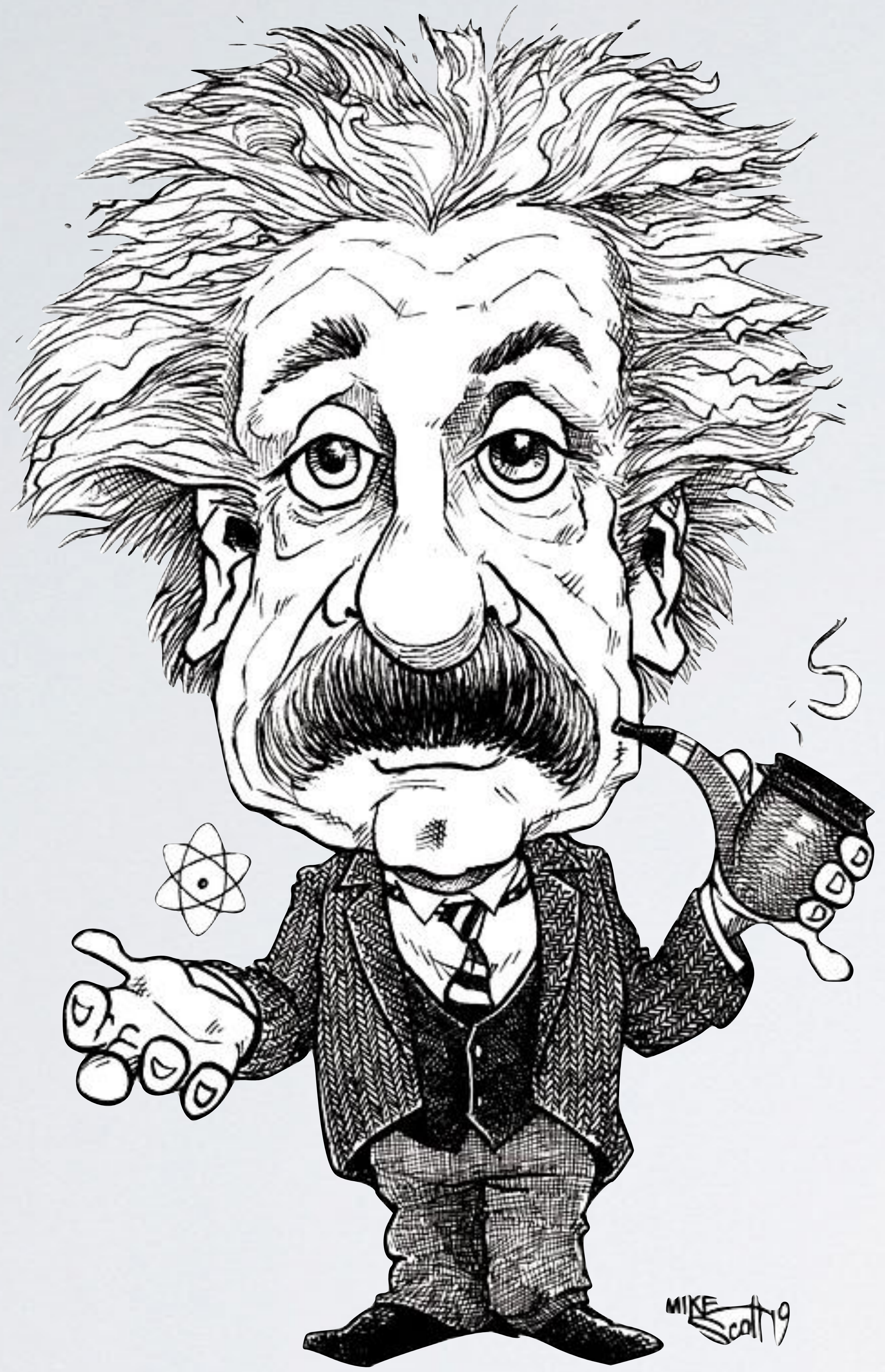
Who's troubled by Bell inequalities, or what is the weight of locality and free choice?

Paweł Błasiak

Institute of Nuclear Physics Polish Academy of Sciences, Kraków, Poland

City, University of London, United Kingdom

IFJ PAN Seminar, 9 Jun 2022

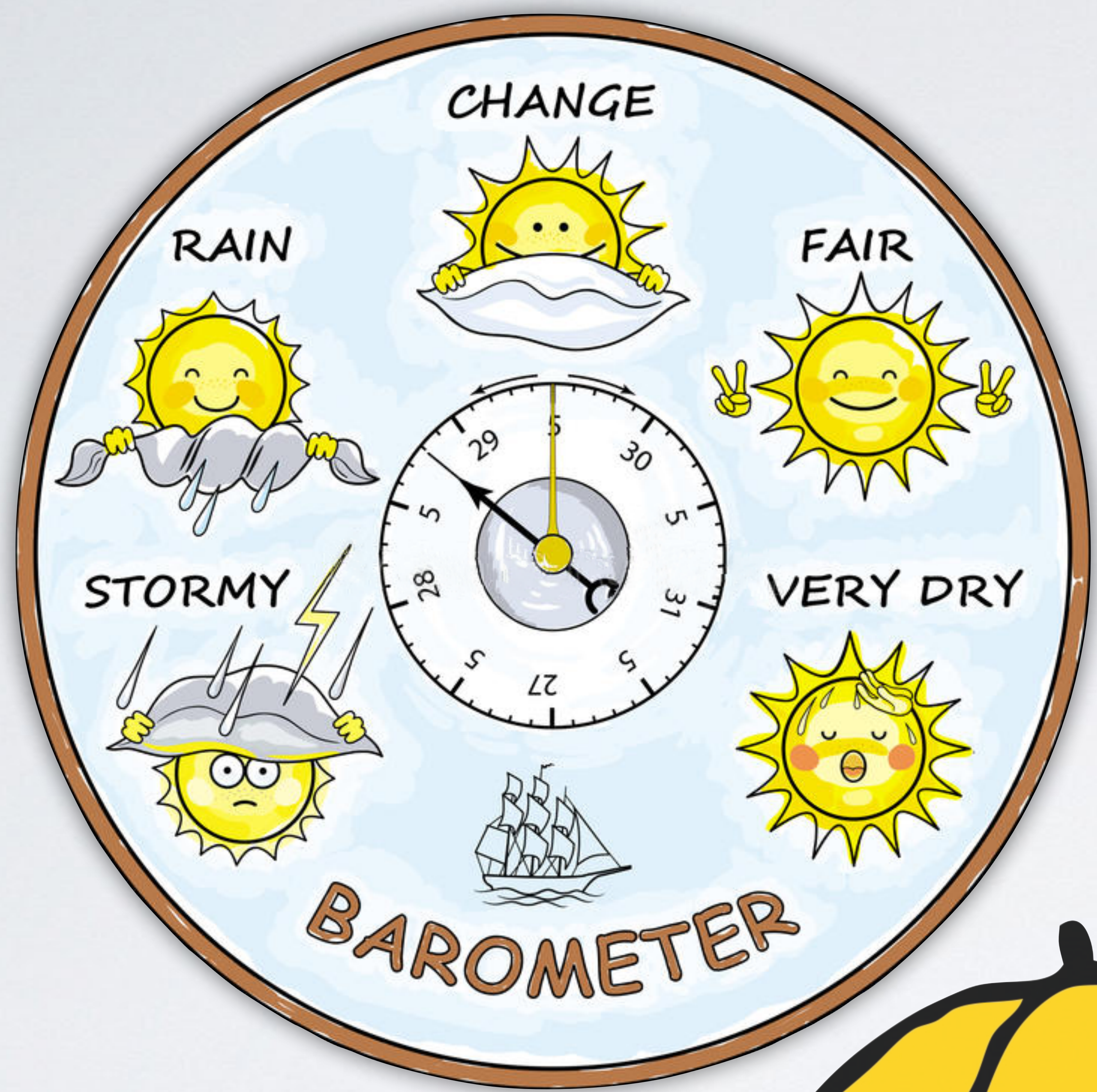


Albert EINSTEIN
(1879 - 1955)

*"Development of Western science is based on two great achievements: the invention of the **formal logical system** (in Euclidean geometry) by the Greek philosophers, and the discovery of the possibility to find out **causal relationships** by systematic experiment (during the Renaissance)."*

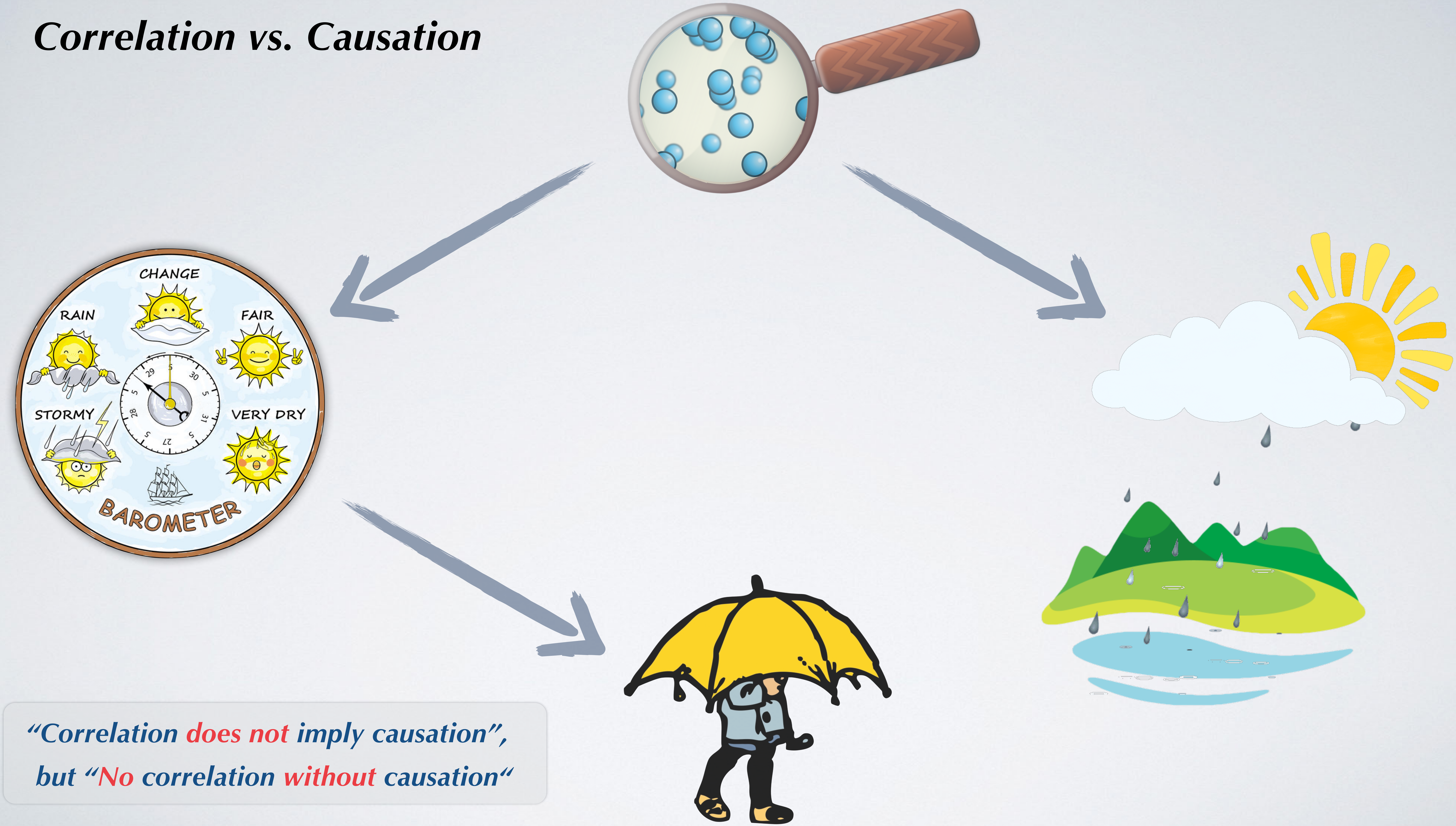
— Albert Einstein (1953)

Correlation vs. Causation



"Correlation does not imply causation",
but "No correlation without causation"

Correlation vs. Causation



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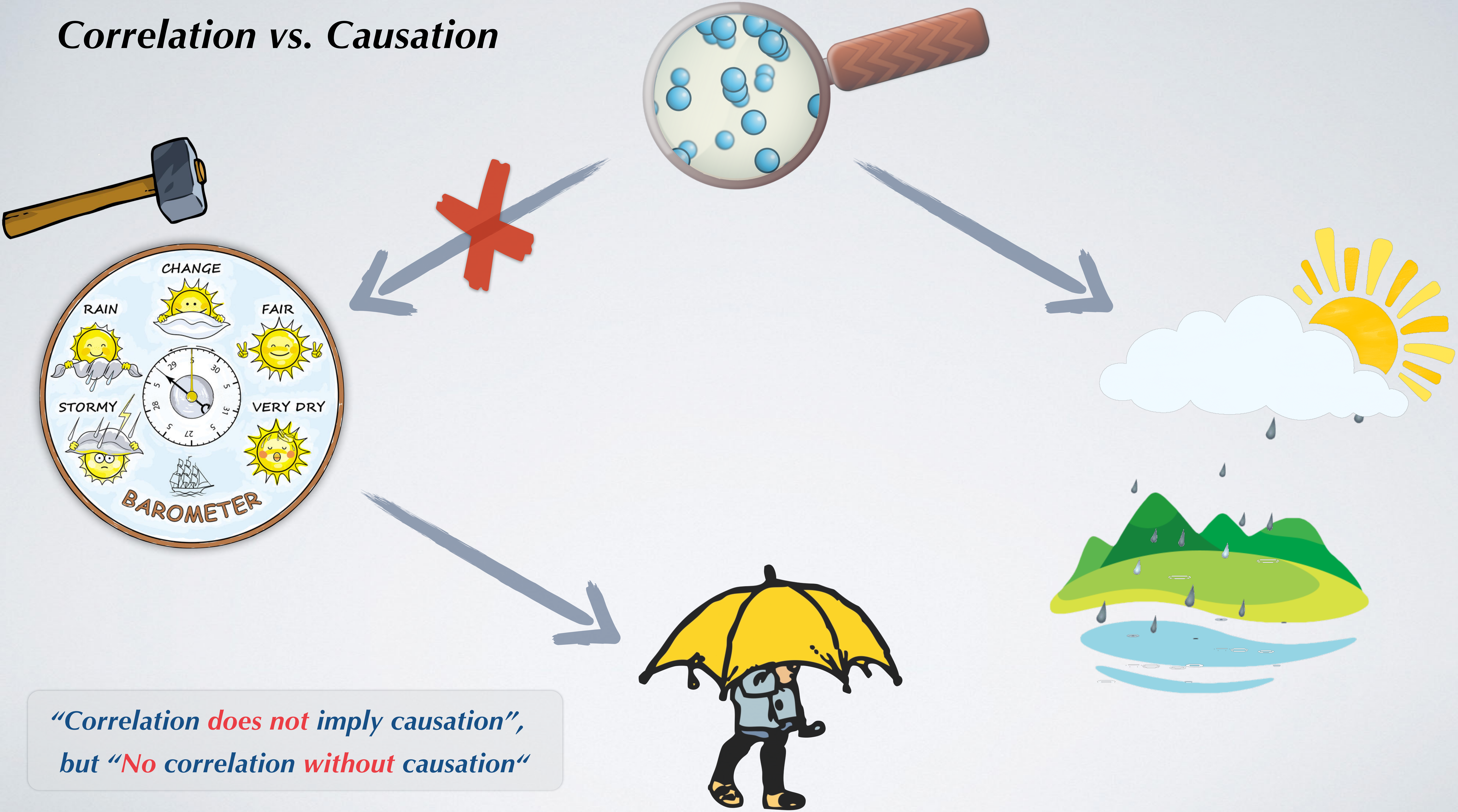

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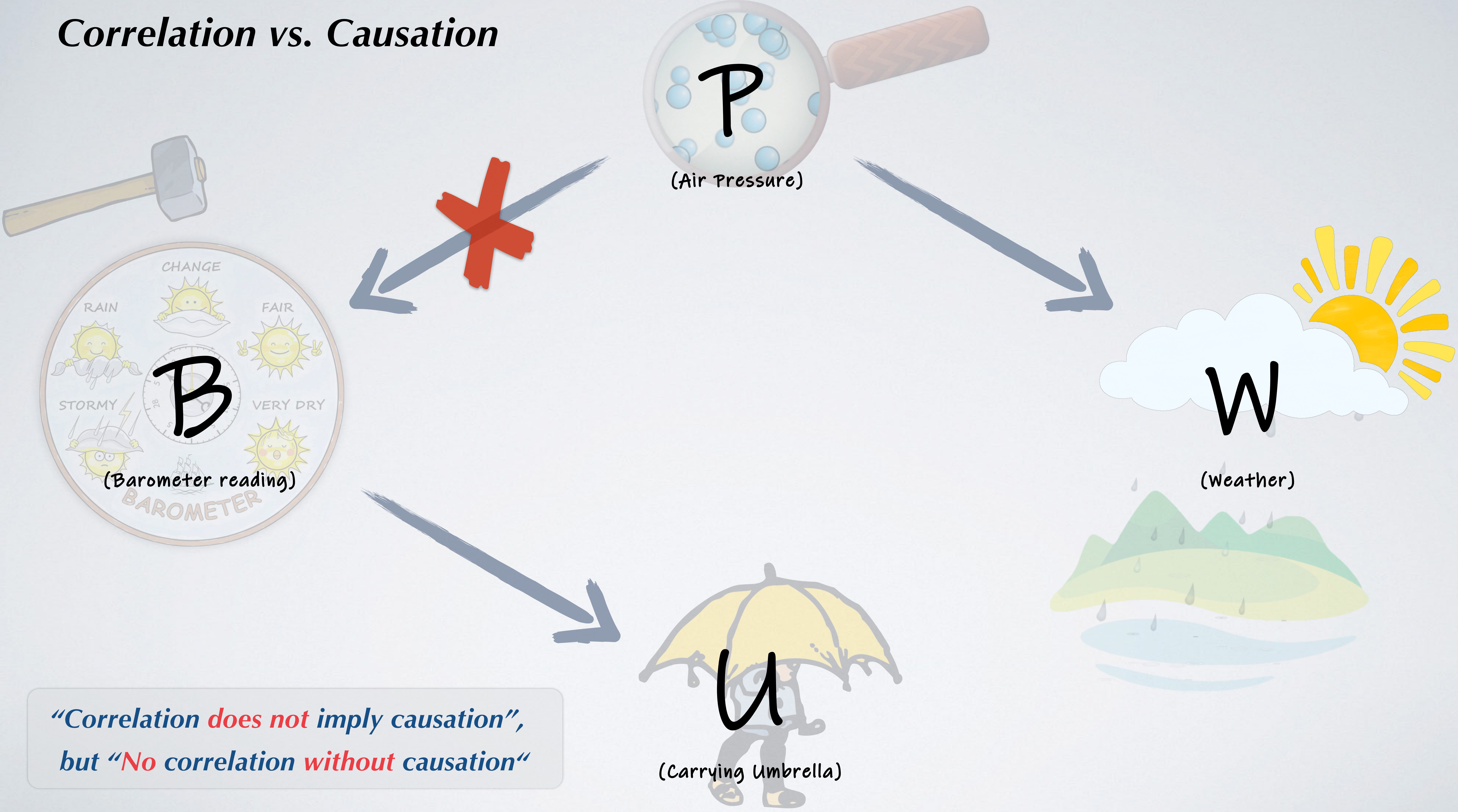

"Correlation does not imply causation",

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Correlation vs. Causation

Statistical correlation

$$P(B|A) \neq P(B)$$

Occurrence of ***A changes*** *the chance that* ***B*** *happens*

Observed from data

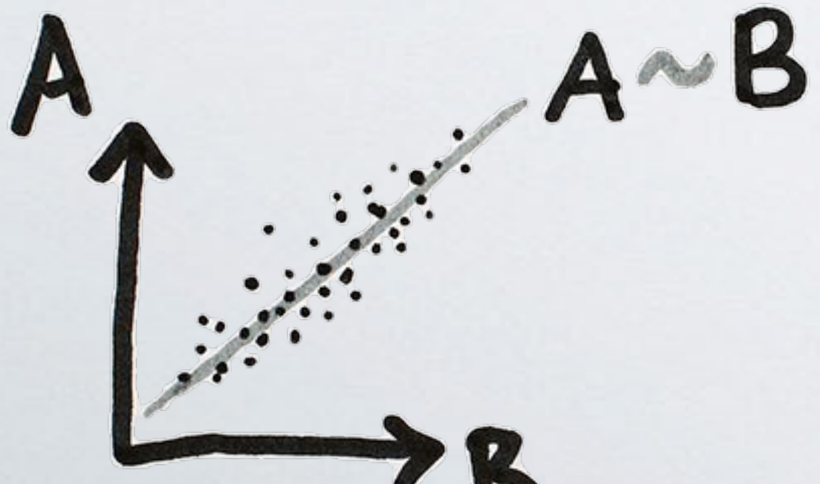
e.g. correlation between symptoms

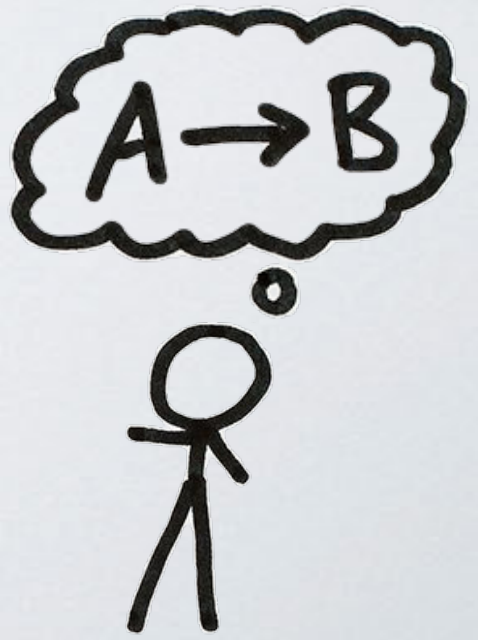
Causal relation

Mechanism in which ***A makes B*** *happen*

Physical model (explanation)

e.g. model of gene propagation



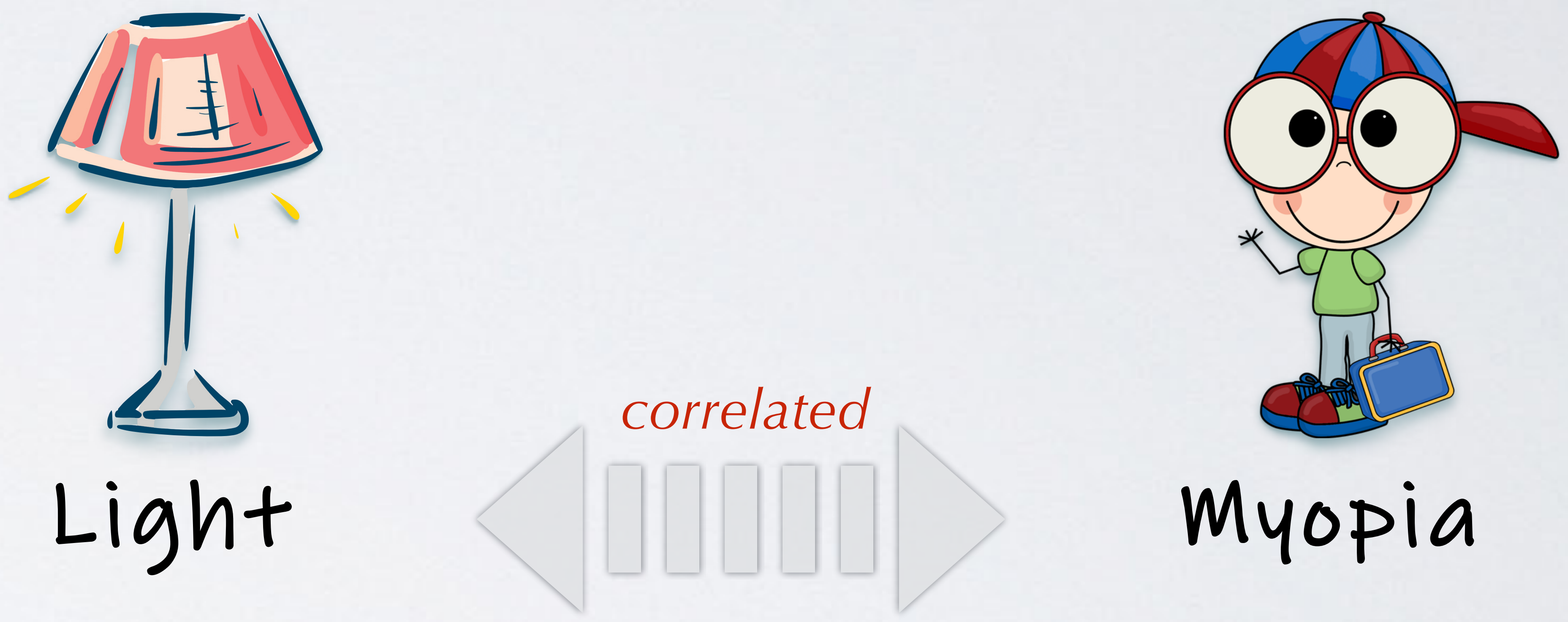


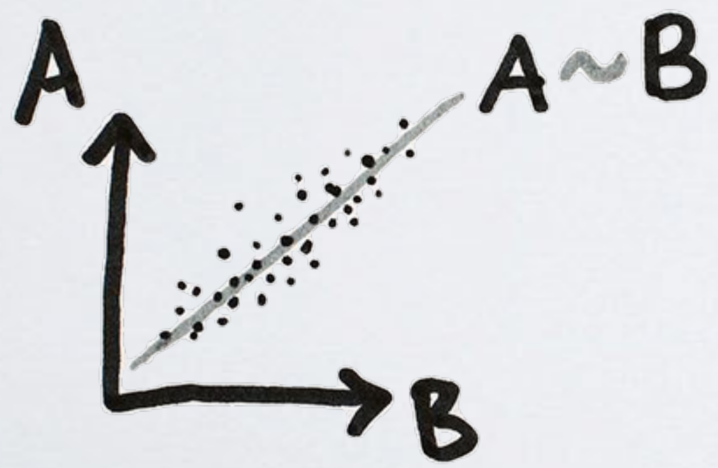
"Correlation does not imply causation",
but "No correlation without causation"

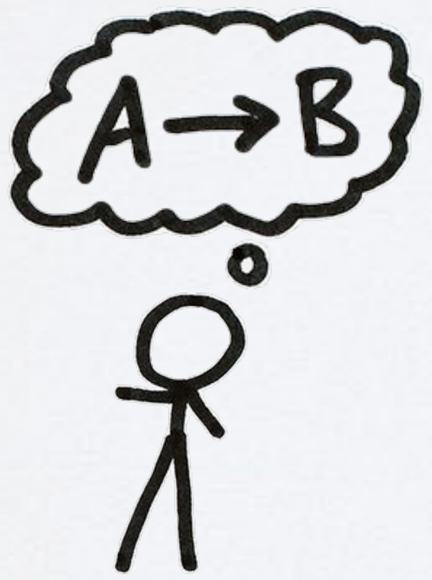
An example

Observation:
Young children who sleep with the ***light on*** *are much* ***more likely*** *to be diagnosed with* ***myopia*** *in later life.*

Conclusion *(?)*:
Does switching off the light at night ***prevent*** *my child from developing myopia in later life?*







Quinn et al. "Myopia and ambient lighting at night", Nature ***399*** *113 (1999)*

