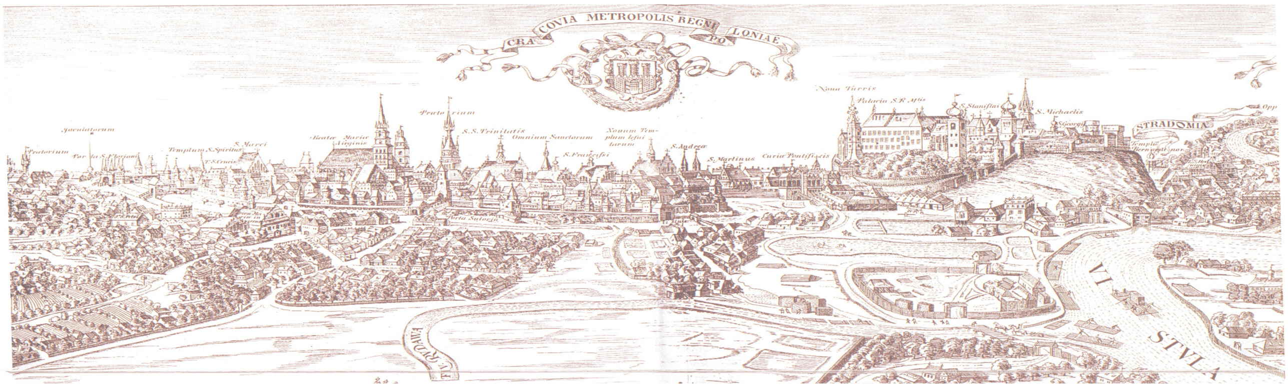

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BOOK OF ABSTRACTS

PARTICIPANTS

Page

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Fractional derivatives with respect to another function in modeling anomalous diffusion processes

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ABSTRACT:
Fractional calculus is widely used in modeling processes occurring in nature, for example in a description of anomalous diffusion. In particular, equations with fractional Caputo and Riemann-Liouville derivatives are often used to describe “ordinary” subdiffusion. In general, replacing a fractional derivative in a subdiffusion equation with a derivative of a different kind changes the properties of the equation; this equation describes a different subdiffusion process. Recently, a more general form of fractional derivatives has been considered, namely fractional derivatives with respect to another function g (fractional g-derivatives). We show the applications of various fractional g-derivatives in subdiffusion modelling, in particular the Caputo and Riemann-Liouville g-derivatives [1-6]. We describe the properties of the derivatives and the method of solving the g-subdiffusion equations. This method is based on the Laplace transform with respect to another function g. We also consider stochastic models describing g-subdiffusion processes and their physical interpretation. Our considerations concern "parabolic" as well as "hyperbolic" versions of the g-subdiffusion equations.

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Sub-, stagnated and negative diffusion of excitons in 2D materials

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ABSTRACT:
The diffusion of excitons in perovskites and transition metal dichalcogenides shows clear anomalous behaviour in experiments. We develop a non-Markovian mobile-immobile model which provides an explanation of this behaviour through paired theoretical and simulation approaches. The simulation model is based on a random walk on a 2D lattice with randomly distributed deep traps such that the trapping time distribution involves slowly decaying power-law asymptotics. The theoretical model uses coupled diffusion and rate equations for free and trapped excitons, respectively, with an integral term responsible for trapping. The model provides a good tting of the experimental data, thus, showing a way for quantifying the exciton diffusion dynamics.

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Stochastic reseting and multimodality in single-well potentials

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ABSTRACT:
Properties of stochastic systems are defined by the noise type and deterministic forces acting on the system. In out-of-equilibrium setups, e.g., for motions under action of Levy noises, the noise determines if for a given potential well, a stationary state exists. Single-well, power-law, potentials need to be steeper than

parabolic in order to assure existence of stationary states. The existence of stationary states, in sub-harmonic potential wells, can be restored by stochastic resetting, which is the protocol of starting over at random times. We demonstrate that the combined action of Levy noise and Poissonian stochastic resetting can result in the phase transition between non-equilibrium stationary states of various multimodality in the overdamped system in super-harmonic potentials. Stochastic resetting not only assures existence of stationary states, but importantly the fine-tuned resetting rate can increase the modality of stationary states. For a low stochastic resetting rate, the stationary state is independent of the restarting, while for high rates it becomes inevitably unimodal. Finally, we discuss various mechanisms of taming Levy flights.

