

Particle Physics for Specialists

Electron–Proton Scattering

Rafał Staszewski

20 January 2022

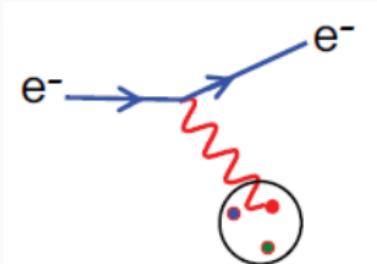
Introduction

Organizational matters

- 3x45 minutes for two topics
- split into two lectures
- today: ep scattering
- next week: forward physics

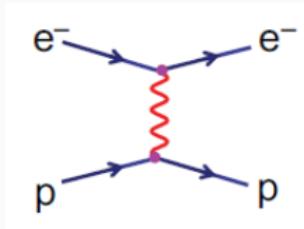
Motivation for interest in ep scattering

- proton is a composite particle
- probing proton structure with a small probe
- photon resolution $\sim 1 / \text{virtuality}$

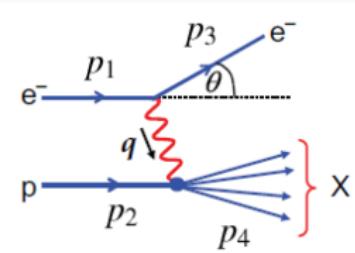


Types of ep scattering

Elastic ep scattering



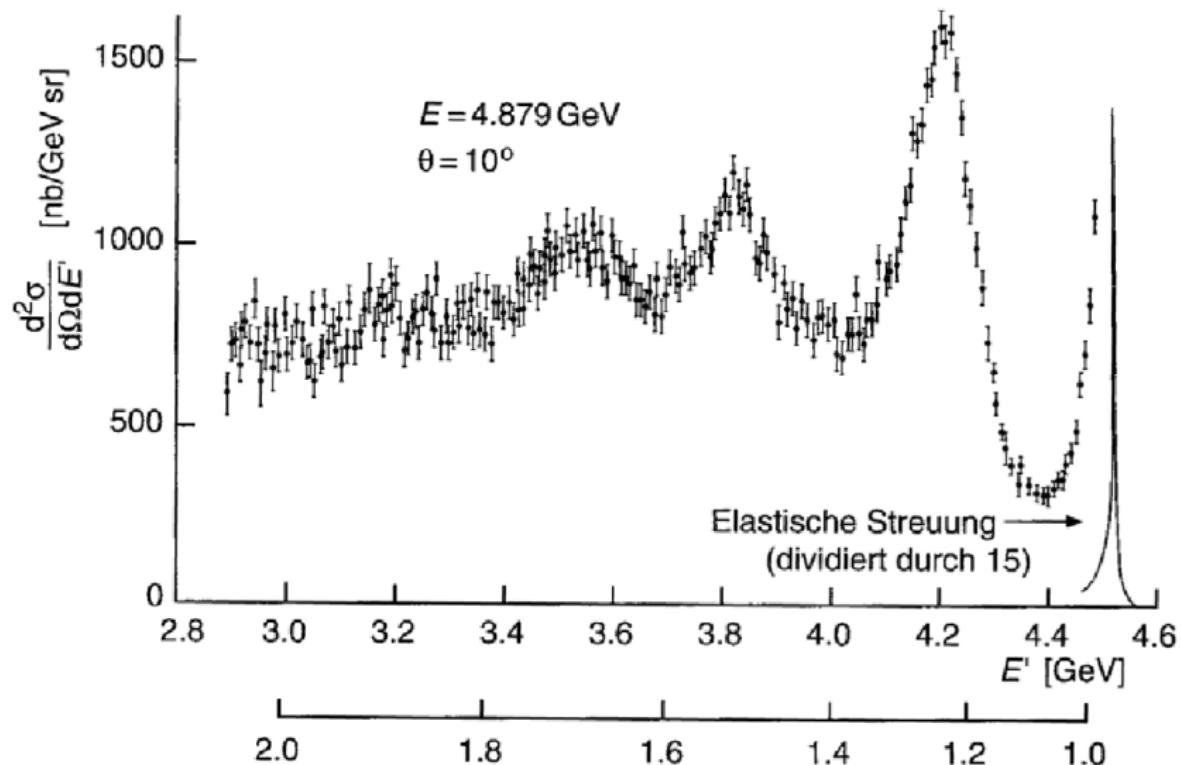
Inelastic ep scattering



- 4 particles in the process:
16 4-momentum components
- 3 DoF: boost of the reference frame
- 2 DoF: orientation of the ref. frame
- 4 constraints from fixed masses
- 4 constraints from (E, \vec{p}) conserv.
- 3 remaining DoF, for example:
 - centre-of-mass energy
 - azimuthal scattering angle – trivial distribution (when no polarization)
 - **polar scattering angle**

