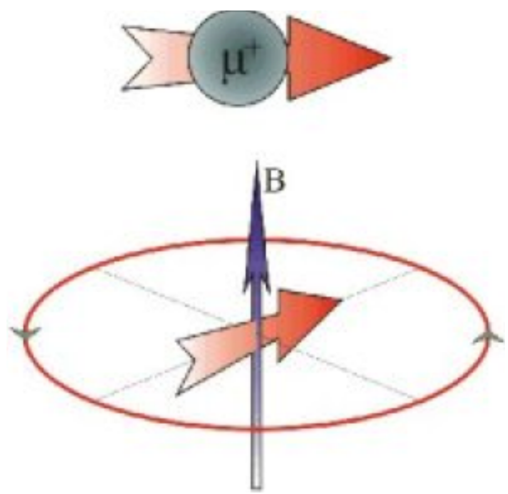
g-2 : anomalous magnetic moments of leptons



IFJ PAN Particle Physics Summer Student Programme

July 22, 2022

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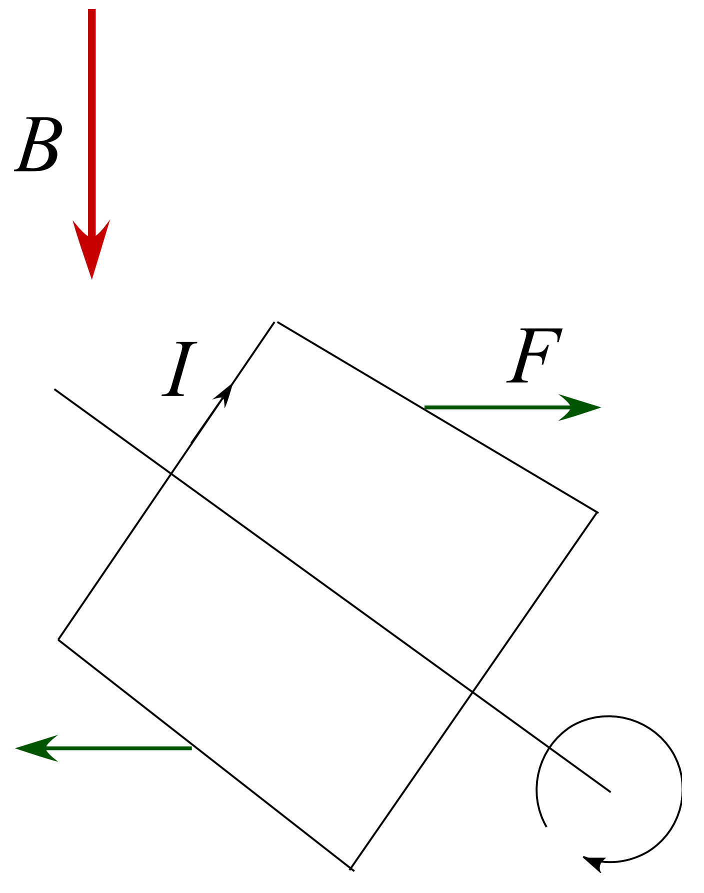
Outline

- What is “g” in “g-2”?
- How does the muon’s spin evolve and how can we observe it?
- What g-2 does the Standard Model predict?
- How does the measurement differ from SM?
- What other experiments can be done? (other muonic properties; g-factors of other things)

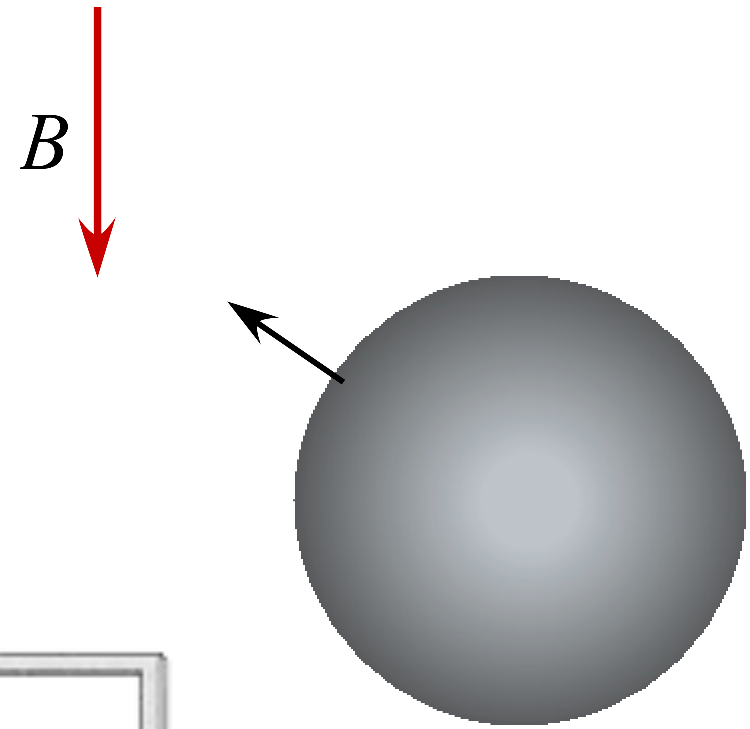
Magnetic moment

A loop with a current
in a magnetic field:

torque acts along
the axis



What happens when a charged particle rotates?