ACKNOWLEDGEMENT:
Computer simulations have been performed using computing infrastructure of PLGrid (HPC Centers: ACK Cyfronet AGH) under computational grant no. PLG/2023/016175. The support from Research Area DigiWorld under the Strategic Programme Excellence Initiative at Jagiellonian University is also acknowledged.

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Detailed study of McKay I_ν Bessel distribution

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Common work with Katarzyna Górska, Andrzej Horzela and Tibor K. Pogány

ABSTRACT:
The probability distributions involving Bessel functions have caught the interest of many mathematicians and first results about this topic can be traced back to the early work of A. T. McKay in 1932 and R. G. Laha in 1954 who considered two classes of continuous distributions called Bessel function distributions, one, which building block is the modified Bessel function of the first kind I_ν of the order ν, and the another which is de ned by the modified Bessel function of the second kind K_μ. This talk is devoted the first type distribution.

Bearing in mind various applications of random variable distributed according to the McKay I_ν Bessel law the appropriate cumulative distribution functions (CDF) has been considered in mathematical literature and the main aim of this talk is to present several new formulas for such CDF.

First set of formulae are given in terms of incomplete generalized Fox-Wright function while the other expressions include Exton generalized hypergeometric X function of two variables, and also the incomplete Lipschitz-Hankel integral for the modi ed Bessel function of the first kind. The connection between CDF of the McKay I_ν Bessel random variable and another important CDF which belongs to the non-central chi-square law will be presented as well. Among others, we will observe results concerning log-concavity for the McKay I_ν law and we resolve the M-determinacy, unimodality and some mode results of the considered distribution.

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Burgers model - fractionalization and wave propagation

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ABSTRACT:
Classical Burgers model of viscoelastic body is generalized by considering the corresponding rheological scheme containing springs and dash-pots, having the dash-pots replaced by the Scott-Blair elements, where the stress is connected with strain through the fractional derivative. The obtained model is analyzed for the thermodynamical consistency and restrictions on model parameters are obtained implying the existence of eight thermodynamically acceptable models grouped into two model classes. Models are theoretically tested in creep and stress relaxation experiments and it is concluded that the thermodynamical restrictions must be narrowed if creep compliance and relaxation modulus are desired to be a Bernstein function and a completely monotonic function, respectively. These properties play an essential role in proving the dissipativity of fractional wave equations. Finally, fractional Burgers models are used for examining the wave propagation phenomena on both infinite and finite domains.

The results are obtained in collaboration with Aleksandar Okuka, Ljubica Oparnica, and Slađan Jelić.

The Physics of Odd Systems

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ABSTRACT:
Odd systems exhibit unusual and novel behavior with no counterpart in normal systems. They are characterized by transverse responses to perturbations. Classical examples would be the Hall effect in conductors and the Magnus effect in fluids. Therefore, in recent years, the physics community has developed an interest in odd physics which arises in the continuous descriptions of matter, such as odd viscosity, odd elasticity, or odd diffusion. Specifically odd-diffusive systems, which, for example, are realized in Brownian systems under Lorentz force, active chiral particles, or diffusing skyrmions are of special interest to us. Particle interactions in these systems result in novel diffusive behavior, the most surprising of which is that collisions enhance the self-diffusion rather than decreasing it. The origin of such a counterintuitive behavior can be traced back to the unusual dynamics in odd systems, which are encoded in correlation tensors with non-zero off-diagonal elements, even though the systems are isotropic. Furthermore, the dynamical enhancement can be traced to the strongly non-Gaussian distribution of the particles' displacements, which, in contrast to ordinary systems, are characterized by heavy tails.