- Proton breaks up into state X
- $M_X > M_p$ (because of baryon number conservation)
- *Inclusive* analysis – state X only described by its invariant mass, the exact composition is not taken into account
- Then, one more DoF than in the elastic ep scattering
- Considering composition of the X state not covered in this lecture

Elastic and inelastic scattering

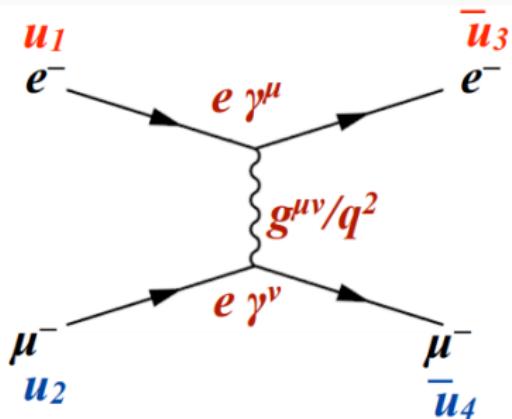


Contents

Elastic scattering

Inelastic scattering

Electron-muon scattering



$$\mathcal{M} = e^2 \frac{g^{\mu\nu}}{q^2} J_{13}^\mu J_{24}^\nu$$

Vertex Couplings

$$\mathcal{M} = e^2 \frac{g^{\mu\nu}}{q^2} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\nu u_2)$$

Photon propagator,

Electron current

Muon current

$$|\mathcal{M}|^2 = \frac{e^4}{q^4} \frac{1}{(2S_1 + 1)(2S_2 + 1)} \sum_{S_3, S_4} (\bar{u}_3 \gamma^\mu u_1) (\bar{u}_3 \gamma^\nu u_1)^* (\bar{u}_4 \gamma^\mu u_2) (\bar{u}_4 \gamma^\nu u_2)^*$$

Electron-muon scattering

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{e^4}{q^4} \frac{1}{(2S_1+1)(2S_2+1)} \sum_{S_3, S_4} (\bar{u}_3 \gamma^\mu u_1) (\bar{u}_3 \gamma^\nu u_1)^* (\bar{u}_4 \gamma^\mu u_2) (\bar{u}_4 \gamma^\nu u_2)^* \\ &= \frac{e^4}{q^4} \left(\frac{1}{(2S_1+1)} \sum_{S_3} (\bar{u}_3 \gamma^\mu u_1) (\bar{u}_3 \gamma^\nu u_1)^* \right) \left(\frac{1}{(2S_2+2)} \sum_{S_4} (\bar{u}_4 \gamma^\mu u_2) (\bar{u}_4 \gamma^\nu u_2)^* \right) \\ &= \frac{e^4}{q^4} L_e L_m \end{aligned}$$

$$\begin{aligned} L_e &= \frac{1}{(2S_1+1)} \sum_{S_3} (\bar{u}_3 \gamma^\mu u_1) (\bar{u}_3 \gamma^\nu u_1)^* \\ &= 2 [p_3^\mu p_1^\nu + p_3^\nu p_1^\mu - (p_3 \cdot p_1 - m_e^2) g^{\mu\nu}] \end{aligned}$$

$$L_\mu = 2 [p_4^\mu p_2^\nu + p_4^\nu p_2^\mu - (p_4 \cdot p_2 - m_\mu^2) g^{\mu\nu}]$$

Scattering cross sections

Rutherford scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_K \sin^4 \Theta/2}$$

- Low energy approximation ($E_K \ll m_e$)
- E_K – electron kinetic energy
- Θ – scattering angle in the lab frame

Mott scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E \sin^4 \Theta/2} \cos^2 \frac{\Theta}{2}$$

- High energy approximation ($m_e \approx 0$)
- E – electron energy (at high energy, equivalent to E_K)
- $\cos^2 \Theta/2$ factor is related to the electron helicity

In both formulae:

- proton is treated as a point-like spin-half particle
- proton recoil is neglected

