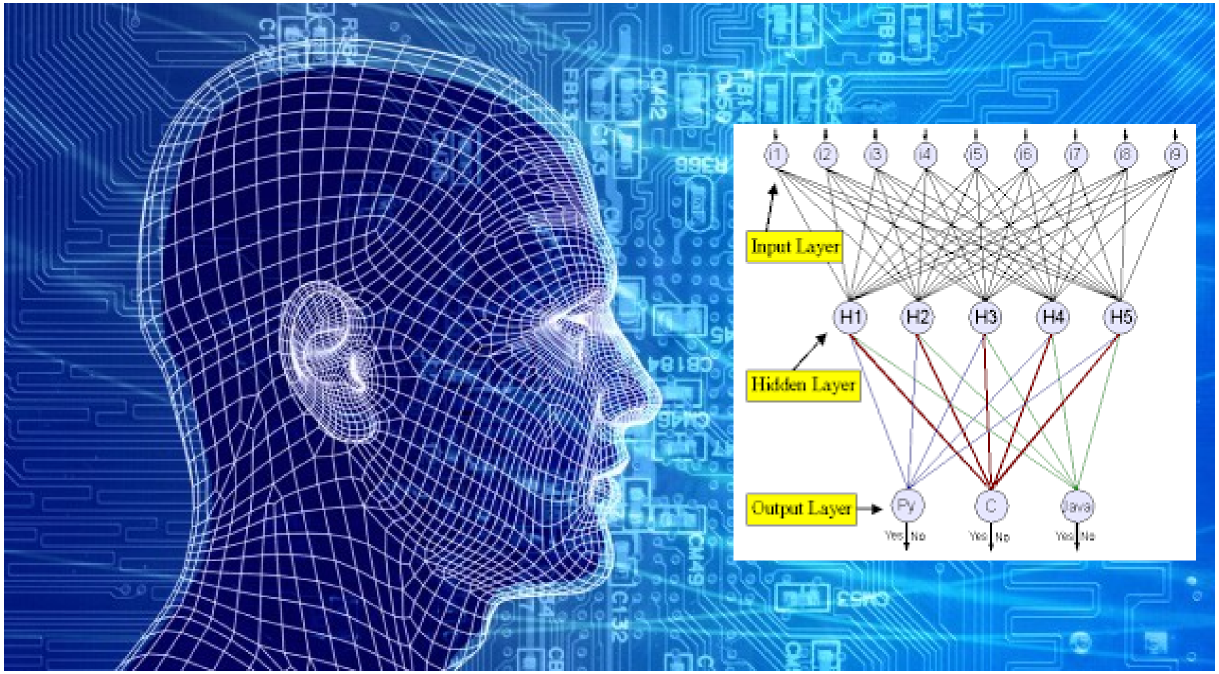
Machine learning
Lecture 5



- Neural Networks
- Radial Base Function (RBF) Networks
- Bayesian Neural Networks
- Deep Neural Networks