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Fractionality in discrete time: theory and applications

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ABSTRACT:
Fractionality in continuous time is usually achieved by suitable random time-changes and it is commonly seen as a tool to extend Markov processes to models in which the presence of persistent memory is taken into consideration. Interestingly enough, these models are non-Markov, still they represent a class of processes retaining a certain mathematical tractability. Even though the literature on continuous-time fractional

processes is vast and growing, only few studies on their discrete-time counterparts have appeared so far. In this talk we present a theory for processes in discrete time admitting in some cases persistent memory. This is achieved by considering discrete infinite divisibility of random variables and defining time-changes resembling and actually converging to inverse subordinators. An example of a discrete-time renewal process having as a scaling limit the time-fractional Poisson process is described. Finally, we will present some applications of the theory.

The talk collects joint works with Angelica Pachon, Costantino Ricciuti, Thomas M. Michelitsch and Alejandro P. Riascos.

Asymptotic estimates for multi-term time fractional problem

Marcello D'Abbicco [1] and Giovanni Girardi [2]

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ABSTRACT:
We consider the Cauchy-type problem associated to the time fractional partial differential equation:

$$\partial_t u + \partial_t^{\frac{1}{2}} u - \Delta u = g(t, x)$$

for $t \geq 0, x \in \mathbb{R}^n$; here, the fractional derivative $\partial_t^{\frac{1}{2}}$ is in Caputo sense. It is known from the literature (see (1)) that the fundamental solution to this equation is the distribution of a telegraph process with Brownian time.
We provide a sufficient condition on the right-hand term g to obtain the well-posedness in classical Sobolev spaces H^s. Moreover, we exploit a dissipative-smoothing effect which allows to describe the asymptotic profile of the solution in low space dimension: under suitable assumption on g the solution to our problem behaves asymptotically like the fundamental solution to the fractional equation

$$\partial_t^{\frac{1}{2}} u - \Delta u = 0;$$

this latter is the density of an iterated Brownian motion.

As a corollary of this result, we investigate a class of nonlinear perturbations of the problem, for which global-in-time small data solutions exist and we show that their asymptotic profile is independent on the nonlinear perturbation.

The presentation is based on the results obtained in (2).

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A generalized model based on rotational diffusion with resetting for investigation of the non-Debye relaxation processes

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ABSTRACT:
We consider the rotational diffusion equation with a generalised memory kernel in the context of dielectric relaxation processes in a medium composed of polar molecules. We give an overview of existing models on non-exponential relaxation and introduce an exponential resetting dynamic in the corresponding process. We provide a detailed analysis of the autocorrelation function and complex susceptibility. It is shown that stochastic resetting leads to a saturation of the autocorrelation function to a constant value, in contrast to the case without resetting, for which it decays to zero. The behaviour of the autocorrelation function, as well as the complex susceptibility in the presence of resetting, confirms that the dielectric relaxation dynamics can be tuned by an appropriate choice of the resetting rate.

ACKNOWLEDGEMENT:
This work was supported by the German Science Foundation (DFG, Grant number ME 1535/12-1).

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Hypergeometric closed forms of numbers of constrained set partitions

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ABSTRACT:
As it is well known the function $F(t) = \exp(\exp(t) - 1)$ is the exponential generating function (egf) of the (integer) Bell numbers $B(n)$, i.e. $F(t) = \sum_{n=0}^{\infty} B(n)t^n/n!$ with $B(n) = 1, 1, 2, 5, 15, 52, 203, \ldots$, $n = 0, 1, \ldots$. The \$B(n)'s\$ count the numbers of all partitions into „boxes” of a set of \$n\$ distinguishable objects. The purpose of this work is to investigate some properties of integers describing constrained partitions (CP) resulting from deformations of the exponent in $F(t)$ in following two ways: either (A) we substruct a finite numbers of terms from the power series $\exp(t) - 1$, or (B) we retain only a finite number of terms from $\exp(t) - 1$. In case A the so obtained egf counts all CP without ``boxes'' of certain types. In case B the egf counts the CP with only ``boxes'' of certain types.