Proton recoil

Mott formula

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E\sin^4\Theta/2} \cos^2 \frac{\Theta}{2}$$

Taking into account proton recoil

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{\alpha^2}{4E_1 \sin^4 \Theta/2}}_{\text{Mott}} \frac{E_3}{E_1} \left(\underbrace{\cos^2 \frac{\Theta}{2}}_{\text{Mott}} + \cancel{2\tau \sin^2 \frac{\Theta}{2}} \right), \quad \tau = -q^2/4M^2$$

- $\frac{E_3}{E_1}$ factor related to recoil
- $2\tau \sin^2 \frac{\Theta}{2}$ term corresponds to spin-spin interaction

Form factors

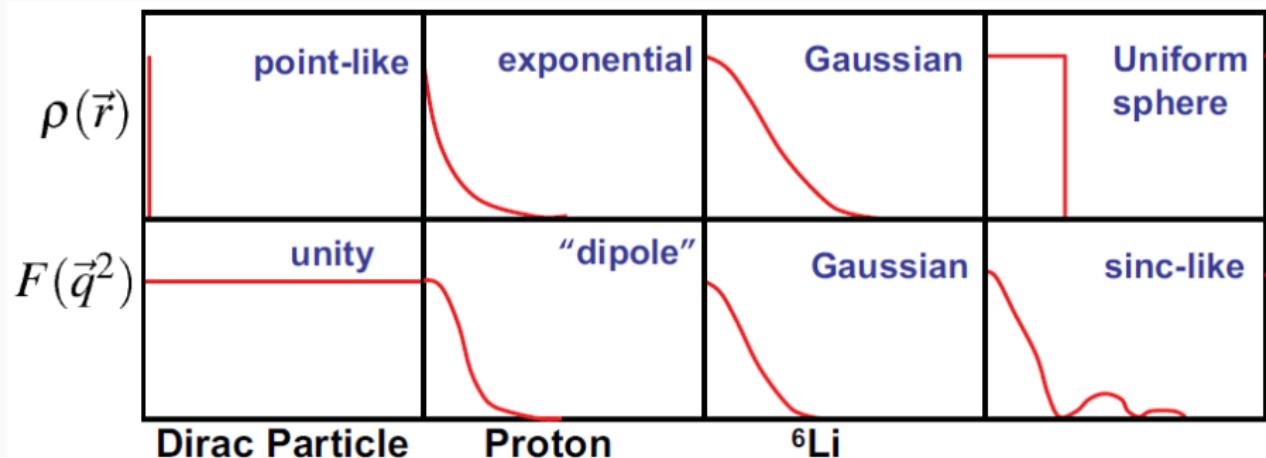
For an extended charge distribution:

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \rightarrow \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} |F(\vec{q})|^2$$

- \vec{q} – momentum of the virtual photon, related to electron: $q^2 = -4p^2 \sin^2 \Theta/2$
- $F(\vec{q})$ – form factor, Fourier transform of the charge distribution

$$F(\vec{q}) = \int \rho(\vec{r}) \exp(i\vec{q} \cdot \vec{r}) d^3\vec{r}$$

- $\rho(\vec{r})$ – charge distribution (normalised to unity: $\int \rho(\vec{r}) d^3\vec{r} = 1$)



Proton recoil and proton structure

Point-like proton

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \Theta/2} \frac{E_3}{E_1} \left(\cos^2 \frac{\Theta}{2} + 2\tau \sin^2 \frac{\Theta}{2} \right), \quad \tau = -q^2/4M^2$$

Finite-size proton

- In Feynman diagram calculation, proton vertex:

$$\gamma^\mu \rightarrow F_1(q^2)\gamma^\mu + \frac{\kappa}{2M}F_2(q^2)i\sigma^{\mu\nu}q_\nu, \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

- Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \Theta/2} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \frac{\Theta}{2} - 2\tau G_M^2 \sin^2 \frac{\Theta}{2} \right)$$

$$G_E = F_1 + \frac{\kappa q^2}{4M^2} F_2 \quad G_M = F_1 + \kappa F_2$$

- G_E and G_M can be interpreted as Fourier transform of charge and magnetic moment distributions *only* for very small scattering angles
- $G_E(0) = 1$, $G_M(0) = 2.79$
- Only one (relevant) DoF describing the final state: Θ , E_3 and q^2 are dependent