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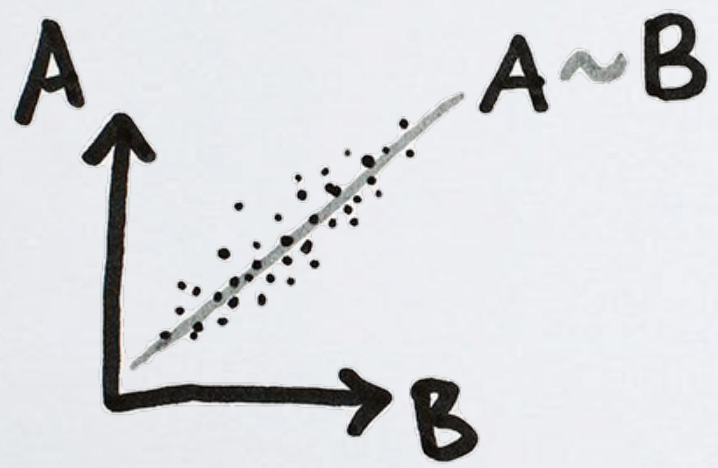
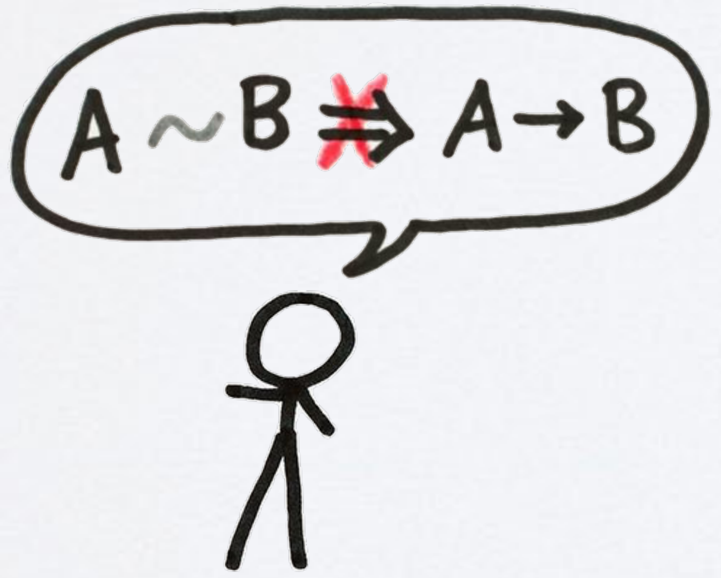
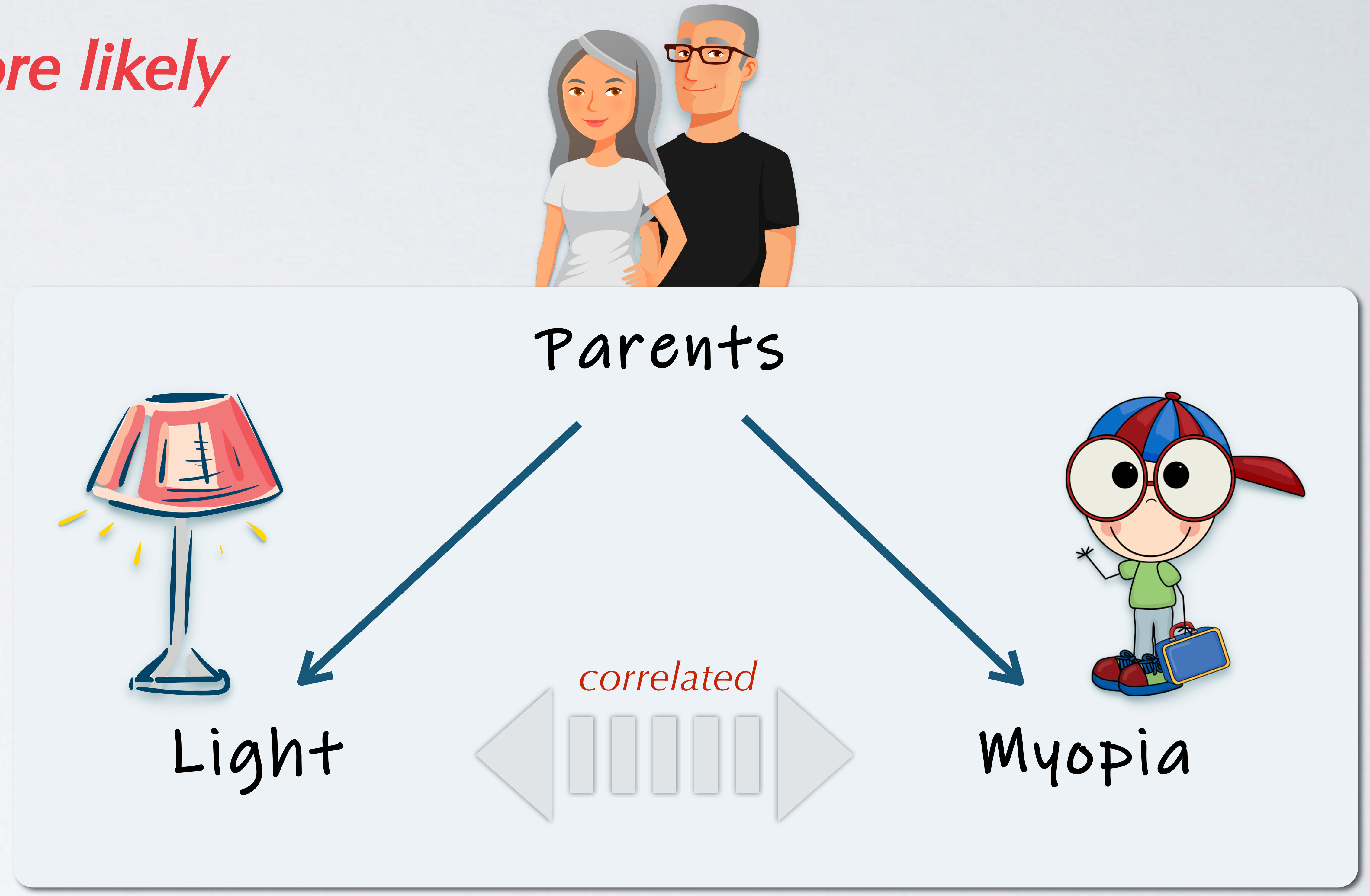






$L \not\perp\!\!\!\perp M$, but $L \perp\!\!\!\perp M \mid P$
correlated — uncorrelated

Confounding bias (common cause principle)

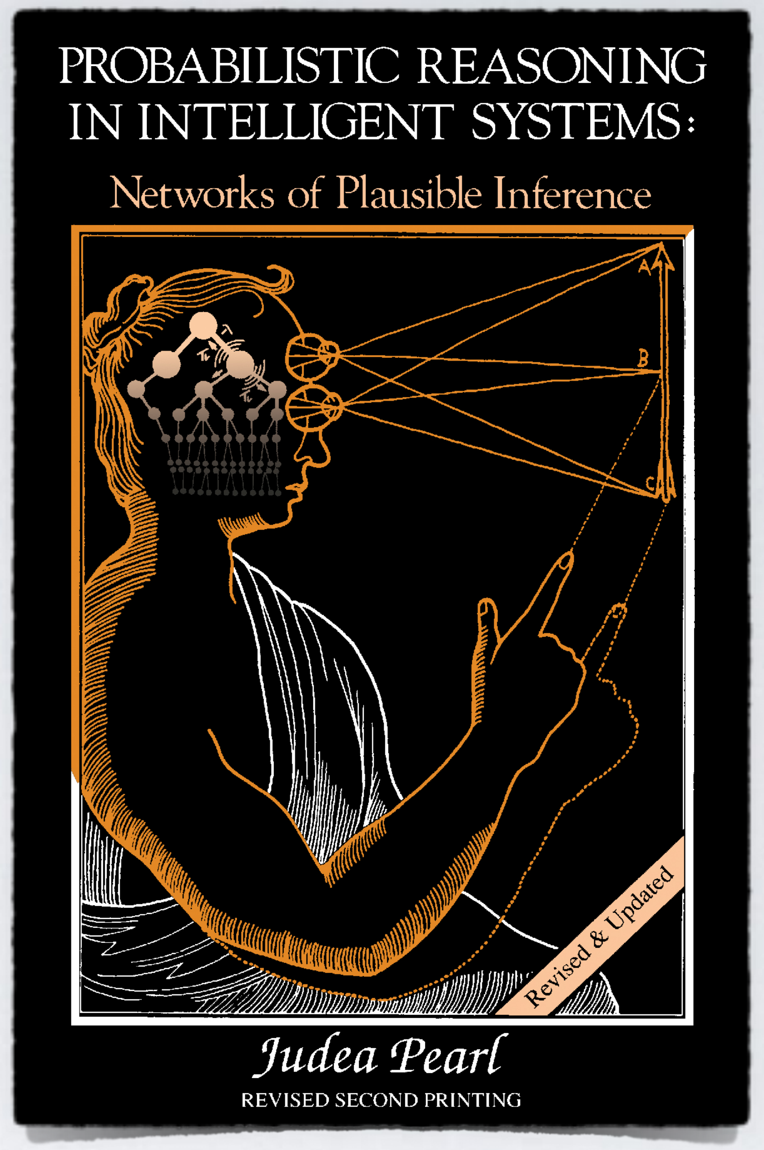
Quinn et al. "Myopia and ambient lighting at night", Nature ***399*** *113 (1999)*

Rigorous approach to causal inference

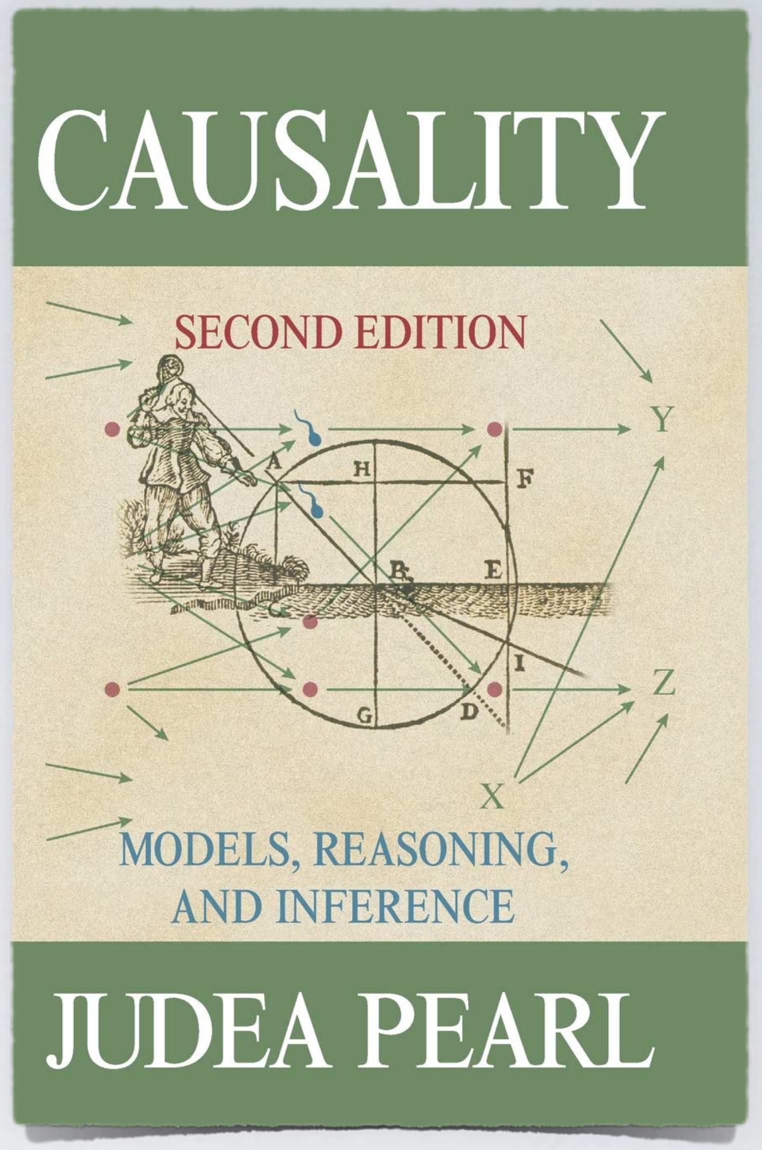
Judea Pearl
(1936)

Turing Award (2011)

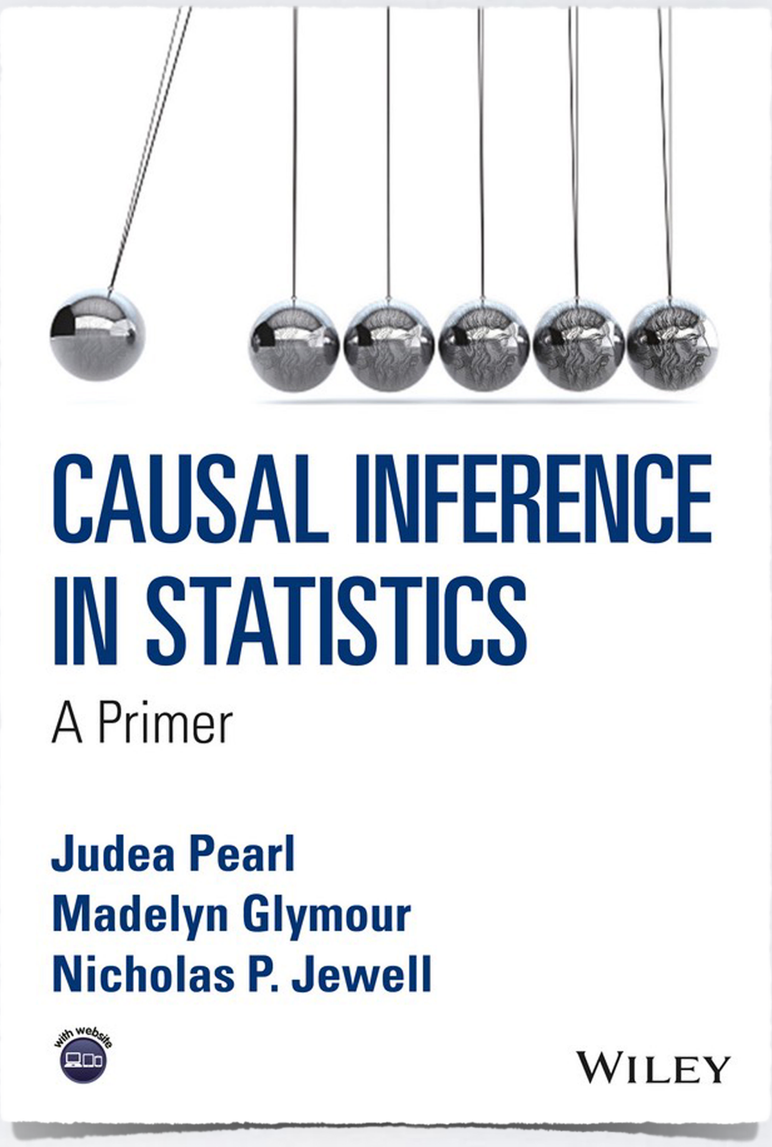
for "fundamental contributions to artificial intelligence through the development of a ***calculus for probabilistic and causal reasoning****"*



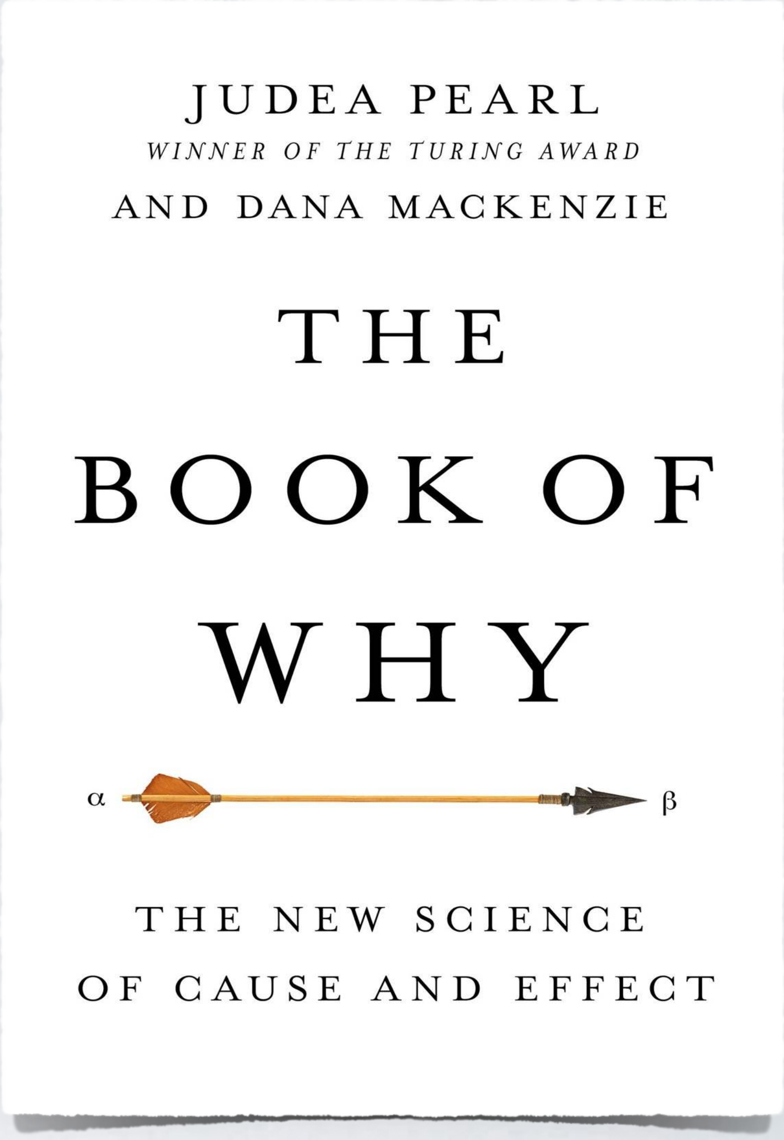
(1988)



(2000, 2009)



(2016)



(2018)

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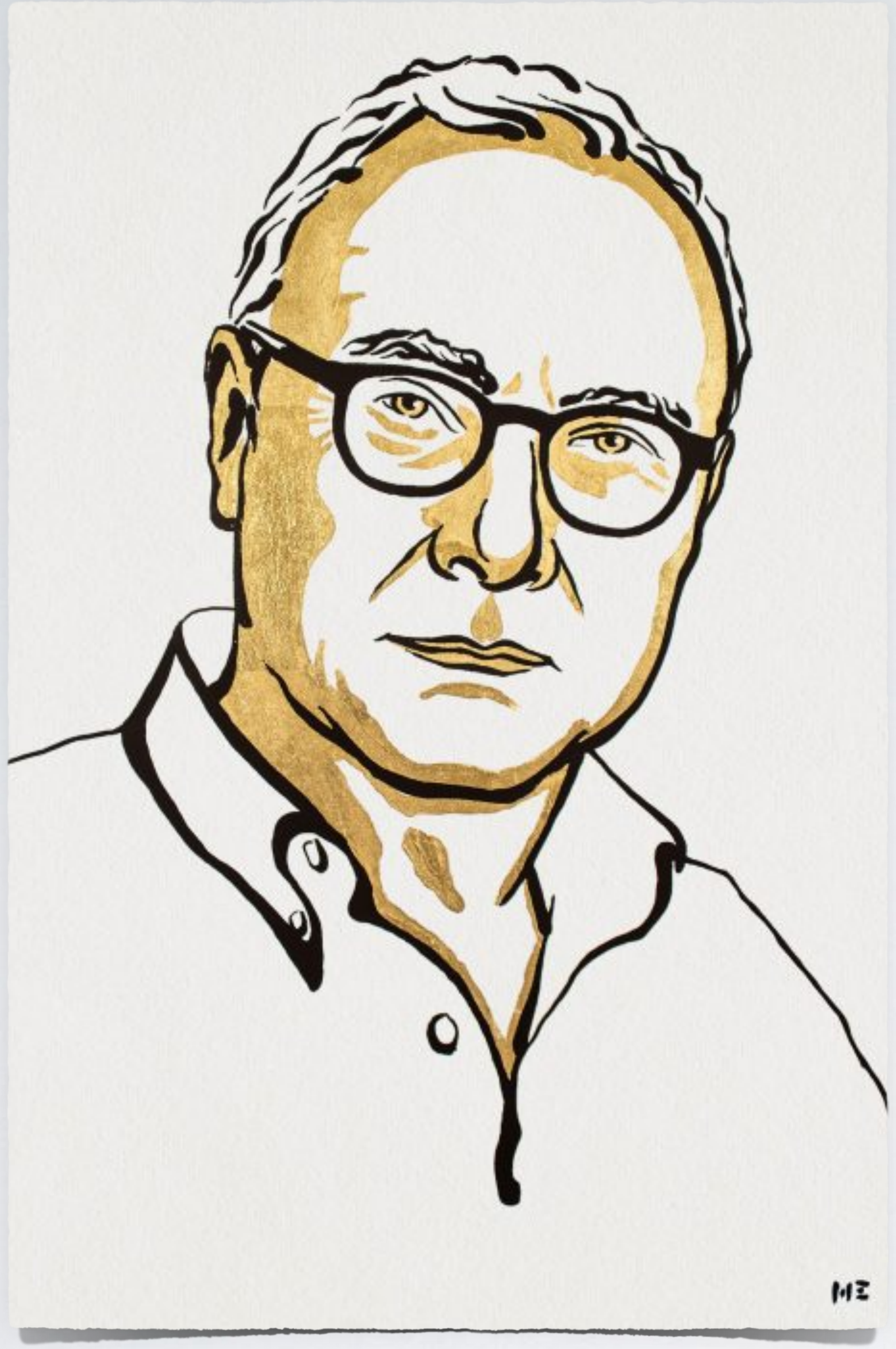
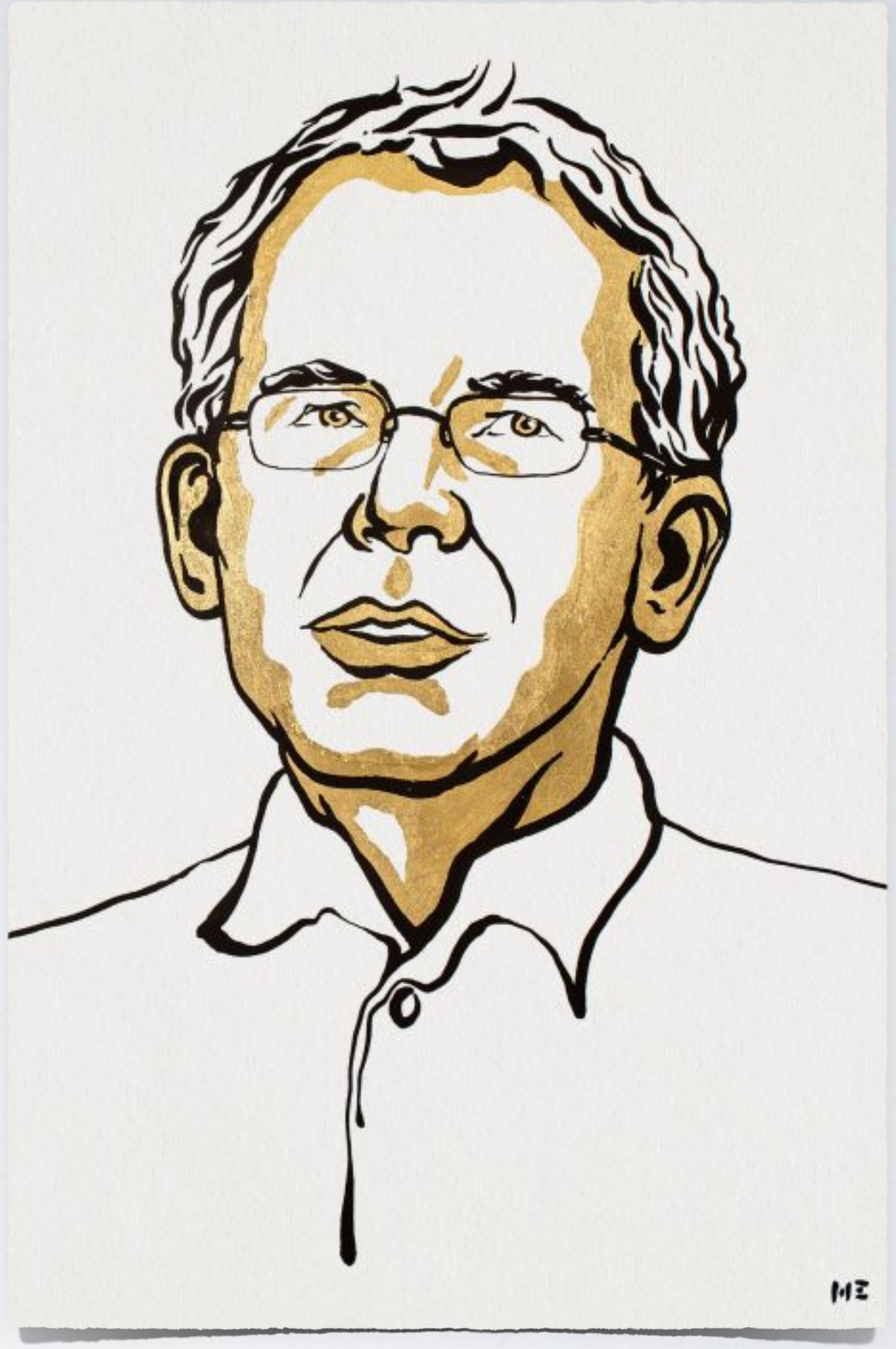
Nobel Prize in Economics 2021



David Card

Joshua D. Angrist

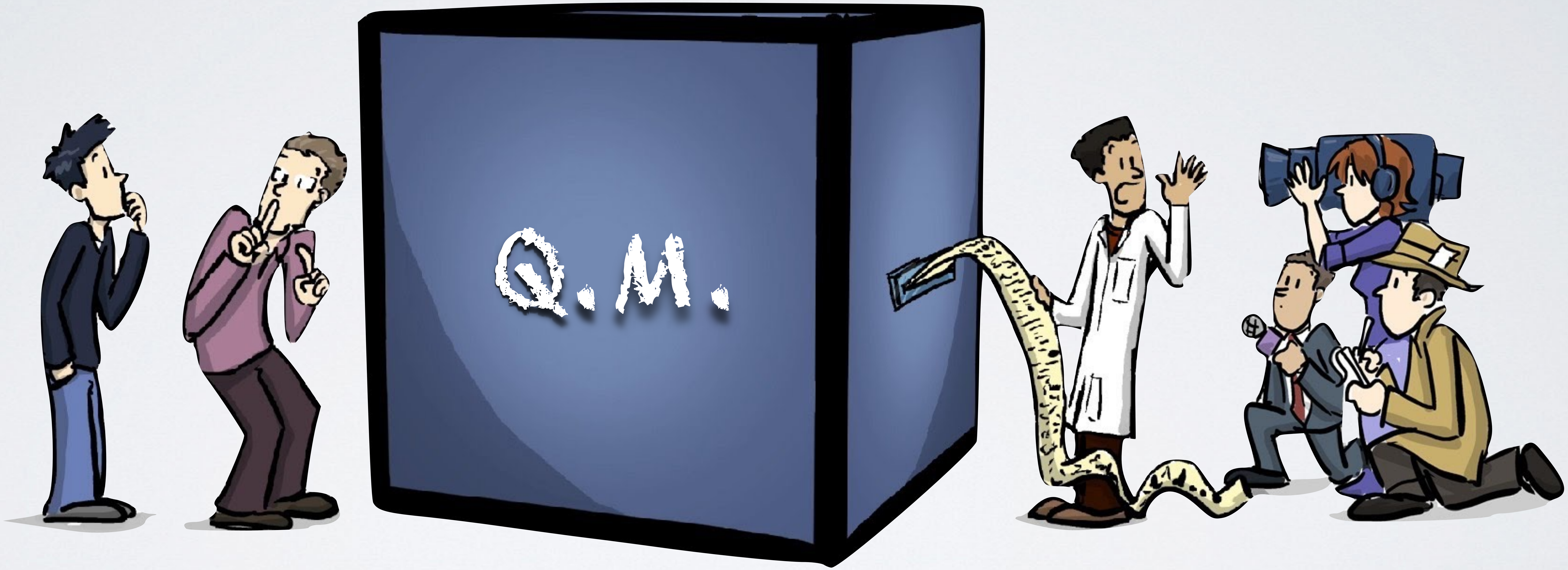


Guido W. Imbens

"for his empirical contributions to labour economics."