Rotation of charges equivalent to currents;

Magnetic field exerts torque

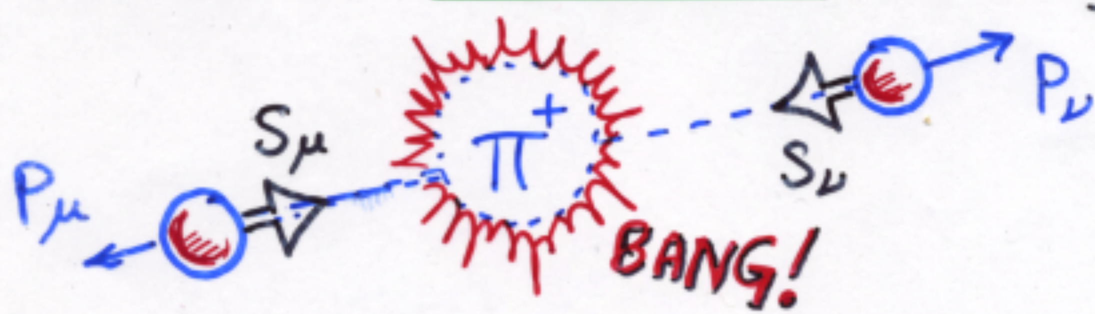
Precession results

Production of muons

- Pions produced by $p+p \rightarrow \pi^+ + p + n$
- Pion decays in 26 ns

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

- This is a TWO-BODY decay

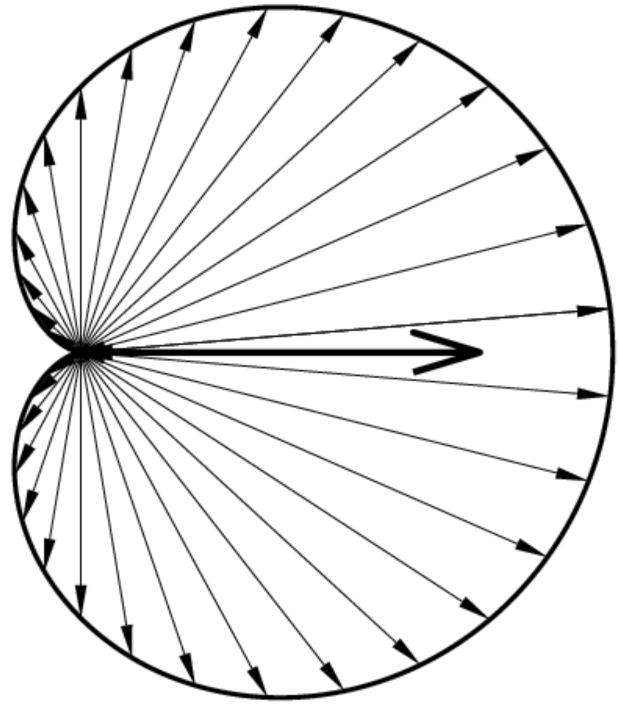


from Steve Blundell

Muon's decay reveals its spin

$$\mu^+ \longrightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

MUON POSITRON NEUTRINOS



Positron emission: asymmetric
with respect to the muon's
spin: parity violation (weak interaction)

from Steve Blundell

Magnetic moments

Spinning charged particle: magnetic dipole

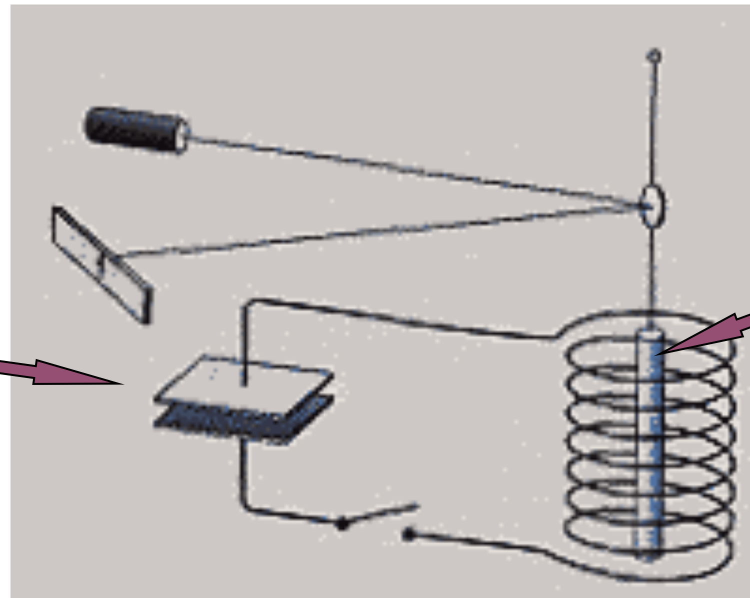
$$\vec{\mu} = g \frac{q}{2m} \vec{s}$$

For elementary fermions (electron, muon),
Dirac equation predicts $g = 2$

This is a special value;
if g were exactly 2,
spin and velocity would stay parallel.

If no spin, only orbital angular momentum: g = 1.

First measurement of g_e: Einstein-de Haas 1915



capacitor

ferrite rod

capacitor discharge → current → magnetisation of the rod

But the angular momentum of the rod stays zero →

the rod rotates to compensate the spin flip

Einstein-de Haas 1915 result

They got g = 1 and were very proud :

Mag auch die Güte der Übereinstimmung auf Zufall beruhen, da wir unserer Bestimmung wohl etwa 10 Proz. Unsicherheit beilegen müssen; jedenfalls ist erwiesen, daß das am Anfang geschilderte Ergebnis der Theorie der kreisenden Elektronen auch quantitativ mindestens annähernd durch den Versuch bestätigt wird.

https://einsteinpapers.press.princeton.edu/vol6-doc/197

This agreement might be fortuitous, since we must ascribe an accuracy
of about 10% to our measurements; nevertheless we have shown
that the result of circular motion of the electrons described at the beginning
of our article is quantitatively confirmed by experiment, at least approximately

translation by Frenkel, Sov. Phys. Usp. 22 (1979) 580

Precession of velocity and of spin

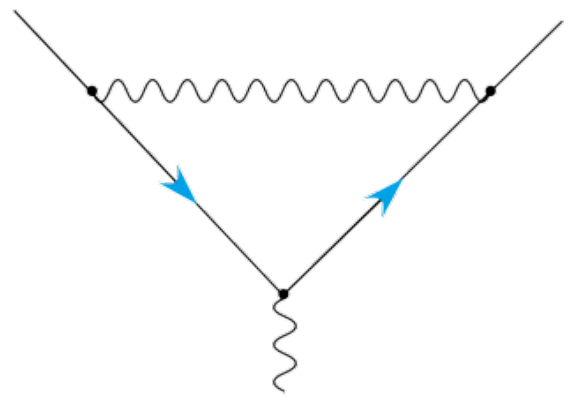
$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{qB}{\gamma m}$$

$$\frac{1}{s}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{qB}{m}\left(\frac{g}{2} - \frac{\gamma - 1}{\gamma}\right)$$

$$\frac{1}{s}\frac{\mathrm{d}s}{\mathrm{d}t} - \frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{qB}{m}\left(\frac{g}{2} - 1\right)$$

