Gamow Shell Model description of stretched resonant states

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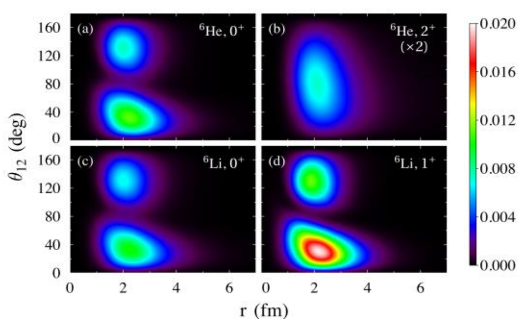
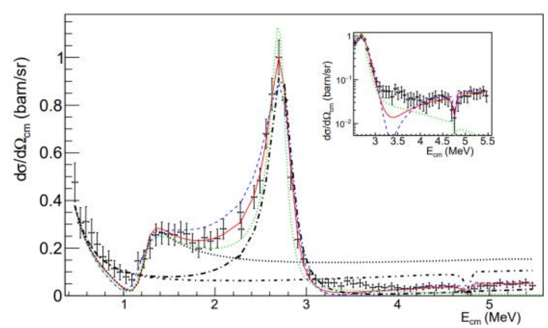
Gamow Shell Model description of stretched resonant states

The Gamow Shell Model (GSM) is an **open quantum system** extension of the traditional Shell Model.
It is designed to describe a variety of **structure (bound + unbound)** and **reaction** observables.

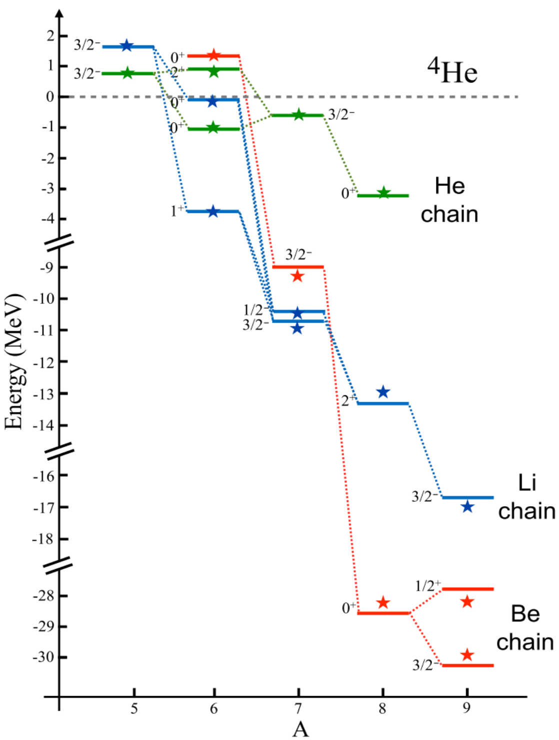
Collaborations with experimentalists are just starting. The interplay between theory and experiments can help improve the interaction and make better predictions.



Correlation densities in ^{6}He, ^{6}Li



GSM-CC ^{14}O(p,p') excitation function



A quantified GSM interaction for *p*-nuclei

The Framework: The Gamow Shell Model

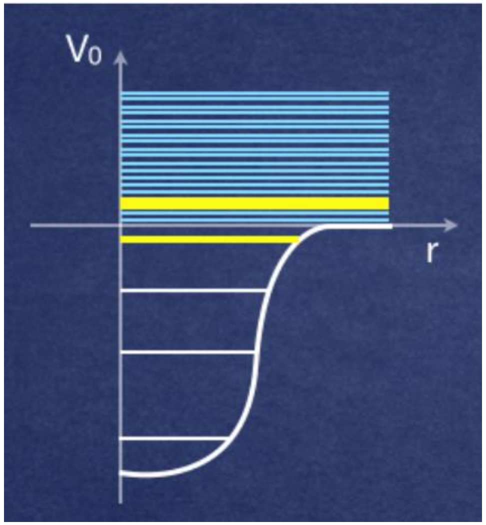
- **Open-quantum system** extension of the traditional Shell Model

