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Jet-evolution in a medium via coherent emissions and scatterings

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based on:

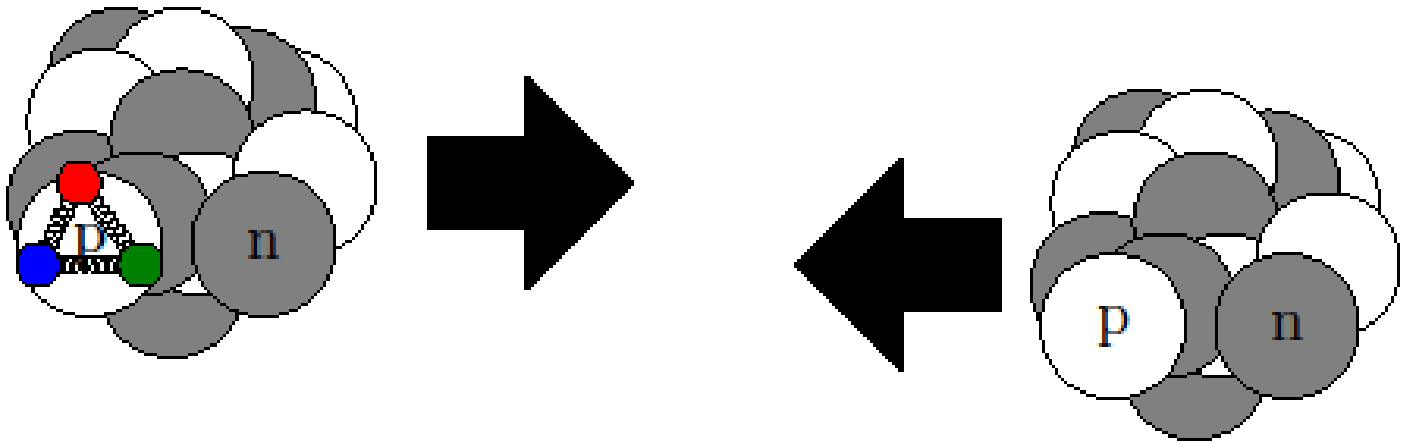
[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918], [MR, arxiv: 2111.00323] (quark+gluon jets; Monte-Carlo)

[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014] (k_T broadening in gluon jets)

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317] (Monte Carlo for gluon fragmentation functions)

QGP tomography via hard probes

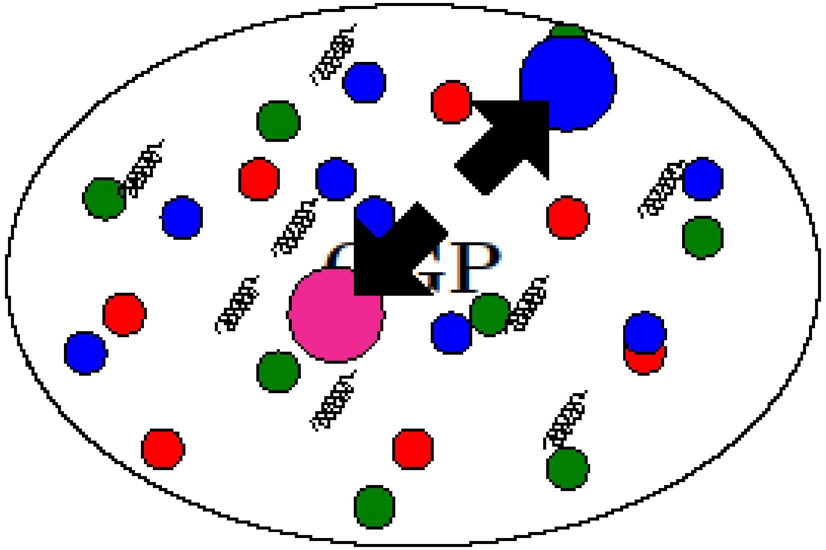
Color degrees of freedom (quarks and gluons):
Confined within proton and neutrons



…collision experiments for heavy ions e.g. at the LHC at CERN or at RHIC at BNL

QGP tomography via hard probes

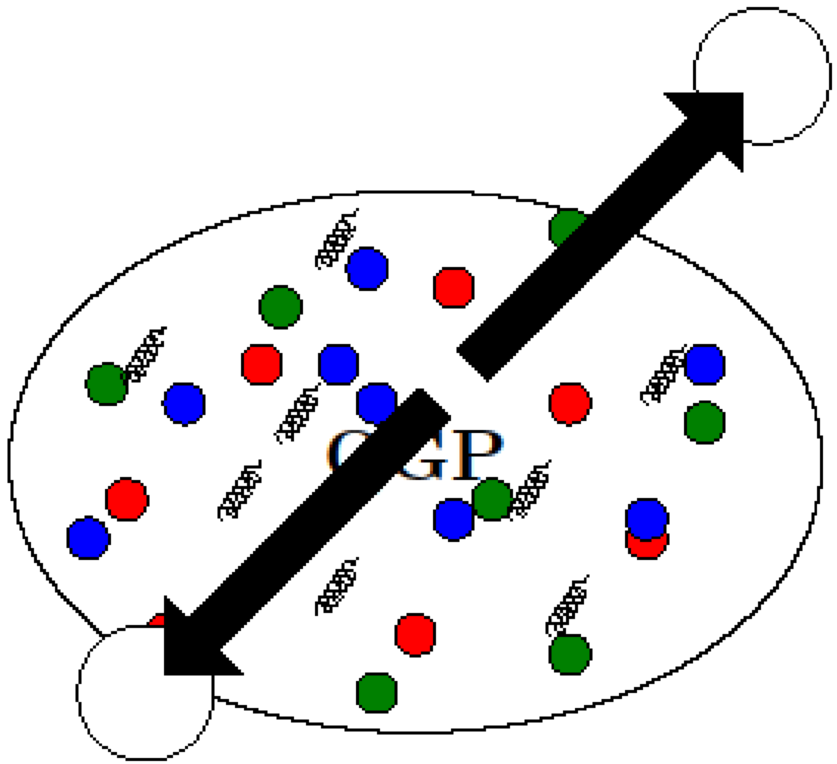
…liquid state, Quark Gluon Plasma (QGP)



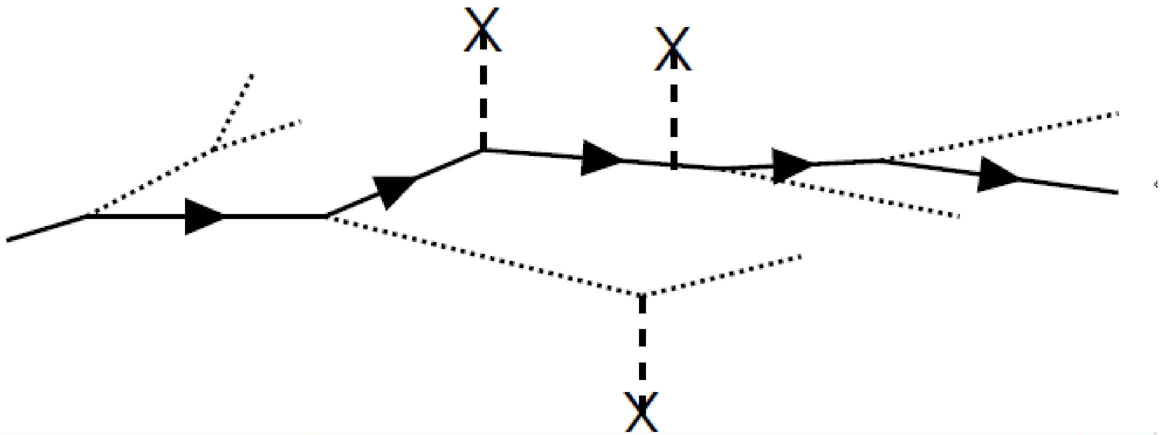
…tested via hard probes:

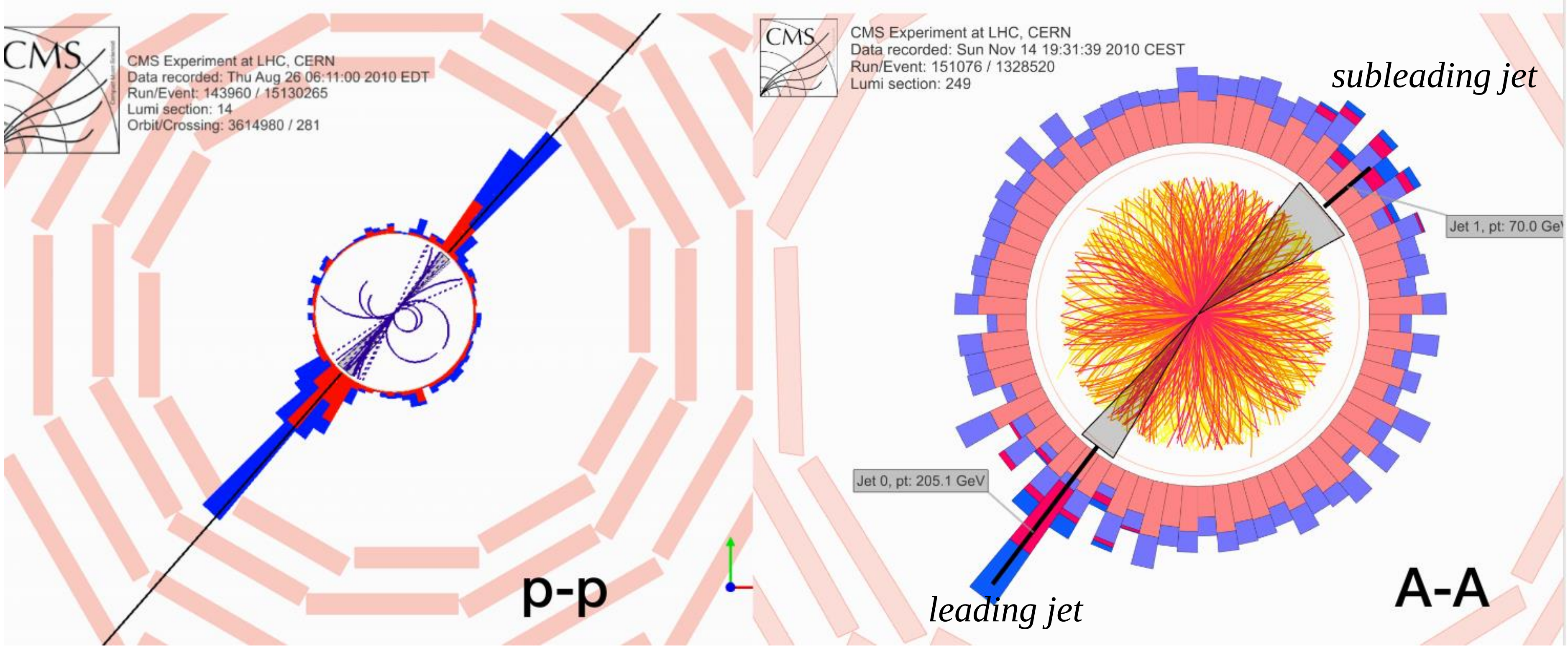
highly energetic, strongly interacting particles

QGP tomography via hard probes



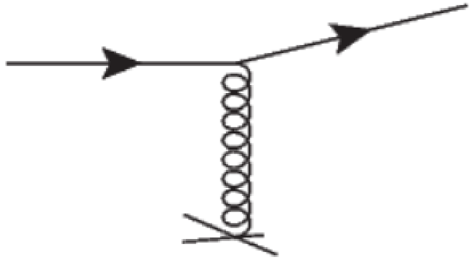
Jet Quenching





Processes in jets in the medium

scattering…



Momentum transfer!

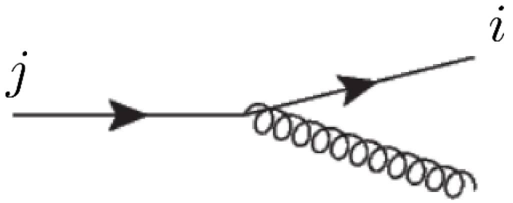
$$p \to p + Q$$

Scattering Kernel:

$$\frac{\partial^3 \mathcal{P}_{\text{scat}}}{\partial t \partial^2 \mathbf{Q}} = \frac{1}{(2\pi)^2} w(\mathbf{Q})$$

Average transfer: $\hat{q}$

…splitting…



Bremsstrahlung as
in vacuum.

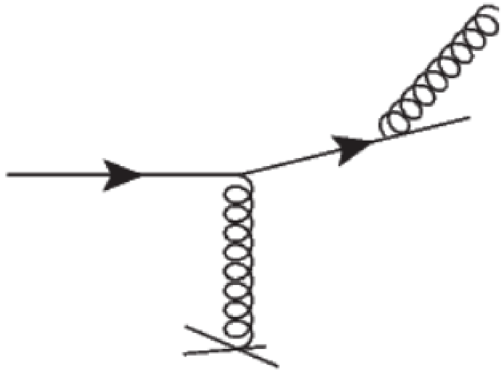
Momentum distribution:

$$p \to zp$$

DGLAP-Kernel:

$$\frac{\partial^2 \mathcal{P}_{\text{split}}}{\partial \hat{t} \partial z} \propto \frac{1}{\hat{t}} P_{ij}(z)$$

…induced radiation



Momentum distribution:

$$p \to zp$$

+Momentum transfer:

$$p \to zp + Q$$

Kernel: $\frac{\partial^4 \mathcal{P}_{\text{split}}}{\partial t \partial z \partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2} \mathcal{K}(\mathbf{Q}, z, p_+)$

This talk: combination of scattering and induced radiation processes!

