Future Compact star observations that could confirm the existence of a Critical End Point in the QCD phase diagram

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DUBNA

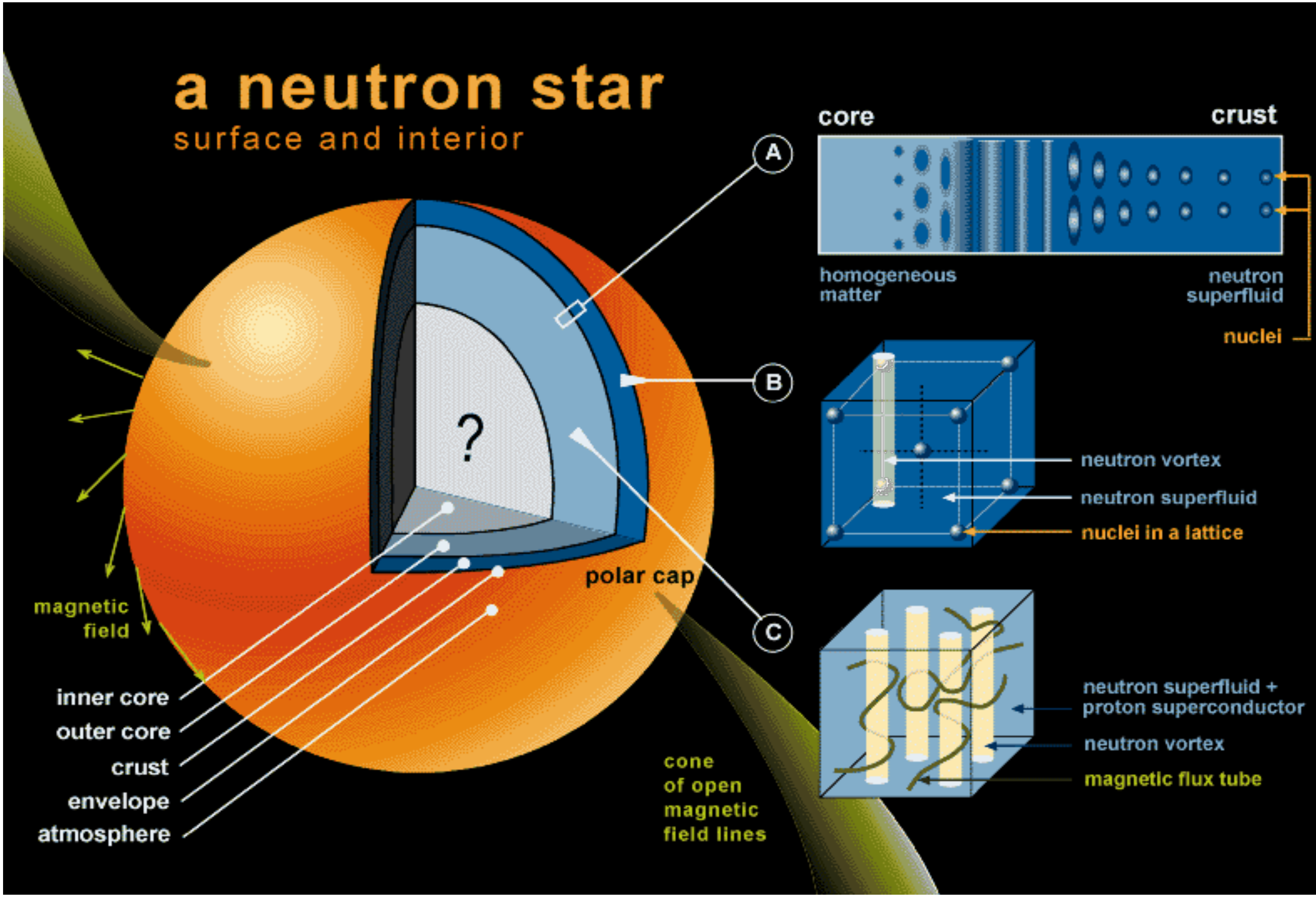
PHAROS
THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

comp star

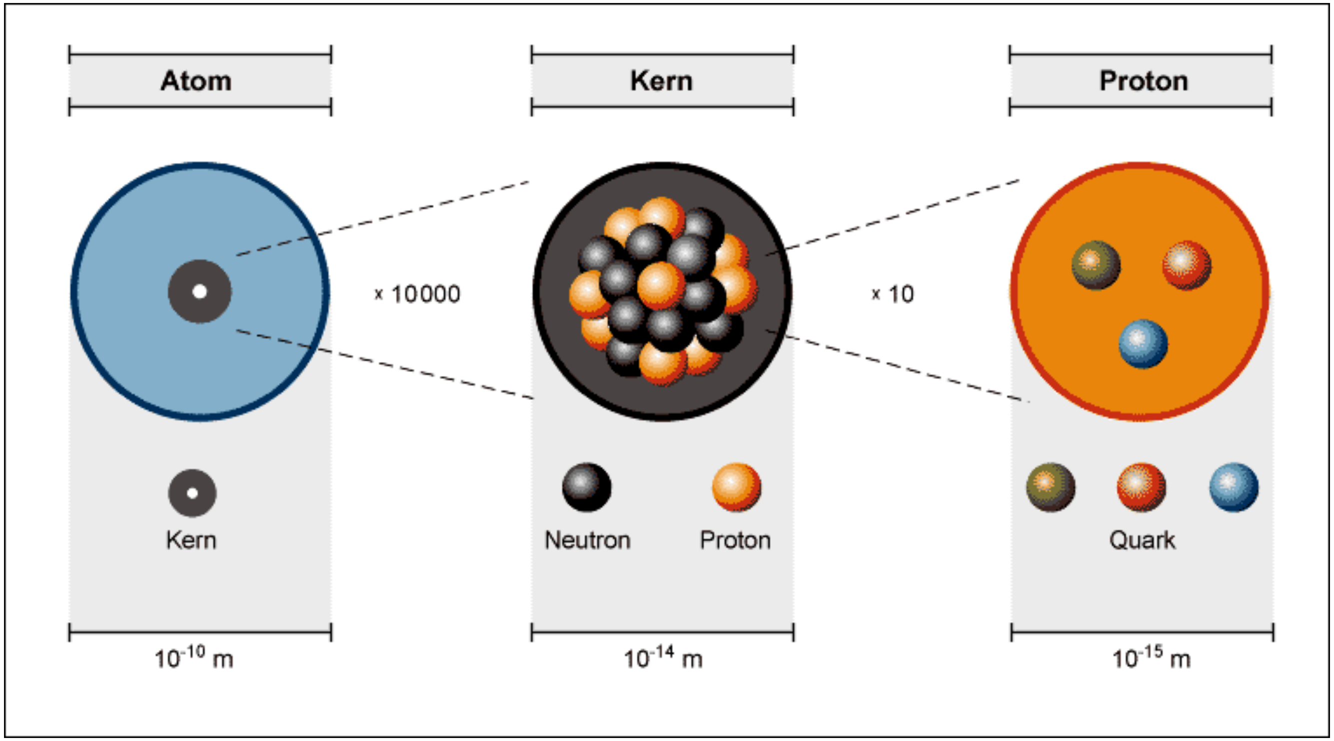
Outline

- A brief introduction to the neutron star matter equation of state (EoS), its location within the QCD phase diagram and related compact star properties.
- The mass twins compact stars scenario and the possibility of probing deconfined quark matter.
- Multi-messenger astronomy measurements that could prove the mass twins hypothesis.
- Introduction of Bayesian methods for model parameter estimation and statistical inference.

Superdense objects – what is inside?



Superdense objects – what is inside?



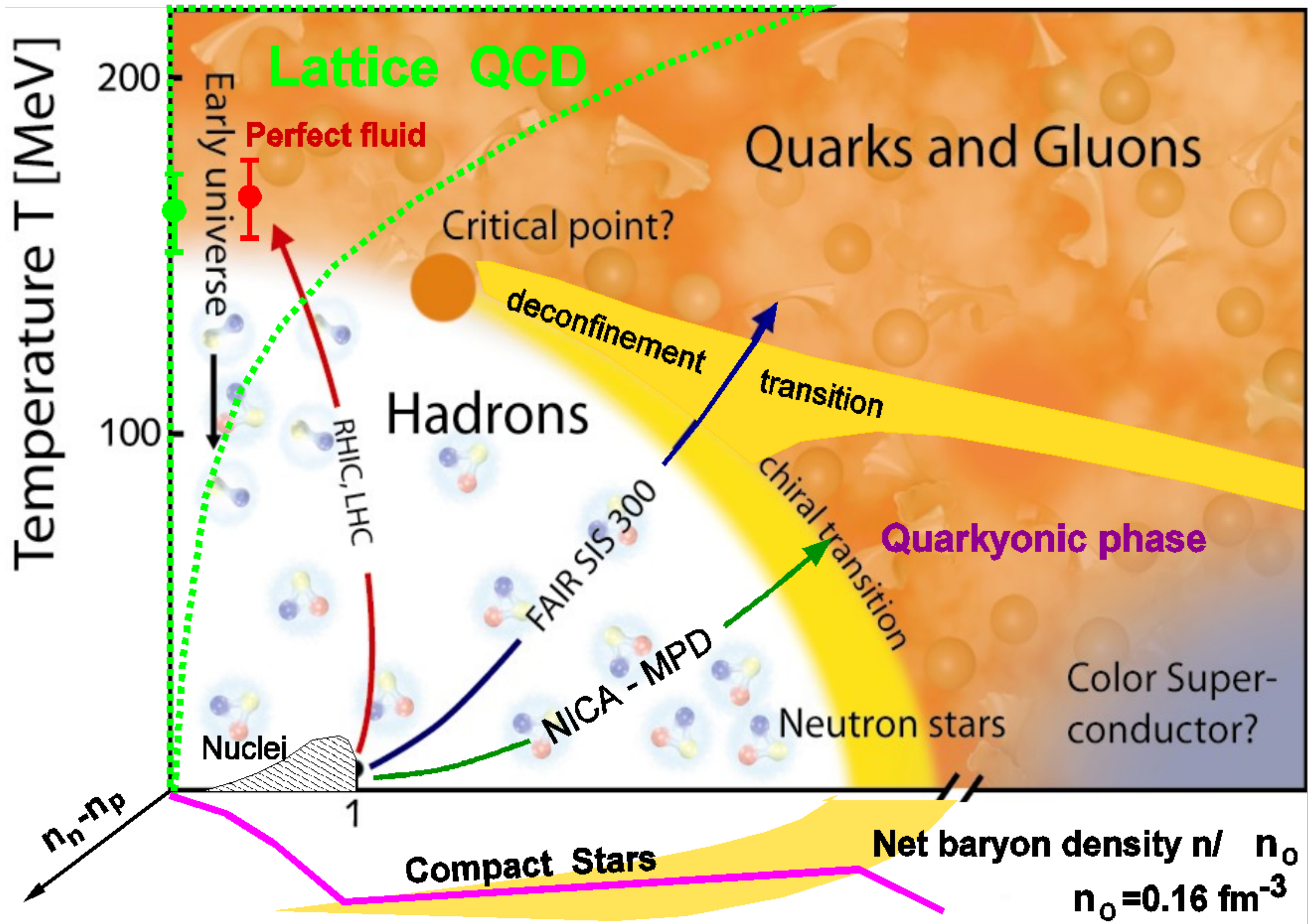
Nucleus, A nucleons: R_A=1.2 10^{-13} cm $A^{1/3}$; ρ_0=A1.67 10^{-24} g/(4π/3 $R_A{}^3$)= 2.3 10^{14} g/cm^3

Neutron star: R= 10 km; ρ= 2 Mo/(4π/3 R^3) = 4 10^{33} g/(4 10^{18} cm^3)= 10^{15} g/cm^3 = 4 ρ_0

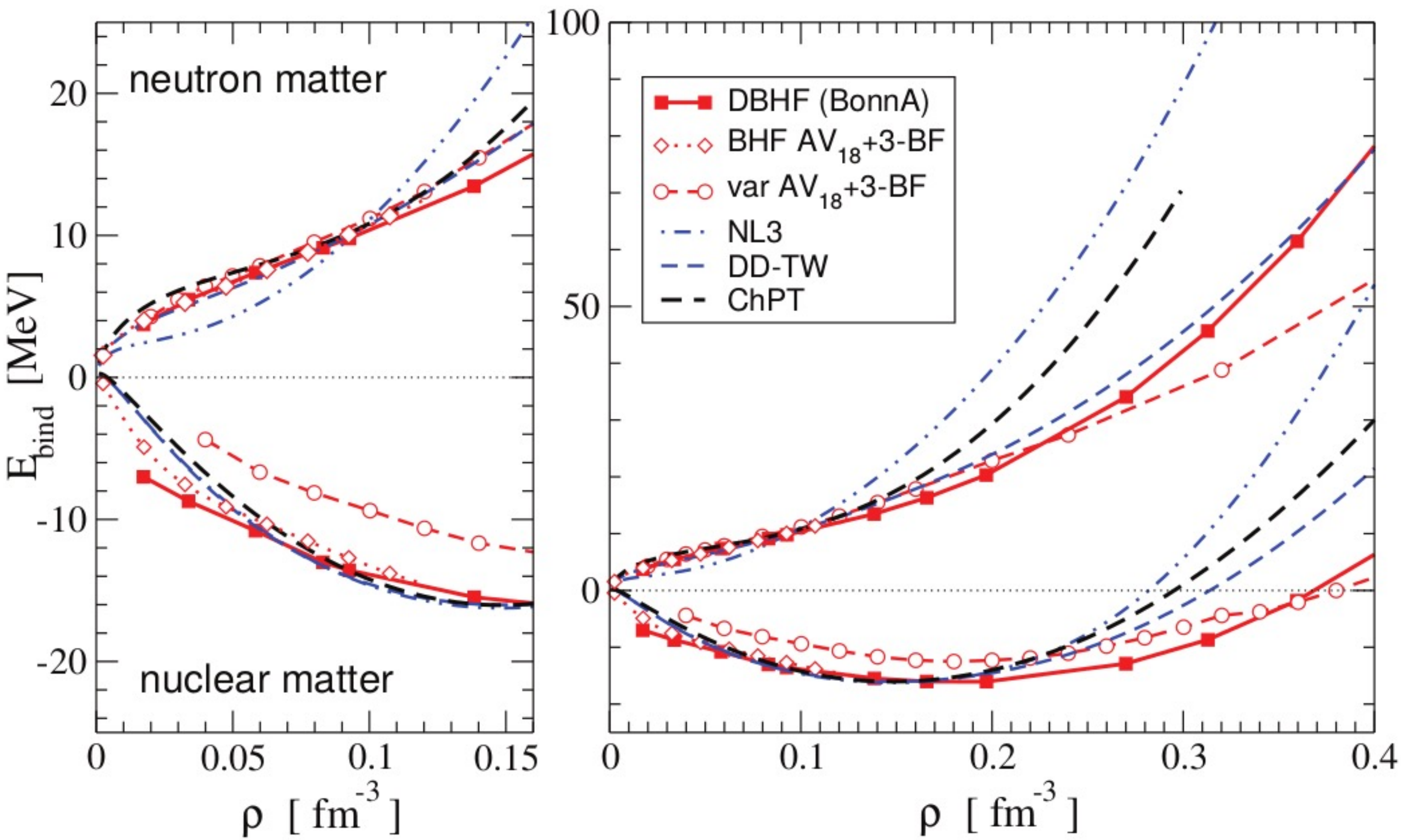
Motivation

- Neutron stars and pulsars are astrophysical laboratories for nuclear and particle physics.
- Recent observations have set constraints on their masses and radii.
- The largest mass measurements probe the highest densities in their interiors. The lowest mass measurements allow to study their progenitors and formation mechanisms.
- Multiple mass and radius measurements can reveal the properties of the nuclear symmetry energy, which is also studied in laboratory experiments.
- The topology of the mass-radius relations can indicate the presence of phase transitions and the existence of deconfined quark matter in compact star interiors.

Critical Endpoint in QCD



Nuclear Matter



C. Fuchs, H.H. Wolter, EPJA 30(2006)5

Neutron Star Equation of State

The energy per nucleon in neutron star core matter is given by:

$$\begin{aligned}
E_{\mathrm{tot}}(n,\{x_i\}) &= E_{\mathrm{b}}(n,x_p) + E_{\mathrm{lep}}(n,x_e,x_\mu)\ , \\
E_{\mathrm{b}}(n,x_p) &= E_0(n) + S(n,x_p) \\
E_{\mathrm{lep}}(n,x_e,x_\mu) &= E_e(n,x_e) + E_\mu(n,x_\mu)\ ,
\end{aligned}$$

where $n = n_p + n_n$ is the total baryon density and $x_i = n_i/n$, $i = p, e, \mu$ are the fractions of protons, electrons and muons, respectively. The baryonic part is very well described by the parabolic approximation w.r.t. the asymmetry

$$\alpha = \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,$$

resulting in $S(n,x_p) = (1-2x_p)^2 E_s(n)$. The leptonic contribution is a sum of the Fermi gas expressions for the contributing leptons $l = e, \mu$

$$E_l(n,x_l) = \tfrac{1}{n}\tfrac{p_{F,l}^4}{4\pi^2}\left[\sqrt{1+z_l^2}\left(1+\tfrac{z_l^2}{2}\right) - \tfrac{z_l^4}{2}\mathrm{Arsinh}\left(\tfrac{1}{z_l}\right)\right]\ ,$$

where $z_l = m_l/p_{F,l}$. For massless leptons ($z_l \to 0$), this expression goes over to

$$E_l(n,x_l)\big|_{m_l=0} = \frac{1}{n}\frac{p_{F,l}^4}{4\pi^2} = \frac{3}{4}\left(3\pi^2 n\right)^{1/3} x_l^{4/3}\ .$$

Charge neutrality and β-equillibrium

Under neutron star conditions charge neutrality holds,

$$x_p = x_e + x_\mu \ .$$

The $\beta-$ equilibrium with respect to the weak interaction processes $n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$ (and similar for muons), for cold neutron stars (temperature T below the neutrino opacity criterion $T < T_\nu \sim 1$ MeV) implies

$$\mu_n - \mu_p = \mu_e = \mu_\mu \ .$$

The chemical potentials are defined as

$$\mu_i = \frac{\partial \varepsilon_i}{\partial n_i} = \frac{\partial}{\partial x_i} E_i(n, \{x_j\}) \ , \quad i, j = n, p, e, \mu \ ,$$

where $\varepsilon_i = n \ E_i(n, \{x_j\})$ is the partial energy density of species i in the system. From the above equations:

$$\mu_e = 4(1 - 2x) E_s(n) \ .$$

Since electrons in neutron star interiors are ultrarelativistic,

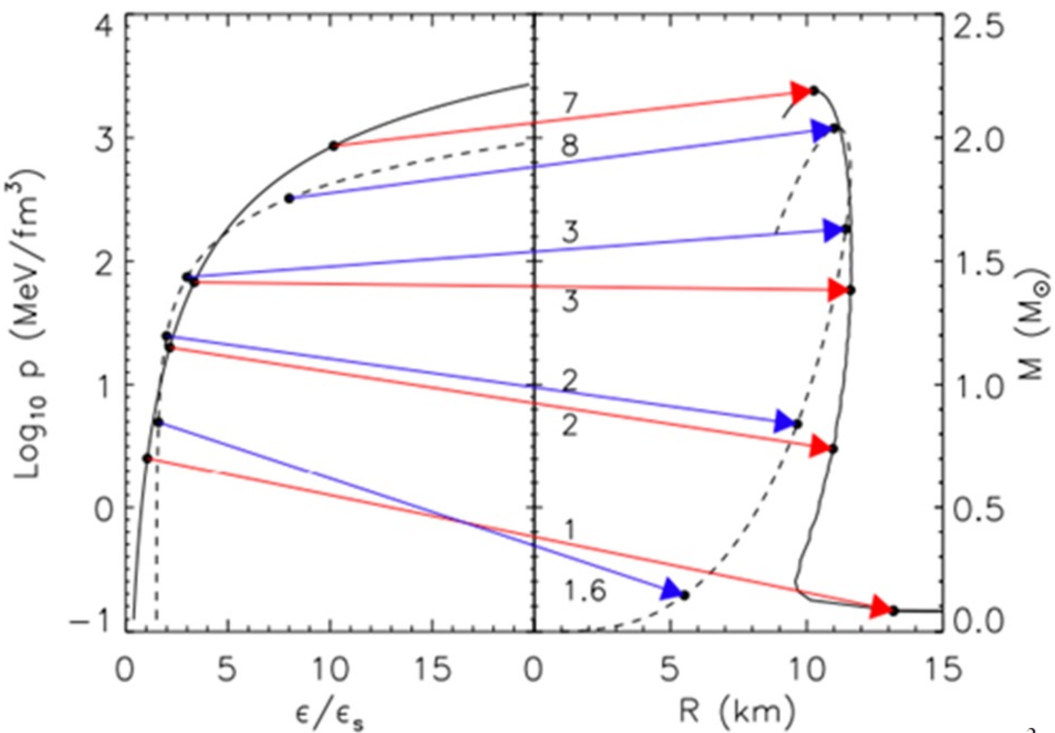
$$\mu_e = \sqrt{p_{F,e}^2 + m_e^2} \approx p_{F,e}, \text{ and } p_{F,e} = (3\pi^2 n_e)^{1/3} = (3\pi^2 n)^{1/3} (x - x_\mu)^{1/3} \ ,$$

$$\frac{x - x_\mu}{(1 - 2x)^3} = \frac{64 E_s^3(n)}{3\pi^2 n} \ , \qquad (x - x_\mu)^{2/3} - x_\mu^{2/3} = \frac{m_\mu^2}{(3\pi^2 n)^{2/3}} \ .$$

The total pressure is then given as $P(n) = n^2 \left(\frac{\partial E_{\rm tot}}{\partial n} \right)$.