- HO is replaced by a **finite-depth** potential:
  - Woods-Saxon
  - Gamow Hartree-Fock

- S.p. states are the numerical solutions of the one-body radial Schrödinger equation:

$$u''_{\alpha\ell j}(r) = \left[\frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}U(r) - k^2\right] u_{\alpha\ell j}(r)$$

$$U(r) = V_{WS}(r) + V_{so}(r)\vec{\ell}\cdot\vec{s} + U_C(r)$$



N. Michel, W. Nazarewicz, M. Płoszajczak, T. Vertse, J. Phys. G: Nucl. Part. Phys. 36, 013101 (2009)

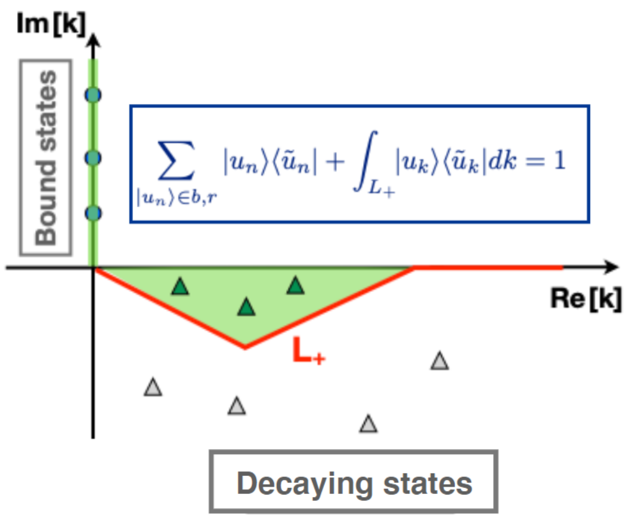
- Specific **boundary conditions**:
  - **Bound states, resonances**: $u_{n\ell j}(r) \sim C_+ H^+_\ell(\eta, kr), \quad r \to +\infty$
  - **Scattering states** (continuum): $u_{k\ell j}(r) \sim C_+ H^+_\ell(\eta, kr) + C_- H^-_\ell(\eta, kr), \quad r \to +\infty$

The Framework: The Gamow Shell Model

- Both **correlations** and **continuum effects** are treated on the same footing (**Berggren ensemble**)

- The discretization of the contours gives rise to a large basis, in which the Hamiltonian *H* is diagonalized

- GSM Cluster Orbital Shell Model (COSM):



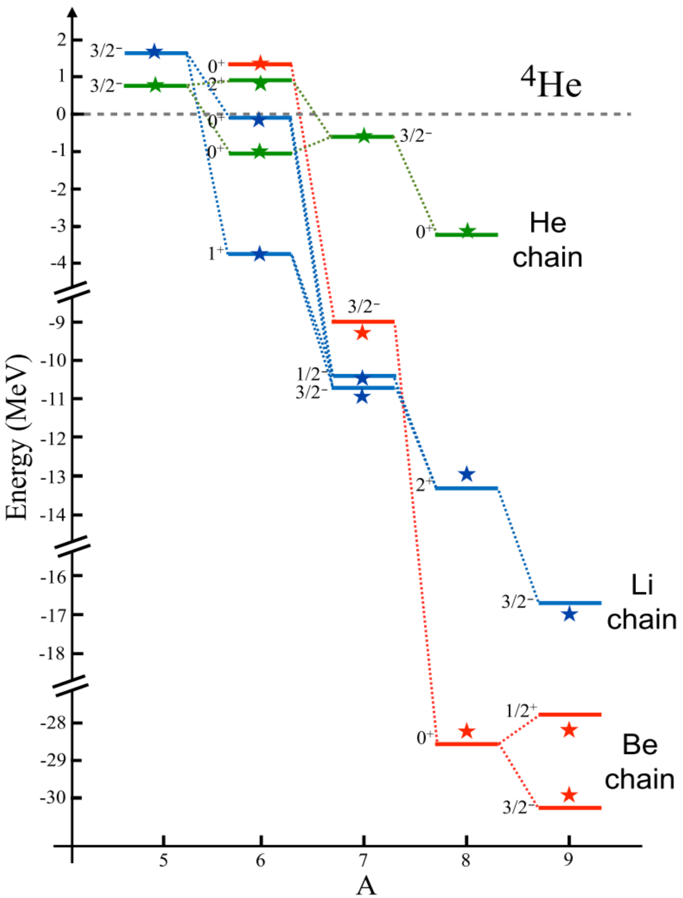
T. Berggren, Nucl. Phys. A 109, 265 (1968)