Coherent emission

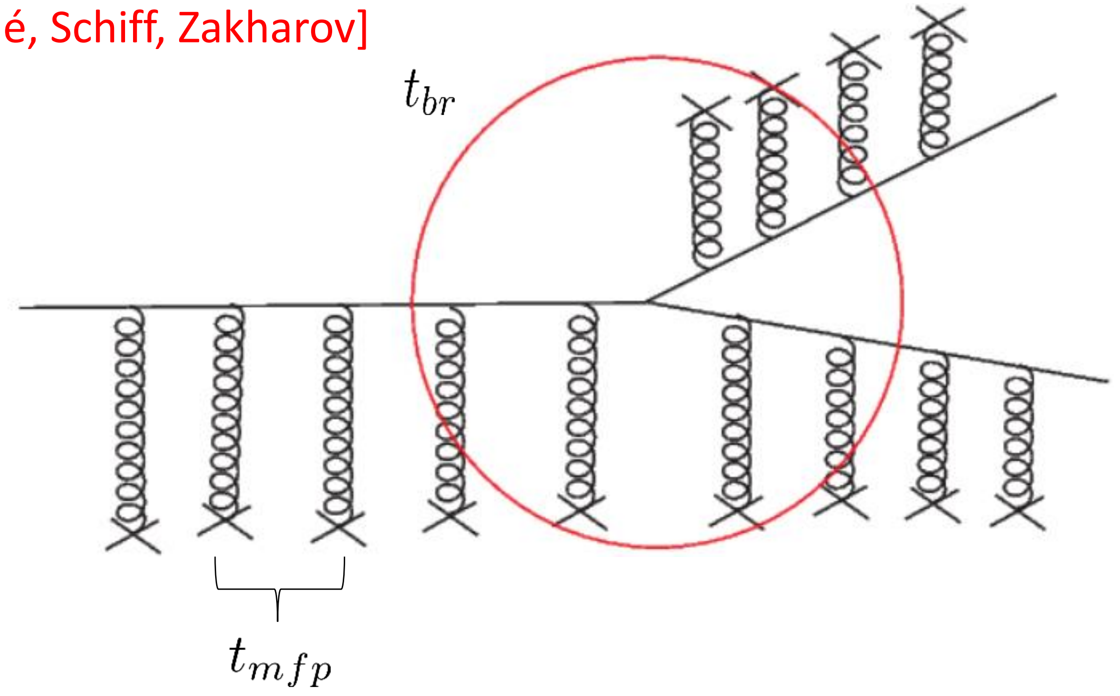
…à la BDMPS-Z [Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov]

$$t_{br} \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

$t_{br} \sim t_{mfp}$: one scattering + radiation
…Bethe-Heitler spectrum

$t_{br} \gg t_{mfp}$: coherent radiation

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_{br}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$



Look at range: $\omega_{BH} < \omega < \omega_c$

need effective kernel: $\mathcal{K}(z, k_T)$

cf. [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]
[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Splitting Kernels for Quarks and Gluons

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918], [Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1301 (2013) 143]

$$\mathcal{K}_{ij}(\boldsymbol{Q}, z, p_0^+) = \frac{2P_{ij}(z)}{z(1-z)p_0^+} \sin\left(\frac{\boldsymbol{Q}^2}{2k_{\rm br}^2}\right) \exp\left(-\frac{\boldsymbol{Q}^2}{2k_{\rm br}^2}\right)$$

$$k_{\rm br}^2 = \sqrt{z(1-z)p_0^+ f_{ij}(z)\frac{\hat{q}}{N_c}}$$

$$f_{gg}(z) = (1-z)C_A + z^2 C_A$$

$$f_{qg}(z) = C_F - z(1-z)C_A\,,$$

$$f_{gq}(z) = (1-z)C_A + z^2 C_F$$

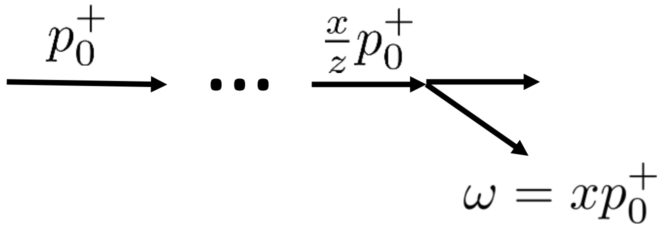
$$f_{qq}(z) = zC_A + (1-z)^2 C_F$$

$$\frac{\partial^4 \mathcal{P}_{\rm split}}{\partial t\,\partial z\,\partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2}\mathcal{K}(\mathbf{Q}, z, p_+)$$

$$\frac{\partial^5 \mathcal{P}_{\rm split}}{\partial t\,\partial x\,\partial z\,\partial^2 \mathbf{Q}} = \frac{\alpha_s}{(2\pi)^2}\mathcal{K}(\mathbf{Q}, z, \tfrac{x}{z}p_0^+)$$

$$\downarrow \int_0^\infty d^2\mathbf{Q}\times$$

$$\frac{\partial^3 \mathcal{P}_{\rm split}}{\partial t\,\partial x\,\partial z} = \frac{2\pi}{\sqrt{x}t^*}\sqrt{z}\mathcal{K}(z)$$



$$\frac{\partial^2 \mathcal{P}_{\rm split}}{\partial t\,\partial x} = 2\pi\frac{1}{\sqrt{x}t^*}\int dz\sqrt{z}\mathcal{K}(z) \qquad \frac{1}{t^*} = \frac{\alpha_s}{\pi}\sqrt{\frac{\hat{q}}{p_0^+}}$$

$$\sqrt{x}t^* \propto t_{\rm br}$$

Generalization of BDMPS-Z approach

Scattering Kernels

Used right now:

$$w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n_{\mathrm{med}}}{\mathbf{q}^4}$$

$$w_g(\mathbf{q}) = \frac{g^2 m_D^2 T}{\mathbf{q}^2(\mathbf{q}^2 + m_D^2)}\,, \qquad g^2 = 4\pi\alpha_s$$

$n_{\mathrm{med}} \ldots$ density of scatterers
$m_D \ldots$ Debye mass
$T \ldots$ medium temperature

$$w_q(\mathbf{q}) = \frac{C_F}{C_A} w_g(\mathbf{q})$$

Sudakov factors

Probabilities of interaction:

$$\Phi_g(x) = \alpha_s \int_\epsilon^{1-\epsilon} dz \int_{q>0} \frac{d^2\mathbf{q}}{(2\pi)^2} \left[\mathcal{K}_{gg}(\mathbf{q}, z, xp_+) + \mathcal{K}_{qg}(\mathbf{q}, z, xp_+)\right] + \int_{q>q_{\min}} \frac{d^2\mathbf{q}}{(2\pi)^2} w_g(\mathbf{q}) ,$$

$$\Phi_q(x) = \alpha_s \int_\epsilon^{1-\epsilon} dz \int_{q>0} \frac{d^2\mathbf{q}}{(2\pi)^2} \mathcal{K}_{qq}(\mathbf{q}, z, xp_+) + \int_{q>q_{\min}} \frac{d^2\mathbf{q}}{(2\pi)^2} w_q(\mathbf{q}) ,$$

Probability of no interaction for particle A over time (t_2-t_1):

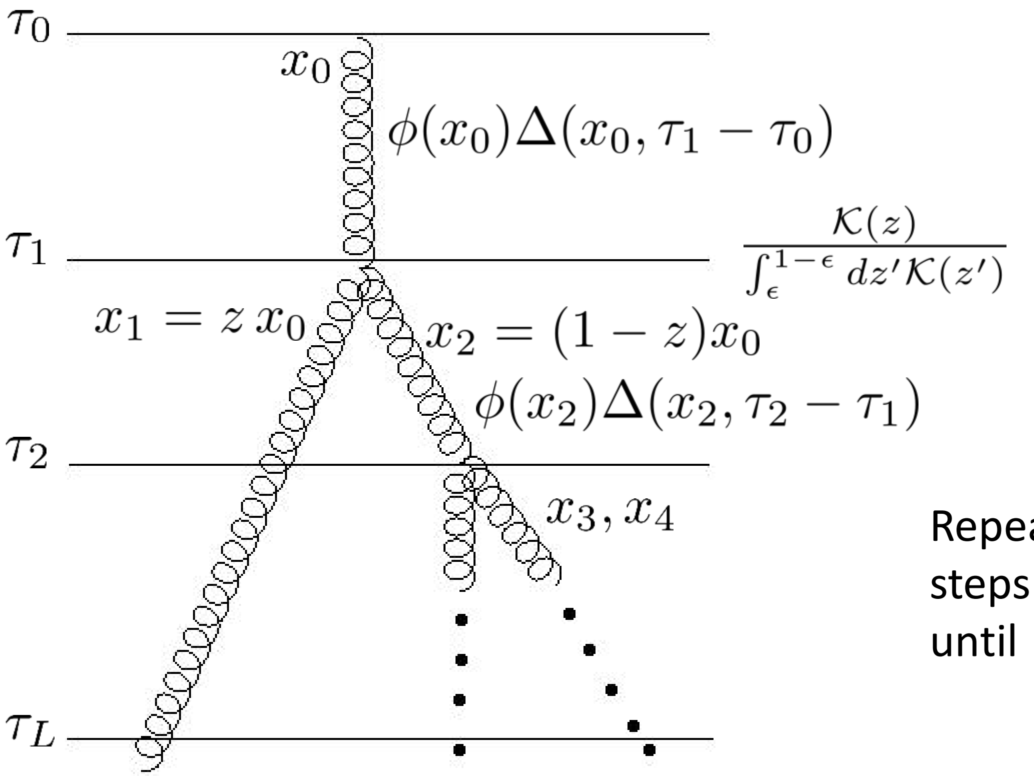
$$\Delta_A(x, t_2 - t_1) = \exp\left(-\Phi_A(x)(t_2 - t_1)\right)$$

… Sudakov factor

Monte-Carlo algorithms for jets

Other codes implementing BDMPS-Z spectra:

MARTINI, JEWEL, QPYTHIA, …



Analogous for the k_T dependent equation in x, k_T, and, τ and system of equations!

Repeat for all steps in τ and x until $\tau > \tau_L$

TMDICE code: [MR, arxiv: 2111.00323]

- Written in C++
- Source code available at https://github.com/Rohrmoser/TMDICE

Link to Evolution equations

[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Collinear evolution: $\mathcal{K}(z)$, $w(\mathbf{q}) = 0$

$$D(x,\tau) = x\frac{dN}{dx}$$

$$D(x,\tau) = e^{-\phi(x)(\tau-\tau_0)}D(x,\tau_0) + \int_{\tau_0}^{\tau} d\tau' \int_{\epsilon}^{1-\epsilon} dz \int_0^1 dy \delta(x-zy)\sqrt{\frac{z}{x}} z\mathcal{K}(z) e^{-\phi(x)(\tau-\tau')}D(y,\tau')$$

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

$$\tau = \frac{t}{t^*}$$

Monte-Carlo algorithm that solves these evolution equations:

MINCAS
[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Exist direct methods: Chebyshev method, Runge Kutta... [Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

System of Equations for quarks and gluons

[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

$$\frac{\partial}{\partial t} D_g(x,\boldsymbol{k},t) = \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\alpha_s\Big\{2\mathcal{K}_{gg}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z},\boldsymbol{q},t\right) + \mathcal{K}_{gq}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right)\sum_i D_{q_i}\left(\frac{x}{z},\boldsymbol{q},t\right)$$
$$-\left[\mathcal{K}_{gg}(\boldsymbol{q},z,xp_0^+) + \mathcal{K}_{qg}(\boldsymbol{q},z,xp_0^+)\right] D_g(x,\boldsymbol{k},t)\Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2} C_g(\boldsymbol{l})\, D_g(x,\boldsymbol{k}-\boldsymbol{l},t),$$