Experimental result

- Proton well described by a dipole form factor

$$G_E = \frac{1}{(1 + q^2/0.71 \text{ GeV}^2)^2}$$

$$G_M = 2.79 G_E$$

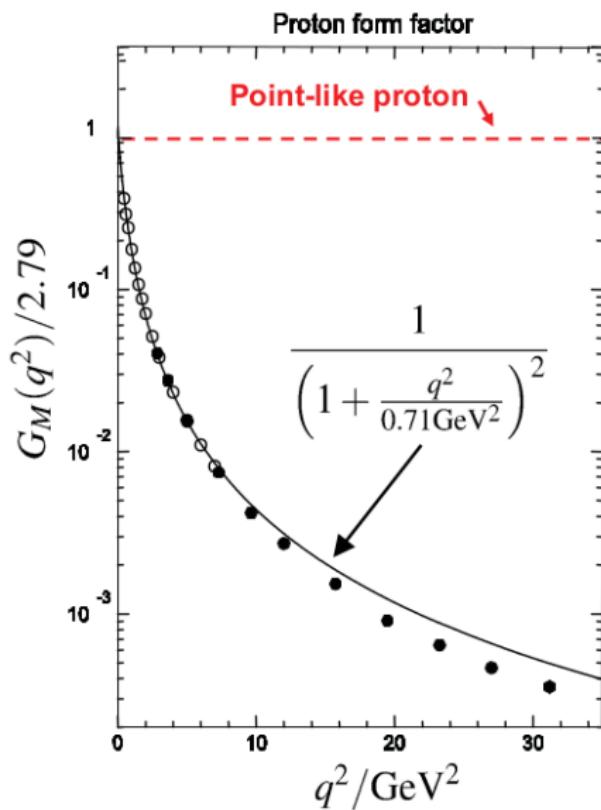
- Corresponds to exponential charge distribution

$$\rho(\vec{r}) = \rho_0 e^{-r/a}$$

with $a = 0.24 \text{ fm}$

- This corresponds to proton radius

$$\langle r \rangle = 0.8 \text{ fm}$$



Contents

Elastic scattering

Inelastic scattering

Kinematics

- Photon virtuality

$$Q^2 = -q^2$$

- Bjorken x

$$x = \frac{Q^2}{2p_2 \cdot q}$$

- Invariant mass of X :

$$W = M_X$$

- Electron energy loss

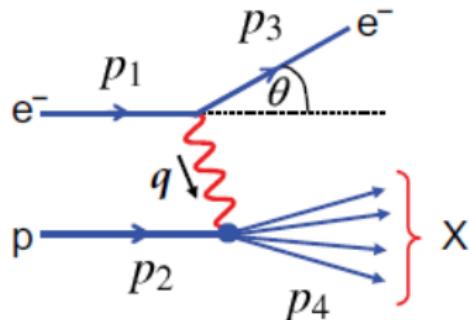
$$\nu = \frac{p_2 \cdot q}{m_p}$$

$$\nu = E_1 - E_3 \text{ (in the proton rest frame)}$$

- Inelasticity

$$y = \frac{p_2 \cdot q}{p_2 \cdot p_1}$$

$$y = \frac{\nu}{E_1} \text{ (in the proton rest frame)}$$



Structure functions

- Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \Theta/2} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \frac{\Theta}{2} - 2\tau G_M^2 \sin^2 \frac{\Theta}{2} \right)$$

G_E and G_M are functions of q^2

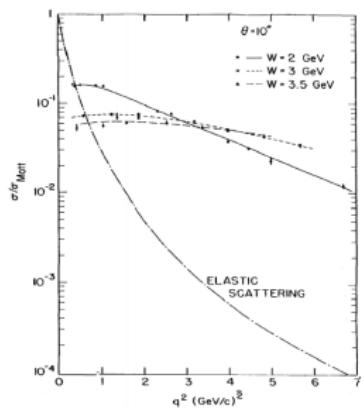
- Inelastic scattering

$$\frac{d\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \Theta/2} \frac{E_3}{E_1} \left(W_2(\nu, q^2) \cos^2 \frac{\Theta}{2} - 2W_1(\nu, q^2) \sin^2 \frac{\Theta}{2} \right)$$