Here we provide closed forms for many enumerating sequences of type A and B in terms of generalized hypergeometric functions. Several explicit examples will be given. These formulas can be efficiently handled by computer algebra systems.

On the role of fractional calculus in Network Theory

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ABSTRACT:
Network theory is currently applied in multiple disciplines, including biology, computer science, business, economics, particle physics, operations research and, most commonly, in sociology.

The interest is usually in identifying the most important entities within the network and, to this purpose, many measures have been introduced. Here we discuss a recently de ned measure which is based on the Mittag-Leffer functions and interpolate between degree and eigenvector centrality.

In particular, these measures require computing the Mittag-Leffer functions in matrix arguments that are very large. Effective numerical strategies tailored to this task are presented, together with numerical tests that show their effectiveness.

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Fractional boundary value problems

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ABSTRACT:
We associate fractional boundary value problems (involving fractional dynamic boundary conditions) with sticky diffusions spending an infinite mean time (and finite time) on a lower-dimensional boundary. Such a behavior can be associated with a trap effect in the macroscopic point of view. Once the process hits the boundary, then it starts again after a random amount of time. While on the boundary it can move according to dynamics that are different from those in the interior. Such processes may be characterized by a time-derivative appearing in the boundary condition for the governing problem. We use time changes obtained by right-inverses of suitable processes in order to describe fractional sticky conditions and the associated boundary behaviors.

Time-fractional Caputo derivative versus other integro-differential operators in generalized Fokker-Planck and generalized Langevin equations

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ABSTRACT:
Fractional diffusion and Fokker-Planck equations are widely used tools to describe anomalous diffusion in a large variety of complex systems. The equivalent formulations in terms of Caputo or Riemann-Liouville fractional derivatives can be derived as continuum limits of continuous time random walks and are associated with the Mittag-Leffler relaxation of Fourier modes, interpolating between a short-time stretched exponential and a long-time inverse power-law scaling. More recently, a number of other integro-differential operators have been proposed, including the Caputo-Fabrizio and Atangana-Baleanu forms. Moreover, the conformable derivative has been introduced. We here study the dynamics of the associated generalized Fokker-Planck equations from the perspective of the moments, the time averaged mean squared displacements, and the autocovariance functions. We also study generalized Langevin equations based on these generalized operators. The differences between the Fokker-Planck and Langevin equations with different integro-differential operators are discussed and compared with the dynamic behavior of established models of scaled Brownian motion and fractional Brownian motion. We demonstrate that the integro-differential operators with exponential and Mittag-Leffler kernels are not suitable to be introduced to Fokker-Planck and Langevin equations for the physically relevant diffusion scenarios discussed in our paper. The conformable and Caputo

Langevin equations are unveiled to share similar properties with scaled and fractional Brownian motion, respectively.

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General approach to stochastic resetting

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ABSTRACT:

We address the effect of stochastic resetting on diffusion and subdiffusion process. For diffusion we find that mean square displacement relaxes to a constant only when the distribution of reset times possess finite mean and variance. In this case, the leading order contribution to the probability density function (PDF) of a Gaussian propagator under resetting exhibits a cusp independent of the specific details of the reset time distribution. For subdiffusion we derive the PDF in Laplace space for arbitrary resetting protocol. Resetting at constant rate allows evaluation of the PDF in terms of H function. We analyze the steady state and derive the rate function governing the relaxation behavior. For a subdiffusive process the steady state could exist even if the distribution of reset times possesses only finite mean.

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Long-range correlated processes: confinement, heterogeneity, & tempering

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ABSTRACT:
Stochastic processes driven by Gaussian yet power-law correlated noise, such as Mandelbrot's fractional Brownian motion (FBM) are ubiquitous in a range of complex systems, e.g., for the motion of tracers in viscoelastic environments, in "rough" financial data, or for the persistent motion of animals. FBM is an ergodic yet strongly non-Markovian process, with often surprising behaviour. In this talk I will briefly introduce these processes and demonstrate that in strong confinement their probability density may assume multimodal shapes, while in soft external potentials no steady state exists. An application to brain fibre growth is discussed. In heterogeneous environments the dynamics of a diffusing test particle may become a (random or deterministic) function of time. For these cases I will introduce novel extensions of FBM such as memory-multimodal FBM and FBM with a "diffusing diffusivity". Finally, I will discuss tempering of the power-law correlations, to emulate systems with a finite correlation time. This description is a good description for observed dynamics of lipid molecules in model membranes.