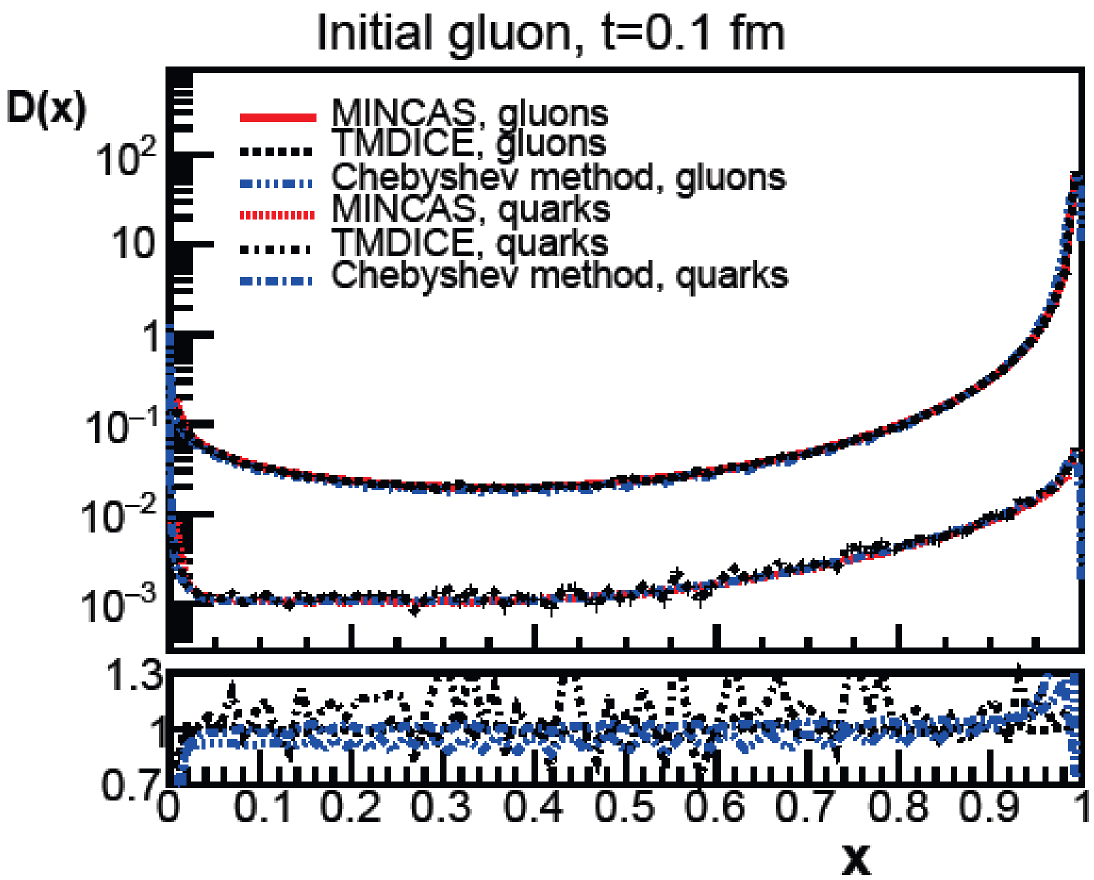
$$\frac{\partial}{\partial t} D_{q_i}(x,\boldsymbol{k},t) = \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\alpha_s\Big\{\mathcal{K}_{qq}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right) D_{q_i}\left(\frac{x}{z},\boldsymbol{q},t\right) + \frac{1}{N_F}\mathcal{K}_{qg}\left(\boldsymbol{Q},z,\frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z},\boldsymbol{q},t\right)$$
$$-\mathcal{K}_{qq}(\boldsymbol{q},z,xp_0^+)\, D_{q_i}(x,\boldsymbol{k},t)\Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2} C_q(\boldsymbol{l})\, D_{q_i}(x,\boldsymbol{k}-\boldsymbol{l},t)\,,$$

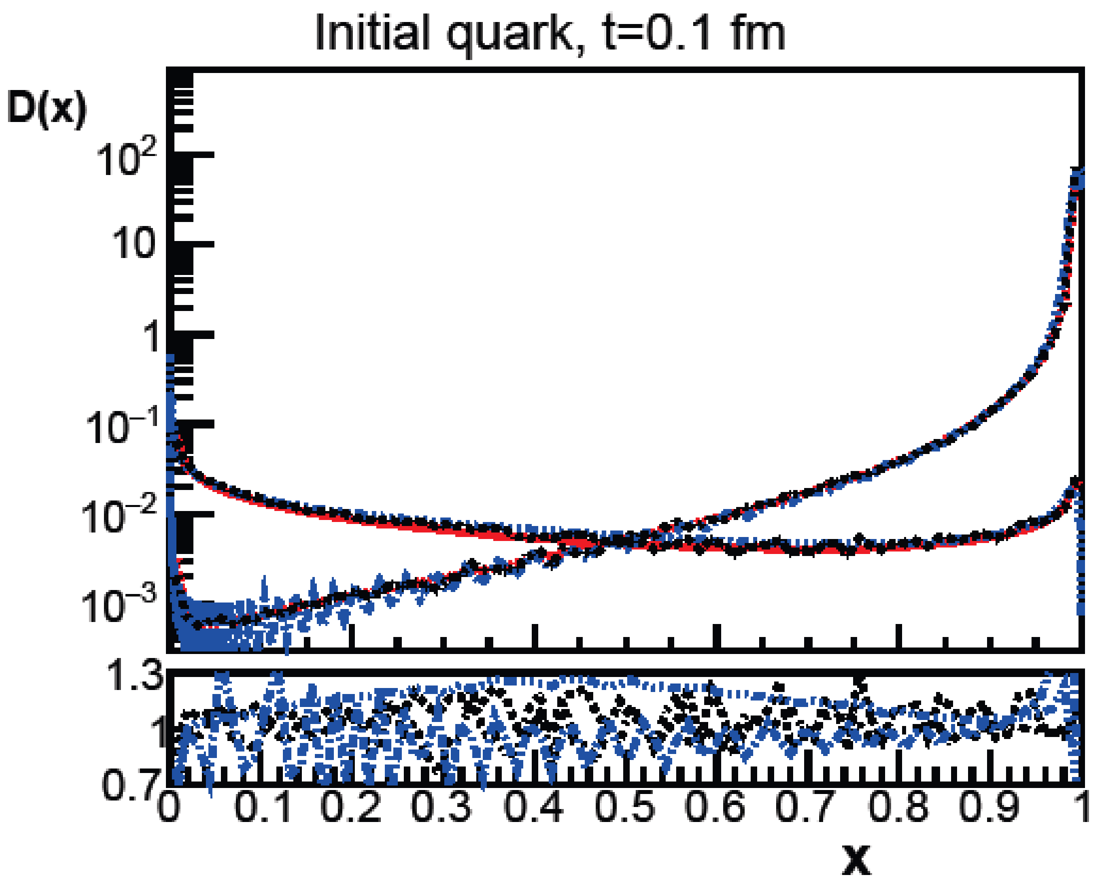
$$C_{q(g)}(\boldsymbol{l}) = w_{q(g)}(\boldsymbol{l}) - \delta(\boldsymbol{l})\int \mathrm{d}^2\boldsymbol{l}'\, w_{q(g)}(\boldsymbol{l}')$$

Fragmentation functions (1/4)

$\mathcal{K}_{ij}(\mathbf{Q}, z, xp_0^+)$

$w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

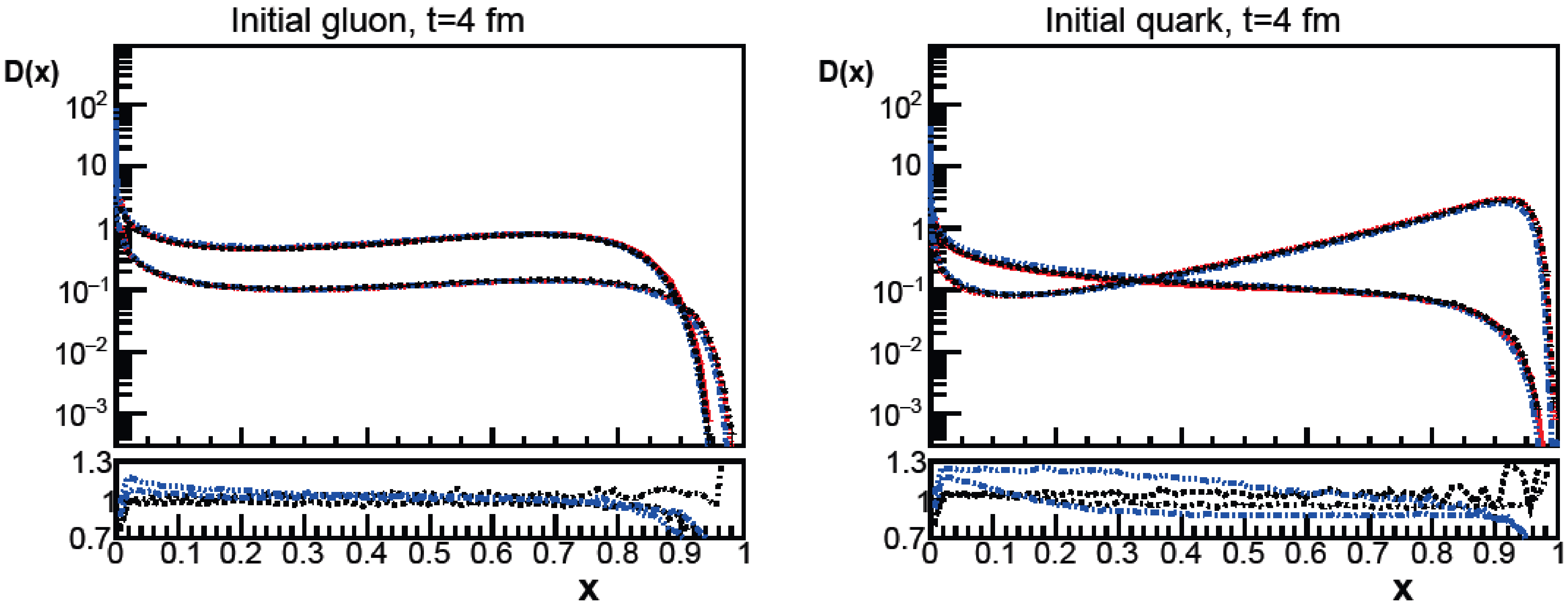




Fragmentation functions (2/4)

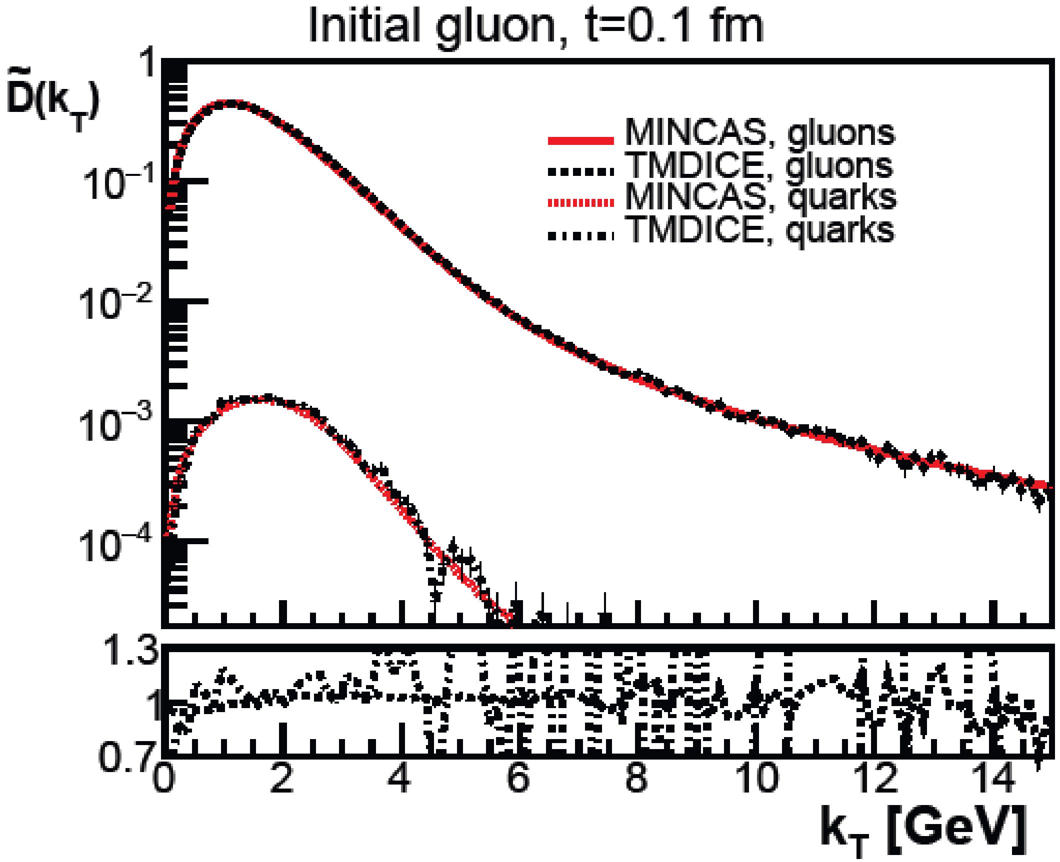
$\mathcal{K}_{ij}(\mathbf{Q}, z, xp_0^+)$