$a = \frac{g}{2} - 1 = \frac{g-2}{2}$ anomalous magnetic moment

g-2: first results



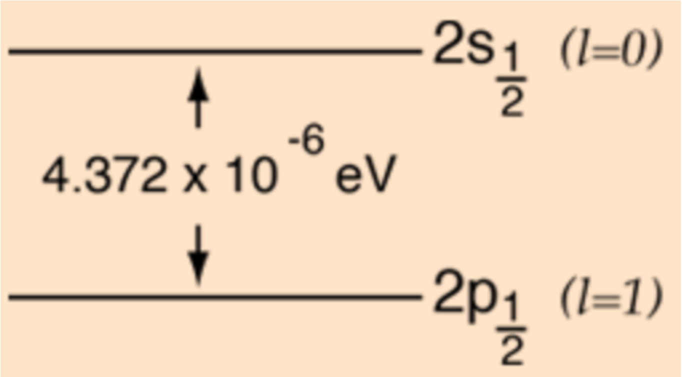
Schwinger (1948)

$$a\,(\text{theory}) \equiv \frac{g-2}{2} = \frac{\alpha}{2\pi} \simeq 0.00116$$

Kusch & Foley (1948)

$$a\,(\text{exp}) = 0.001188\,(22)$$

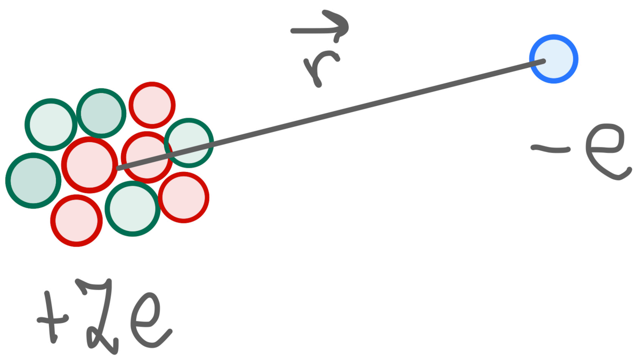
Origin of radiative corrections: start with the Lamb shift

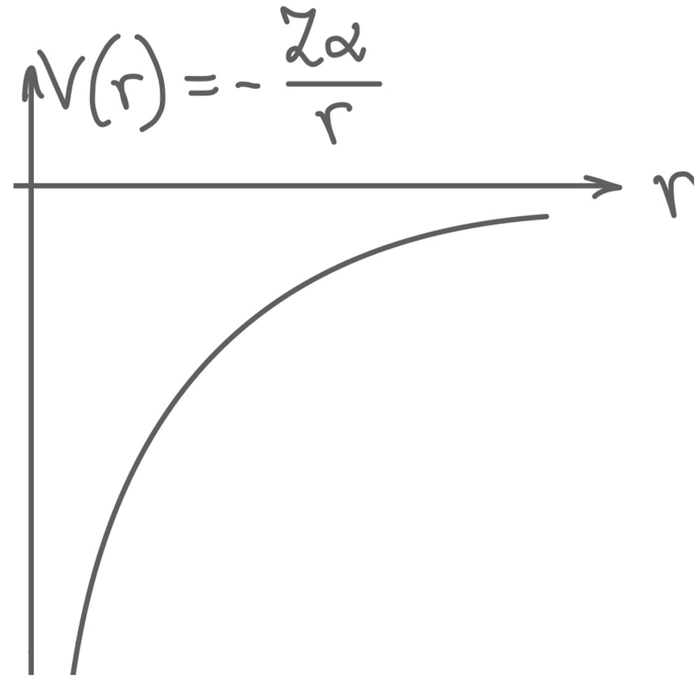


~ 1 GHz

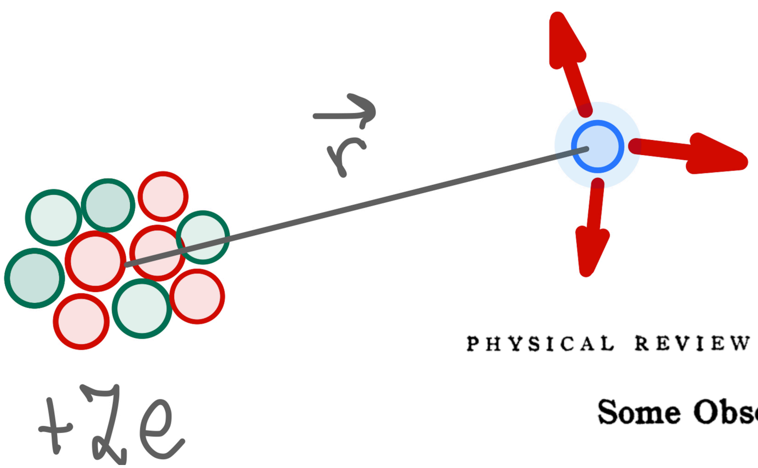
Note: we shall see that only S-states are affected.
They get **less strongly** bound.

Qualitative explanation of radiative corrections: Lamb shift





Effect of vacuum fluctuations:



Electron moved in the potential,
binding energy changed.

PHYSICAL REVIEW VOLUME 74, NUMBER 9 NOVEMBER 1, 1948

Some Observable Effects of the Quantum-Mechanical Fluctuations of the Electromagnetic Field

THEODORE A. WELTON*

Interpretation of the Lamb shift (Welton)

Vacuum energy in one mode $E_k \exp ikr$

$$\boxed{\frac{\hbar\omega}{2} = 2 \cdot V \cdot \frac{\epsilon_0}{2} E_k^2}$$

Change of the potential energy due to fluctuations

$$\delta U = \langle U_c(\boldsymbol{r} + \boldsymbol{q}) \rangle - U_c(r) \simeq \frac{1}{6} \langle q^2 \rangle \nabla^2 U_c = \boxed{\frac{1}{6} \langle q^2 \rangle \alpha \hbar c 4\pi \delta^3(\boldsymbol{r})}$$

Note: the Dirac delta appears because the Coulomb potential is a "harmonic function"

Equation of motion of the electron, decomposed into modes: $\boxed{q_k = \frac{e}{m\omega^2} E_k}$

Total disturbance, summed over modes: $\langle q^2 \rangle = 2 \int \left(\frac{e}{m\omega^2} \right)^2 \frac{V \mathrm{d}^3 k}{(2\pi)^3} E_k^2$

$$\langle \delta U \rangle_{2S} = \frac{1}{6\pi^2 \hbar} \alpha^5 \ln \frac{1}{\alpha} \cdot mc^2 \sim 1000 \text{ MHz}.$$

Origin of the Dirac delta function

As the electron vibrates under the impact of vacuum fluctuations,
it probes regions with a stronger and with a weaker potential.