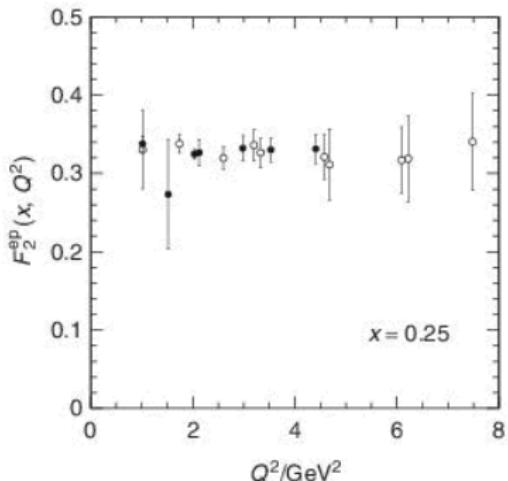
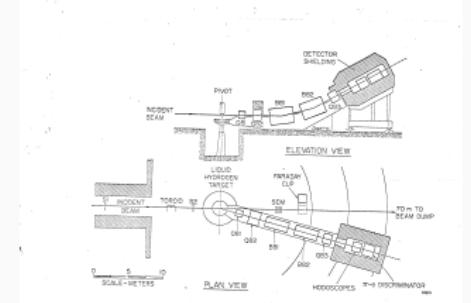
- Equivalent formulation in the limit of high Q^2 :

$$\frac{d\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{1-y}{x} F_2(x, Q^2) + y^2 F_1(x, Q^2) \right]$$

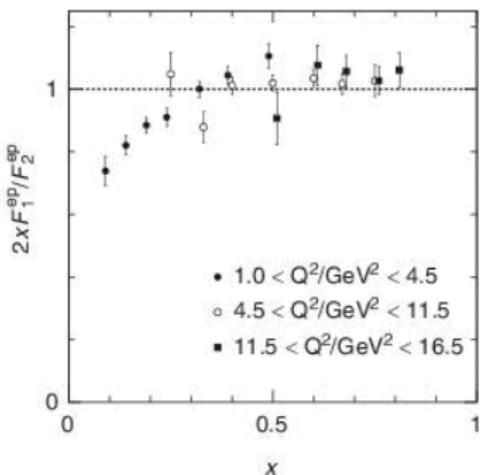
- At high Q^2 , inelastic scattering dominates



Experimental measurements at SLAC

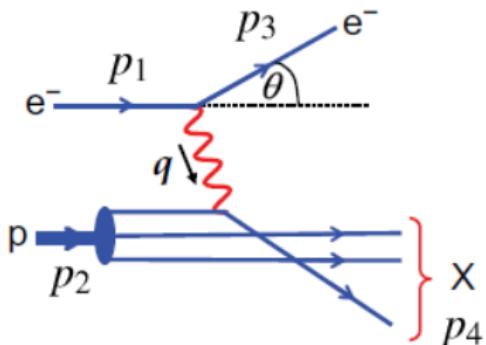
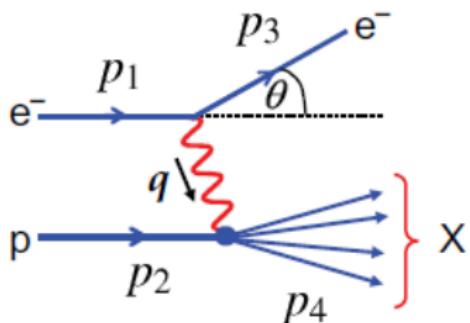


Bjorken scaling



Callan-Gross relation

Quark–Parton Model



Electron–quark scattering:

- quark momentum before scattering: p_{quark}
- quark momentum after scattering: $p'_{\text{quark}} = p_{\text{quark}} + q$
- negligible quark mass and no breaking up: $p_{\text{quark}}^2 = p'^2_{\text{quark}} = m_{\text{quark}}^2 \approx 0$

$$(p_{\text{quark}} + q)^2 = 0 \quad \rightarrow \quad Q^2 = 2 p_{\text{quark}} \cdot q$$

$$x = \frac{Q^2}{2 p_2 \cdot q} \quad \rightarrow \quad p_{\text{quark}} = x p_2$$

- x is the fraction of proton momentum carried by the quark
(in a frame where the proton is very fast)

Parton Distribution Functions

- electron–quark scattering (neglecting masses):

$$\frac{d\sigma_{eq}}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4} \left[1 + (1 - y)^2 \right]$$

- $q(x)$ – probability density of scattering off a given quark flavour, then

$$\frac{d\sigma_{ep}}{dQ^2 dx} = q(x) \frac{d\sigma_{eq}}{dQ^2}$$

- Comparing with

$$\frac{d\sigma_{eq}}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{1-y}{x} F_2(x, Q^2) + y^2 F_1(x, Q^2) \right]$$