$$H = \sum_{i=1}^{N_v}\left[\frac{\vec{p}_i^{\,2}}{2\mu_i} + U(i)\right] + \sum_{i<j=1}^{N_v}\left[V_{res}(i,j) + \frac{\vec{p}_i\cdot\vec{p}_j}{M_c}\right]$$

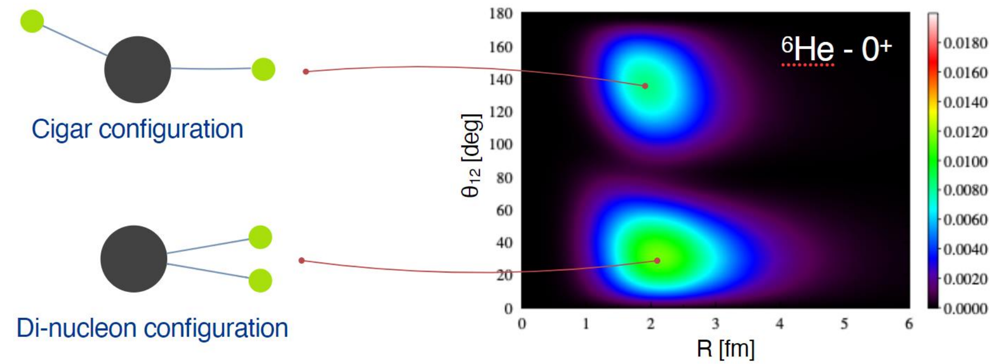
- Translational invariance but approximate antisymmetry in the laboratory frame
- Exact treatment of the Coulomb interaction

- Diagonalization of *H* gives the ***A*-body bound states** and **resonances.**

The Background: A Quantified Gamow Shell Model Interaction for the *p*-shell Nuclei



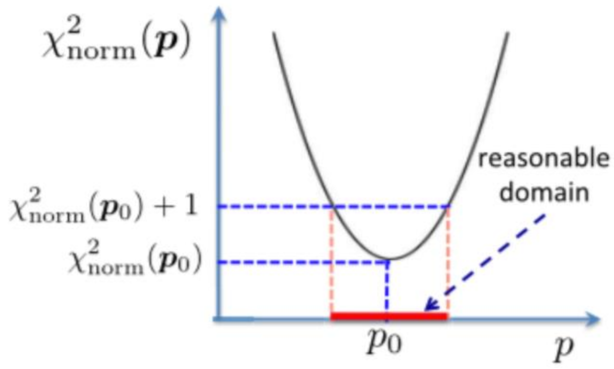
Y. Jaganathen, R. Id Betan, N. Michel, W. Nazarewicz, M. Płoszajczak, Phys. Rev. C 96, 054316 (2017)



- An effective N+NN interaction was developed, as a good starting point to describe *p*-nuclei within the *psd*-model space.
- A first application?

The Background: Uncertainty Quantification

- ”Remember that all models are wrong. The practical question is how wrong can they be to still be useful”. (G.E. Box)
- In Nuclear Theory, **all models are phenomenological**.
- **Systematic model errors** (imperfect modeling/missing physics) are often difficult to assess, but the **statistical model uncertainties** (reasonable domain) are usually quantifiable.