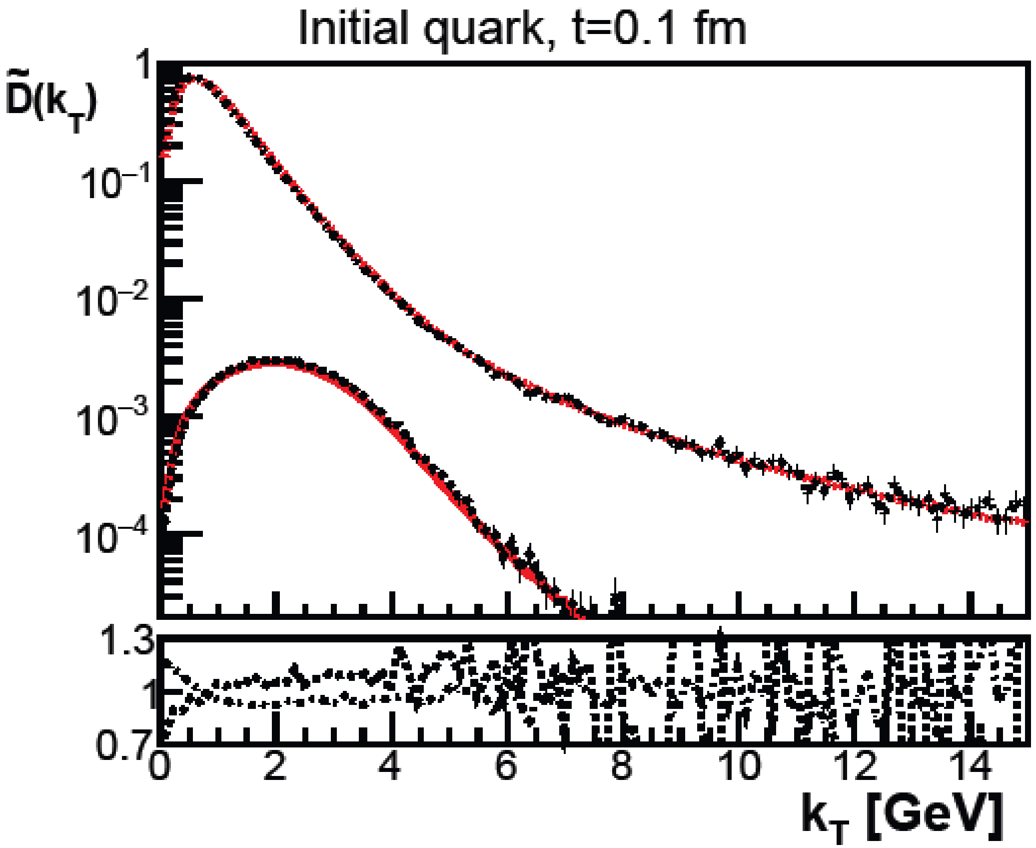
$w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$



Fragmentation functions (3/4)

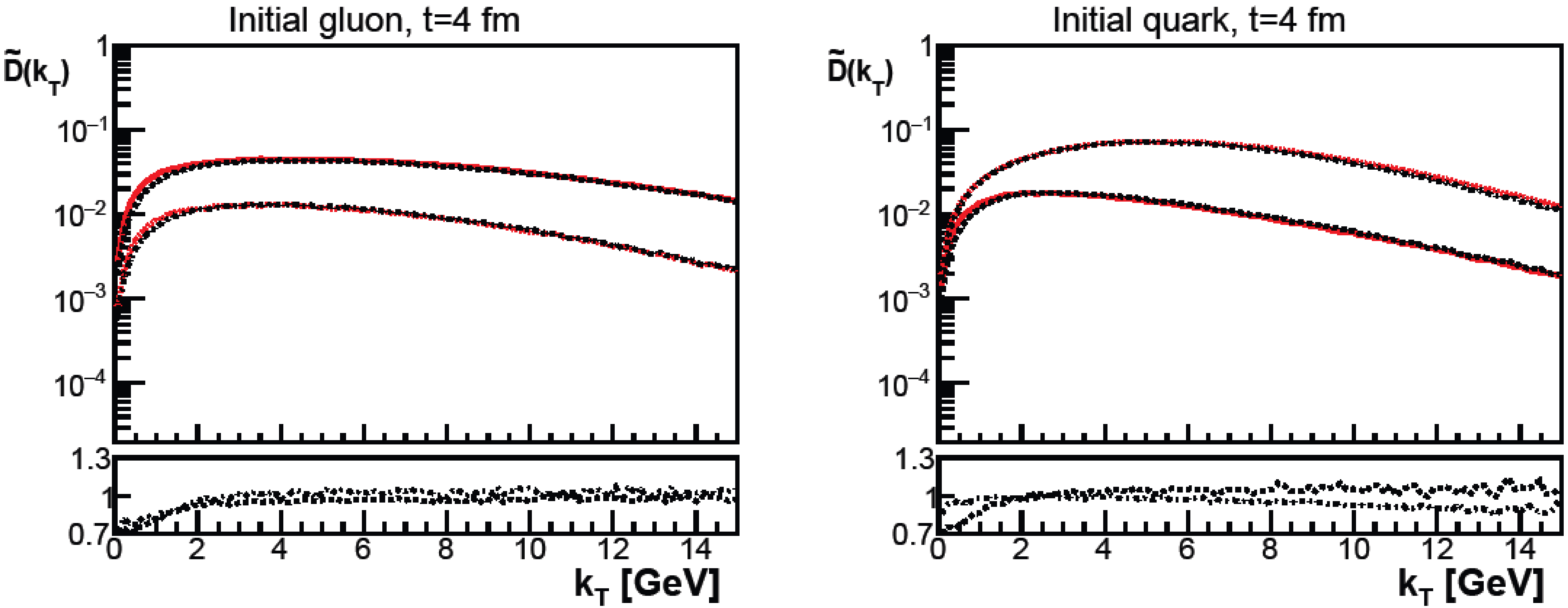
$$\mathcal{K}_{ij}(\mathbf{Q}, z, xp_0^+) \qquad w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$$





Fragmentation functions (4/4)

$$\mathcal{K}_{ij}(\mathbf{Q}, z, xp_0^+) \qquad w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$$



Different models

- Broadening in branching: $\mathcal{K}_{ij}(\mathbf{Q}, z, xp_0^+)$
  - No scattering
  - Scattering: $w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
  - Scattering: $w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$
- No broadening in branching: $\mathcal{K}_{ij}(z)$
  - Scattering: $w_g(\mathbf{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\mathbf{q}^4}$
  - Scattering: $w_g(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$
- Gaussian broadening:

  x given by collinear evolution without scattering via $\mathcal{K}_{ij}(z)$

  $\boldsymbol{k}$ given by Gaussian distribution with variance $\sigma^2 \sim \hat{q}L$

All models yield the same k_T averaged splitting kernel $\mathcal{K}_{ij}(z)$!

Constant medium parameters: L, $\hat{q}$, n, m_D

Departure from Gaussian broadening

$$\frac{\partial}{\partial t}D(x,\boldsymbol{k},t) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} C(\boldsymbol{q}) D(x, \boldsymbol{k}-\boldsymbol{q}, t)$$
$$+\frac{1}{t^*}\int_0^1 dz\mathcal{K}(z)$$
$$\left[\frac{1}{z^2}\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t\right)\theta(z-x) - \frac{z}{\sqrt{x}} D(x,\boldsymbol{k},t)\right]$$

always same distribution for changes $p \to p + \boldsymbol{q}$
→central limit theorem

Splitting à la $p \to zp$
→ perturbations of different sizes
→non Gaussian behavior

Virtual emissions

For example:

$$p \to z_1 p \to z_1 p + \boldsymbol{q}_1$$
$$\to z_1 p + \boldsymbol{q}_1 + \boldsymbol{q}_2$$
$$\to z_2(z_1 + \boldsymbol{q}_1 + \boldsymbol{q}_2) \to \ldots$$

$\mathcal{K}(z)$ $\quad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

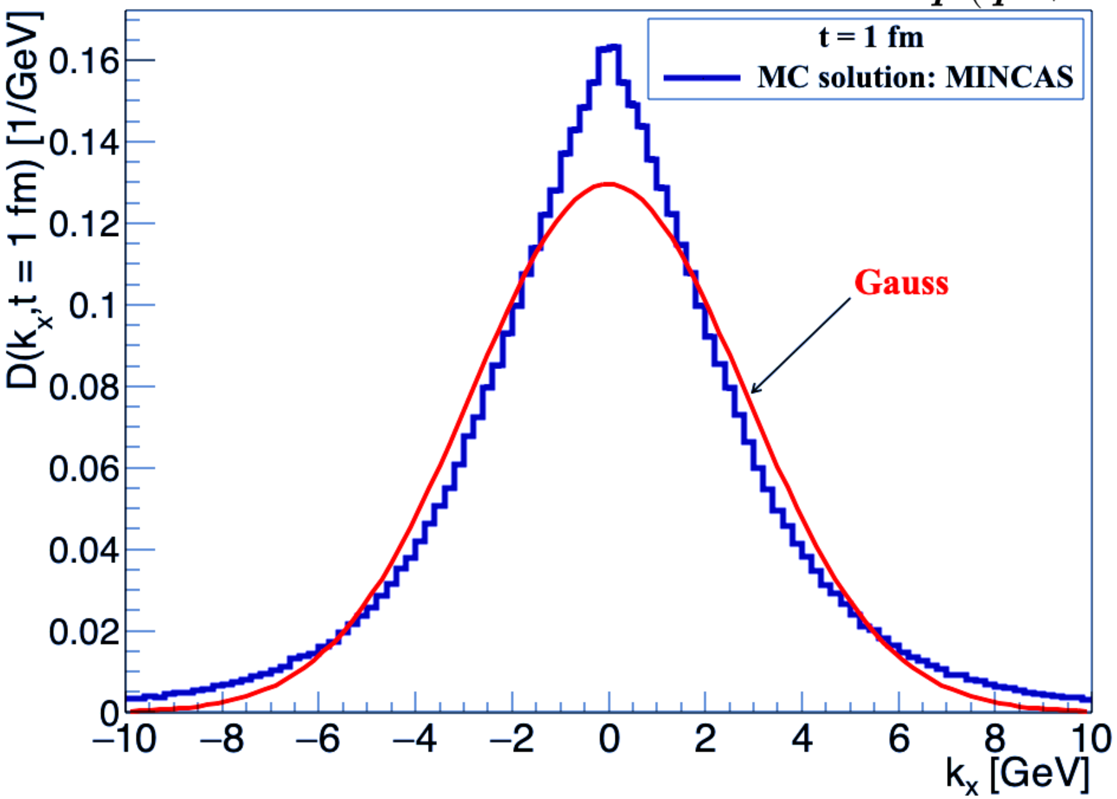
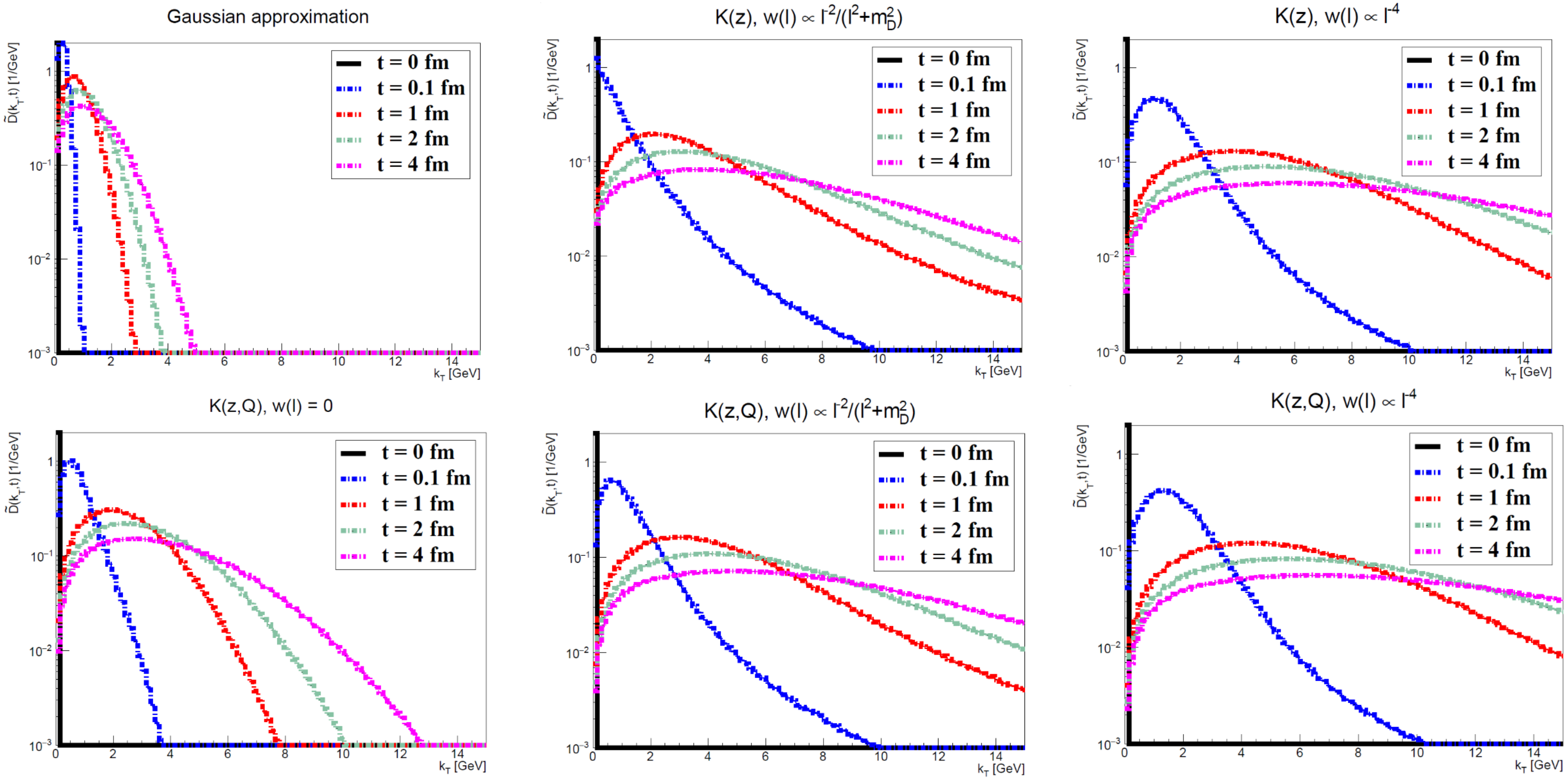