The total change of its potential energy ~ difference between
the average potential in the neighborhood and in the actual electron position.

This difference in turn ~ Laplacian of the potential.

But the Laplacian vanishes except if there is a charge density.

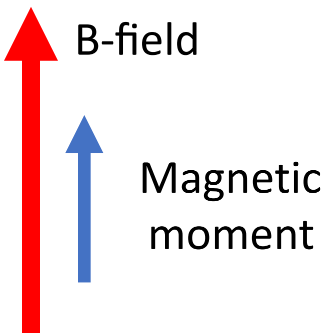
If the nucleus' size is neglected: charge density ~ Dirac delta!

How do fluctuations modify the g-factor?

We have found the change of the electron-nucleus interaction
due to vacuum fluctuations.

How is the electron's spin interaction
with an external B-field modified?

Easy to see that the sign will be wrong: g will be smaller, not larger than 2:



Fluctuation disturb the spin direction
away from the best. Decrease magnitude of H,
and thus of g.

$$H = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

$$\boldsymbol{\mu} = g \frac{q}{2m} \boldsymbol{s}$$

Similarly in the Lamb shift:
fluctuations decrease the binding energy,
lift the S-state.

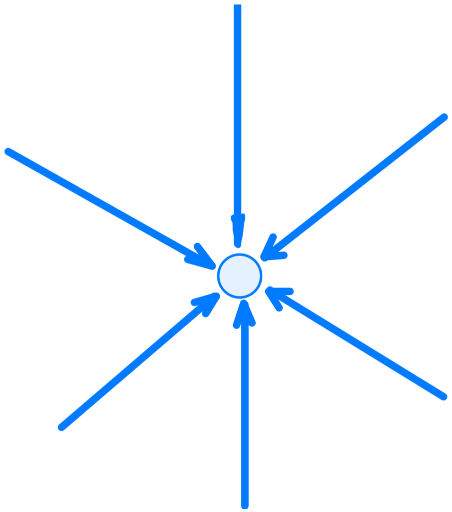
How can g be increased? By increasing (renormalizing) the mass m!

Mass renormalization: inertia of the E-field

$$\boldsymbol{\mu} = g\frac{q}{2m}\boldsymbol{s} \longrightarrow g = \frac{2m}{q}\frac{\|\boldsymbol{\mu}\|}{\|\boldsymbol{J}\|}$$

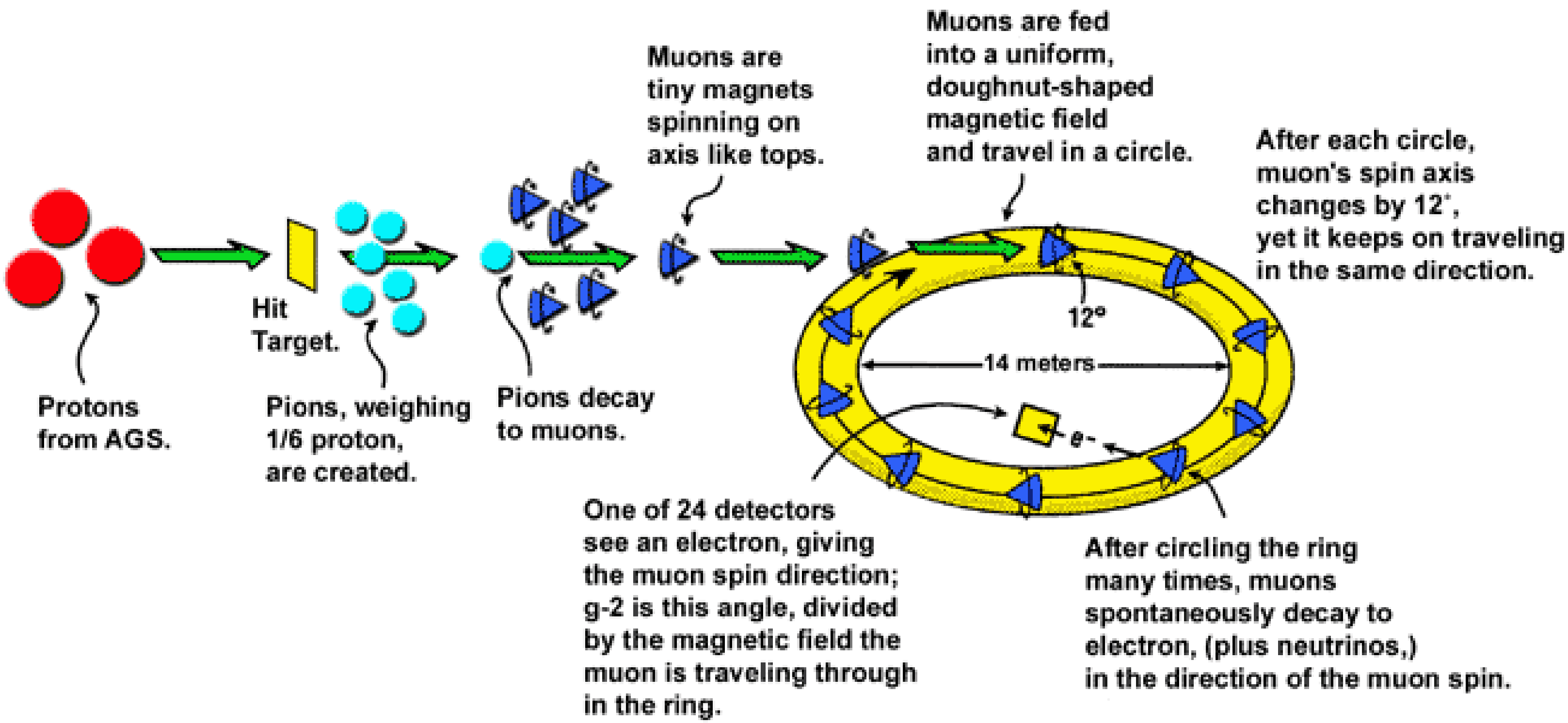
$$g \longrightarrow g\left(1 + \frac{\delta m}{m}\right)$$

g increases if the mass change is positive; it is!