The Background: Uncertainty Quantification

- Within the **linear regression** approximation, UQ is based on the covariance matrix:

$$\mathcal{C} = (J^T J)^{-1}\ , \quad J_{i\alpha} = \frac{1}{\delta\mathcal{O}_i}\left.\frac{\partial \mathcal{O}_i}{\partial p_\alpha}\right|_{\mathbf{p}_0}$$

Uncertainties on parameters

$$\Delta p_\alpha = \sqrt{\mathcal{C}_{\alpha\alpha}}$$

Uncertainties on observables

$$\overline{\Delta A}^2 = \sum_{\alpha,\beta=1}^{N_p} \left.\frac{\partial A}{\partial p_\alpha}\right|_{\mathbf{p}_0} \mathcal{C}_{\alpha\beta} \left.\frac{\partial A}{\partial p_\beta}\right|_{\mathbf{p}_0}$$

Correlation coefficients between physical quantities

$$\overline{\Delta A\, \Delta B} = \sum_{\alpha,\beta=1}^{N_p} \left.\frac{\partial A}{\partial p_\alpha}\right|_{\mathbf{p}_0} \mathcal{C}_{\alpha\beta} \left.\frac{\partial B}{\partial p_\beta}\right|_{\mathbf{p}_0}$$

Uncertainties as in ‘reasonable domain’

The Background: Uncertainty Quantification

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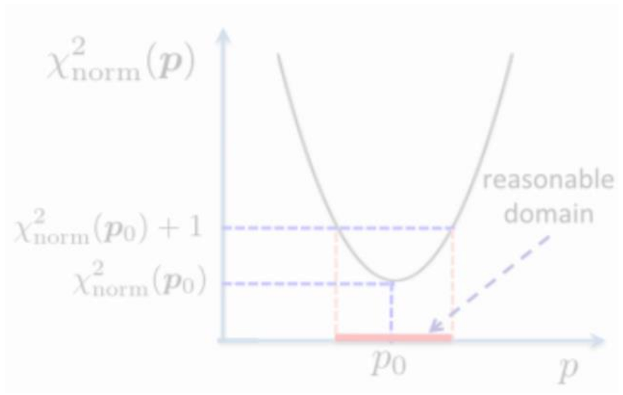
- In Nuclear Theory, **all models are phenomenological**.

- **Systematic model errors** (imperfect modeling/missing physics) are often difficult to assess, but the **statistical model uncertainties** (reasonable domain) are usually quantifiable.

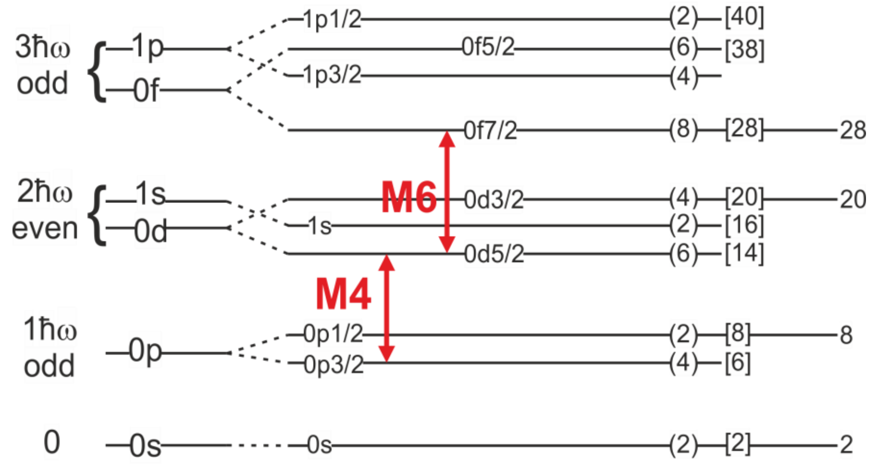
- Linear regression model applied to the interaction showed that **many parameters are sloppy**. Can different experimental data constrain these parameters?