Compact Star Sequences

(M-R ⇔ EoS)



Lattimer,
Annu. Rev. Nucl. Part. Sci. 62,
485 (2012)
arXiv: 1305.3510

- TOV Equations

$$\frac{dp}{dr} = -\frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

- Equation of State (EoS)

$$p(\varepsilon)$$

Flow Constraint

Klaehn et al. PhysRev C74 (2006)
P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002)

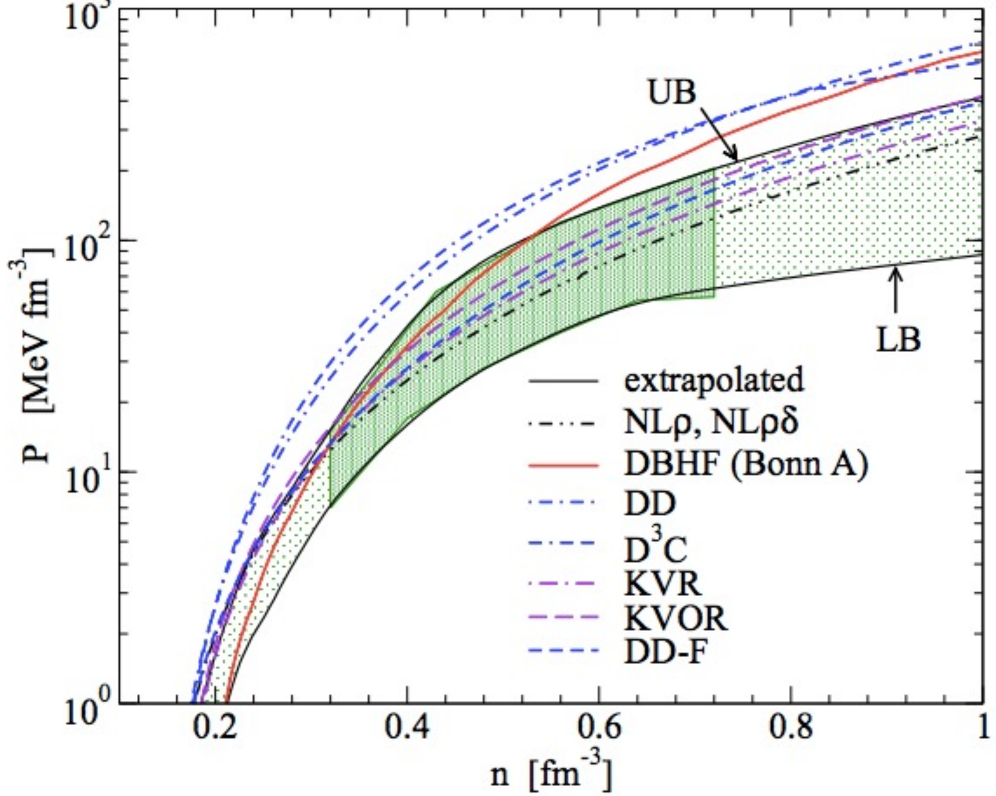
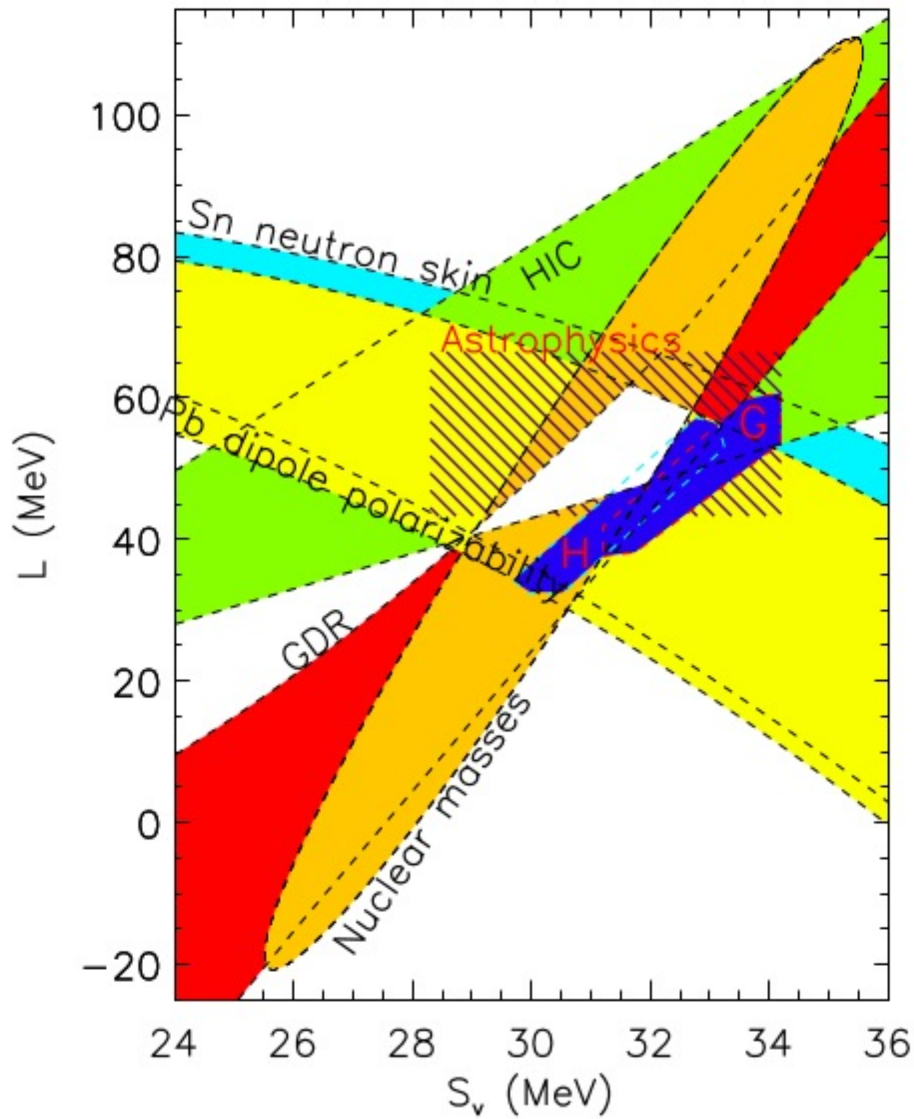


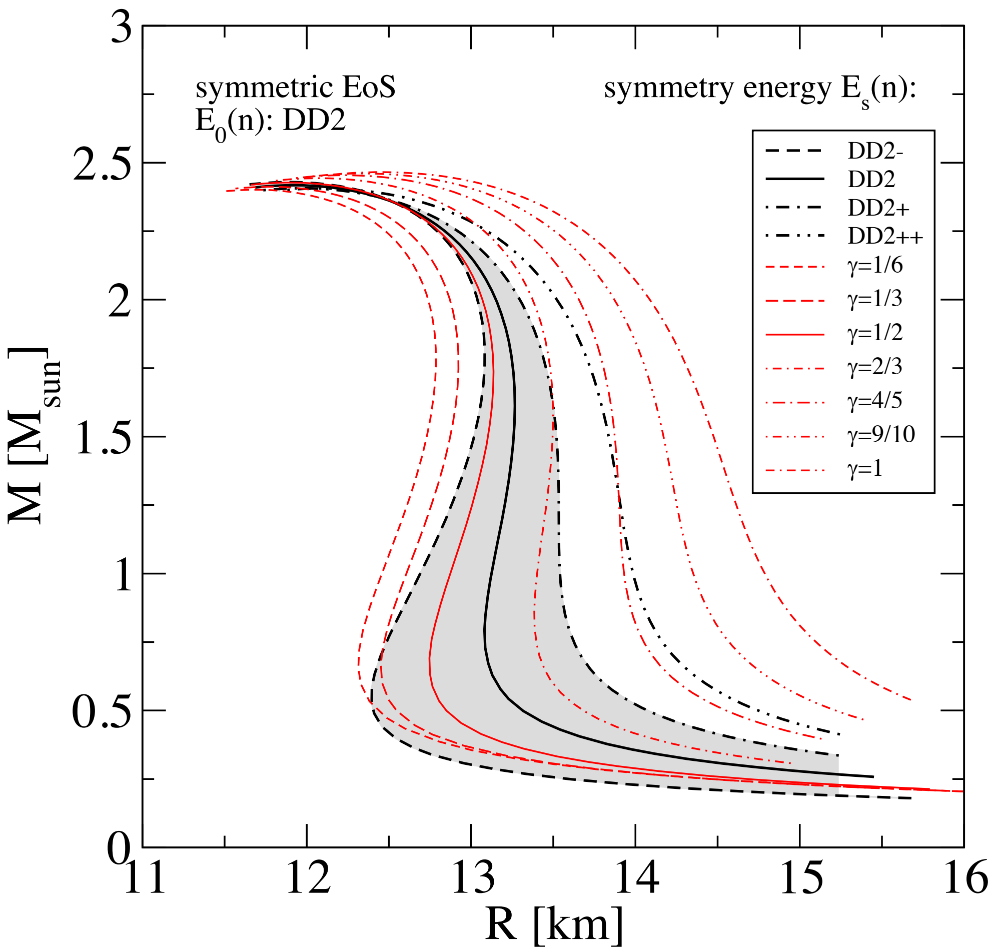
FIG. 6: Pressure region consistent with experimental flow data in SNM (dark shaded region). The light shaded region extrapolates this region to higher densities within an upper (UB) and lower border (LB).

Measuring the symmetry energy



Lattimer and Lim (2013) ApJ 771 51

Symmetry energy effects



model	J [MeV]	L [MeV]	K_{sym} [MeV]
γ =1/6	27.27	13.64	-34.09
γ =1/3	28.94	28.94	-57.89
γ =1/2	30.72	46.08	-69.11
γ =2/3	32.60	65.20	-65.20
γ =8/10	34.19	82.05	-49.23
γ =9/10	35.43	95.66	-28.67
γ =1	36.71	110.14	0
DD2-	30.17	40.14	-54.41
DD2	31.78	55.19	-93.33
DD2+	33.13	70.25	-93.12
DD2++	34.38	85.40	-64.59

Massive Neutron Stars

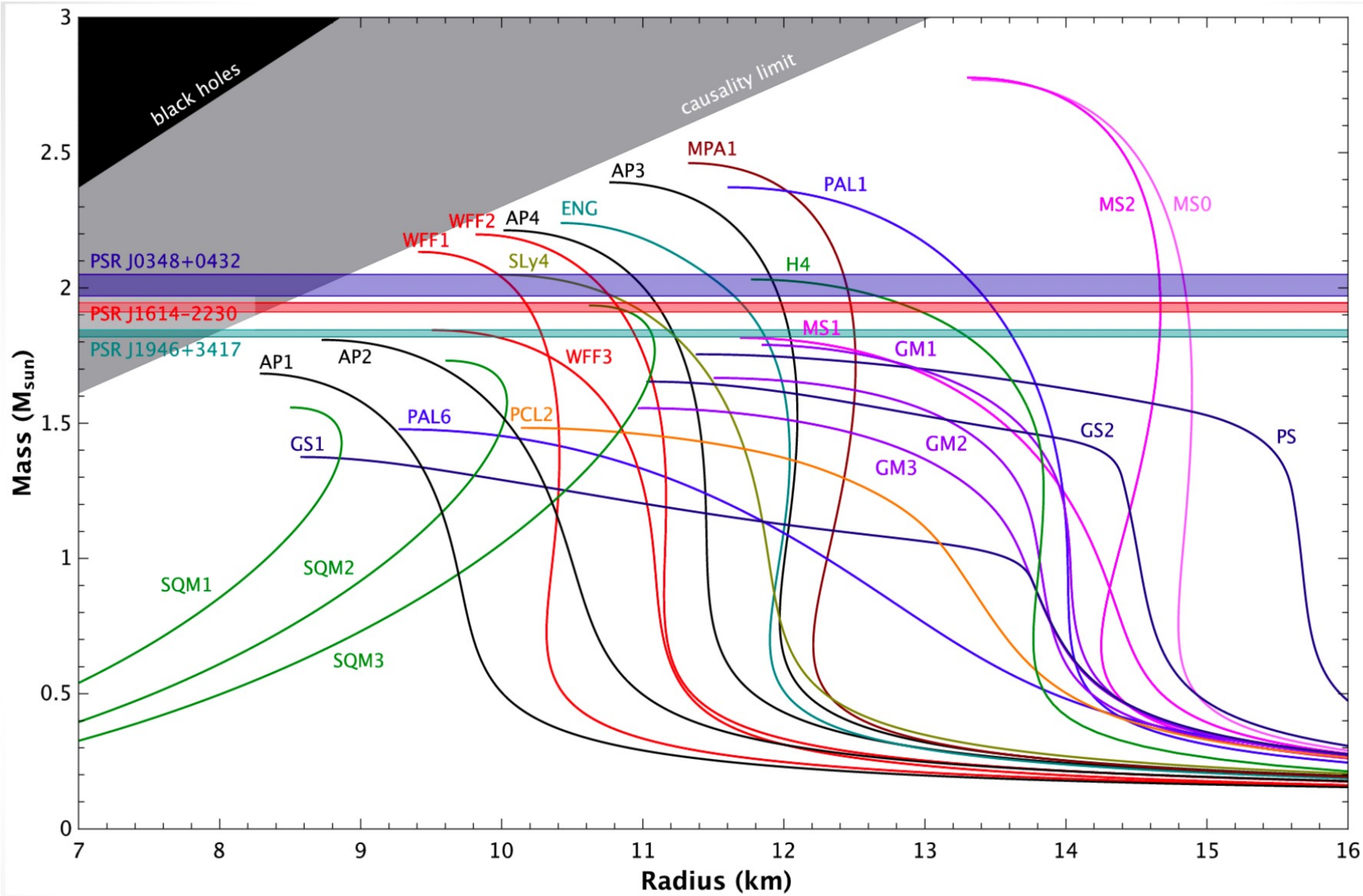


Figure created by Norbert Wex. EoS tabulated in Lattimer & Prakash (2001) and provided by the authors.

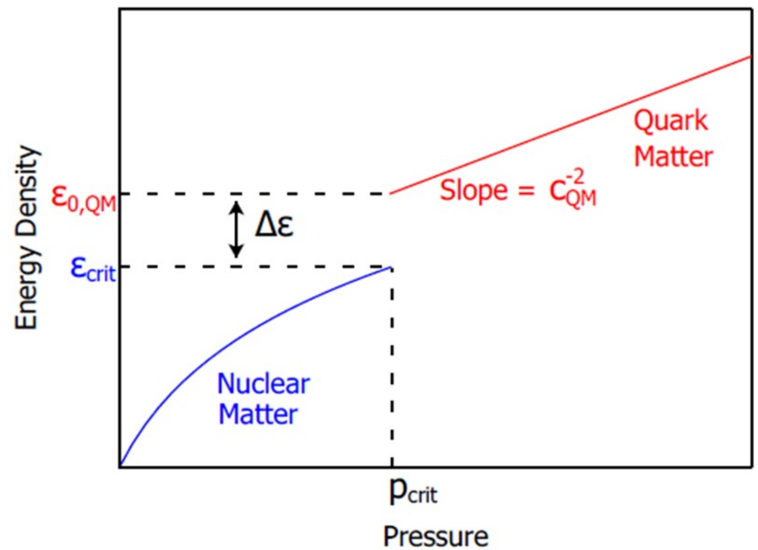
Neutron Star Twins

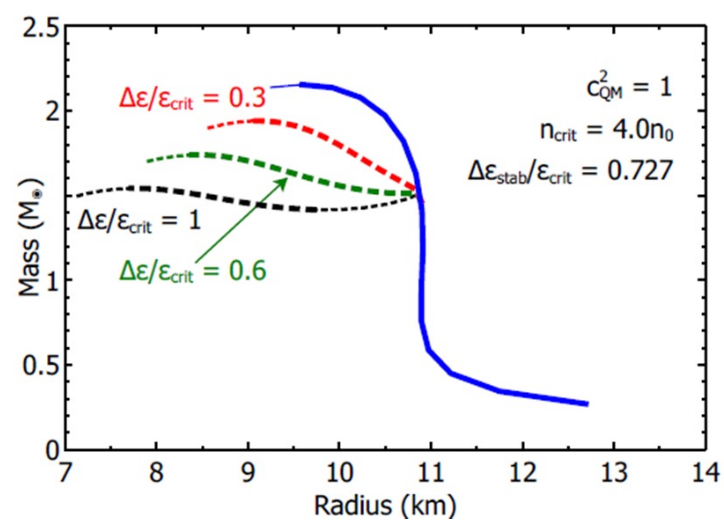
Compact Star Mass Twins and the AHP scheme

- First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → “**third family of CS**”.

- Measuring two **disconnected populations** of compact stars in the M-R diagram would represent **the detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!

