ?$$

- A precise prediction of the antineutrino flux from the nuclear reactor is crucial because we need to compare the measured neutrino spectrum with the predicted one.
- Detailed calculation of the antineutrino flux from the nuclear reactor is challenging (summing up the spectra of beta decays)
    - Fission processes of four main isotopes involves thousands(!) of beta decay branches → The main problem of reactor neutrino experiments is the neutrino flux calculation
    - Many improvements in the reactor neutrino flux calculations recently.
- Currently operating reactor neutrino experiments:



Name	Reactor power (GW_{th})	Baseline (km)	Detector mass (t)	Year
KamLAND	various	180 (ave.)	1,000	2001–
Double Chooz	4.25×2	1.05	8.3	2011–2018
Daya Bay	2.9×6	1.65	20×4	2011–
RENO	2.8×6	1.38	16	2011–
JUNO	26.6 (total)	53	20,000	

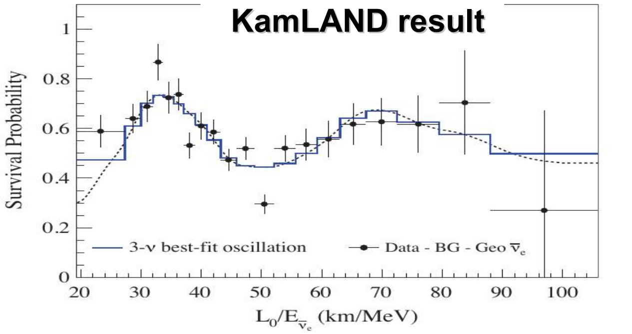
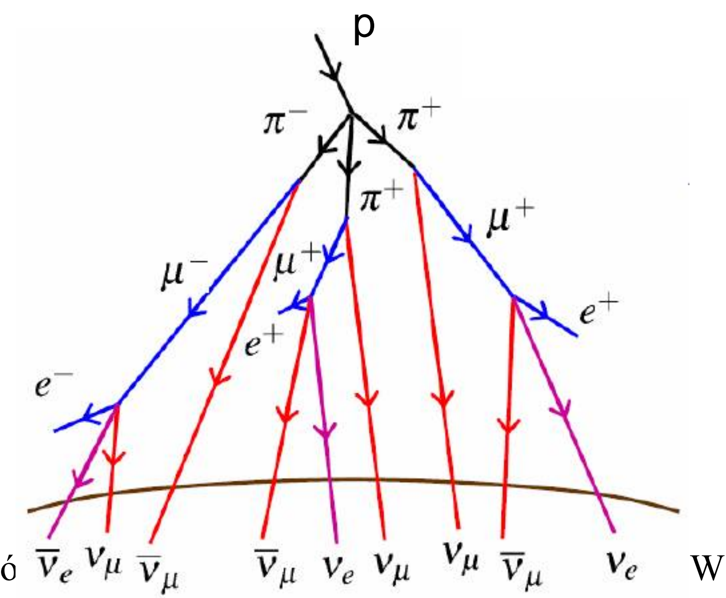


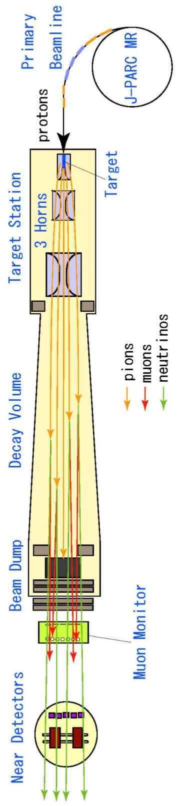
Figure 14.7: Ratio of the observed $\bar{\nu}_e$ spectrum to the expectation for no-oscillation versus L_0/E for the KamLAND data. $L_0 = 180$ km is the flux-weighted average reactor baseline. The 3-ν histogram is the best-fit survival probability curve from the three-flavour unbinned maximum-likelihood analysis using only the KamLAND data. This figure is taken from [150].

Artificial neutrino sources: accelerators

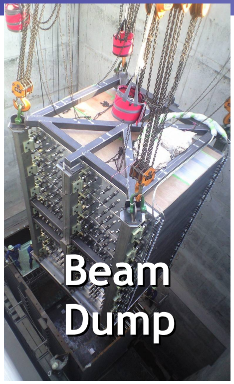
- The main principle behind the production accelerator neutrinos is an analogy to atmospheric neutrino production mechanism
  - Accelerate protons to high energies with the accelerator
  - Collide protons with the target (eg. graphite) and produce secondary particles, mainly pions
  - Let the pions decay and produce neutrinos

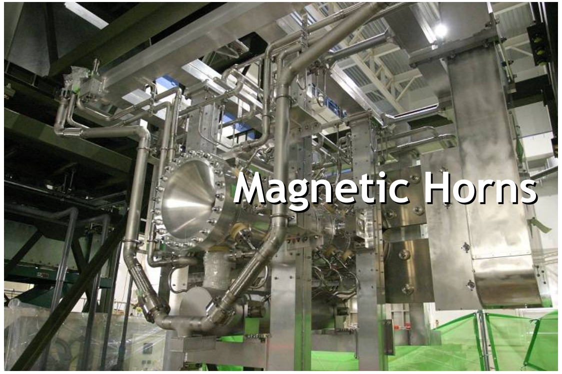


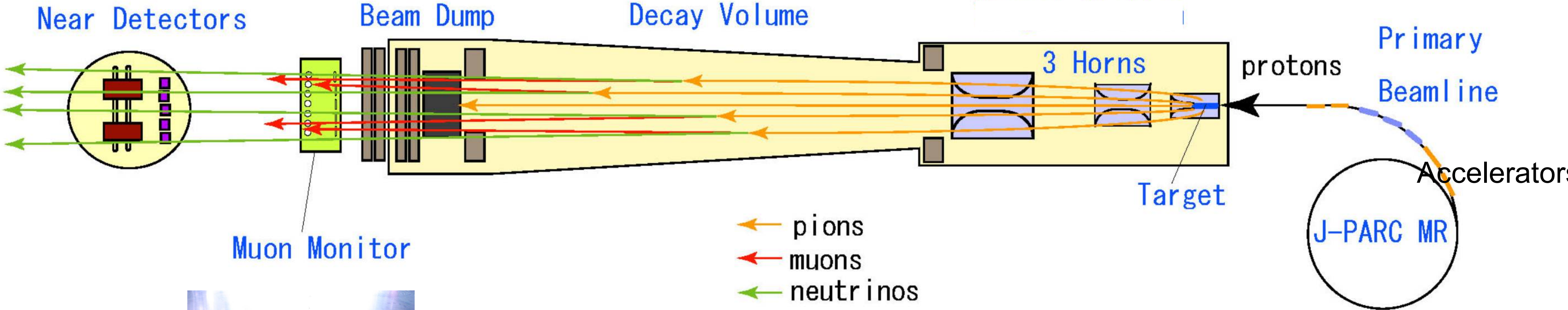
- Some extra ideas in accelerator neutrino production:
  - Focus charged pions with the same sign (and deflect pions with opposite sign) with magnetic horns (toroidal magnetic field).
    - Ultimately able to get either neutrino (positive pions are focused) or antineutrino (negative pions are focused) beam.
    - Can change the polarity of the horns → one experiment can operate in two modes: neutrino or antineutrino mode
  - Stop muons produced in the charged pion decay using a block of graphite and iron → beam dump.