"for their methodological contributions to the analysis of ***causal relationships****."*

Quantum mechanics … as we have it

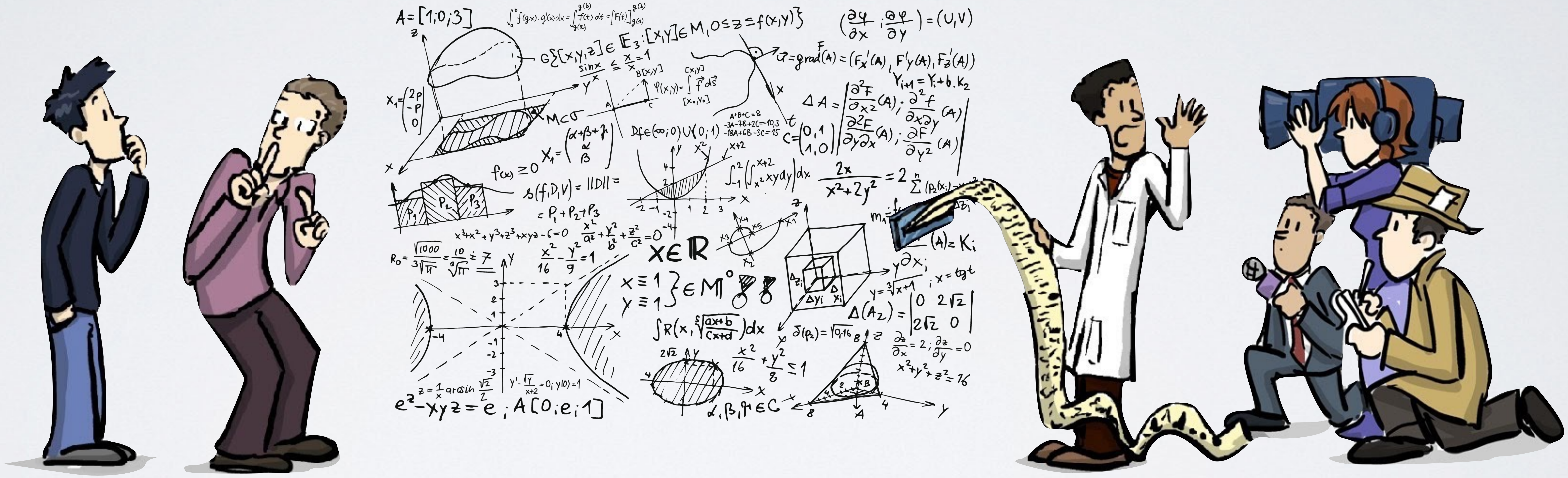


In a strict sense, quantum theory is a ***set of rules*** *allowing the computation of* ***probabilities for the outcomes*** *of tests which follow specified preparations.*

Asher Peres in "Quantum Theory: Concepts and methods" (1995)

Quantum mechanics … as we have it

Mathematical formalism ✔



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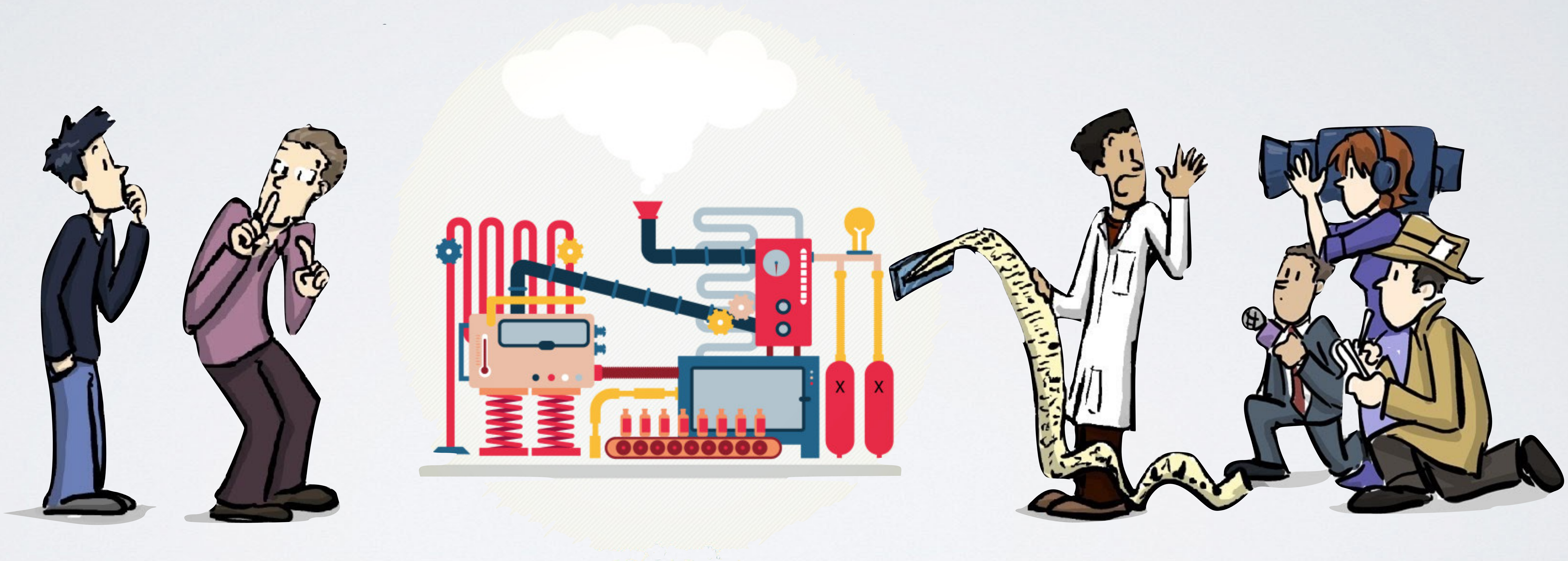
Mathematical formalism ✓

Operational description ✓



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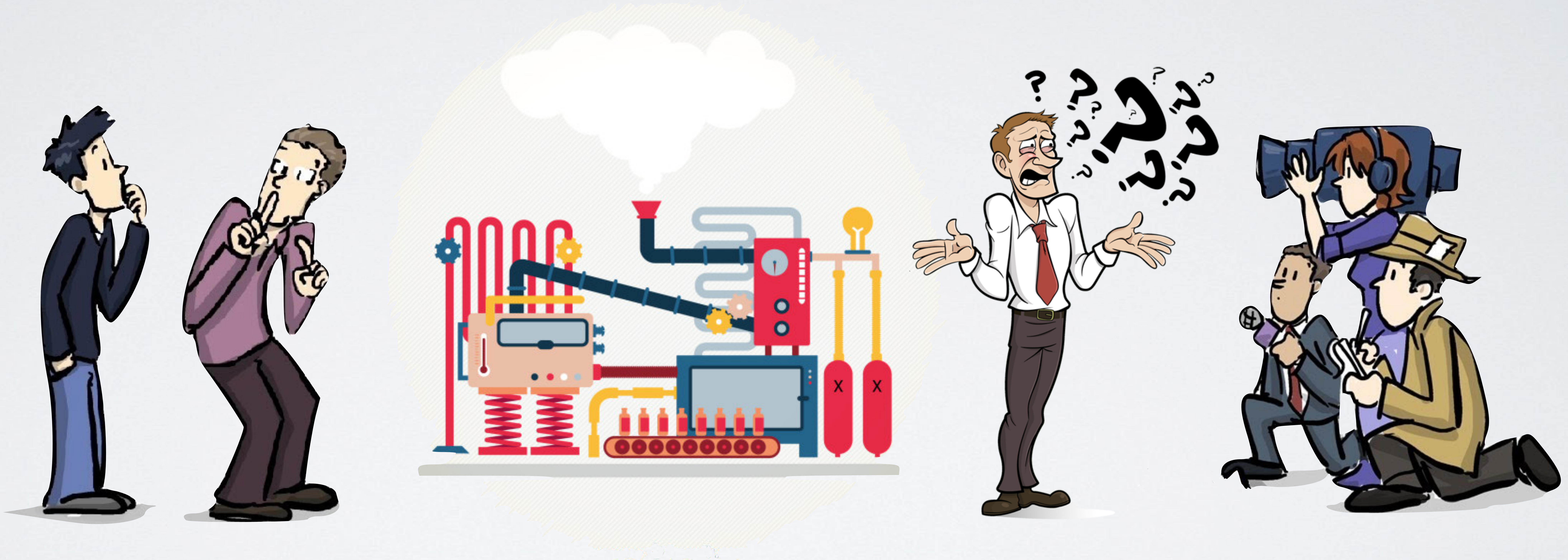
Quantum mechanics … as we have it

Mathematical formalism ✔

Operational description ✔

… but what is the ontology ❓



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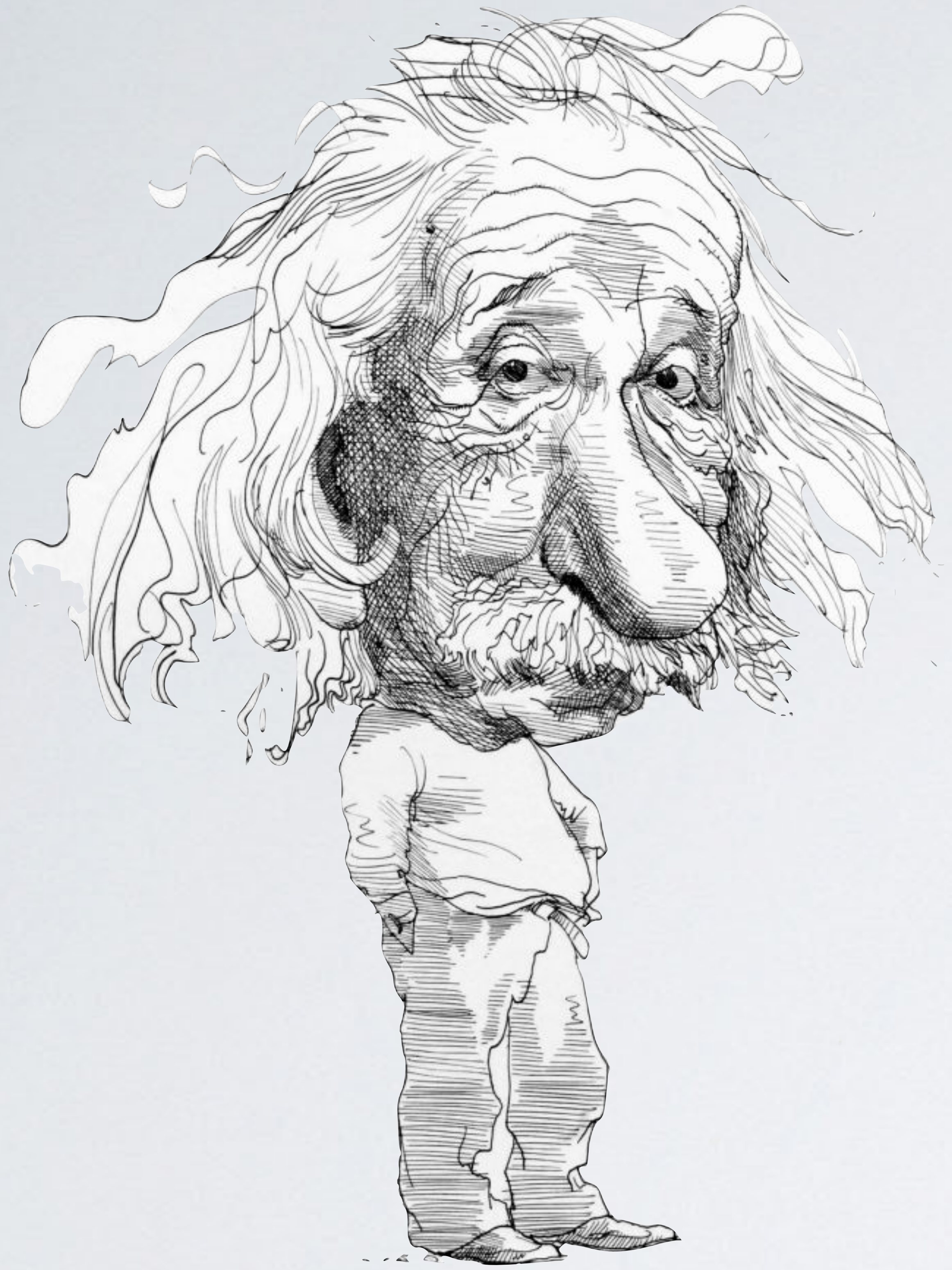
Quantum foundations debate

*I am a **quantum engineer**, but on Sundays I have **principles**.*

*Niels … Get **real** !!!*



Albert EINSTEIN
(1879 - 1955)

VS.

*Albert … it's all built of **unreal** stuff !!!*

Niels BOHR
(1885 - 1962)

John Stewart BELL
(1928 - 1990)

x=0,1
a= ±1
y=0,1
b= ±1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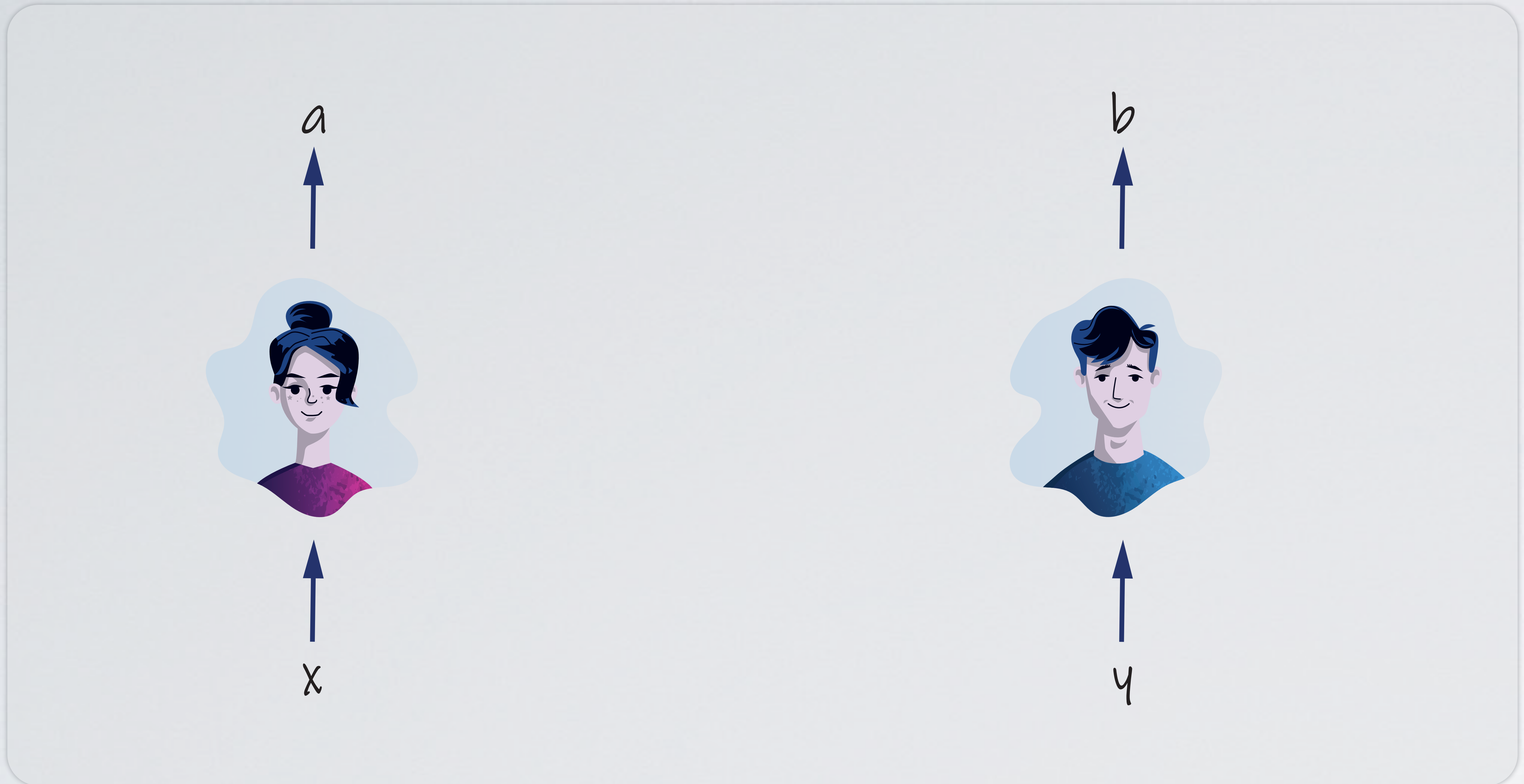
Bell experiment — causal model

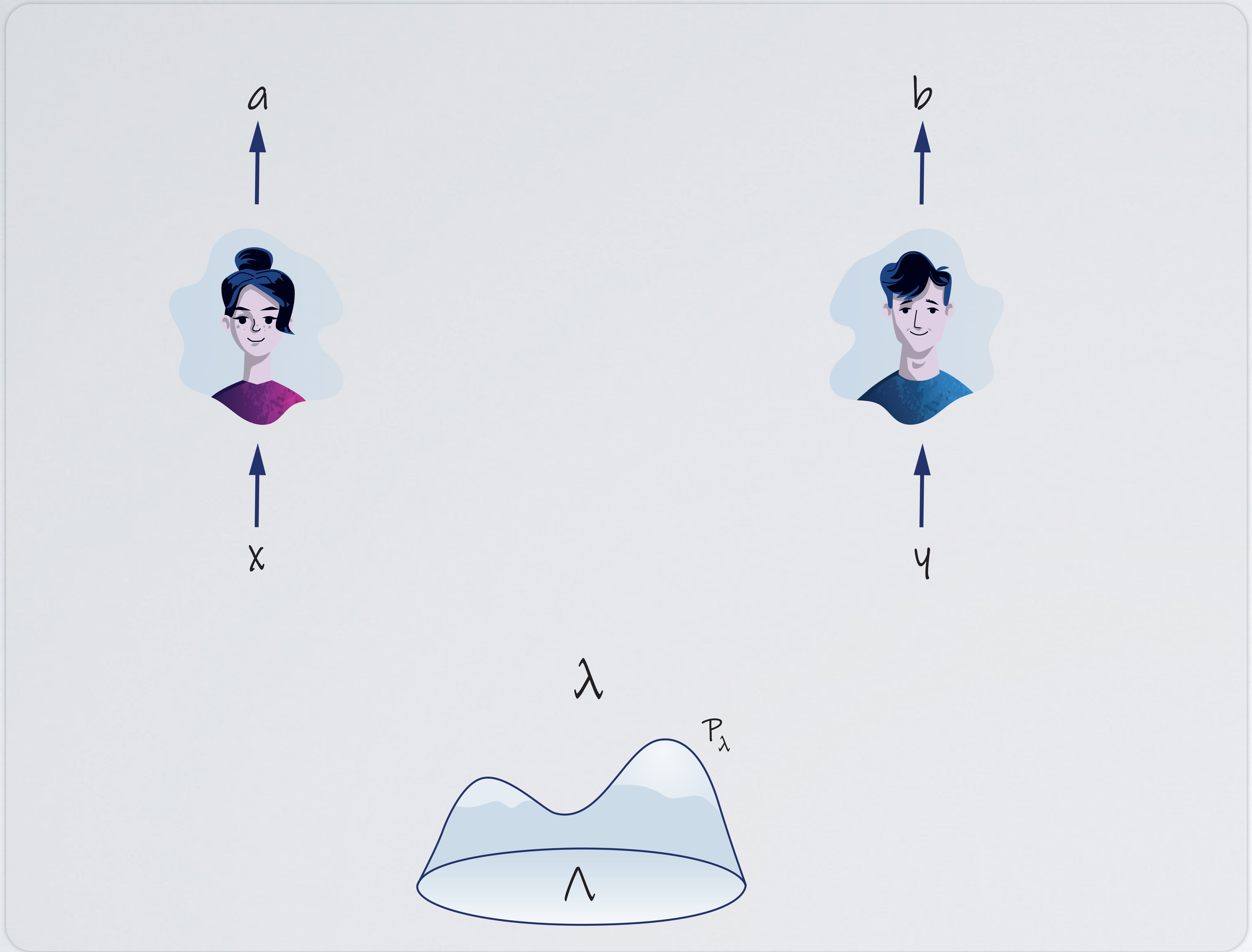


After many trials Alice and Bob collect experimental statistics for ***outcomes*** *a,b under different* ***choices of settings*** *x,y.*

- *Observed* $\{P_{ab|xy}\}_{xy}$ ← *Experimental behaviour*

  P_{xy} ← *Distribution of settings*

Bell experiment — causal model



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- *Realist (causal) framework*

$$P_{ab|xy} = \sum_{\lambda \in \Lambda} P_{ab|xy\lambda} \cdot P_{\lambda|xy}$$

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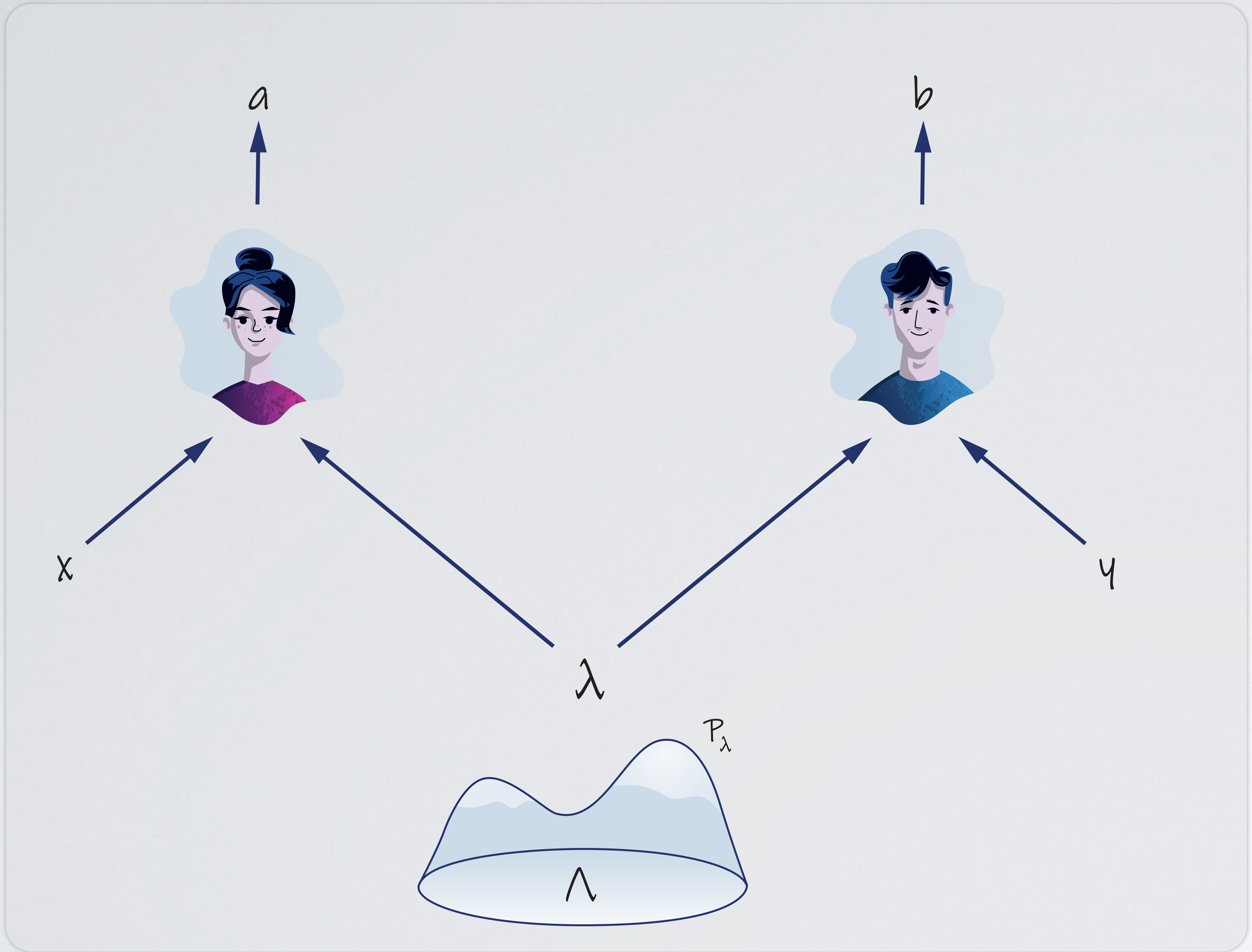
Hidden variable model

Bell experiment — causal model



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- ***Arrow of time*** *(no retro-causality)*

- ***Locality*** $P_{ab|xy\lambda} = P_{a|x\lambda} \cdot P_{b|y\lambda}$

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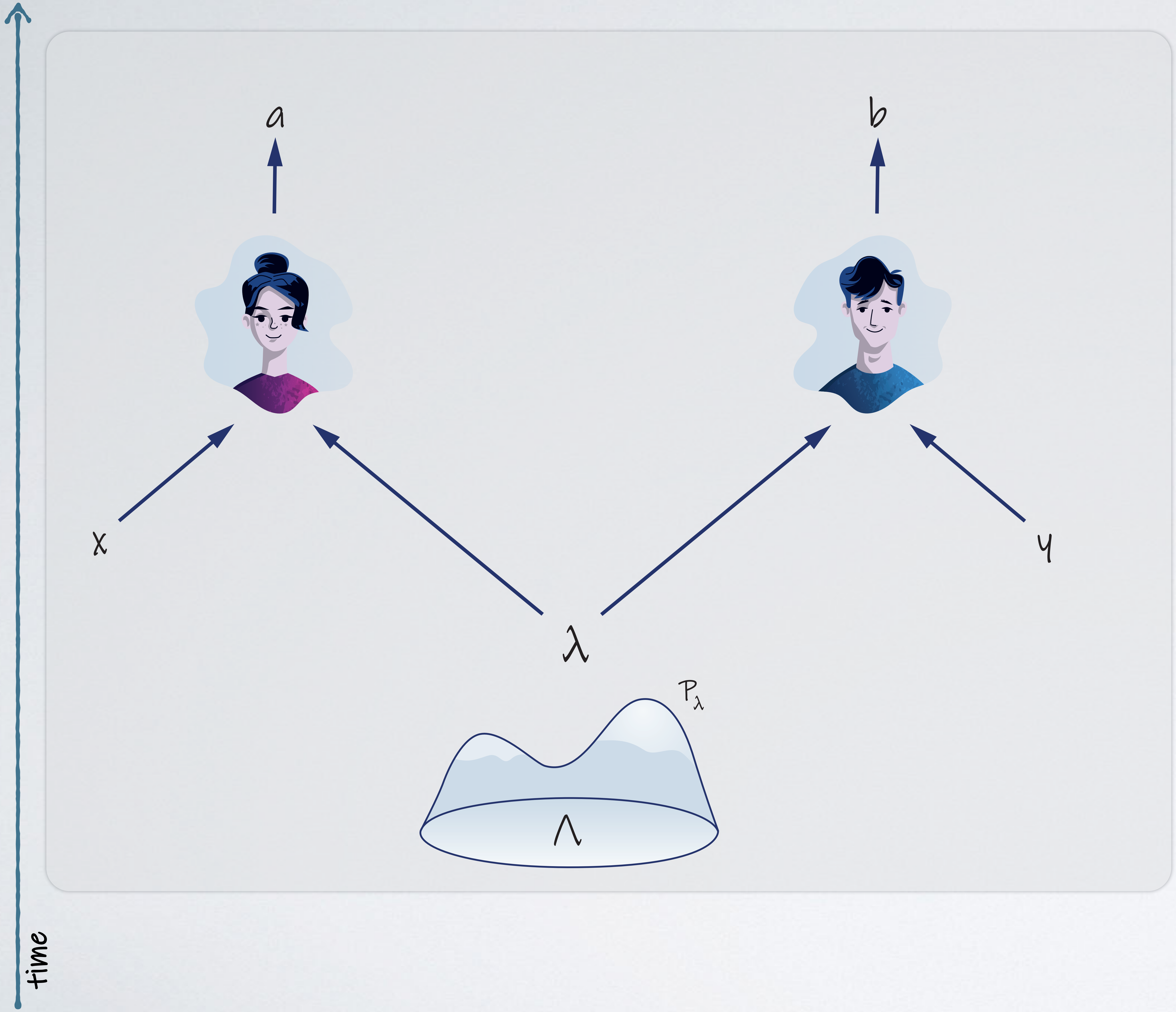