$$\delta m = \frac{\epsilon_0}{2c^2}\int \mathrm{d}\boldsymbol{r}\,\boldsymbol{E}_e^2 > 0$$

Principle of the g-2 experiment



The idea pioneered in CERN ~ 50 years ago,
continued in Brookhaven, moved to Fermilab

Move to Fermilab

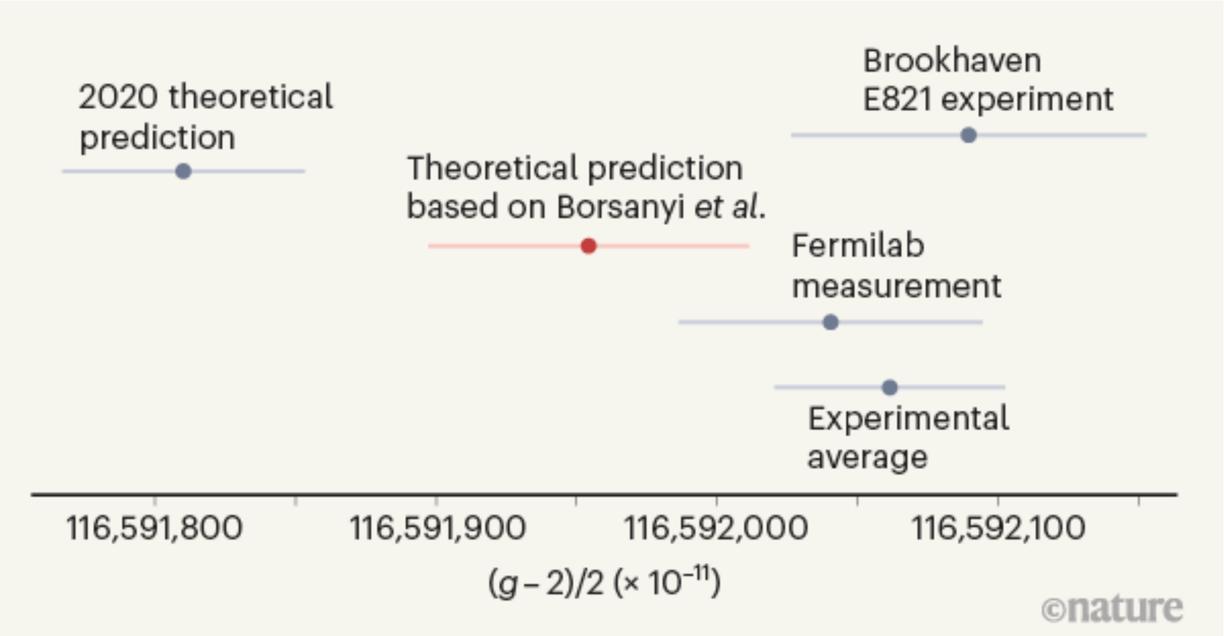
Experimental result

a(FNAL) = 116 592 040(54) × 10^{-11} (0.46 ppm)   (2021)

Together with 2006 Brookhaven,

a(FNAL+BNL) = 116 592 061(41) × 10^{-11} (0.35 ppm)

This exceeds theory by 4.2 sigma



Note: one lattice collaboration reports (2021) better agreement with their hadronic result.

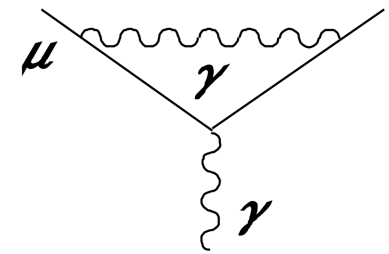
Anomalous magnetic moment in the Standard Model

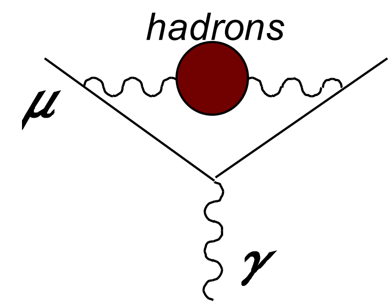
Muon *g-2*: Standard Model

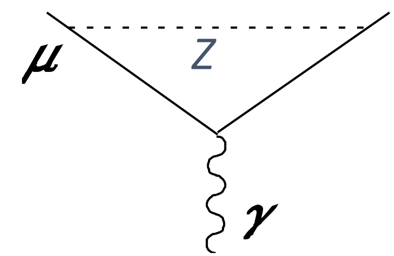
Units: 10^{-11}

QED		116 584 719	(1)
Hadronic			
	LO	6 931	(40)
	NLO,NNLO	– 86	(1)
	LBL	92	(20)
Electroweak		154	(1)
Total SM		116 591 810	(43)

Experiment – SM Theory = 251 (59) (4.2σ deviation)



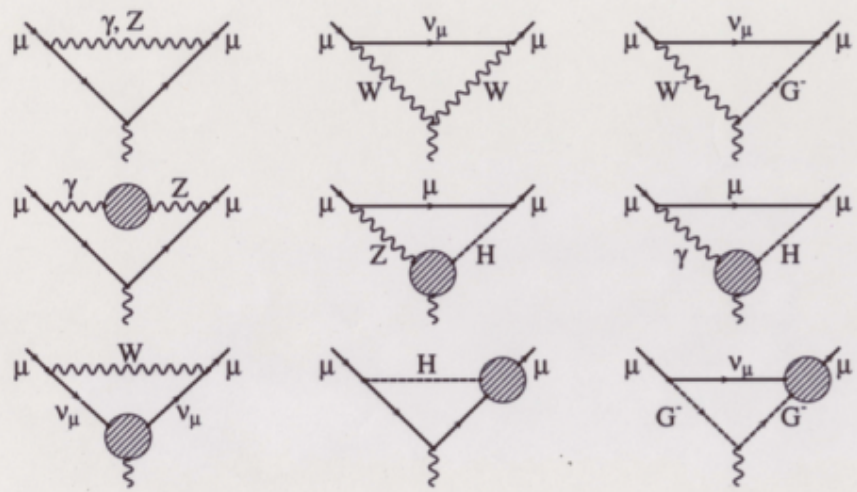




+SuSy
+Z'
+leptoquarks?
+SuSy?