Figure: [Kutak, Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

k_T Broadening (1/3)

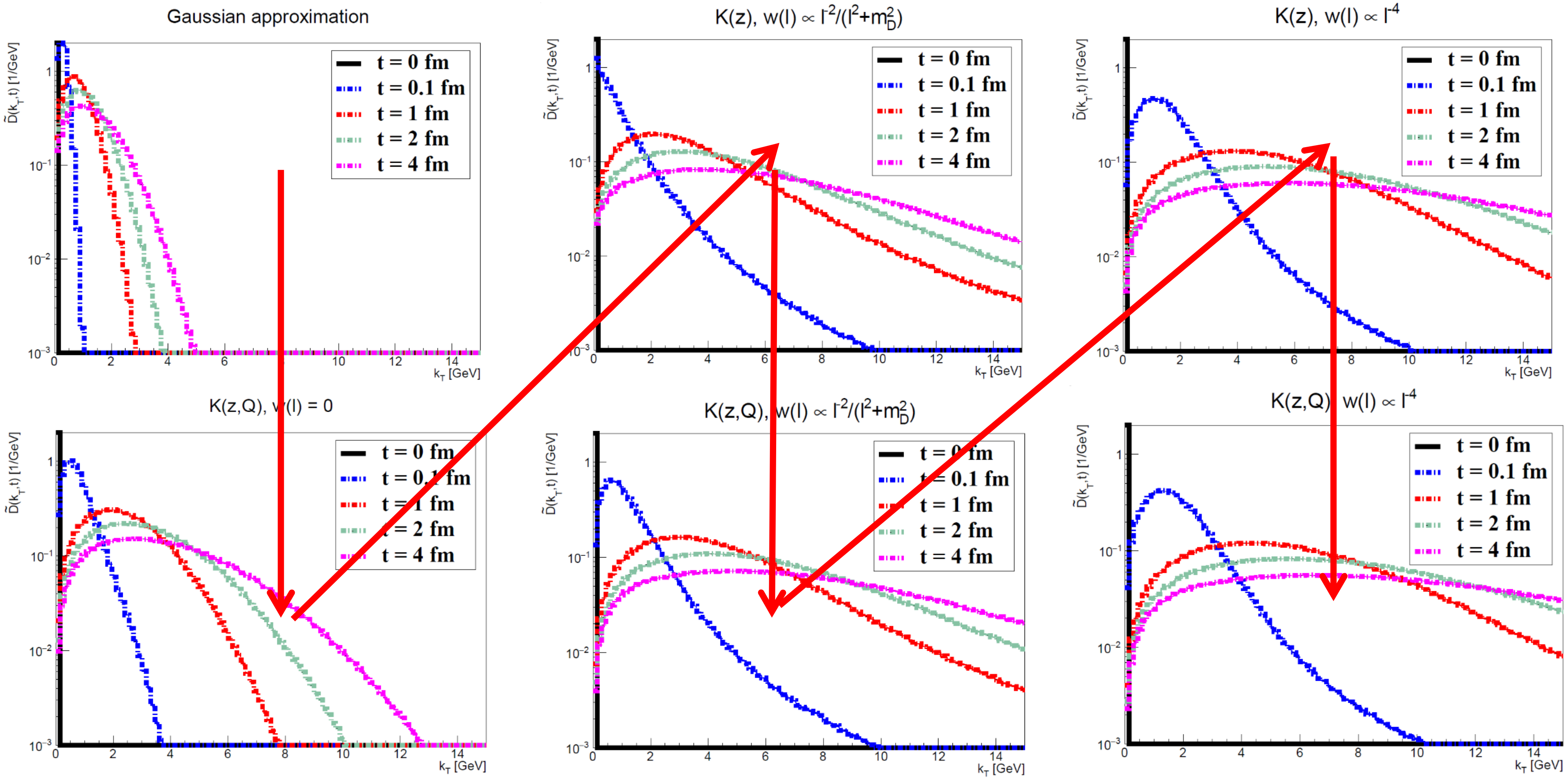
$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

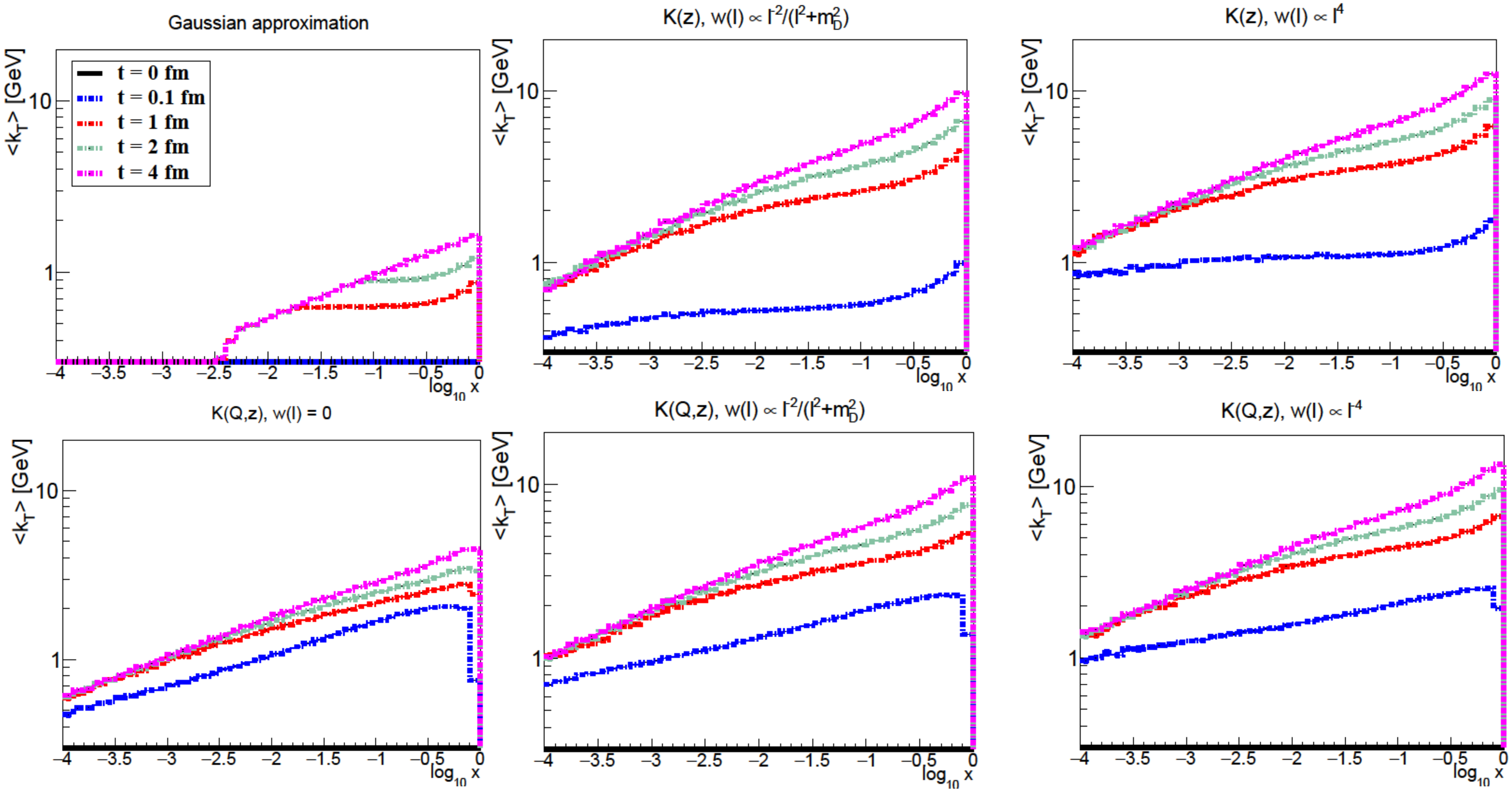
k_T Broadening (1/3)

$$\tilde{D}(x, k_T, t) = 2\pi k_T D(x, k_T, t)$$



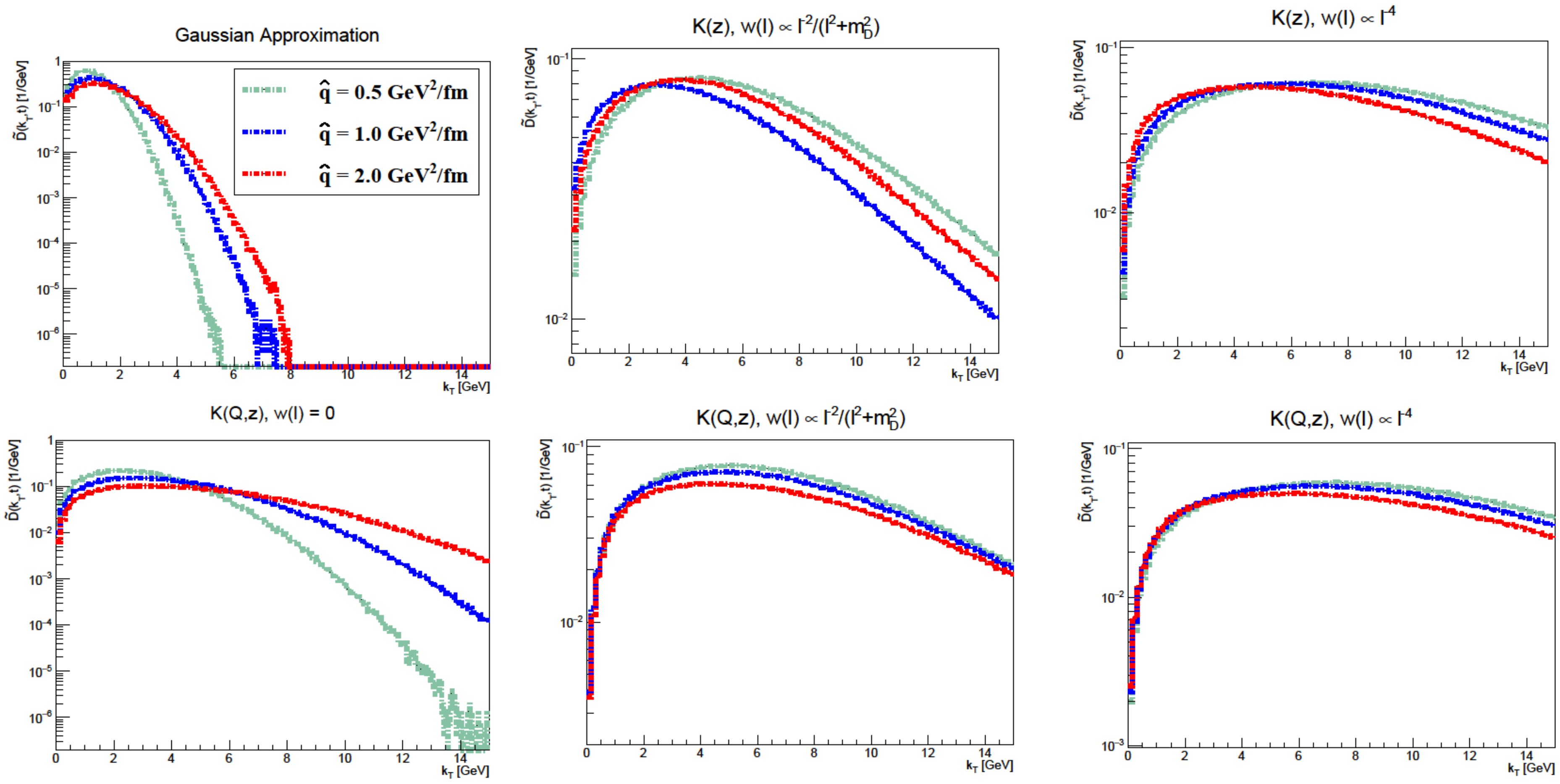
[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

k_T Broadening (2/3)