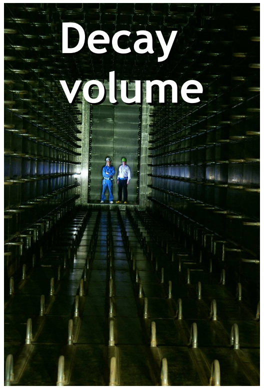


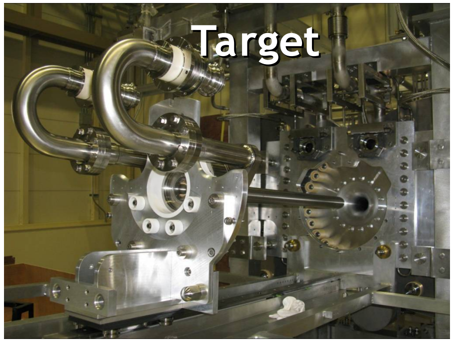
Accelerator neutrinos



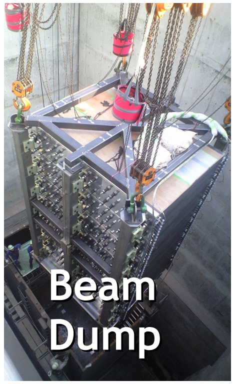


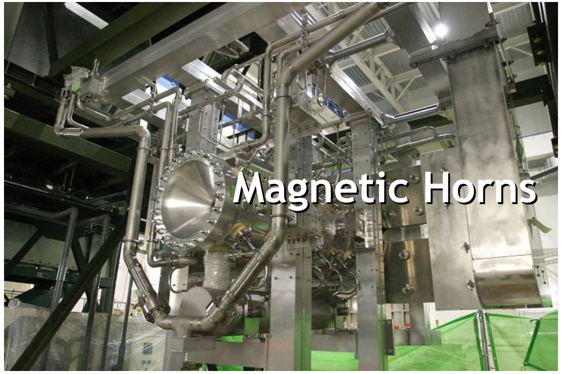


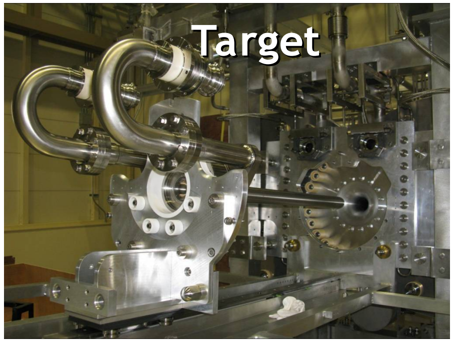


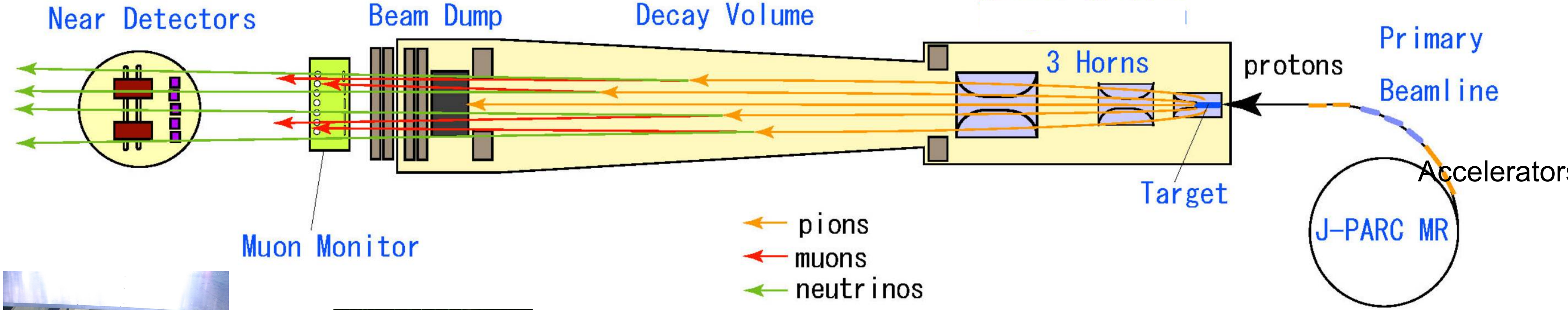


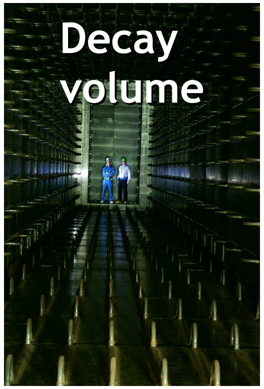
Accelerator neutrinos

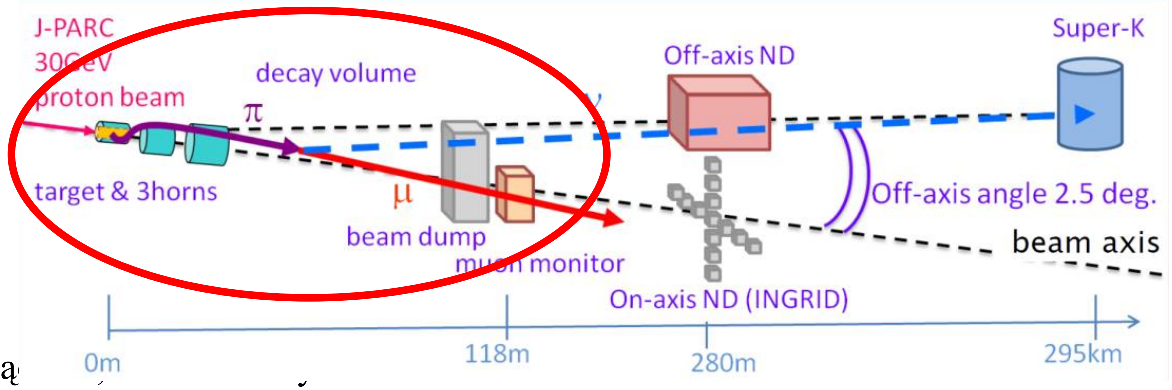






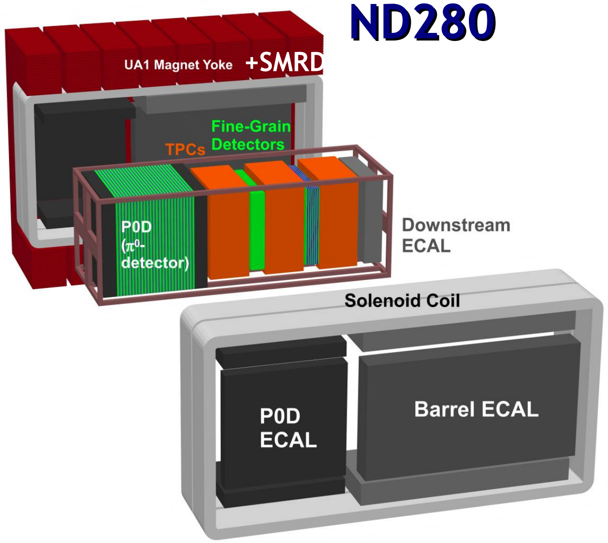




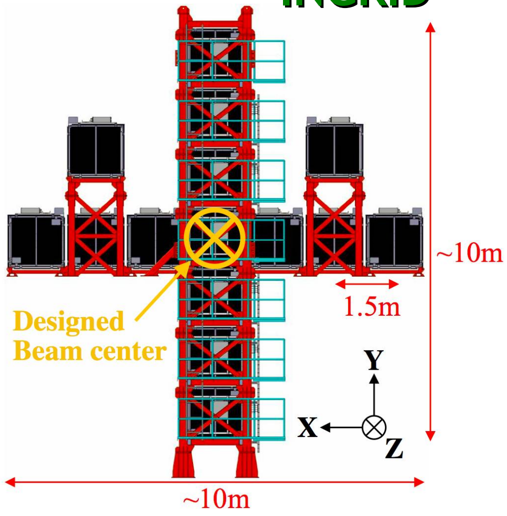


Near detector example (T2K)

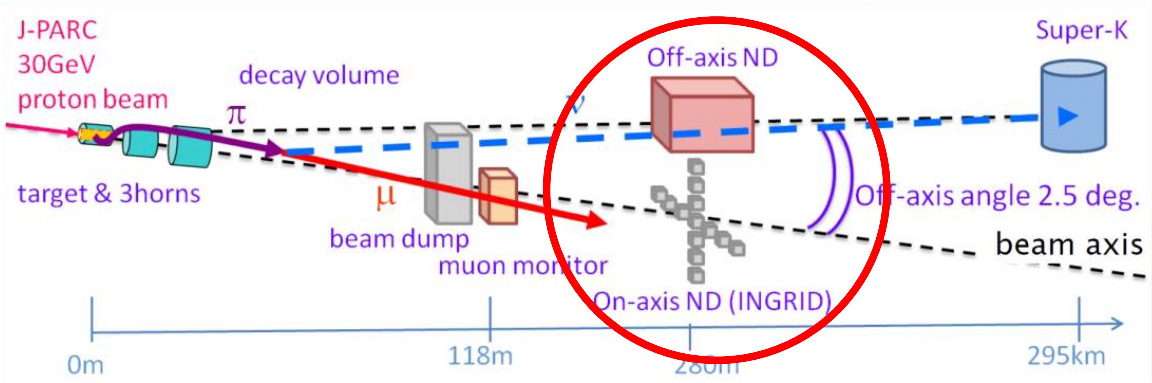
ND280



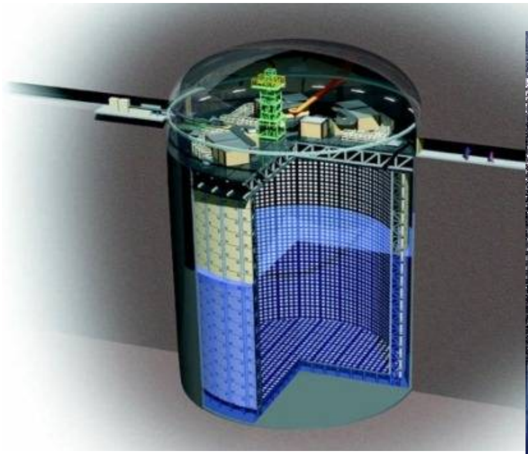
INGRID

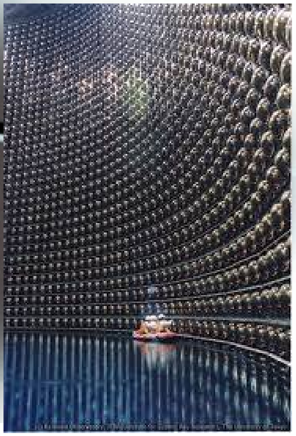


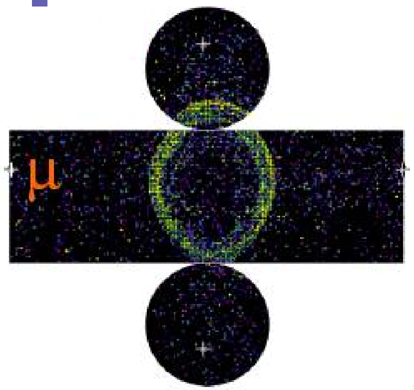
- Off-axis detector:
  - Several sub-detectors in 0.2T magnetic field:
    - Tracker (TPC + FGD), pizero detector (P0D), electromagnetic calorimenter (ECAL), muon ranger (SMRD)
  - <u>Measures the neutrino flux before the oscillations occur</u>
  - Measures intrinsic ν_e contamination
  - Measures neutrino interaction cross sections
- On-axis detector (INGRID):
  - 16 iron-scintillator modules form the cross
  - Monitoring flux, direction and stability of the neutrino beam
  - Neutrino cross section measurements

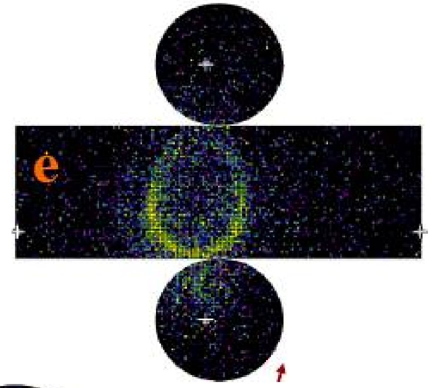


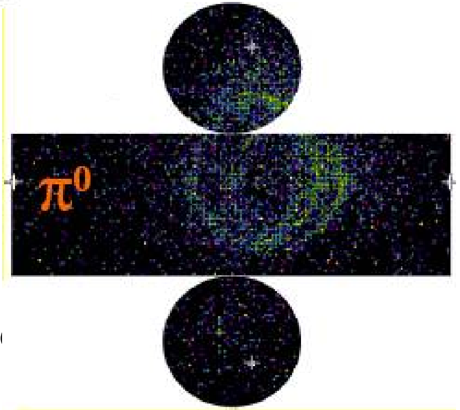
Far detector (T2K)



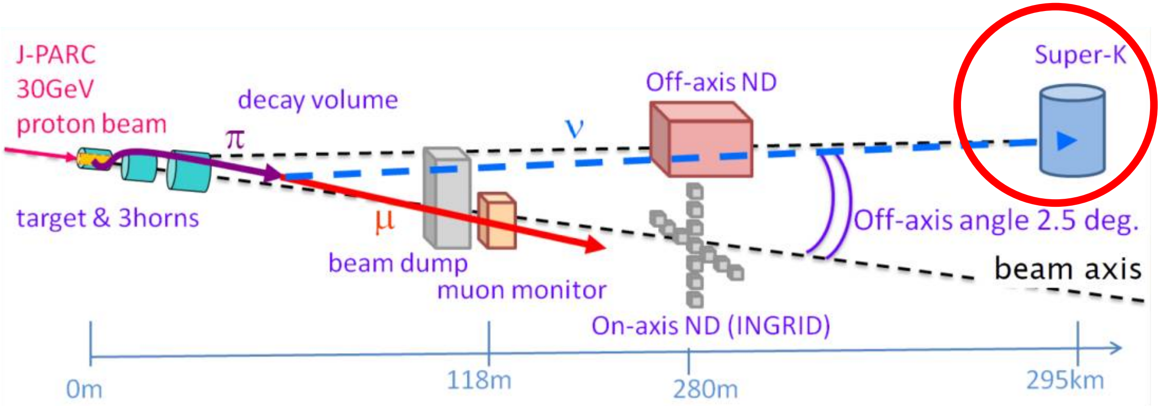




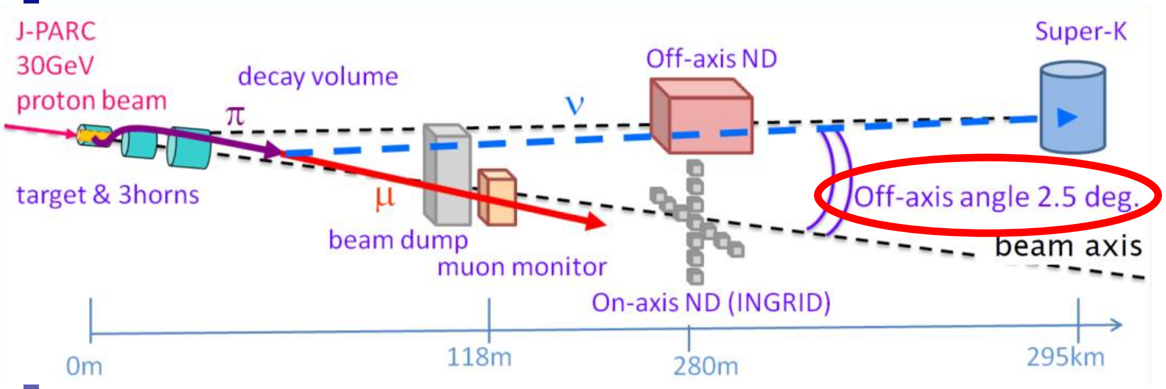


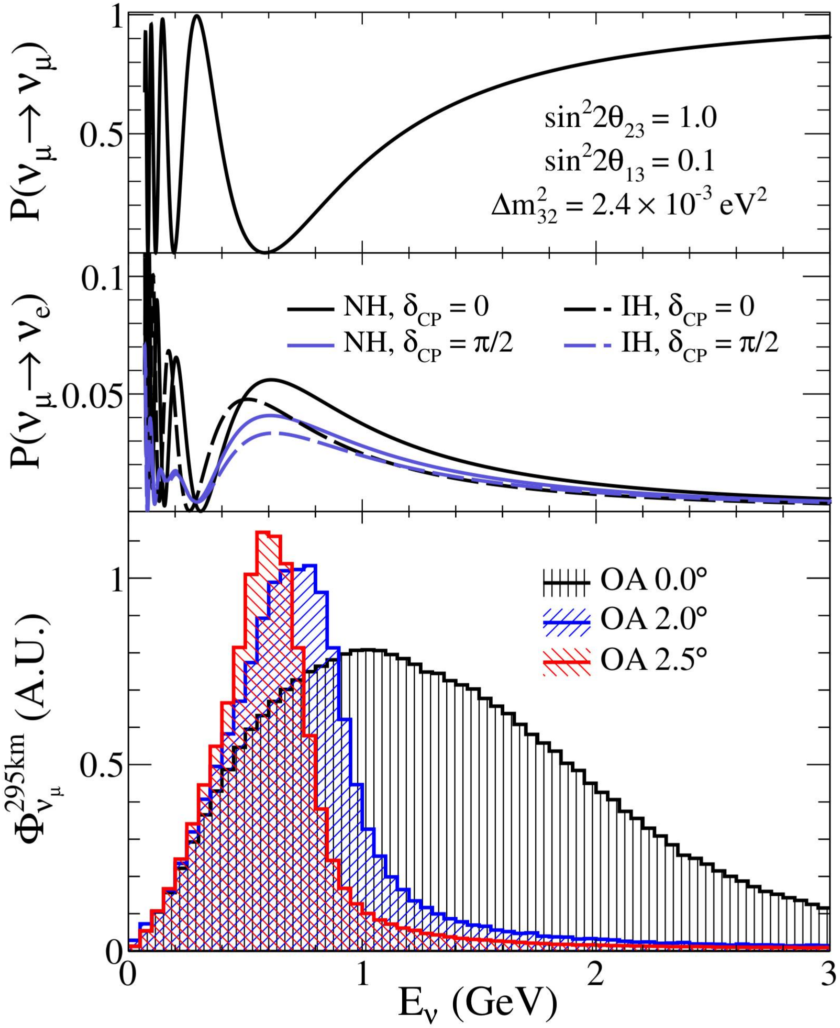


- <u>Measures the oscillated neutrino spectrum</u>
- Super-Kamiokande (operating since 1996):
  - Water Cherenkov (50 kt, 22.5 kt fiducial volume).
  - 11 000 (inner) + 2000 (outer) photomultipliers
  - Neutrino energy resolution ~10%
  - Particle identification:
    - Good electron-muon discrimination (<1% muons identified as electrons)
    - Neutral pion detection (rejecting background from neutrino interactions with π^0).