Large, universal two-loop electroweak correction

BOSONIC LOOPS: ~ 1600 DIAGRAMS



Logs dominate: $\dfrac{\Delta a_\mu^{\rm bos}(\text{2-loop})}{a_\mu^{\rm EW}(\text{1-loop})} \approx -\dfrac{107}{30}\dfrac{\alpha}{\pi}\ln\dfrac{M_W^2}{m_\mu^2}$ → −23% reduction

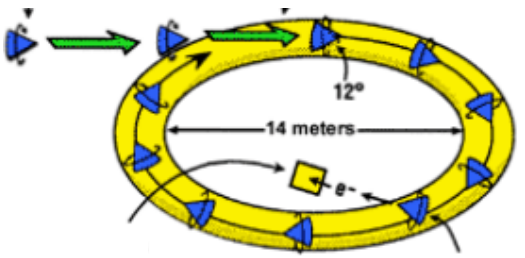
(A.C., B. Krause, and W. Marciano, Phys. Rev. Lett. 76 (1996) 3267)

First "full" two-loop EW calculation.

A record QED
correction!
Also affects
mu -> e gamma

AC and Jankowski, hep-ph/0106237, PRD65 (2002) 113004

Other dipole moments



$$a_\mu^{\rm NP} = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 251 \cdot 10^{-11}$$

This is rather large when compared with other bounds on New Physics:

Muon MDM $$d_\mu \sim \frac{e}{2m_\mu} a_\mu^{\rm NP} \sim 3 \cdot 10^{-22}\, e \cdot {\rm cm}$$

Muon-electron transition moment $$|d_{\mu\to e}| < 4 \cdot 10^{-27}\, e \cdot {\rm cm}$$ MEG 2013

Electron EDM $$|d_e| < 1.1 \cdot 10^{-29} e \cdot {\rm cm}$$ ACME 2018

How can g_μ-2 be checked?

More data expected from Fermilab; current 0.46 ppm → 0.14 ppm

New experimental concept at J-PARC → 0.07 ppm?

g_e-2, other muonic observables?

New approach to g_μ-2 at J-PARC

Slower muons 300 MeV (instead of the "magic" 3.1 GeV)

Ultracold muons; no electric focusing!

Smaller ring r = 33 cm (instead of 7 m)

$$r\,[\text{in meters}] \simeq \frac{\gamma}{3B\,[\text{in Tesla}]}$$

Strong, very precisely controlled magnetic field.

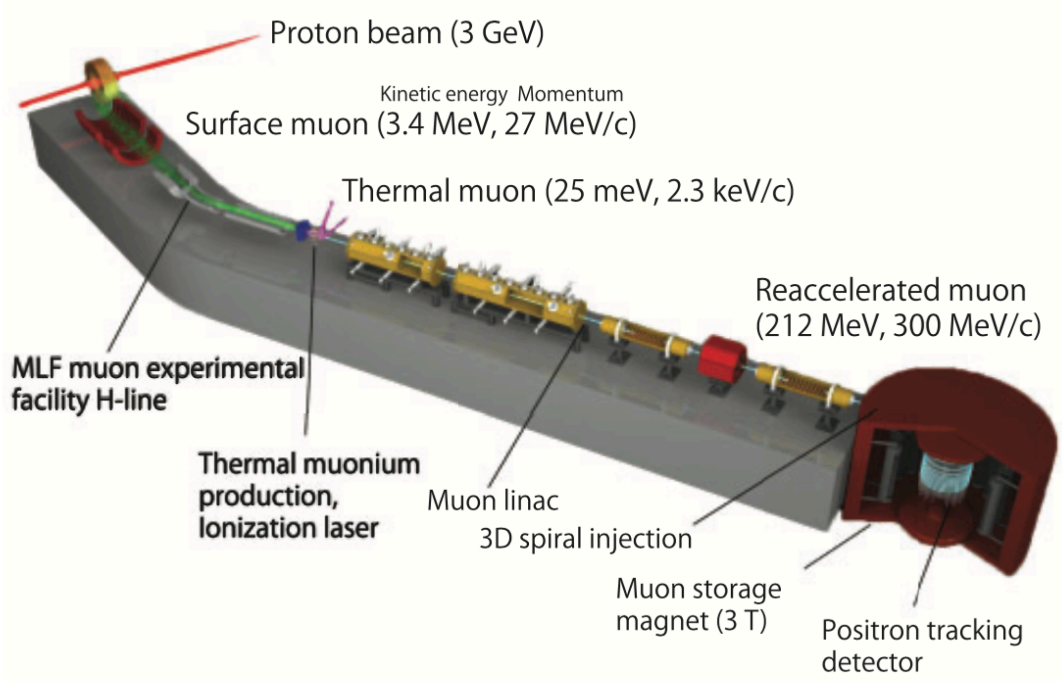
~ 10 times more muons than at Fermilab (compensates shorter lifetime).