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

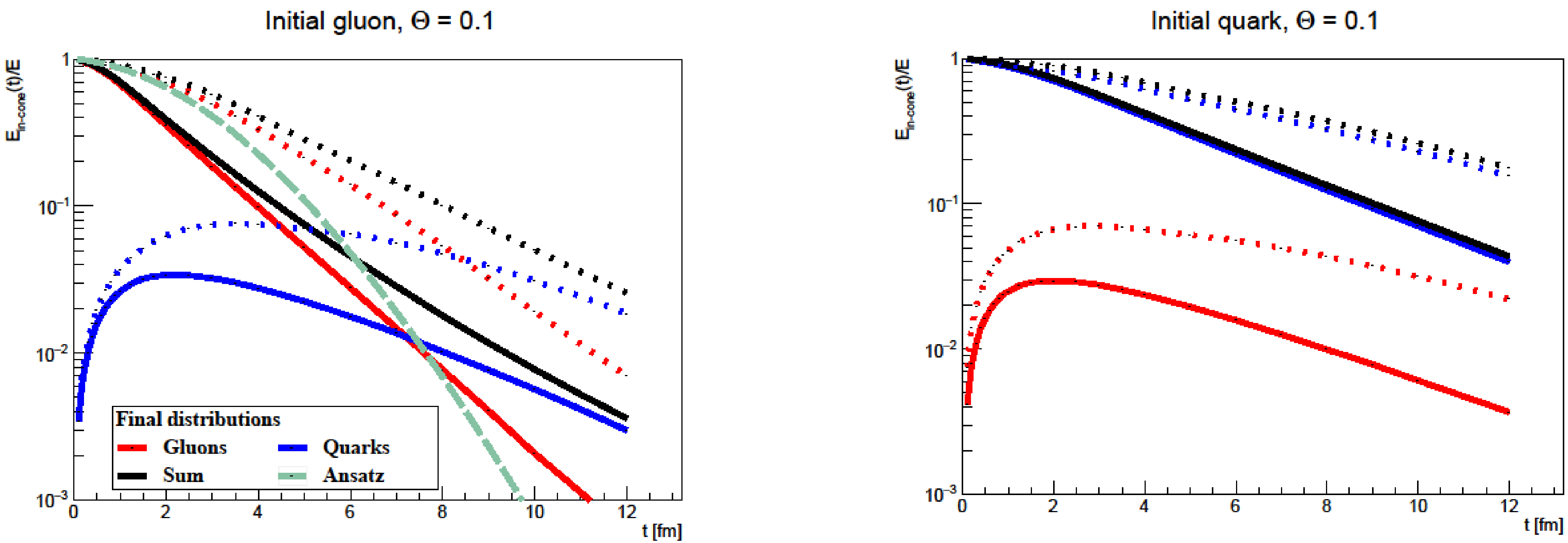
k_T Broadening (3/3)



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

In cone energy

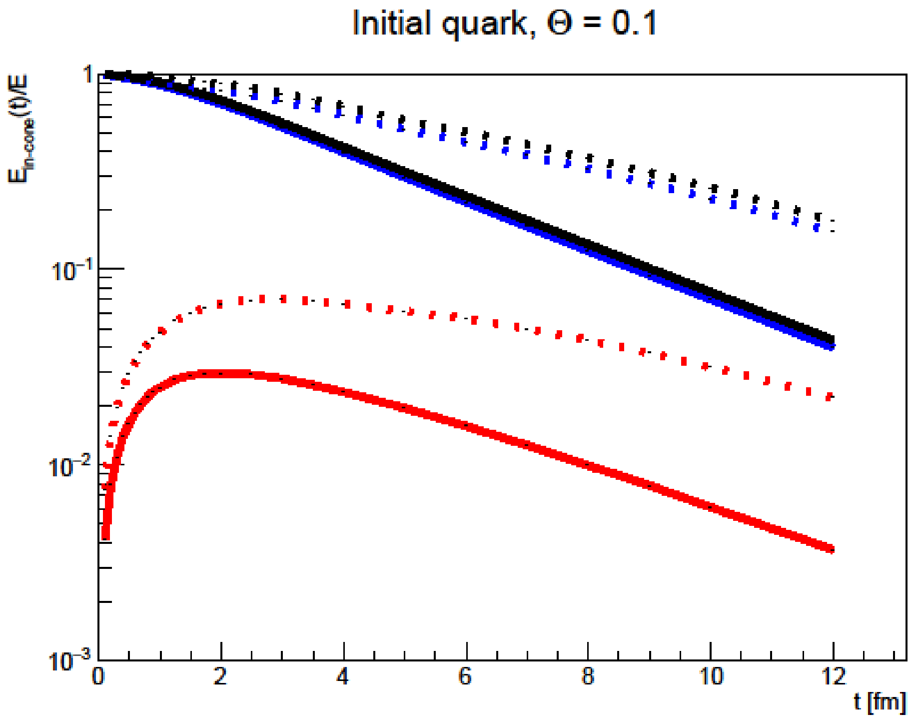
$$E_{\rm in-cone}(\Theta) = \int_0^1 {\rm d}x \int_0^{2\pi} d\varphi \int_0^{xE\sin\Theta} {\rm d}k_T k_T\, D(x, \mathbf{k}, t)$$

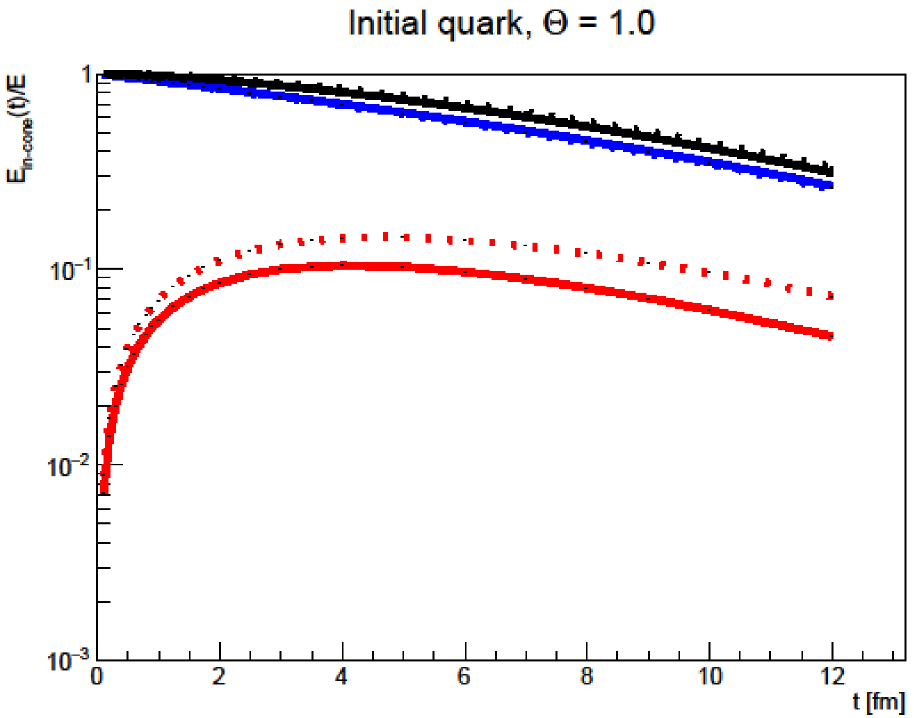


[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

In cone energy

$$E_{\text{in}-\text{cone}}(\Theta) = \int_0^1 \mathrm{d}x \int_0^{2\pi} d\varphi \int_0^{xE\sin\Theta} \mathrm{d}k_T k_T \, D(x, \mathbf{k}, t)$$





[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Summary & Outlook

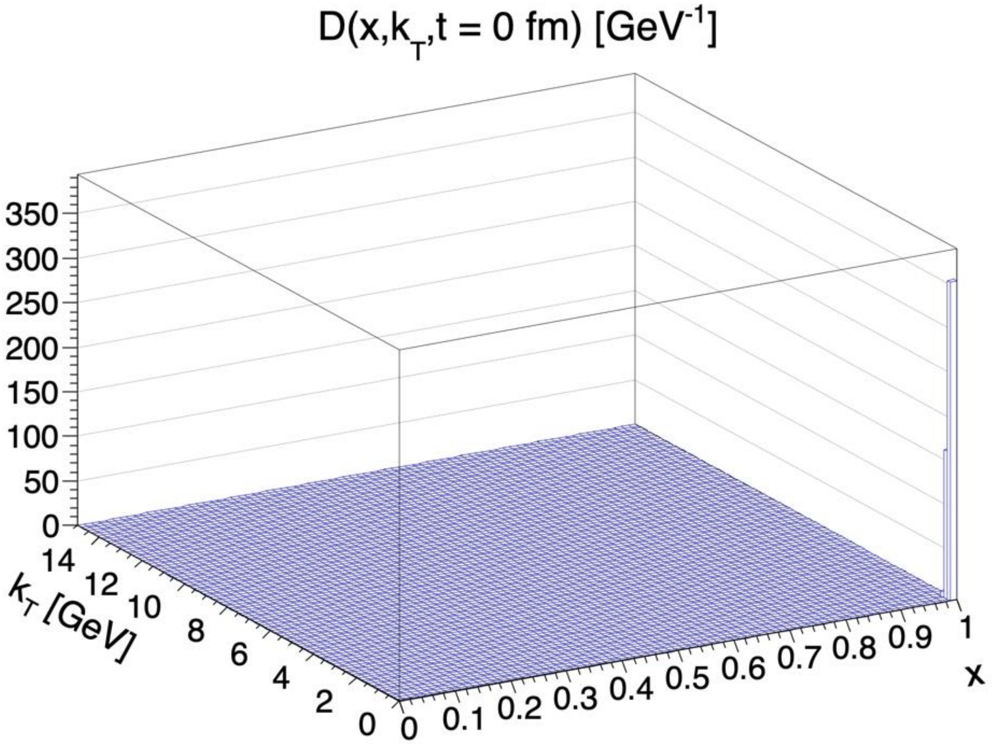
- Monte-Carlo algorithms **TMDICE** and **MINCAS** based on coherent emission and scattering for quarks and gluons
- Transverse momentum broadening differs from Gaussian distribution
- Gaussian distribution: smallest k_T broadening
- Clear ordering of broadening effects
- Quark jets keep more energy inside a jet cone.
- Covers regime of coherent emissions: requires underlying events (hard cross section) and cascade evolution outside of medium (vacuum like emissions)

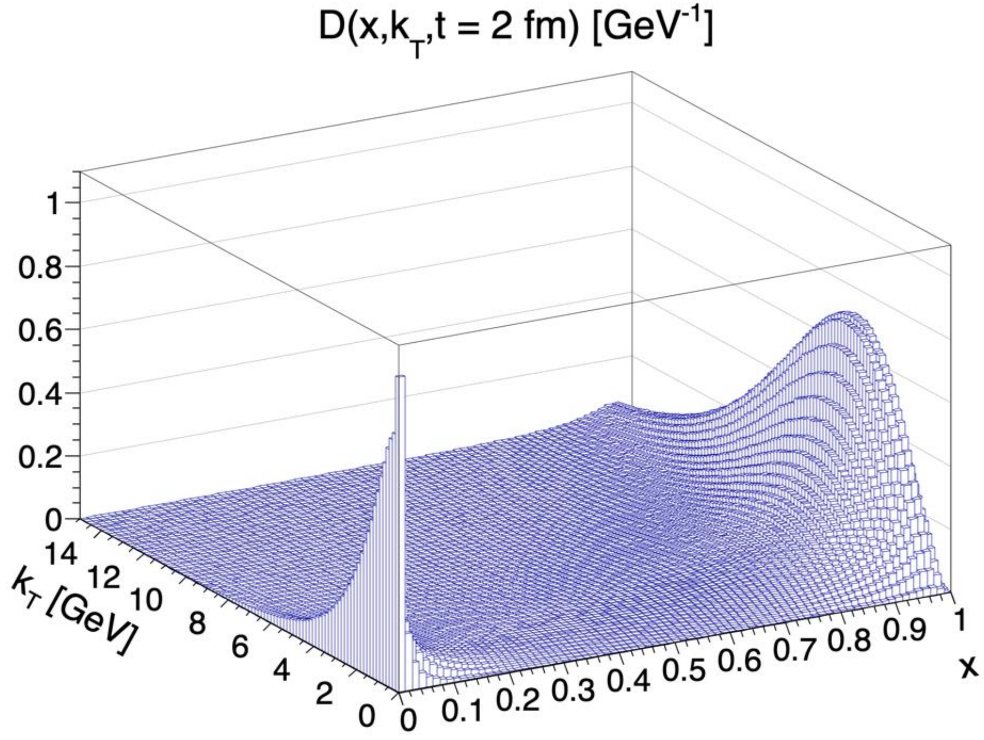
Thank you for your attention!

