Theoretical Frameworks for Non-Fermi Liquids

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Condensed Matter Physics

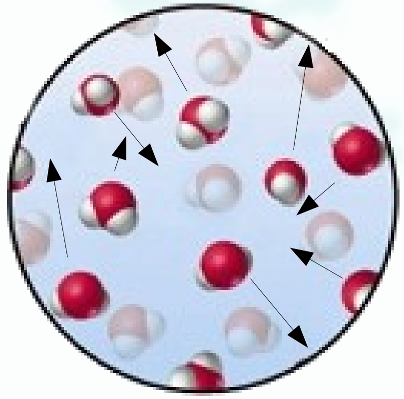
Study of complex behaviour of a large number of interacting particles

Collective behaviour gives rise to

Emergent Properties

Strongly Correlated Systems

Gases
☛ Weakly interacting

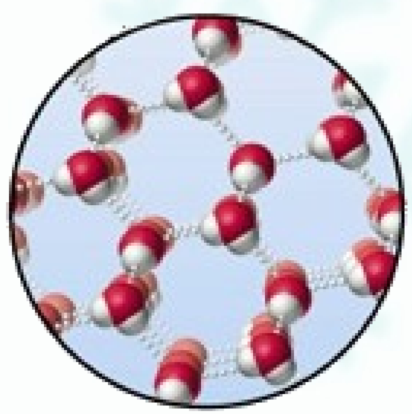


Motion hardly depends on position / motion of others

Increase in Complexity

Lower Temperature
☛ motions get more correlated

Below certain T crystal forms
☛
Strongly Correlated



Excitations
☛ collective motion of many atoms e.g. Phonons

Strategy: Effective Field Theory

- Position + motion of each electron (**e**$^{-}$) correlated with those of all the others ☛ hard to describe

- Number / density of **e**$^{-}$'s ~10^{23} ☛ brute force (direct computation) fails

- One way of approach ☛ understand long wavelength / macroscopic properties using a low-energy (IR) Effective Field Theory (EFT)

- Long wavelength ⟺ short distance information averages out / microscopic details irrelevant

- A "tractable" EFT
  ☛ enables to identify Universality Phenomena
  ☛ simpler than original microscopic models + relate to experiments

Non-Fermi Liquids (NFL)

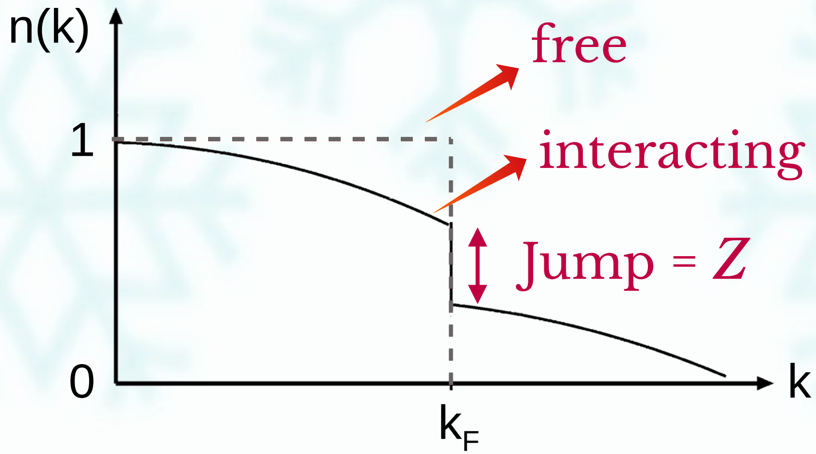
Theory of Normal Metals

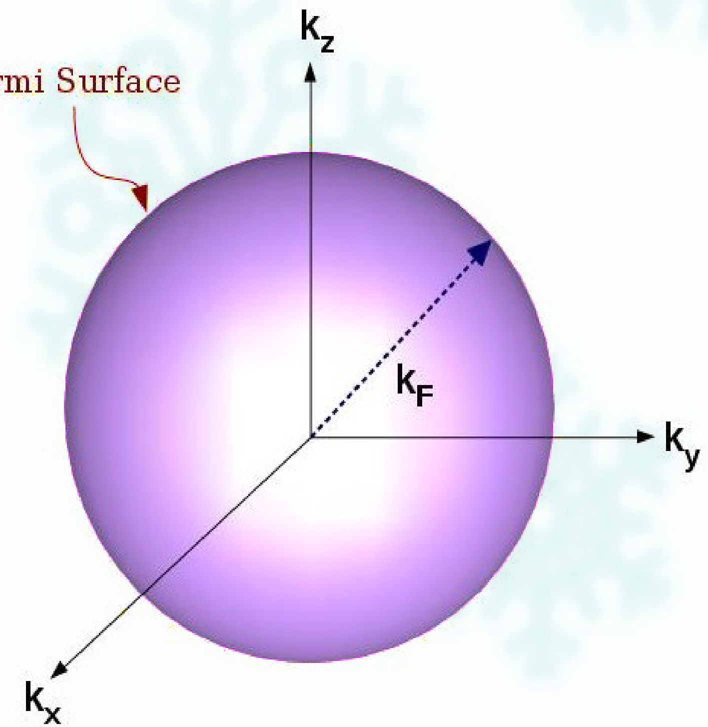
Landau

Fermi Liquid (FL) ☛ A finite density of weakly interacting fermions does not depend on specific microscopic details

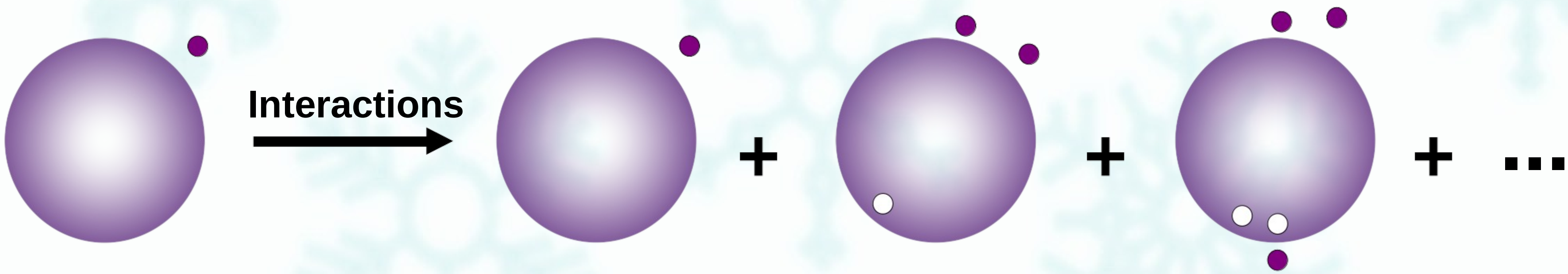
- **Ground state** ☛ characterized by a sharp Fermi surface (FS) in momentum space
- **Quasiparticles** ☛ low energy excitations near FS

FS ☛ jump in fermion occupation number n(k) at T=0





Quasiparticles: Emergent Entities in FL



Coherent part of
single-particle Green's function

$$G_R(\omega, \mathbf{k}) = \frac{Z}{\omega - v_F\, k_\perp - k_\parallel^2 + i\,\Gamma}$$

$$\omega = E - E_F\,, \quad k_\perp = k - k_F$$

Quasiparticles

☛ collective low energy quantum oscillations

- ✓ Quasiparticle lifetime diverges close to FS ☛ Decay rate $\boldsymbol{\Gamma \sim \omega^2}$
- ✓ Overlap between elementary excitations of free and interacting systems ☛ quasiparticle weight $\mathbf{0 < Z \leq 1}$

Manifestation of FL

- Ground state adiabatically connected to the non-interacting problem
- Quasiparticles $\xleftrightarrow{\text{one-to-one}}$ free system excitations
- Temperature (T) dependence of thermodynamic & transport properties similar to free fermions

> - Resistivity $\rho \propto T^2$
> - Specific heat $\propto T$
> - Im (self energy Σ) $\propto \omega^2$