Off-axis beam concept



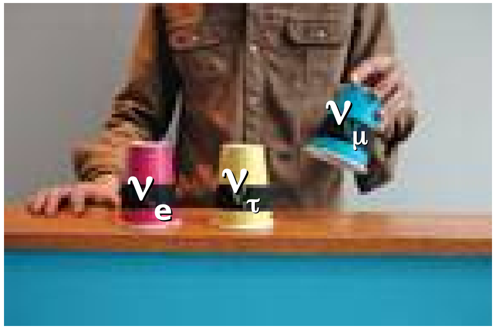


- Currently two world leading accelerator experiments (T2K, NOvA) use ’off-axis’ beam idea:
  - Pion decay kinematic effect
  - Thin energy spectrum with the mean energy tuned to the neutrino oscillation probability
  - Lower background from high energy interactions that are difficult to reconstruct

Accelerator neutrino oscillations

Disappearance of muon neutrinos/antineutrinos from the beam

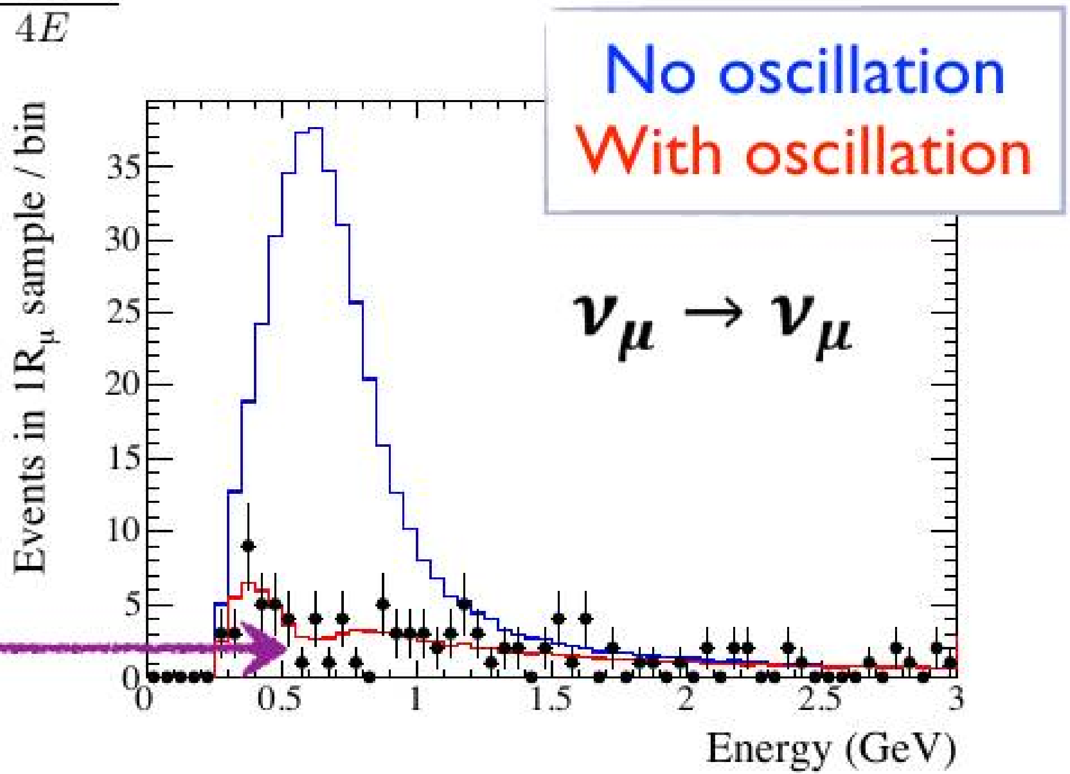
(ν_μ/anti-ν_μ disappearance)



$$P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu) \simeq 1 - 4\cos^2\theta_{13}\sin^2\theta_{23} \times [1-\cos^2\theta_{13}\sin^2\theta_{23}]\sin^2\frac{\Delta m^2_{32}L}{4E} + (\text{solar, matter effect terms})$$

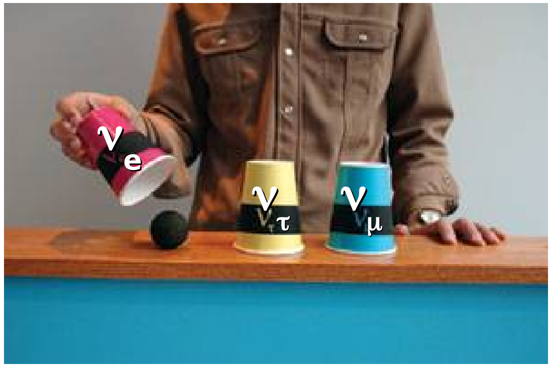
Location of min: Δm^2_{32}

Depth of min: $\sin^2 2\theta_{23}$



Accelerator neutrino oscillations

Electron neutrino/antineutrino appearance in the muon neutrino/antineutrino beam (ν_e/anti-ν_e appearance)



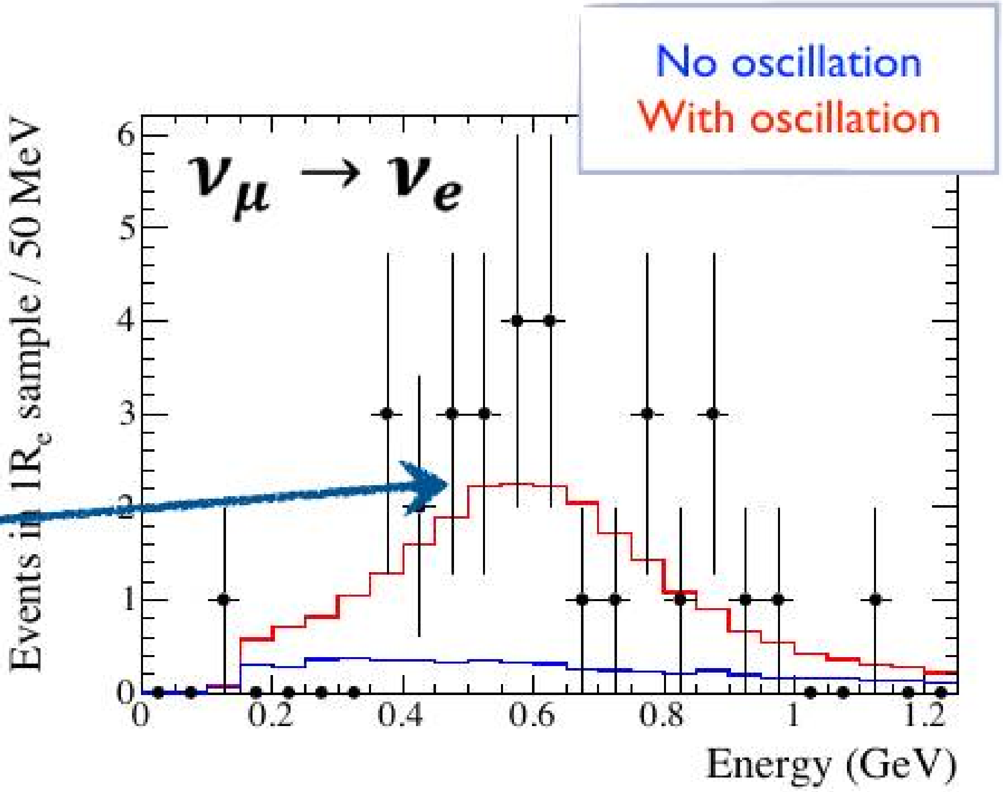
$$P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\frac{\Delta m^2_{32}L}{4E}$$

$$(+)-\left[\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13} \times \sin\frac{\Delta m^2_{21}L}{4E}\sin^2\frac{\Delta m^2_{32}L}{4E}\sin\delta_{CP}\right]$$

$$+\ (\text{CP-even, solar, matter effect terms})$$

Magnitude of the peak

$\sin^2\theta_{23}$, $\sin^2 2\theta_{13}$, δ_{CP}



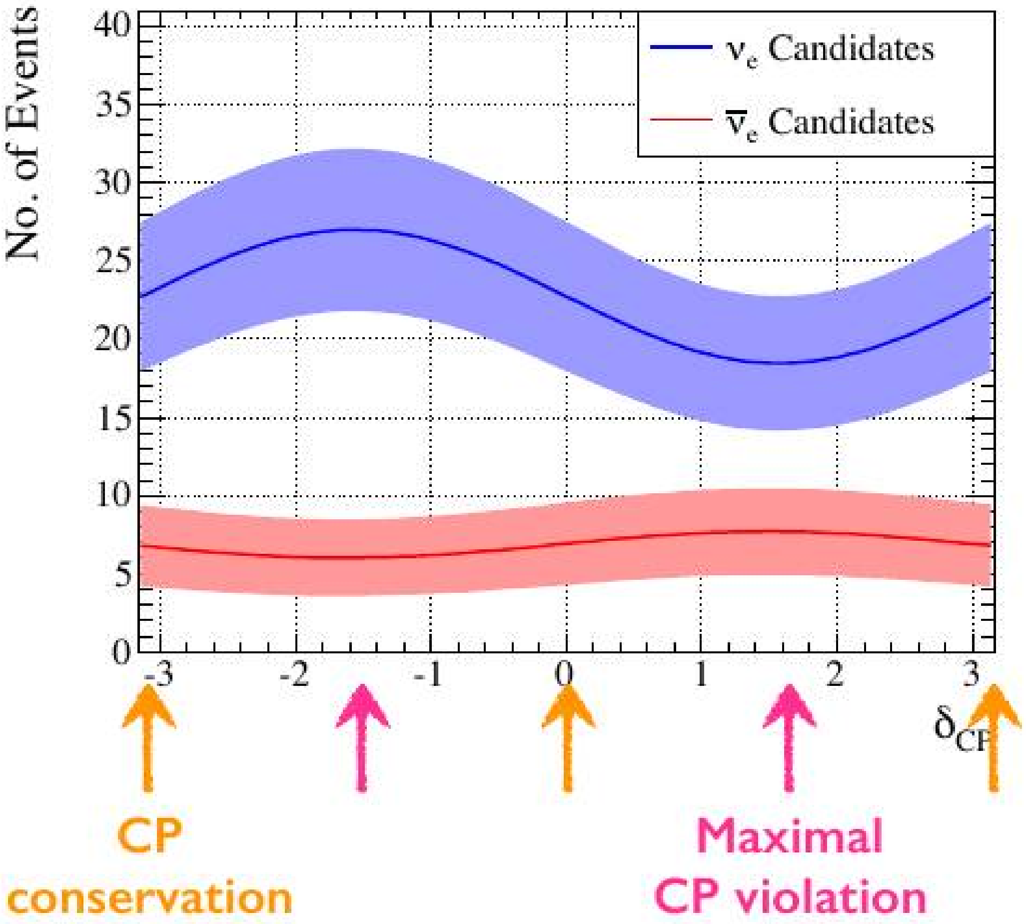
CP violation in neutrino sector?