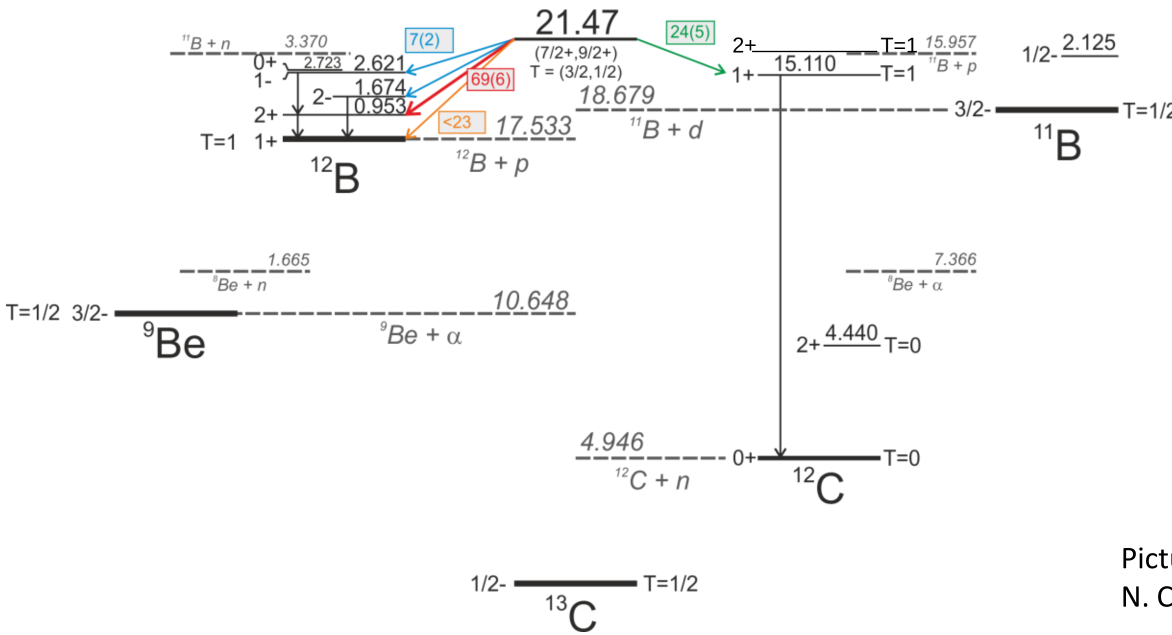
Parameter		Value
central	S=1 , T=1	-3.2 ± 22.0
	S=1 , T=0	**-5.1 ± 1.0**
	S=0 , T=0	**-21.3 ± 6.6**
	S=0 , T=1	**-5.6 ± 0.5**
spin-orbit	S=1 , T=1	-540 ± 1240
tensor	S=1 , T=1	-12.1 ± 79.5
	S=1 , T=0	**-14.2 ± 7.1**

”Stretched” states

- High-energy states dominated by a **single particle-hole component** between 2 distant shells
- The excited particle and the hole are coupled to the maximal possible spin value $J_{max} = J_p + J_h$
- A certain purity of the state is expected as other configurations require at least 2ħω excitations.
- Good states to test the interaction?



The 21.466 MeV M4-resonant state in ^{13}C

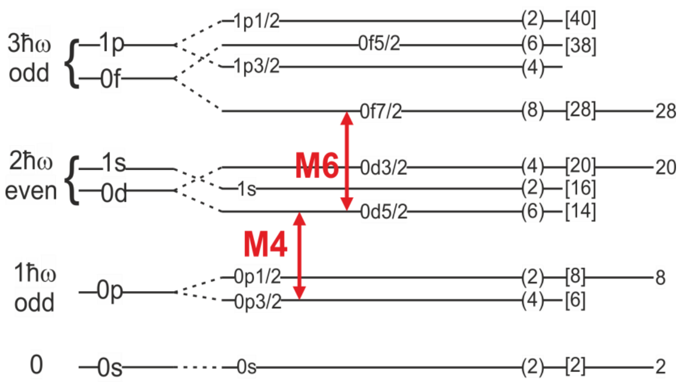


Picture courtesy of
N. Cieplicka-Oryńczak

- J = (7/2$^+$, 9/2$^+$), T = (3/2, 1/2)
- Decays by emission of protons and neutrons, Γ = 270 ± 20 keV.
- **Recently been measured at the Cyclotron Centre Bronowice** (CCB IFJ PAN), and is being analyzed by N. Cieplicka-Oryńczak (IFJ)
- Challenging for the Gamow Shell Model.