A new variable-order approach to fractional calculus

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ABSTRACT:
In the past a number of approaches to define fractional derivatives and integrals of variable-order have been proposed. In this talk we present a different approach based on some seminal ideas presented in 1972 by the Italian engineer G. B. Scarpi. Instead of generalizing the usual definitions of constant-order operators by means of a direct replacement in the time domain, new operators are derived in the Laplace transform domain. In this way, it is simpler to fulfill a Sonin condition and frame new operators within the theory of General Fractional Calculus developed first by Kochubei and hence by Luchko. Some aspects related to the numerical treatment of differential equations with the obtained variable-order operators are also presented.

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Towards Unification of Fractional Calculus

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ABSTRACT:
Fractional integrals and derivatives are traditionally defined based on widely different mathematical and physical interpretations (see [1] for an overview). It is thus of interest to unify different definitions by determining maximal domains on which they agree. This can be achieved through suitable generalizations. Recent results [2] indicate that "unification through generalization" is feasible for some translation invariant and causal fractional derivatives and integrals on the real axis [3].

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Active fractal networks with stochastic force-monopoles and force-dipoles

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ABSTRACT:
Motivated by recent experiments detecting the anomalous diffusion of various chromatin segments both normal and ATP-depleted cells, we study the dynamics of elastic fractals with embedded, stochastically driven, active force-monopoles and force-dipoles. The forces are assumed to be independent of each, to follow the random telegraph process with a given correlation time τ, and to be distributed randomly within the fractal network. We use Langevin dynamics in which the hydrodynamic drag is accounted for either locally, i.e., a Rouse-type model. Adopting a general analytical framework, we compute the mean square displacement of an arbitrary network node, $\langle(\vec{u}(t)-\vec{u}(0))^2\rangle$. We find, at short-time, $t \ll \tau$, super-diffusive behavior. Remarkably, at long times, $t \gg \tau$, the Rouse model with force-monopoles yields an anomalous diffusion exponent identical to that of the Rouse thermal system, $\nu = 1 - d_s/2$, with d_s standing for the spectral dimension, albeit with a different amplitude. In the case of force-dipoles with Rouse model, the anomalous sub-diffusion regime is usually absent. We back up the results, for the case of the Rouse model, by Langevin dynamics simulations on Sierpinski gasket. Moreover, force dipoles may lead to the rotational motion of the whole object, and to network collapse at large forces. Implications for chromatin dynamics are elucidated.

Fractional Brownian motion with stochastic Hurst exponent

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ABSTRACT:
Fractional Brownian motion, a Gaussian, non-Markovian, self-similar process with stationary long-correlated increments, has been identified to give rise to the anomalous diffusion behavior in a great variety of physical systems. The correlation and diffusion properties of this random motion are fully characterized by its index of self-similarity or the Hurst exponent. However, recent single-particle tracking experiments in biological cells revealed highly complicated anomalous diffusion phenomena that cannot be attributed to a class of self-similar random processes. Inspired by these observations, we generalize fractional Brownian motion to include (a) Hurst index that randomly changes from trajectory to trajectory but remains constant along a given trajectory, and (b) Hurst index that varies stochastically in time along a trajectory. We provide a general mathematical framework for analytical, numerical, and statistical analysis for both (a) and (b). Considering generic examples of the cases (a) and (b), we present results for the mean squared displacement and probability density function. We also provide an algorithm to distinguish between three classes of

random motions, namely the canonical FBM and its generalizations (a) and (b), and further demonstrate the applicability of this algorithm by analyzing real-world examples for all the three classes.