Back-up slides

Evolution of D(x,k_T,t) (1/2)

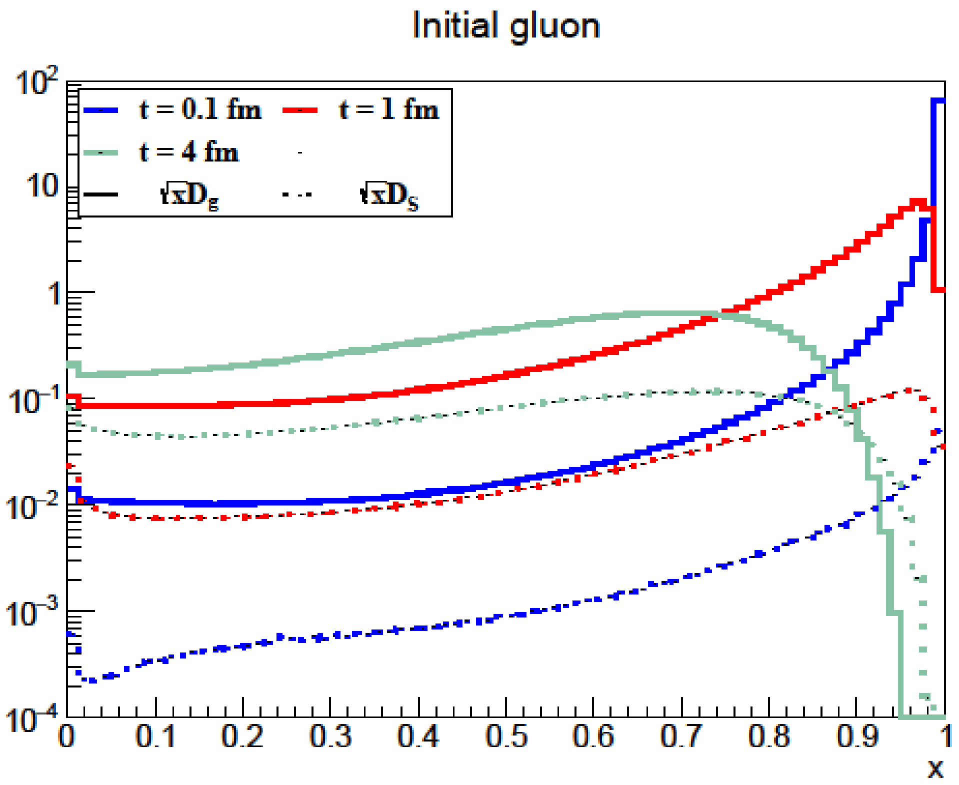
$\mathcal{K}(z) \quad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

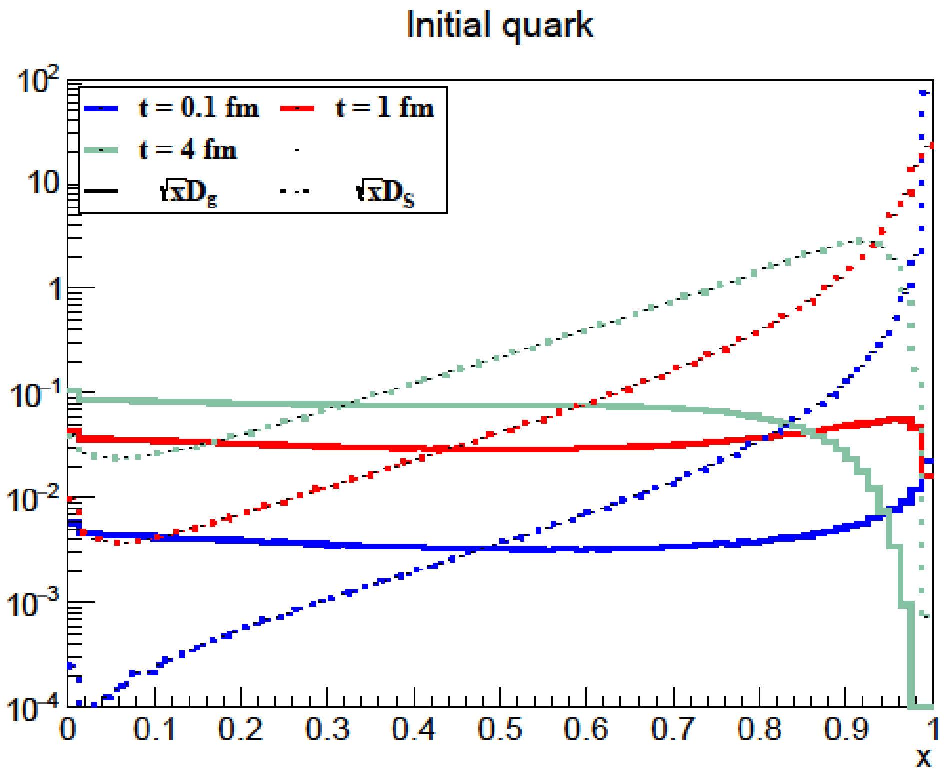




[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Evolution in x

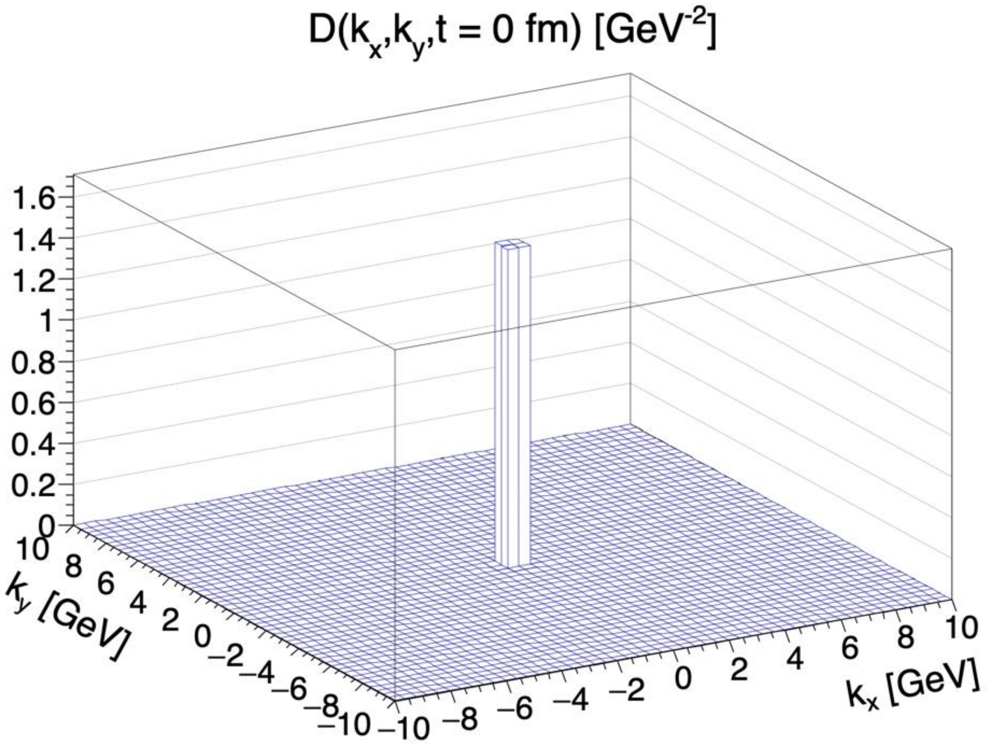


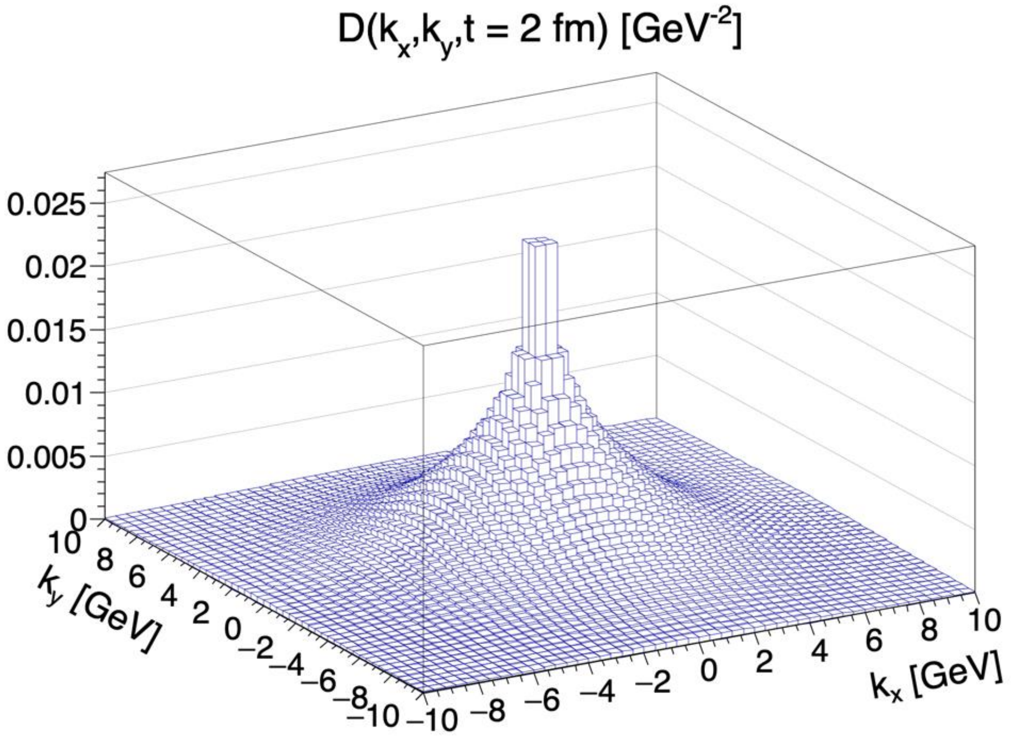


[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Evolution of D(x,k_T,t) (2/2)

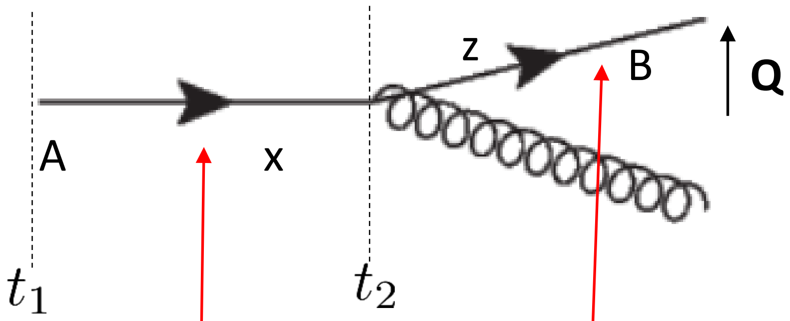
$\mathcal{K}(z) \quad w(\boldsymbol{q}) = \frac{16\pi^2\alpha_s^2 N_c n}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$





[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Probabilities for interactions

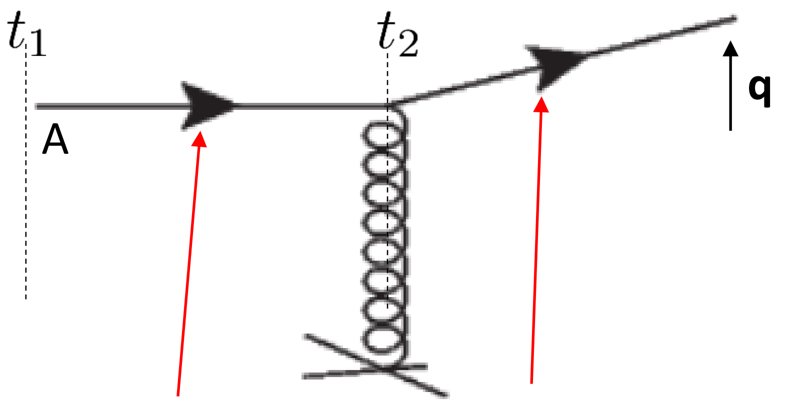


$$\Delta_A(x, t_2 - t_1)\left(\frac{\alpha_s}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{Q}, z, xp_+)\right)$$

$$(\Delta_A(x,t_2-t_1)\phi_A(x))\left(\frac{\sum_B \rho_{BA}(x)}{\phi_A(x)}\right)\left(\frac{\rho_{BA}(x)}{\sum_B \rho_{BA}(x)}\right)\left(\frac{1}{\rho_{BA}(x)}\frac{\alpha_s}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{Q}, z, xp_+)\right)$$

Splitting probability

Probability for splitting into B



$$\Delta_A(x, t_2 - t_1)\left(\frac{1}{(2\pi)^2}w_A(\mathbf{q})\right)$$

$$(\Delta_A(x,t_2-t_1)\phi_A(x))\left(\frac{W_A}{\phi_A(x)}\right)\left(\frac{1}{W_A}\frac{1}{(2\pi)^2}w_A(\mathbf{q})\right)$$

Scattering probability

$$\rho_{BA}(x) = \alpha_s \int_{\epsilon}^{1-\epsilon} dz \int_{q>0} \frac{d^2q}{(2\pi)^2}\mathcal{K}_{BA}(\mathbf{q}, z, xp_+)$$

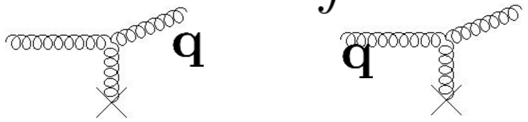
$$W_A = \int \frac{d^2\mathbf{q}}{(2\pi)^2} w_A(\mathbf{q})$$

BDIM Equation for Gluons

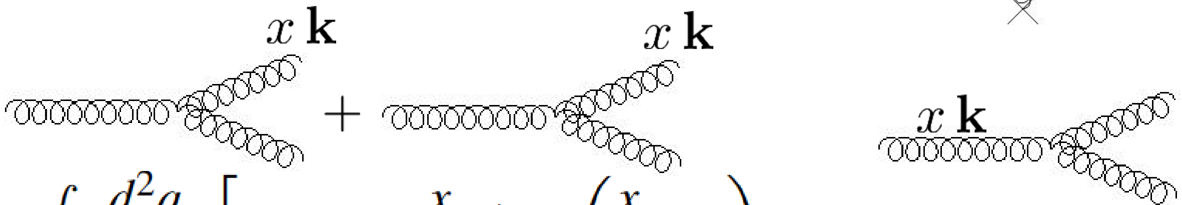
[Blaizot, Dominguez, Iancu, Mehtar-Tani: JHEP 1406 (2014) 075]

Scattering:

$$C(\boldsymbol{q}) = w(\boldsymbol{q}) - \delta(\boldsymbol{q}) \int d^2\boldsymbol{q}' w(\boldsymbol{q}')$$



$$D(x, \mathbf{k}, t) = x \frac{\partial^3 N(x,\mathbf{k},t)}{\partial x \partial^2 \mathbf{k}}$$



For gluon-jets:

$$\frac{\partial}{\partial t} D(x,\mathbf{k},t) = \alpha_s \int_0^1 dz \int \frac{d^2q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x,\mathbf{k},t) \right]$$
$$+ \int \frac{d^2\mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k}-\mathbf{l}, t).$$