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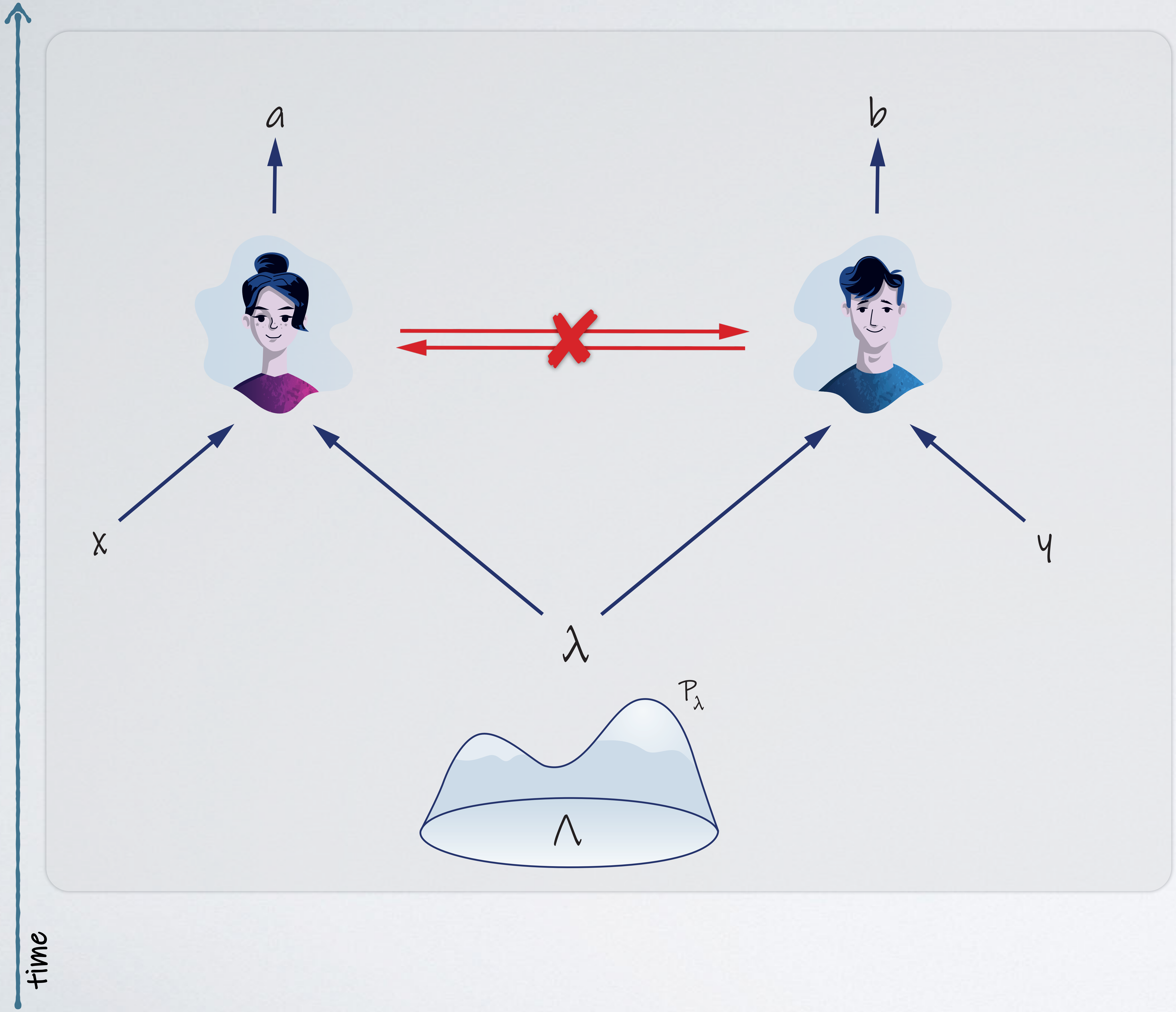

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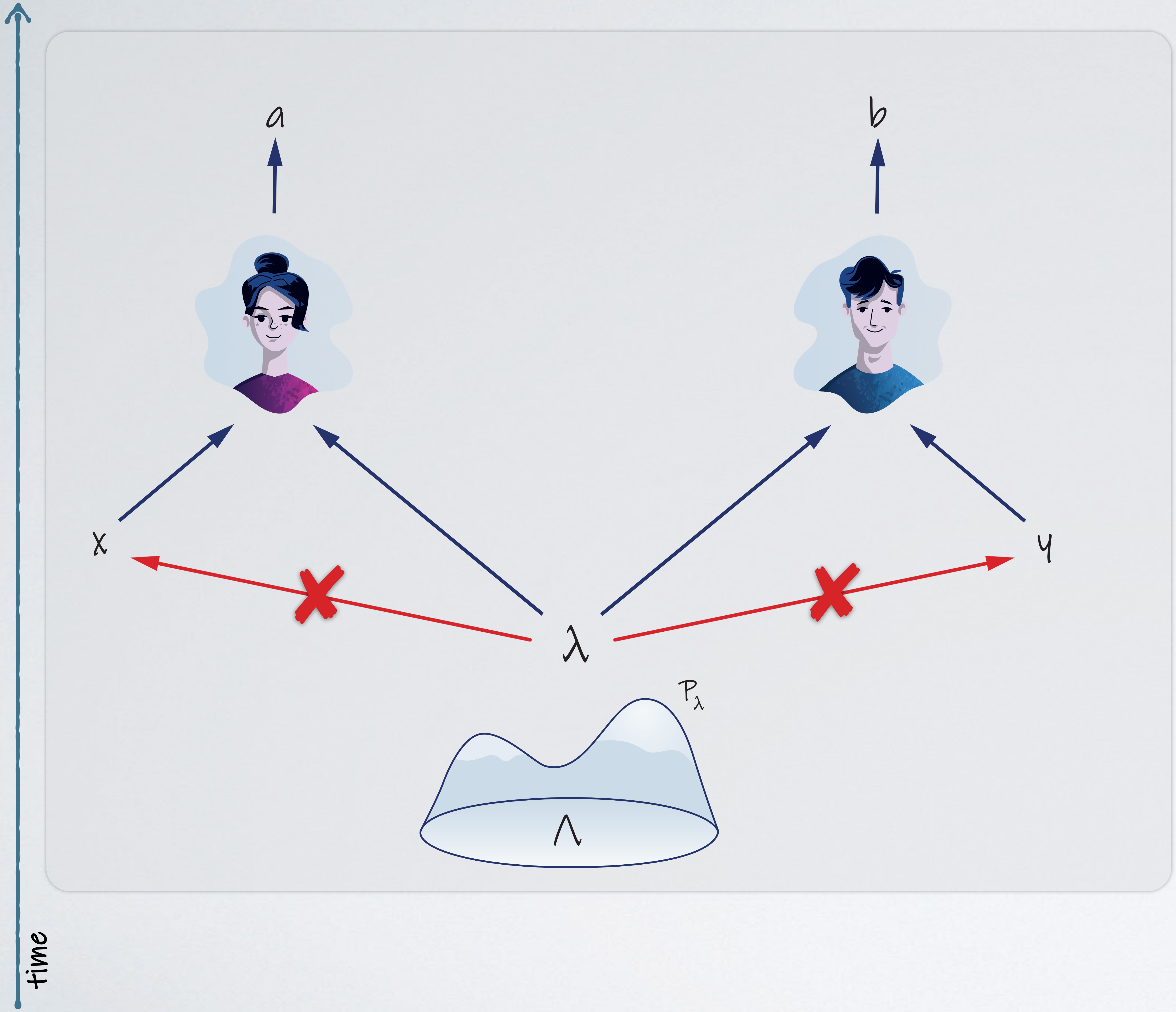

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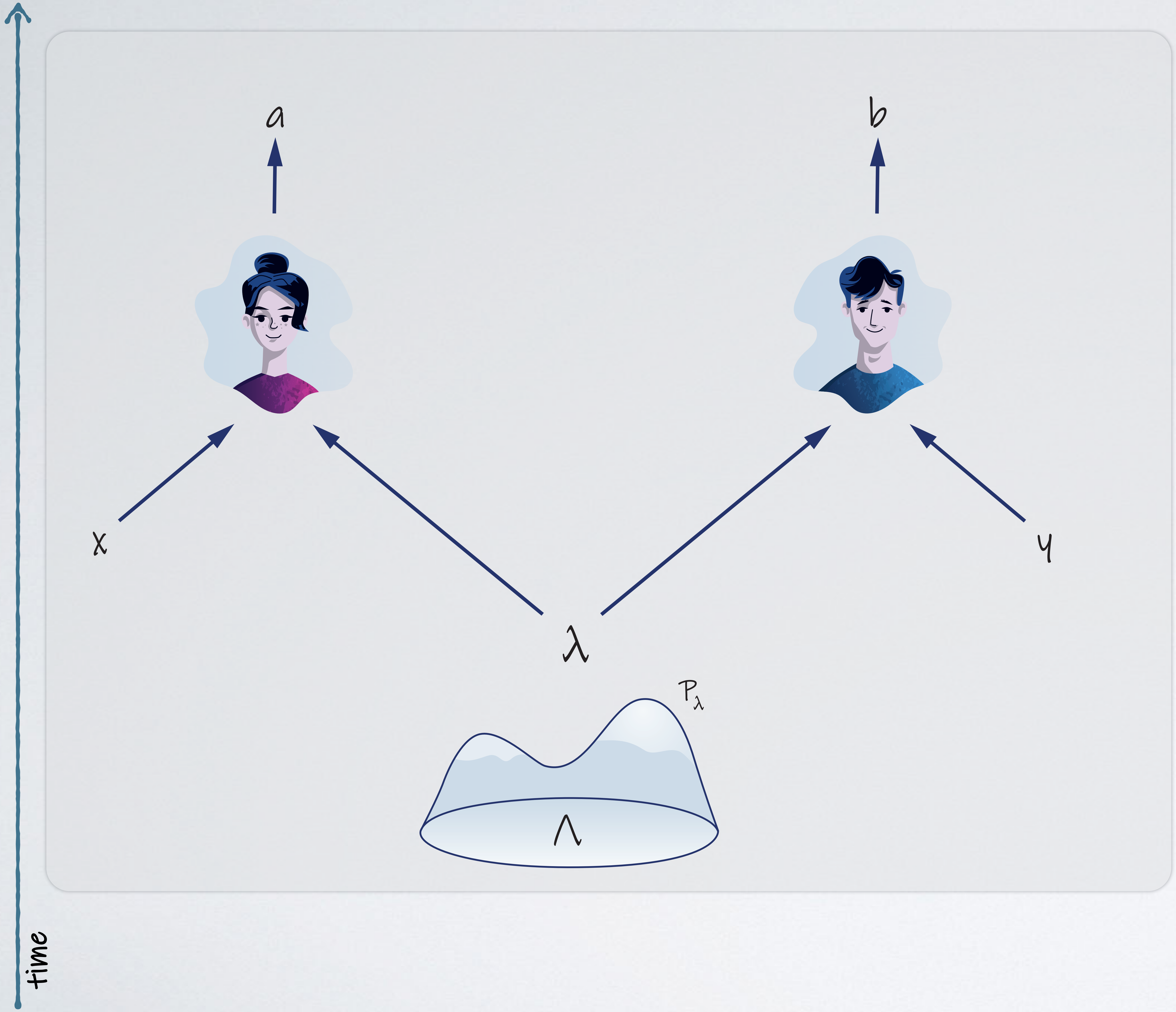

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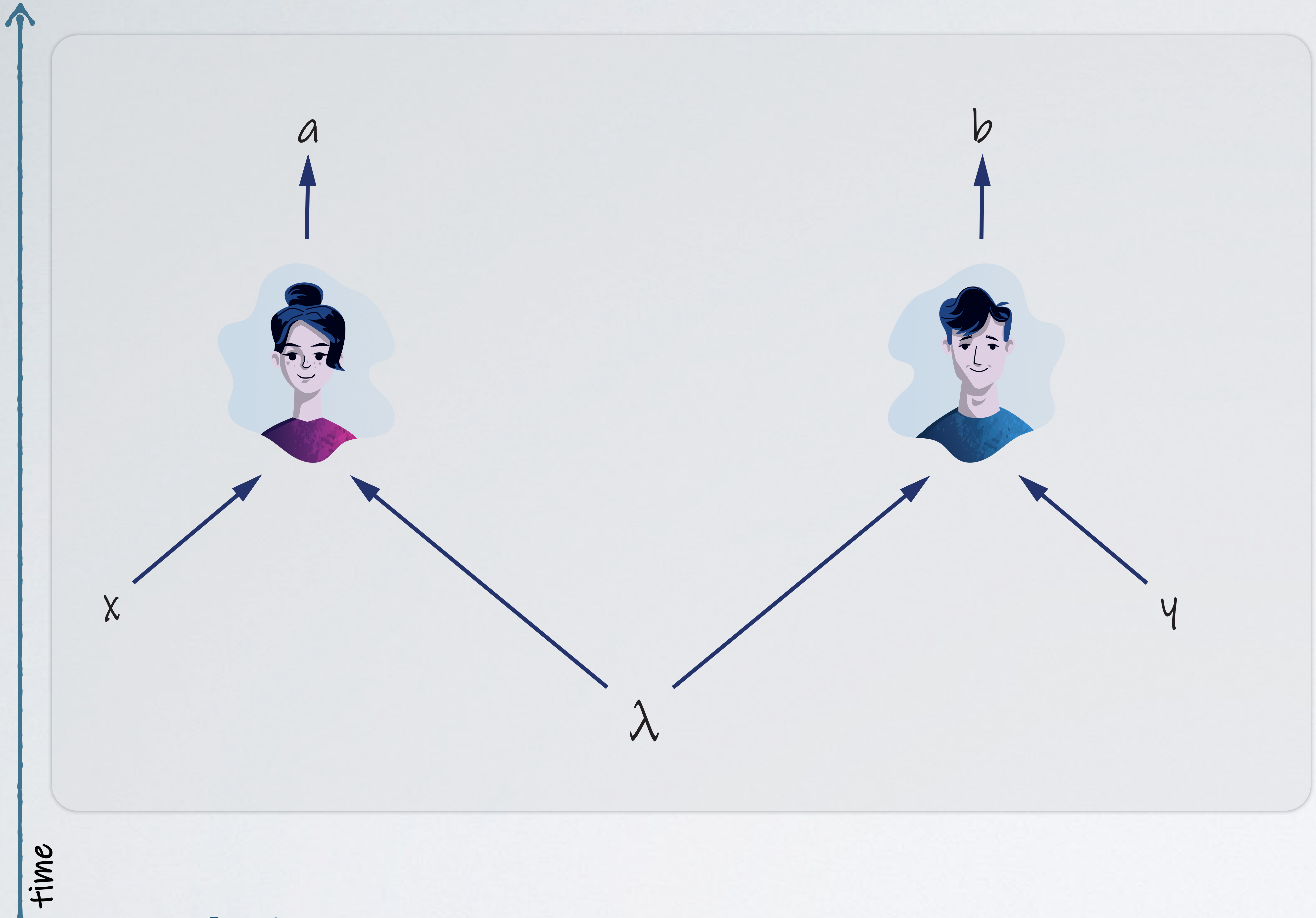
Bell inequalities



- *Bell-CHSH expressions*

$$
\begin{aligned}
S_1 &= \langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} - \langle ab\rangle_{11} \\
S_2 &= \langle ab\rangle_{00} + \langle ab\rangle_{01} - \langle ab\rangle_{10} + \langle ab\rangle_{11} \\
S_3 &= \langle ab\rangle_{00} - \langle ab\rangle_{01} + \langle ab\rangle_{10} + \langle ab\rangle_{11} \\
S_4 &= -\langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} + \langle ab\rangle_{11}
\end{aligned}
$$

where: $\langle ab\rangle_{xy} = \sum_{a,b} ab\, P_{ab|xy}$

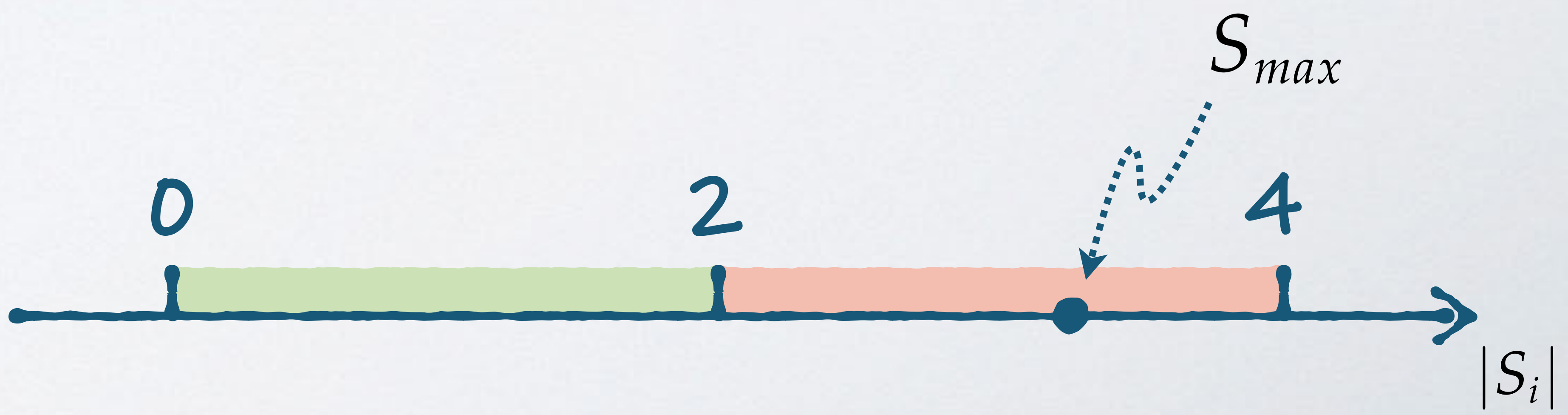
Bell's theorem

Realism + Arrow of time + Locality + Free choice:

$$|S_i| \leqslant 2$$

whereas in QM it can be: $|S_i| \leqslant 2\sqrt{2} \leqslant 4$

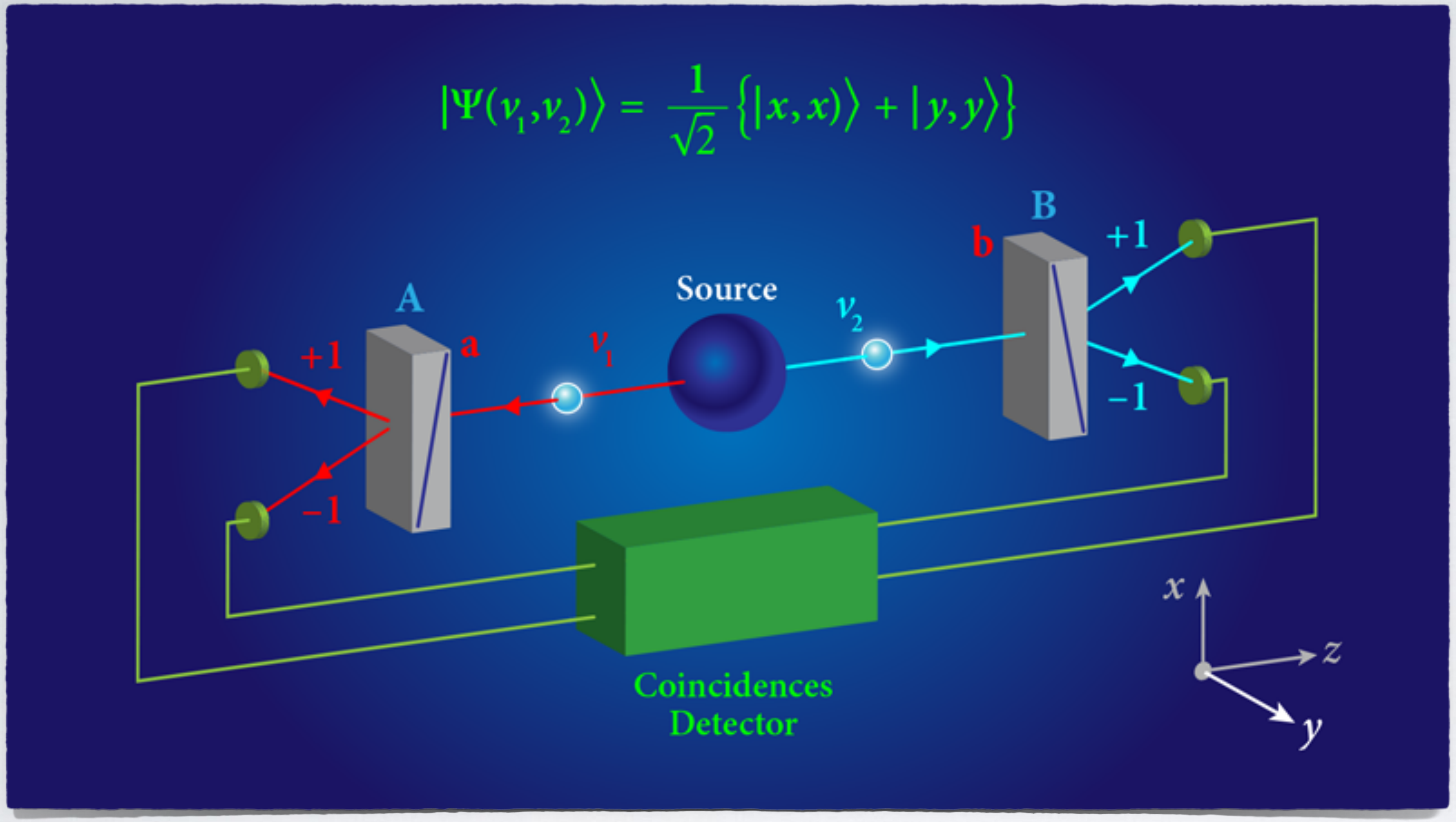
Tsirelson PR- box



Conclusion:

Those assumptions entail ***testable constraints*** *on correlations called* ***Bell inequalities*** *which are* ***violated in QM****.*

Bell experiment — in reality



Alain ASPECT
(1947)

Experimental violation of Bell's inequality:

$$2 < |S_i| \leqslant 2\sqrt{2}$$

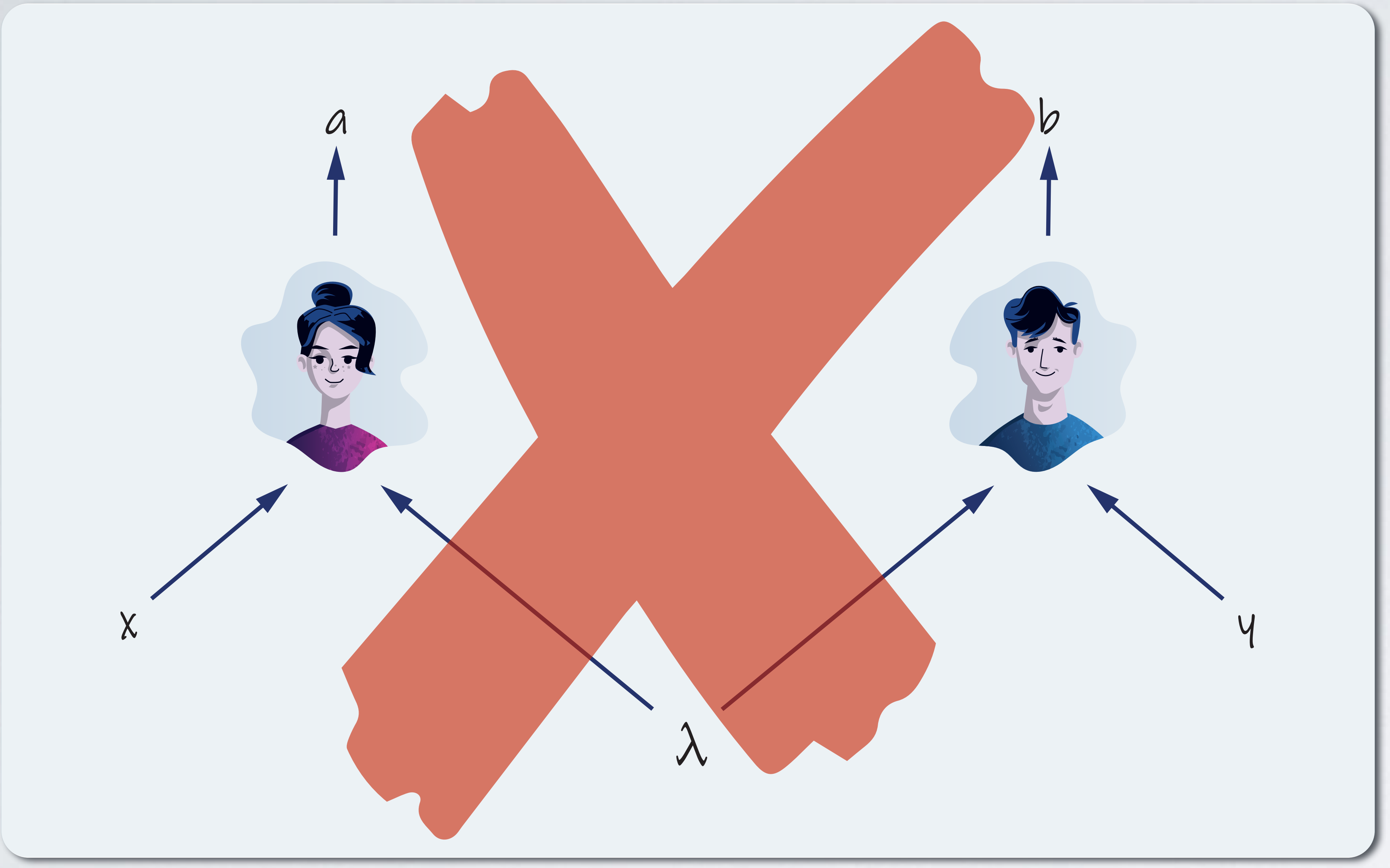
(*) Closed locality loophole (exp. with photons)
(**) Locality + fair sampling loophole closed in 2015 (exp. with electrons)
(***) Superdeterminism loophole

Democritus
(460 - 370 BC)

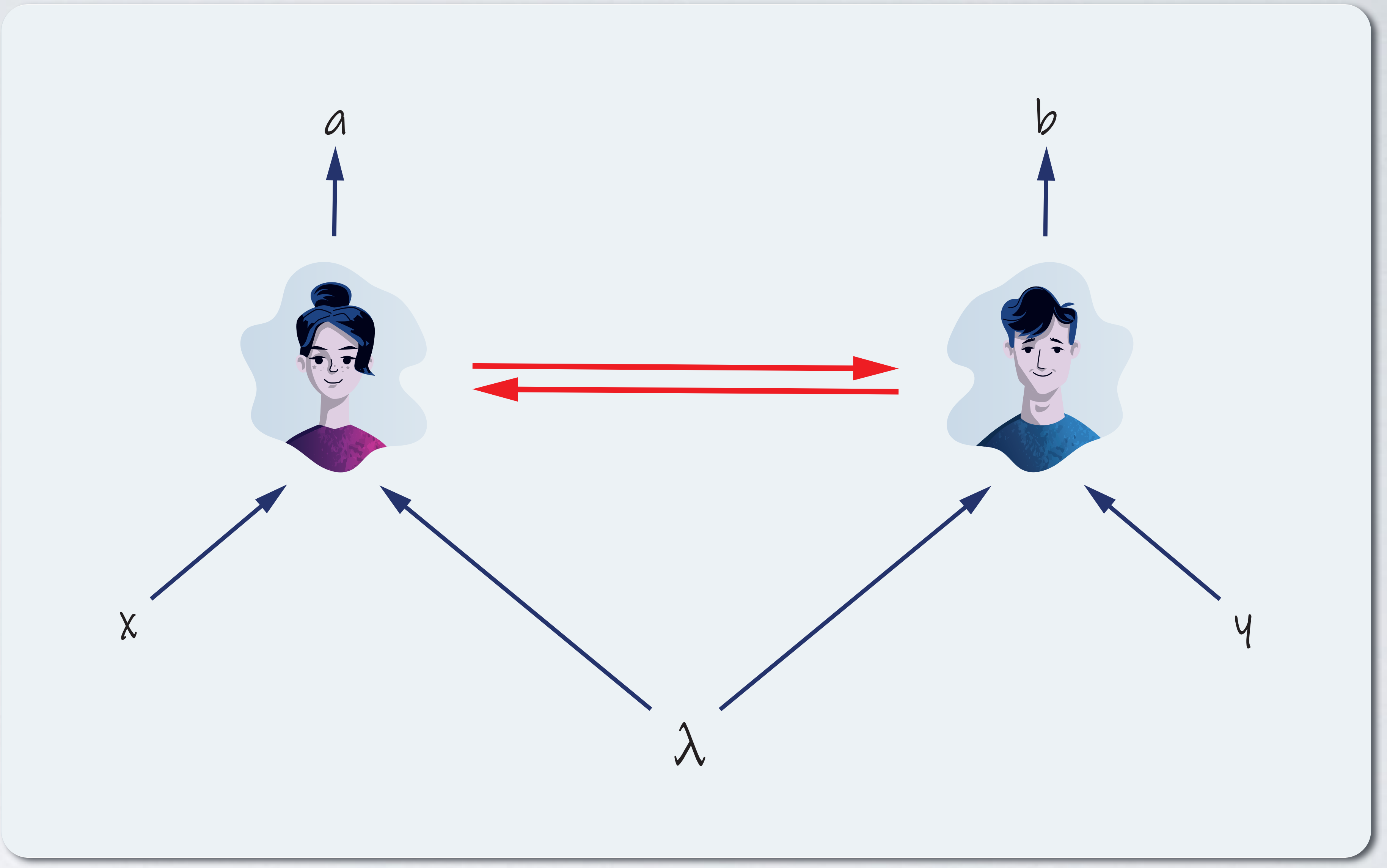
"I would rather discover one true cause than gain the kingdom of Persia."