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IFJ PAN

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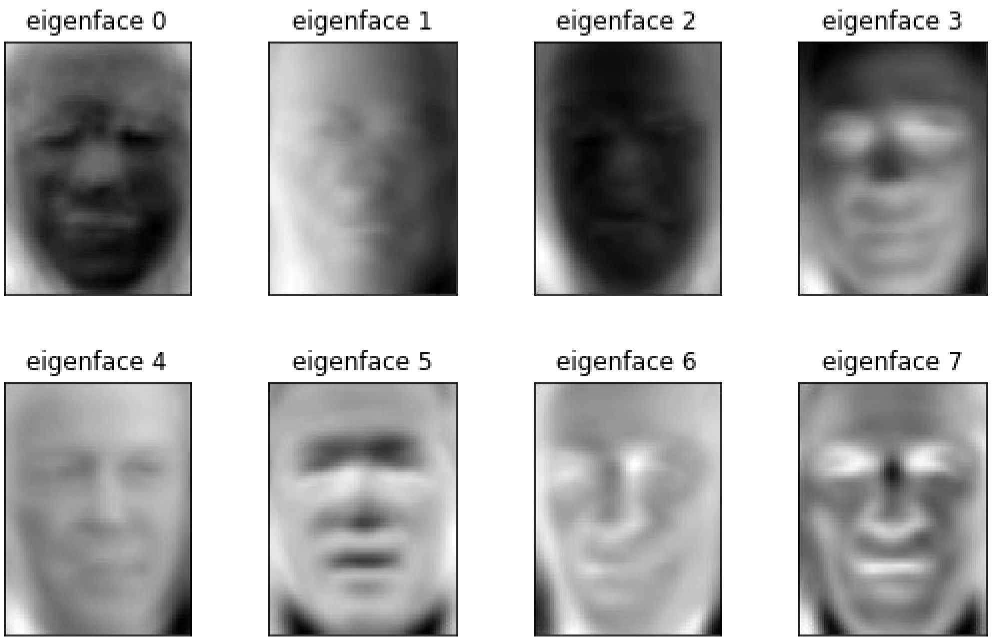
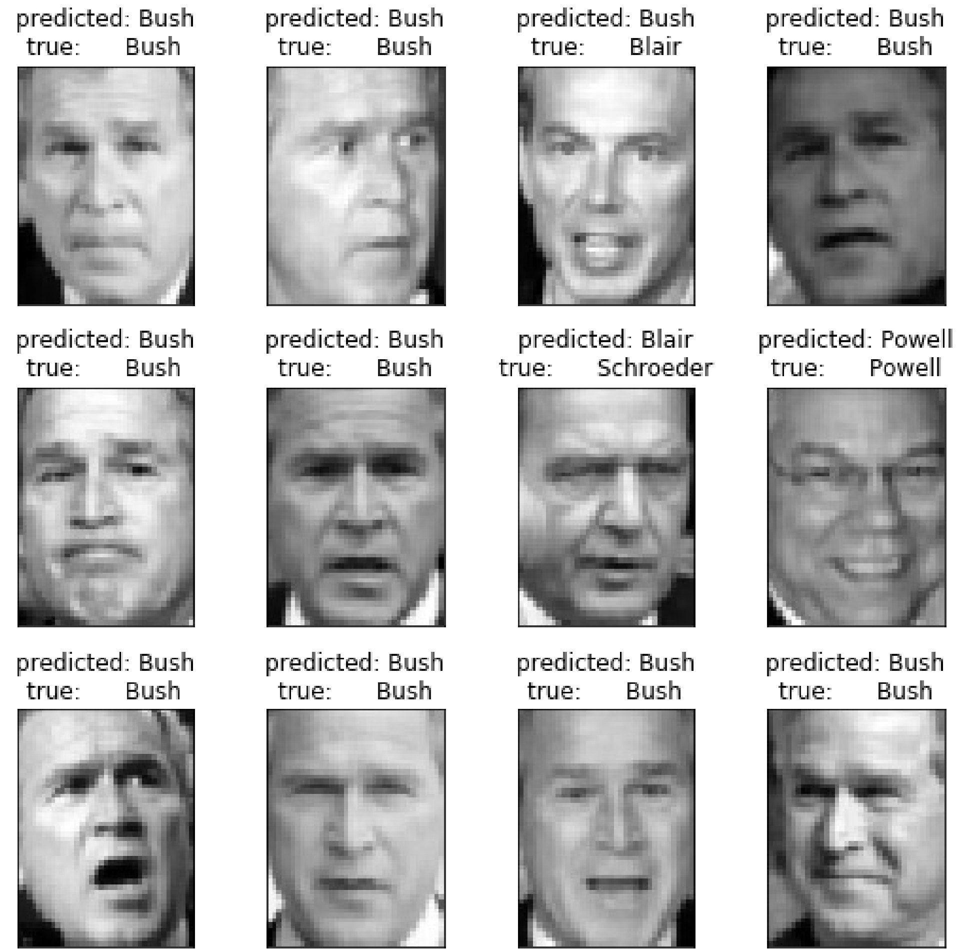
Last lecture – PCA

Any questions?