Alford, Han, Prakash,
Phys. Rev. D 88, 083013 (2013)
arxiv:1302.4732



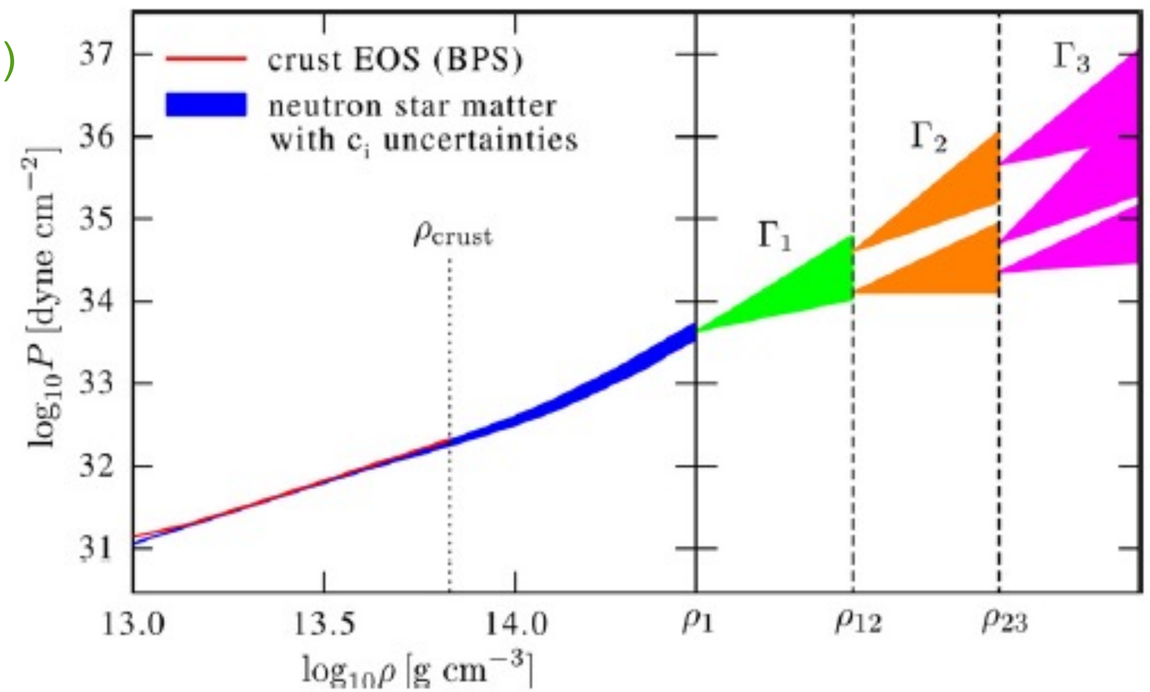


Piecewise polytrope EoS

Hebeler et al., ApJ 773, 11 (2013)

$$P_i(n) = \kappa_i n^{\Gamma_i}$$

$$i = 1 \; : \; n_1 \leq n \leq n_{12}$$
$$i = 2 \; : \; n_{12} \leq n \leq n_{23}$$
$$i = 3 \; : \; n \geq n_{23} \; ,$$



Here, 1st order PT in region 2:

$$\Gamma_2 = 0 \text{ and } P_2 = \kappa_2 = P_{\rm crit}$$

$$P(n) \;=\; n^2 \frac{d(\varepsilon(n)/n)}{dn},$$
$$\varepsilon(n)/n \;=\; \int dn \, \frac{P(n)}{n^2} = \int dn \, \kappa n^{\Gamma-2} = \frac{\kappa \, n^{\Gamma-1}}{\Gamma - 1} + C,$$
$$\mu(n) \;=\; \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa \, \Gamma}{\Gamma - 1} n^{\Gamma-1} + m_0,$$

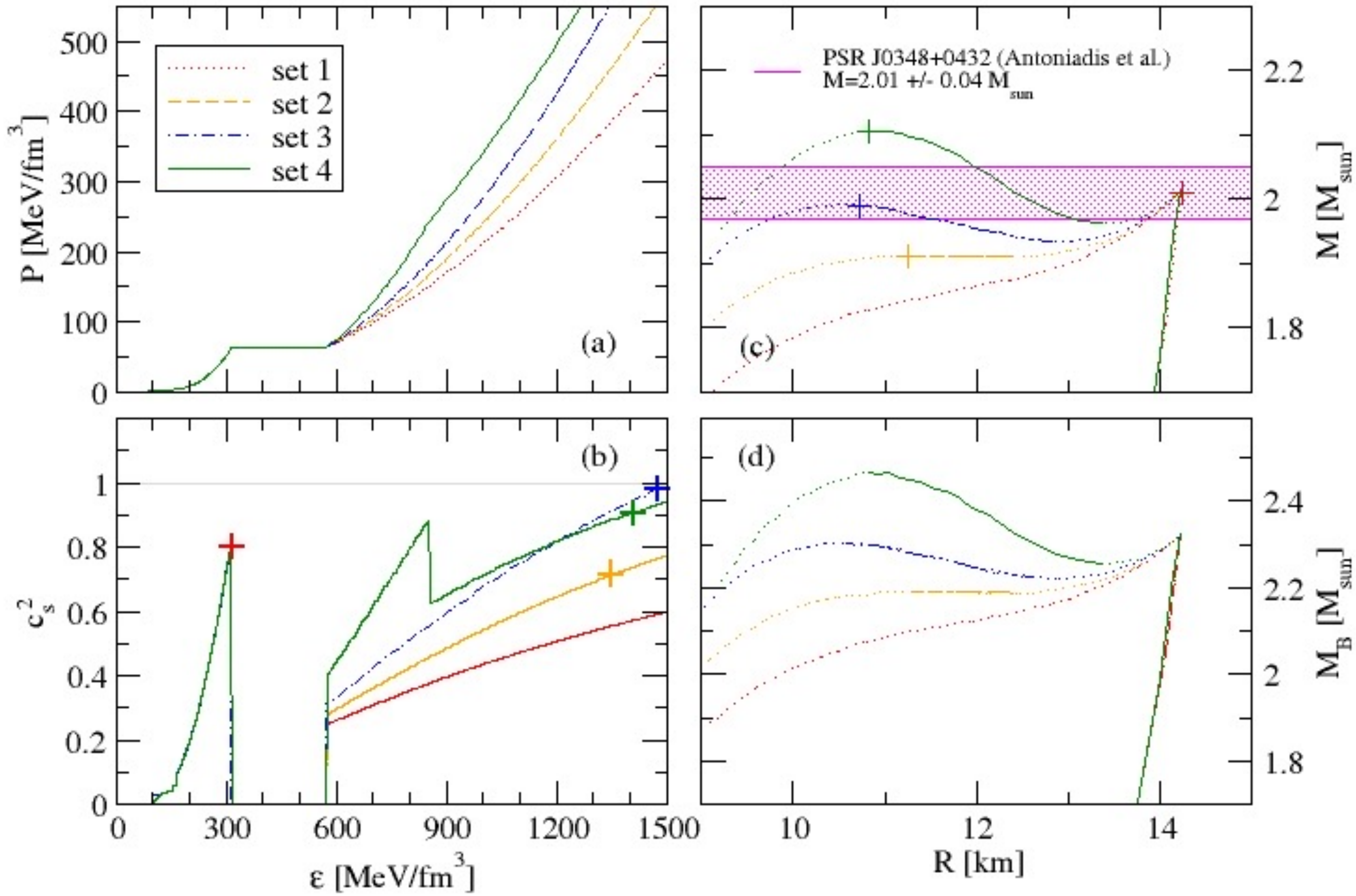
$$n(\mu) \;=\; \left[(\mu - m_0)\frac{\Gamma - 1}{\kappa\Gamma}\right]^{1/(\Gamma-1)}$$
$$P(\mu) \;=\; \kappa \left[(\mu - m_0)\frac{\Gamma - 1}{\kappa\Gamma}\right]^{\Gamma/(\Gamma-1)}$$

Maxwell construction:

$$P_1(\mu_{\rm crit}) \;=\; P_3(\mu_{\rm crit}) = P_{\rm crit}$$
$$\mu_{\rm crit} \;=\; \mu_1(n_{12}) = \mu_3(n_{23})$$

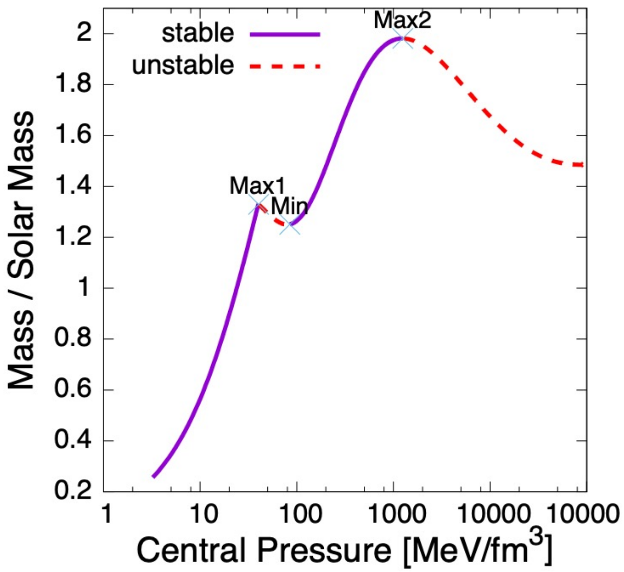
Seidov criterion for instability: $$\frac{\Delta\varepsilon}{\varepsilon_{\rm crit}} \geq \frac{1}{2} + \frac{3}{3}\frac{P_{\rm crit}}{\varepsilon_{\rm crit}}$$

Compact Star Twins



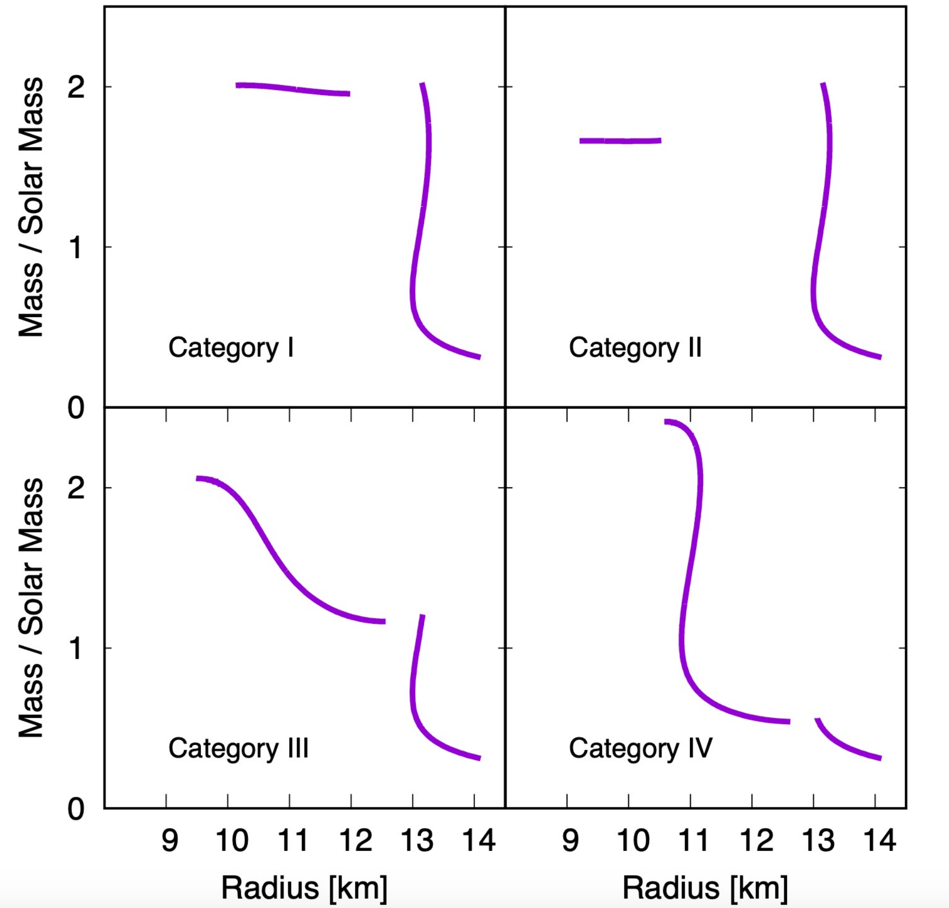
Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
arXiv: 1703.02681v2, Phys. Rev. C 96, 045809 (2017)

Classification



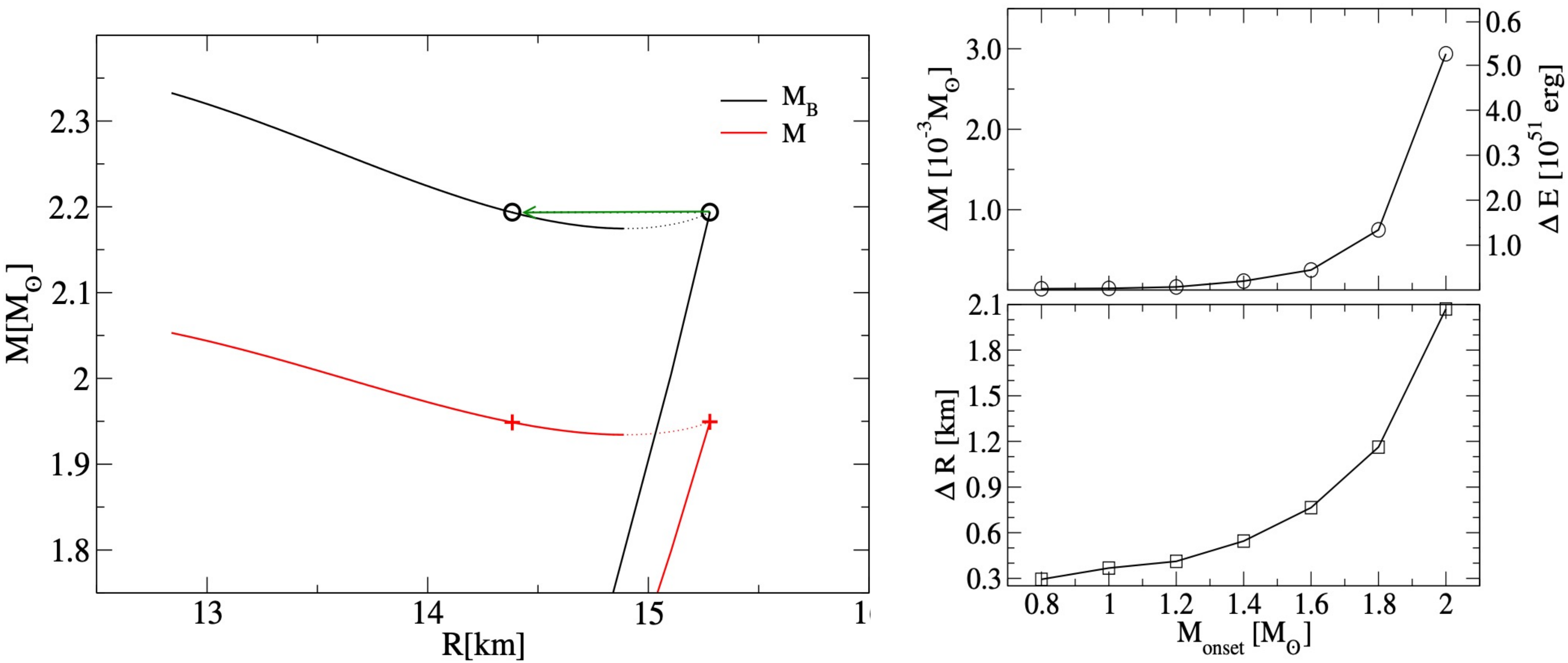
	Low p_{trans}	High p_{trans}	Low $\Delta\epsilon$	High $\Delta\epsilon$
Category I	118	184	214	375
Category II	118	136	375	725
Category III	24	117	214	368
Category IV	7	23	150	425

TABLE I. The four categories of twin stars defined by the masses of their maxima. All entries are given in units of MeV/fm^3. "High" and "Low" describes the upper or lower limit of p_{trans} and $\Delta\epsilon$ of the category.



Christian, J., Zacchi, A. & Schaffner-Bielich, J.
Eur. Phys. J. A **54,** 28 (2018).

Mass Twins – Energy Released



DD2MEV-CSS EoS

D. A-C, Astronomischen Nachrichten (2021) 1–6, arXiv: 2011.11145

EoS & Neutron Star Structure

Alternative to the “standard” QCD phase diagram:

Alternative phase transition constructions:

(a) “normal”

- Maxwell construction exists
- Mixed phase construction
  Mimics “pasta” phases

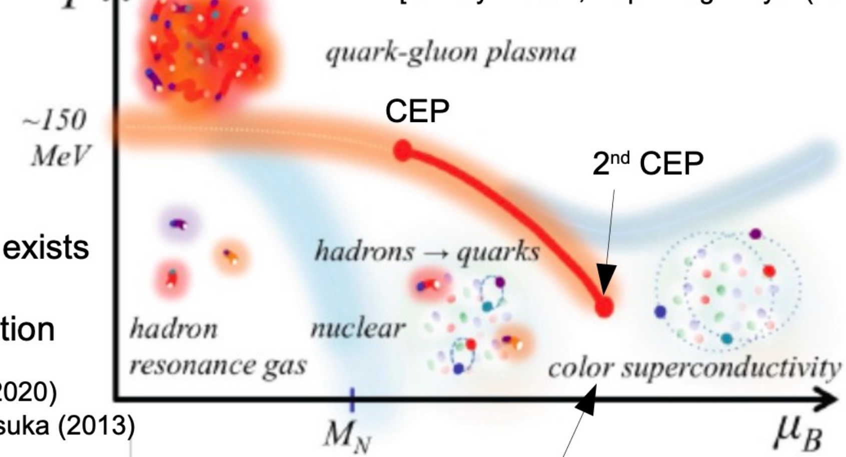
A. Ayriyan et al., PRC 97, 054802 (2018)
K. Maslov et al. PRC 100, 025802 (2019)

(b) “anomalous”

- No Maxwell construction exists
- Interpolation
  Mimics “crossover” transition

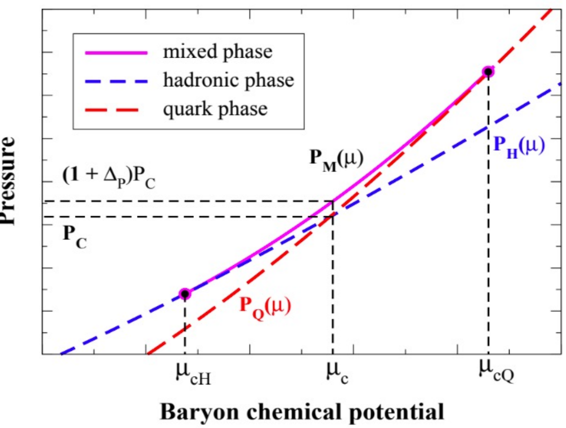
A. Ayriyan et al., in preparation (2020)
K. Masuda, T. Hatsuda, T. Takatsuka (2013)

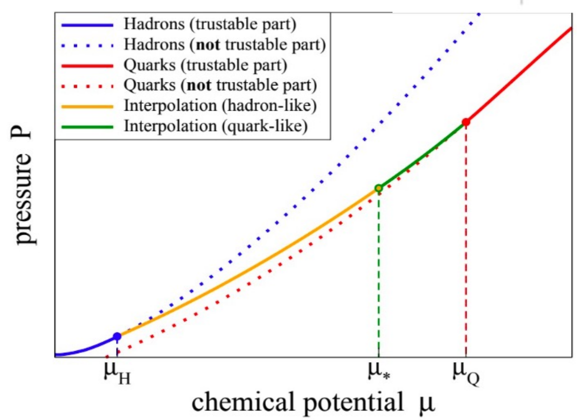
[G. Baym et al., Rep. Prog. Phys. (2018)]



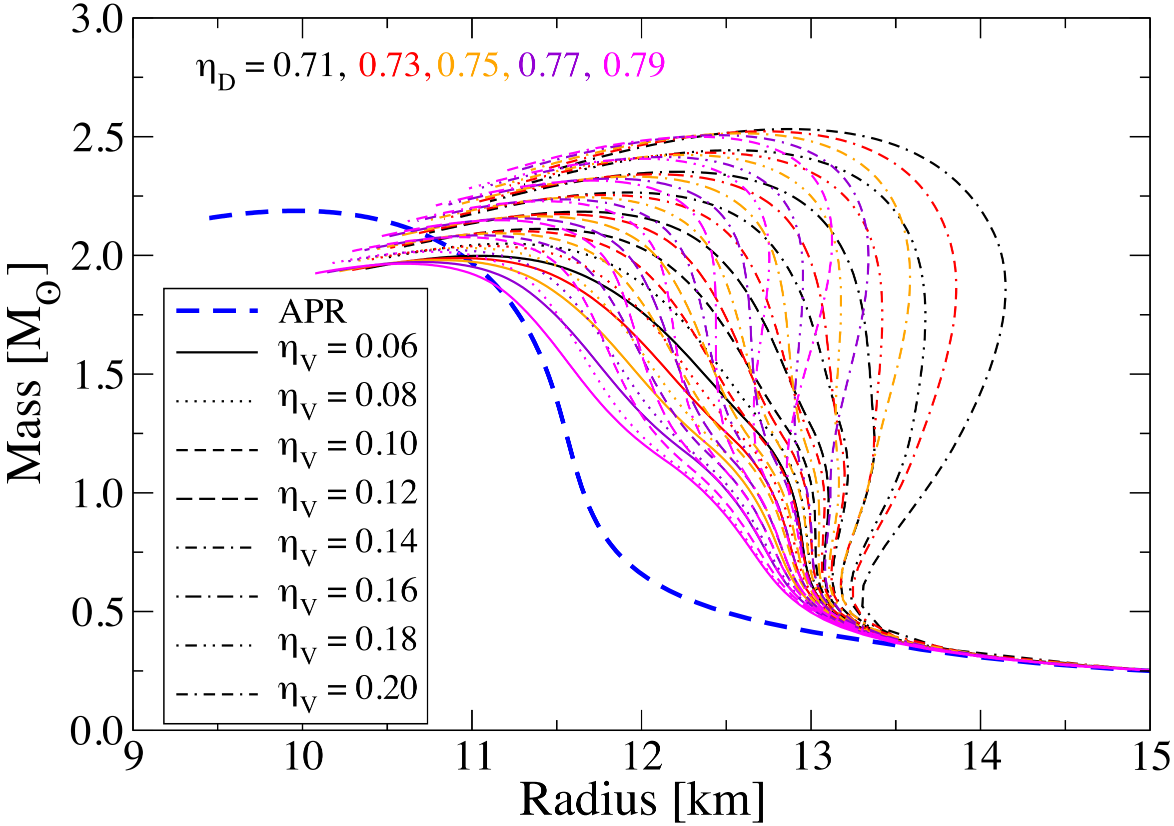
Quark-hadron continuity:

T. Schaefer & F. Wilczek,
Phys. Rev. Lett. 82 (1999) 3956
C. Wetterich,
Phys. Lett. B 462 (1999) 164
T. Hatsuda, M. Tachibana, T. Yamamoto & G. Baym,
Phys. Rev. Lett. 97 (2006) 122001





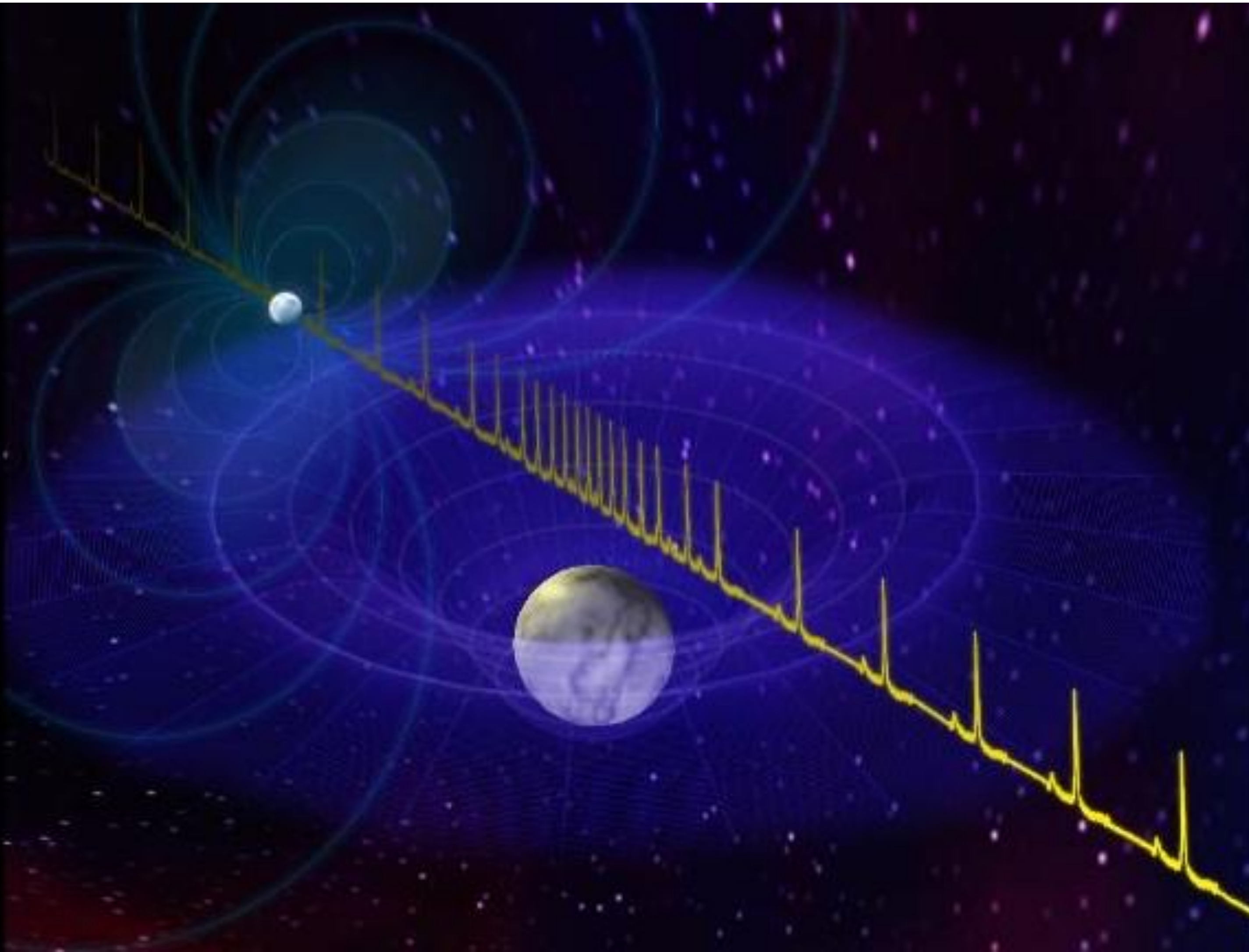
Interpolation



A. Ayriyan et al. - arXiv: 2102.13485

Multi-messenger Astronomy

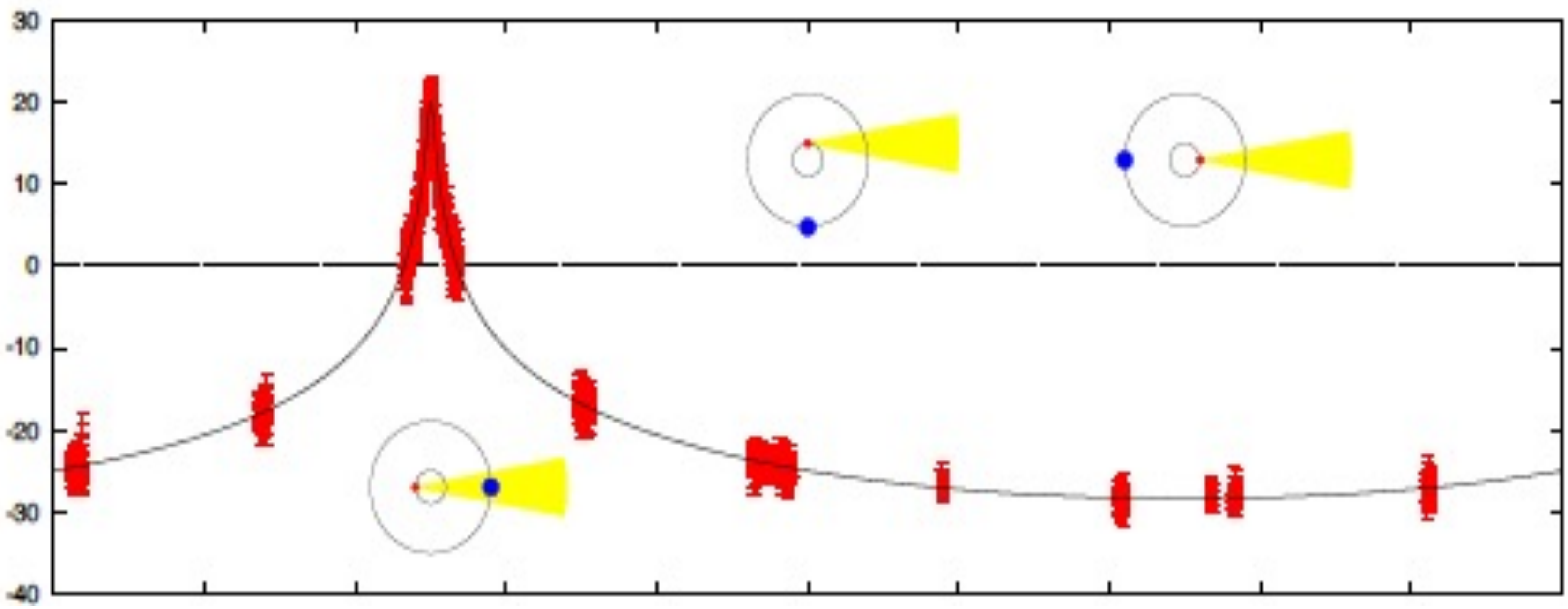
Massive Neutron Stars



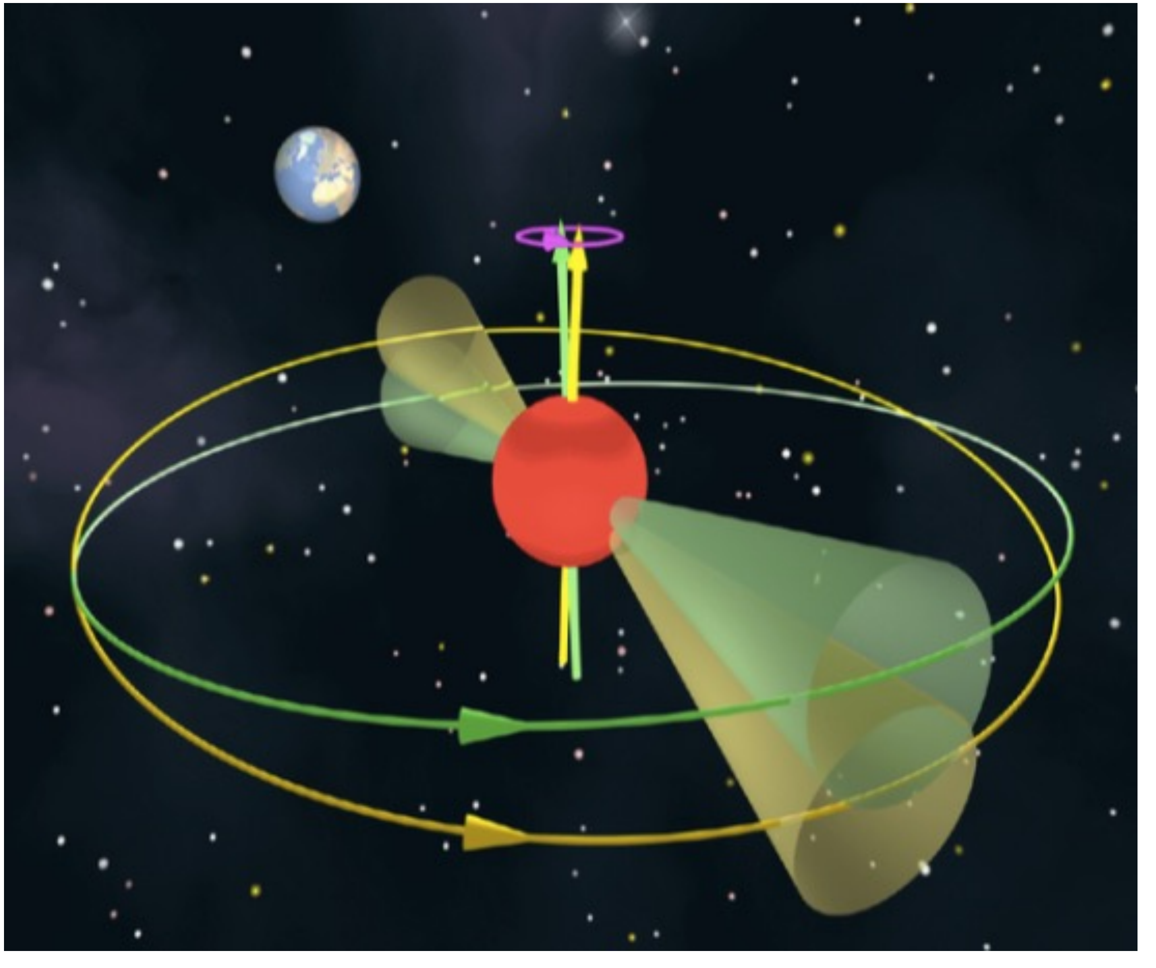
PSR J1614-2230

A precise AND large mass measurement

Shapiro delay:



Measurement of spin precession of a Pulsar



Moments of Inertia

J.M. Lattimer, M. Prakash / Physics Reports 442 (2007) 109–165

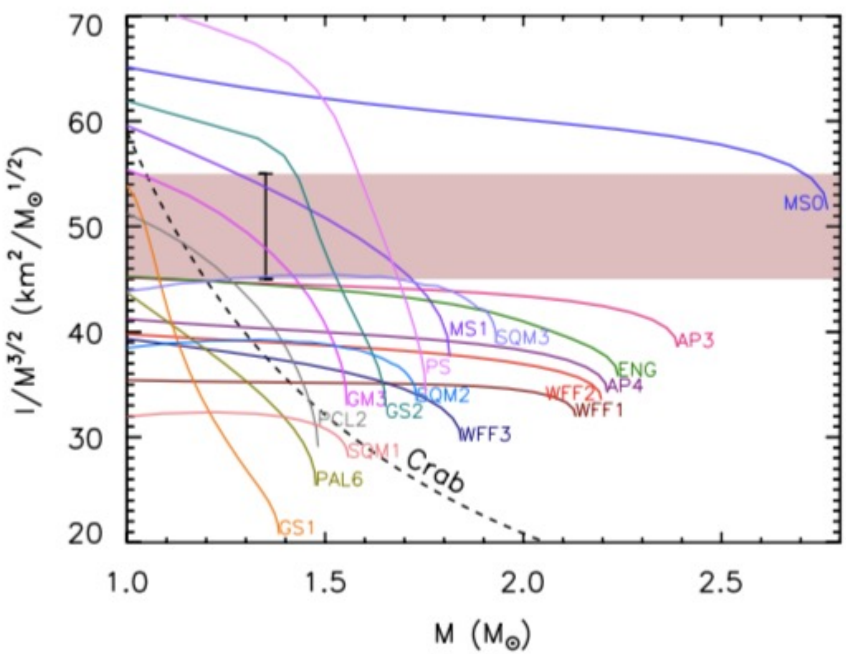
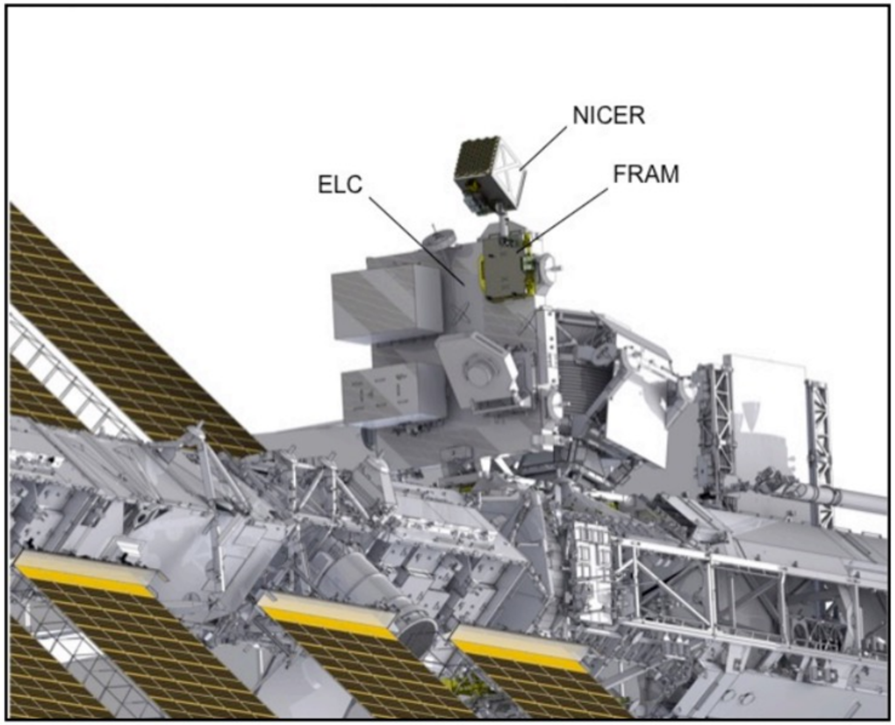
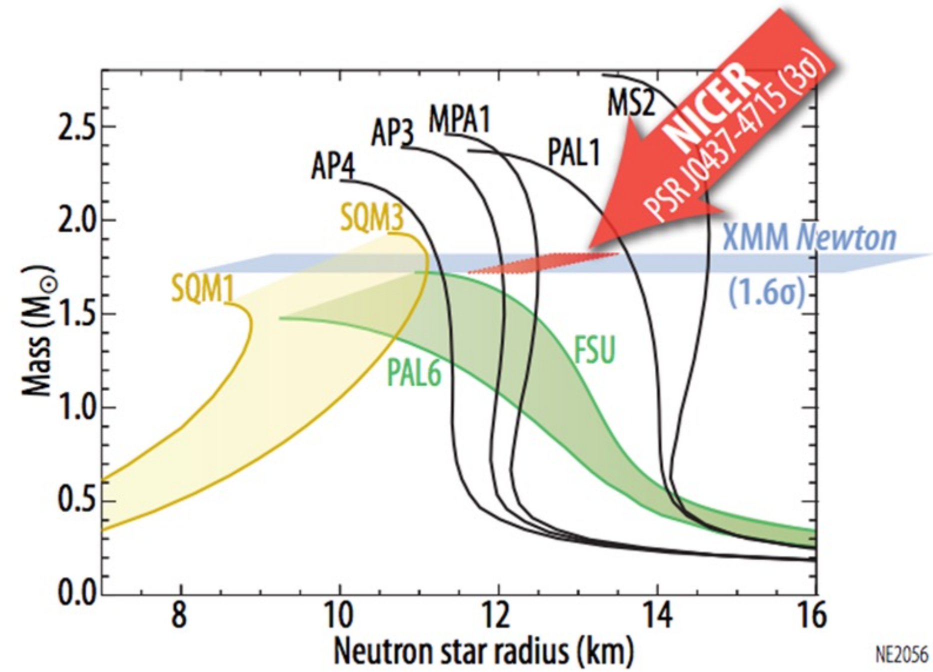


Fig. 9. The moment of inertia scaled by $M^{3/2}$ as a function of stellar mass M for EOSs described in [6]. The shaded band illustrates a $\pm 10\%$ error on a hypothetical $I/M^{3/2}$ measurement with centroid $50\,\mathrm{km}^2\,\mathrm{M}_\odot^{-1/2}$; the error bar shows the specific case in which the mass is $1.34\,\mathrm{M}_\odot$ with essentially no error. The dashed curve labelled "Crab" is the lower limit derived by [123] for the Crab pulsar.