"When you have eliminated the impossible, whatever remains, however improbable, must be the truth."
– Sherlock Holmes
(Sir Arthur Conan Doyle)

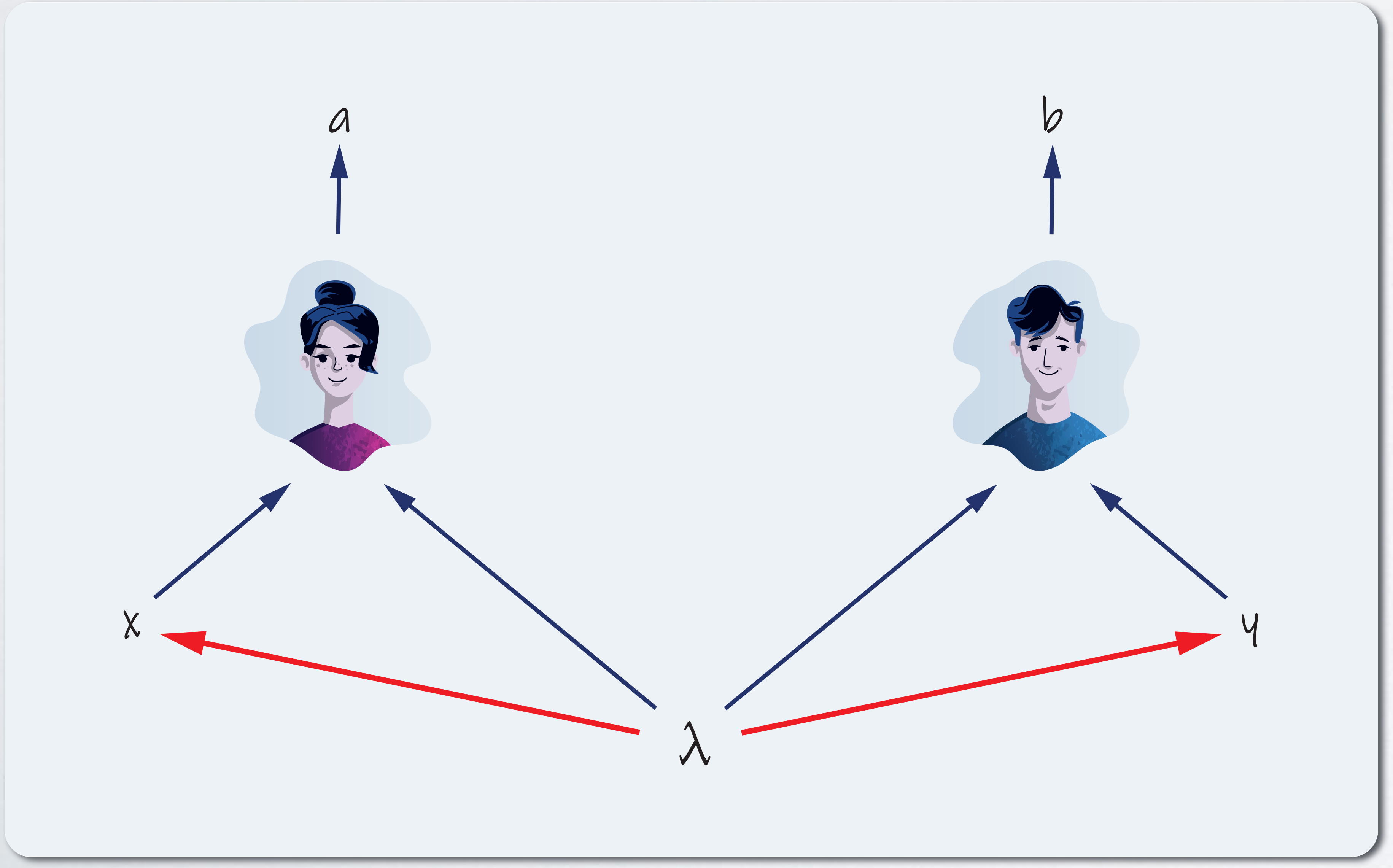
Locality + Free choice + Arrow of Time



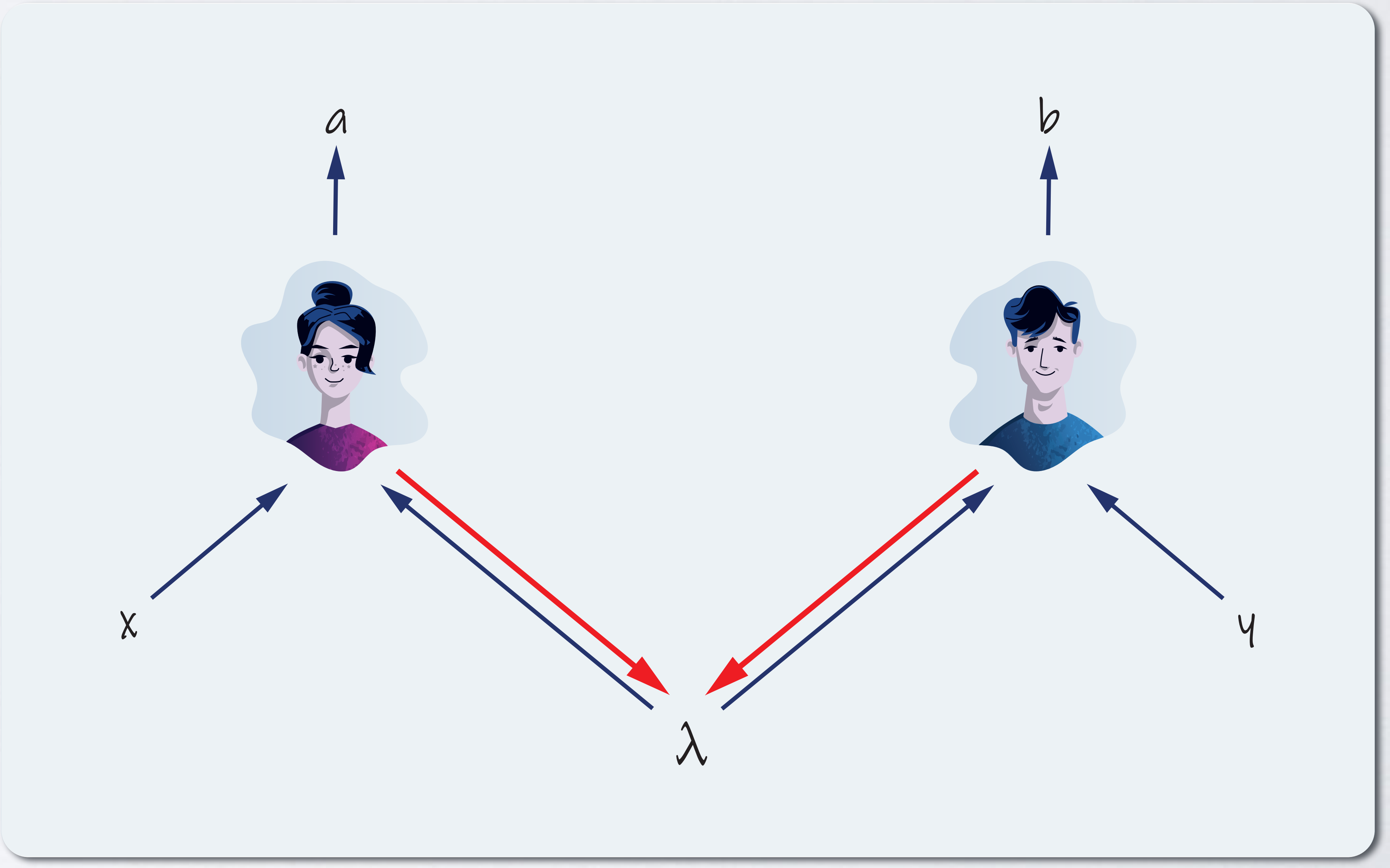
Violation of Locality



Violation of Free choice



Violation of Arrow of time



Conclusions (to be continued …)

Causal assumptions:

Locality + ***Free choice*** + ***Arrow of time***

*entail **testable constraints** on correlations called **Bell inequalities** which are **violated in QM**.*

Dropping either of those causal assumptions is problematic …

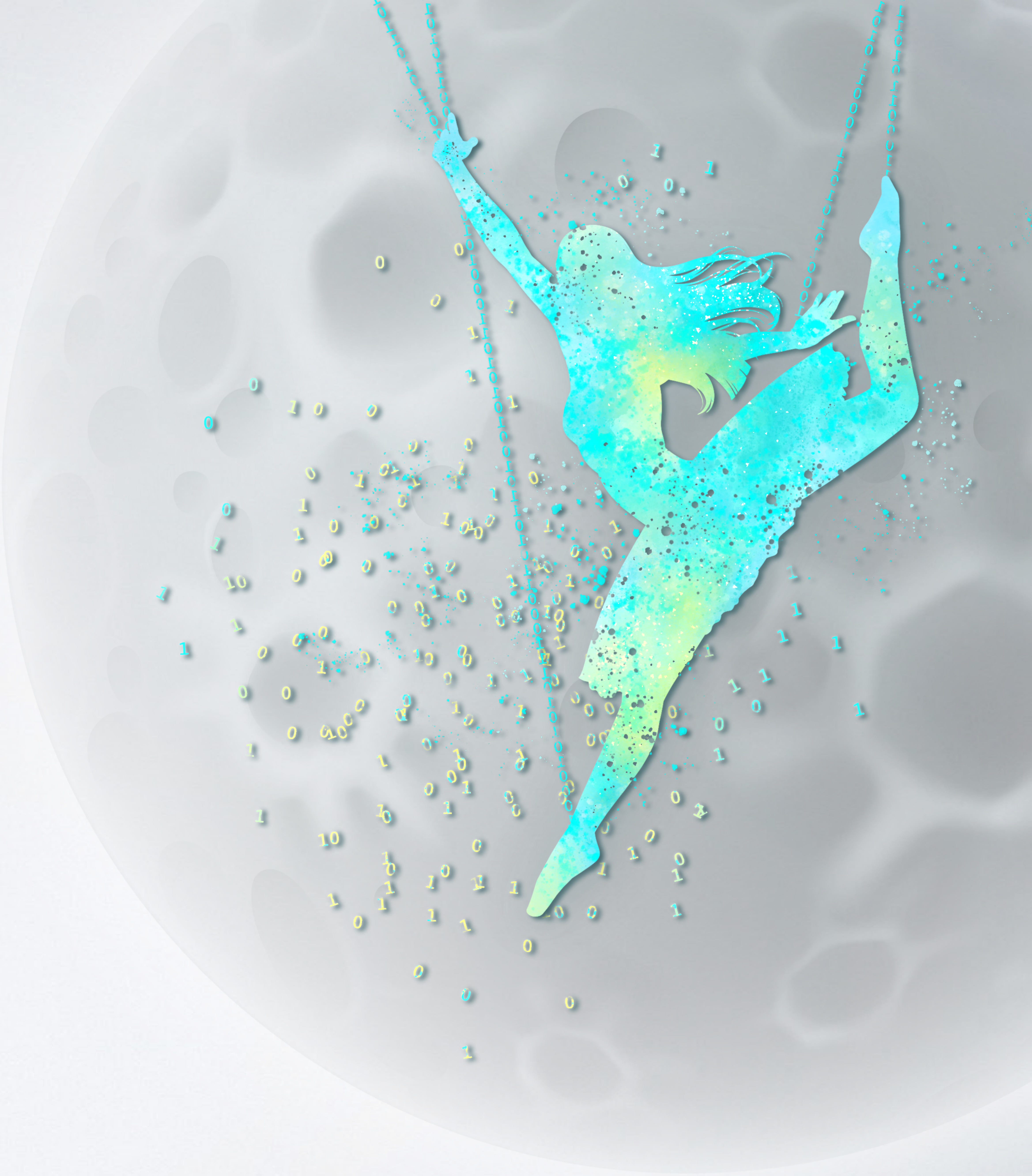
*… would require **rethinking the notion of space-time** …*

*… and/or the **role of observers**.*

Its always healthy to …

***DRAW** your assumptions*

***BEFORE** your conclusions.*



PNAS Proceedings of the National Academy of Sciences of the United States of America

Violations of locality and free choice are equivalent resources in Bell experiments

Pawel Blasiak[a,b,1], Emmanuel M. Pothos[b], James M. Yearsley[b], Christoph Gallus[c], and Ewa Borsuk[a]

[a]Division of Theoretical Physics, Institute of Nuclear Physics Polish Academy of Sciences, PL-31342 Krakow, Poland; [b]Psychology Department, City, University of London, London EC1V 0HB, United Kingdom; and [c]THM Business School, Technische Hochschule Mittelhessen, D-35390 Giessen, Germany

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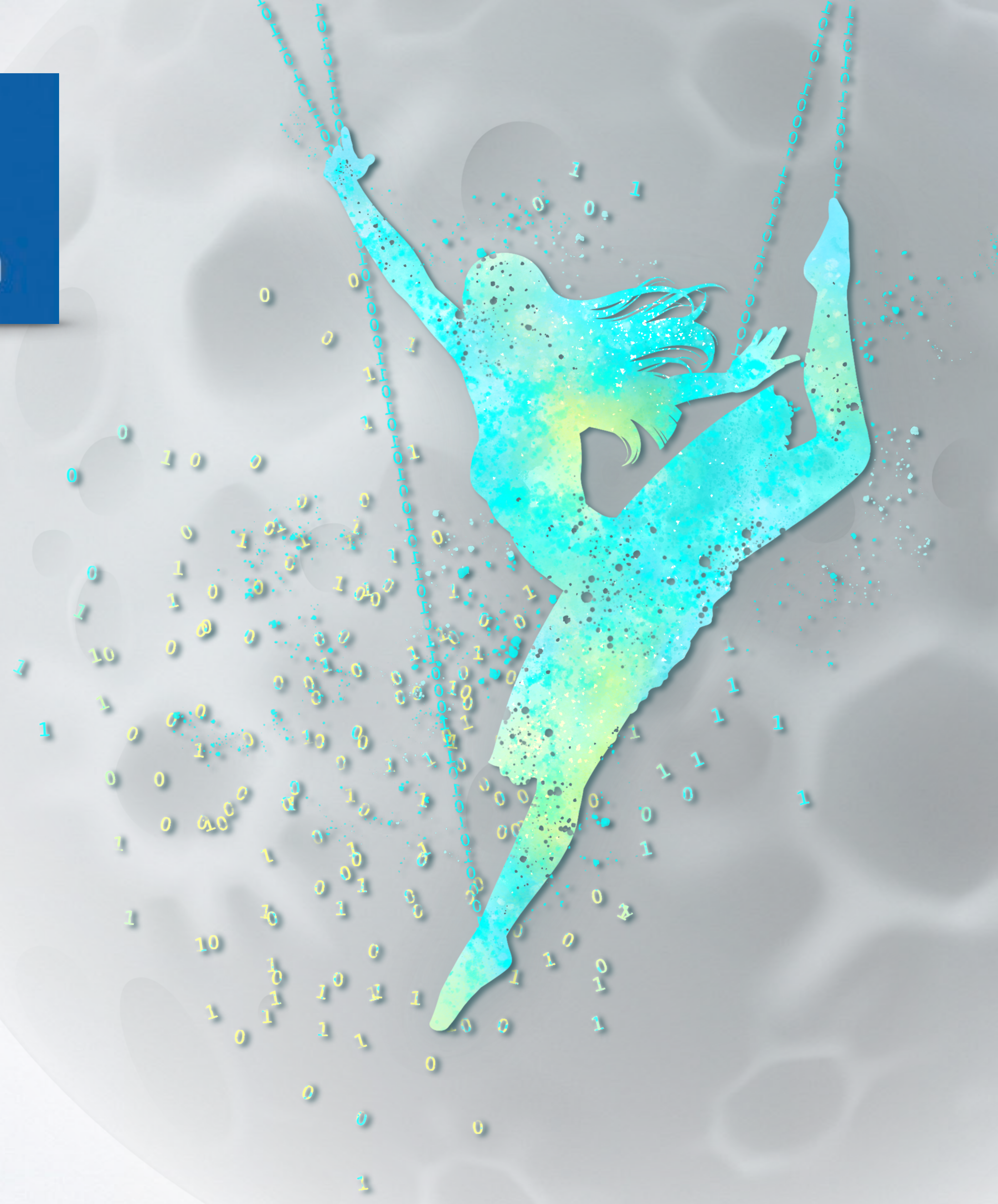
EurekAlert!

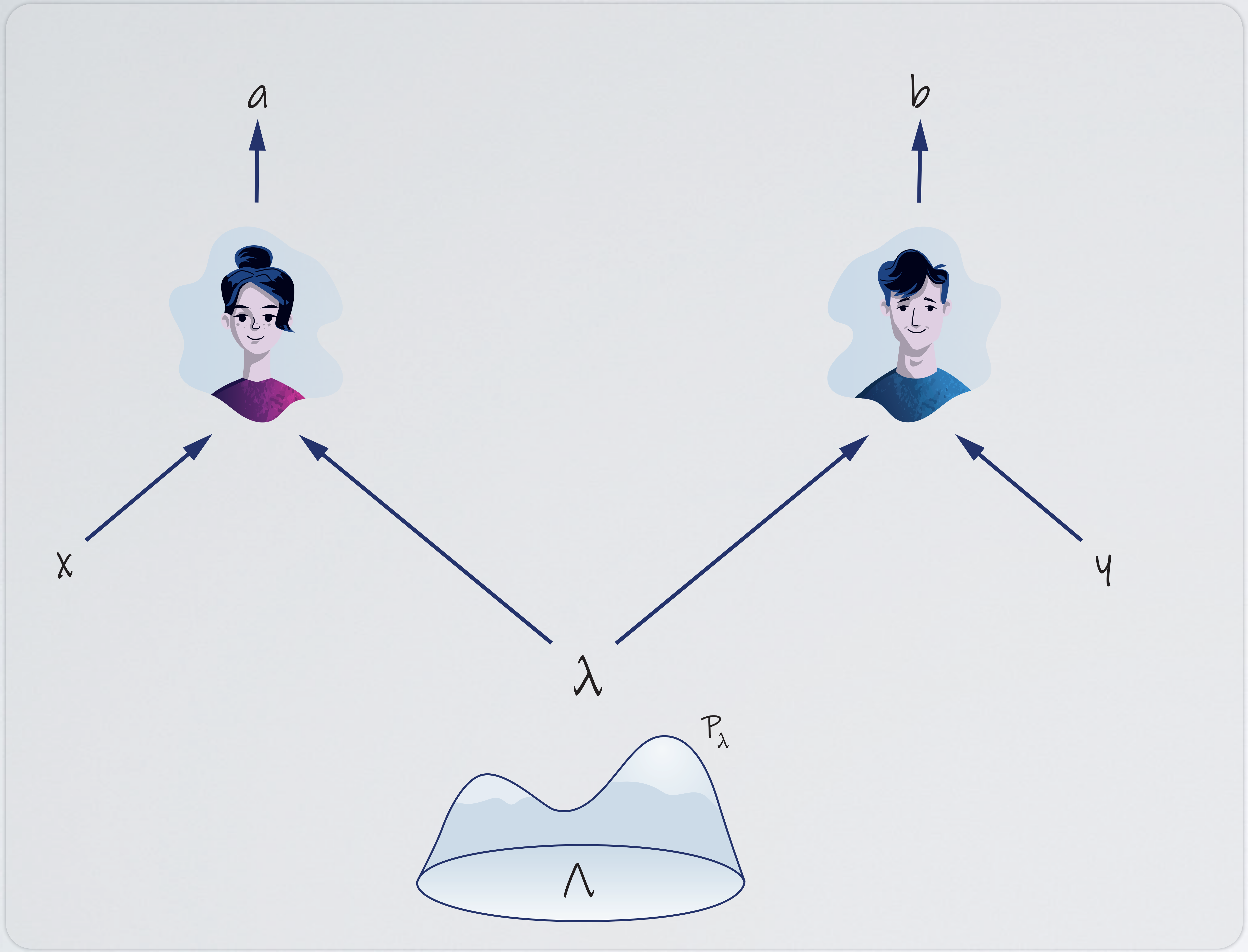
News Release: 20 May 2021



Take aways

- *Which is **more costly** **locality** or **free choice**?*
- ***Calculate** and **compare** measures of both in **QM** and **not only**.*

Bell experiment — recap (I)



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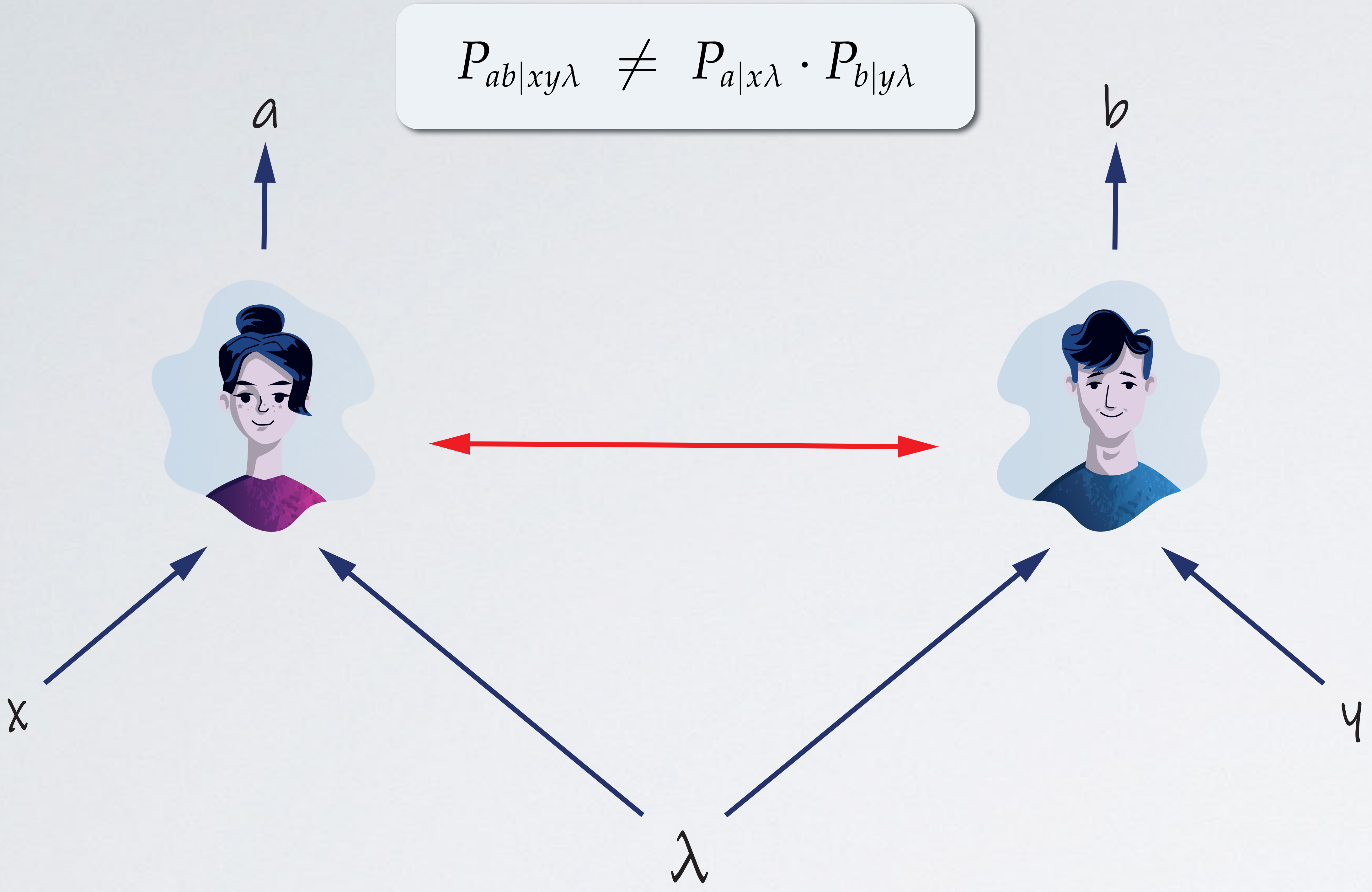
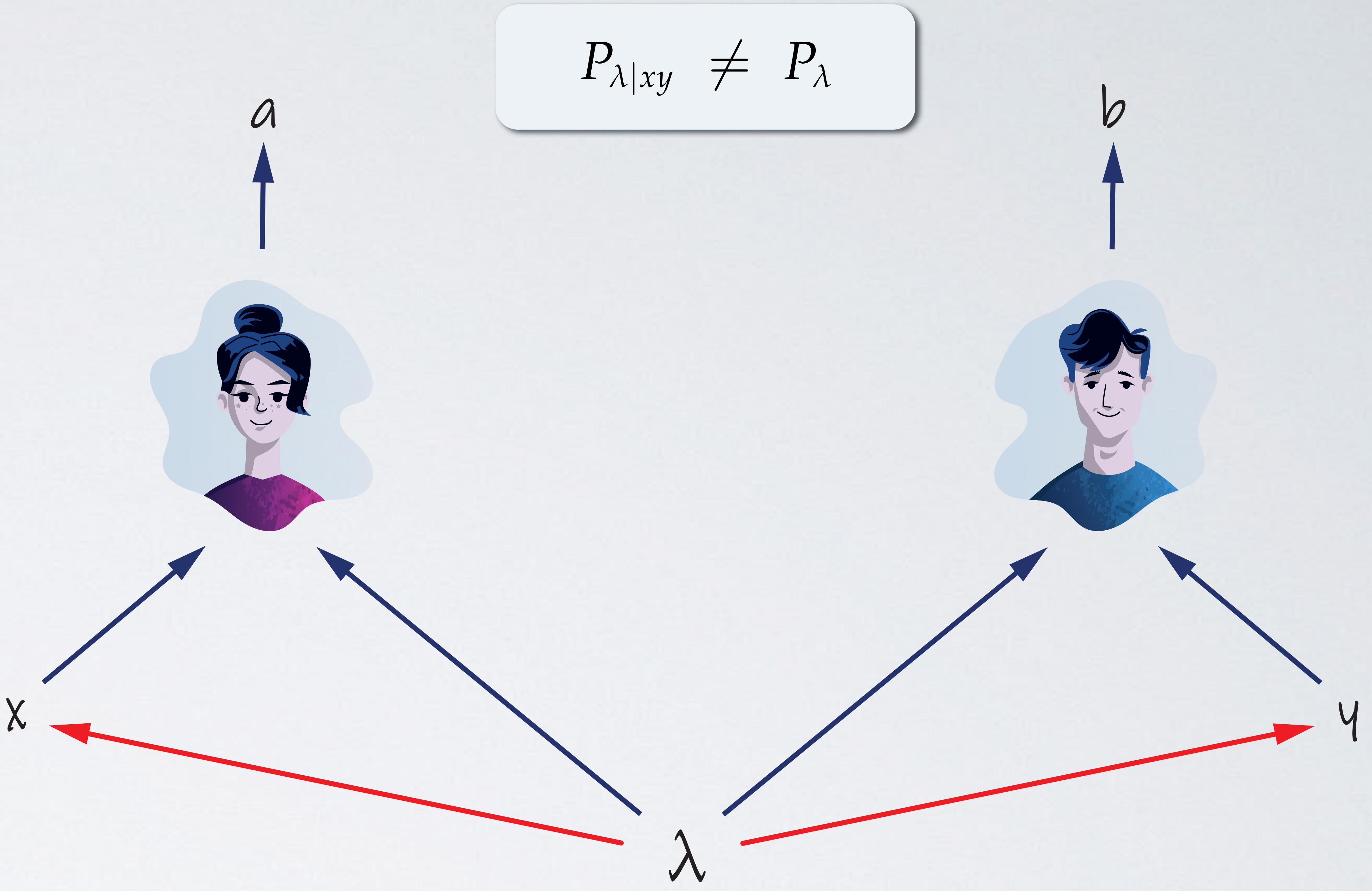
- *Arrow of time (no retro-causality)*