The Model Space

- Model space **specifically adapted** to describe the M4 state:
  - An **effective** ^{4}He core modeled by a Woods-Saxon + spin-orbit + Coulomb
  - 3 effective holes max. in the ^{12}C core
  - *psd* model space, as the *f* shells seem to be negligeable to describe the M4 state
  - An effective finite-range NN potential with **central, spin-orbit and tensor terms** similar to the one used within the p-shell.



- The depths of the one-body potential and the 8 parameters of the NN interaction were adjusted to the low-lying spectra of ^{12}B, ^{12}C, ^{12}N, ^{13}C, ^{13}N, as well as ^{14}C, ^{14}N, ^{14}O
- R.m.s deviation of ~ 450 keV.

Results – Parameters and Uncertainties

Parameter		Value
Central	S=1, T=1	**-6.5 ± 10.0**
	S=1, T=0	**-8.9 ± 0.8**
	S=0, T=0	**-8.7 ± 5.0**
	S=0, T=1	**-2.4 ± 0.5**
Spin-orbit	S=1, T=1	**3400 ± 800**
	S=1, T=0	1900 ± 3000
Tensor	S=1, T=1	44 ± 50
	S=1, T=0	**-47.4 ± 8.2**

- Like in the interaction in the *p*-shell, some parameters remain sloppy.
- They can probably be constrained by **experimental data of different kinds** (charge/matter radii, EM moments).

Results – Spectra and Configuration Probabilities

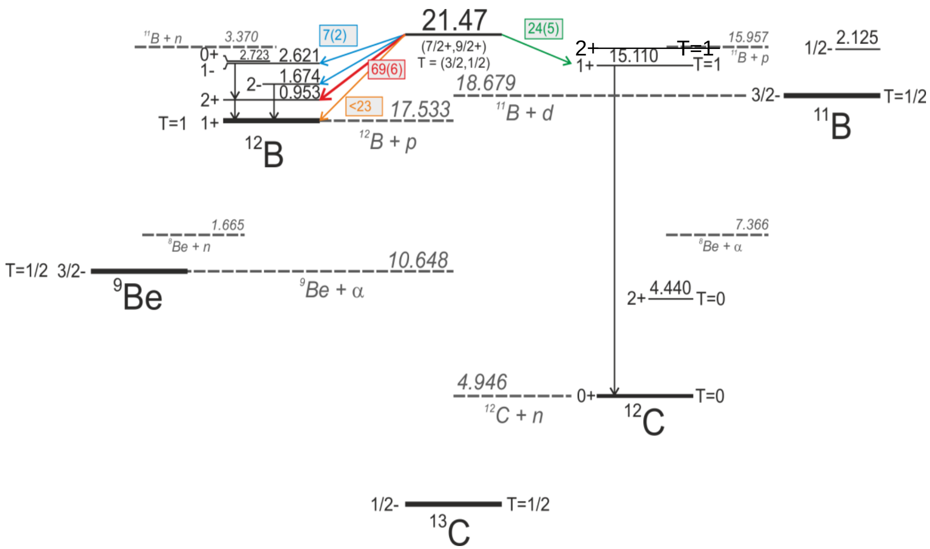
State	E_{calc} (MeV)	E_{exp} (MeV)	Main configurations
^{12}B, 1-, T=1	-48.832*	-48.659	67%: P: $(0p_{3/2})^3$ - N : $(0p_{3/2})^4$ $(1s_{1/2})^1$
^{12}B, 2-, T=1	-50.014*	-49.607	70%: P: $(0p_{3/2})^3$ - N: $(0p_{3/2})^4$ $(1s_{1/2})^1$
^{12}B, 2+, T=1	-50.546*	-50.327	67%: P: $(0p_{3/2})^3$ - N: $(0p_{3/2})^4$ $(0p_{1/2})^1$
^{12}B, 1+, T=1	-51.420*	-51.288	63%: P: $(0p_{3/2})^3$ - N: $(0p_{3/2})^4$ $(0p_{1/2})^1$
^{12}C, 2+, T=1	-47.1(4)	-47.761	34%: P: $(0p_{3/2})^3(0p_{1/2})^1$ - N: $(0p_{3/2})^4$ 34%: P: $(0p_{3/2})^4$ - N: $(0p_{3/2})^3(0p_{1/2})^1$
^{12}C, 1+, T=1	-48.2(4)	-48.757	34%: P: $(0p_{3/2})^3(0p_{1/2})^1$ - N: $(0p_{3/2})^4$ 32%: P: $(0p_{3/2})^4$ - N: $(0p_{3/2})^3(0p_{1/2})^1$
^{12}C, 2+, T=0	-59.057*	-59.427	33%: P: $(0p_{3/2})^3(0p_{1/2})^1$ - N: $(0p_{3/2})^4$ 33%: P: $(0p_{3/2})^4$ - N: $(0p_{3/2})^3(0p_{1/2})^1$