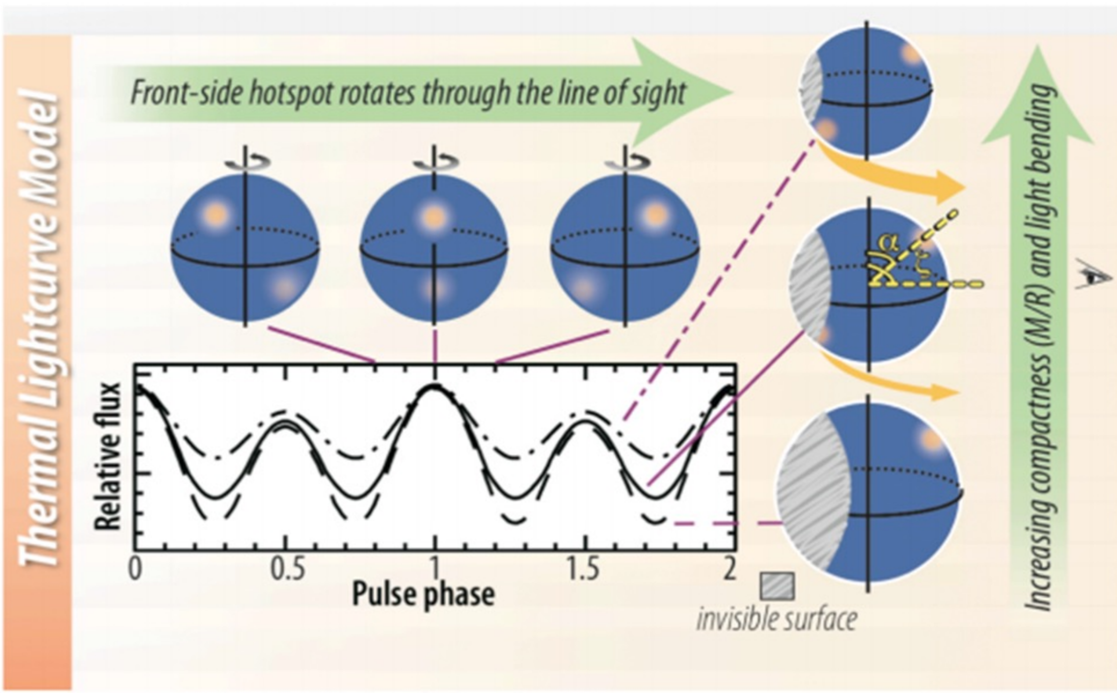
$$I \simeq \frac{J}{1+2GJ/R^3c^2}, \quad J = \frac{8\pi}{3}\int_0^R r^4\left(\rho + \frac{p}{c^2}\right)\Lambda dr, \quad \Lambda = \frac{1}{1-2Gm/rc^2}$$

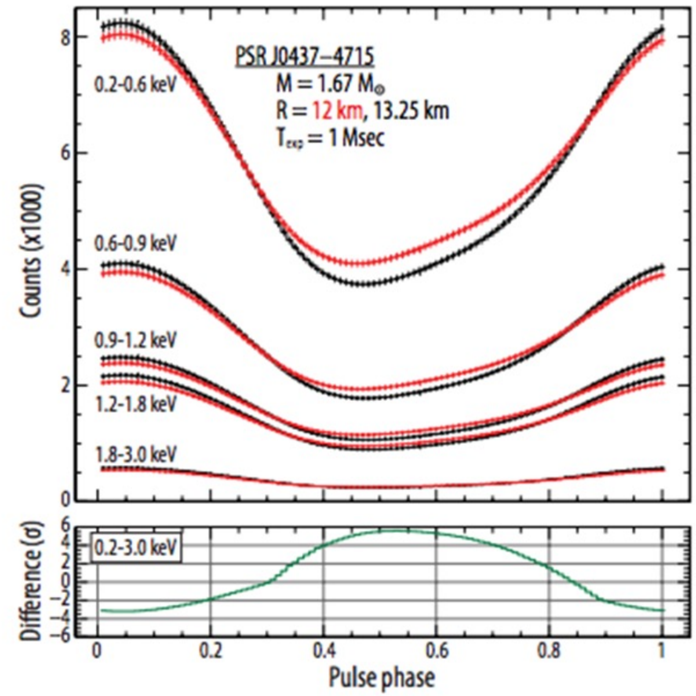

NICER
Neutron star Interior Composition ExploreR


NICER
ELC
FRAM


NICER
PSR J0437-4715 (3σ)
MS2
AP3
MPA1
PAL1
AP4
SQM3
SQM1
XMM Newton
(1.6σ)
FSU
PAL6
Mass ($M_\odot$)
0.0
0.5
1.0
1.5
2.0
2.5
8
10
12
14
16
Neutron star radius (km)
NE2056


NICER
Neutron star Interior Composition ExploreR


Thermal Lightcurve Model
Front-side hotspot rotates through the line of sight
Increasing compactness (M/R) and light bending
Relative flux
Pulse phase
0
0.5
1
1.5
2
invisible surface

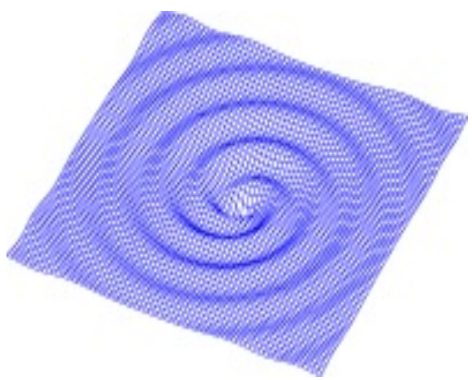

PSR J0437−4715
M = 1.67 M$_\odot$
R = 12 km, 13.25 km
T$_{exp}$ = 1 Msec
0.2-0.6 keV
0.6-0.9 keV
0.9-1.2 keV
1.2-1.8 keV
1.8-3.0 keV
Counts (x1000)
Difference (σ)
0.2-3.0 keV
Pulse phase

Hot Spots

Hulse-Taylor pulsar – binary system

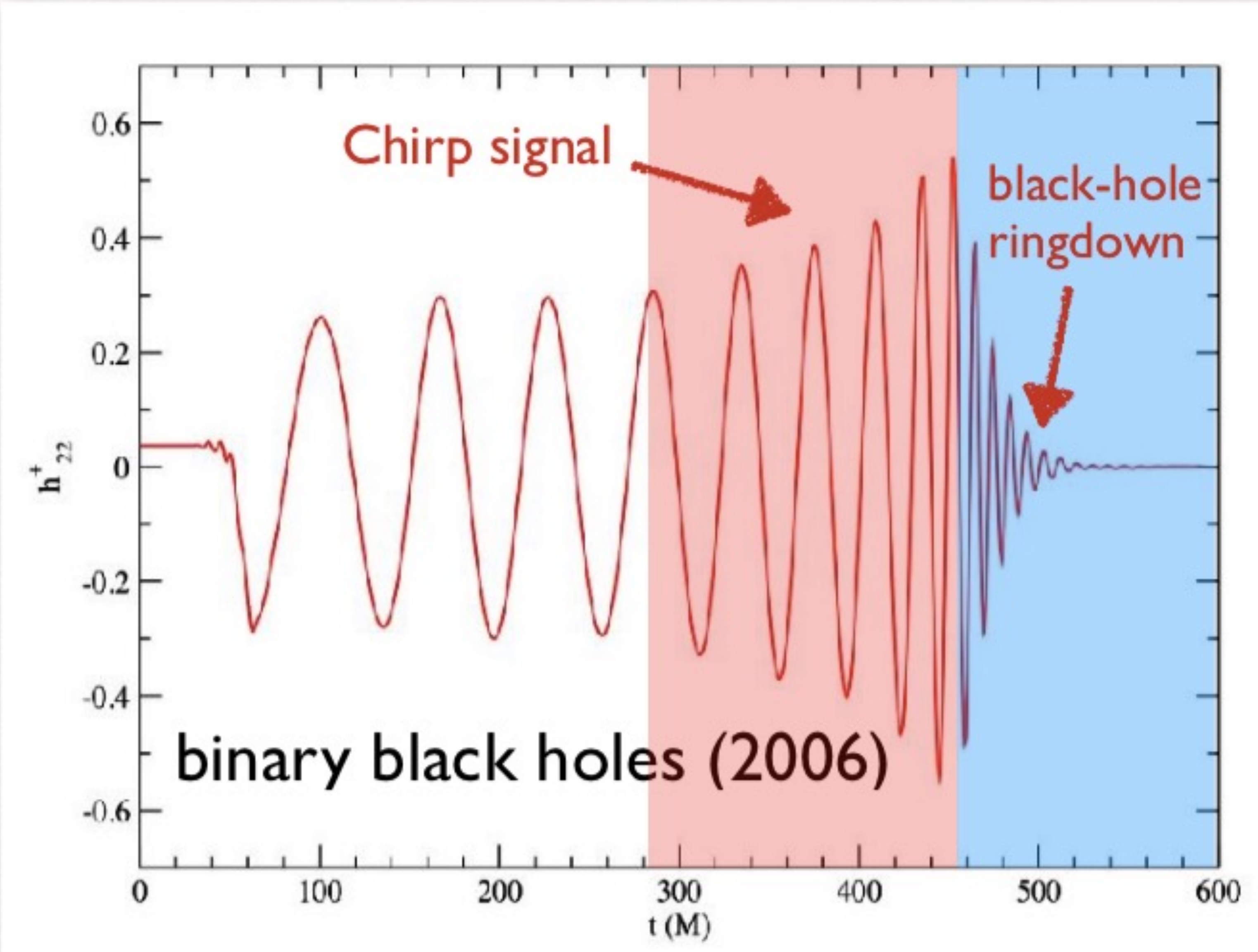
PSR B1913+16 (now J1915+1606)

Nobel Prize for Hulse and Taylor (1993)



Excellent confirmation of Einstein theory of GW emission by observation of period decay

Anatomy of the GW signal



Direct measurement of gravitational waves – merging of two massive black holes (2015)

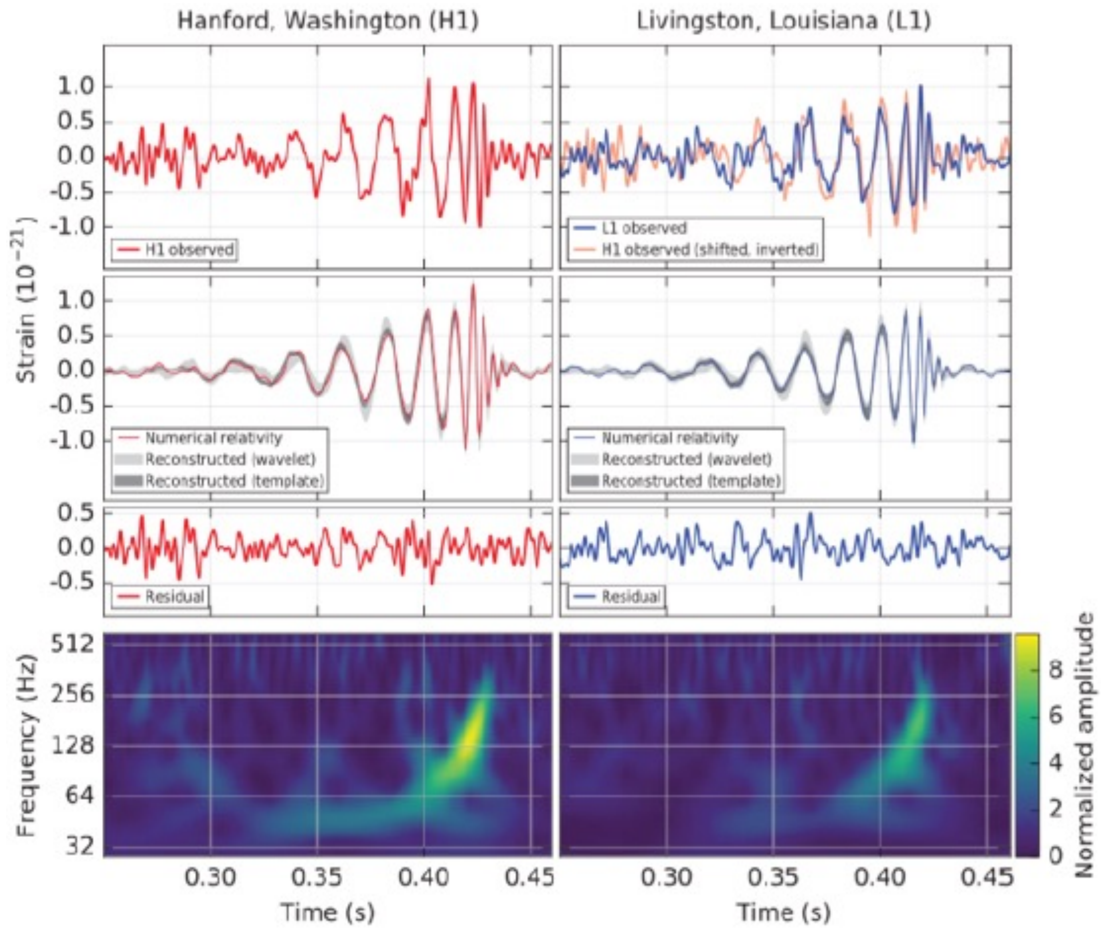
First detection of gravitational waves
September 14, 2015 at 5:51 a.m. EDT
(LIGO Collaboration)

Source at 410(18) Mpc [z=0.09(4)]

Initial black hole masses:
36(5) Mo and 29(4) Mo
Final black hole mass:
62(4) Mo

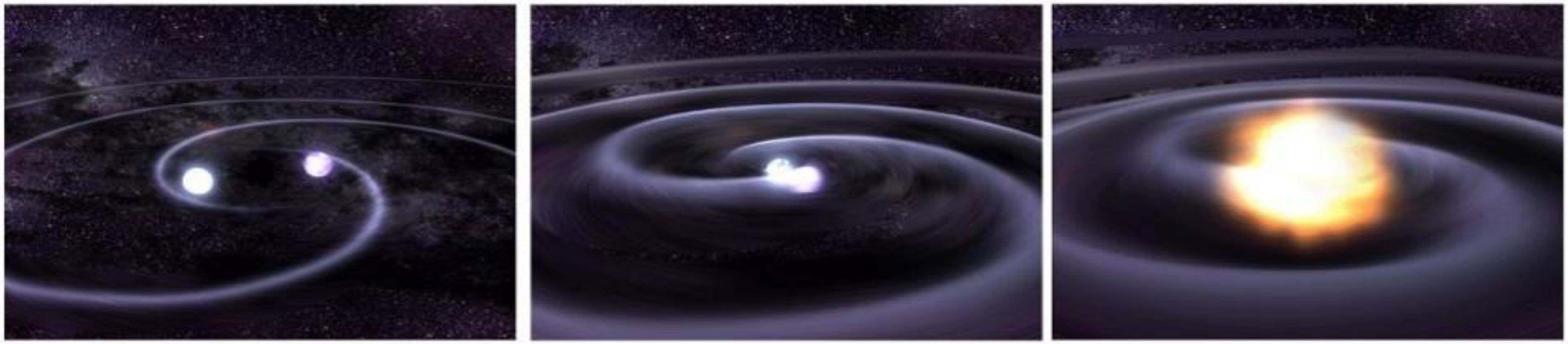
Energy release in gravitational waves
3.0(5) Mo c^2

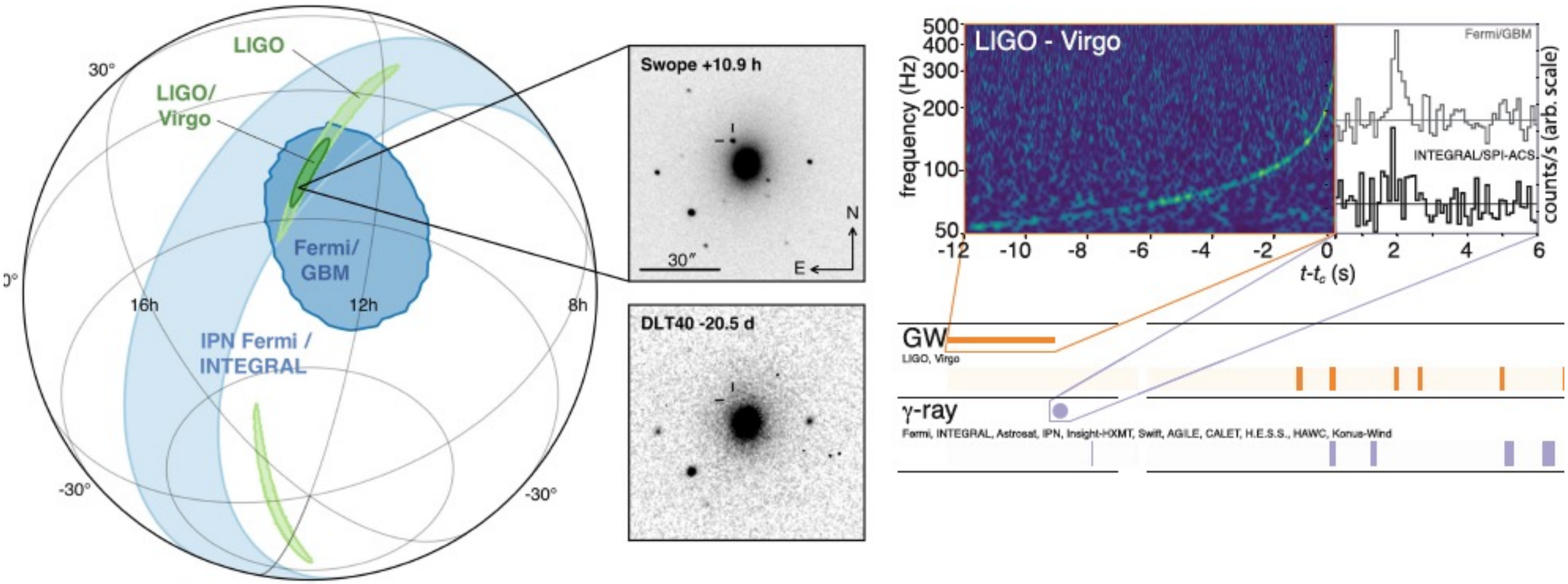
Phys. Rev. Lett. 116, 061102 (2016)



Nobel Prize Physics 2017 !! Rainer Weiss, Barry C. Barish, Kip Thorne

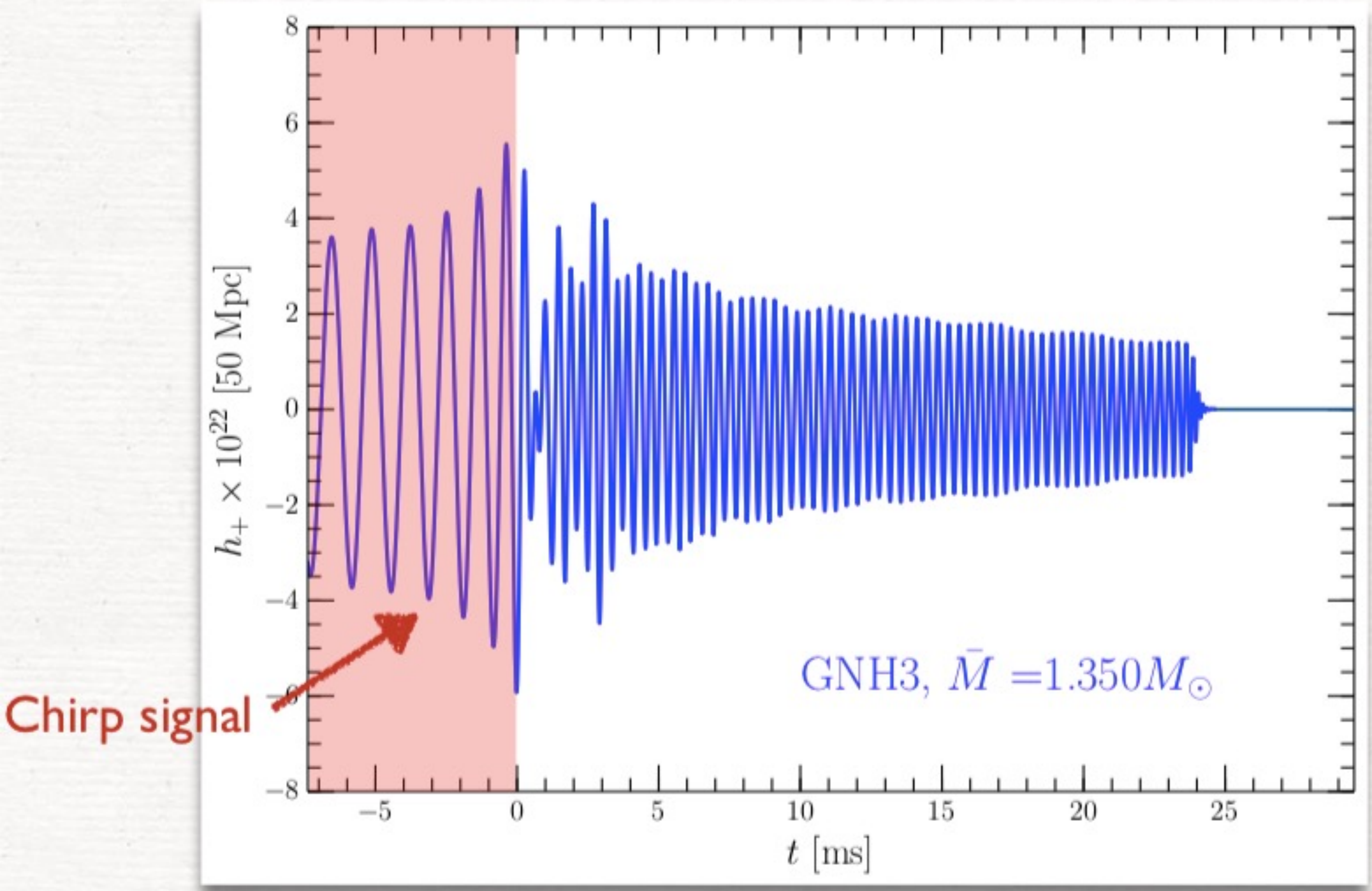
GW170817: Neutron Star Merger



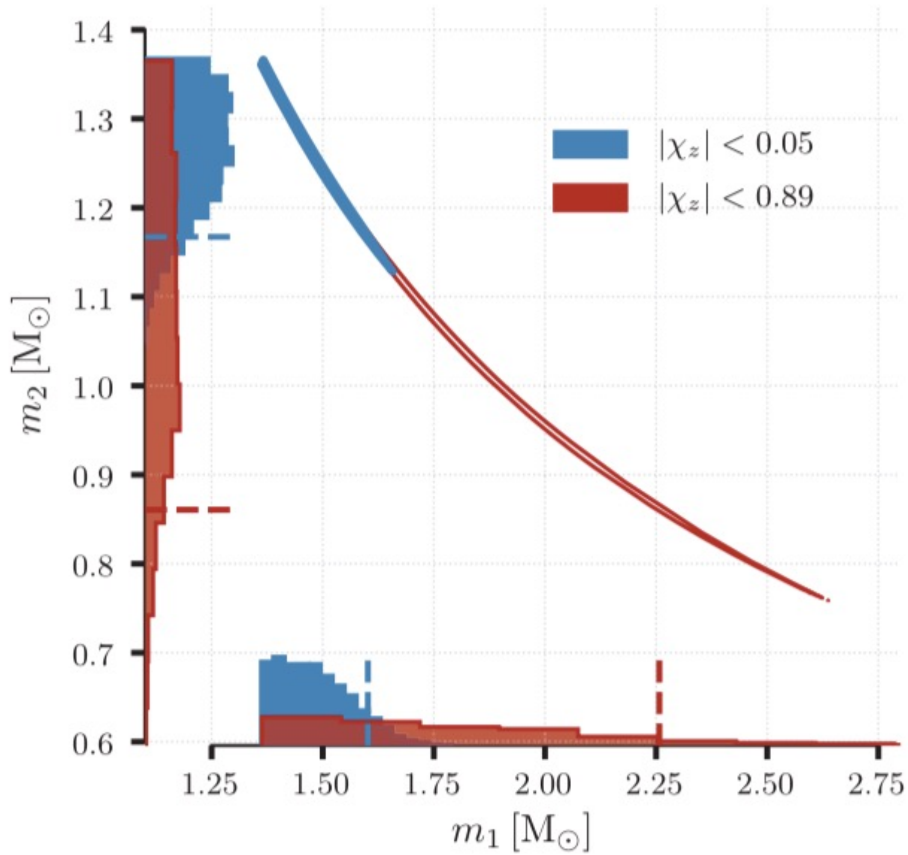


*) B.P. Abbott et al. [LIGO/Virgo Collab.], PRL 119, 161101 (2017); ApJLett 848, L12 (2017)