- Structure function in terms of parton distributions

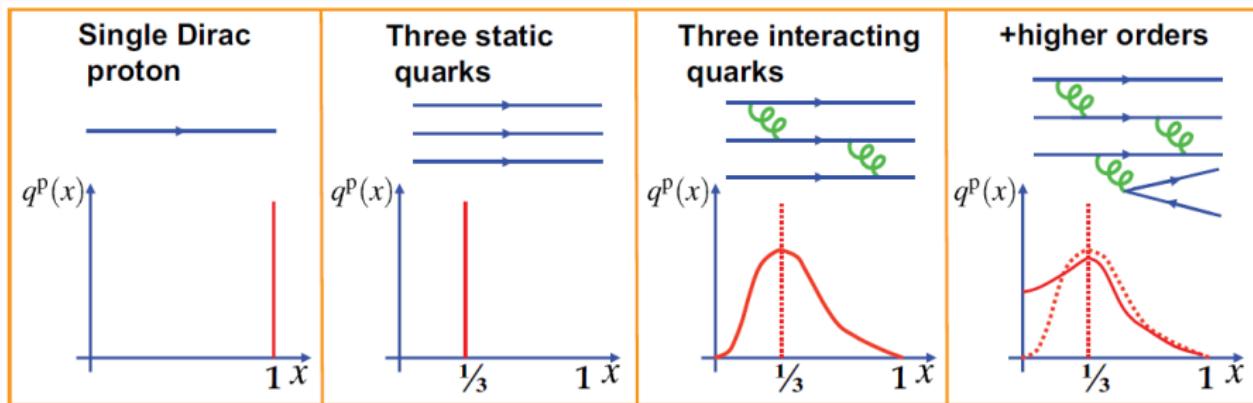
$$F_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 q(x)$$

$$F_2(x, Q^2) = 2x F_1(x, Q^2)$$

For example, including only u and d quarks:

$$\sum_q e_q^2 q(x) = \frac{4}{9} [u(x) + \bar{u}(x)] + \frac{1}{9} [d(x) + \bar{d}(x)]$$

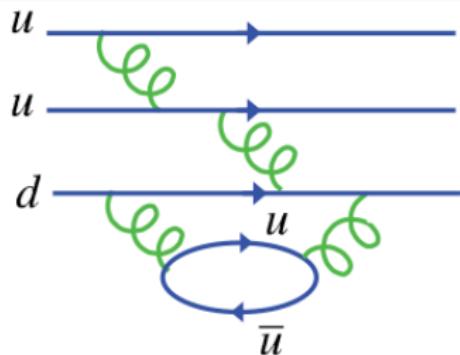
Parton Distribution Functions



Valence quarks, see quarks, gluons

Valence and sea quarks

- proton = $uud \leftarrow$ valence quarks
- sea quarks – quantum fluctuations
eg. $g \rightarrow q\bar{q}$
- $u(x) = u_v(x) + u_s(x)$, $\bar{u}(x) = u_s(x)$
- $d(x) = d_v(x) + d_s(x)$, $\bar{d}(x) = d_s(x)$
- similar masses of u and d : $u_s(x) = d_s(x)$



Momentum fraction

- $q(x)$ – quark density
- $xq(x)$ – momentum density
- $\int xq(x)dx = f_q$ – proton momentum fraction carried by q
- $F_2 = \sum_q xe_q^2 q(x) \rightarrow \int F_2(Q^2, x)dx = \frac{4}{9}f_u + \frac{1}{9}f_d$
- neutron: $u \leftrightarrow d \rightarrow \int F_2^{\text{neutron}}(Q^2, x)dx = \frac{4}{9}f_d + \frac{1}{9}f_u$
- $f_u \approx 0.36$, $f_d \approx 0.018$
- ~50% of proton momentum carried by neutral partons (gluons)