A complicated problem reduced to a simpler one

Breakdown of FL Theory

Long-lived quasiparticles

☛ FL

Massive

Electrons + Bosons
↓
Low energy QFT

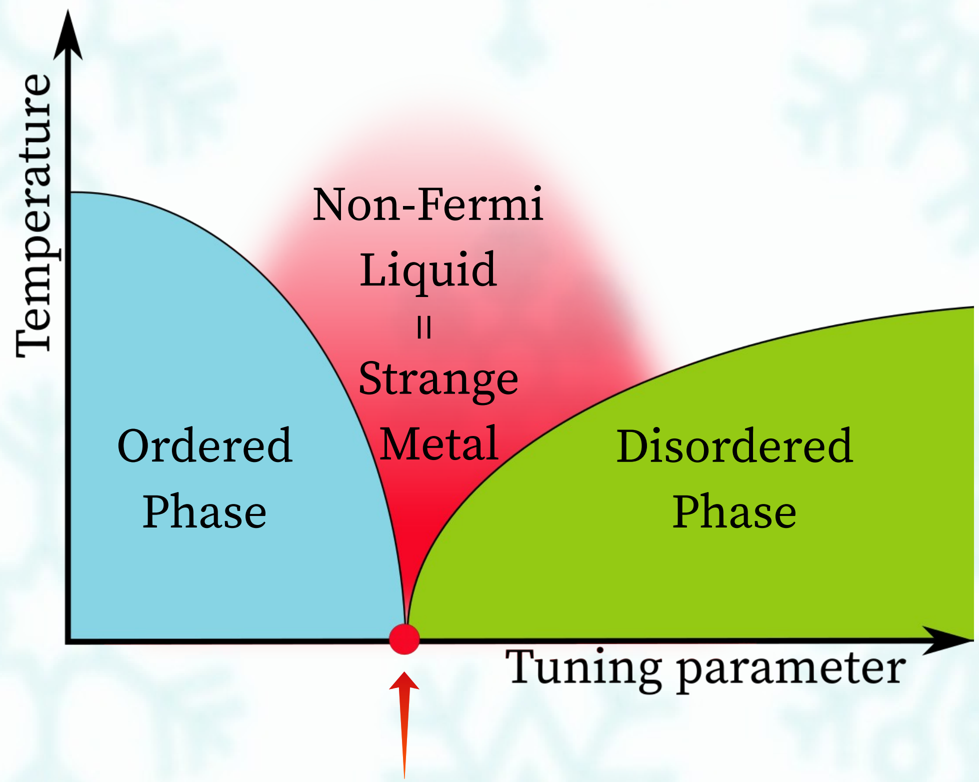
Massless

No quasiparticle

☛ non-Fermi liquid (**NFL**)

Boson

Boson



Bosonic order parameter massless at Quantum Critical Point (**QCP**)

mediates

strong interactions

e.g. Antiferro order,
nematic order,
charge density order,
gauge fields ...

Critical Fermi Surface

States with

❶ Sharp FS

+

❷ No Landau quasiparticles

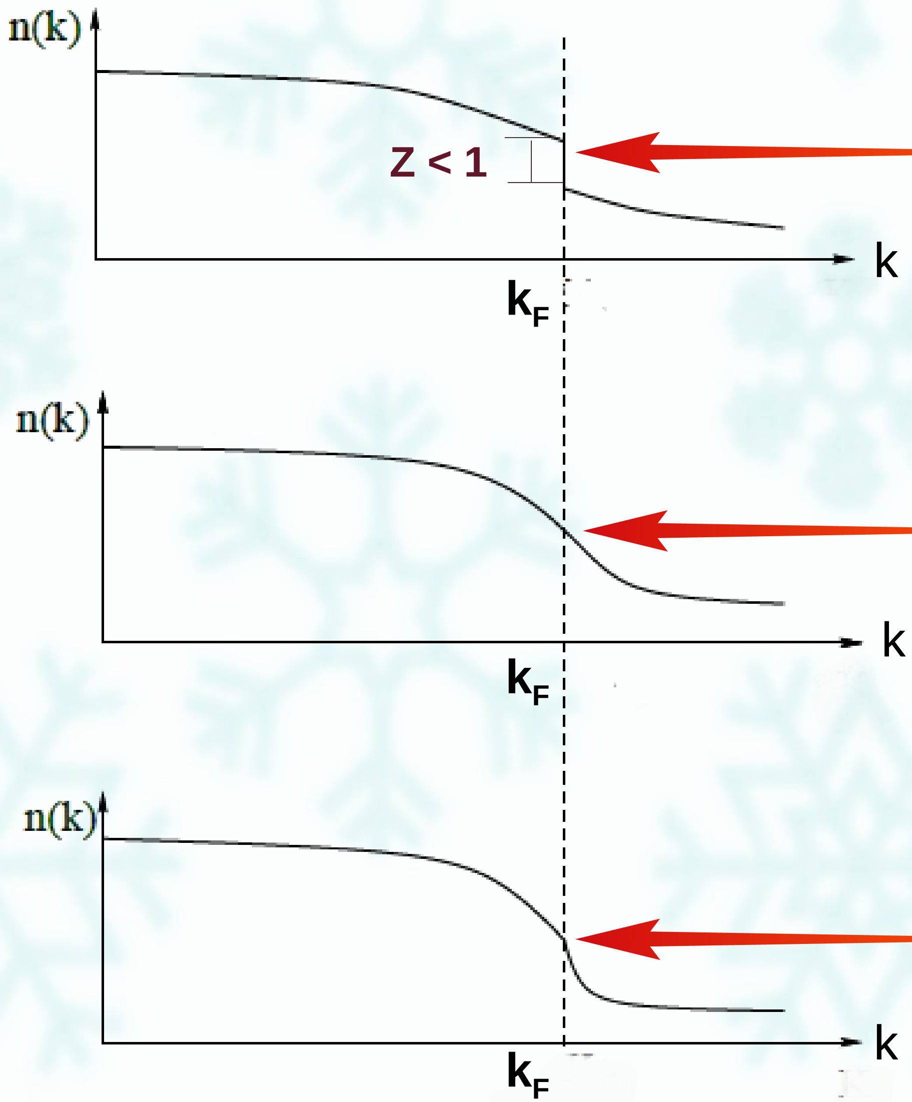
QCPs associated with onset of order, emergent gauge fields, ...

FS Disappears at QCP

Origin of critical FS ☛ Z vanishes continuously everywhere on FS

[Senthil (2008)]

Ground state momentum distribution



Interacting Landau FL

☛ discontinuity 0 < Z < 1 at k_F

Phase where FS disappears

☛ n(k) smooth everywhere

QCP

☛ n(k) continuous at k_F discontinuity replaced by kink singularity

FS + Massless Boson

Recipe for NFL as fermion-boson coupling becomes strong in 2d, even if bare coupling is weak ☛ $Z \to 0$

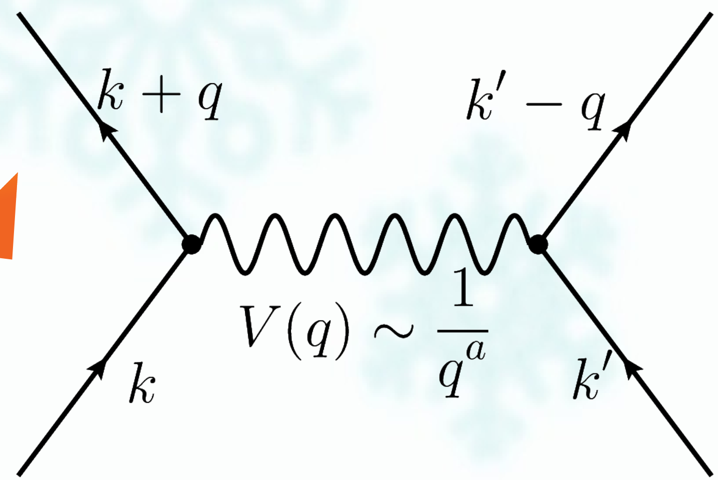
☛ quasiparticles destroyed

FL **FS away from QCP**

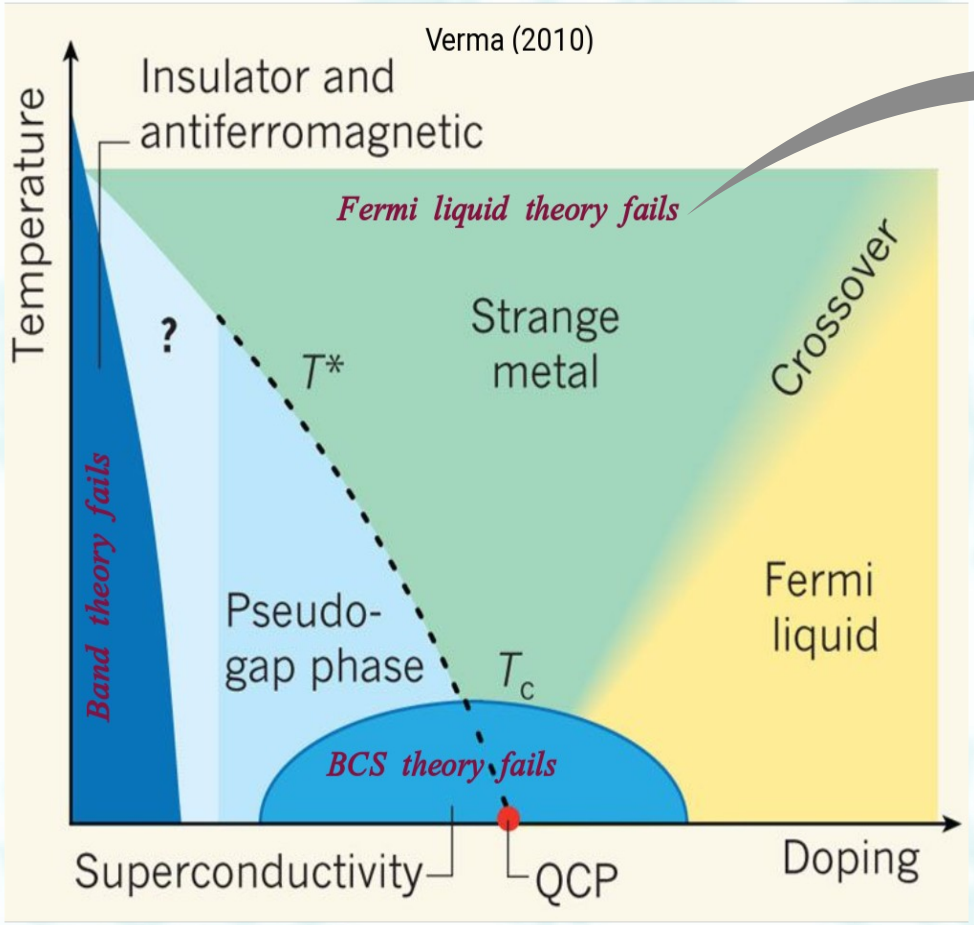
NFL **FS at QCP**

Attempt to describe NFL as

FS + Gapless Order Parameter fluctuations



Where do NFLs Appear?