G-subdiffusion equations with fractional Caputo time derivative with respect to another function

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ABSTRACT:

The Caputo derivative with respect to another function g (the g-Caputo derivative) is more general than the "ordinary" fractional Caputo derivative. The commonly used subdiffusion equation involves the "ordinary" fractional Caputo time derivative. In the g-subdiffusion equation, the "ordinary" Caputo derivative is replaced by the g-Caputo derivative. This equation can be derived by means of the modified continuous time random walk model (the g-CTRW model) [2]. An interpretation of g-subdiffusion is that it is a subdiffusion process with a changed time scale controlled by the function g. The g-subdiffusion equation offers different possibilities for modeling anomalous diffusion [1-6], the examples are processes in which a type of diffusion evolves continuously over time. We use the g-subdiffusion equation to describe continuous transitions from "ordinary" subdiffusion to: ultraslow diffusion [1], "ordinary" subdiffusion with changed parameters [3], and superdiffusion [6]. In the transition from subdiffusion to superdiffusion, in the long time limit the Green's function for the g-subdiffusion equation takes the form of Green's function for superdiffusion described by the equation with fractional Riesz spatial derivative. Thus, for a sufficiently long time the g–subdiffusion equation describes superdiffusion well, despite a different stochastic interpretation of the processes. We show that the g-subdiffusion equation can be also used to describe anomalous diffusion of antibiotic (colistin) in a system consisting of packed gel (alginate) beads immersed in water, the function g has been derived from empirical data [4]. We mention that experimental results show that this process cannot be described by the "ordinary" subdiffusion equation with constant parameters.

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On the probability density in Rice--Middleton model

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Common work with Katarzyna Górska, Andrzej Horzela and Dragana Jankov Maišrević

ABSTRACT:
The probability density function of the single sinusoidal signal combined with Gaussian noise is expressed in three mutually independent ways: firstly, with the aid of an integral representation of the modified Bessel function of the first kind of integer order; secondly, by a hyperbolic cosine differential operator and thirdly, applying the Grunwald--Letnikov fractional derivative. The cumulative distribution functions are also obtained in all these cases, and separately, using the Nuttall Q - function description. An associated, seemingly new, probability distribution is introduced which cumulative distribution function and the raw moments of general real order are obtained whilst the characteristic function's power series form is inferred. The exposition ends with a discussion in which by--product summations are given for the considered Neumann series of the second type built by modified Bessel functions of the second kind having integer order.

On the trail of the Doppler-like effect in the generalized Cattaneo-Vernotte equation with the power-law memory function

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ABSTRACT:
The Doppler effect is a physical phenomenon discovered in 1842 that relies on a change in the observed frequency of a wave which is caused by the relative motion of the source and observer. This phenomenon is utilized, e.g. in astronomical measurements, radar systems, or even in medicine. An important aspect of the Doppler effect is its occurrence for any type of wave, making it possible to use it as a test of solutions to equations aspiring to describe wave motion. During the presentation, the generalized Cattaneo-Vernotte equation with the power-law memory function, its solution as well as the problem of the existence of the Doppler-like effect for the obtained solution will be presented.

Subordination approach to heterogeneous diffusion and telegrapher's processes with resetting

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ABSTRACT:
We analyze diffusion processes with infinite and finite propagation speed in a non-homogeneous medium in presence of stochastic resetting in terms of the heterogeneous Fokker-Planck and telegrapher's equations. To analyze the corresponding models, we will use the subordination approach, which is a powerful technique for solving various fractional and generalized diffusion and Fokker-Planck equations.

On some operational properties and applications of the special functions of fractional calculus

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ABSTRACT:
The developments in theoretical and applied sciences have always required a knowledge of the properties of the "Mathematical Functions" (in terms of Hary Bateman, on whose notes the basic old 3 volumes handbook on "Higher Transcendental Functions" has been edited posthumously): from the elementary trigonometric functions to the variety of Special Functions (SF), appearing in studies of natural and social phenomena, in formulation of engineering problems, and models of dynamical systems. The well known "Classical SF" (SF of Mathematical Physics, or Named SF) appear as solutions of differential and integral equations of integer order, mainly of 2nd order, but also of higher (integer) ones. Their theory has been developed for centuries and presented in many well-known "old" handbooks.

