
Heavy flavour physics

Lecture 1

Marcin Kucharczyk

IFJ PAN, Kraków
01 December 2022

Contents

Lecture 1

- What is flavour physics
- CKM mechanism
- SM flavour sector
- Flavour sector beyond the SM

What is flavour physics?

Fermions ("matter")	Bosons ("forces")
$ \left\{ \begin{array}{l} \text{Quarks} \\ uuu \quad ccc \quad ttt \\ ddd \quad sss \quad bbb \\ \\ \text{Leptons} \\ e \quad \mu \quad \tau \\ \nu_e \quad \nu_\mu \quad \nu_\tau \end{array} \right\} \times \left\{ \begin{array}{l} \text{MATTER} \\ \text{ANTIMATTER} \end{array} \right\} $	$ \begin{array}{l} gggggggg \\ \gamma \\ W^+ \\ W^- \\ Z \\ \\ H \end{array} $

Flavour physics:

- transitions between different kinds of quarks
- its all about weak interactions...
- strong interactions as a "background"

Parameters of the Standard Model

- 3 gauge couplings: α_{EM} , α_{weak} , α_{strong}
- 2 Higgs parameters: v , m_H

- 6 quark masses:
- 3 quark mixing angles + 1 phase (*CKM matrix*)
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase) (*PMNS matrix*)

() = with Dirac neutrino masses

Open questions in flavour physics

- Why are there so many different fermions?
- What is responsible for their organisation into generations / families?
- Why are there 3 generations / families each of quarks and leptons?
- Why are there flavour symmetries?
- What breaks the flavour symmetries?
- What causes matter–antimatter asymmetry?

Flavour physics issues

Families / generations

3 pairs of quarks (are we sure?)

3 pairs of leptons (are we sure?)

Hierarchies

$m(t) > m(c) > m(u)$ $m(b) > m(s) > m(d)$

$m(\tau) > m(\mu) > m(e)$ $m(\nu_\tau) > m(\nu_\mu) > m(\nu_e) ?$

Mixings & couplings

hierarchy in quark mixings

what about lepton mixings?

Flavour physics issues

Mixings & couplings

universality

(no) flavour changing neutral currents (at tree level in the Standard Model)

Symmetry principles & their violation

P violation / C violation

CP violation / T violation

baryon asymmetry of the universe

lepton flavour violation

Unification

Why is heavy flavour physics interesting?

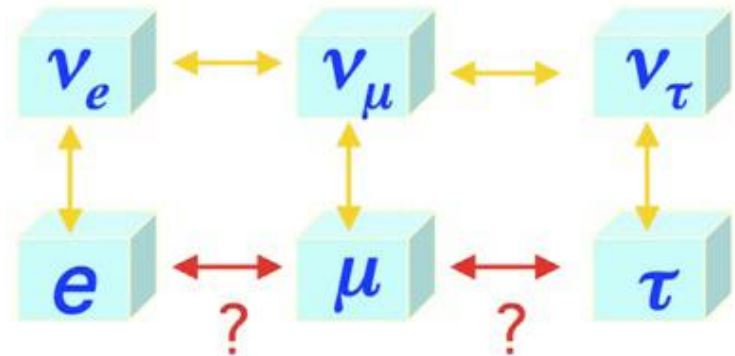
- Hope to learn something about the mysteries of the flavour structure of the Standard Model
- CP violation and its connection to the matter–antimatter asymmetry of the Universe
- Discovery potential far beyond the energy frontier via searches for rare or SM forbidden processes

What breaks the flavour symmetries?

- In the Standard Model, the vacuum expectation value of the Higgs field breaks the electroweak symmetry
- Fermion masses arise from the Yukawa couplings of the quarks and charged leptons to the Higgs field (*taking $m_\nu = 0$*)
- The CKM matrix arises from the difference between weak eigenstates and mass eigenstates
- Consequently, the only flavour-changing interactions are the charged current weak interactions
 - *no flavour-changing neutral currents (GIM mechanism)*
 - *not generically true in most extensions of the SM*
 - *flavour-changing processes provide sensitive tests*

Lepton flavour violation

- No right-handed neutrinos in the SM, implies they are massless
 - neutrinos only left-handed (chirality)
 - antineutrinos only right-handed (chirality)
- Neutrino oscillations show they have small but finite masses
 - **where are the right-handed neutrinos?**
 - charged lepton flavour violation
 - physics beyond the Standard Model
- Why do we not observe the decay $\mu \rightarrow e \gamma$?
 - exact (but accidental) lepton flavour conservation in the SM with $m_\nu = 0$
 - SM loop contributions suppressed by $(m_\nu / m_W)^4$
 - LFV - a mechanism beyond the SM needed



What causes matter–antimatter asymmetry?

- The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks

$$V_{CKM} = U_u U_d^\dagger$$

(U - diagonalisation of mass matrices)

- It is a 3x3 complex **unitary** matrix
 - described by 9 (real) parameters
 - 5 can be absorbed as phase differences between the quark fields
 - 3 can be expressed as (Euler) mixing angles
 - the fourth makes the CKM matrix complex (i.e. gives it a phase)
 - weak interaction couplings differ for quarks and antiquarks
 - CP violation

Reminder - 1964: CP violation

Both $K^0 \rightarrow \pi\pi$ and $\text{anti-}K^0 \rightarrow \pi\pi$ occur

- K^0 may turn into its antiparticle, so are not mass eigenstates

The mass eigenstates are:

$$|K_S^0\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle)$$

CP operator gives:

$$\mathbf{CP}|K^0\rangle = |\bar{K}^0\rangle, \mathbf{CP}|K_S\rangle = +|\bar{K}_S\rangle, \mathbf{CP}|K_L\rangle = -|\bar{K}_L\rangle$$

Thus:

$$\text{only } K_S \rightarrow \pi\pi, \text{ but } K_L \rightarrow 3\pi$$

Under CP symmetry:

K_S (CP=+1): can only decay (hadronically) to 2π 's (CP=+1)

K_L (CP=-1): can only decay (hadronically) to 3π 's (CP=-1)

If CP conserved, should *not* see the decay $K_L \rightarrow 2\pi$'s

Reminder - Cronin-Fitch experiment

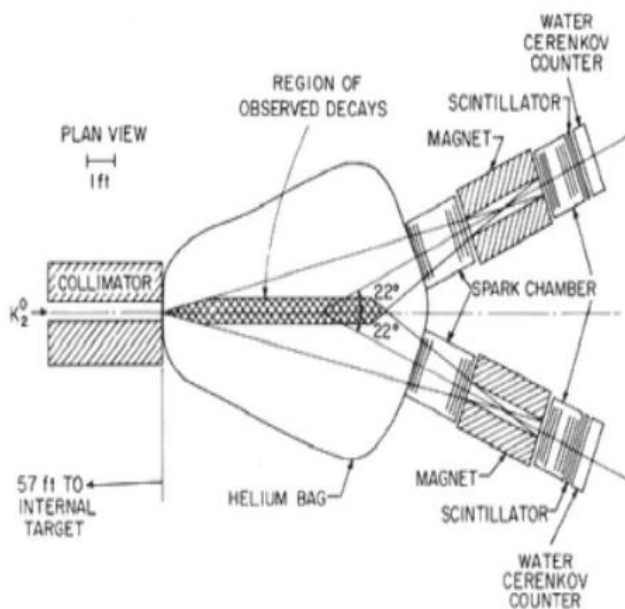
Observation of $K_2 \rightarrow \pi^+ \pi^-$ → Christenson, Cronin, Fitch, Turlay (1964)

The experiment shoot protons on a target to produce K^0 , after a long enough trip in a vacuum pipe, they achieved a pure K_2 beam.

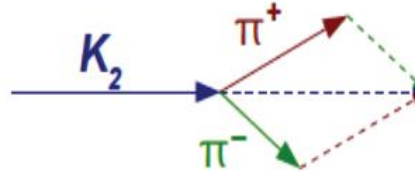
Experimentally use invariant mass (energy conservation) and angle between K_2 and $\pi^+ \pi^-$ (momentum conservation).