Phase diagram of high-T_c cuprates

Non-Fermi Liquid (NFL) metals

- $\rho \propto T$
- $\mathrm{Im}\,\Sigma \propto \omega$ in ARPES
- No Landau quasiparticles

NFLs: How to Explain Theoretically?

We developed a novel analytical framework

QFT

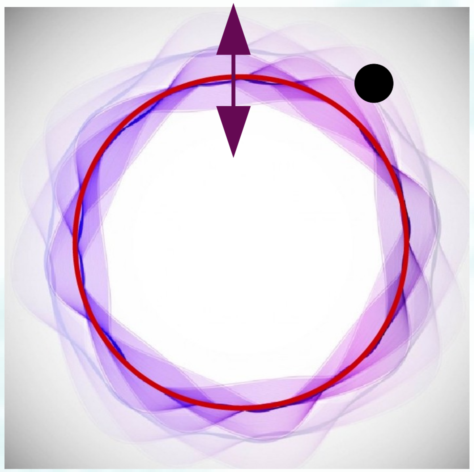
Dimensional Regularization + Renormalization Group

[**IM** & S-S Lee, *PRB (2015)*]

Our controlled approximation allowed to compute critical exponents, optical conductivity, …

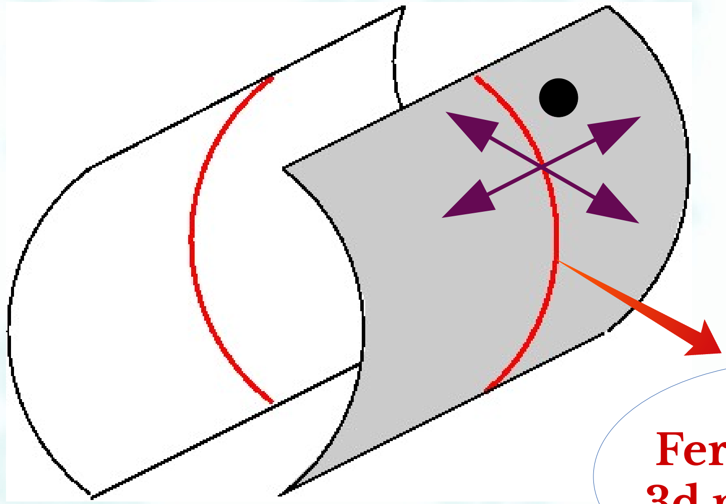
How to Explain Theoretically?

A controlled approx. to determine critical scalings by dimensional regularization



FS of 2d metal

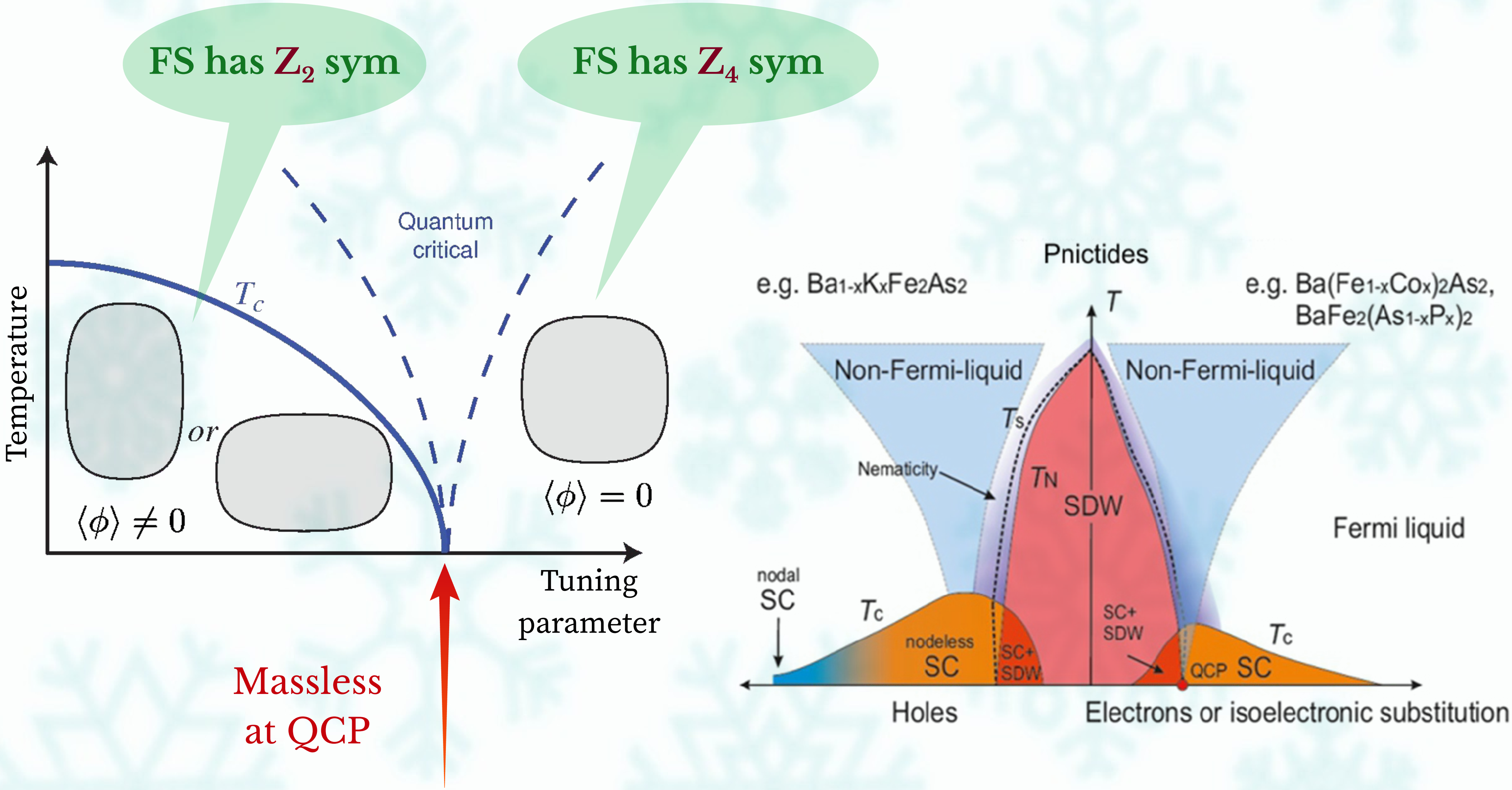
Adding extra dimensions ⊥ FS suppress qtm fluctuations



Fermi line in 3d mom space

- Find upper critical dimension $\mathbf{d_c}$
  ☛ well-known tool from Statistical Mechanics / QFT
- $\mathbf{d} > \mathbf{d_c}$ described by mean-field theory (FL)
- $\mathbf{d_{phys}} \leq \mathbf{d_c}$ ☛ mean-field theory inapplicable
  ☛ perturbative expansion in $\epsilon = d_c - d_{phys}$

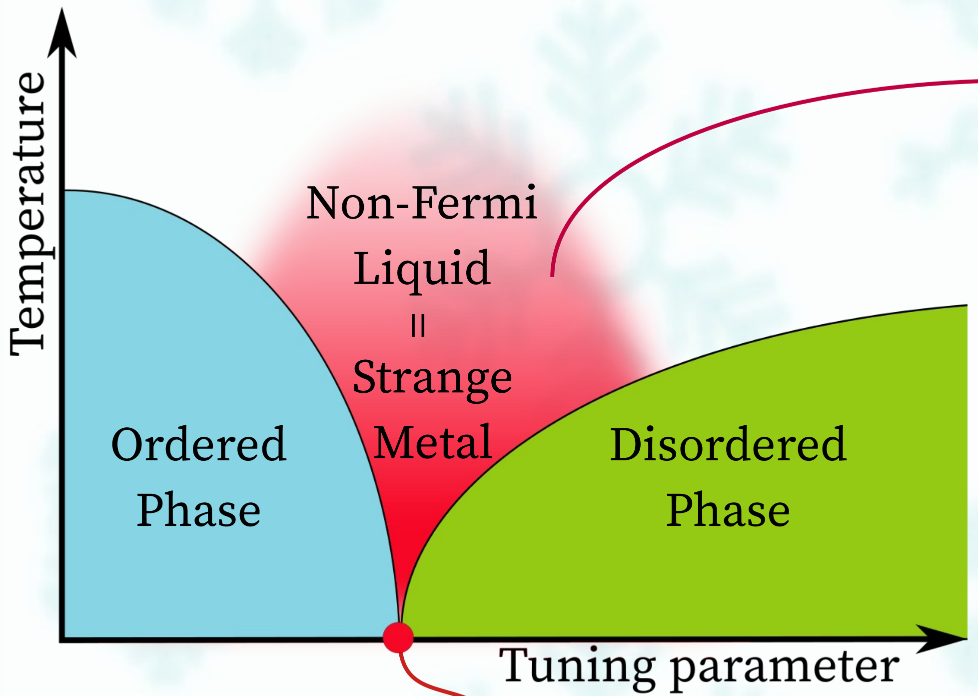
Applications: (1) Ising-Nematic QCP



Order Parameter ϕ ☛ Real Scalar Boson

[**$YBa_2Cu_3O_y$** (Cuprate), **$Sr_3Ru_2O_7$** (Ruthenate), **Pnictides**]