With the revival of the Fractional Calculus (FC: theory of the operators of integration and differentiation of arbitrary, i.e. not integer order) as not only an exotic theory extending the Calculus, and the recognition that the fractional order models can describe better the dynamical systems, the solutions of the fractional order differential and integral equations and systems gained their important place and became unavoidable operational tools. These are the so-called "Special Functions of Fractional Calculus" (SF of FC), in the general case as Fox H-functions. Among them the basic role have the generalized Wright hypergeometric functions ${}_p\Psi_q$ and in particular, the Mittag-Leffler (M-L) functions, and their various extensions (with multi-indices).

In this talk we make a short survey on the author's results and recent publications on the operational rules and applications of a very long list of SF of FC. The main references are to start and end by the works as:

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Anomalous diffusion, nonergodicity, non-Gaussianity, and aging of fractional Brownian motion with nonlinear clocks

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ABSTRACT:
How do nonlinear clocks in time or/and space affect the fundamental properties of a stochastic process? Specifically, how precisely may ergodic processes such as fractional Brownian motion (FBM) acquire predictable nonergodic and aging features being subjected to such conditions? We address these questions in the current study. To describe different types of non-Brownian motion of particles—including power-law anomalous, ultraslow or logarithmic, as well as superfast or exponential diffusion—we here develop and analyze a generalized stochastic process of scaled-fractional Brownian motion (SFBM). The time- and space-SFBM processes are, respectively, constructed based on FBM running with nonlinear time- and space-clocks. The fundamental statistical characteristics such as non-Gaussianity of particle displacements, nonergodicity as well as aging are quantified for time- and space-SFBM by selecting different clocks. The latter parameterize power-law anomalous, ultraslow and superfast diffusion. The results of our computer simulations are fully consistent with the analytical predictions for several functional forms of clocks. We thoroughly examine the behaviors of the probability-density function, the mean-squared displacement, the time-averaged meansquared displacement, as well as the aging factor. Our results are applicable for rationalizing the impact of nonlinear time- and space-properties superimposed onto the FBM-type dynamics. SFBM offers a general framework for a universal and more precise

model-based description of anomalous, nonergodic, non-Gaussian, and aging diffusion in single-molecule-tracking observations.

Operational calculus for the regularized general fractional derivatives and its applications

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ABSTRACT:
In this talk, we first address the general fractional integrals and derivatives with the special Sonin kernels. Then an operational calculus of the Mikusinski type for the regularized general fractional derivatives is constructed. Finally, we present an application of this operational calculus for analytical treatment of some initial value problems for the fractional differential equations with the regularized general fractional derivatives. The solutions to these problems are provided in terms of the convolution series that are a far-reaching generalization of the power series.

Cracks in the framework of fractional thermoelasticity

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ABSTRACT:
The time-nonlocal generalization of the Fourier law with the ‘long-tail’ power kernel can be interpreted in terms of fractional calculus and leads to the time-fractional heat conduction equation with the Caputo derivative. The theory of thermal stresses based on this equation was proposed in the paper (1) and summarized in the book (2). The pioneering paper (3) laid the foundation for study of thermoelastic problems for solids with cracks and slits.
We discuss problems for solids with cracks in the framework of fractional thermoelasticity. The first papers on fractional heat conduction in solids with cracks (4, 5) were published by the author.

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Volterra-Prabhakar function and Applications

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ABSTRACT:
We introduce the Volterra-Prabhakar function (1), which is involved in the definition of the distributed order Prabhakar function (2). This function is considered as a memory integral kernel of the generalized Fokker-Planck equation. Some integral representations (generalizations of the Ramanujan integrals (3)), integral transforms, convolutional properties and complete monotonicity results are also given. We will present some PDFs associated to Volterra-Prabhakar function, as well as some statistical results like mean square displacement, higher moments, standard deviations, skewness and kurtosis. Applications will be given to examine the generalized Langevin equation approach to anomalous diffusion of a free particle driven by distributed order Volterra noises.

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