Find excess of ~ 56 events in the signal region: **$BF(K_2 \rightarrow \pi^+ \pi^-) \sim 2 \times 10^{-3}$**

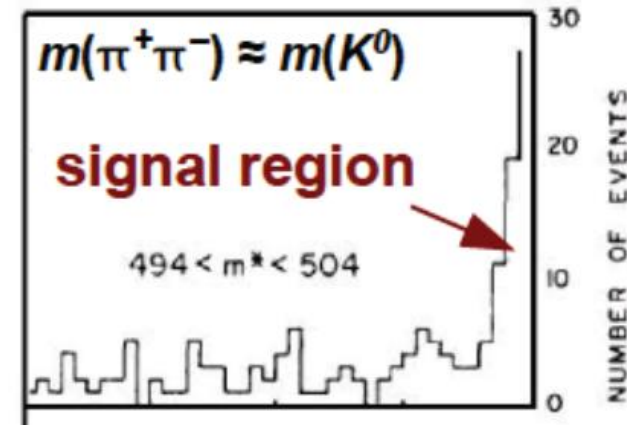
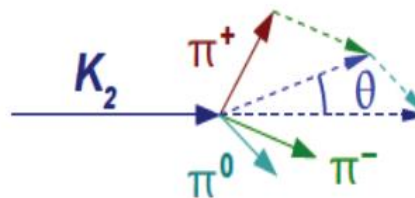
→ **CP violation!**



2-body decay (signal):



3-body decay (background):



Reminder - 1963: Cabibbo mixing

The weak coupling did not look to be universal:

$$s \rightarrow u \quad \text{e.g.} \quad K^+ \rightarrow \mu^+ \nu_\mu$$

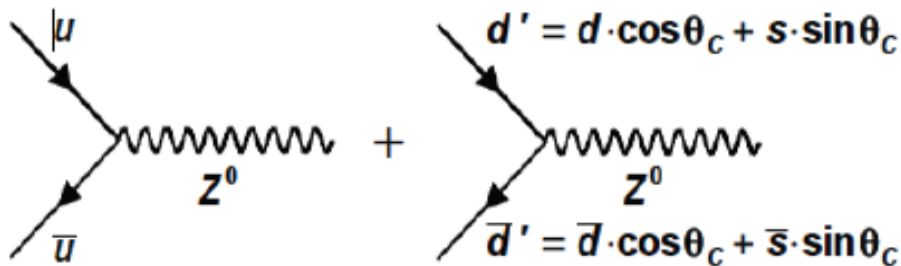
$$d \rightarrow u \quad \text{e.g.} \quad \pi^+ \rightarrow \mu^+ \nu_\mu$$

$s \rightarrow u$ transitions suppressed by a factor ~ 20

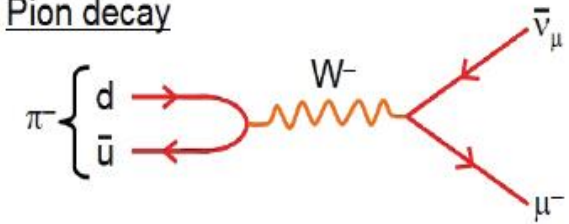
Cabibbo (1963): **weak interactions couples to a linear combination:**

$$\mathbf{d}' = \mathbf{d} \cdot \cos \theta_c + \mathbf{s} \cdot \sin \theta_c$$

$\sin \theta_c = 0.22$ (empirically)

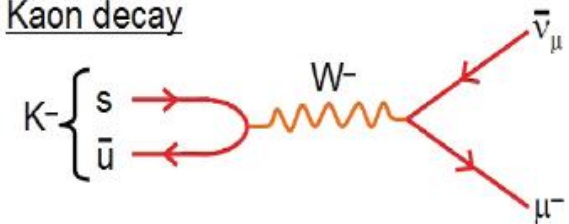


Pion decay



$$\pi^-(d\bar{u}) \rightarrow \mu^- + \bar{\nu}_\mu$$

Kaon decay



$$K^-(s\bar{u}) \rightarrow \mu^- + \bar{\nu}_\mu$$

But, if the neutral weak currents also couple to d' expect large FCNC
 Experimentally, however, $BR(K \rightarrow \mu\mu) \sim 7 \times 10^{-9}$

Reminder - 1970: GIM mechanism

$K^+ \rightarrow \mu^+ \nu_\mu$ so why not

$K^0 \rightarrow \mu^+ \mu^-$?

$K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ so why not

$K^0 \rightarrow \pi^0 \mu^+ \mu^-$?

$BR(K_L \rightarrow \mu^+ \mu^-) \sim 7 \cdot 10^{-9}$

$BR(K_L \rightarrow e^+ e^-) \sim 10^{-11}$

$BR(K^0 \rightarrow \pi^0 \mu^+ \mu^-) < \sim 10^{-10}$

GIM (Glashow, Iliopoulos, Maiani) mechanism (1970)

assume a **new (not yet observed) quark**
in SU(2) quark doublets

no tree level flavour changing neutral currents

suppression of FCNC via loops

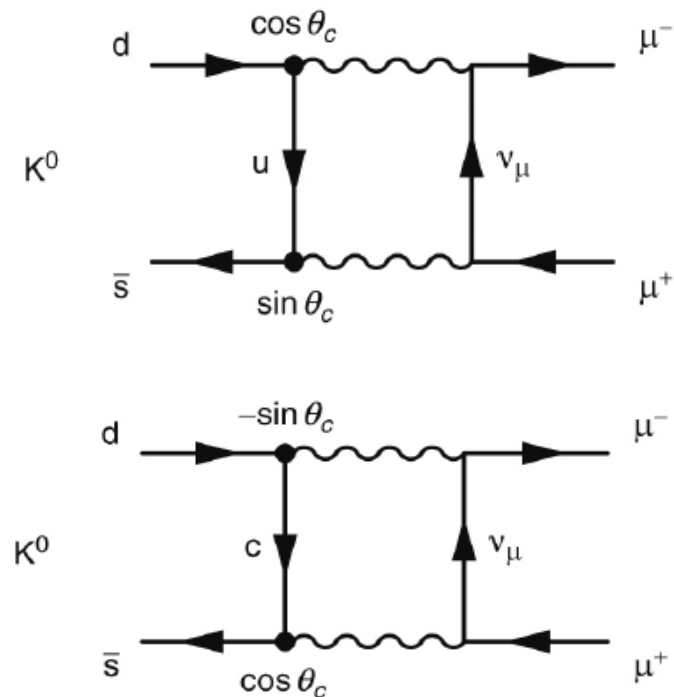
Requires that quarks come in pairs (doublets)

prediction of a **2nd up-type quark**

additional Feynman graph cancels the „u-box graph”

prediction of $m(c) \approx 1.5 \text{ GeV}$

→ *Gaillard and Lee (1974)*



Reminder - 1973: The CKM mechanism

1973: **Kobayashi & Maskawa** demonstrate that CP violation arises naturally from quark mixing if there are **3 generations of quarks**



CKM Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

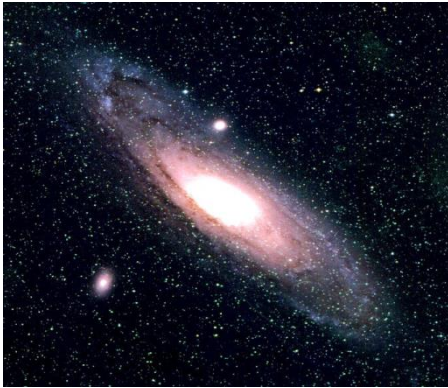


3x3 matrix of complex numbers

- \Rightarrow 18 parameters
- unitary \Rightarrow 9 parameters
- quark fields absorb unobservable phases \Rightarrow 4 parameters

3 mixing angles and 1 phase allowing for CP violation

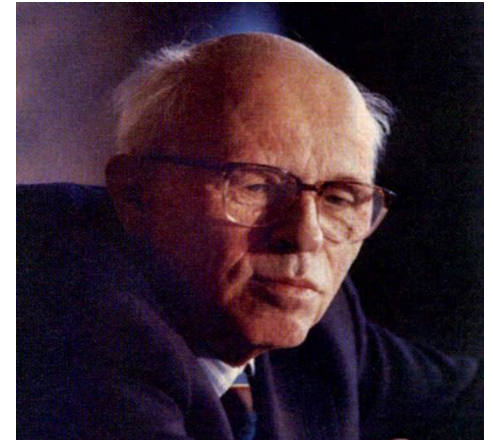
Matter-antimatter asymmetry



- We know that the matter – anti-matter symmetry in the Universe is broken: the Universe consists of matter.
- But, shortly after the Big Bang, there should have been equal amounts of matter and anti-matter
→ how did the Universe develop a preference of matter?