Bell's theorem

*Those assumptions entail **testable constraints** on correlations called Bell inequalities (**violated in QM**).*

Violation of Locality vs. Violation of Free choice



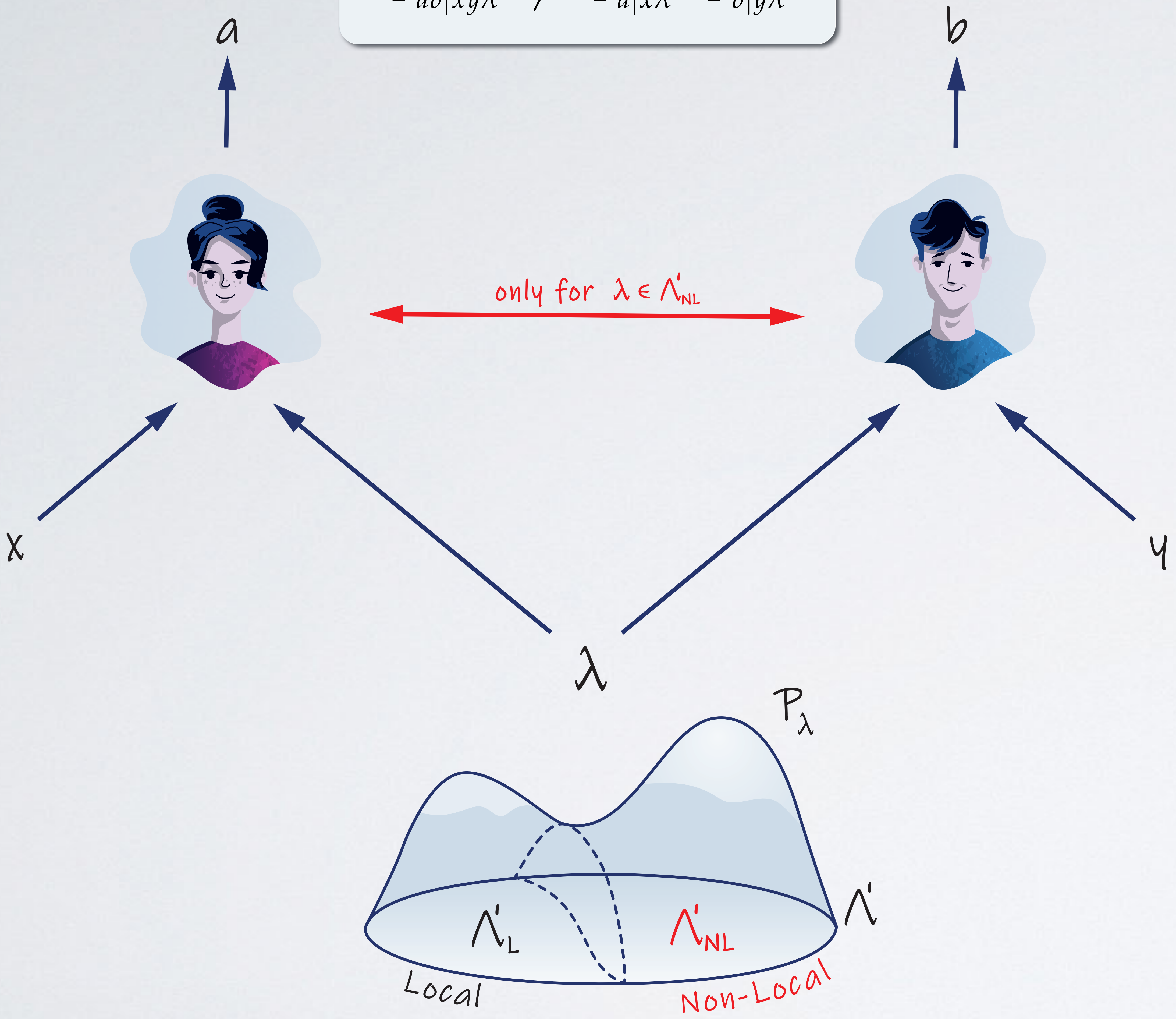


Heuristic idea

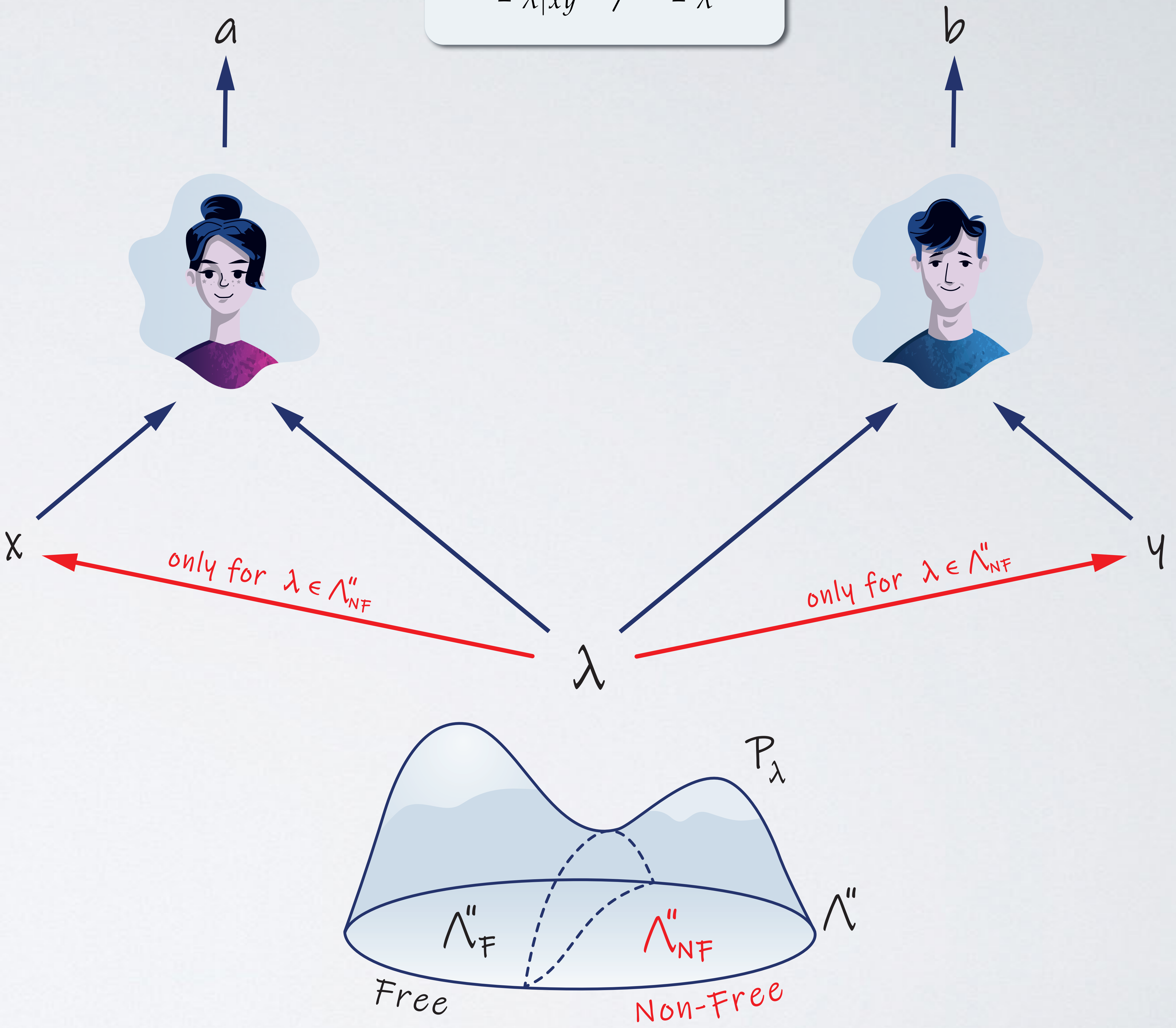
How often *a given assumption, i.e. **locality** or **free choice**, can be **retained**, while **safeguarding the other assumption**, in order to fully reproduce some given experimental statistics within a standard causal (or realist) approach?*

Violation of Locality vs. Violation of Free choice

$$P_{ab|xy\lambda} \neq P_{a|x\lambda} \cdot P_{b|y\lambda}$$



$$P_{\lambda|xy} \neq P_\lambda$$



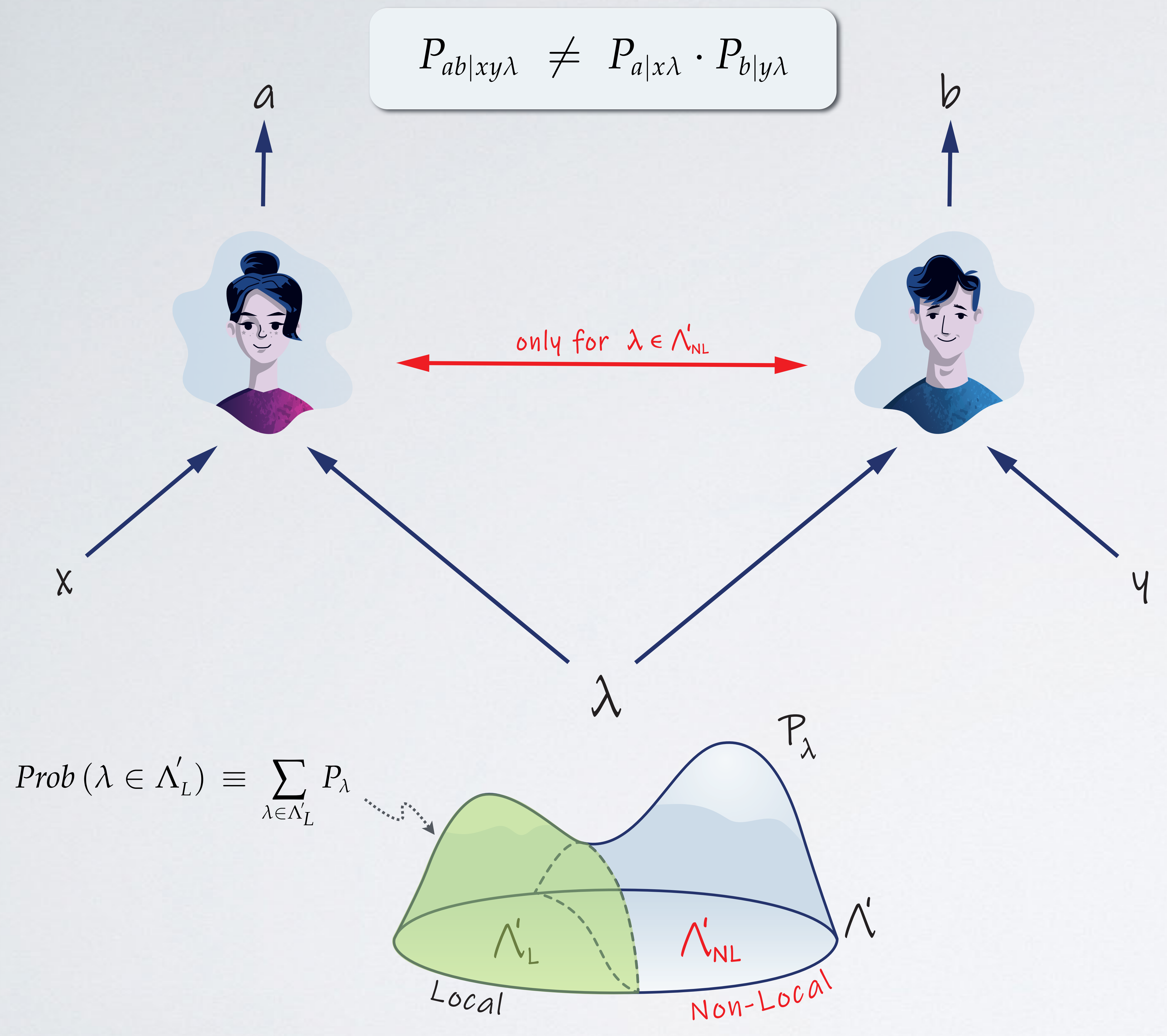
$\lambda \in \Lambda'_L \quad \Leftrightarrow \quad$ **locality holds** for *all* x, y

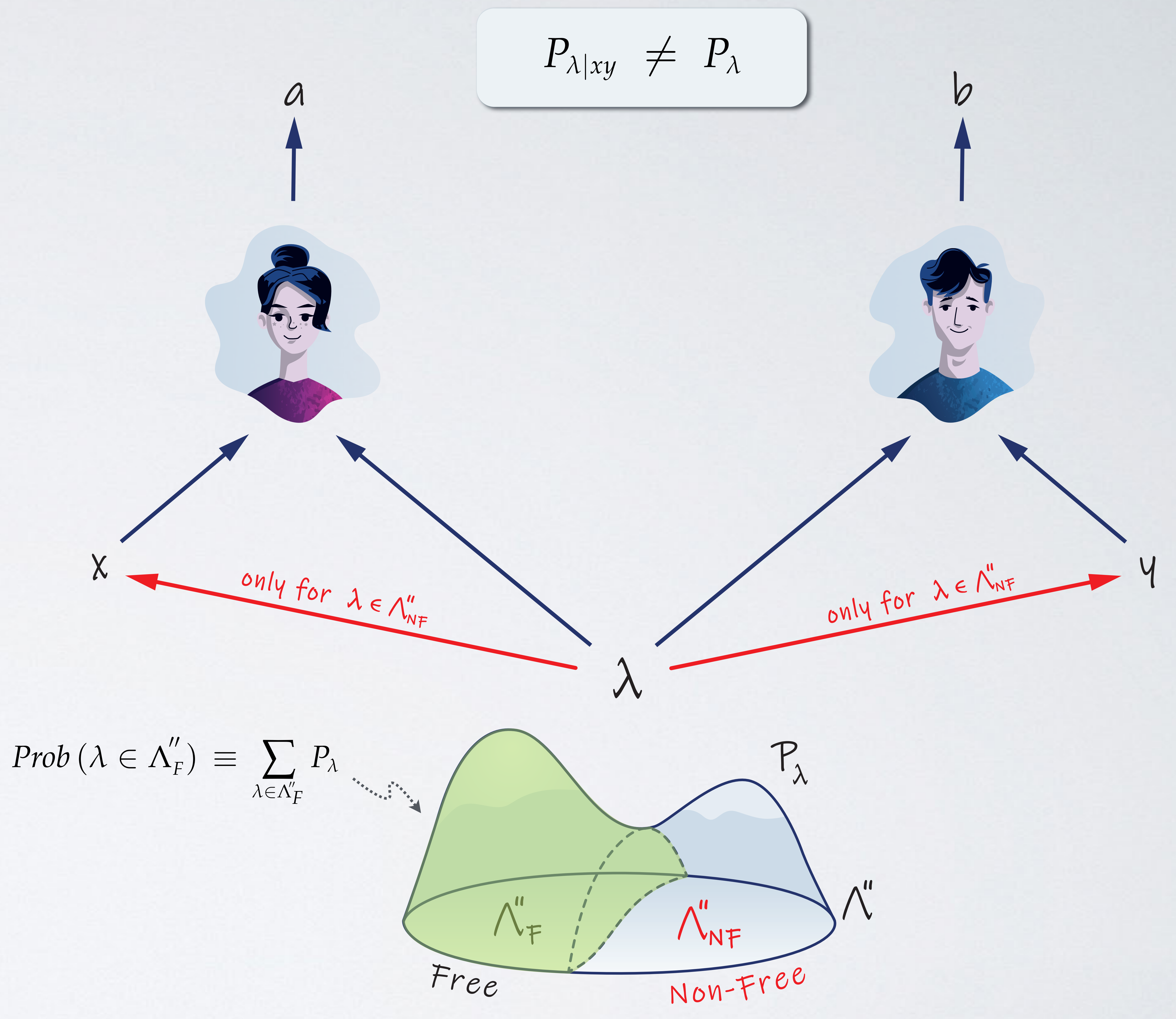
$\lambda \in \Lambda'_{NL} \quad \Leftrightarrow \quad$ **locality fails** for *some* x, y

$\lambda \in \Lambda''_F \quad \Leftrightarrow \quad$ **free choice holds** for *all* x, y

$\lambda \in \Lambda''_{NF} \quad \Leftrightarrow \quad$ **free choice fails** for *some* x, y

Violation of Locality vs. Violation of Free choice





$\lambda \in \Lambda'_L \quad \Leftrightarrow$ **locality holds** for *all* x, y

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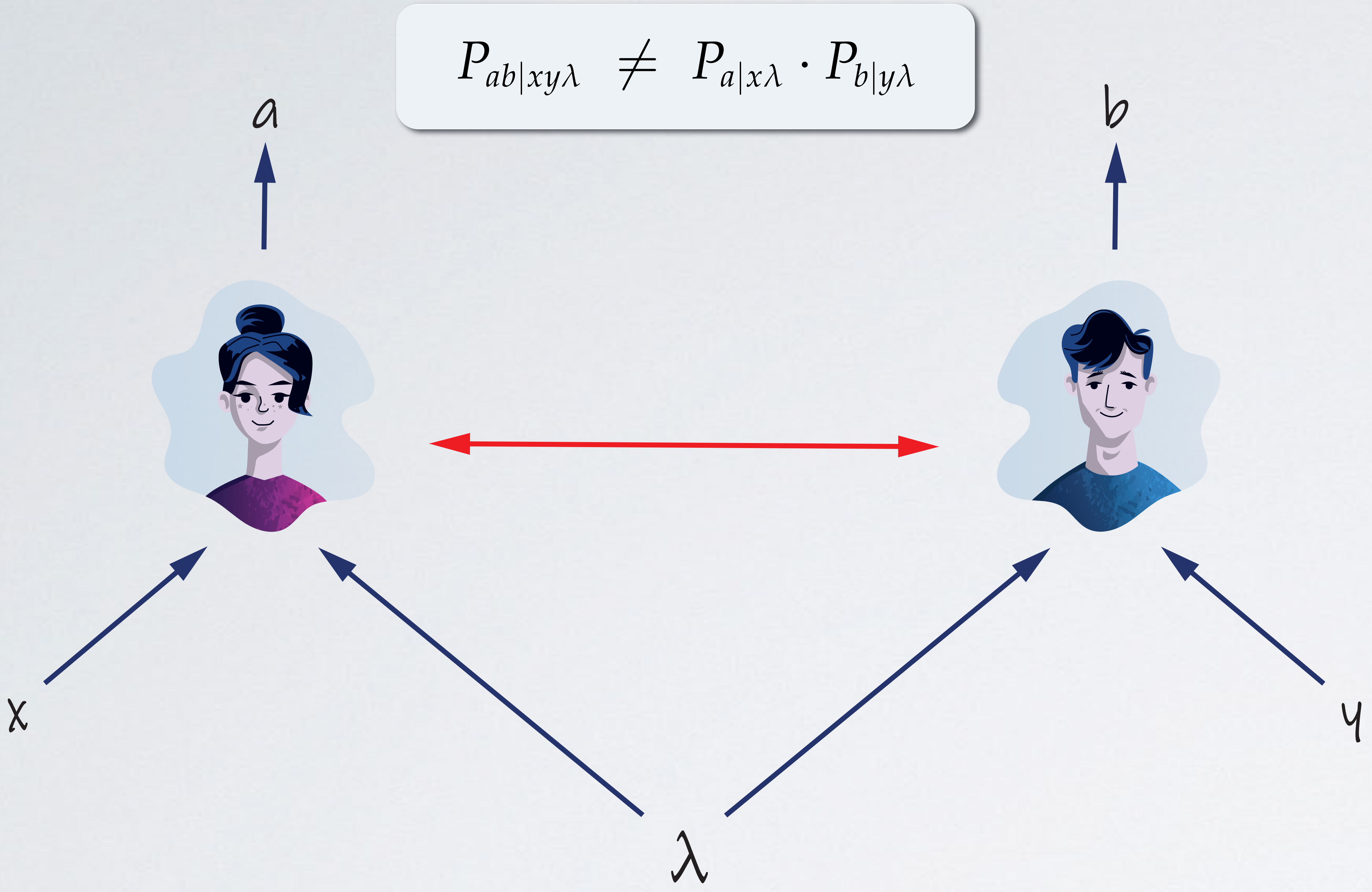
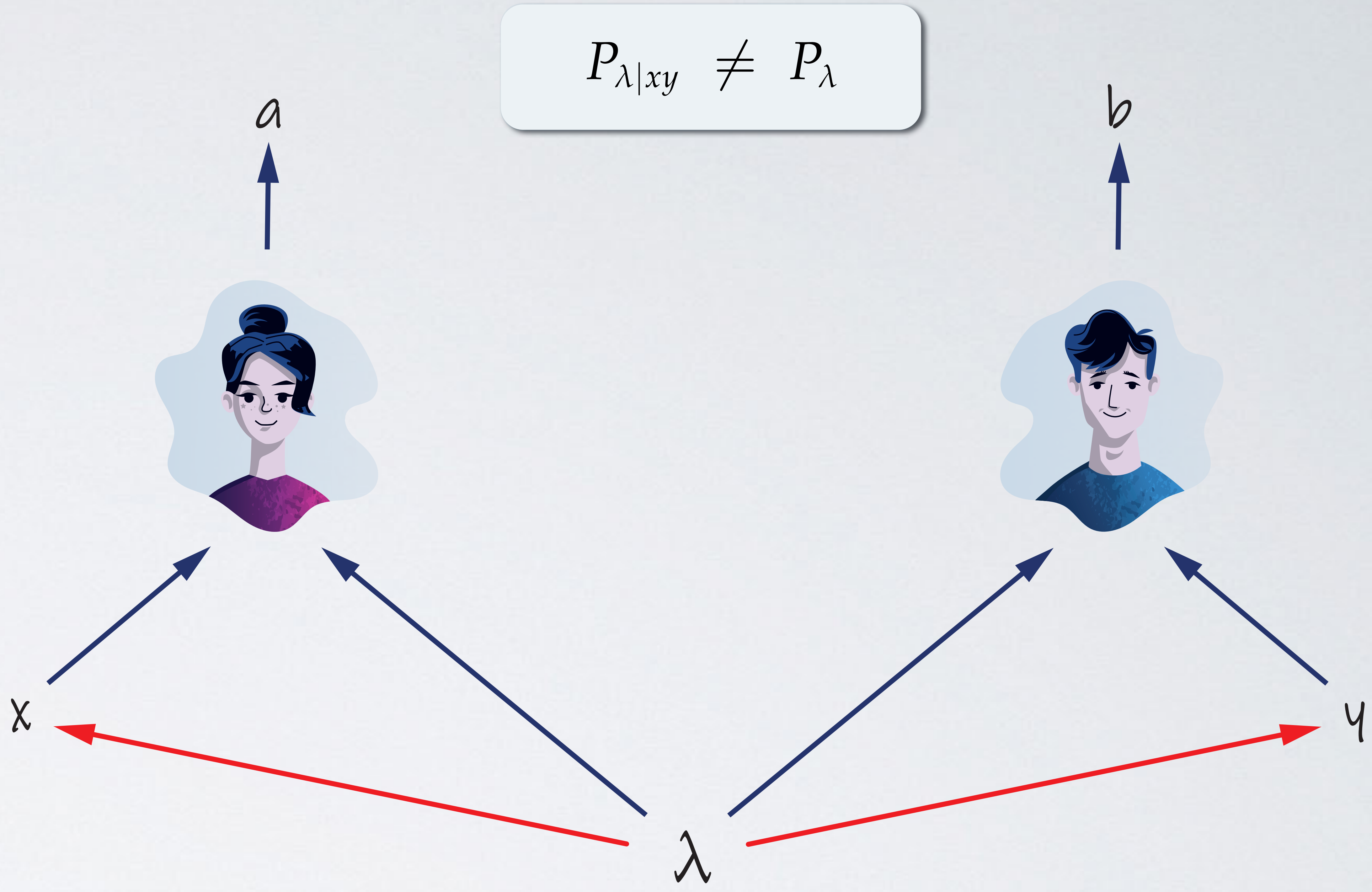
Violation of Locality vs. *Violation of Free choice*





Measure of locality

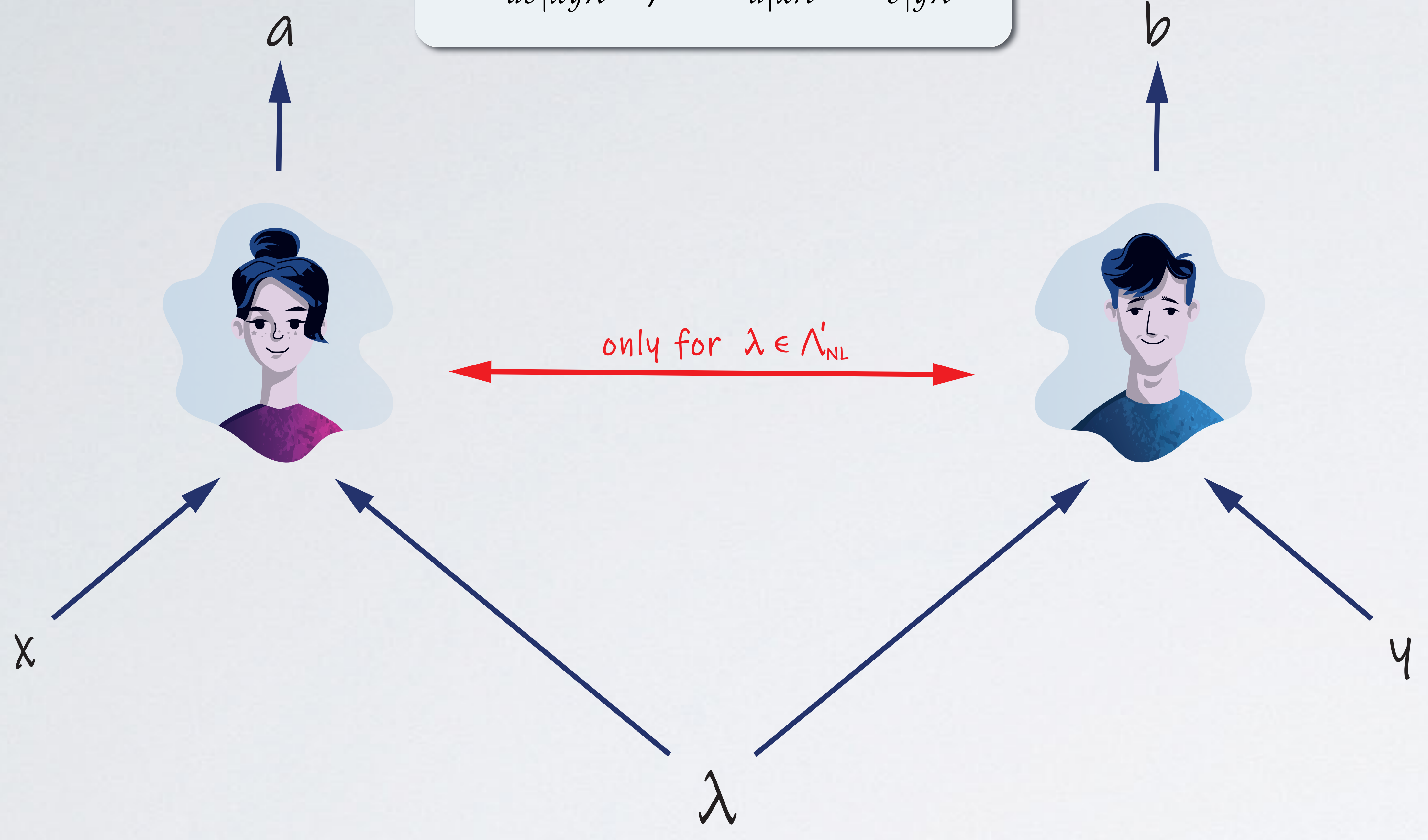
*the **maximal fraction** of trials in which Alice and Bob **do not** need to **communicate** trying to simulate a given behaviour $\{P_{ab|xy}\}_{xy}$ for <u>any</u> distribution of settings P_{xy}, optimised over <u>all</u> conceivable strategies with **freely chosen** settings.*