Results: 2d Ising-Nematic QCP



1d FS fluctuations
effectively local

Optical conductivity

$\boldsymbol{\sigma(\omega) \sim \omega^{-2/3}}$

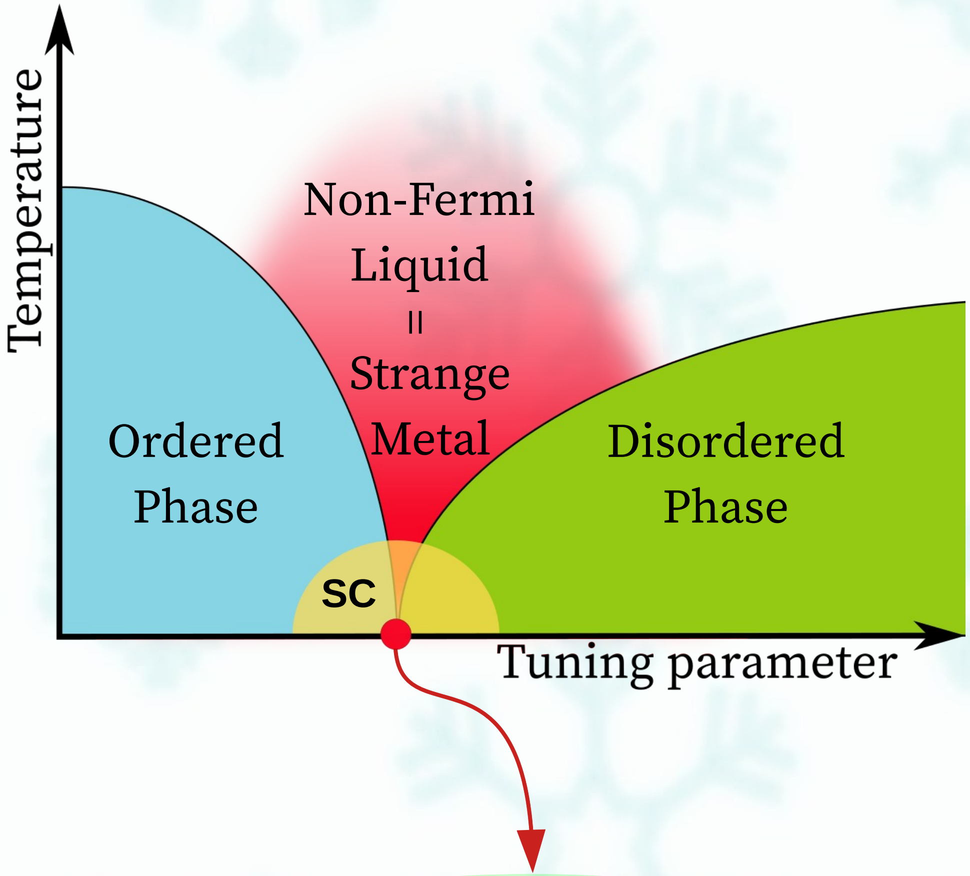
☛ close to $\boldsymbol{\omega^{-0.65}}$ found in expts.
on optimally doped cuprates

[A. Eberlein, **IM**, & S. Sachdev, *PRB (2016)*]

Massless Boson at

Quantum Critical Point

Results: 2d Ising-Nematic QCP



Competition between
non-Fermi liquid phase
&
pairing instability at T=0

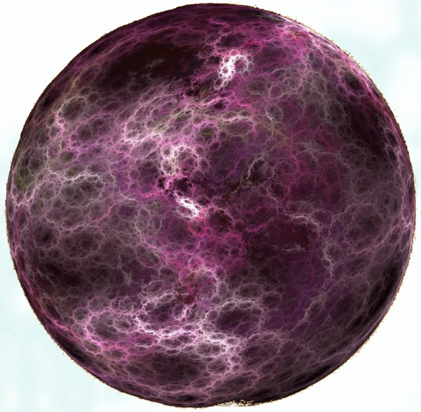
☛ Superconductivity (SC) wins

[**IM**, *PRB (2016)*]

Quantum Critical Point
masked by superconducting
(SC) dome

Results: 3d Ising-Nematic QCP

2d FS fluctuations
non-local
☛ entangled all over the FS



☛ **Fluctuations less violent than 1d FS**

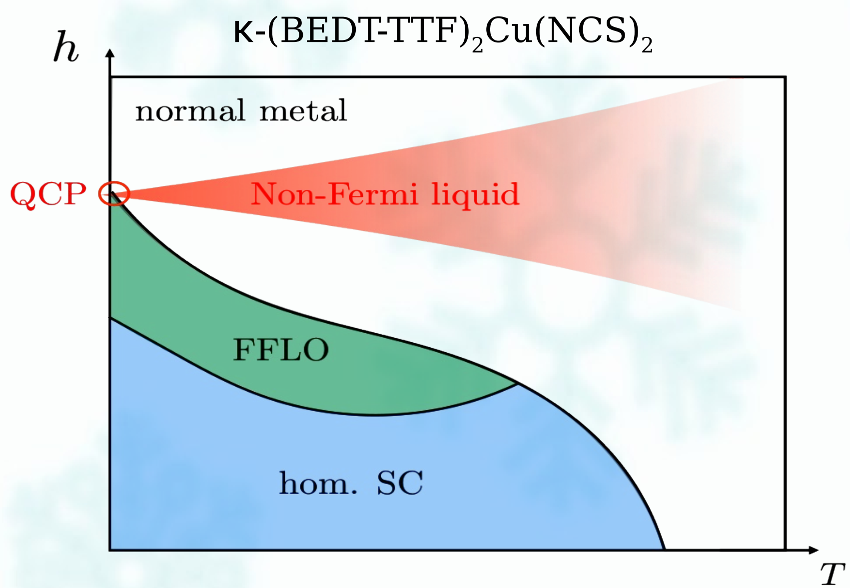
☛ **Ultraviolet / Infrared mixing**

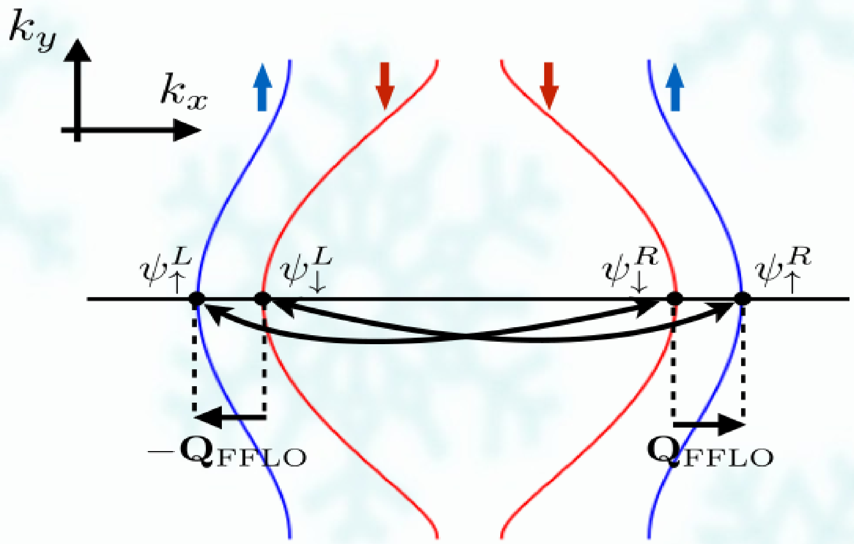
☛ **≥ 2-loop corrections vanish**

[**IM** & S-S Lee, *PRB (2015)*]

[**IM**, *EPJB (2016)*]

Applications: (2) FFLO-Normal Metal QCP





Magnetic field splits FS's

☛ QCP between 2d metal & FFLO phase

FFLO ☛ Cooper pair with finite momentum $\mathbf{Q}_{\text{FFLO}}$

[F. Piazza, W. Zwerger, P. Strack, PRB 93, 085112 (2016)]

Potentially naked / unmasked QCP ☛ scaling regime observable down to arbitrary low T