In 1966, Andrei Sakharov showed that necessary for evolution of matter dominated universe, from symmetric initial state, are:

- (1) baryon number violation
 - (2) C & CP violation
 - (3) thermal inequilibrium
- No significant amounts of antimatter observed!
 - $(N(\text{baryon}) - N(\text{antibaryon})) / N_\gamma \sim 10^{-10}$



Standard Model *CPV* cannot explain matter asymmetry in the universe

→ the *only* CP violating phase in SM leads to $10^{-17} \Delta N_B / N_\gamma$

SM flavour sector

Flavour in Standard Model

- Higgs field was introduced to give masses to W^+ , W^- and Z^0 bosons (after SBB)
- Since we have a Higgs field we can add (ad-hoc) interactions between the Higgs field ϕ and the fermions in a gauge invariant way (Yukawa couplings):

$$-L_{Yukawa} = Y_{ij} \left(\overline{\psi_{Li}} \phi \right) \psi_{Rj} + h.c.$$

↑ doublets
↑ singlet

- The quark flavour structure within the SM is described by 6 couplings and 4 CKM params
- It is convenient to move the CKM matrix from Yukawa sector to the weak current sector
- We can diagonalize the Y_{ij} matrices, such that we arrive in the „mass basis”

However, then the Lagrangian of the charged weak current should also be rewritten:

$$-L_{W^+} = \frac{g}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t})_L \left(V_{CKM} \right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W_\mu^+$$

CKM matrix (rotation matrix)

V_{CKM} originates from the diagonalization of the Yukawa couplings

Weak interactions in the SM

- After SSB, the charged current of a W^- exchange can be written as:

$$J^{\mu-} = (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\mu V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

- Weak interaction only couples to left-handed field
 - left-handed quarks or right-handed anti-quarks
 - manifestly violates parity

The weak eigenstates are related to the mass eigenstates by the CKM matrix:

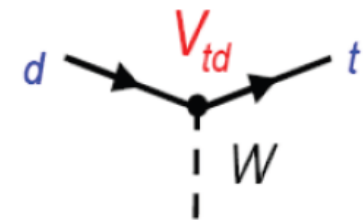
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak eigenstates Mass eigenstates

CP transformation & the weak interaction

Quarks

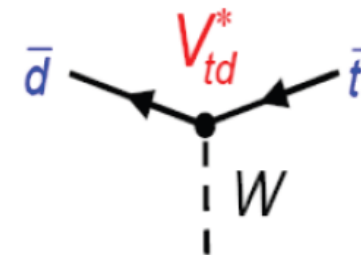
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



----- CP -----

Anti-quarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V_{ud}^* & V_{us}^* & V_{ub}^* \\ V_{cd}^* & V_{cs}^* & V_{cb}^* \\ V_{td}^* & V_{ts}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$



CP violation requires complex matrix elements

Relative phases

Q: How many parameters does the CKM matrix have?

18 parameters (9 complex numbers):

9 unitary conditions: $V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1$

5 relative phases of the quark fields

4 parameters (*)

(*) 3 (real) **Euler angles** and 1 **phase** (single source of CP violation in the SM)

- with 2 generations there is only one real (Euler) angle: **the Cabibbo angle**
- CP violation requires 3 generations!

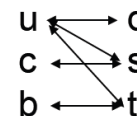
When I do a phase transformation of the (left-handed) quark fields:

$$u_{Lj} \rightarrow e^{i\phi_j} u_{Lj} \quad d_{Lk} \rightarrow e^{i\phi_k} d_{Lk}$$

And a simultaneous transformation of the CKM matrix:

$$V \rightarrow \begin{pmatrix} e^{-i\phi_u} & & & & & \\ & e^{-i\phi_c} & & & & \\ & & e^{-i\phi_t} & & & \\ & & & e^{-i\phi_d} & & \\ & & & & e^{-i\phi_s} & \\ & & & & & e^{-i\phi_b} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \text{or} \quad V_{jk} \rightarrow \exp(-i(\phi_j + \phi_k)) V_{jk}$$

There are only 5 relative phases (+ one overall phase)



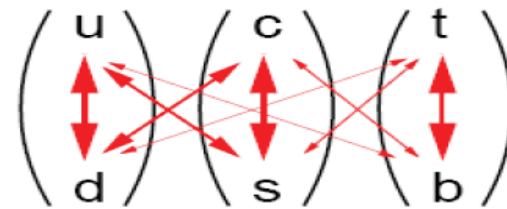
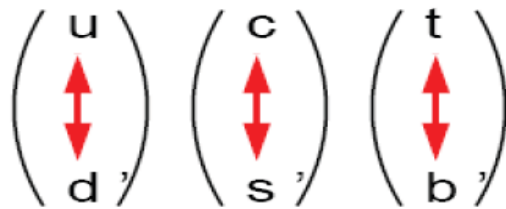
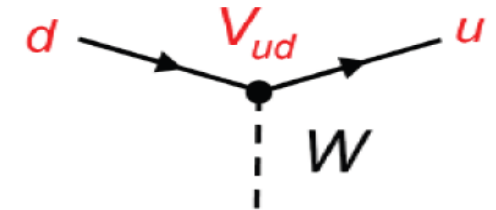
The charged current (i.e. the physics) remains invariant:

$$J_{CC}^\mu = \overline{u_{Li}} \gamma^\mu V_{ij} d_{Lj}$$

In other words, I can always absorb the 5 relative phases by redefining the quark fields
 → **these 5 phases are unobservable**

Hierarchy in quark mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} u & \color{red}{\square} & \color{red}{\square} & \color{red}{\cdot} \\ c & \color{red}{\square} & \color{red}{\square} & \color{red}{\cdot} \\ t & \color{red}{\cdot} & \color{red}{\square} & \color{red}{\square} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



- Diagonal elements of CKM matrix are close to one
- Only small off-diagonal contributions
- Mixing between quark families is „CKM suppressed”

Wolfenstein parametrization

Makes use of the fact that the off-diagonal elements are small compared to the diagonal elements

→ expansion in $\lambda \approx V_{us}$, $A \approx V_{cb} / \lambda^2$ and ρ, η

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{matrix} \lambda \sim 0.22 \text{ (=sin}\theta_C, \text{ sine of Cabibbo angle)} \\ A \sim 1 \text{ (actually 0.80)} \\ \rho \sim 0.14 \\ \eta \sim 0.34 \end{matrix}$$

$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

$$A^2\lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

$$\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4}(\bar{\rho} + i\bar{\eta})}{\sqrt{1 - \lambda^2} [1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}$$

CKM angles and unitarity triangle

Writing the complex elements explicitly:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & \sim \lambda^3 e^{-i\gamma} \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ \sim \lambda^3 e^{-i\beta} & \sim -\lambda^2 e^{-i\beta_s} & 1 \end{pmatrix} + O(\lambda^4)$$

Definition of the angles:

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

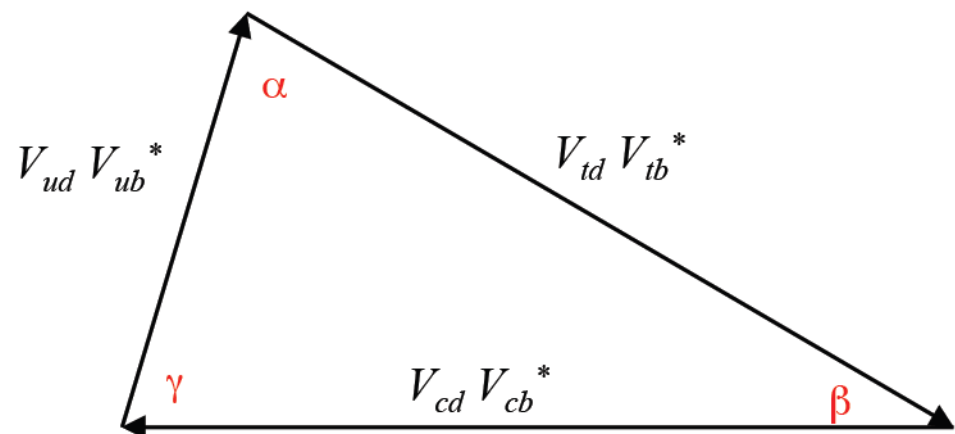
$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