Use accelerator neutrinos and measure:

$$CPV: \quad P(\nu_\mu \rightarrow \nu_e) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$$

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) \simeq\ & \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} \\ & (+)-\Big[\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \\ & \times \sin \frac{\Delta m^2_{21} L}{4E} \sin^2 \frac{\Delta m^2_{32} L}{4E} \sin\delta_{CP} \Big] \\ & + (\text{CP-even, solar, matter effect terms}) \end{aligned}$$

$\sin\delta_{CP}$ occurs in ν_e and $\bar{\nu}_e$ appearance probability with opposite sign



Complicated measurement because the sensitivity to measure δ_{CP} depends on:

δ_{CP} true value, θ_{23} true value, mass ordering

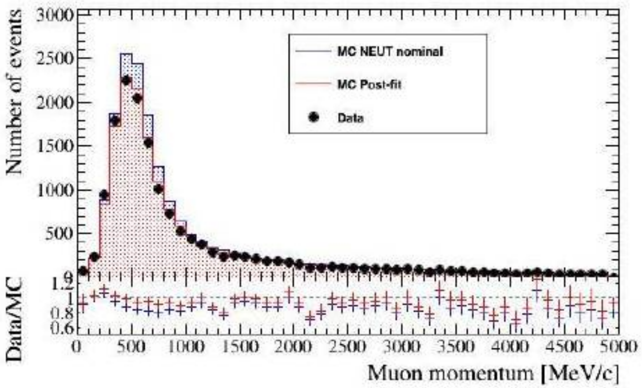
Need to have control over all systematic effects → precision measurements in neutrino oscillations

Accelerator neutrino oscillation analysis

Neutrino flux model:
Monte Carlo simulations + external data from other experiments (eg. NA61)

Neutrino interaction models and their uncertainties:
Monte Carlo simulations + external data (MINERvA, MiniBooNE experiments)

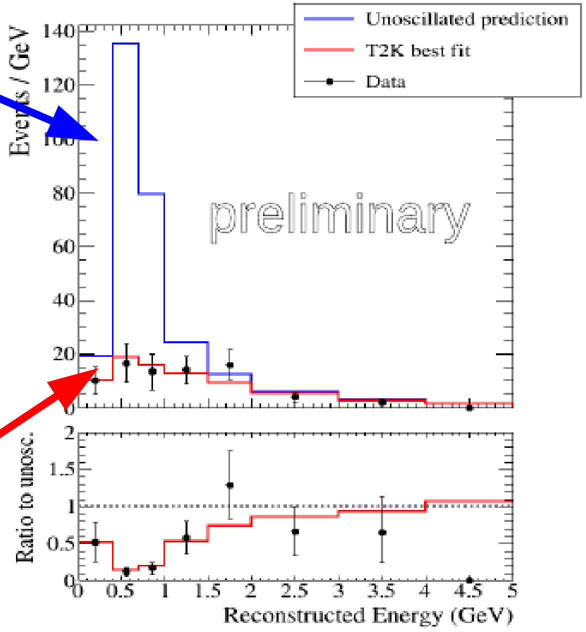
Fit to the data from the near detector



From near detector fit:
Flux model parameters,
Neutrino interaction model parameters,
Backgrounds in far detector

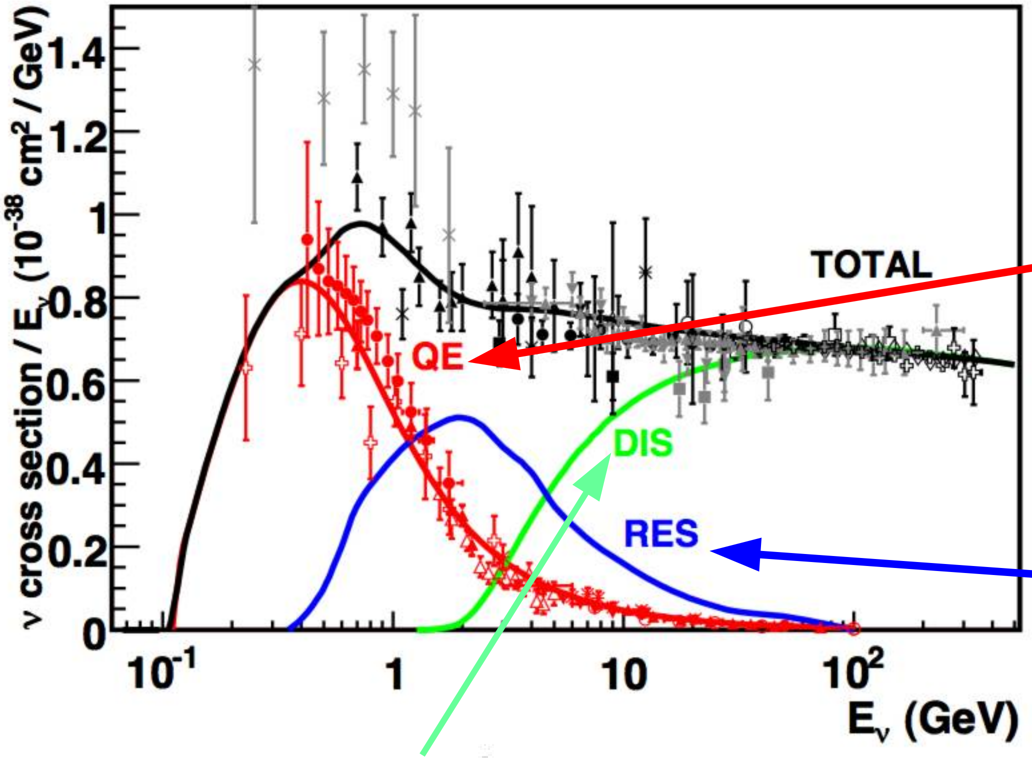
Expected neutrino spectrum
In far detector

Fit to the data from far detector (with PMNS model)

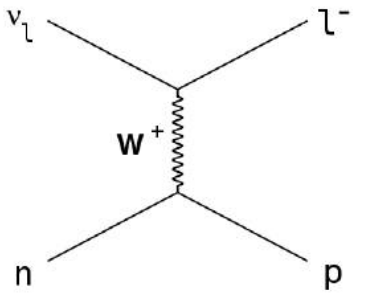


Neutrino interaction modelling

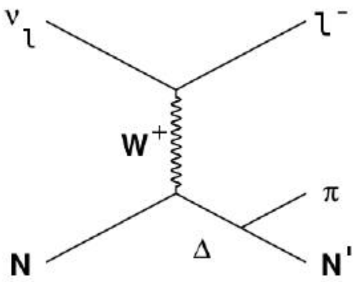
Charged-current interactions

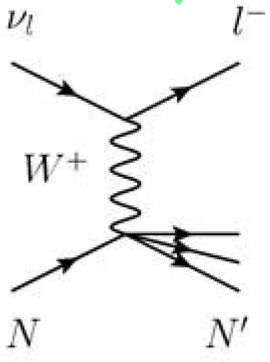


Quasi-elastic scattering – CCQE (dominant in T2K)



Resonance pion production – RES (dominant in NOvA)



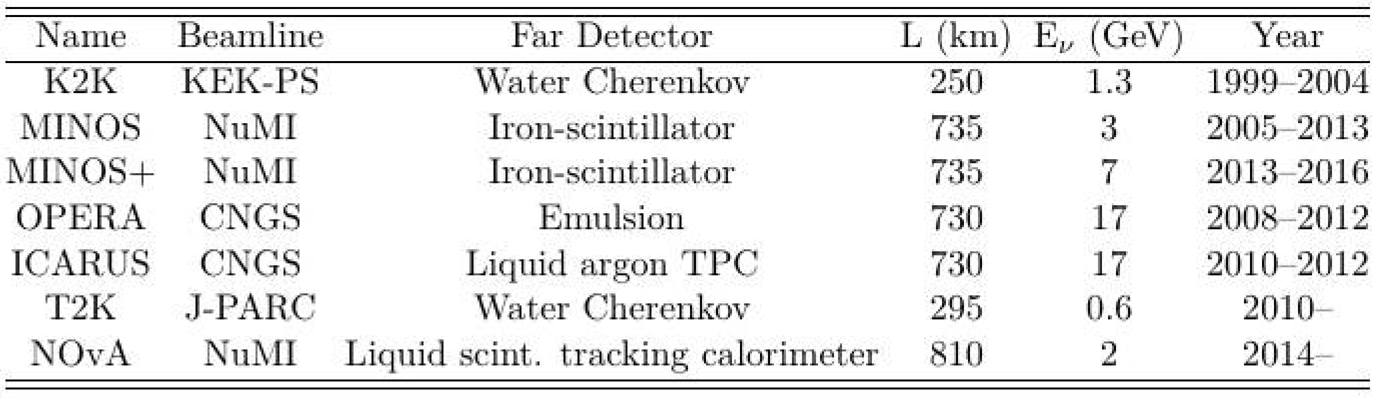


Deep Inelastic Scattering (DIS) (NOvA)

\+ additional complications:

- Nuclear ’initial-state’ models
- Nuclear re-interactions models (Final State Interactions)
- ...

Accelerator neutrino experiments



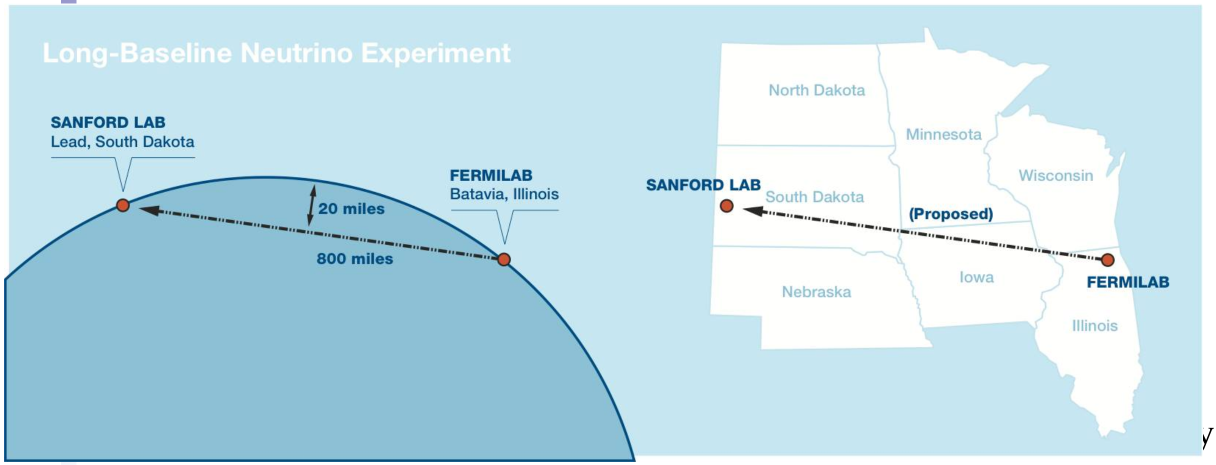
Name	Beamline	Far Detector	L (km)	E_ν (GeV)	Year
K2K	KEK-PS	Water Cherenkov	250	1.3	1999–2004
MINOS	NuMI	Iron-scintillator	735	3	2005–2013
MINOS+	NuMI	Iron-scintillator	735	7	2013–2016
OPERA	CNGS	Emulsion	730	17	2008–2012
ICARUS	CNGS	Liquid argon TPC	730	17	2010–2012
T2K	J-PARC	Water Cherenkov	295	0.6	2010–
NOvA	NuMI	Liquid scint. tracking calorimeter	810	2	2014–

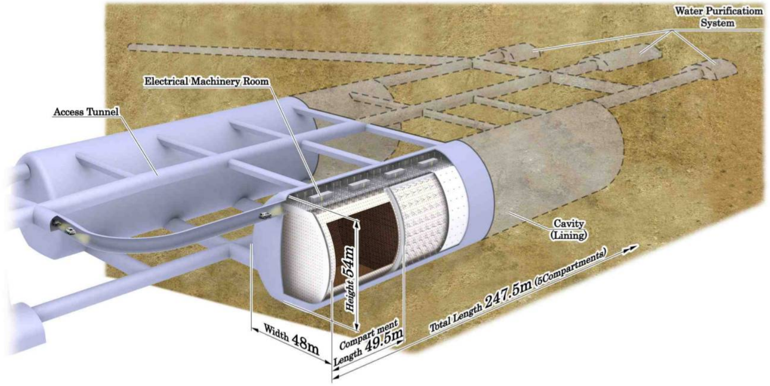
- History of accelerator neutrino studies and major results:
  - K2K (Japan): confirmed atmospheric neutrino oscillations discovered by Super-Kamiokande
  - MINOS, MINOS+ (USA): measured atmospheric neutrino oscillations Δm^2_{32}, θ_{23}
  - OPERA (Italy): confirmed $\nu_\mu \rightarrow \nu_\tau$ oscillations
  - ICARUS (Italy): first neutrino detector using liquid argon Time Projection Chambers technique
  - T2K (Japan): θ_{13} angle measurement, Δm^2_{32} measurement, first measurement of δ_{CP} phase
  - NOvA (USA): θ_{13} angle measurement, δ_{CP} phase measurement, mass ordering

Future experiments

- DUNE (USA)
  - USA flagship accelerator neutrino project.
  - Neutrino beam produced in Fermilab (1.2 MW and 2.4 MW power after upgrade)
  - 10 kton or larger liquid argon detector in South Dakota (1300 km from Fermilab) 1.5 km underground.
  - Two prototype far detectors are at the European research center CERN. The first started taking data in September 2018 and the second is under construction.
  - Should be able to measure δ_{CP} ~ 20 degree accuracy (~10 degrees after beam upgrade).
  - Start ~2028

- Hyper-Kamiokande (Japan)
  - T2K experiment extension with larger far detector.
  - High-intensity neutrino beam (1.7 MW) from J-PARC complex
  - Far detector – 0.5 kton water Cherenkov 300 km from J-PARC.
  - Measuring δ_{CP} with accuracy of 18 degrees but depends on mass ordering measurements.
  - Start ~2027