Measure of free choice

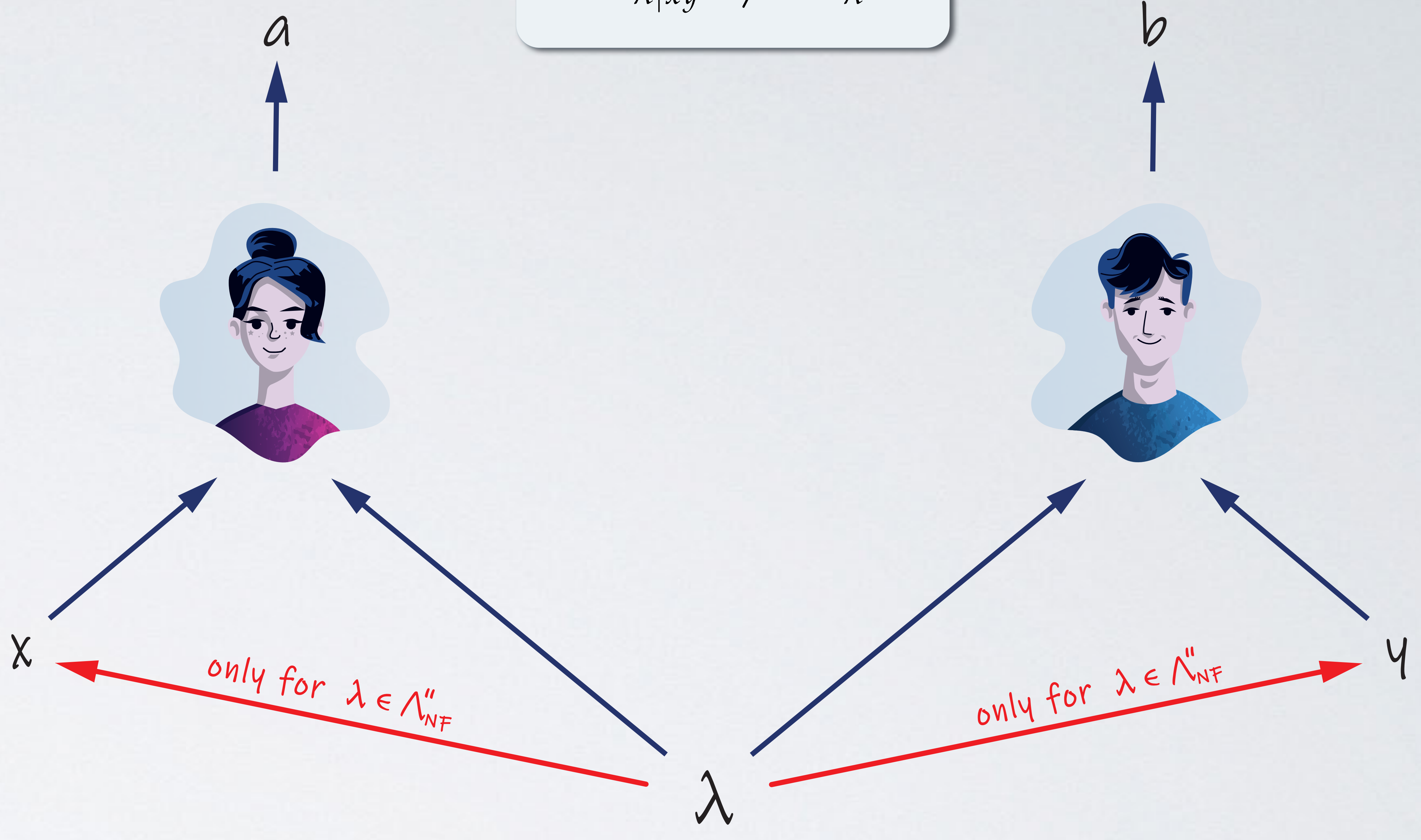
*the **maximal fraction** of trials in which Alice and Bob **can grant freedom of choice** of settings in trying to simulate a given behaviour $\{P_{ab|xy}\}_{xy}$ for <u>any</u> distribution of settings P_{xy}, optimised over <u>all</u> conceivable **local strategies**.*

*Violation of **Locality*** vs. *Violation of **Free choice***

$$P_{ab|xy\lambda} \neq P_{a|x\lambda} \cdot P_{b|y\lambda}$$



$$P_{\lambda|xy} \neq P_{\lambda}$$



*Measure of **locality***

$$\mu_L := \min_{P_{xy}} \max_{FHV} \sum_{\lambda \in \Lambda_L} P_\lambda$$

*Hidden variable model with **free choice***

*Measure of **free choice***

$$\mu_F := \min_{P_{xy}} \max_{LHV} \sum_{\lambda \in \Lambda_F} P_\lambda$$

*Hidden variable model with **locality***

Comparison

Theorem:

For a given behaviour $\{P_{ab|xy}\}_{xy}$ the degree of **locality** and **free choice are the same**, i.e.

$$\mu_L = \mu_F$$

- <u>Any</u> number of settings and outcomes
- Readily extends to <u>any</u> number of parties $\{P_{abc...|xyz...}\}_{xyz...}$

- How to prove:

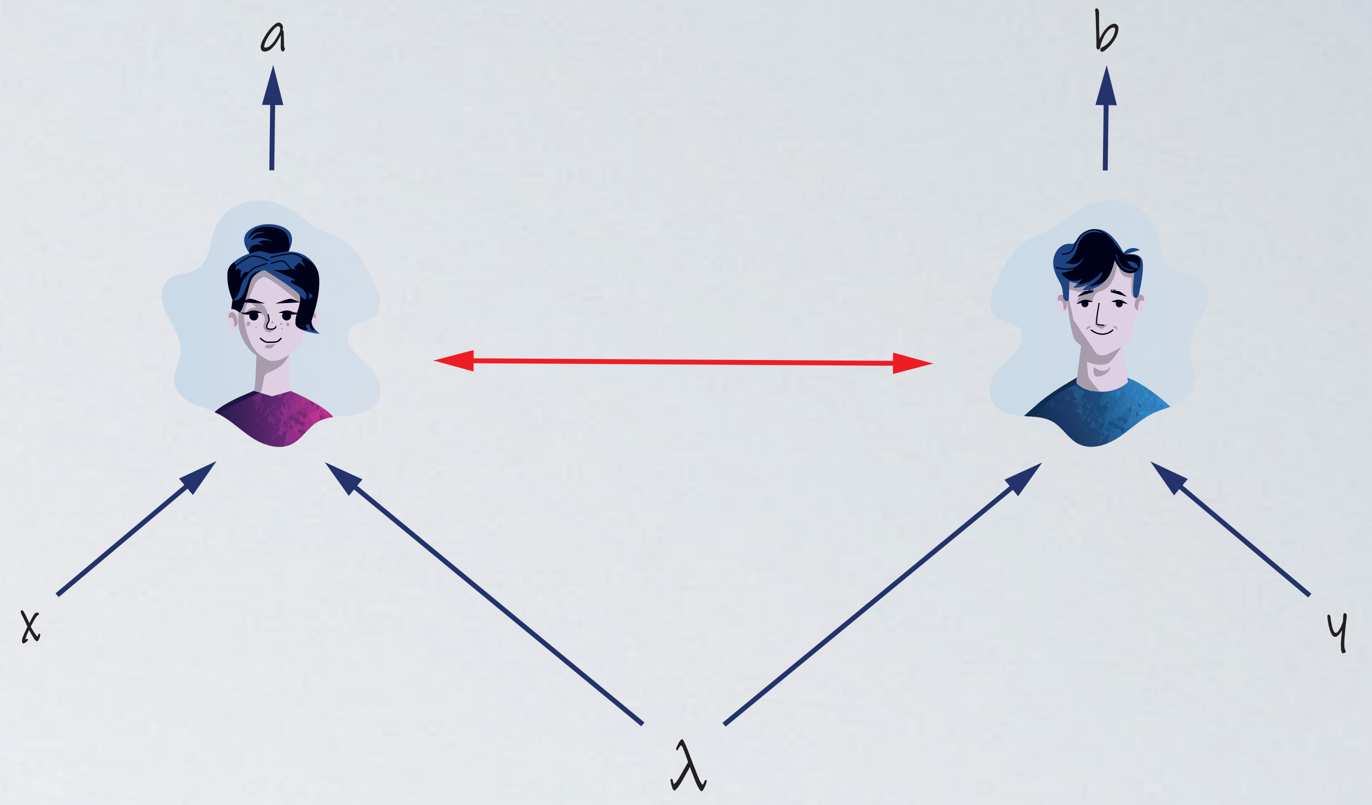
$$P_{ab|xy} = \sum_{\lambda\in\Lambda} P_{ab|xy\lambda} \cdot P_{\lambda|xy}$$

$$P_{xy} = \sum_{\lambda\in\Lambda} P_{xy|\lambda} \cdot P_{\lambda}$$

Hidden variable model

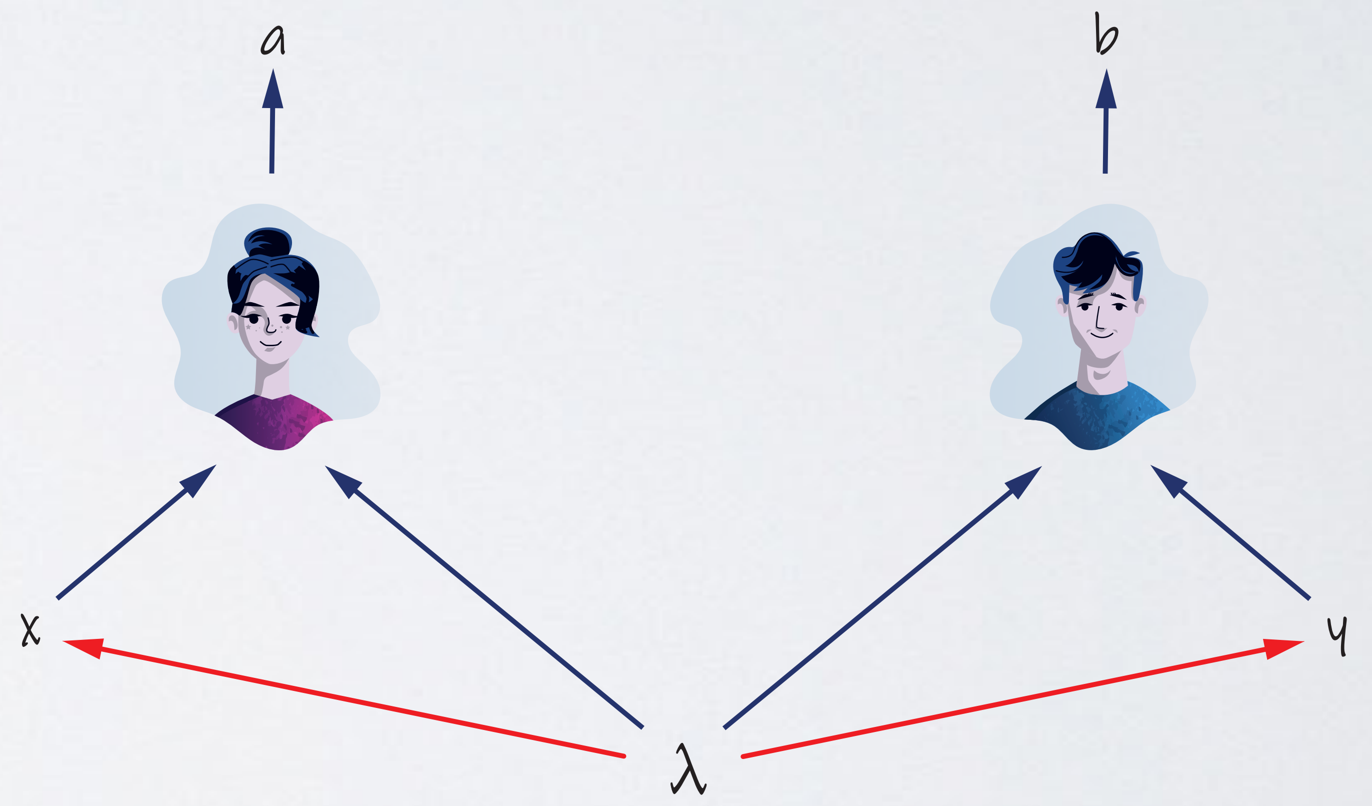
- Get rid of the mins (warning)

- Bijective construction



Measure of locality

$$\mu_L := \min_{P_{xy}} \max_{FHV} \sum_{\lambda\in\Lambda_L} P_\lambda$$



Measure of free choice

$$\mu_F := \min_{P_{xy}} \max_{LHV} \sum_{\lambda\in\Lambda_F} P_\lambda$$

Comparison

Theorem:

*For a given behaviour $\{P_{ab|xy}\}_{xy}$ the degree of **locality** and **free choice are the same**, i.e.*

$$\mu_L = \mu_F$$

- *<u>Any</u> number of settings and outcomes*
- *Readily extends to <u>any</u> number of parties $\{P_{abc...|xyz...}\}_{xyz...}$*

- *How to prove:*

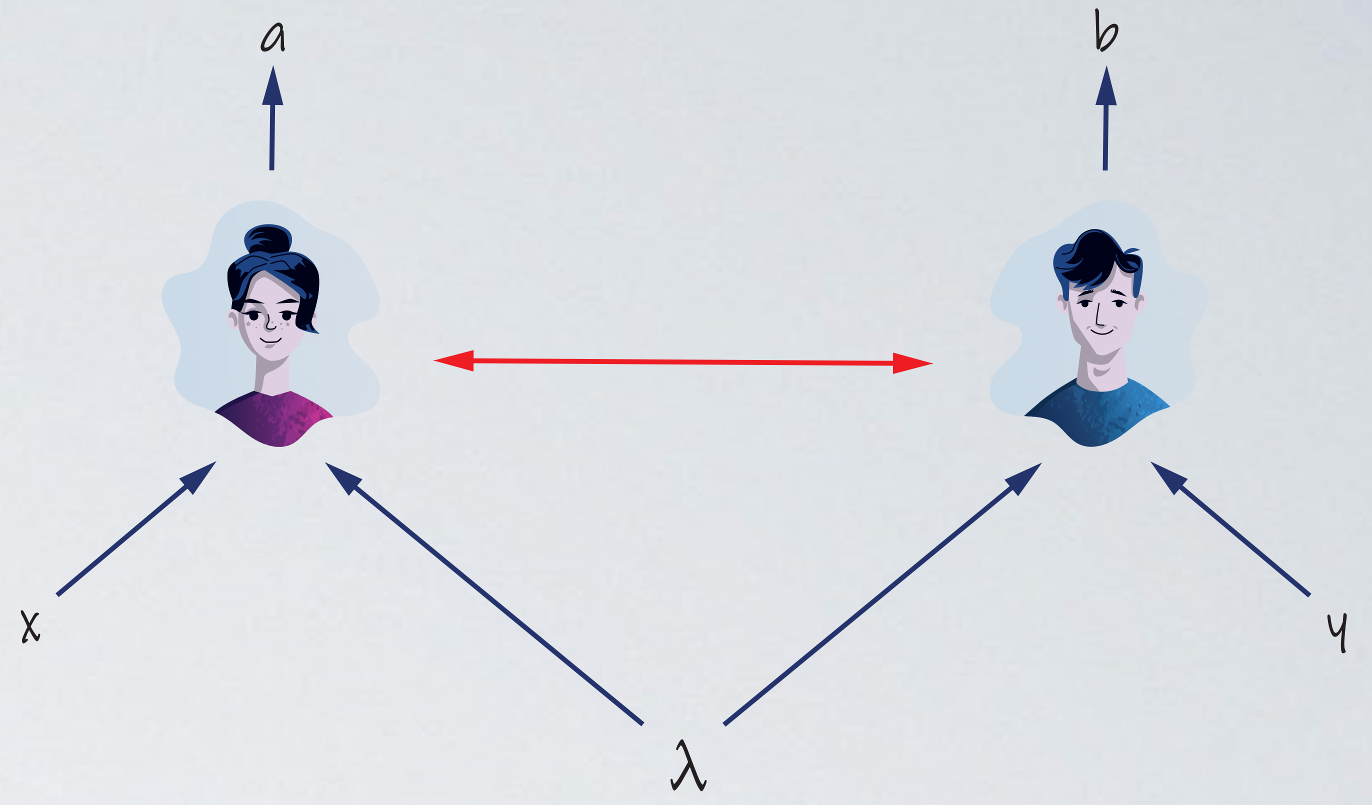
$$P_{ab|xy} = \sum_{\lambda\in\Lambda} P_{ab|xy\lambda} \cdot P_{\lambda|xy}$$

$$P_{xy} = \sum_{\lambda\in\Lambda} P_{xy|\lambda} \cdot P_\lambda \quad \Rightarrow \quad \tilde{P}_{xy} = \sum_{\lambda\in\Lambda} \tilde{P}_{xy|\lambda} \cdot P_\lambda$$

- Get rid of the mins (warning)

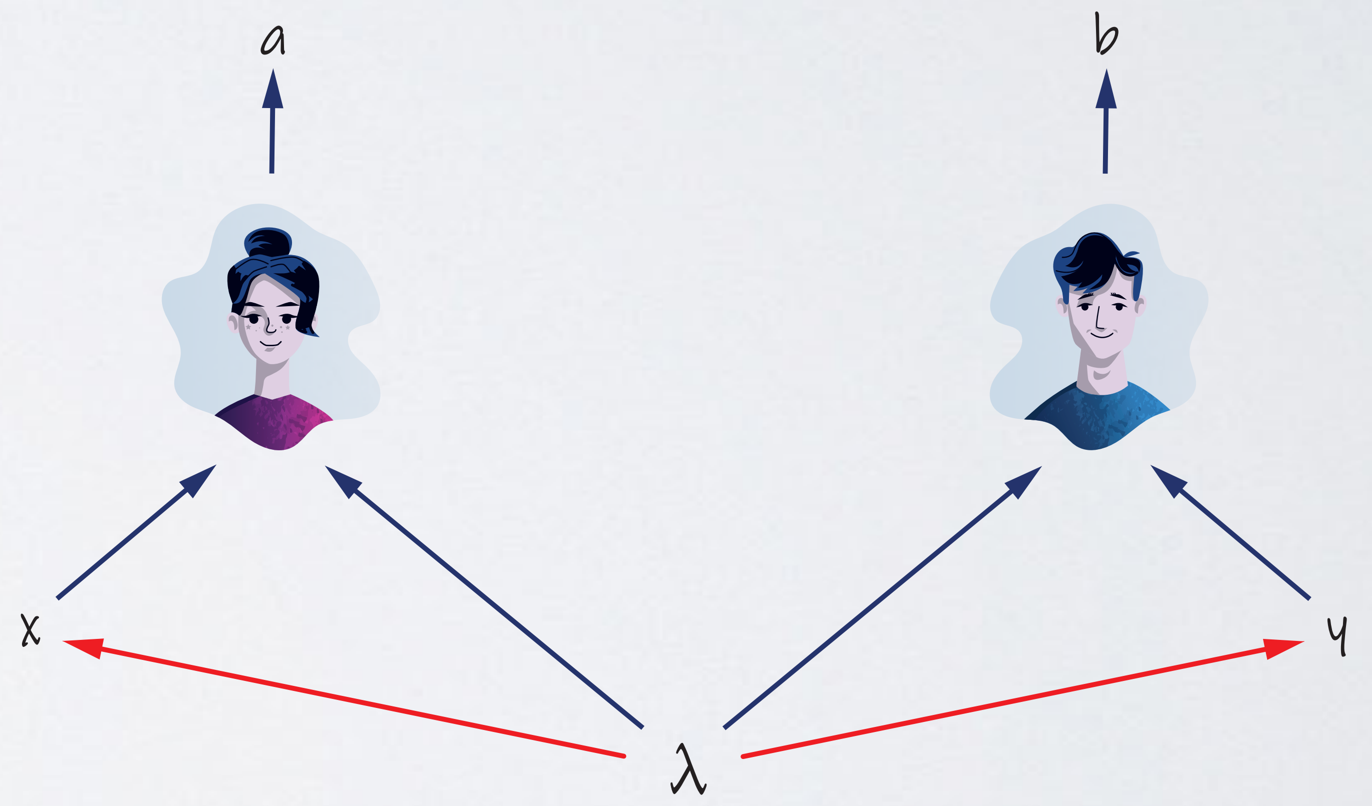
- Bijective construction

$$\tilde{P}_{xy|\lambda} = \frac{P_{\lambda|xy} \cdot \tilde{P}_{xy}}{P_\lambda} \overset{?}{\leqslant} 1$$



Measure of locality

$$\mu_L := \cancel{\min_{P_{xy}}} \max_{FHV} \sum_{\lambda\in\Lambda_L} P_\lambda$$



Measure of free choice

$$\mu_F := \cancel{\min_{P_{xy}}} \max_{LHV} \sum_{\lambda\in\Lambda_F} P_\lambda$$

Democritus
(460 - 370 BC)

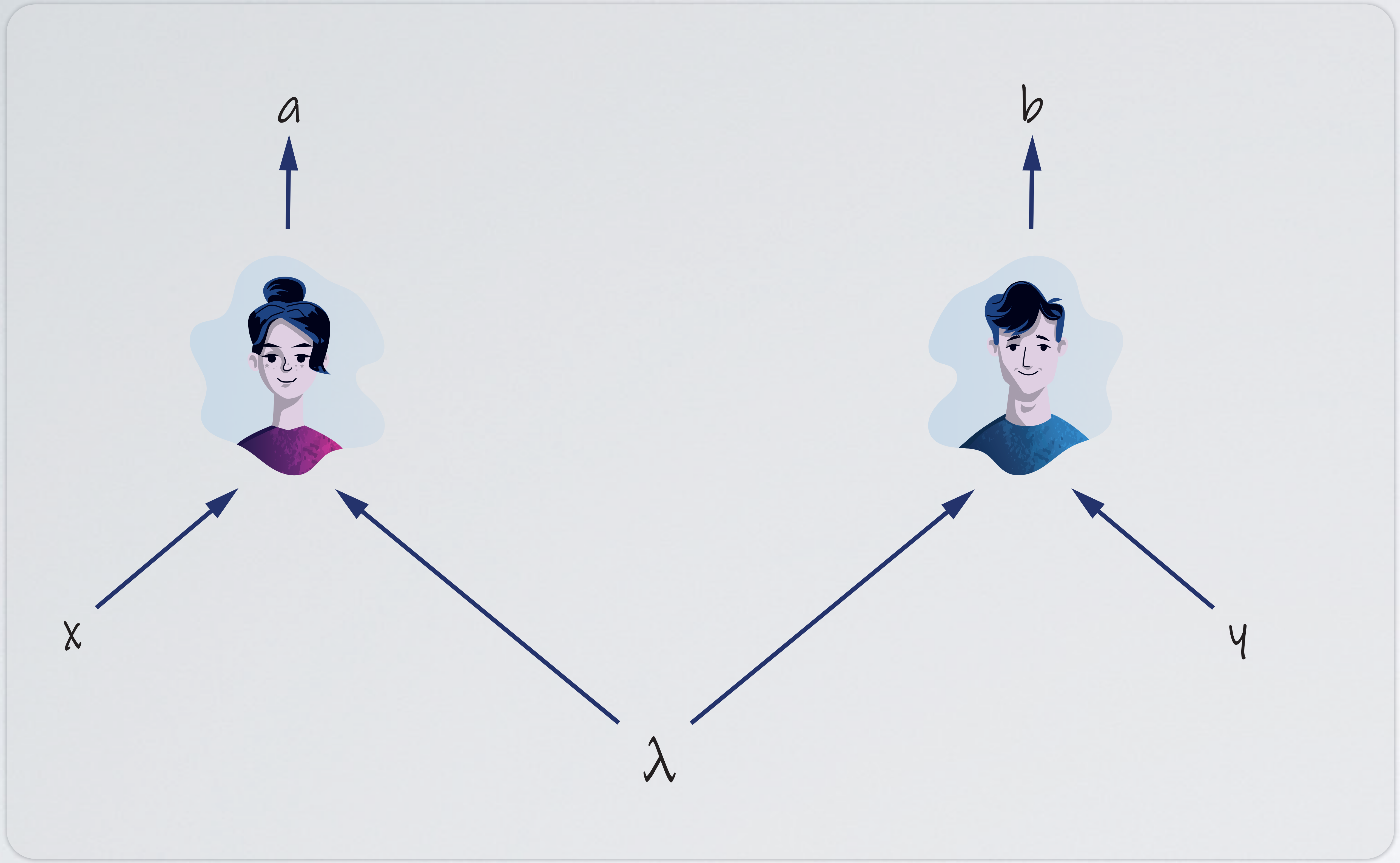
"I would rather discover one true cause than gain the kingdom of Persia."

$$\mu_L = \mu_F = ?$$

"Measure what is measurable, and make measurable what is not so."