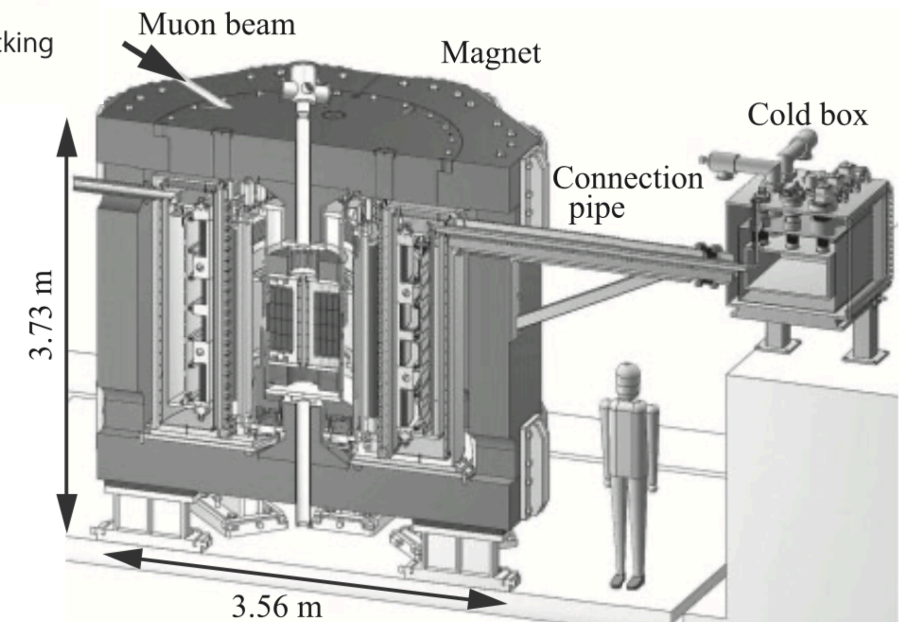
	Brookhaven	Fermilab	J-PARC
Muon momentum	3.09 GeV/c		0.3 GeV/c
gamma	29.3		3
Storage field	B=1.45 T		3.0 T
Focusing field	Electric quad		None
# of detected μ+ decays	5.0E9	1.8E11	1.5E12
# of detected μ- decays	3.6E9	-	-
Precision (stat)	0.46 ppm	0.1 ppm	0.1 ppm

$$\simeq \sqrt{\frac{2\pi}{\alpha}}$$

From N. Saito

g-2 in Japan (2025?)





Magnetic moment of the electron

$$a_e = \frac{g_e - 2}{2}$$

Measured with relative error $25 \cdot 10^{-11}$

Phys. Rev. Lett. 100, 120801 (2008)

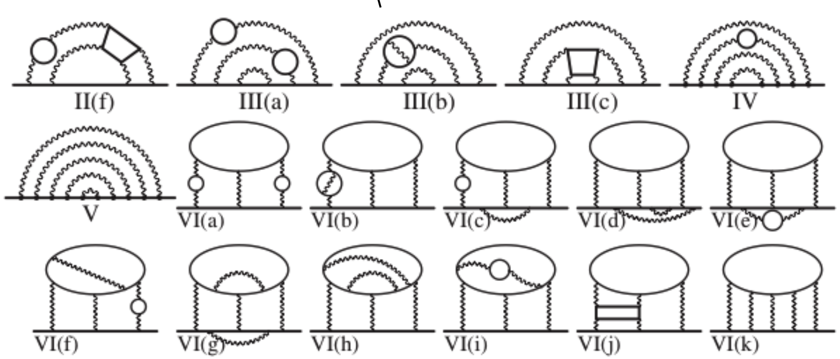
Provides the fine structure constant with the same precision,

$$\alpha^{-1}(a_e) = 137.035\,999\,1736\,(331)\,(86)$$

Phys. Rev. Lett. 109, 111807 (2012)

Experimental error dominates (for now)

Numerical errors from 4- and 5-loop diagrams



How to use g_e-2 to check g_μ-2?

If the muon anomaly is due to New Physics, the expected effect for the electron is likely smaller by $\frac{m_e^2}{m_\mu^2} \sim \frac{1}{43000}$

$$\Delta a_\mu \sim 250 \cdot 10^{-11} \Rightarrow \Delta a_e \sim 7 \cdot 10^{-14}$$

This means relative uncertainty $\frac{\Delta a_e}{a_e} \sim 7 \cdot 10^{-11}$

and requires a factor of 4 improvement of the latest measurement.

In addition, an independent determination of the fine structure constant is needed, with matching precision.

Muon vs electron: comments

Precision achieved in the studies of magnetic dipole moments

$$\begin{aligned}\Delta\left(a_e^{\rm SM}-a_e^{\rm exp}\right) &\simeq 10^{-12}\\ \Delta\left(a_\mu^{\rm SM}-a_\mu^{\rm exp}\right) &\simeq 10^{-9}\end{aligned}$$

Sensitivity to new physics scales (in general) like the lepton mass squared,

$$a_f^{\rm NP} \sim \frac{m_f^2}{\Lambda^2}$$

So muon is a more sensitive probe but the electron is becoming relevant,

$$\frac{\Lambda_\mu}{\Lambda_e} \sim \frac{m_\mu}{m_e}\sqrt{\frac{\Delta a_e}{\Delta a_\mu}} \sim 6$$

How to use g_e-2 to check g_μ-2?

Nature 442, 516 (2006)
PRA 89, 052118 (2014)

The second best determination of alpha:
from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Needed precision:

$$\alpha^2 = \frac{R_\infty}{2} \cdot \frac{u}{m_e} \cdot \frac{M_X}{u} \cdot \frac{h}{M_X}$$

$14 \cdot 10^{-11}$

$7 \cdot 10^{-12}$

$8 \cdot 10^{-11}$

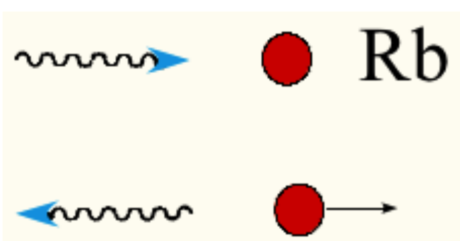
Nature 2014
Sturm et al.