Computed critical properties of the stable NFL

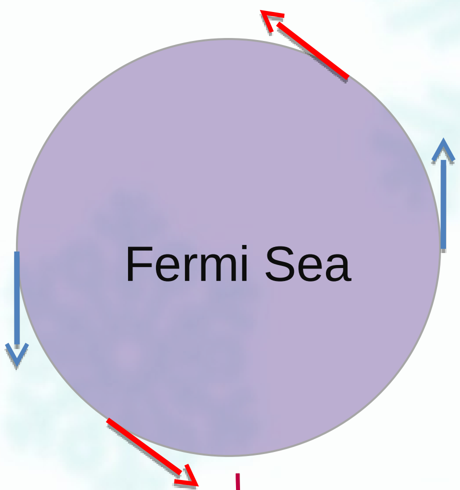
[D. Pimenov, IM, F. Piazza, M. Punk, PRB 98, 024510 (2018)]

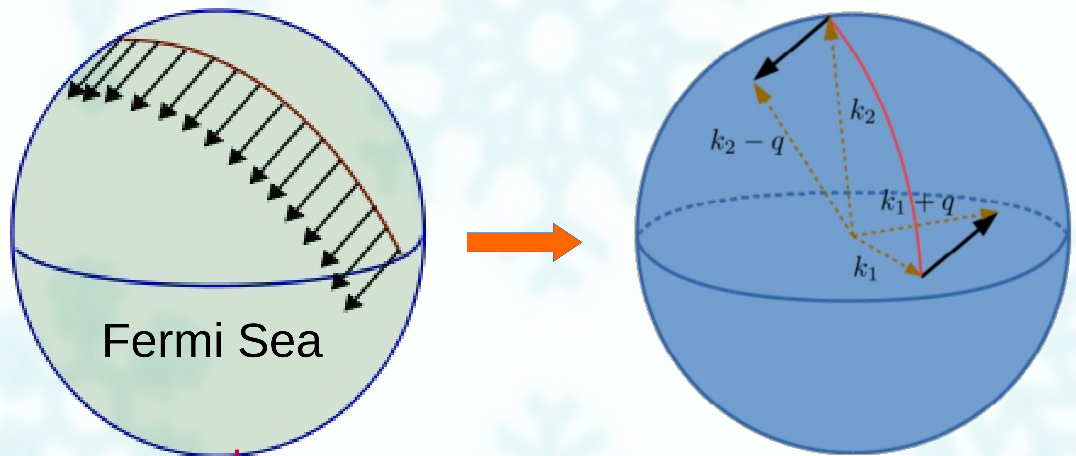
Technical Details for Ising-Nematic QCP

Generalize to (m-dim FS) + Q=0 Scalar Boson

Low energy limit ☛ Fermions scatter tangentially

Time-Reversal Invariance assumed





Circular FS (m=1)

☛ fermions in different patches

decoupled except

antipodal points

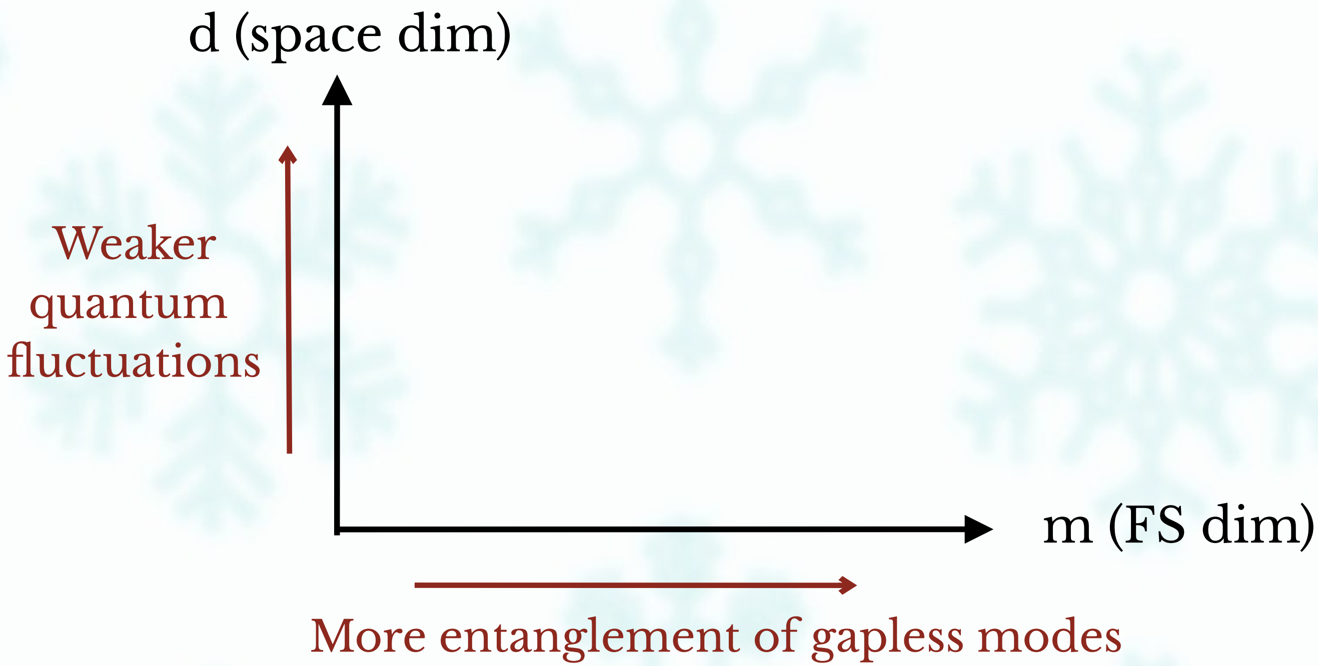
Not true for

m-dim FS

with m > 1

Size of FS (k_F) enters as a dimensionful parameter

Significance of m



- m = 1 ☛ observables local in mom space (e.g. Green's fns) can be extracted from local patches ☛ emergent locality

  [D. Dalidovich & S-S. Lee, *Phys. Rev. B 88, 245106 (2013)*]

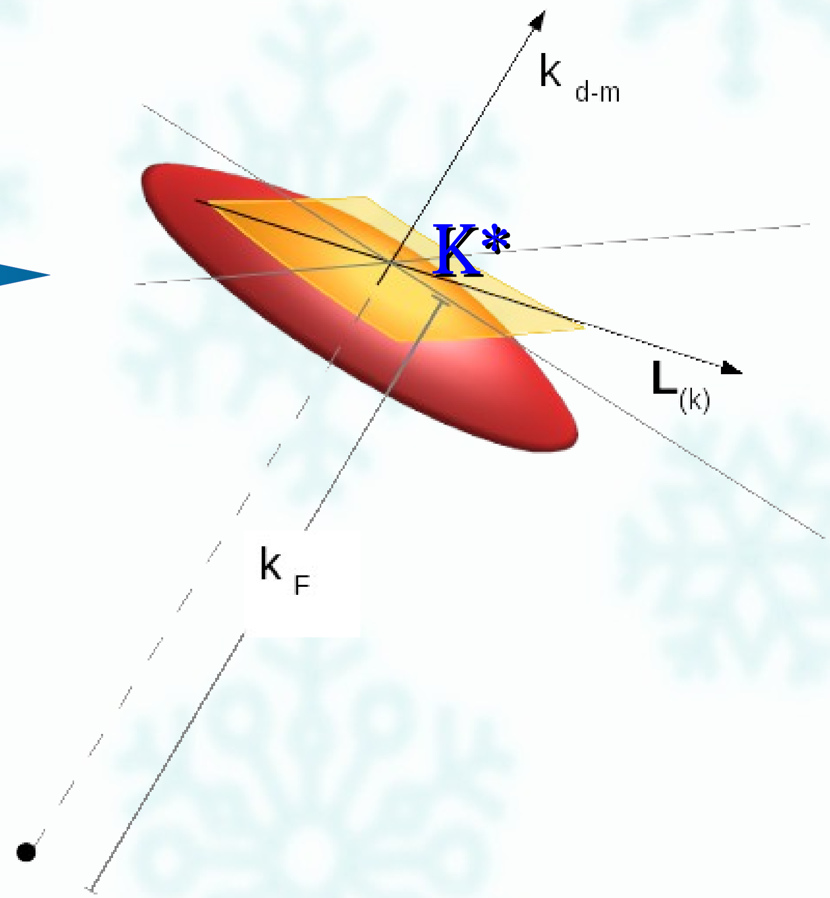
- m > 1 ☛ UV/IR mixing ☛ low-energy physics affected by gapless modes on entire FS ☛ size of FS (k_F) modifies naive scaling coming from patch description ☛ k_F becomes a 'naked scale'

  [**IM** & S-S Lee, *Phys. Rev. B 92, 035141 (2015)*]