DGLAP – evolution of PDFs

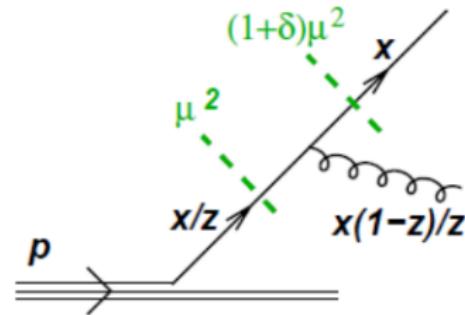
- DGLAP = Dokshitzer, Gribov, Lipatov, Altarelli, Parisi
- Ambiguity in splitting between proton structure and quark-electron process
- But F_2 is well defined and measurable
- Renormalization group equations
- Simplified case ($P_{qq}(z)$ – splitting function):

$$\frac{dq(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 q(y, Q^2) P_{qq} \left(\frac{x}{y} \right) \frac{1}{y} dy$$

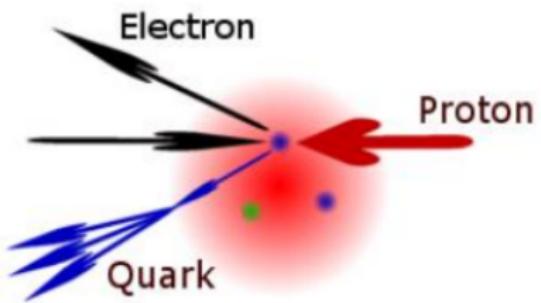
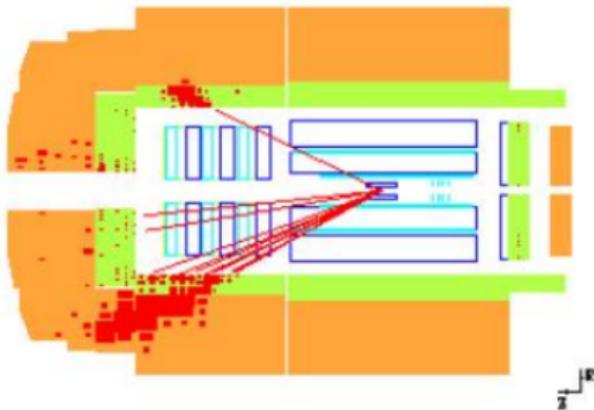
- In reality, also gluons are involved:

$$\frac{d}{d \log Q^2} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

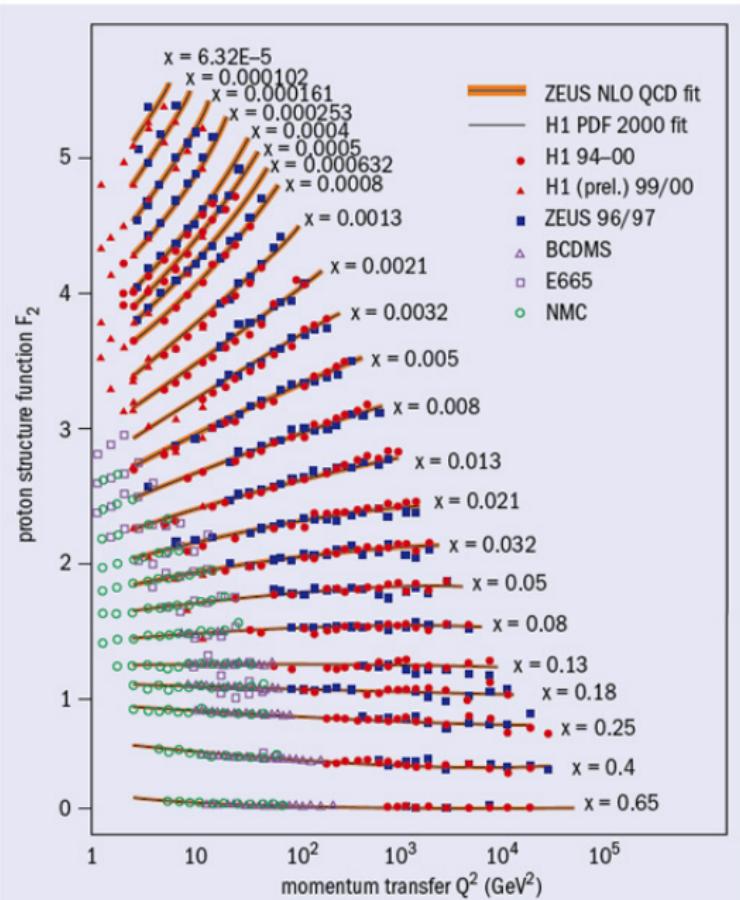
- DGLAP → evolution of PDFs in Q^2 predicted by QCD