$12 \cdot 10^{-11}$

for Rb
(better for He)

$124 \cdot 10^{-11}$

improvement
needed by
factor ~10

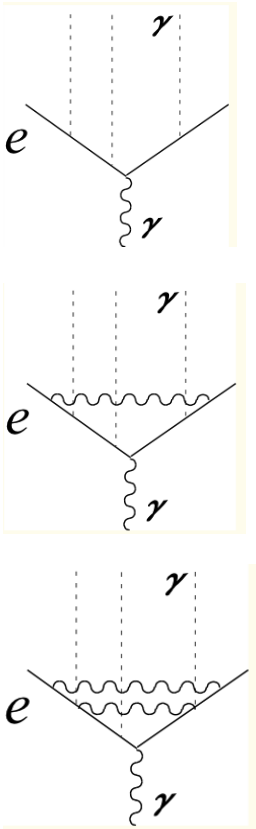


gives h/m

$$1/\alpha = 137.035\,999\,206(11) \quad [8.1 \cdot 10^{-11}]$$

Mission accomplished!
Morel, Yao, Cladé, Guellati-Khélifa
2020

Bound-electron g-2: theory needed for u/m_e



$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots$$
$$+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \ldots\right]$$
$$+\left(\frac{\alpha}{\pi}\right)^2 \underbrace{\left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \ldots\right]}_{\text{two-loop corrections}}$$

$$b_{41} = \frac{28}{9}$$
$$b_{40} = -16.4$$

Pachucki,
AC
Jentschura,
Yerokhin

Lepton flavor violation

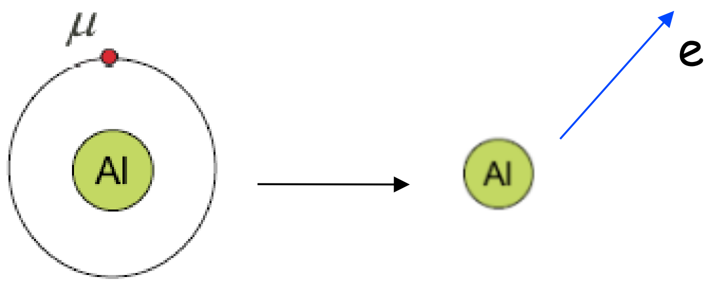
and the muon decay in orbit

Muon-electron conversion: probes various types of interactions

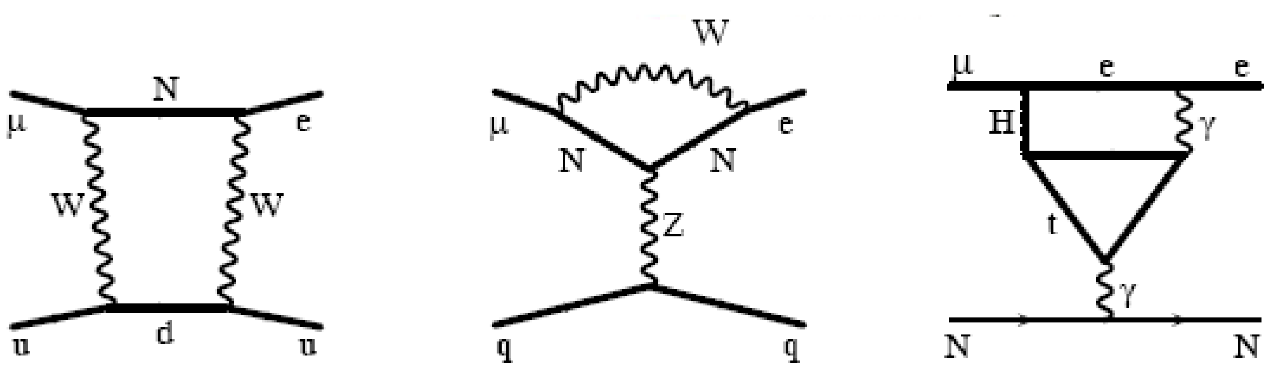
Non-dipole interactions are not (directly) probed by processes with external photons, by gauge invariance requirements.

New process: muon-electron conversion

(as well as mu --> eee)

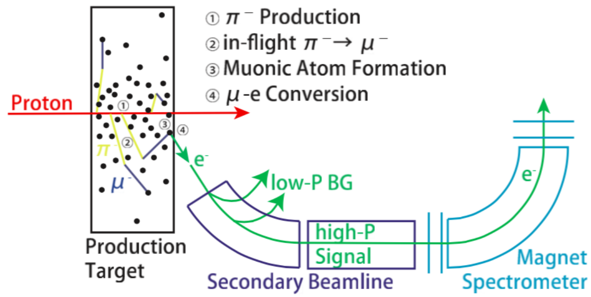


Variety of mechanisms:



Muon-electron conversion plans (The Next Big Thing)

DeeMe
J-PARC

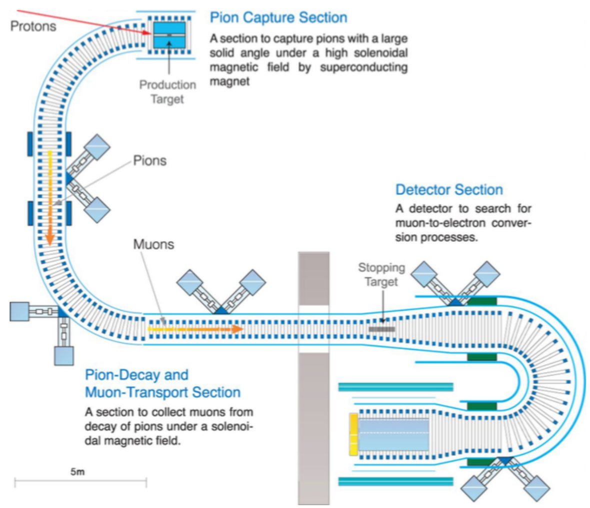


starts 2016;
aims for 1e-13 (graphite target),
followed by 1e-14 (SiC target)

COMET
Phase 1
J-PARC

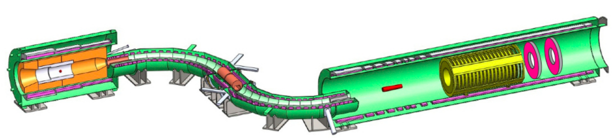
7e-15

COMET
Phase 2
J-PARC



2.6e-17

Mu2e
Fermilab



2e-17

Summary

Discrepancy in the muon g-2 at present 4.2 sigma.

Probes energy scales … - 100 MeV – 100 GeV - …

At least a factor of 3 improvement expected on the experimental side

An independent experiment in Japan will be the ultimate test
(the first new experimental concept in 50 years)

Several other experiments with muons:
- muon-electron conversion (Fermilab, J-Parc)
- rare muon decays (PSI)
- hadronic decays into muons (LHCb Run 3, Belle II)