Using one of the 9 unitarity relations: $V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1$

Multiply first „d” column with last „b” column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



CKM angles and unitarity triangle

Writing the complex elements explicitly:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & \sim \lambda^3 e^{-i\gamma} \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ \sim \lambda^3 e^{-i\beta} & \sim -\lambda^2 e^{-i\beta_s} & 1 \end{pmatrix} + O(\lambda^4)$$

Definition of the angles:

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

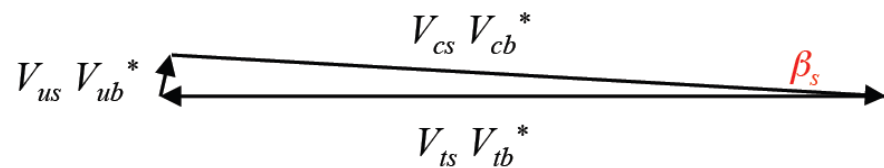
$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{tb}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

Using another unitarity relation: $V_{\text{CKM}} V_{\text{CKM}}^\dagger = 1$
 Multiply second „s” column with last „b” column:

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$



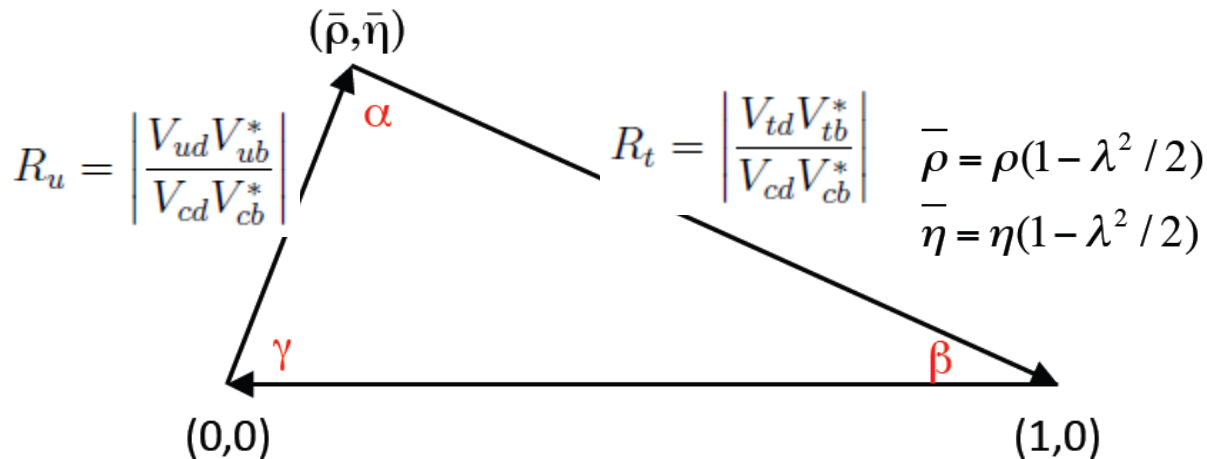
„Squashed unitarity triangle”

CKM angles and unitarity triangle

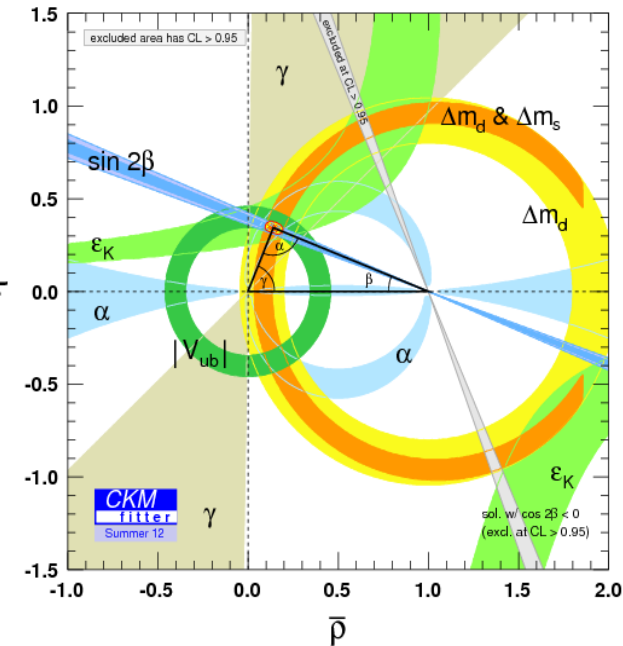
- Imposing unitarity to the CKM matrix results in six equations that can be seen as the sum of three complex numbers closing a triangle in the complex plane
- Two of these triangles are relevant for study of CP-violation in B-physics and define the angles

Normalized CKM triangle:

→ divide each side by $V_{cd} V_{cb}^*$



Current knowledge of UT (from CKMFitter)

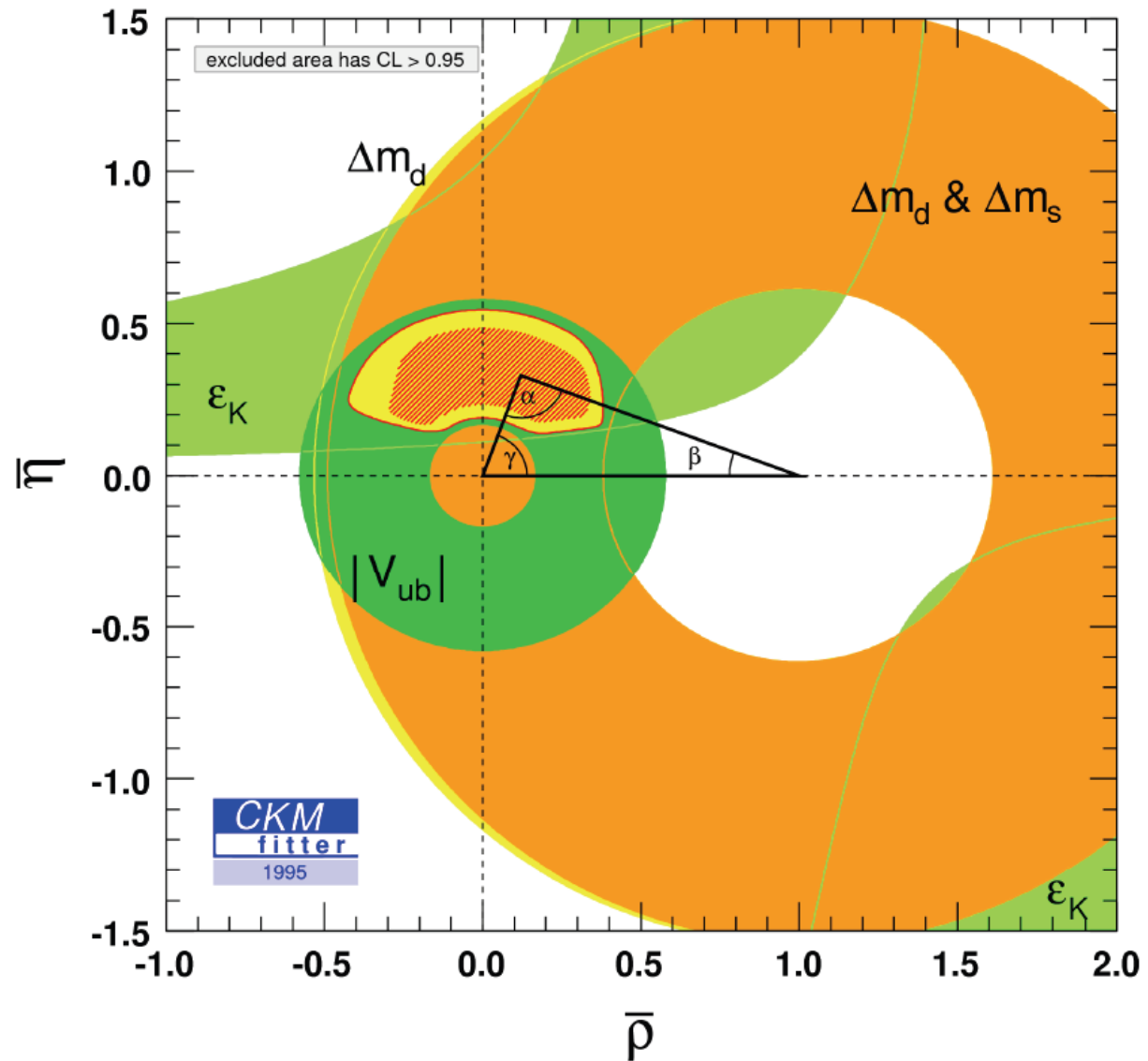


The unitarity triangle:

- Shows the size of the CP violation (no CPV means no triangle!)
- Presents our knowledge of CKM parameters
- Shows how consistent the measurements are!