HERA Accelerator at DESY

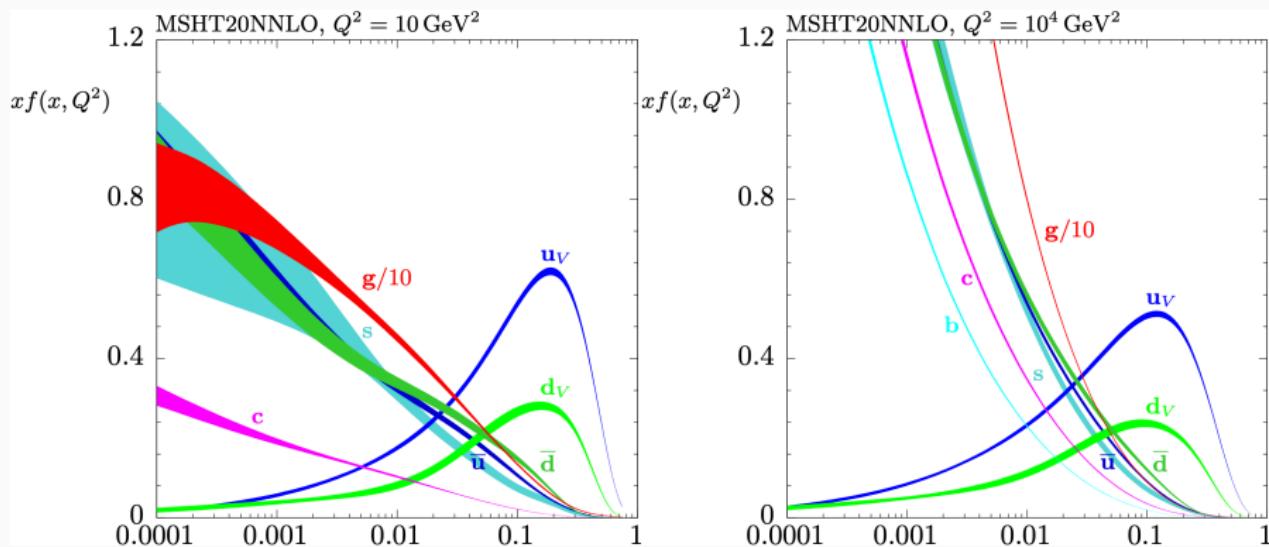


F_2 measurement at HERA

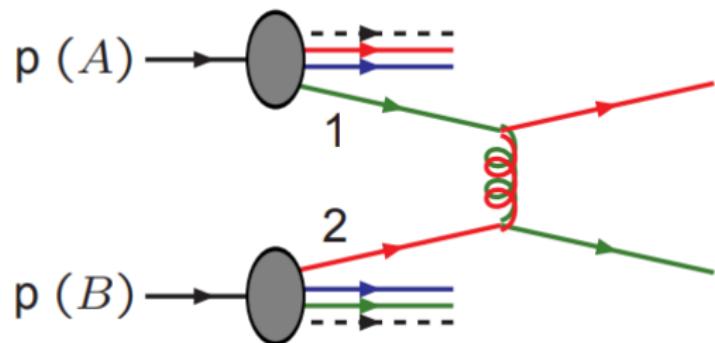


Global PDF fits

- Assume functional form of the x dependence of PDFs at a fixed Q_0
- A simplistic example: $u(x, Q_0^2 = 1 \text{ GeV}^2) = A \cdot x^B \cdot (1 - x)^C$
- Evolve PDFs to the needed Q^2 using DGLAP
- Compute the observable and compare with the experimental measurement
- Build a global χ^2 function, find parameters (A, B, C, \dots) minimising the χ^2



Proton–proton collisions



$$s = (p_A + p_B)^2$$

$$x_1 \approx E_1/E_A$$

$$x_2 \approx E_2/E_B$$

$$\hat{s} = x_1 x_2 s$$

$$\sigma = \sum_{i,j} \iiint dx_1 dx_2 d\hat{t} f_i^{(A)}(x_1, Q^2) f_j^{(B)}(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

Outlook

Related topics, not covered in this lecture

- NLO, NNLO
- higher twist effects
- BFKL
- saturation
- Other QCD factorisations
- More general parton distributions: GPD, TMD, ...
- Spin-dependent proton structure
- Nuclear PDFs
- Multi-parton structure
- PDFs from lattice

Acknowledgements

This lecture was inspired by:

- Halzen and Martin *Quarks and leptons*
- Prof. Mark Thomson's lectures (available online)