Galileo GALILEI
(1564 - 1642)

Bell experiment — recap (II)



Bell's theorem

Realism + Locality + Free choice: $|S_i| \leqslant 2$

whereas in QM it can be: $|S_i| \leqslant 2\sqrt{2} \leqslant 4$

Tsirelson *PR- box*

- *Bell-CHSH expressions*

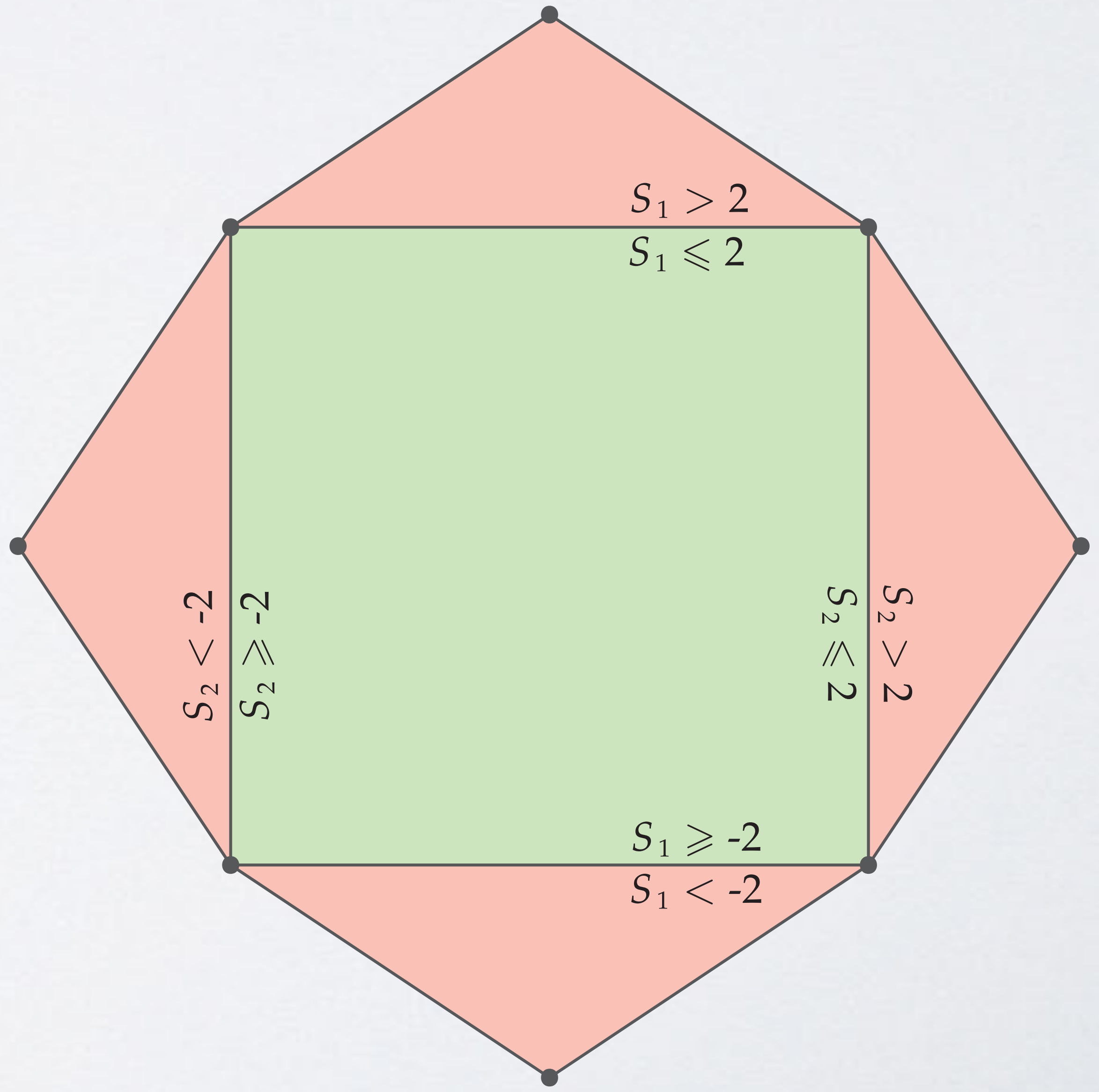
$$\begin{aligned} S_1 &= \langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} - \langle ab\rangle_{11} \\ S_2 &= \langle ab\rangle_{00} + \langle ab\rangle_{01} - \langle ab\rangle_{10} + \langle ab\rangle_{11} \\ S_3 &= \langle ab\rangle_{00} - \langle ab\rangle_{01} + \langle ab\rangle_{10} + \langle ab\rangle_{11} \\ S_4 &= -\langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} + \langle ab\rangle_{11} \end{aligned}$$

where: $\langle ab\rangle_{xy} = \sum_{a,b} ab\, P_{ab|xy}$

- *Non-signalling*

$$\begin{aligned} P_{b|0y} &= P_{b|1y} \quad \text{for all } b, y \\ P_{a|x0} &= P_{a|x1} \quad \text{for all } a, x \end{aligned}$$

- *Non-signalling polytope (free choice assumed)*



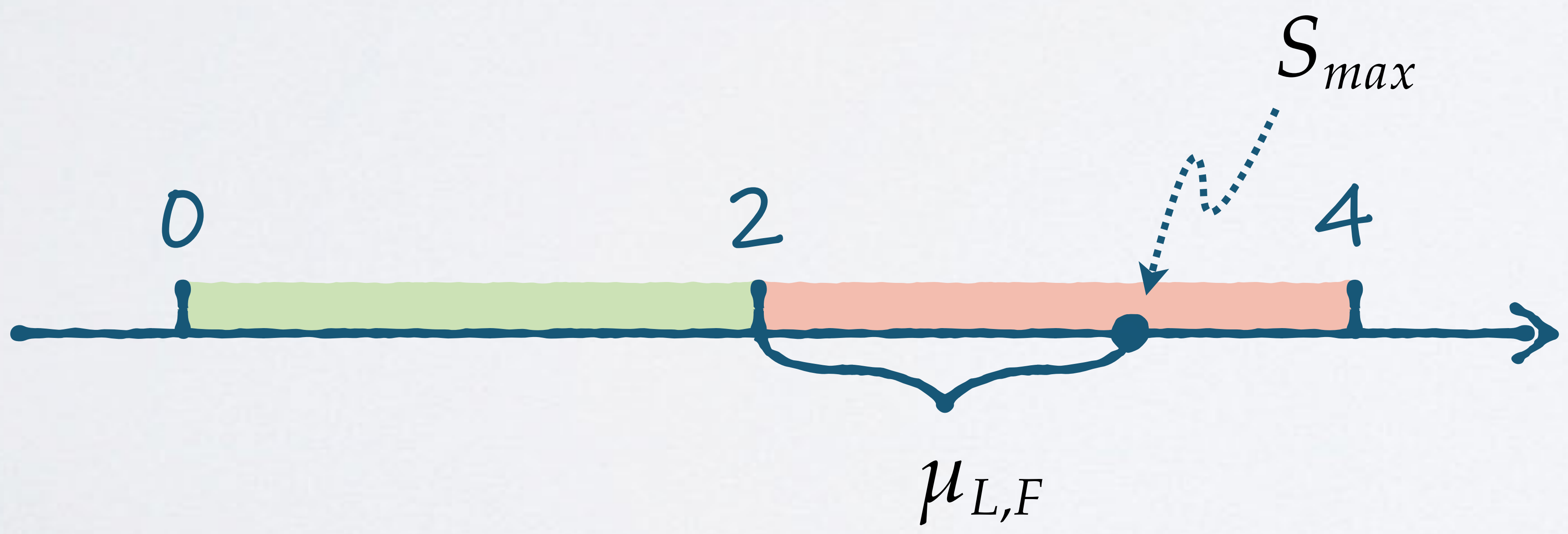
Explicit measure for Bell scenario

Theorem:

For a given ***non-signalling*** *behaviour* $\{P_{ab|xy}\}_{xy}$ *with* ***binary settings*** *both measures of locality* μ_L *and free choice* μ_F *are equal to*

$$\mu_L \;=\; \mu_F \;=\; \begin{cases} \frac{1}{2}(4 - S_{max})\,, & \text{if } S_{max} > 2\,, \\ 1\,, & \text{otherwise}\,, \end{cases}$$

where $S_{max} = \max\{|S_i| : i = 1, \dots, 4\}$ *is the maximum absolute value of the four CHSH expressions.*



- *Convex decomposition*

$$P_{ab|xy} \;=\; p_L \cdot P^L_{ab|xy} + (1 - p_L)\cdot P^{NL}_{ab|xy}$$

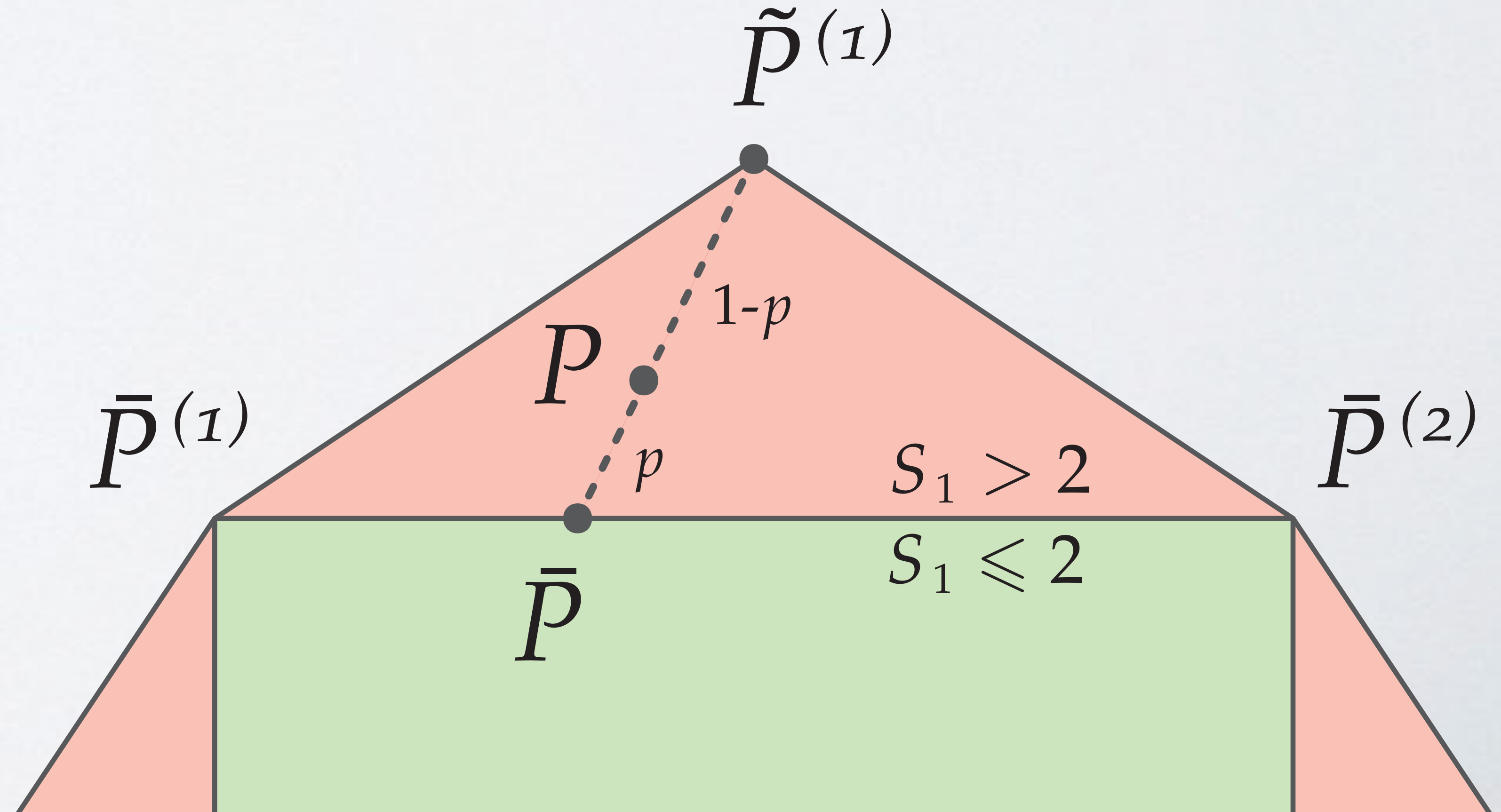
$$\mu_L \;=\; \max_{\text{decomp.}} \; p_L$$

- *Upper bound*

$$p_L \;\leqslant\; \tfrac{1}{2}(4 - |S_i|) \quad \Rightarrow \quad \mu_L \;\leqslant\; \tfrac{1}{2}(4 - S_{max})$$

- *Saturation of the bound*

$$P_{ab|xy} \;=\; \sum_{j=1}^{16} p_j \cdot \bar{P}^{(j)}_{ab|xy} + \sum_{k=1}^{8} q_k \cdot \tilde{P}^{(k)}_{ab|xy}$$



Quantum statistics

- ***Binary settings***

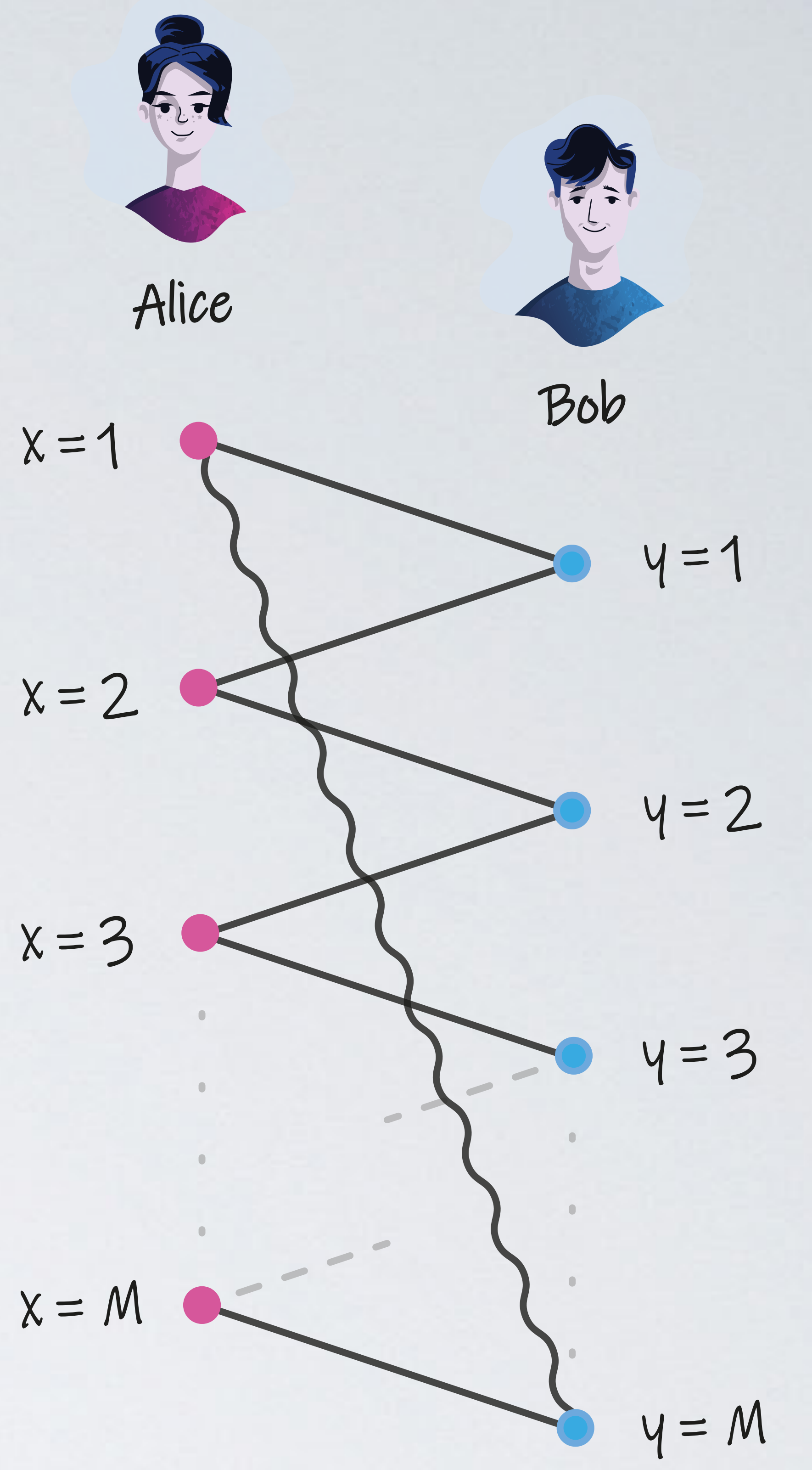
  Tsirelson's bound

$$\mu_L = \mu_F = 2 - \sqrt{2} \approx 0.59$$

- ***Bell state & infinite number of settings***

  A. Elitzur et al., Phys. Lett. A 162, 25 (1992)
  J. Barrett et al., PRL 97, 170409 (2006)

$$\mu_L = \mu_F \xrightarrow[M \to \infty]{} 0$$



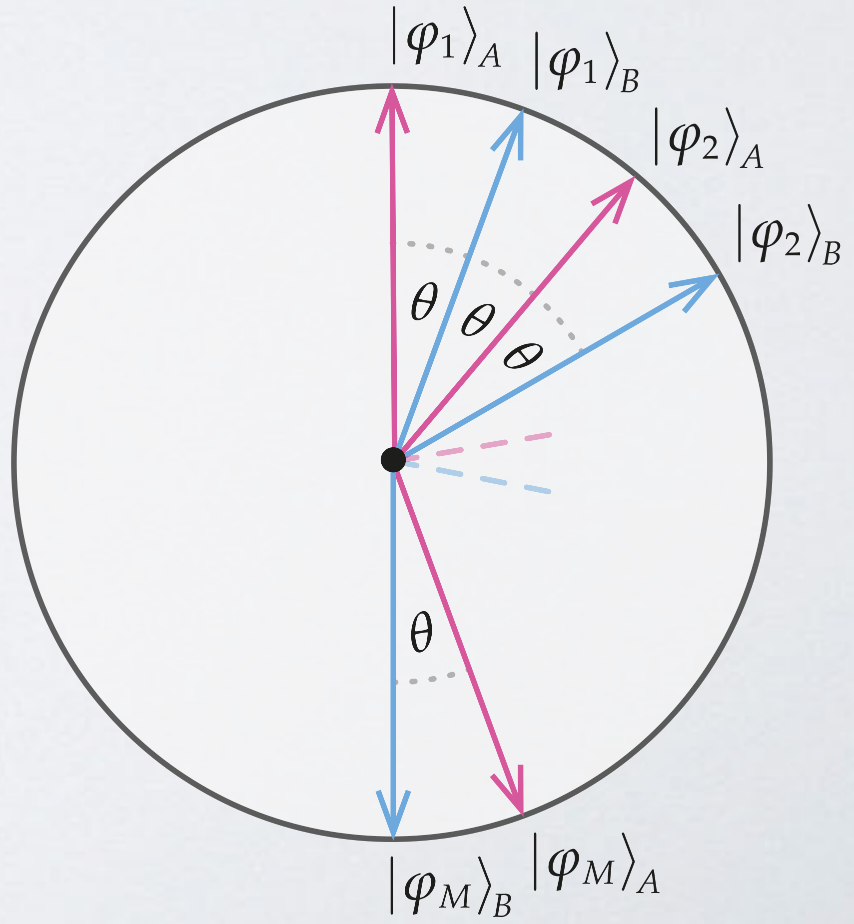
- ***Two-qubit state & arbitrary settings***

$$|\psi\rangle = \cos\tfrac{\theta}{2}\,|00\rangle + \sin\tfrac{\theta}{2}\,|11\rangle$$

$$\theta \in [0, \tfrac{\pi}{2}]$$

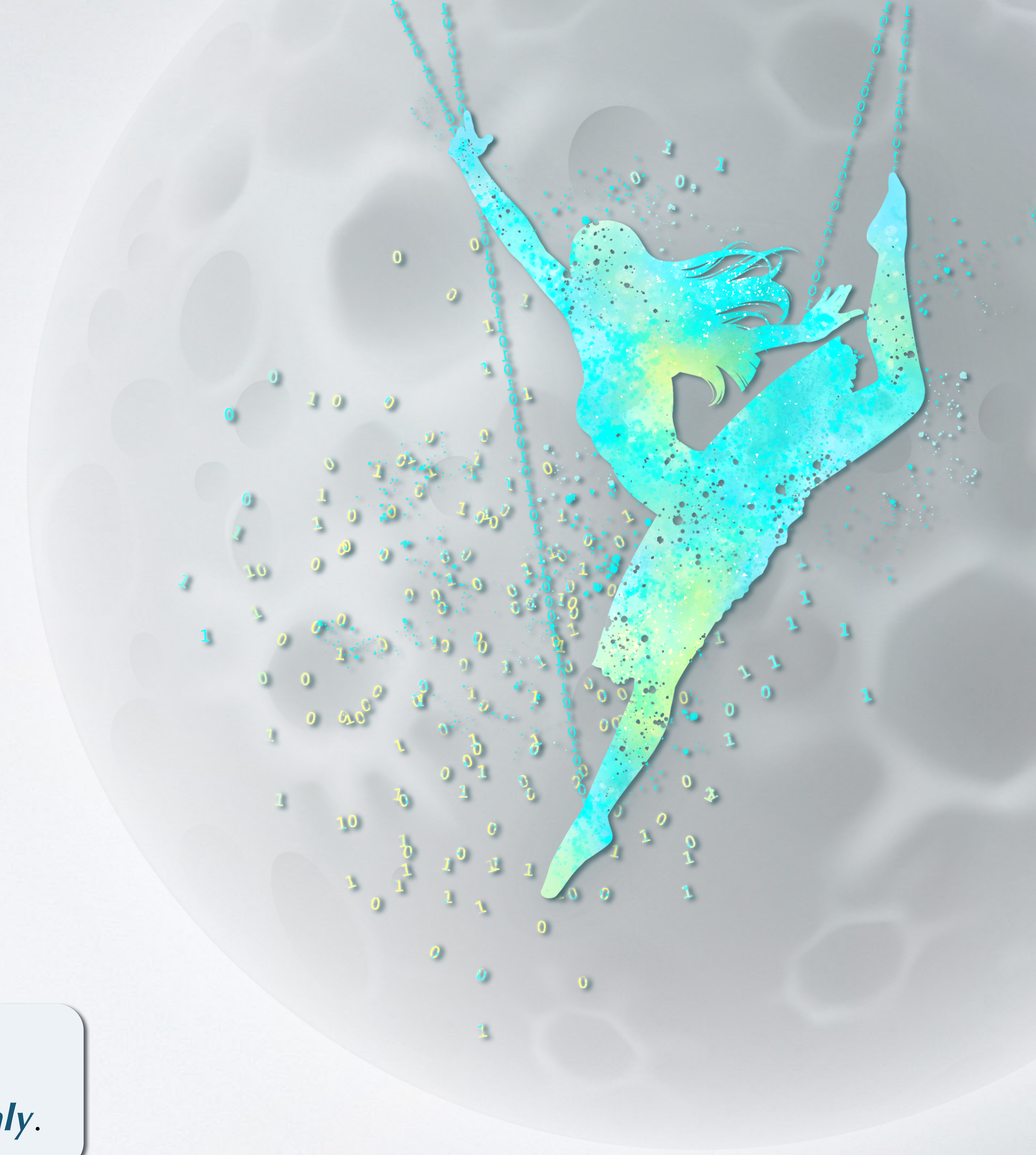
  S. Portman et al., PRA 86, 012104 (2012)

$$\mu_L = \mu_F = \cos\theta$$



Summary of results (… conclusions)

		Locality	Free choice
Any statistics	*any no. settings*	$\mu_L = \mu_F$	
	non-signalling two settings	$\mu_L = \frac{1}{2}(4 - S_{max})$	$\mu_F = \frac{1}{2}(4 - S_{max})$
Quantum statistics	*Bell state infinite no. settings*	$\mu_L \xrightarrow[M \to \infty]{} 0$ (*)	$\mu_F \xrightarrow[M \to \infty]{} 0$
	two-qubit state any no. settings	$\mu_L = \cos\,\theta$ (*)	$\mu_F = \cos\,\theta$



Take aways

- *Which is **more costly locality** or **free choice**?*
- ***Calculate** and **compare** measures of both in **QM** and **not only**.*

Blasiak et al., PNAS 118 (17) e2020569118 (2021)


Thank you for your attention
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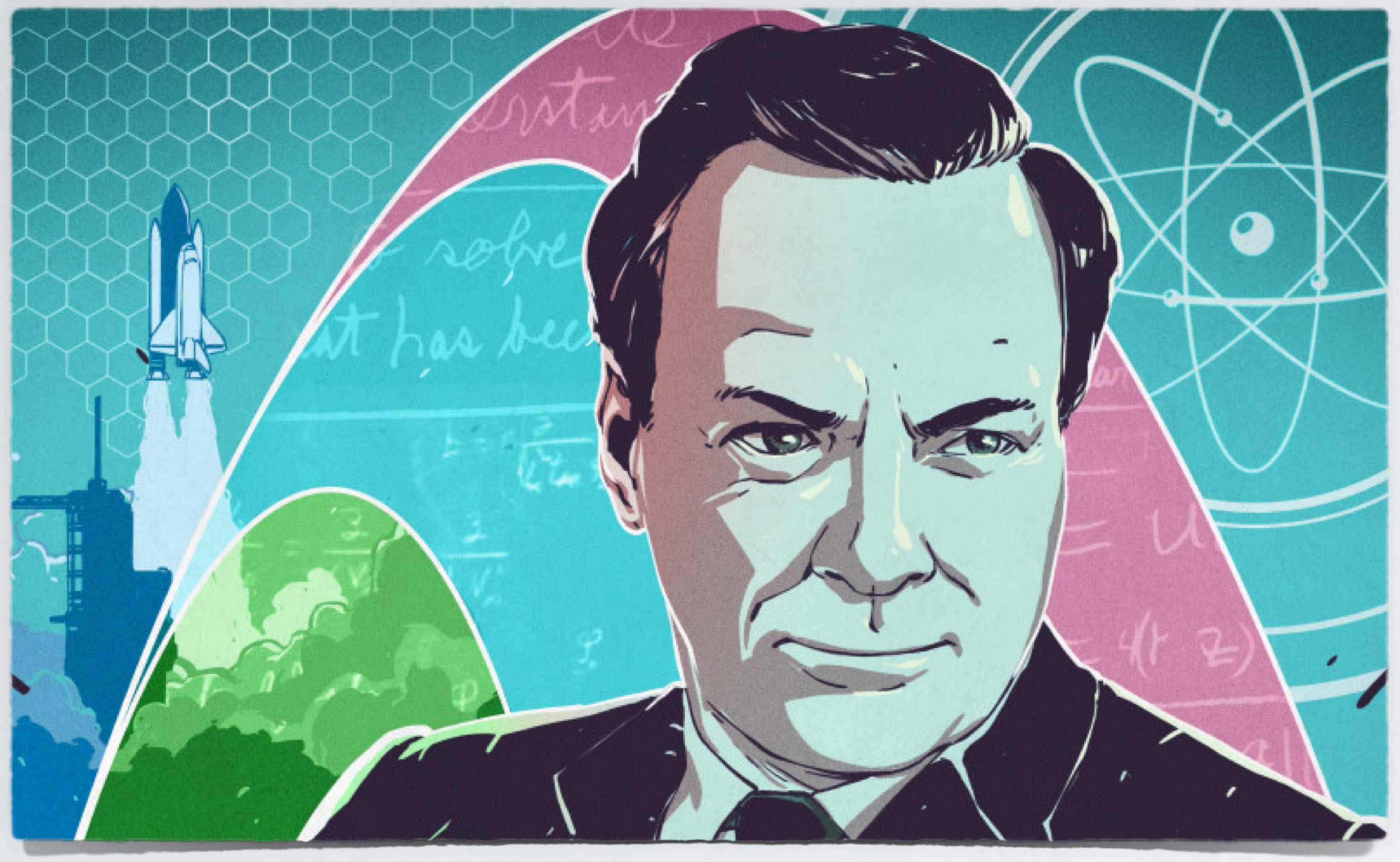
Quantum mechanics … as we have it

"I should begin by expressing my general attitude to ***present-day quantum theory****, by which I mean standard non-relativistic quantum mechanics. The theory has, indeed, two powerful bodies of fact in its favour, and only one thing against it. First, in its favour are all the* ***marvellous agreements*** *that the theory has had with every experimental result to date. Second, and to me almost as important, it is a theory of* ***astonishing and profound mathematical beauty****. The one thing that can be said against it is that* ***it makes absolutely no sense!****"*

Roger Penrose
"Gravity and ***State Vector Reduction****"*
in: "Quantum Concepts in Space and Time" (1986)

Sir Roger PENROSE
(1931)

Feynman on free will



Richard P. FEYNMAN
(1918-1988)

"Now, we say 'this kind of logic;' what other possibilities are there? Perhaps there may be no possibilities, but perhaps there are.
[…]
***We have an illusion that we can do any experiment that we want**. We all, however, come from the same universe, have evolved with it, and **don't really have any 'real' freedom**. For we obey certain laws and have come from a certain past. Is it somehow that **we are correlated to the experiments that we do**, so that the apparent probabilities don't look like they ought to look if you assume they are random. There are all kinds of questions like this […]. In fact the **physicists have no good point of view**."*

Richard P. Feynman
"Simulating physics with computers"
Int. J. Theor. Phys. **21** 467 (1982)