Coordinate Set-up

Patch of m-dim FS of arbitrary shape

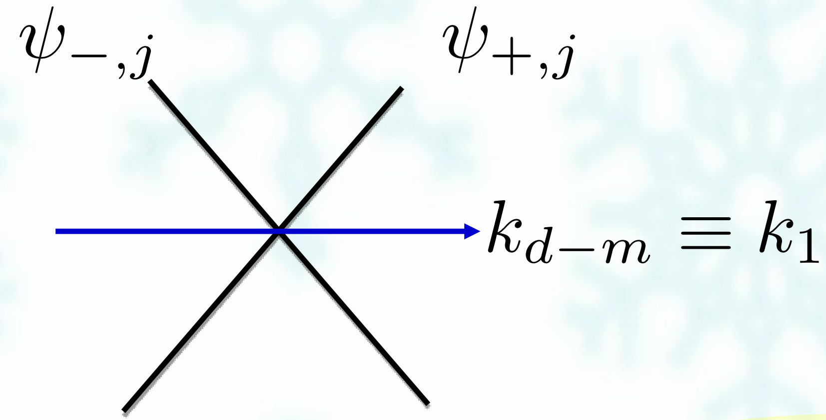
$d = d_{phys} = m + 1$

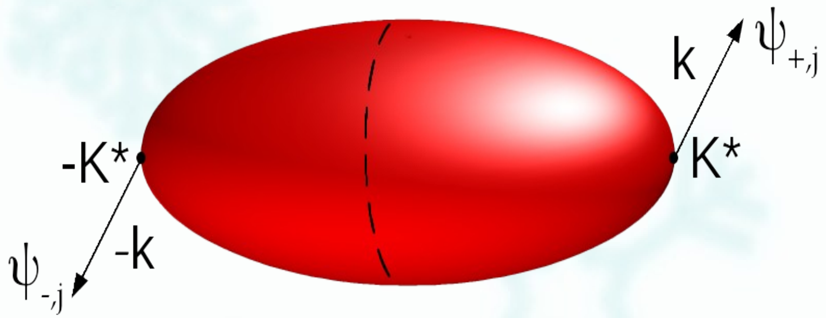


- At a chosen point K^* on FS : $k_{d-m} \perp$ local S^m ☛ its magnitude measures deviation from k_F
- $L_{(k)} = (k_{d-m+1} , k_{d-m+2} ,..., k_d)$ ☛ tangential along the local S^m

Fermions on Antipodal Points

Time-Reversal Invariance assumed





$$\psi_{+,j}\ (\psi_{-,j})$$

right (left) moving fermion
with flavour j=1,2,...,N

Effective Action

2 halves of m-dim FS
+ massless boson
in **d** space
& one time dim

d = d_{phys} = m + 1

$$S = \sum_{s=\pm,j} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \, \psi^\dagger_{s,j}(k) \Big[\mathrm{i}\, k_0 + s\, k_{d-m} + \mathbf{L}^2_{(k)} \cancel{+ \mathcal{O}\big(|\mathbf{L}_{(k)}|^3\big)} \Big] \psi_{s,j}(k)$$

$$+ \frac{1}{2} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \left[k_0^2 + k_1^2 + \mathbf{L}^2_{(k)} \right] \phi(-k)\, \phi(k)$$

$$+ \frac{e}{\sqrt{N}} \sum_{s=\pm,j} \int \frac{d^{m+2}k\, d^{m+2}q}{(2\pi)^{2m+4}} \, \phi(q) \; \psi^\dagger_{s,j}(k+q)\, \psi_{s,j}(k)$$

Action in terms of Dirac Fermions

$$\Psi_j(k) = \begin{pmatrix} \psi_{+,j}(k) \\ \psi^\dagger_{-,j}(-k) \end{pmatrix}$$

Interpret $|\mathbf{L}_{(k)}|$ as a continuous flavour

☛ **Each (m+2)-d spinor can be viewed as a (1+1)-d Dirac fermion**

$$S = \sum_j \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \bar{\Psi}_j(k)\, \mathrm{i} \left[\gamma_0\, k_0 + \gamma_1 \left(k_{d-m} + \mathbf{L}^2_{(k)} \right) \right] \Psi_j(k)$$

$$+ \frac{1}{2} \int \frac{d^{m+2}k}{(2\pi)^{m+2}} \mathbf{L}^2_{(k)}\, \phi(-k)\, \phi(k)$$

$$+ \frac{\mathrm{i}\; e}{\sqrt{N}} \sum_j \int \frac{d^{m+2}k\, d^{m+2}q}{(2\pi)^{2m+4}}\, \phi(q)\, \bar{\Psi}_j(k+q)\, \gamma_{d-m} \Psi_j(k)$$

Embed FS in Higher Dimensions

Add extra spatial dimensions ⊥ $L_{(k)}$

☛ $d > d_{phys}$

$$k_0 \to \mathbf{K} \equiv (k_0, k_1, \ldots, k_{d-m-1})$$

$$\gamma_0 \to \Gamma \equiv (\gamma_0, \gamma_1, \ldots, \gamma_{d-m-1})$$

$$\gamma_1 \to \gamma_{d-m}$$

$$\delta_k \equiv k_{d-m} + \mathbf{L}^2_{(k)}$$

$$S = \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k)\, \mathrm{i} \left[\mathbf{\Gamma} \cdot \mathbf{K} + \gamma_{d-m}\, \delta_k \right] \Psi_j(k)$$

$$+ \frac{1}{2} \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \mathbf{L}^2_{(k)}\, \phi(-k)\, \phi(k)$$

$$+ \frac{\mathrm{i}\, e}{\sqrt{N}} \sum_j \int \frac{d^{d+1}k\, d^{d+1}q}{(2\pi)^{2d+2}} \phi(q)\, \bar{\Psi}_j(k+q)\, \gamma_{d-m} \Psi_j(k)$$

Energy Scales

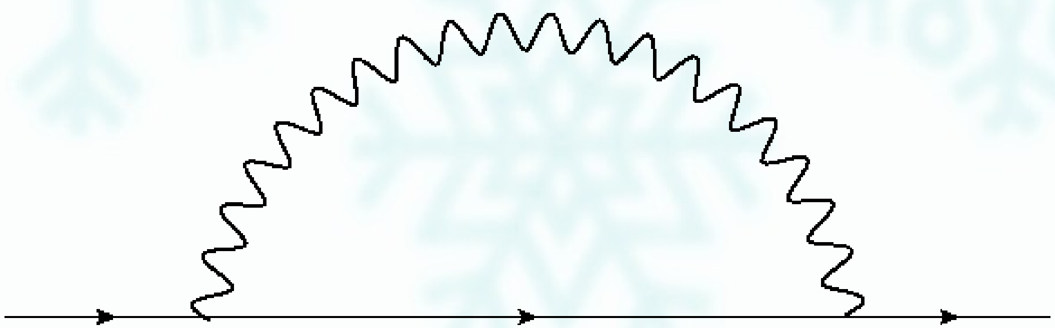
- **Λ** is implicit UV cut-off with **K**, $\mathbf{k_{d-m}} \ll \Lambda \ll \mathbf{k_F}$

- $\mathbf{k_F}$ ☛ sets FS size

  Λ ☛ sets the largest momentum fermions can have ⊥ FS

- RG flow ☛ change **Λ** & require low-energy observables independent of **Λ**

- Fix **m** & tune **d** towards $\mathbf{d_c}$ at which fermion self-energy diverge logarithmically in **Λ** ☛ access NFL perturbatively in $\epsilon = d_c - (m+1)$



Critical Dimension

- Upper critical dim ☛ $d_c = m + \frac{3}{m+1}$

  $d_c = 3$ for ($m = 2$, $d_{phys} = 3$)

  $d_c = 5/2$ for ($m = 1$, $d_{phys} = 2$)

- Scaling dim of e = $1 - d/2 + m/4$

- e has positive scaling dimension at d_c for $1 < m < 2$
  - ☛ cannot be the control parameter in perturbative loop expansions

- $e_{eff} = e^{\frac{2(m+1)}{3}} / \tilde{k}_F^{\frac{(m-1)(2-m)}{6}}$ $\quad \left(\tilde{k}_F = k_F / \Lambda \right)$

  has scaling dimension [m + 3/(m + 1) - d] (m+1)/ 3 that vanishes at d_c
  - ☛ effective coupling that is control parameter in loop expansions

One-Loop Results

Effective coupling
control parameter in loop expansions

$$e_{eff} = e^{\frac{2(m+1)}{3}} / \tilde{k}_F^{\frac{(m-1)(2-m)}{6}}$$

Fixed points of beta-function

$$\tilde{\beta} \equiv \frac{\partial e_{eff}}{\partial \ln \mu} = \frac{(m+1)(u_1\, e_{eff} - N\epsilon)\, e_{eff}}{3N - (m+1)u_1\, e_{eff}} = 0$$