Average Kernels over $\mathbf{Q}$

$$\frac{\partial}{\partial t} D(x, \boldsymbol{k}, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t\right) \theta(z-x) - \frac{z}{\sqrt{x}} D(x, \boldsymbol{k}, t) \right] + \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} C(\boldsymbol{q}) D(x, \boldsymbol{k}-\boldsymbol{q}, t)$$

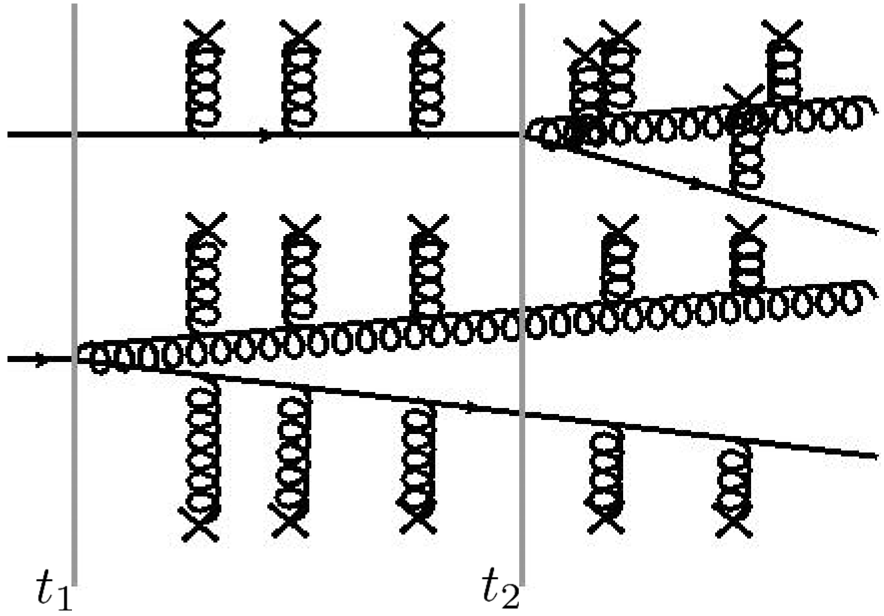
Integrate over $\mathbf{k}$

$$\frac{\partial}{\partial t} D(x,t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) - \frac{z}{\sqrt{x}} D(x,t) \right]$$

$$D(x,t) = \int d^2\mathbf{k} D(x, \mathbf{k}, t)$$

Effective Splitting Kernels

$$\mathcal{K}_{ij}(\mathbf{Q}, z, p_0^+) \sim P_{ij}(z) \times \mathcal{I}_{ij}$$



Assumptions:
Transverse momentum transfer only,
harmonic oscillator approximation,
static medium,
static scattering centers.

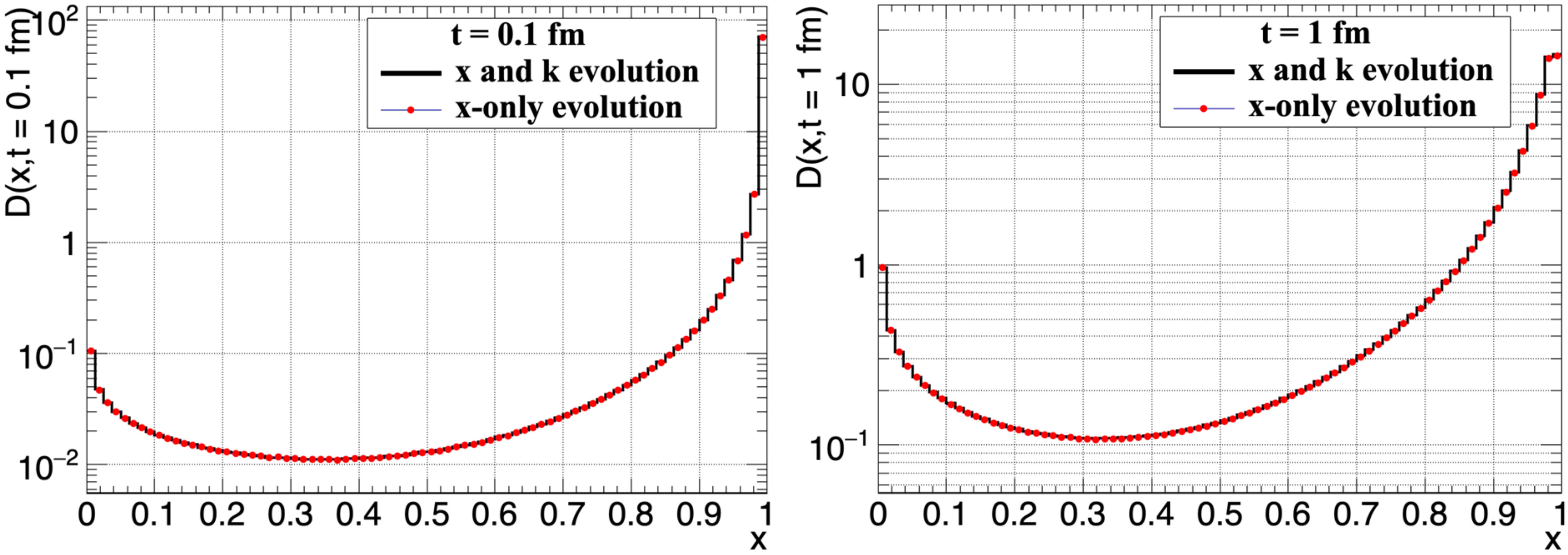
$$\mathcal{I}_{ij}(\boldsymbol{u}_2, t_2; \boldsymbol{u}_1, t_1) = \int_{\boldsymbol{u}(t_1)=\boldsymbol{u}_1}^{\boldsymbol{u}(t_2)=\boldsymbol{u}_2} \mathcal{D}\boldsymbol{u}\, \mathrm{e}^{i\frac{\omega_0}{2}\int_{t_1}^{t_2} \mathrm{d}s\, \dot{\boldsymbol{u}}^2(s) - \int_{t_1}^{t_2} \mathrm{d}s\, n(s)\sigma_{\mathrm{eff}}(\boldsymbol{u}(s), \boldsymbol{v})}$$

$$\boldsymbol{u} = \boldsymbol{r}_i - \boldsymbol{r}_k$$

$$\boldsymbol{v} = z\boldsymbol{r}_i + (1-z)\boldsymbol{r}_k - \boldsymbol{r}_j$$

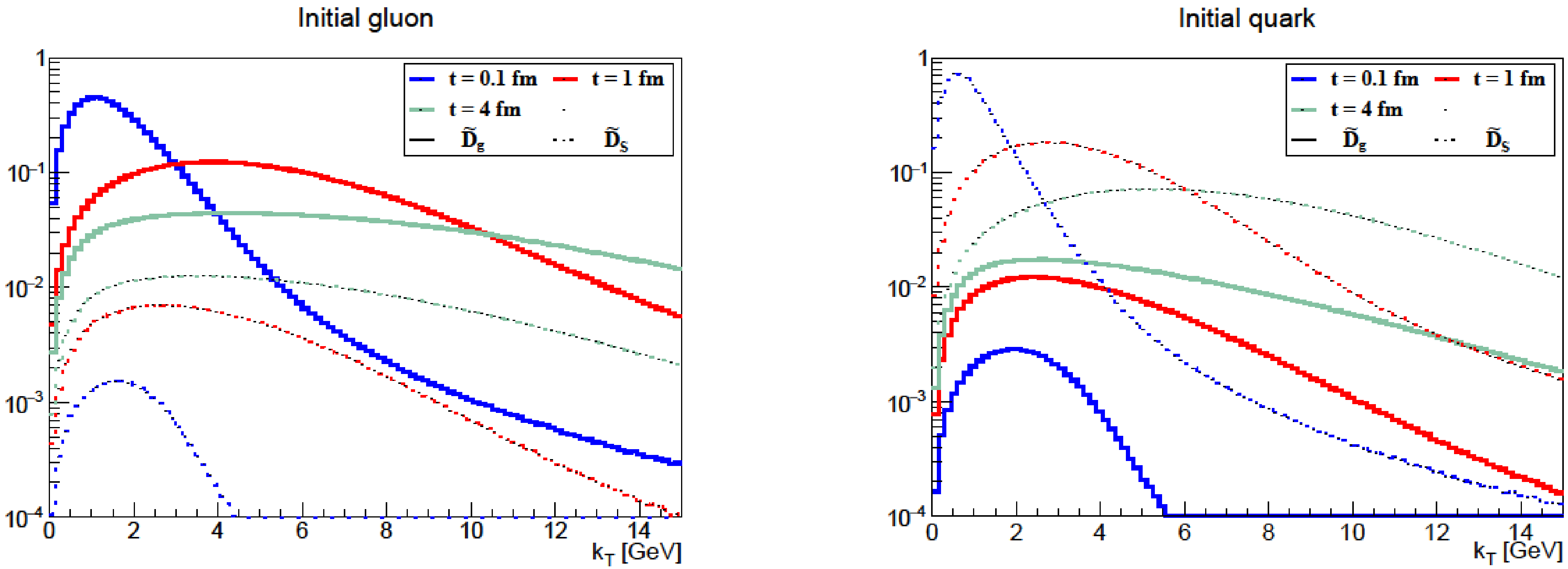
$$\sigma_{\mathrm{eff}}(\boldsymbol{u}, \boldsymbol{v}) = \frac{C_i + C_k - C_j}{2}\,\sigma(\boldsymbol{u}) + \frac{C_i + C_j - C_k}{2}\,\sigma(\boldsymbol{v} + (1-z)\boldsymbol{u}) + \frac{C_k + C_j - C_i}{2}\,\sigma(\boldsymbol{v} - z\boldsymbol{u})$$

Turbulent behavior



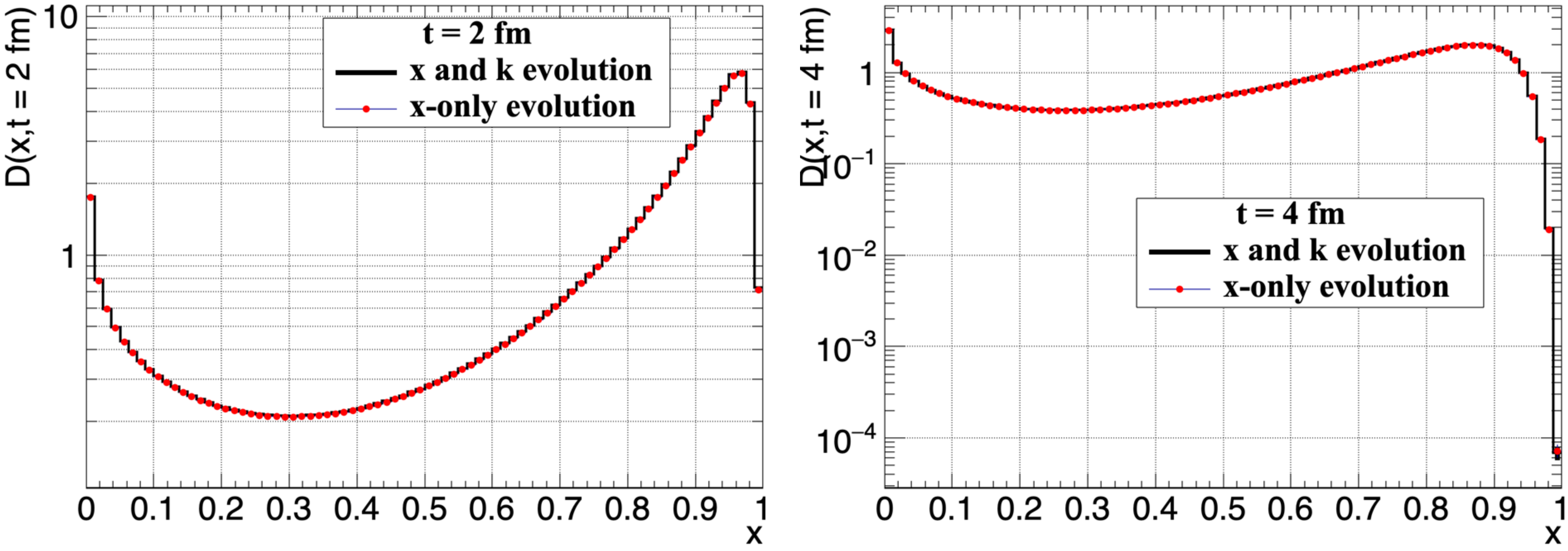
[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Evolution in k_T



[Blanco, Kutak, Płaczek, MR, Tywoniuk,arxiv: 2109.05918]

Turbulent behavior (2/2)



[Kutak,Płaczek, Straka: Eur.Phys.J. C79 (2019) no.4, 317]

Diffusion approximation (1/2)

$$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}\,D\left(\frac{x}{z},\frac{\mathbf{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}\,D(x,\mathbf{k},t)\right] + \int\frac{d^2\mathbf{q}}{(2\pi)^2}\,C(\mathbf{q})\,D(x,\mathbf{k}-\mathbf{q},t),$$

$$\downarrow$$

$$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}\,D\left(\frac{x}{z},\frac{\mathbf{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}\,D(x,\mathbf{k},t)\right] + \frac{1}{4}\hat{q}\nabla_k^2\Big[D(x,\mathbf{k},t)\Big].$$

Diffusion approximation (2/2)