Summary

- Today covered:
    - ✓ Neutrino Oscillations
    - ✓ Artificial neutrino sources: reactor, accelerator
- To remember:
  - ➔ How are reactor neutrinos produced and measured?
  - ➔ Accelerator neutrinos: production, oscillations, CP violation

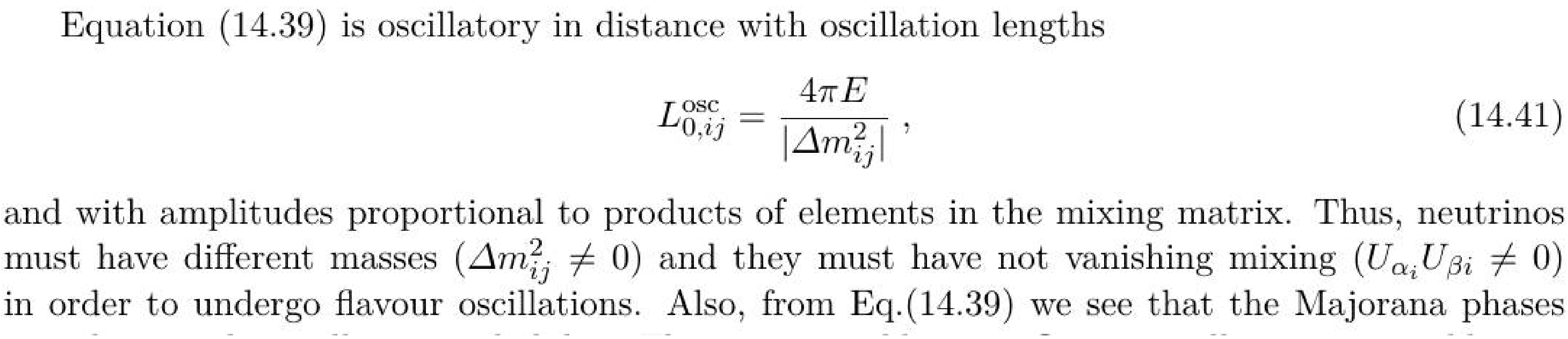
  - ➔ References:
    - ✓ PDG, Neutrino Masses, Mixing and Oscillations review (pdg.lbl.gov)
    - ✓ M. Zito, Neutrino masses and mixing lectures

Backup slides

Table 14.7: 3ν oscillation parameters obtained from different global analyses of neutrino data. In all cases, the numbers labeled as NO (IO) are obtained assuming NO (IO), *i.e.*, relative to the respective local minimum. SK-ATM makes reference to the tabulated χ^2 map from the Super-Kamiokande analysis of their data in Ref. [97].

	Ref. [185] w/o SK-ATM		Ref. [185] w SK-ATM		Ref. [186] w SK-ATM		Ref. [187] w SK-ATM	
NO	Best Fit Ordering		Best Fit Ordering		Best Fit Ordering		Best Fit Ordering	
Param	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.04^{+0.14}_{-0.13}$	$2.65 \to 3.46$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.46^{+0.87}_{-0.88}$	$30.98 \to 36.03$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.58^{+0.20}_{-0.33}$	$4.27 \to 6.09$	$5.63^{+0.18}_{-0.24}$	$4.33 \to 6.09$	$5.51^{+0.19}_{-0.80}$	$4.30 \to 6.02$	$5.47^{+0.20}_{-0.30}$	$4.45 \to 5.99$
$\theta_{23}/^\circ$	$48.3^{+1.2}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$47.9^{+1.1}_{-4.0}$	$41.0 \to 50.9$	$47.7^{+1.2}_{-1.7}$	$41.8 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.241^{+0.066}_{-0.065}$	$2.046 \to 2.440$	$2.237^{+0.066}_{-0.065}$	$2.044 \to 2.435$	$2.14^{+0.09}_{-0.07}$	$1.90 \to 2.39$	$2.160^{+0.083}_{-0.069}$	$1.96 \to 2.41$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.41^{+0.18}_{-0.14}$	$7.9 \to 8.9$	$8.45^{+0.16}_{-0.14}$	$8.0 \to 8.9$
$\delta_{\mathrm{CP}}/^\circ$	222^{+38}_{-28}	$141 \to 370$	221^{+39}_{-28}	$144 \to 357$	238^{+41}_{-33}	$149 \to 358$	218^{+38}_{-27}	$157 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$	$2.449^{+0.032}_{-0.030}$	$2.358 \to 2.544$	$2.454^{+0.029}_{-0.031}$	$2.362 \to 2.544$	$2.419^{+0.035}_{-0.032}$	$2.319 \to 2.521$	2.424 ± 0.03	$2.334 \to 2.524$
IO	$\Delta\chi^2 = 6.2$		$\Delta\chi^2 = 10.4$		$\Delta\chi^2 = 9.5$		$\Delta\chi^2 = 11.7$	
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.03^{+0.14}_{-0.13}$	$2.64 \to 3.45$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$	$33.40^{+0.87}_{-0.81}$	$30.92 \to 35.97$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.63^{+0.19}_{-0.26}$	$4.30 \to 6.12$	$5.65^{+0.17}_{-0.22}$	$4.36 \to 6.10$	$5.57^{+0.17}_{-0.24}$	$4.44 \to 6.03$	$5.51^{+0.18}_{-0.30}$	$4.53 \to 5.98$
$\theta_{23}/^\circ$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$	$48.2^{+1.0}_{-1.4}$	$41.8 \to 50.9$	$47.9^{+1.0}_{-1.7}$	$42.3 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.261^{+0.067}_{-0.064}$	$2.066 \to 2.461$	$2.259^{+0.065}_{-0.065}$	$2.064 \to 2.457$	$2.18^{+0.08}_{-0.07}$	$1.95 \to 2.43$	$2.220^{+0.074}_{-0.076}$	$1.99 \to 2.44$
$\theta_{13}/^\circ$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$	$8.49^{+0.15}_{-0.14}$	$8.0 \to 9.0$	$8.53^{+0.14}_{-0.15}$	$8.1 \to 9.0$
$\delta_{\mathrm{CP}}/^\circ$	285^{+24}_{-26}	$205 \to 354$	282^{+23}_{-25}	$205 \to 348$	247^{+26}_{-27}	$193 \to 346$	281^{+23}_{-27}	$202 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$	$-2.509^{+0.032}_{-0.032}$	$-2.603 \to -2.416$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$	$-2.478^{+0.035}_{-0.033}$	$-2.577 \to -2.375$	$-2.50 \pm^{+0.04}_{-0.03}$	$-2.59 \to -2.39$

Oscillation length



Equation (14.39) is oscillatory in distance with oscillation lengths

$$L^{\text{osc}}_{0,ij} = \frac{4\pi E}{|\Delta m^2_{ij}|} \ , \tag{14.41}$$

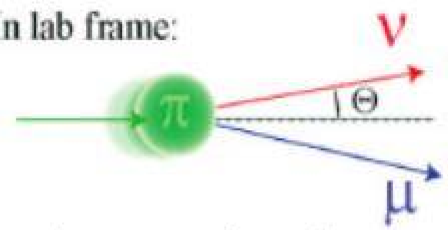
and with amplitudes proportional to products of elements in the mixing matrix. Thus, neutrinos must have different masses ($\Delta m^2_{ij} \neq 0$) and they must have not vanishing mixing ($U_{\alpha_i} U_{\beta i} \neq 0$) in order to undergo flavour oscillations. Also, from Eq.(14.39) we see that the Majorana phases

Off-axis Neutrino Beams

In pion rest frame:

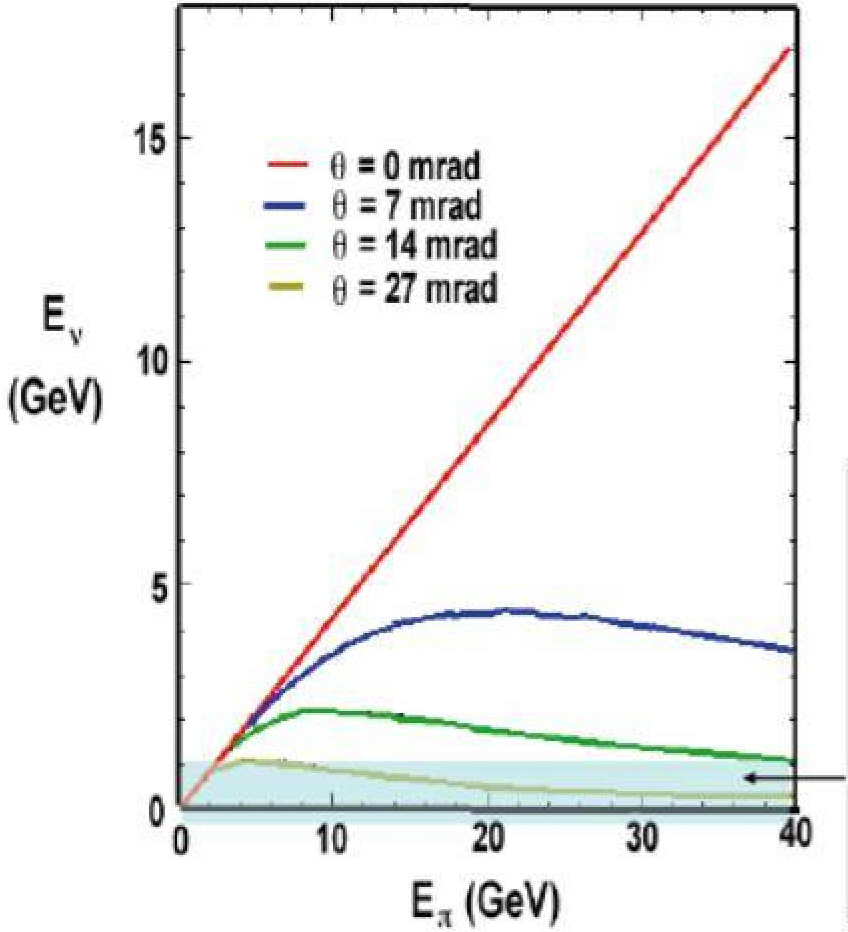
Neutrino and muon energy completely determined

In lab frame:



Neutrino energy depends on boost and angle to boost direction

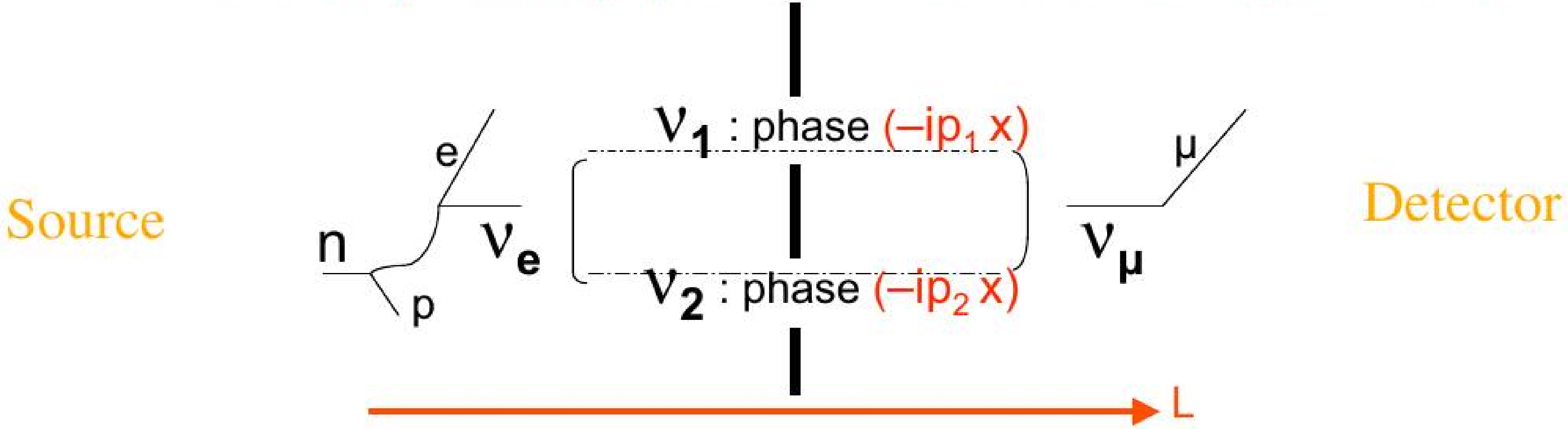
$$E_\nu = \frac{0.43\gamma\, m_\pi}{1+\gamma^2\theta^2}$$



At 14mrad off-axis almost all π produce ν around 2 GeV, i.e. at oscillation maximum

Neutrino oscillations - interference

Two-slits (masses) quantum interference experiments !