Progress in UT

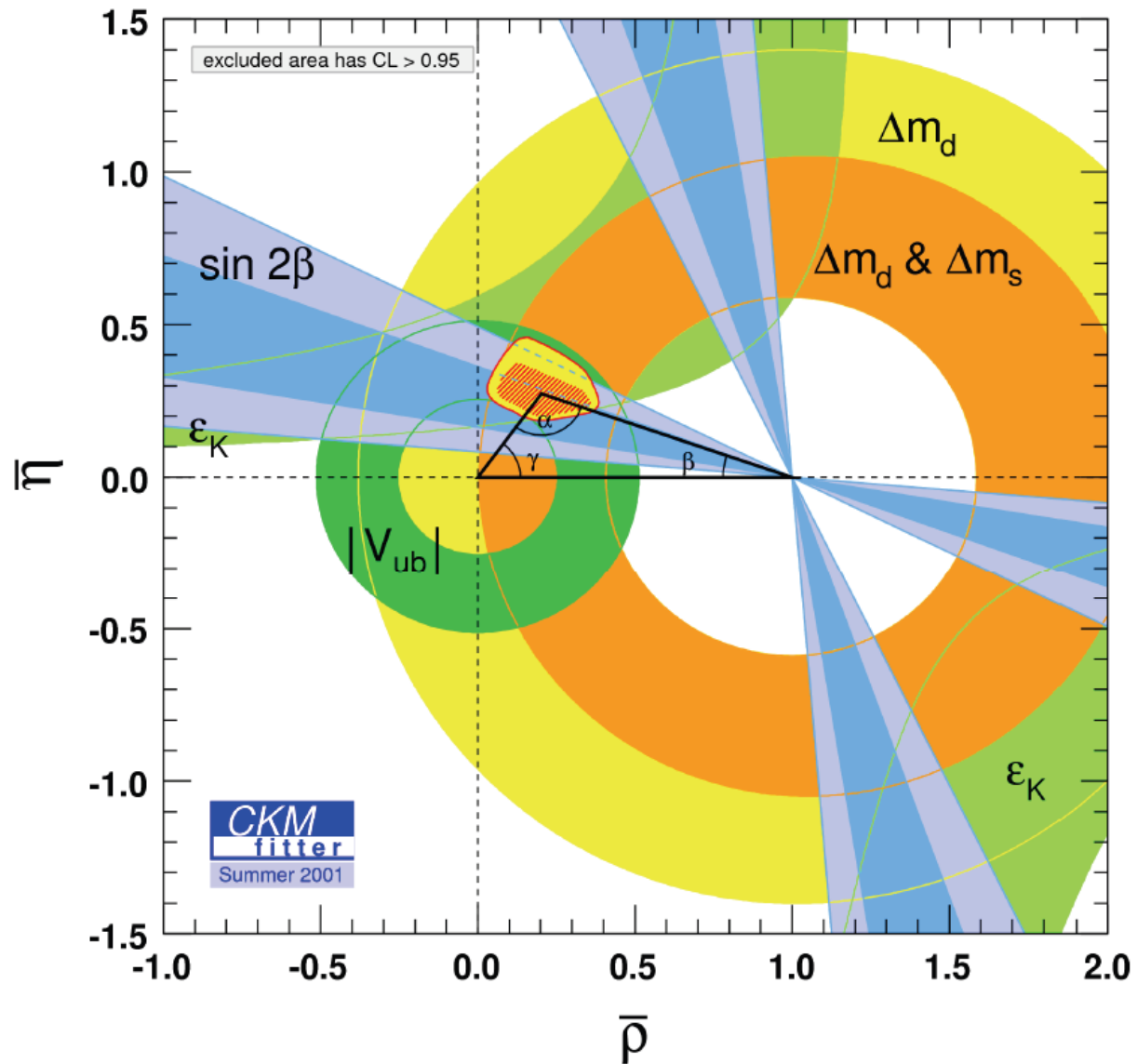
1995



Progress in UT

2001

first observation
of non-kaon CPV



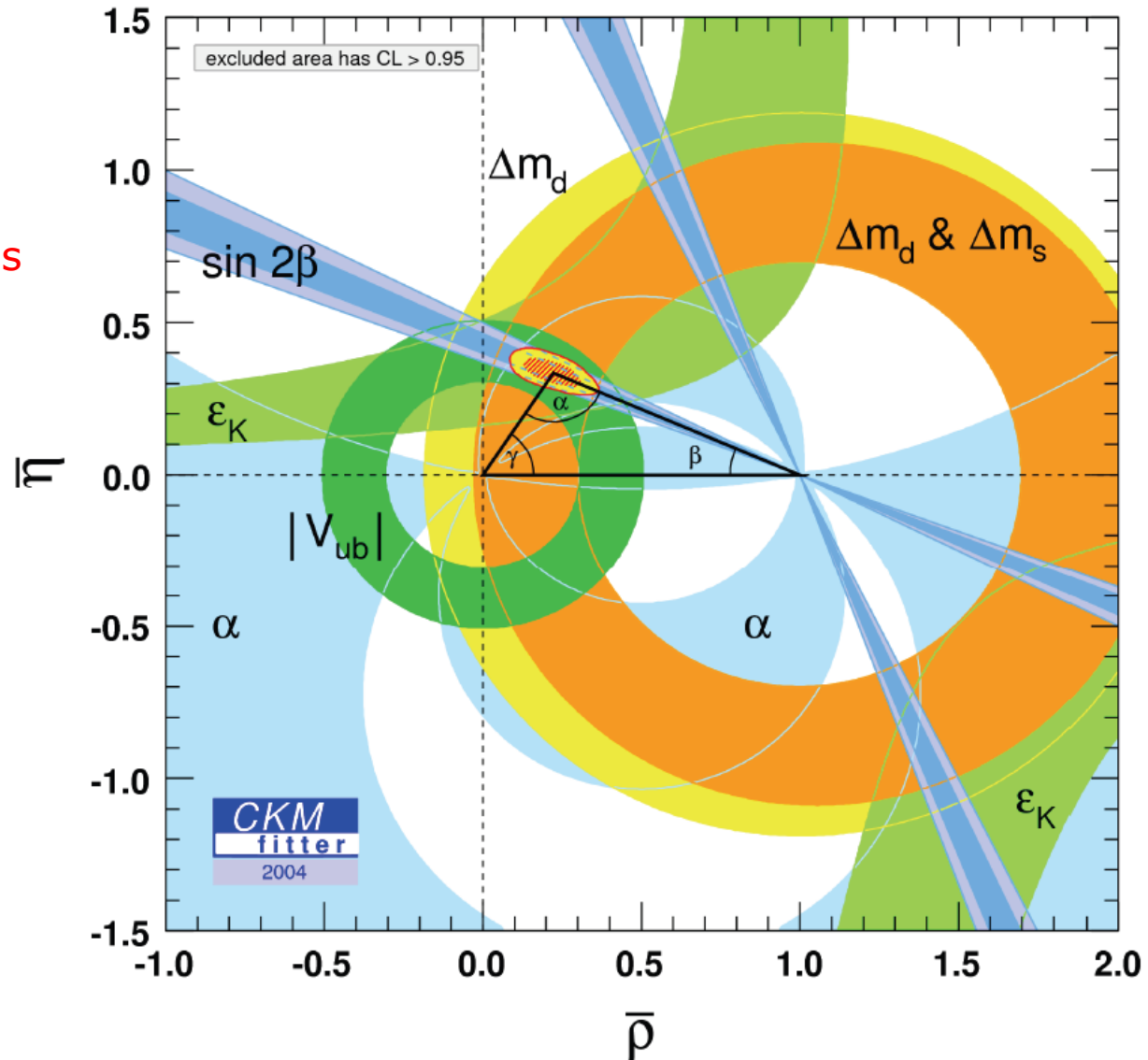
Progress in UT

2004

improvement in
lattice calculations

more data on
 $A_{CP}(B \rightarrow J/\psi K_S)$

first constraints
on the angle α
from $B \rightarrow \rho\rho$



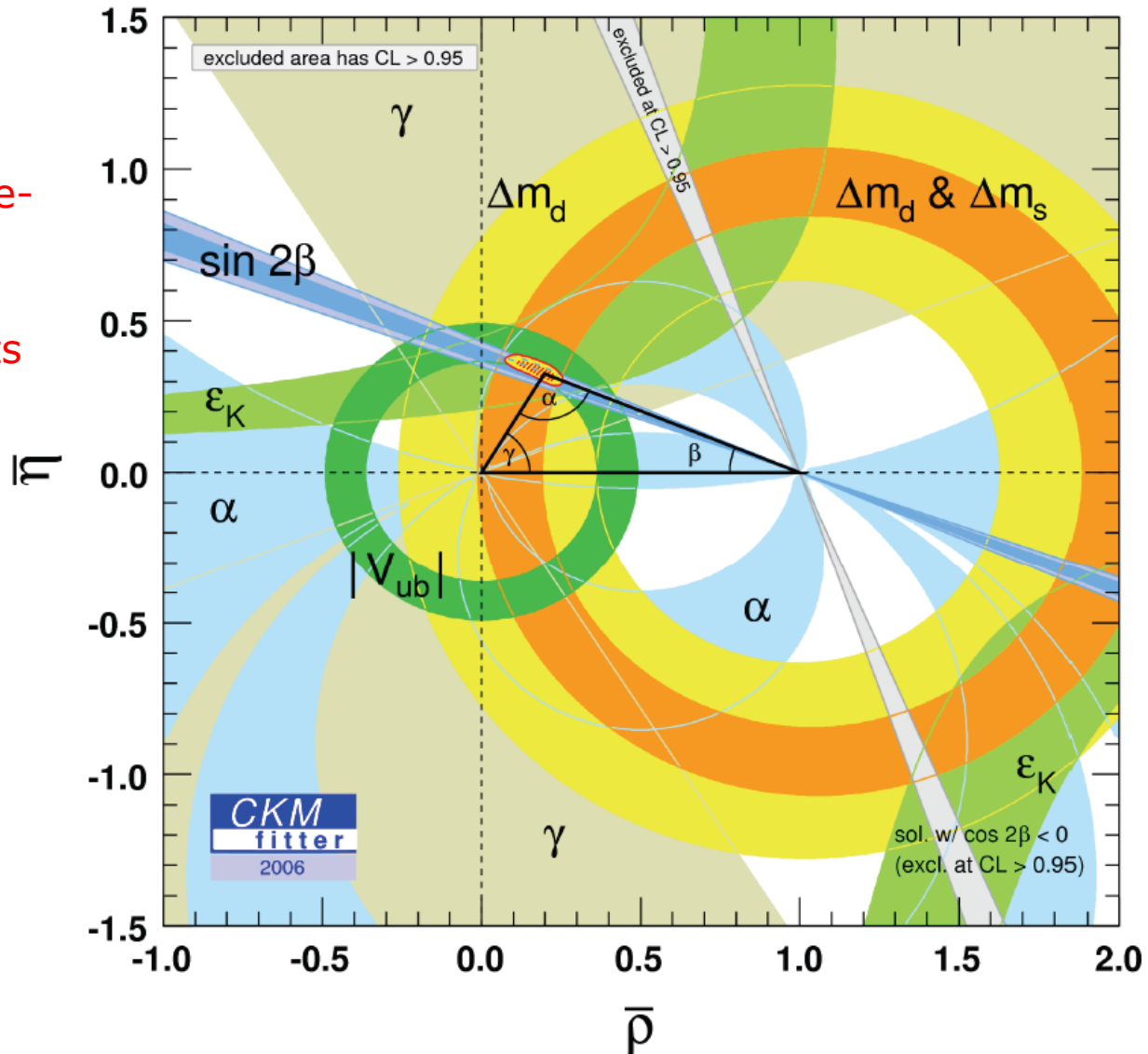
Progress in UT

2006

Tevatron measurement of Δm_s

tighter constraints on α

first constraints on γ from CPV in $B \rightarrow K \pi$

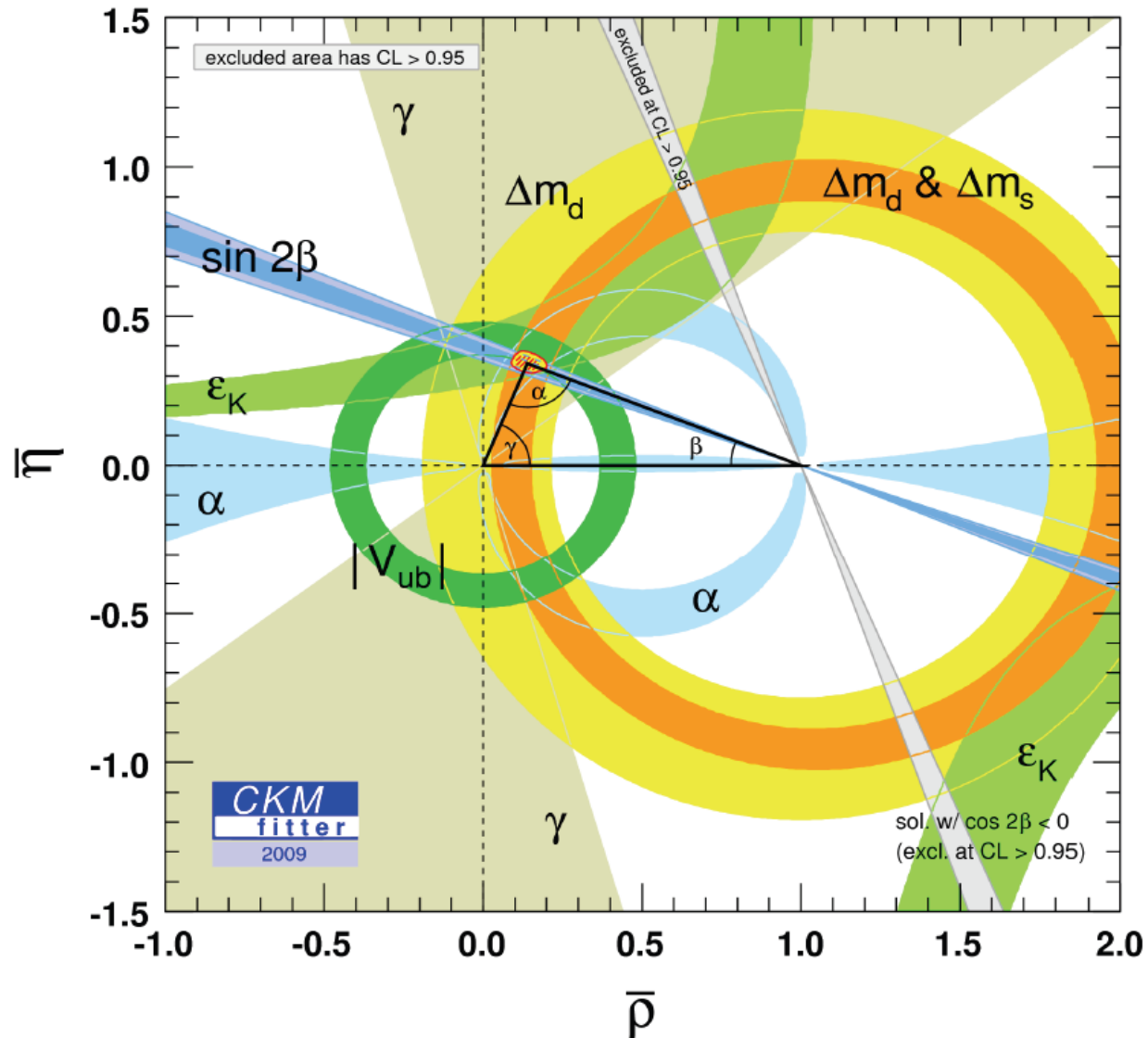


Progress in UT

2009

more constraints
from B mixing

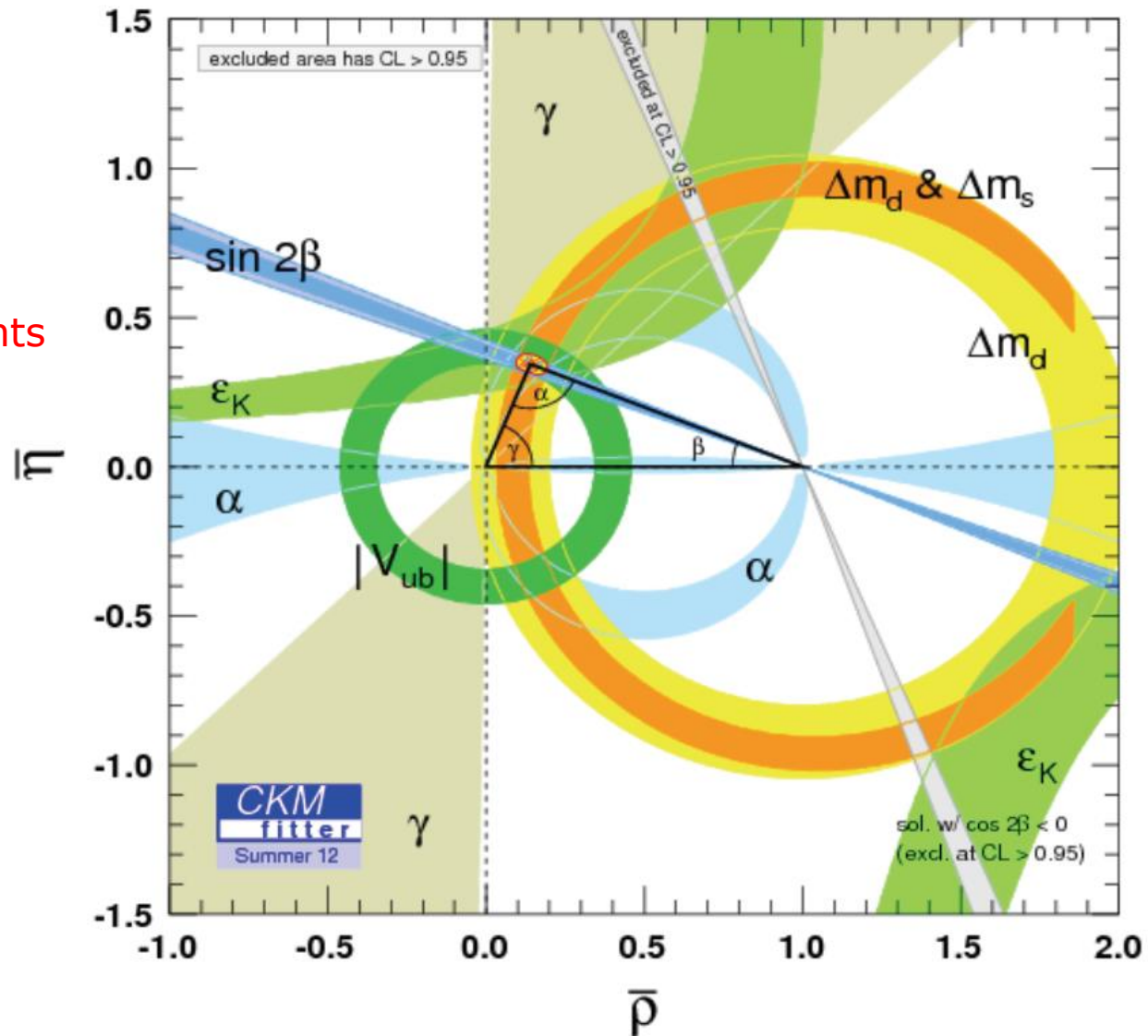
constraints from
BaBar on β
from CPV
in $B \rightarrow \chi_{0c} K_s$



Progress in UT

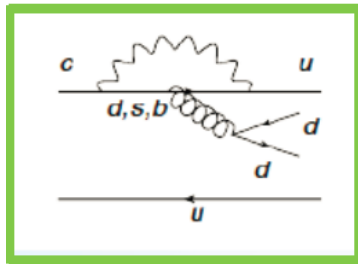