Anatomy of the GW signal

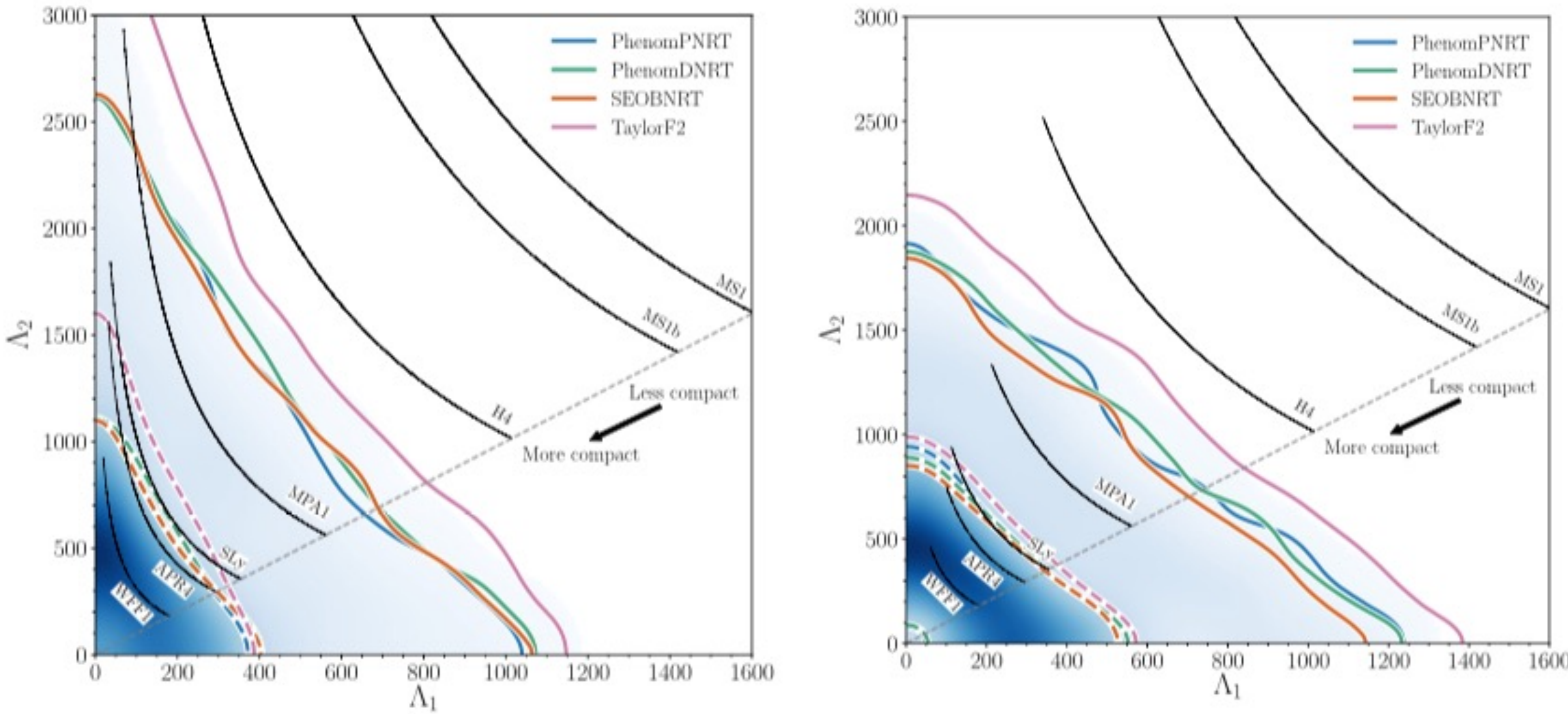


Implications from GW170817



GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral
B.P. Abbott et al. arXiv:1712.00451

Implications from GW170817



Properties of the Binary Star Merger GW170817
B. P. Abbott et al., Phys. Rev. X 9, 011001 (2019)

Computing the love number/tidal deformability

Extension of a standard TOV solver (i.e. numerically an integration of coupled ODEs):

Ansatz for the metric including a l=2 perturbation

$$\begin{aligned} ds^2 &= -e^{2\Phi(r)}\left[1+H(r)Y_{20}(\theta,\varphi)\right]dt^2 \\ &\quad +e^{2\Lambda(r)}\left[1-H(r)Y_{20}(\theta,\varphi)\right]dr^2 \\ &\quad +r^2\left[1-K(r)Y_{20}(\theta,\varphi)\right]\left(d\theta^2+\sin^2\theta d\varphi^2\right) \end{aligned}$$

Following Hinderer et al. 2010

Integrate standard TOV system:

$$\begin{aligned} e^{2\Lambda} &= \left(1-\frac{2m_r}{r}\right)^{-1}, \\ \frac{d\Phi}{dr} &= -\frac{1}{\epsilon+p}\frac{dp}{dr}, \\ \frac{dp}{dr} &= -(\epsilon+p)\frac{m_r+4\pi r^3 p}{r(r-2m_r)}, \\ \frac{dm_r}{dr} &= 4\pi r^2\epsilon. \end{aligned}$$

EoS to be provided ε(p)

And additional eqs. for perturbations:

$$\begin{aligned} \frac{dH}{dr} &= \beta \qquad (11) \\ \frac{d\beta}{dr} &= 2\left(1-2\frac{m_r}{r}\right)^{-1} H\left\{-2\pi\left[5\epsilon+9p+f(\epsilon+p)\right]\right. \\ &\qquad \left. +\frac{3}{r^2}+2\left(1-2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2}+4\pi rp\right)^2\right\} \\ &\qquad +\frac{2\beta}{r}\left(1-2\frac{m_r}{r}\right)^{-1}\left\{-1+\frac{m_r}{r}+2\pi r^2(\epsilon-p)\right\}. \end{aligned}$$

(K(r) given by H(r))

Note: Although multidimensional problem – computation in 1D since absorbed in Y20

Love number

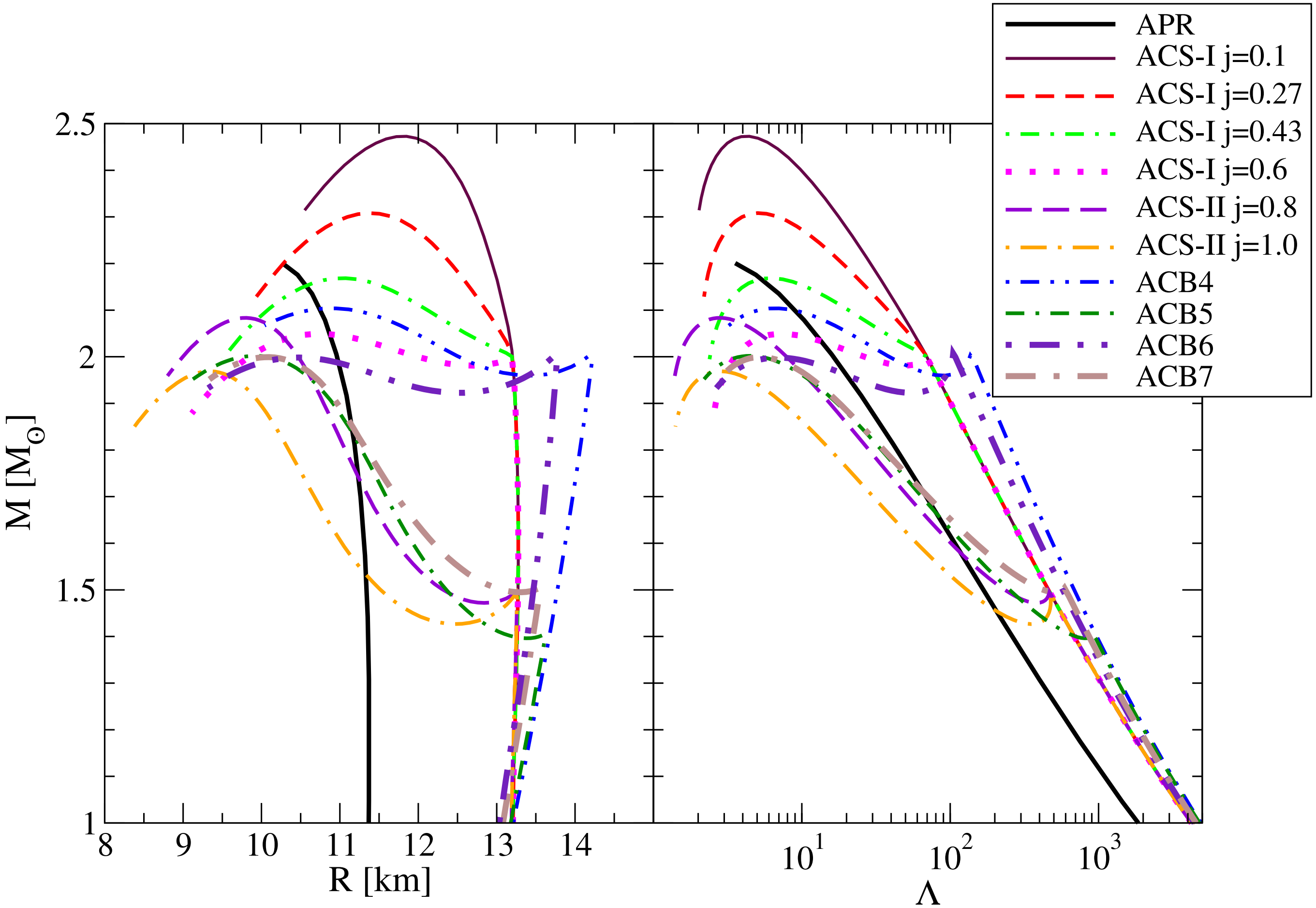
$$y = \frac{R\beta(R)}{H(R)}$$

$$\Lambda \equiv \frac{2}{3} k_2 \left(\frac{R}{M}\right)^5$$

$$\begin{aligned} k_2 \;=\; & \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \\ & \times\Big\{2C[6-3y+3C(5y-8)] \\ & +4C^3[13-11y+C(3y-2)+2C^2(1+y)] \\ & +3(1-2C)^2[2-y+2C(y-1)]\ln(1-2C)\Big\}^{-1} \end{aligned}$$

where $C = M/R$ is the compactness of the star.

Implications from GW170817



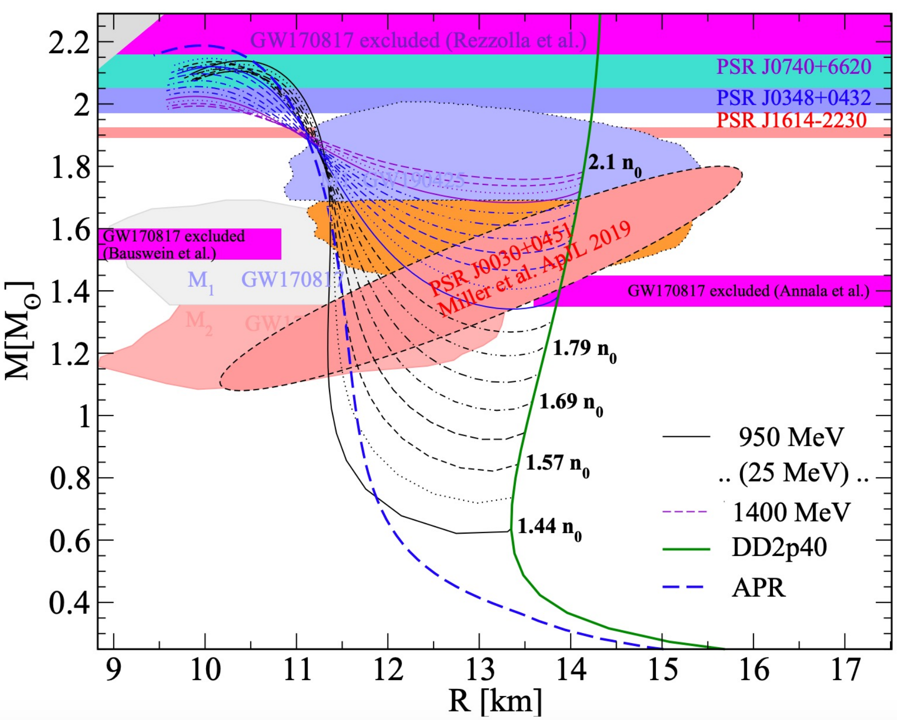
Vasileios Paschalidis, Kent Yagi, David Alvarez-Castillo, David B. Blaschke, Armen Sedrakian
Phys. Rev. D 97, 084038 (2018), arXiv:1712.00451

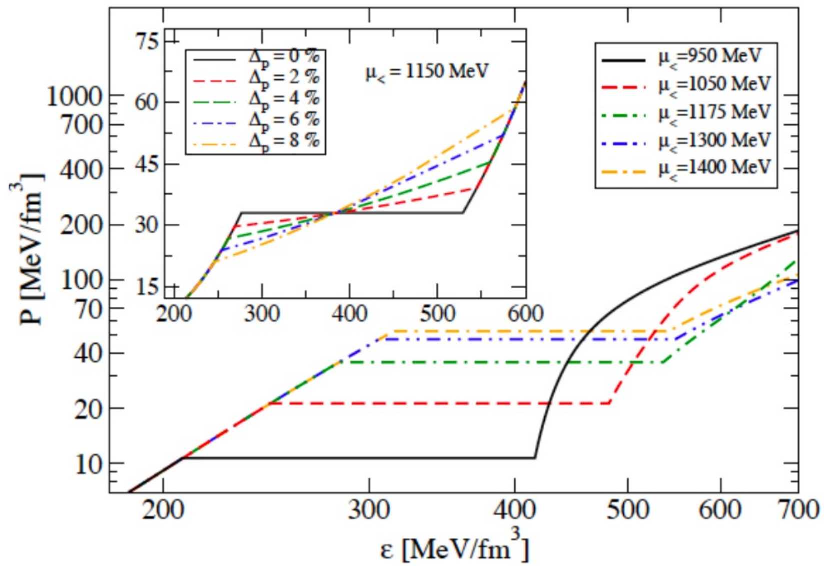
Was GW170817 a canonical neutron star merger?

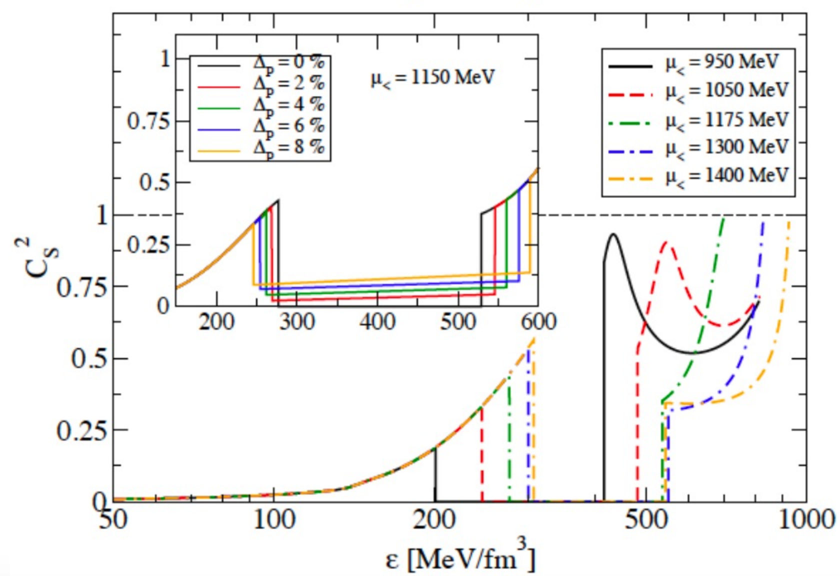
- Mass 2.14+0.10-0.09 M_sun &
- Compactness (tidal deform.) GW170817 &
- Mass+Radius NICER PSR J0030+0451

- New (fictitious) mass+radius measurement "NICER PSR J0030+0451", R→R+1.6 km Gaussian width parameter σ → σ/2

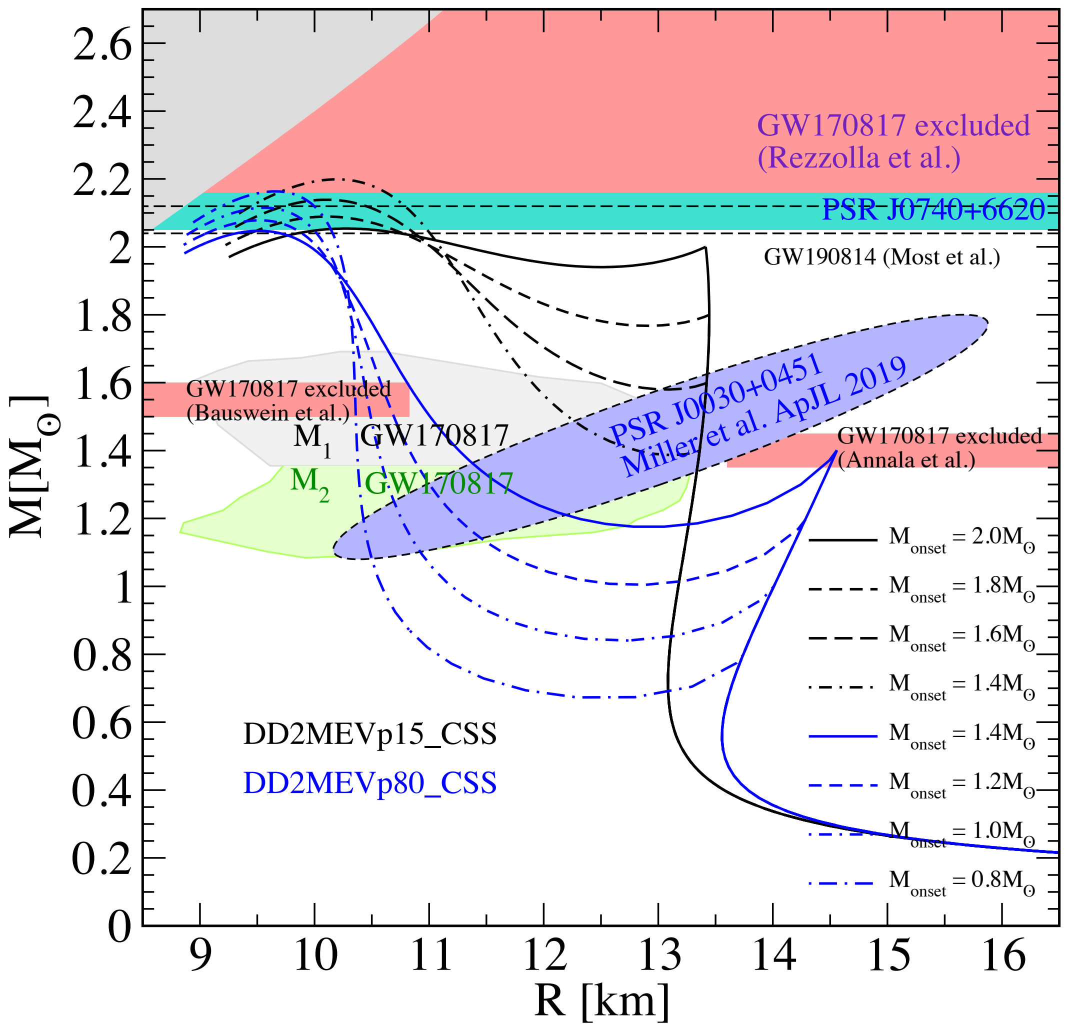
→ Two-parameter family EoS: $\mu_<$, Δ_P