Source

n

e

p

ν_e

ν_1 : phase $(-ip_1 x)$

ν_2 : phase $(-ip_2 x)$

ν_μ

μ

Detector

L

Phase shift $(p_1 - p_2)x \sim \Delta m^2 L/4p$ => **Interference**

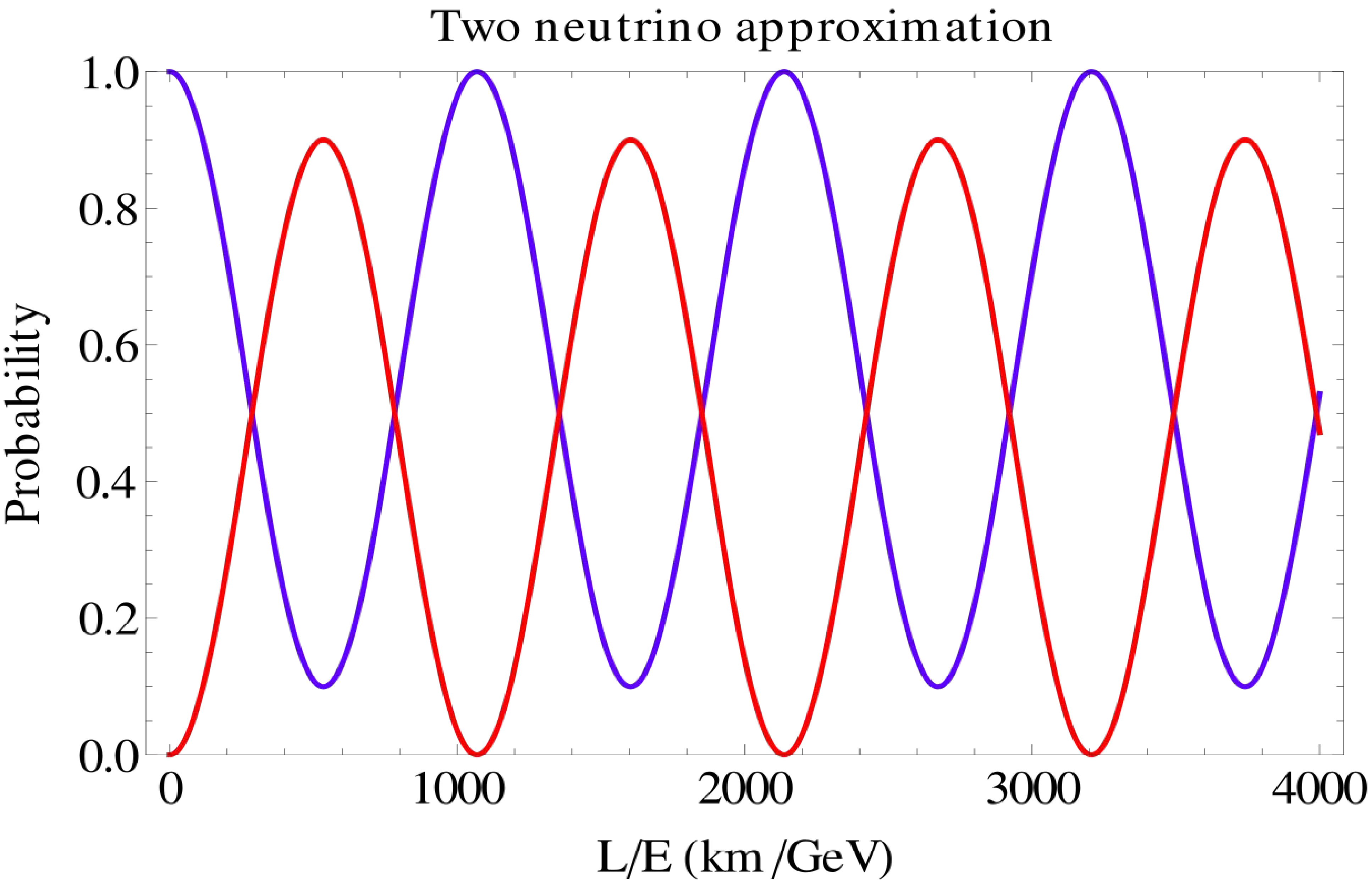
"Neutrino oscillation" is due to the phase shift between the lighter states (in advance with respect to phase) versus the heavier. Out of phase linear superposition means the other flavor eigenstates (not present at t=0) appear during the propagation.

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \overset{\text{“Atmospheric”}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \overset{\text{“Solar”}}{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

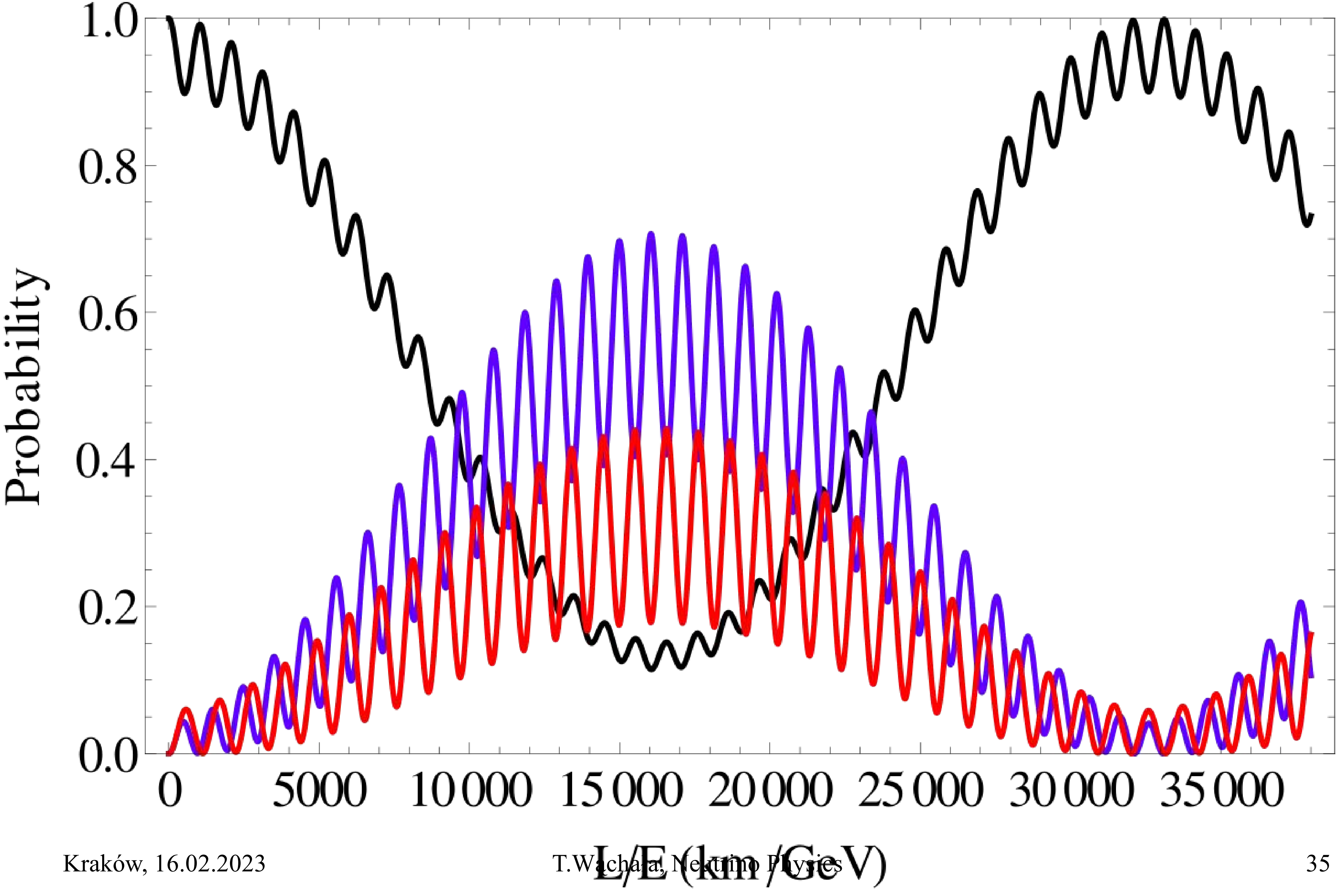
NxN unitary matrix, N(N-1)/2 angles, N(N+1)/2 phases
(2N**2 real parameters, Unitarity: N conditions on the diagonal+ ½N(N-1) Im(V1k V2k*=0) + ½N(N-1) Re(V1k V2k*=0) =>N**2 real parameters)
N phases can be rotated away by redefining the charged lepton fields
N-1 phases can be rotated away by redefining the neutrino fields
However this is not possible if Majorana (not invariant under U(1))

How many physical phases?
(N-1)(N-2)/2 phases in general
(N-1) additional phases if Majorana

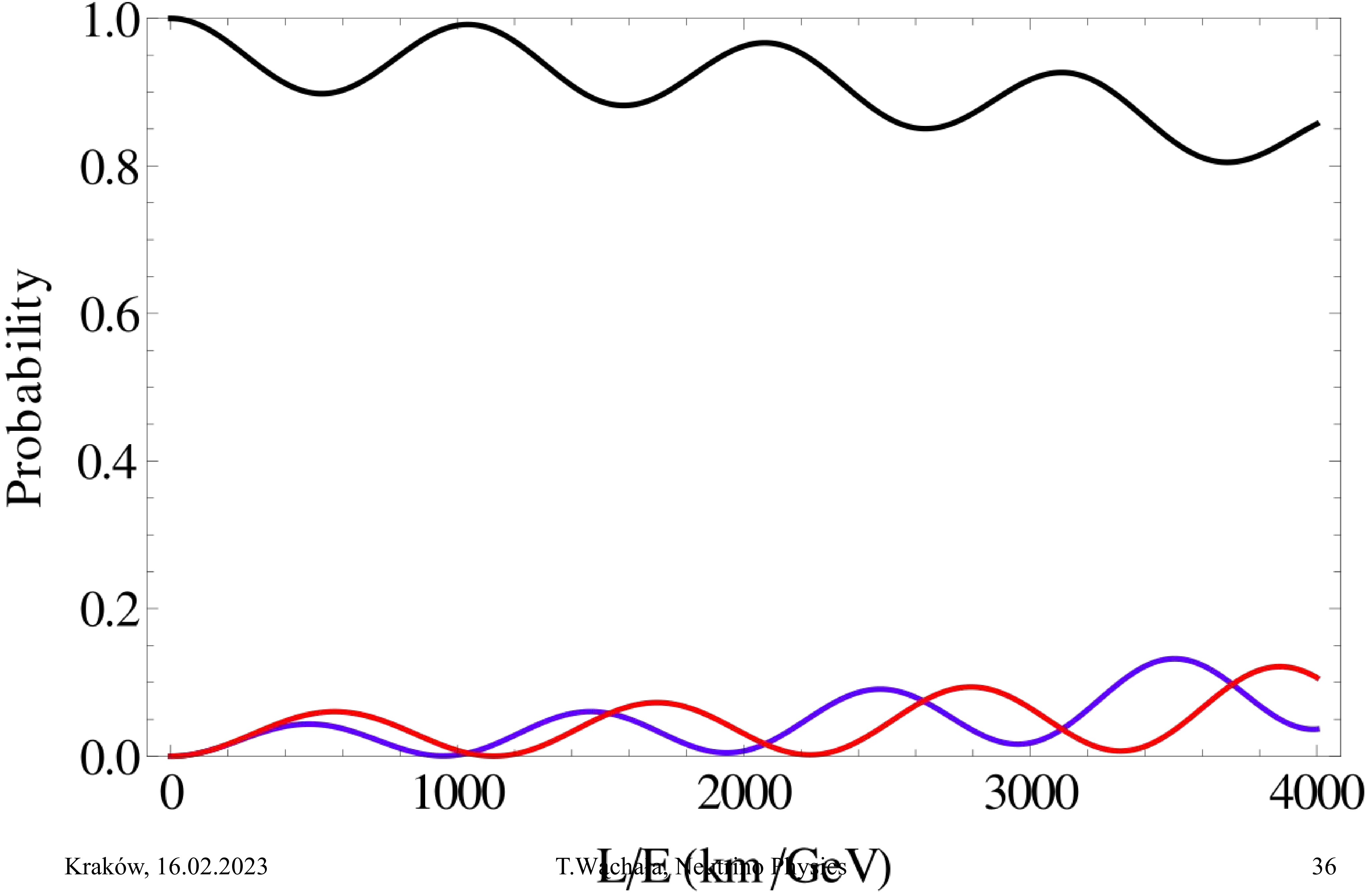
$$U=U'\cdot\begin{pmatrix} 1 & & & \\ & e^{i\phi_2} & & \\ & & \cdots & \\ & & & e^{i\phi_N} \end{pmatrix}$$


Two neutrino approximation
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
L/E (km /GeV)