Interacting Fixed Point

$$e^*_{eff} = \frac{N\epsilon}{u_1}$$

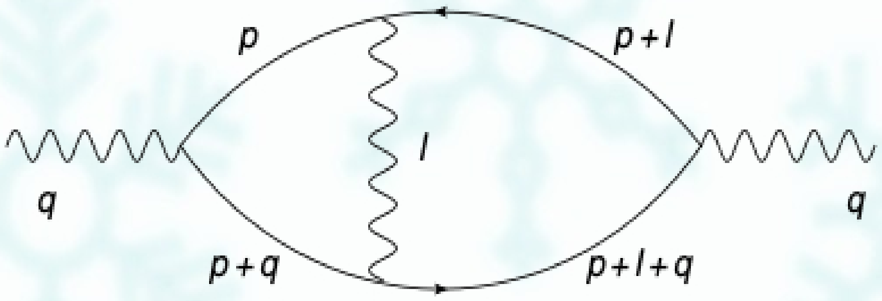
$$z^* = 1 + \frac{(m+1)\,\epsilon}{3}$$

$$\eta^*_\psi = \eta^*_\phi = -\frac{\epsilon}{2}$$

Dynamical critical exponent

Anomalous dimensions for fermions & boson

Two-Loop Boson Self-Energy



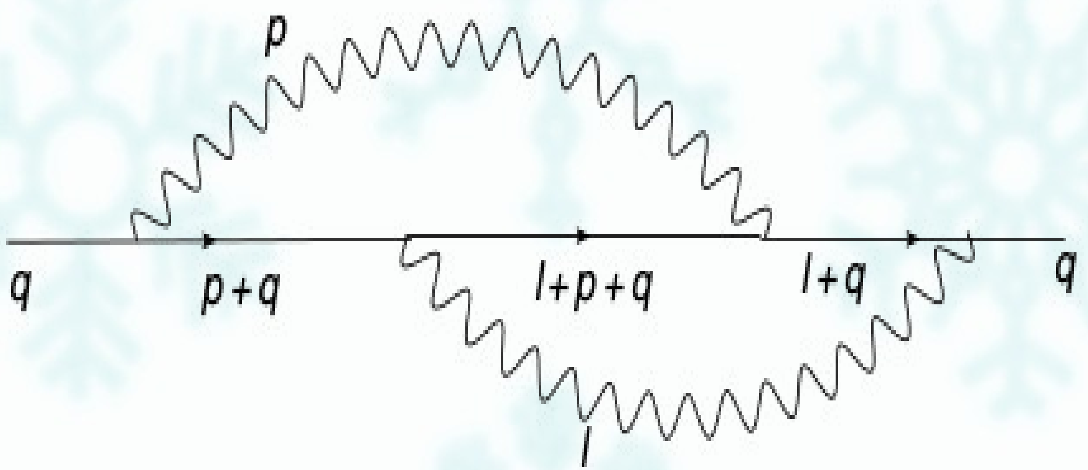
- For m > 1 ☛ k_F suppressed ☛ no correction

$$\Pi_2(q) \sim \frac{e^2 \, k_F^{\frac{m-1}{2}} \, \pi^2}{6\, N\, |\vec{L}_{(q)}|^2 \, \sin\left(\frac{m\,\pi}{3}\right)} \frac{e_{eff}^{\frac{m}{m+1}}}{k_F^{\frac{m-1}{2\,(m+1)}}}$$

- For m = 1 ☛ UV-finite correction

$$\Pi_2(q) \sim \left(\frac{e^2}{N\,|L_{(q)}|}\right) e_{eff}$$

Two-Loop Fermion Self-Energy



- For m > 1 ☛ $\Sigma_2(q) \quad \sim \quad k_F - suppressed$

  ☛ no correction

- For m = 1 ☛ UV-divergent

Pairing Instabilitites of Critical FS

FL unstable to arbitrary weak -ve interaction
in BCS channel leading to Cooper pairs
☛ How about a critical FS ?

[**IM**, *Phys. Rev. B 94, 115138 (2016)*]

Superconducting Instability

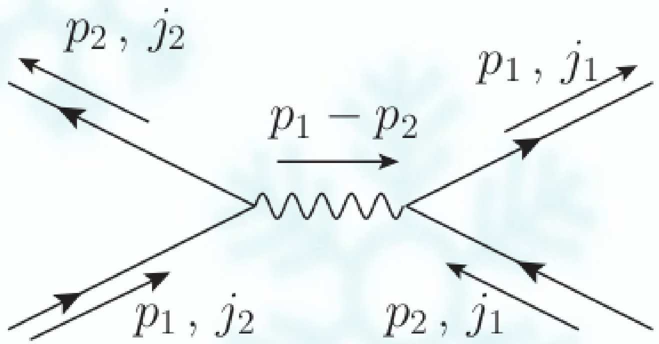
Add relevant 4-fermion terms
For simplicity, we consider s-wave case with 2 flavours

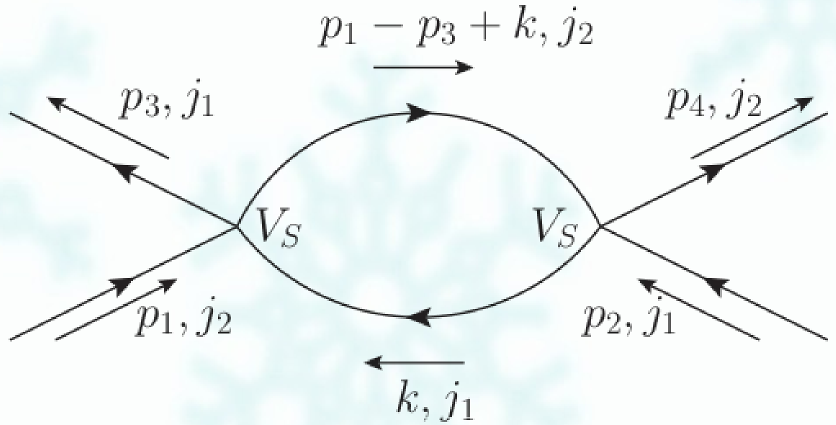
coupling constant

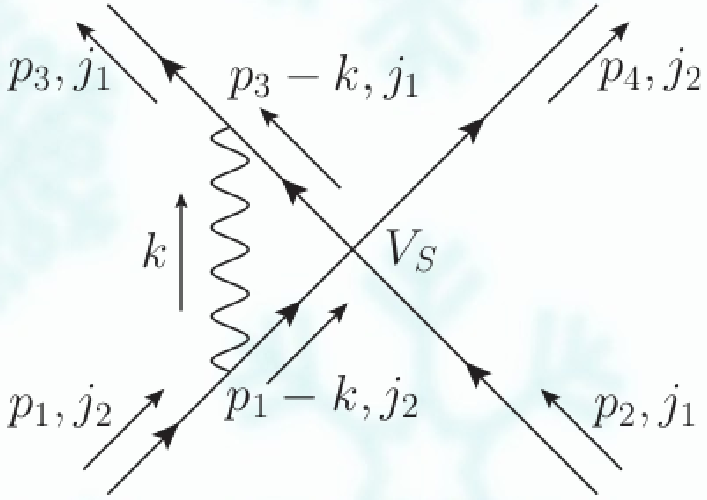
$$S^{\text{SC}} = \frac{\mu^{d_v} V_S}{4} \sum_{j_1, j_2} \int \left(\prod_{s=1}^{4} dp_s \right) (2\pi)^{d+1} \, \delta^{(d+1)}(p_1 + p_2 - p_3 - p_4) \, (\delta_{j_1, j_2} - 1)$$

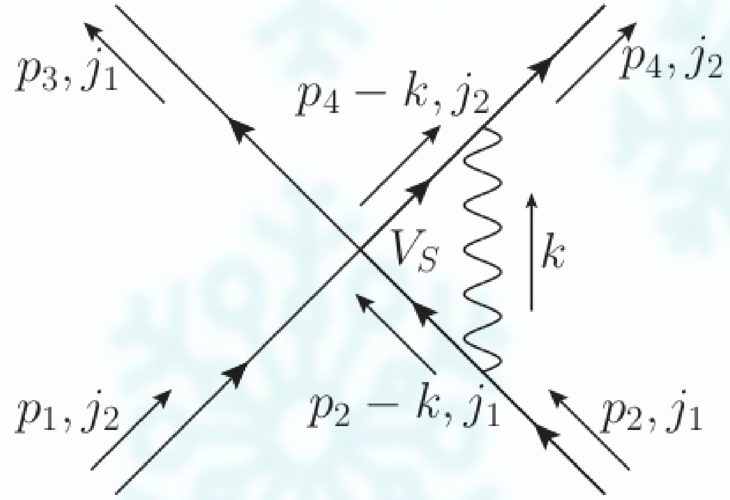
$$\times \left[\{\bar{\Psi}_{j_1}(p_3) \Psi_{j_2}(p_1)\} \{\bar{\Psi}_{j_2}(p_4) \Psi_{j_1}(p_2)\} - \{\bar{\Psi}_{j_1}(p_3) \sigma_z \Psi_{j_2}(p_1)\} \{\bar{\Psi}_{j_2}(p_4) \sigma_z \Psi_{j_2}(p_1)\} \right]$$