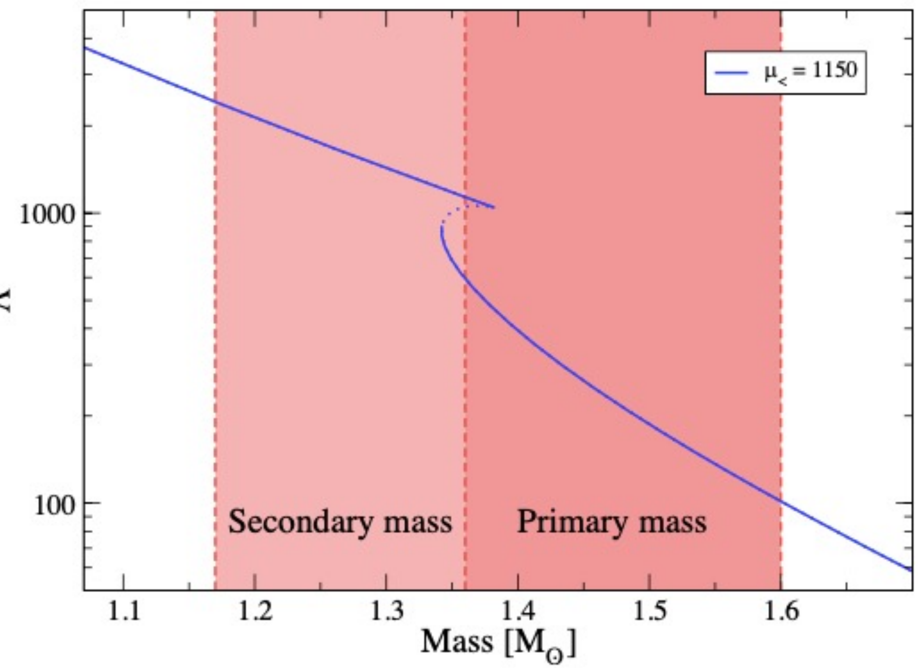


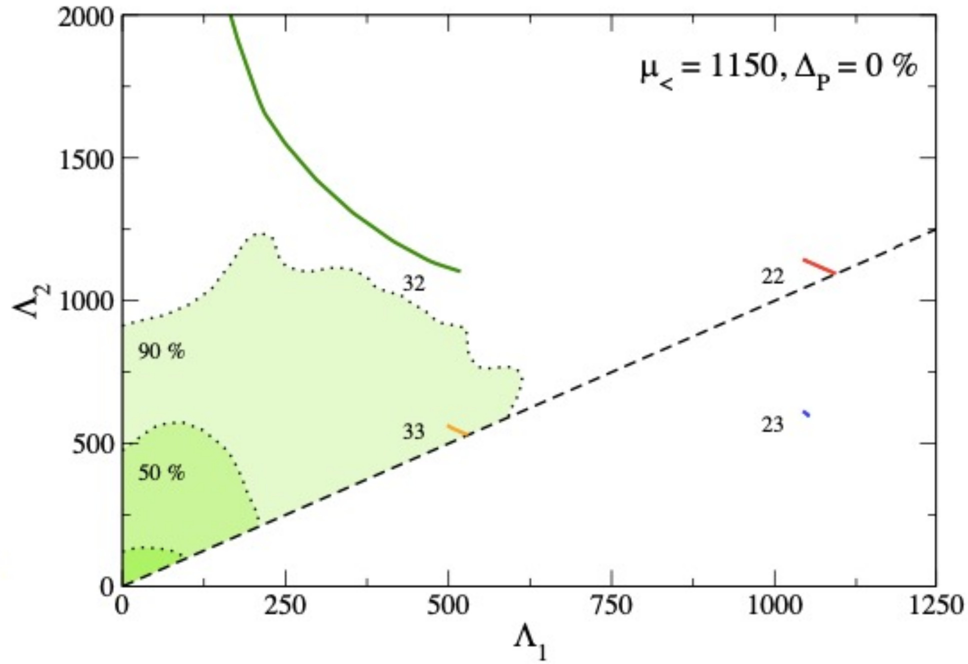


High vs Low Mass Twins



Was GW170817 a canonical neutron star merger?





A. Ayriyan, D. Alvarez-Castillo, D. Blaschke and H. Grigorian,
Universe 6, 81 (2020)

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

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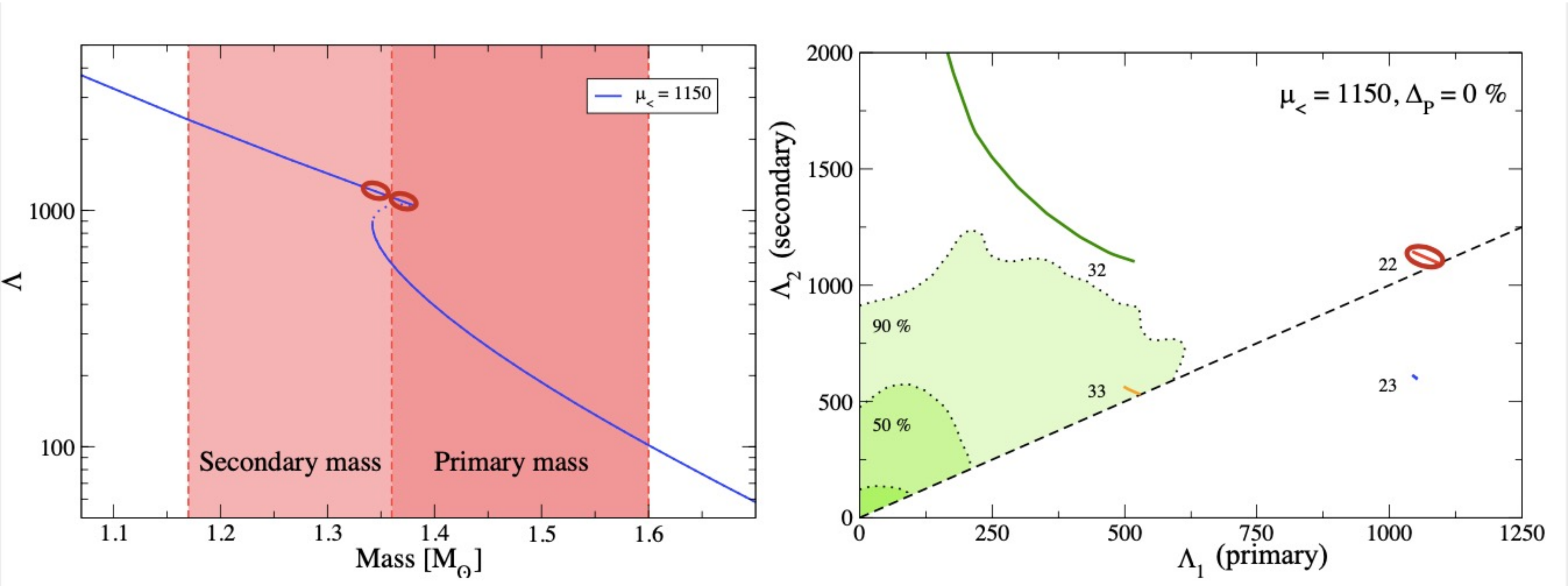

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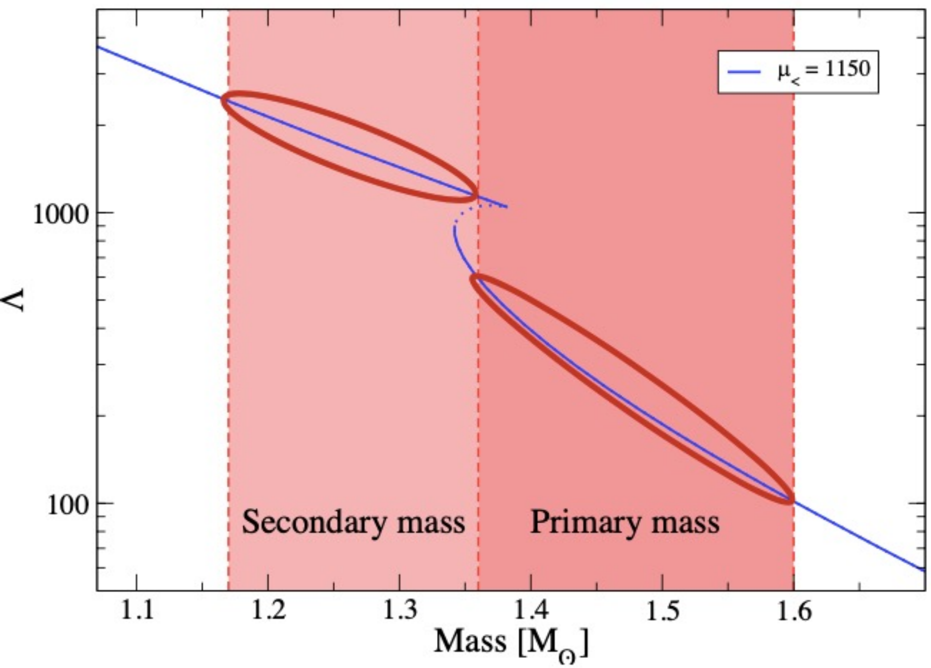

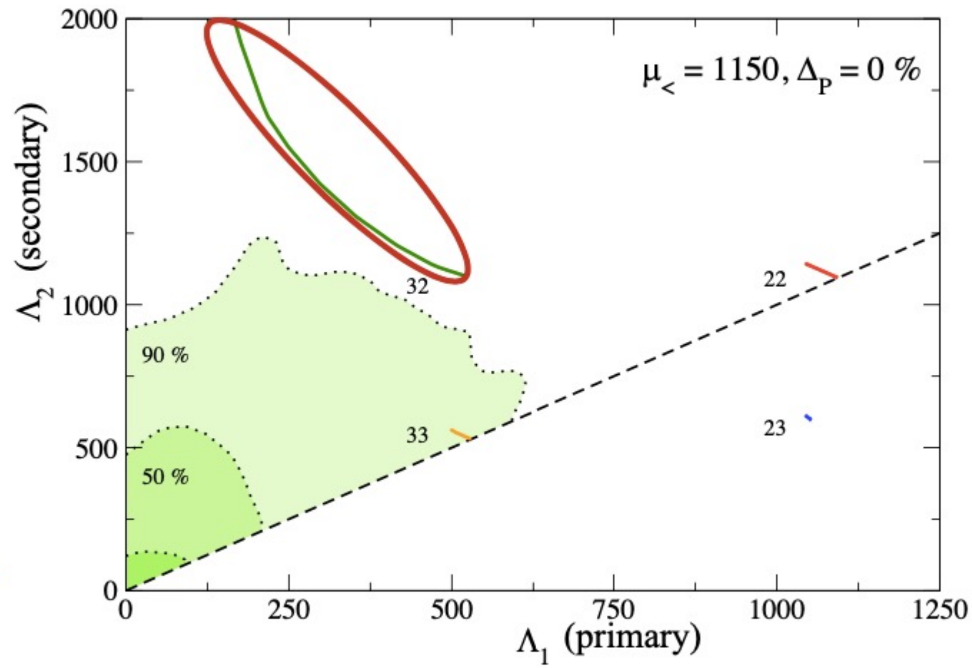

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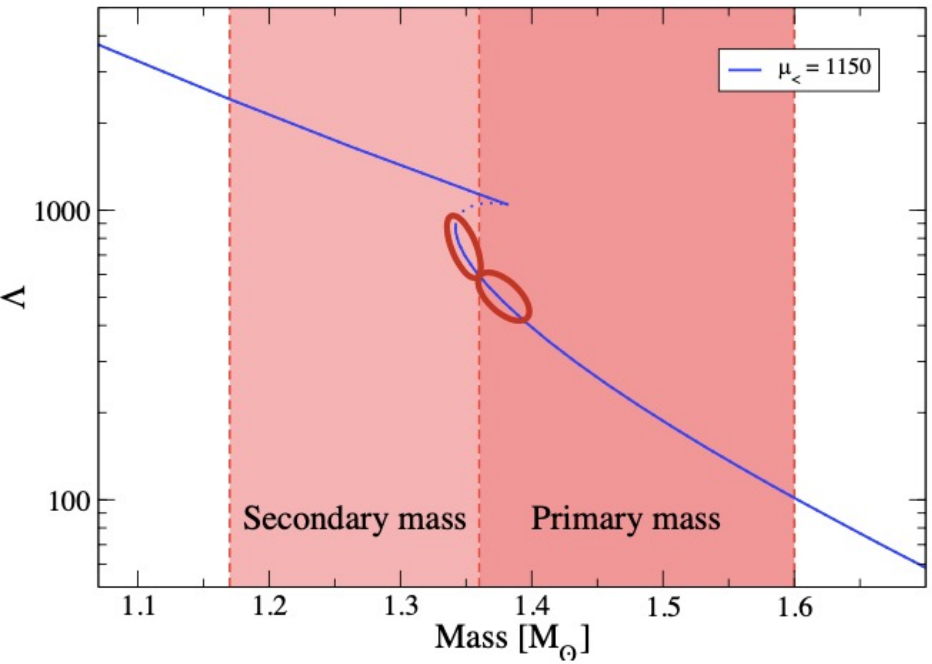

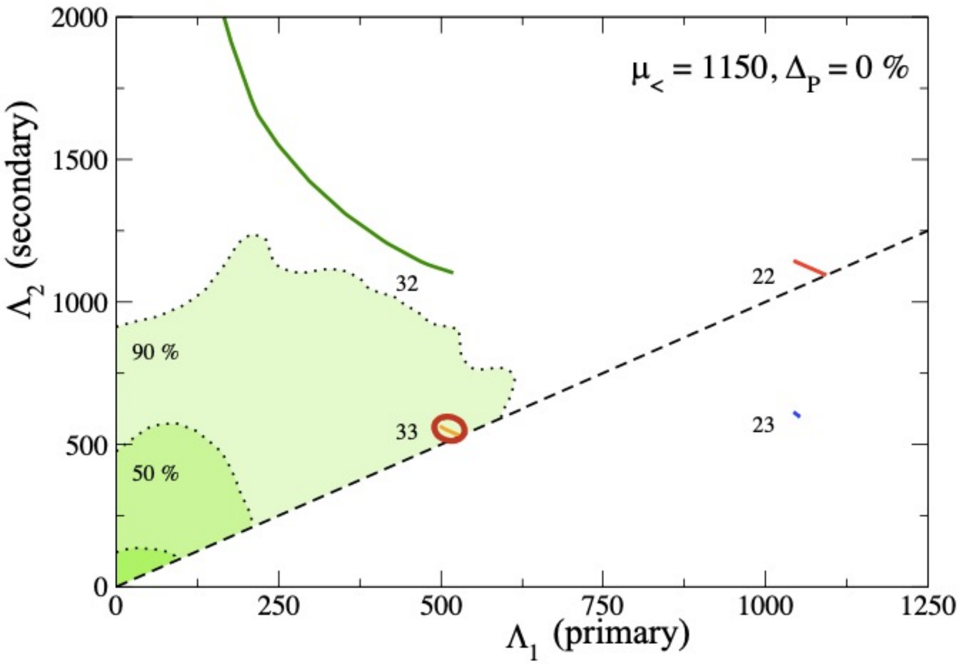

A. Ayriyan, D. Alvarez-Castillo, D. Blaschke and H. Grigorian, Universe 6, 81 (2020)

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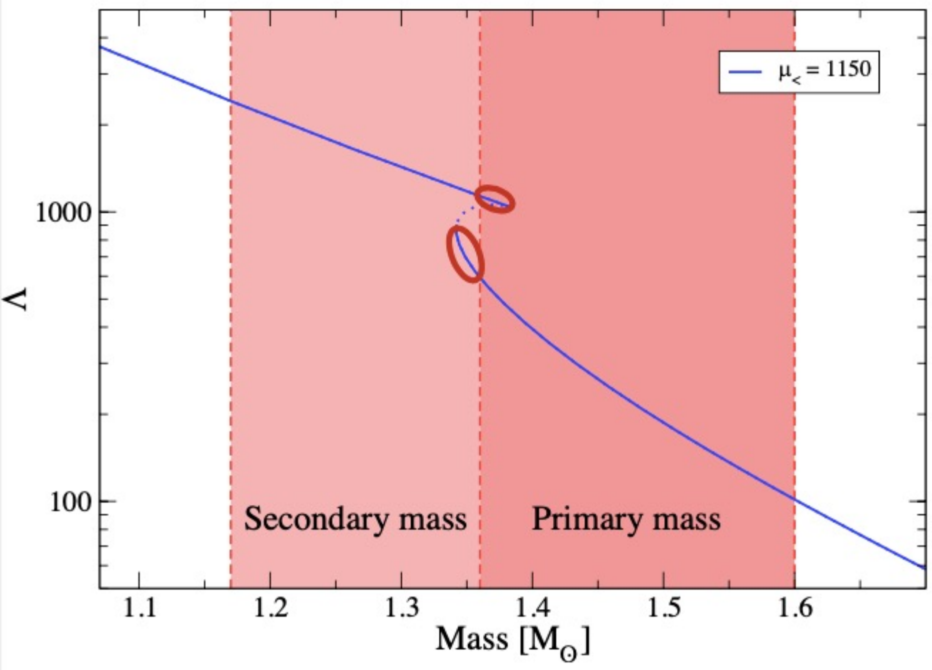
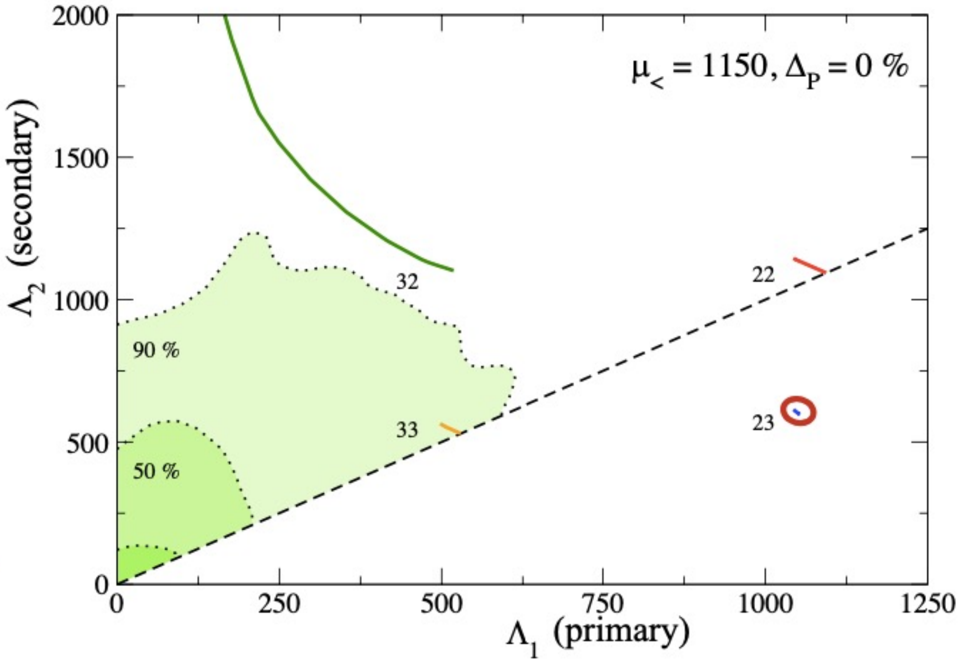




A. Ayriyan, D. Alvarez-Castillo, D. Blaschke and H. Grigorian, Universe 6, 81 (2020)

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

Bayesian Analysis

Bayesian inference

$$P\left(E|\boldsymbol{\pi}_q\right) = \prod_{\alpha} P\left(E_\alpha|\boldsymbol{\pi}_q\right),$$

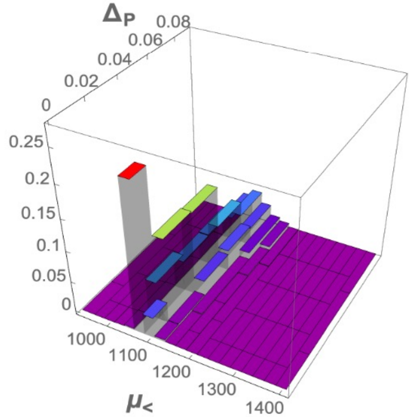
$$P\left(\pi_q|E\right) = \frac{P\left(E|\boldsymbol{\pi}_q\right) P\left(\boldsymbol{\pi}_q\right)}{\sum\limits_{p=0}^{N-1} P\left(E|\boldsymbol{\pi}_p\right) P\left(\boldsymbol{\pi}_p\right)},$$

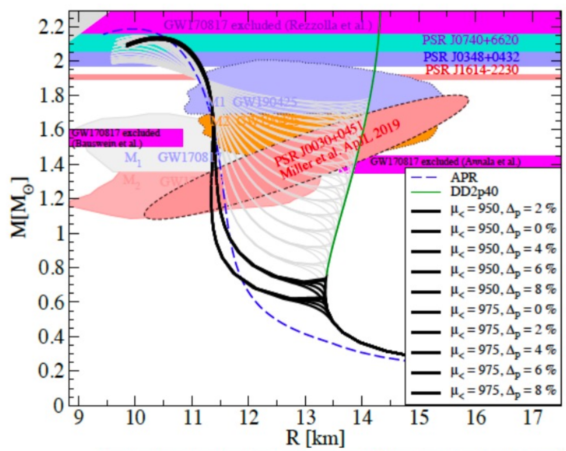
where the factor $P\left(\boldsymbol{\pi}_q\right)$ is the prior of a given model.