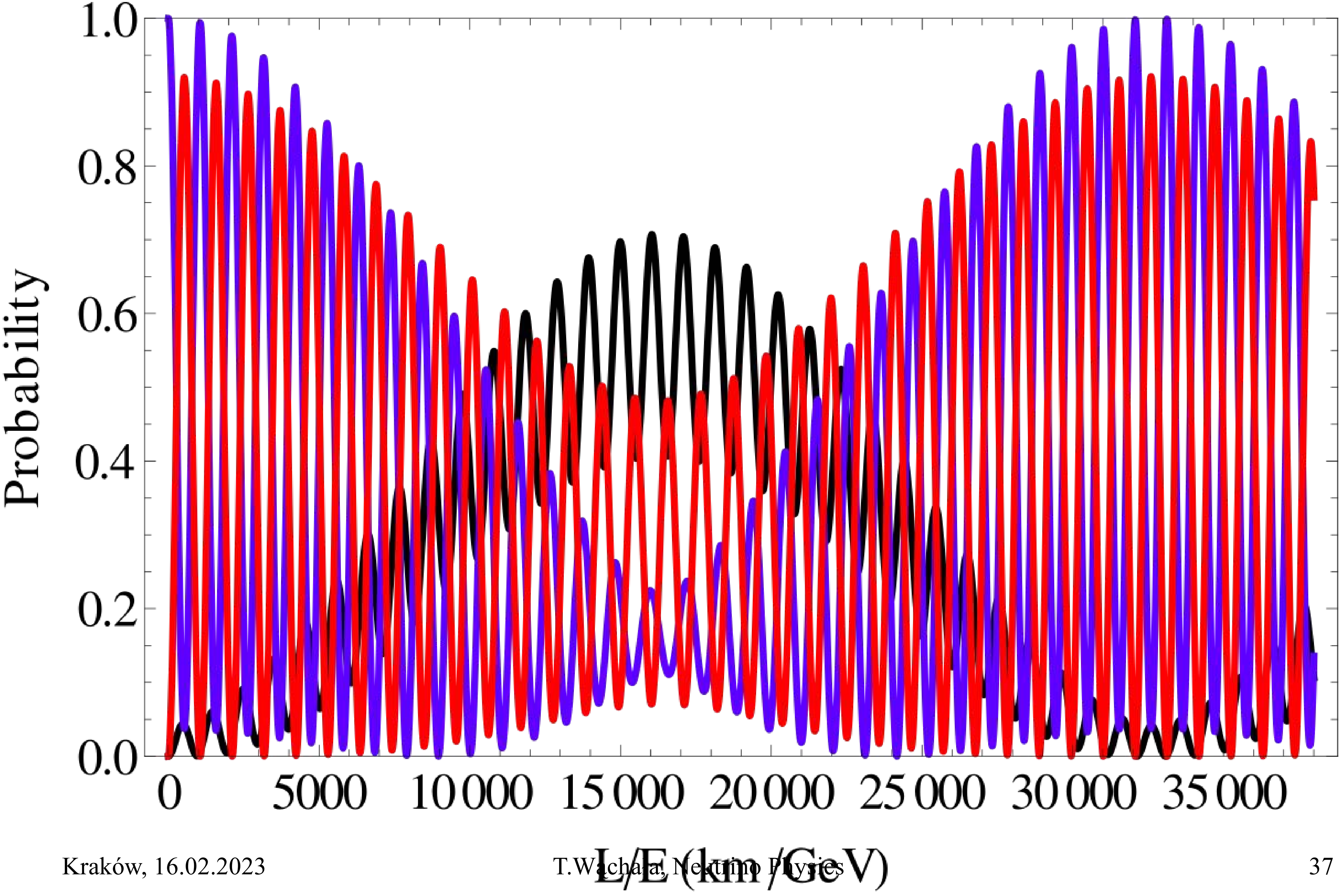
Oscillation probabilities for an initial electron neutrino



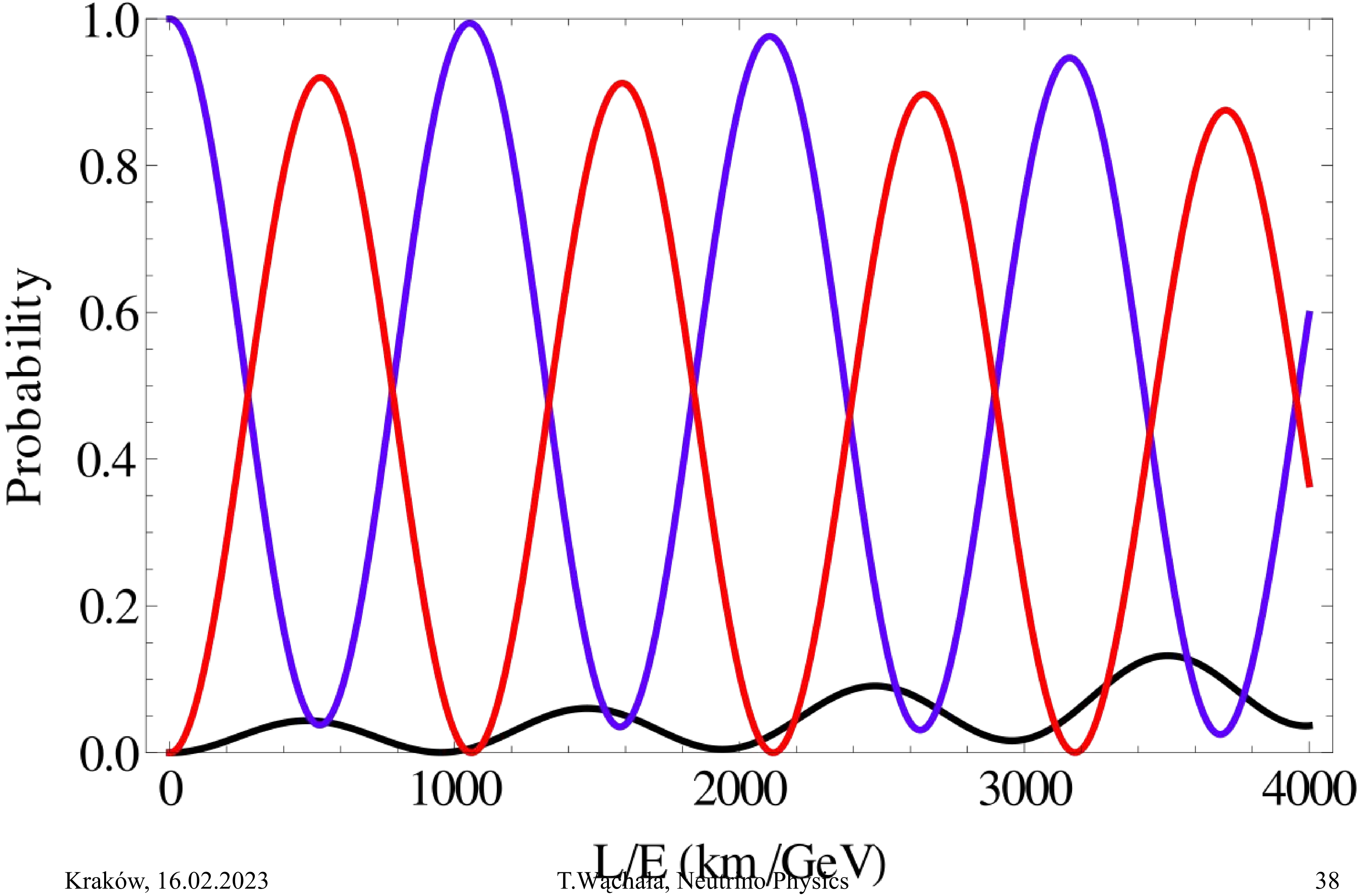
Oscillation probabilities for an initial electron neutrino



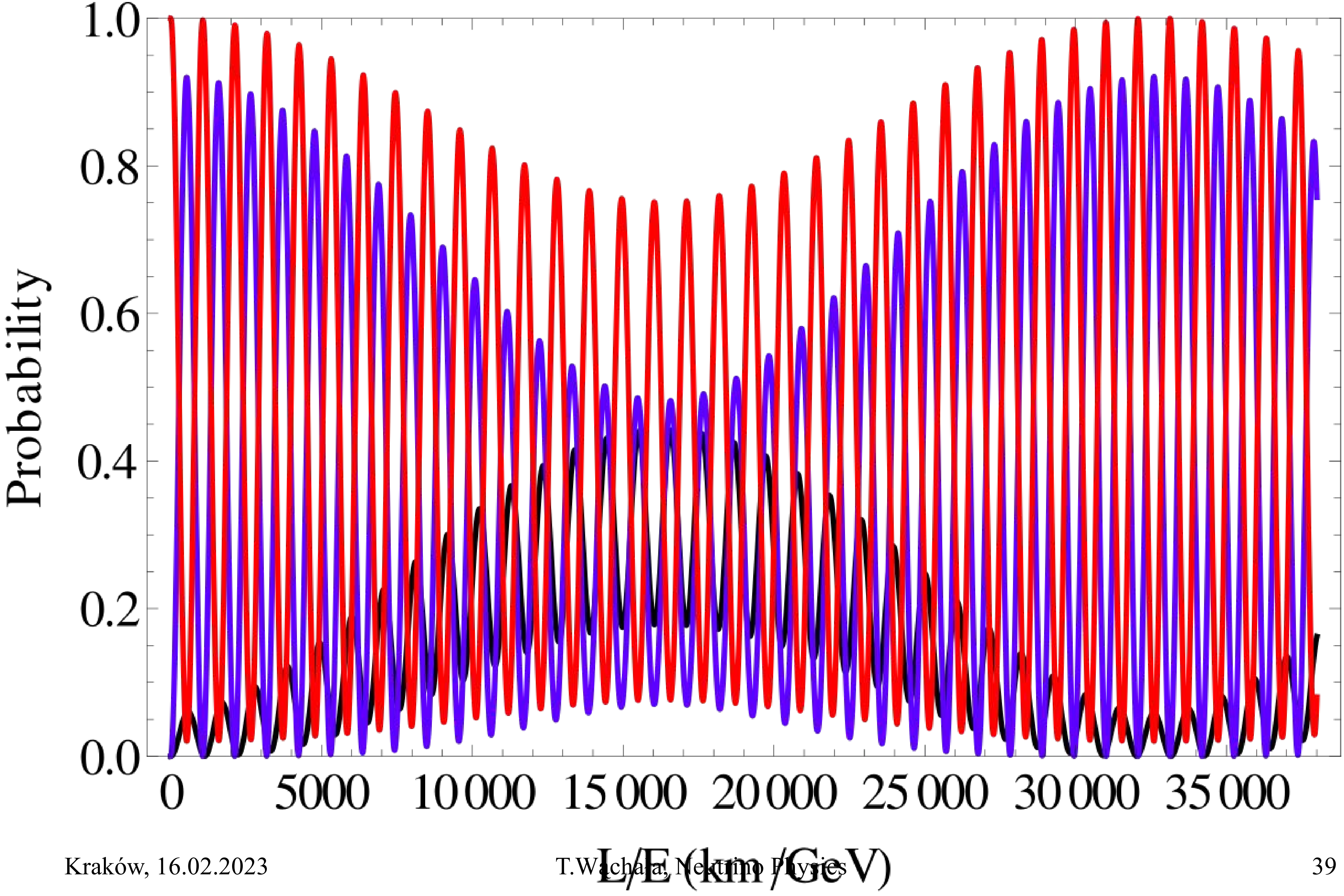
Oscillation probabilities for an initial muon neutrino