2012

LHCb measurement of Δm_s
tighter constraints on γ

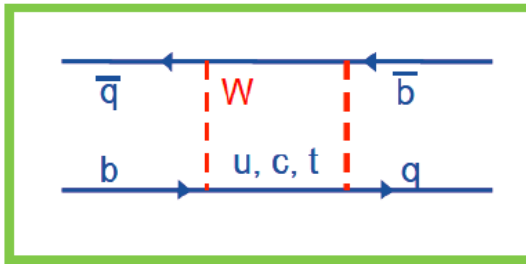


FCNC loops in the SM

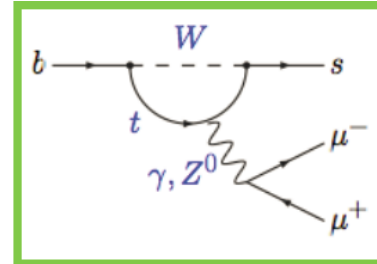
Map of flavour transitions and type of loop processes



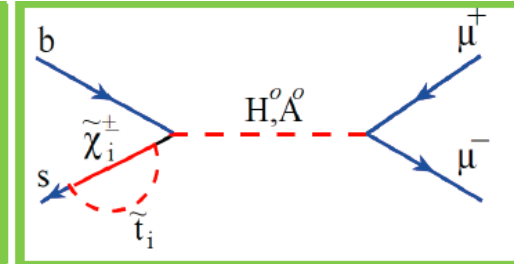
QCD penguin



$\Delta F=2$ box



EW penguin



Higgs penguin

	$b \rightarrow s$	$b \rightarrow d$	$c \rightarrow u$	$s \rightarrow d$
QCD penguin	$A_{CP}(B_s \rightarrow hhh)$	$A_{CP}(B^0 \rightarrow hhh)$	$\Delta a_{CP}(D \rightarrow hh)$	$K \rightarrow \pi^0 ll$ $\varepsilon' / \varepsilon$
$\Delta F=2$ box	ΔM_{B_s} $A_{CP}(B_s \rightarrow J/\psi \phi)$	ΔM_{B_d} $A_{CP}(B^0 \rightarrow J/\psi K_s)$	$x, y, q/p$	ΔM_K ε_K
EW penguin	$B \rightarrow K^{(*)} \mu \mu$ $B \rightarrow X_s \gamma$	$B \rightarrow \pi \mu \mu$ $B \rightarrow X \gamma$	$D \rightarrow X_u ll$	$K \rightarrow \pi^0 ll$ $K \rightarrow \pi^\pm \nu \nu$
Higgs penguin	$B_s \rightarrow \mu \mu$	$B^0 \rightarrow \mu \mu$	$D \rightarrow \mu \mu$	$K^0 \rightarrow \mu \mu$

Flavour sector beyond the SM

Yukawa mechanism in the lepton sector

- in the SM the lepton Yukawa matrices can be diagonalized independently due to the global G_f symmetry of the Lagrangian, and therefore there are no FCNC

$$-\mathcal{L}_{\text{Yukawa}}^{\text{SM}} = Y_d^{ij} \bar{Q}_L^i D_R^j H + Y_u^{ij} \bar{Q}_L^i U_R^j H_c + Y_e^{ij} \bar{L}_L^i E_R^j H + \text{h.c.}$$

$$\mathcal{G}_q = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}, \quad \mathcal{G}_\ell = SU(3)_{L_L} \otimes SU(3)_{E_R}$$