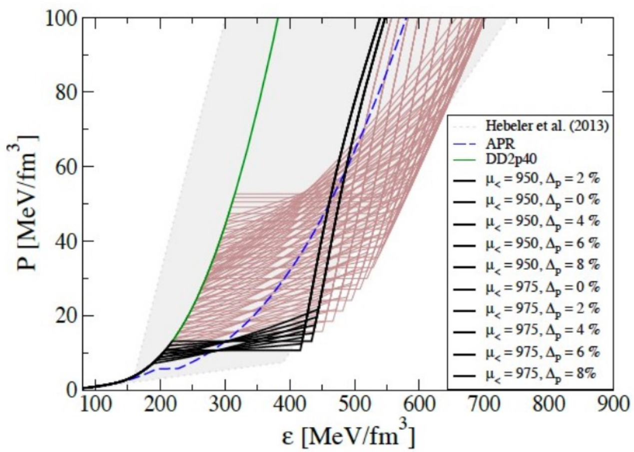
Bayesian inference for hybrid EoS Models

M-R from NICER:

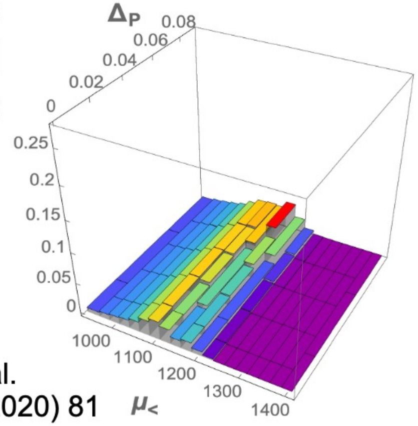
Present Variance →

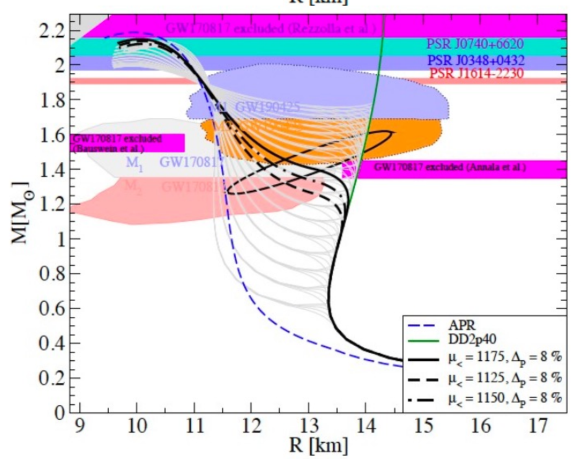


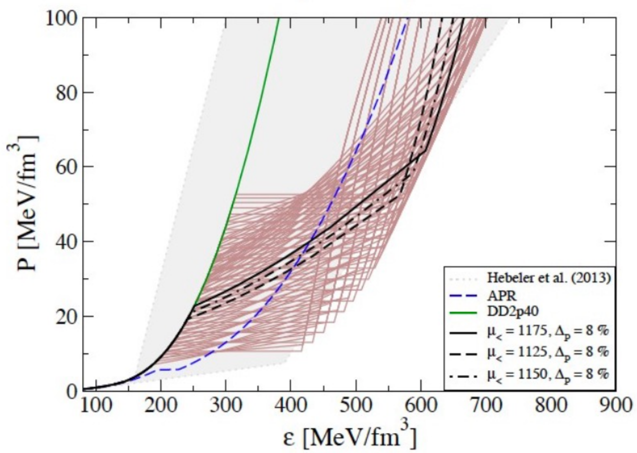




Fictitious Reduced Variance →

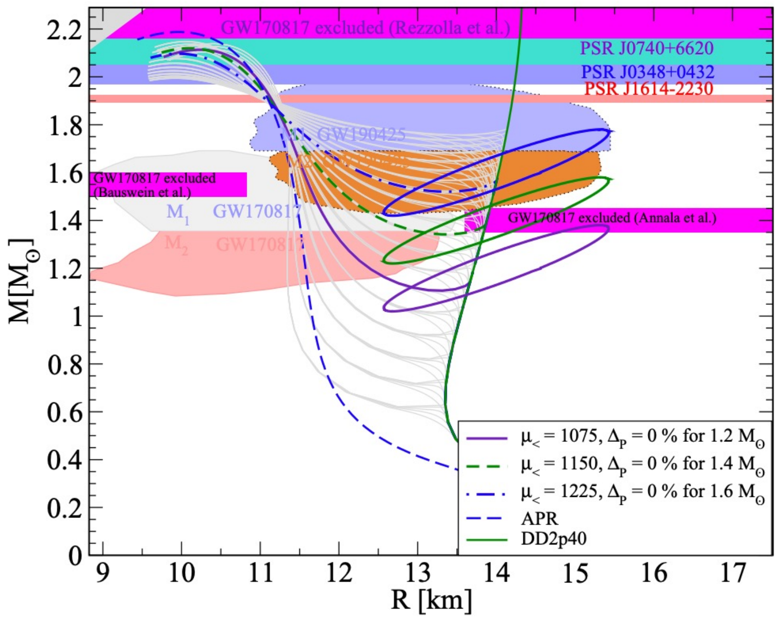


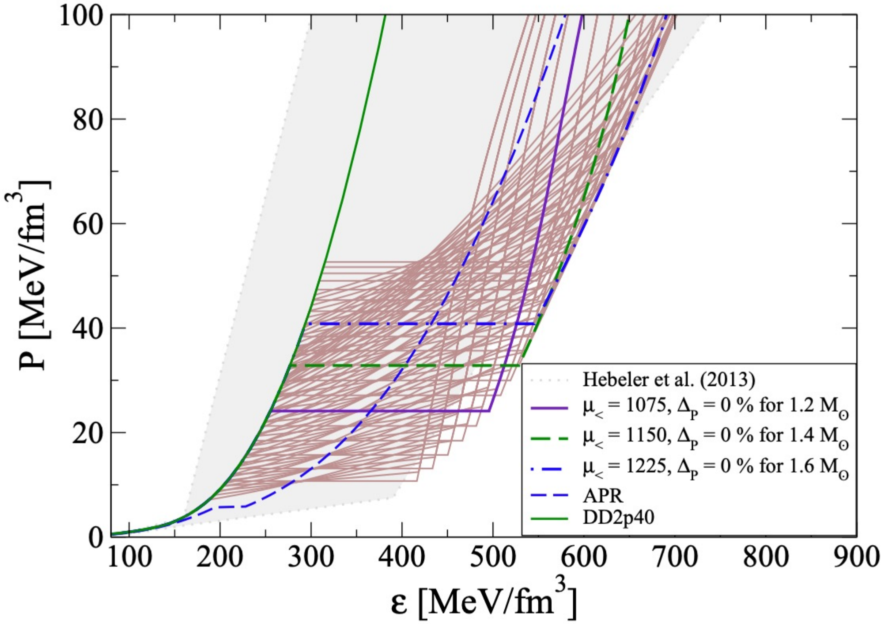


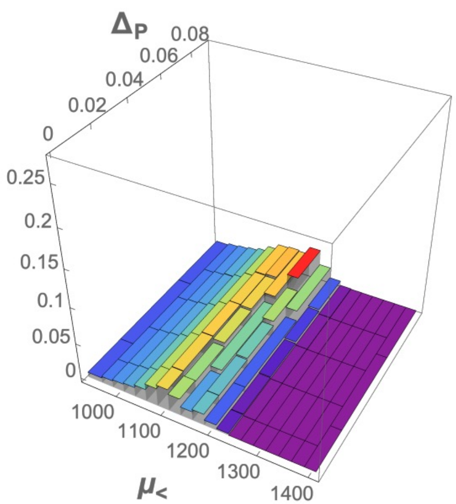


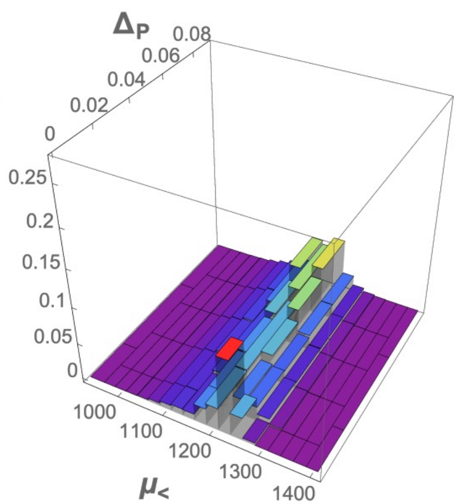
Blaschke et al.
Universe 6 (2020) 81

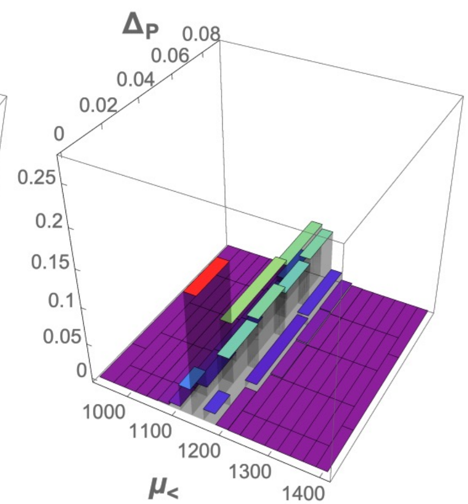
Fictitious Measurements

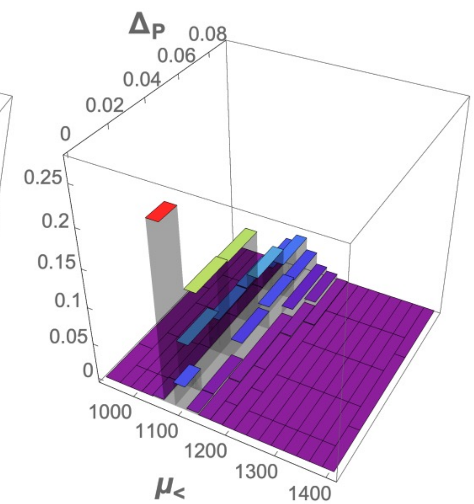




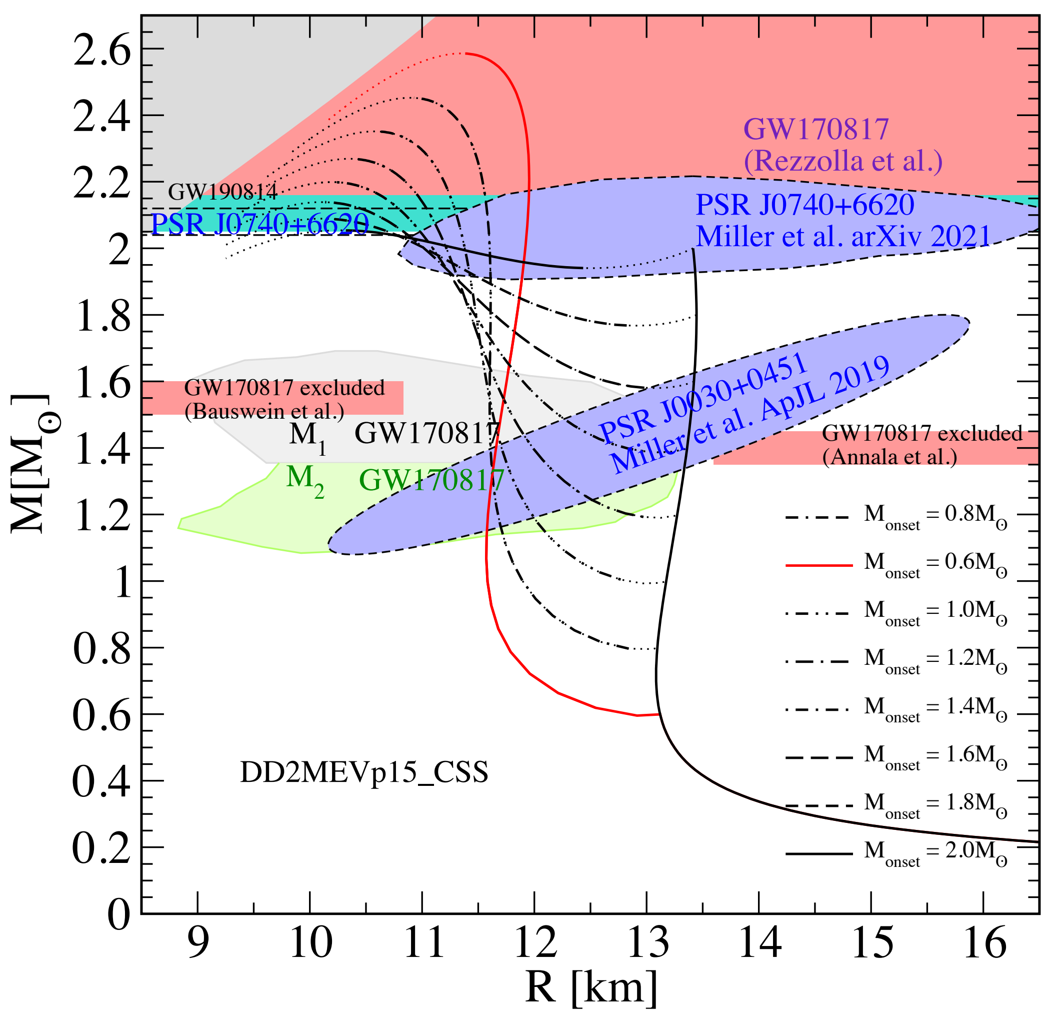








Mass - Radius Constraints



DD2MEV-CSS EoS

D. A-C, Astronomischen Nachrichten (2021) 1–6, arXiv: 2011.11145

Conclusions

- The recent NICER radius measurement of the massive PSR J0740+6620 of has set a constraint on twin compact stars that contain a deconfined quark matter core.
- Radius measurements of about 1.2 solar mass compact stars can probe the stiffness of nuclear matter around 1 to 2 times saturation density therefore probing the low mass twins hypothesis as well as constraining the symmetry energy.
- Bayesian analysis allows for constraining parameters of the high density EoS.
- Detection of signals from dynamical scenarios like star transitional evolution from second to third family may serve as a smoking gun for mass twins.
- The non-existence of compact star twins does not rule out the possibility of a CEP in the QCD phase diagram.

Gracias

Accretion-induced collapse to third family compact stars as trigger for eccentric orbits of millisecond pulsars in binaries

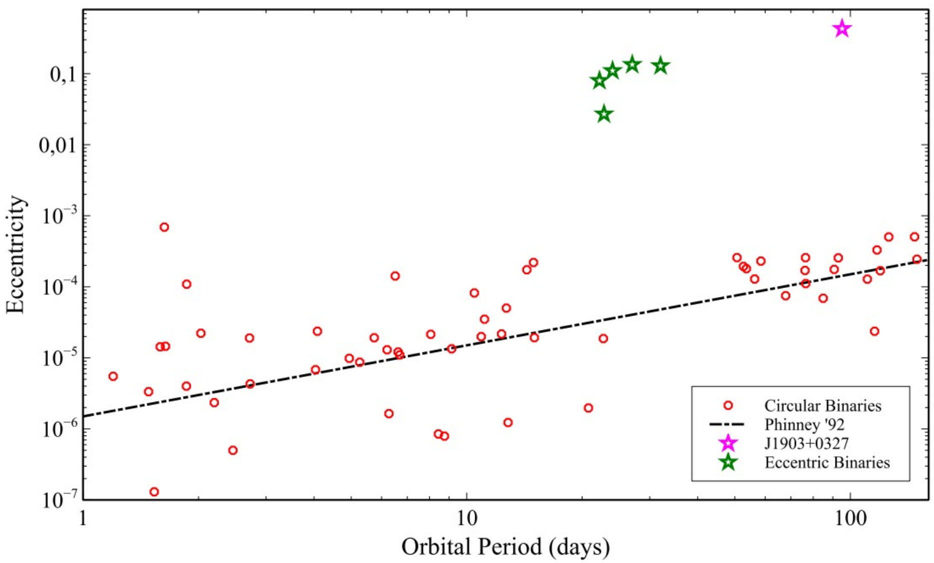
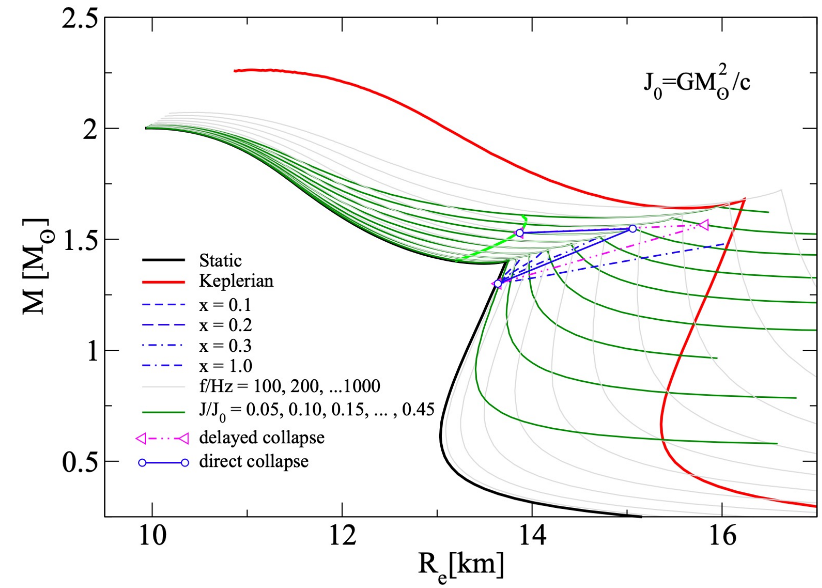


FIGURE 1 Eccentricity vs. orbital period for millisecond pulsars in binaries with white dwarf companions, see (J. Antoniadis, 2014; Stovall, 2019).



David Edwin Alvarez-Castillo, John Antoniadis, Alexander Ayriyan, David Blaschke, Victor Danchev, Hovik Grigorian, Noshad Khosravi Largani, Fridolin Weber.

Astron. Nachr. 2019;340:878-884. Eprint: arXiv: 1912.08782