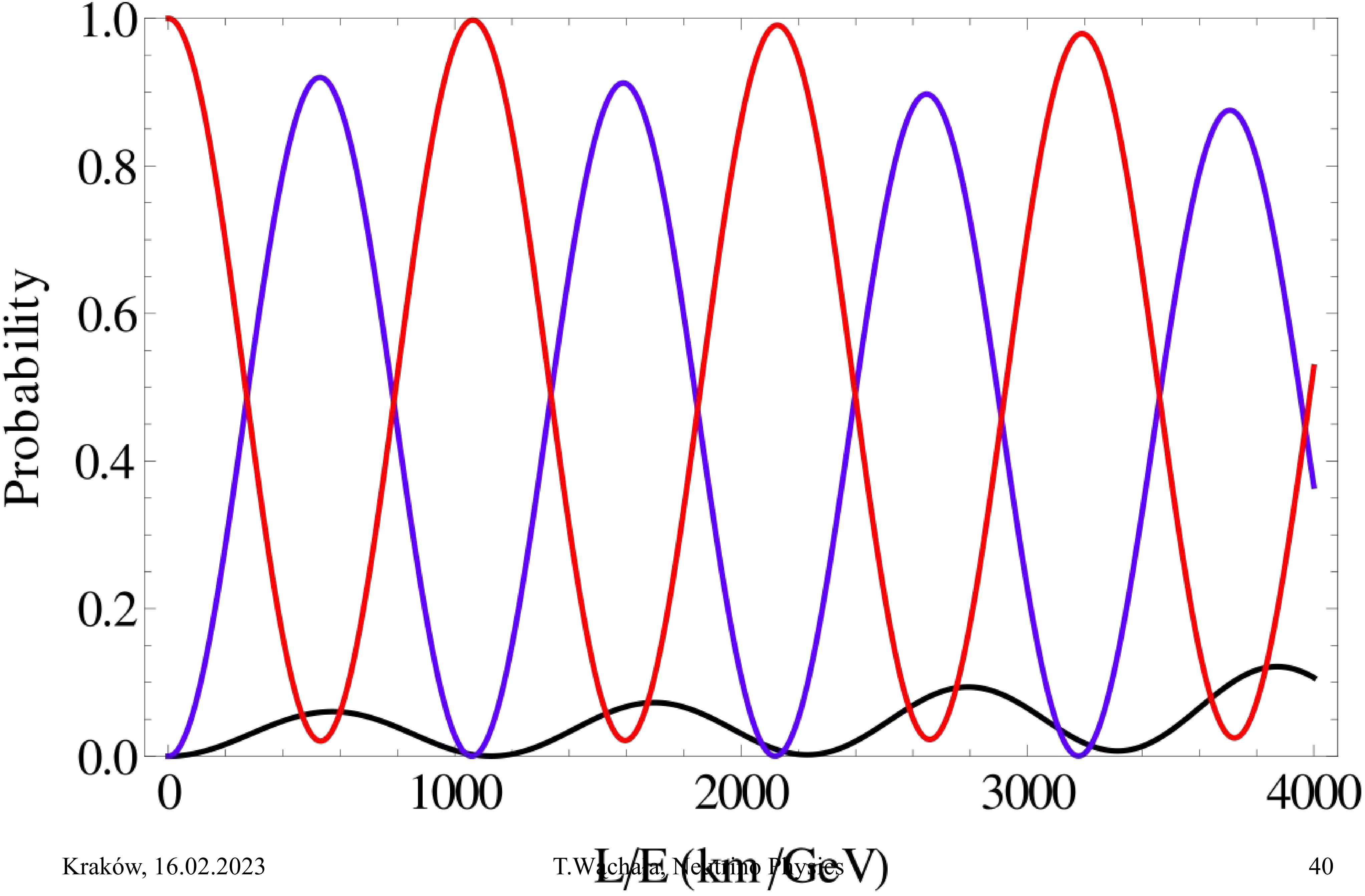
Oscillation probabilities for an initial muon neutrino



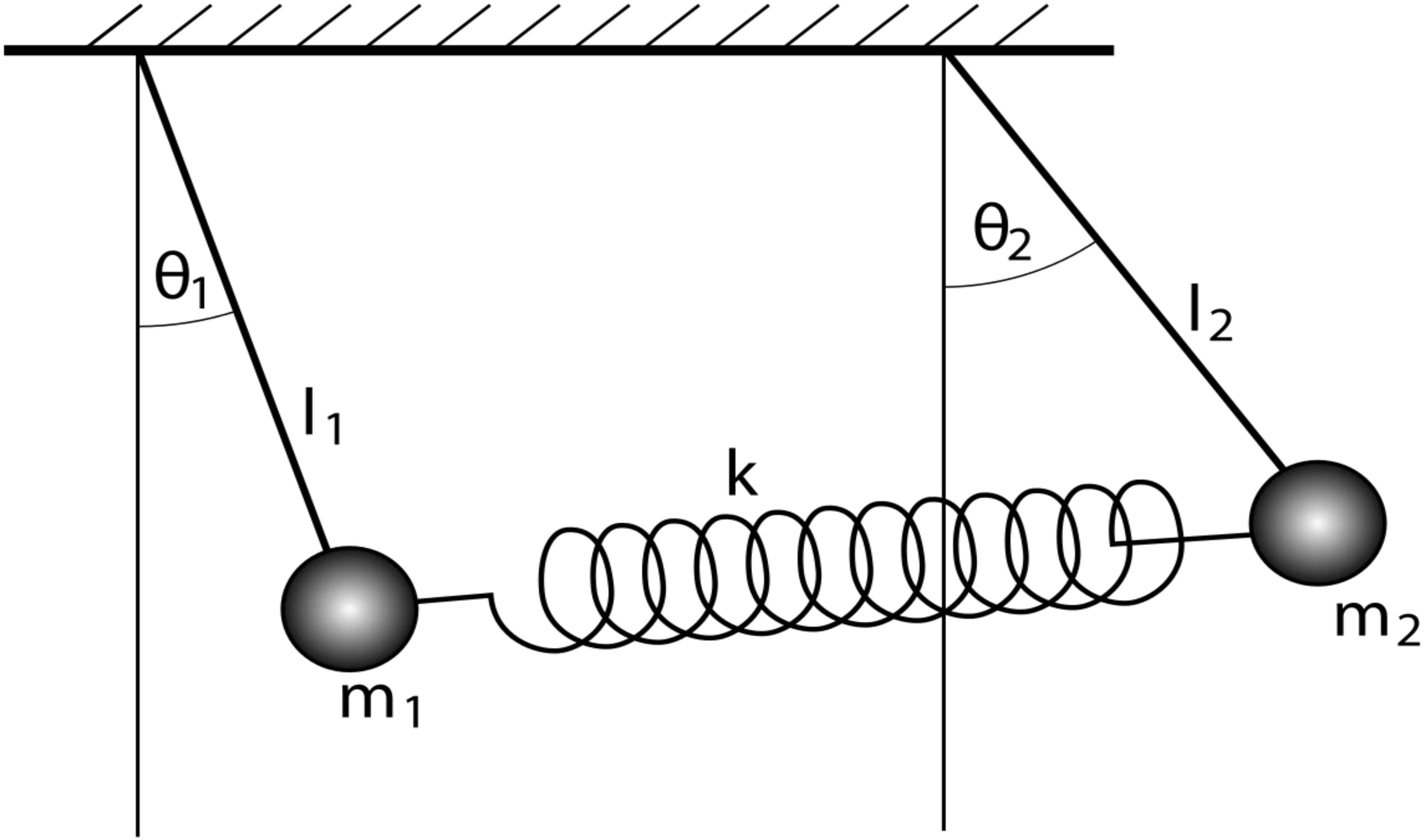
Oscillation probabilities for an initial tau neutrino



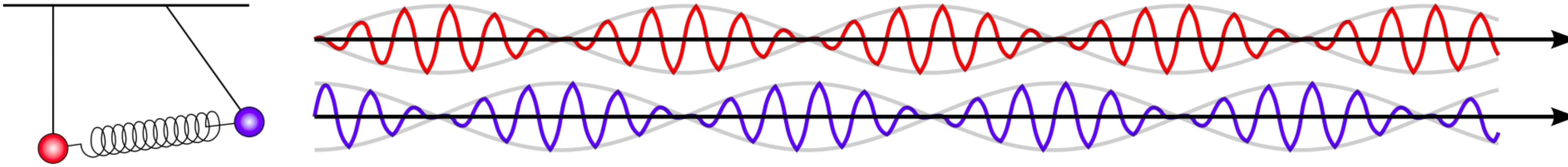
Oscillation probabilities for an initial tau neutrino



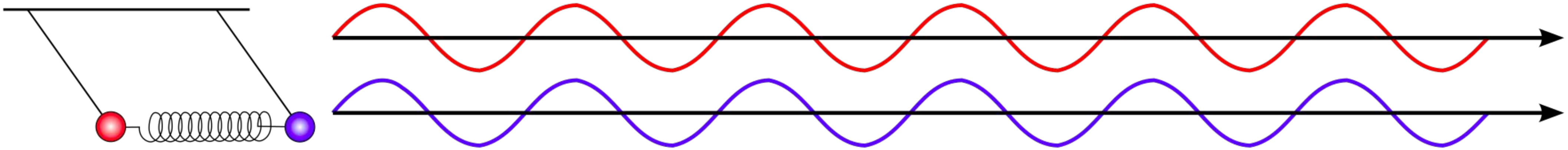
Classical analogy: spring-coupled pendulums



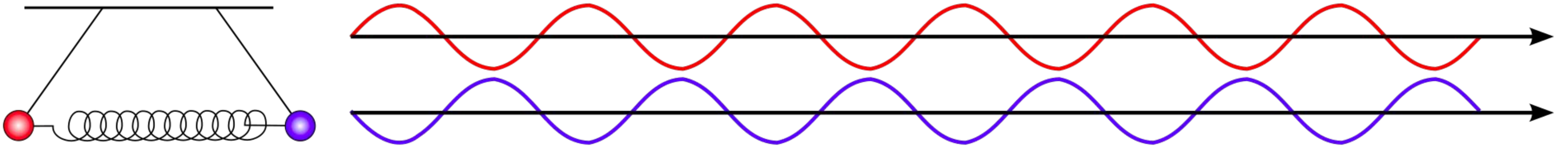
Classical analogy



- Time evolution of the pendulums

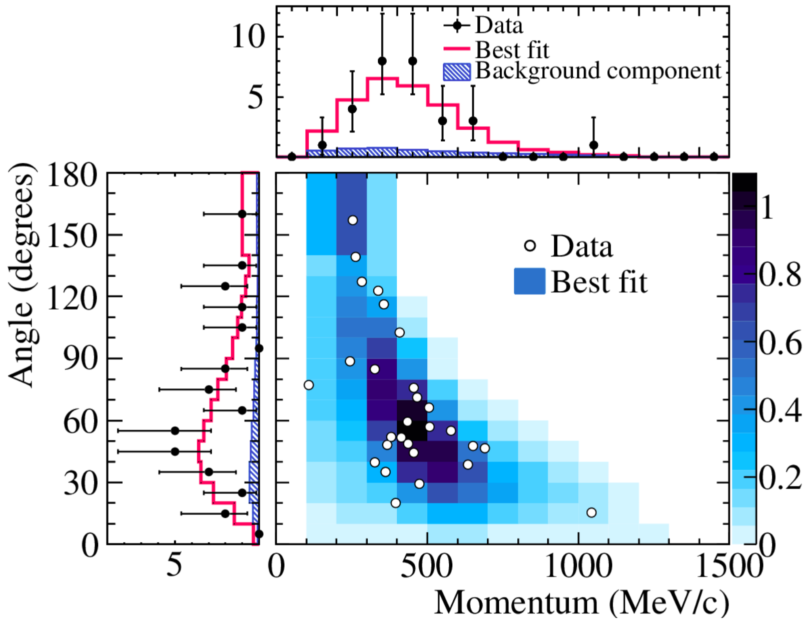


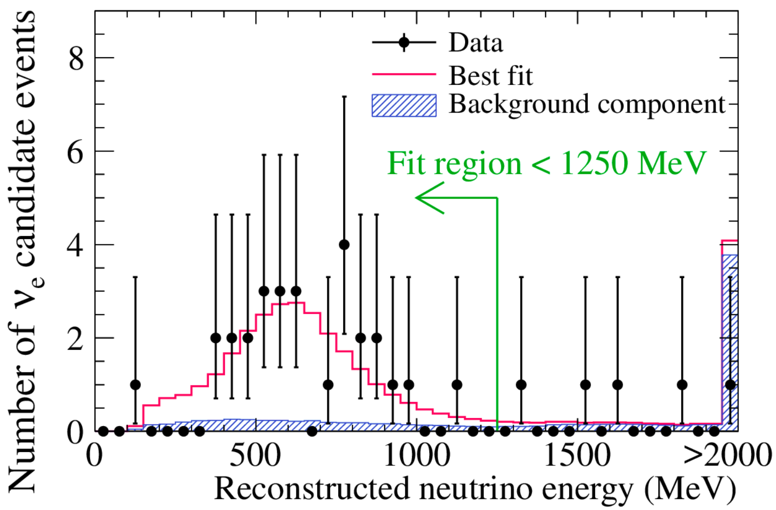
- Lower frequency normal mode



- Higher frequency normal mode

T2K ν_e appearance measurement





Phys.Rev.Lett. 112 (2014) 061802

- 28 electron neutrino candidates in the far detector, 4.92±0.55 candidates predicted for the no-oscillation hypothesis
- Best fit (N.H, δ_{CP}=0):

$$\sin^2 2\theta_{13} = 0.140^{+0.038}_{-0.032}$$

- 7.3σ significance for non-zero θ_{13}.
- Discovery of ν_e appearance in ν_μ beam!

T2K ν_μ disappearance measurement

Phys.Rev.Lett. 112 (2014) 181801

- 120 muon neutrino candidates in the far detector, 446±22.5 candidates predicted for no-oscillation hypothesis
- Best fit value (N.H.):

$$\sin^2\theta_{23} = 0.514\,^{+0.055}_{-0.056}$$

- World’s best θ_{23} measurement!
- T2K prefers maximal mixing (θ_{23} equal 45 degrees)