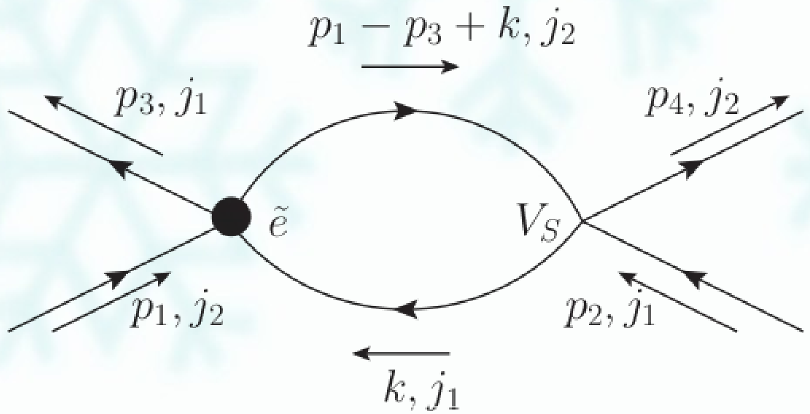
Feynman Diagrams

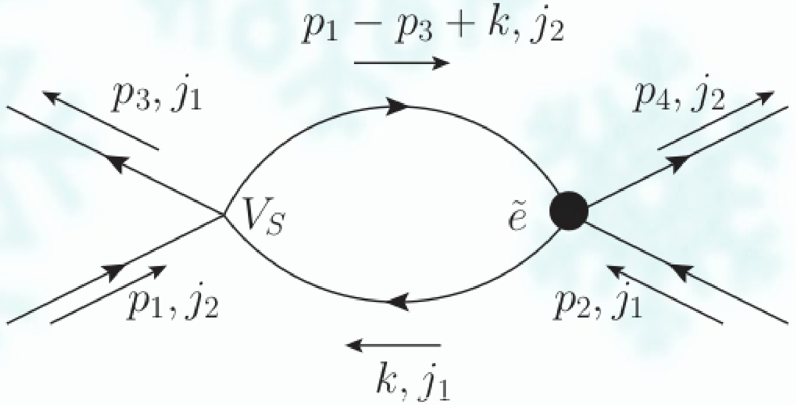












Coupled Beta-Functions for V_S & e_{eff}

- Scatterings in pairing channel enhanced by volume of FS ~ $(k_F)^{m/2}$

- Effective coupling that dictates potential instability :

$$\tilde{V}_S = \tilde{k}_F^{m/2} V_S$$

- $\tilde{V}_S$ marginal at d = m + 1

- Aim ☛ study how **e_{eff}** affects pairing instability

Beta-Functions for V_s & e_{eff}

$$\frac{\partial \tilde{V}_S}{\partial l} = \gamma\,\epsilon\,\tilde{V}_S - v_2\,\tilde{V}_S^2 - v_1\,e_{eff} + v_3\,e_{eff}\,\tilde{V}_S$$

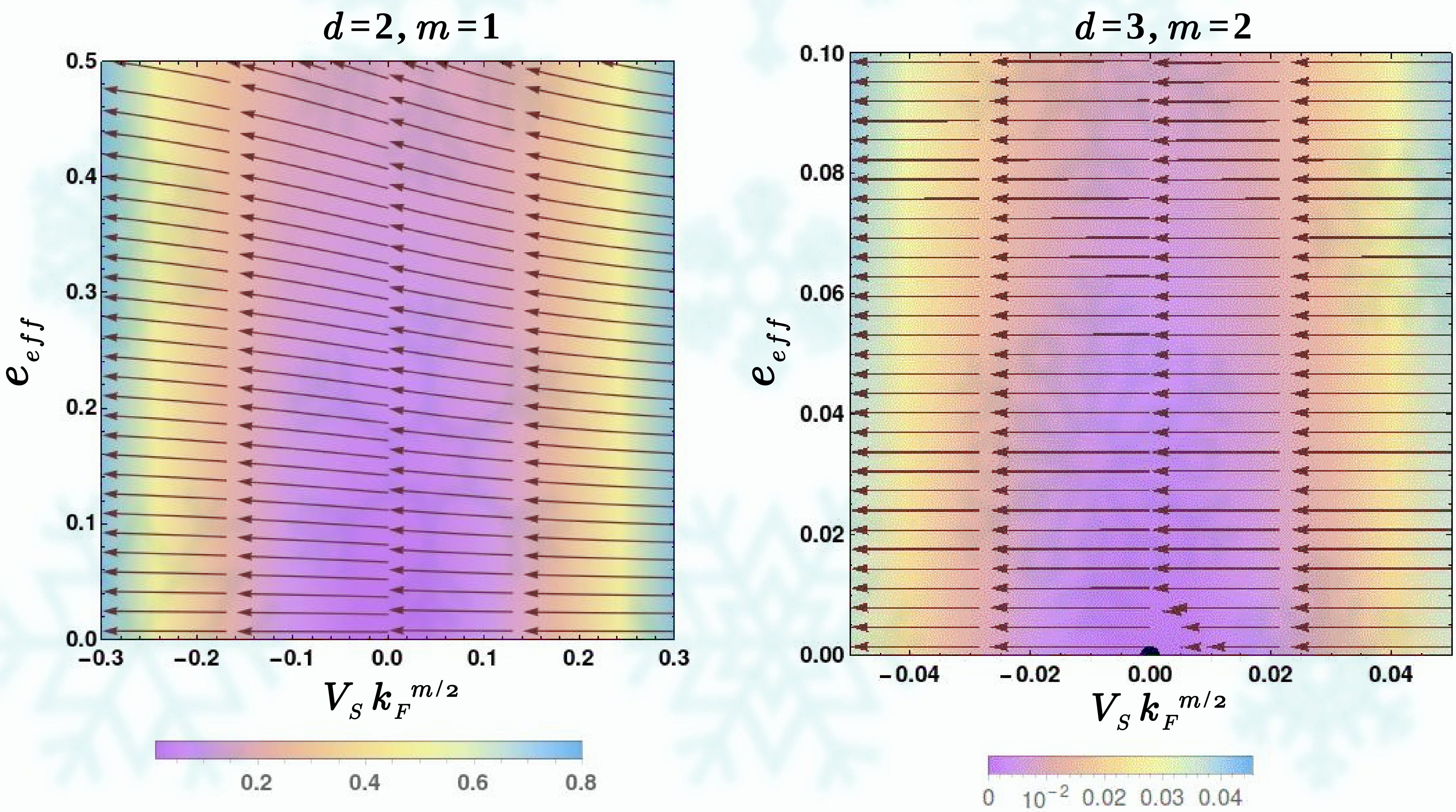
$$d - m = 1 - \gamma\,\epsilon$$

$$\gamma\,\epsilon = \epsilon - \frac{2-m}{m+1}$$

Coupled Beta-Functions for V_S & e_{eff}

RG flows to $V_S \to -\infty$ for any initial V_S

☛ More susceptible to pairing than FL ($e_{eff} = 0$)



Fermi Surface
+
Transverse Gauge Field(s)

Fermi Surface + U(1) Gauge Field

$$S = \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \, \bar{\Psi}_j(k) \, \mathrm{i} \left[\mathbf{\Gamma} \cdot \mathbf{K} + \gamma_{d-m} \, \delta_k \right] \Psi_j(k)$$

$$+ \frac{1}{2} \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \, \mathbf{L}^2_{(k)} \, \phi(-k) \, \phi(k)$$

$$+ \frac{e}{\sqrt{N}} \sum_j \int \frac{d^{d+1}k \, d^{d+1}q}{(2\pi)^{2d+2}} \, \phi(q) \, \bar{\Psi}_j(k+q) \, \gamma_0 \Psi_j(k)$$

Interaction vertex contains

γ_0 instead of $\mathrm{i} \, \gamma_{d\text{-}m}$

Values of $\mathbf{d_c}$ & critical exponents same as Ising-nematic case

[**IM**, *Phys. Rev. Research 2, 043277 (2020)*]

2 Fermion Flavours + $U_c(1) \times U_s(1)$

Model for QCPs for Mott insulator to metal & metal to metal transitions

[L. Zou & D. Chowdhury, *Phys. Rev. Research 2, 023344 (2020)*]

- First fermion couples to the gauge fields a_c & a_s as ($e^c a_c + e^s a_s$)

- Second fermion couples as ($e^c a_c - e^s a_s$)

- At one-loop, beta functions for the effective coupling constants give a **fixed line** ($e^c_{eff} + e^s_{eff}$) $\propto \epsilon$

- m > 1 ☛ fixed line feature survives at generic loops
  [**IM**, Phys. *Rev. Research 2, 043277 (2020)*]

- m = 1 ☛ fixed line feature breaks at three-loop
  [**IM**, *Phys. Rev. Research 2, 043277 (2020)*]

Epilogue

- RG analysis for critical FS ☛ scaling behaviour of NFL states in a controlled approximation

- m-dim FS with its co-dim extended to a generic value ☛ stable NFL fixed points identified using $\boldsymbol{\epsilon = d_c - d_{phys}}$ as perturbative parameter

- Pairing instability as a fn of dim & co-dim of FS
☛ superconductivity masks QCP

- Key point ☛ $\mathbf{k_F}$ enters as a dimensionful parameter unlike in relativistic QFT ☛ modify naive scaling arguments

- Effective coupling constants
☛ combinations of original coupling constants & $\mathbf{k_F}$

Thank you for your attention !