- however, the discovery that neutrinos oscillate (and are massive) implies that Lepton Flavour is not conserved
- the level of neutral Lepton Flavour Violation depends on the mechanism to generate neutrino masses (for instance **seesaw mechanism**)
- it could be just a copy of the quark sector, but it may be different due to the properties of the right-handed neutrino

Seesaw mechanism

Simplification: one family: ν_L and ν_R

- total mass term: Dirac and Majorana mass

$$\mathcal{L}_{mass} = -m(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L) - \frac{1}{2}M(\nu_R^T C\nu_R + \bar{\nu}_R C\bar{\nu}_R^T)$$

- diagonalization of the mass matrix:
→ Majorana mass eigenstates of the neutrinos

for $M \gg m$ we get

$$m_1 \approx \frac{m^2}{M} \quad m_2 \approx M$$

- one very heavy, practically right handed neutrino
- one very light, practically left handed neutrino

At energies small compared to M , Majorana mass term for left handed neutrino:

$$\mathcal{L}_{mass} = -\frac{1}{2} \frac{m^2}{M} (\nu_L^T C\nu_L + \bar{\nu}_L C\bar{\nu}_L^T)$$

Majorana mass is small if $M \gg m$

Seesaw mechanism

- In case of three families: **Neutrino Mixing**
- Compact notation for the Leptons:

$$\mathcal{N}_{L/R} = \begin{bmatrix} \nu_{e,L/R} \\ \nu_{\mu,L/R} \\ \nu_{\tau,L/R} \end{bmatrix} \quad \mathcal{E}_{L/R} = \begin{bmatrix} e_{L/R} \\ \mu_{L/R} \\ \tau_{L/R} \end{bmatrix}$$

- Dirac masses are generated by the Higgs mechanism: (as for the quarks)

$$\mathcal{L}_{DM}^N = -\mathcal{N}_L m^N \mathcal{N}_R + h.c. \quad \mathcal{L}_{DM}^E = -\mathcal{E}_L m^E \mathcal{E}_R + h.c.$$

- m^N : Dirac mass matrix for the neutrinos, m^E : (Dirac) mass matrix for e, μ , τ
- Right handed neutrinos \rightarrow Majorana mass term:

$$\mathcal{L}_{MM} = -\frac{1}{2} (N_R^T M C N_R + \bar{N}_R M C \bar{N}_R^T)$$

- M : (symmetric) **Majorana Mass Matrix**
- this term is perfectly $SU(2)_L \otimes U(1)$ invariant

Implementation of the seesaw mechanism:

- assume that all eigenvalues of M are large

Effective theory at low energies \rightarrow *only light, practically left handed neutrinos*

- effect of right handed neutrino: Majorana mass term for the light neutrinos

$$\mathcal{L}_{mass} = -\frac{1}{2} (N_L^T m^T M^{-1} m C N_L + \bar{N}_L m^T M^{-1} m C \bar{N}_L^T)$$

Lepton mixing: PMNS matrix

- we know there are FCNC in the lepton sector (analogous to the quark sector) because we have observed neutrino oscillations
- therefore the Yukawa couplings in lepton sector do contain also a mixing matrix

Pontecorvo Maki Nakagawa Sakata Matrix

- almost like CKM: Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- a Dirac phase δ and two Majorana phases α_1 and α_2

$$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}, \quad U_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{bmatrix}$$

- PMNS parametrization: $V_{\text{PMNS}} = U_{23} U_\delta^\dagger U_{13} U_\delta U_{12} U_\alpha$

- V_{PMNS} is unitary like the CKM matrix

- left handed neutrinos are Majorana

→ no freedom to rephase these fields!

$$\theta_{12} [^\circ] = 33.36_{-0.78}^{+0.81}$$

$$\theta_{23} [^\circ] = 40.0_{-1.5}^{+2.1} \text{ or } 50.4_{-1.3}^{+1.3}$$

$$\theta_{13} [^\circ] = 8.66_{-0.46}^{+0.44}$$

$$\delta_{\text{CP}} [^\circ] = 300_{-138}^{+66}$$

No hierarchy observed!

Lepton mixing: PMNS matrix

- we know there are FCNC in the lepton sector (analogous to the quark sector) because we have observed neutrino oscillations
- therefore the Yukawa couplings in lepton sector do contain also a mixing matrix

Pontecorvo Maki Nakagawa Sakata Matrix

- almost like CKM: Three Euler angles θ_{ij}

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

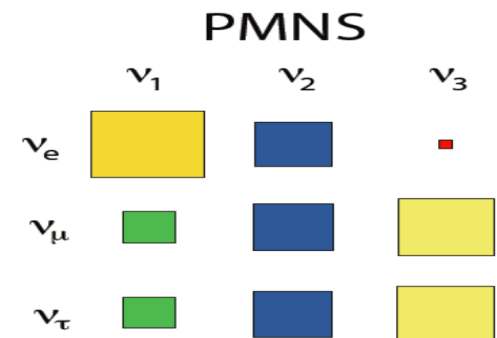
- a Dirac phase δ and two Majorana phases α_1 and α_2

$$U_{\delta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}, \quad U_{\alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{bmatrix}$$

- PMNS parametrization: $V_{\text{PMNS}} = U_{23} U_{\delta}^{\dagger} U_{13} U_{\delta} U_{12} U_{\alpha}$

- V_{PMNS} is unitary like the CKM matrix
- left handed neutrinos are Majorana
→ no freedom to rephase these fields!

No hierarchy observed!



Lepton Flavour Violation

FCNC processes in the leptonic sector:

$$\tau \rightarrow \mu\gamma \quad \mu \rightarrow e\gamma \quad \tau \rightarrow eee \text{ etc.}$$

$$\nu_\tau \rightarrow \nu_e\gamma \quad \nu_\tau - \nu_e \text{ mixing}$$

Lepton Flavour Violation:

- right handed neutrinos are Majorana fermions:
 - no conserved quantum number corresponding to the rephasing of the right handed neutrino fields
- lepton flavour violation could feed via conserved B-L into baryon number violation
- if neutrinos are **Dirac particles**, expect **very small** (far from experimental sens.) **LFV**
- however, if neutrinos are **Majorana particles** and something like the **seesaw mechanism** is at work, **large values** (close to exp. sens.) are favoured
- in general, **any extension of the SM with new states at the TeV scale generates large charged LFV**

Many flavour related open questions

- **Our understanding of Flavour is unsatisfactory:**
 - 22 (out of 27) free parameters of the SM originate from the Yukawa Sector (including Lepton Mixing)
 - Why is the CKM Matrix hierarchical?
 - **Why is CKM so different from the PMNS?**
 - Why are quark masses (except top) so small compared with electroweak VEV?
 - **Why do we have three families?**
- Why is CP Violation in flavour-diagonal processes not observed? (e.g. electric dipole moments of electron and neutron)
- Where is the CP violation needed to explain the matter-antimatter asymmetry of the Universe?

Strong CP remains mysterious

- flavour diagonal CP Violation is well hidden
 - e.g. electric dipole moment of the neutron:

$$\begin{aligned}d_e &\sim e \frac{\alpha_s}{\pi} \frac{G_F^2}{(16\pi^2)^2} \frac{m_t^2}{M_W^2} \text{Im}\Delta \mu^3 \\ &\sim 10^{-32} e \text{ cm} \quad \text{with } \mu \sim 0.3 \text{ GeV} \\ d_{\text{exp}} &\leq 3.0 \times 10^{-26} e \text{ cm}\end{aligned}$$

Many open questions

Standard Model

- does **not describe neutrino masses**
- does not have **a good DM candidate**
- **cannot explain the baryon asymmetry** in the Universe
- no explanation for the **flavour structure**
- does **not include gravity**
- suffers from **fine tuning issues in the Higgs sector**

Possible extensions

- SUSY, extra dimensions, hidden sectors,
- in general, the diagonalization of the mass matrix will not give diagonal Yukawa couplings → **large FCNC**

Needed

- precision measurements of flavour observables are generically sensitive to additions to the Standard Model
- precise measurements of the Higgs boson properties
- precise measurements of FCNC