Potential M4 states	E_{calc}, Γ_{calc}	Main configurations
^{13}C, 7/2+, T=1.48	**E = -46.9(5) MeV, Γ = 400(500) keV**	**38% M4 proton, 27% M4 neutron**
^{13}C, 9/2+, T=1.4	**E = -46.2(7) MeV, Γ = 150(300) keV**	**24% M4 proton, 41% M4 neutron**

(*) fitted

- Exp: (7/2+, 9/2+), T = (3/2, 1/2), **E = -47.347 MeV, Γ = 270 ± 20 keV**
- Another T = 3/2 state, **E = -47.585 MeV, Γ = 18 keV**

Results – Spectroscopic Factors



7/2+ E = 21.832 MeV, Γ = 450 keV, T = 1.48			
12B, 1-, T=1	X		
12B, 2-, T=1	$S_{p3/2}$ = 0.005		
12B, 2+, T=1	$S_{d5/2}$ = 0.08 $S_{d3/2}$ = 0.02	12C, 2+, T=1	$S_{d5/2}$ = 0.10 $S_{d3/2}$ = 0.02
12B, 1+, T=1	$S_{d5/2}$ = 0.14	12C, 1+, T=1	$S_{d5/2}$ = 0.25

9/2+ E = 22.594 MeV, Γ = 150 keV, T = 1.40			
12B, 1-, T=1	X		
12B, 2-, T=1	X		
12B, 2+, T=1	$S_{d5/2}$ = 0.11	12C, 2+, T=1	$S_{d5/2}$ = 0.41
12B, 1+, T=1	X	12C, 1+, T=1	X

- **Spectroscopic Factors** are a decent indicator of the global decay scheme
- According to the GSM calculation, **the M4 state is the 7/2+.**
- Our calculation show that the decay to the 2+ in ^{12}C is possible.
- Radial overlap integrals are currently being calculated to assess the full theoretical decay scheme

Conclusions and Outlook

- The study of the 21.47 M4-resonant state in ^{13}C is a **current collaboration between theory and experiment within the IFJ PAN**.
- While its decay has recently been measured recently at the Cyclotron Centre Bronowice (CCB IFJ PAN), the Gamow Shell Model offers its first theoretical insight to a so-called stretched state.

- GSM predicts the M4 to be a 7/2+, T=3/2 state.
- It also predicts the possibility of a decay to the 2+ in ^{12}C, which demands further experimental investigation.
- Radial overlap integrals are currently being calculated to assess the full theoretical decay scheme.

- This study opens the path for future projects on stretched states in N and/or O, which could be the subject of future experiments.