- PCA – each face can be represented as a combination of a limited number of “eigenfaces”
- https://github.com/marcinwolter/MachineLearning2020/blob/main/plot_face_recognition.ipynb

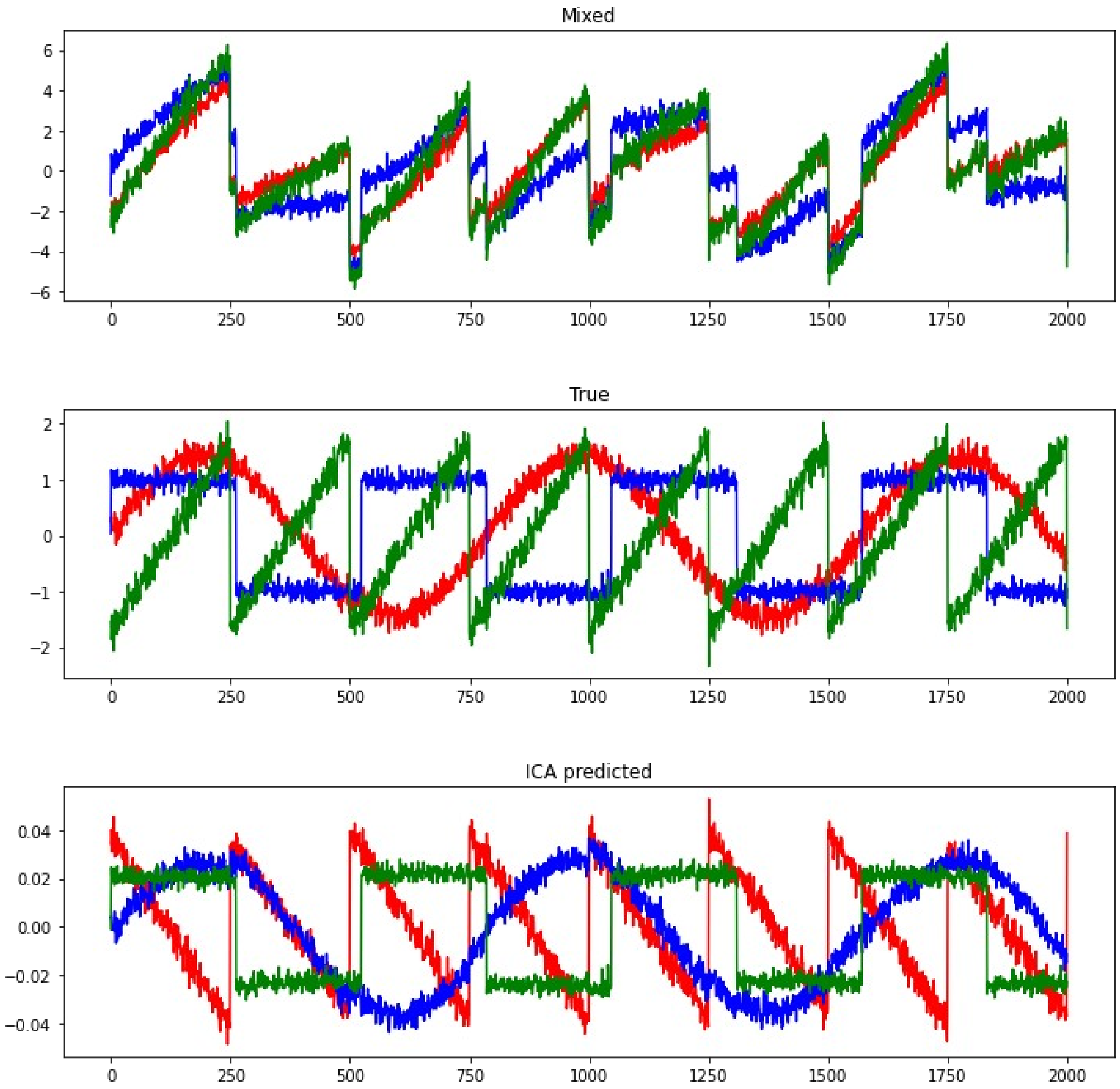




ICA example

Any questions?

- https://github.com/marcinwolter/MachineLearning2020/blob/main/ICA_sklearn.ipynb

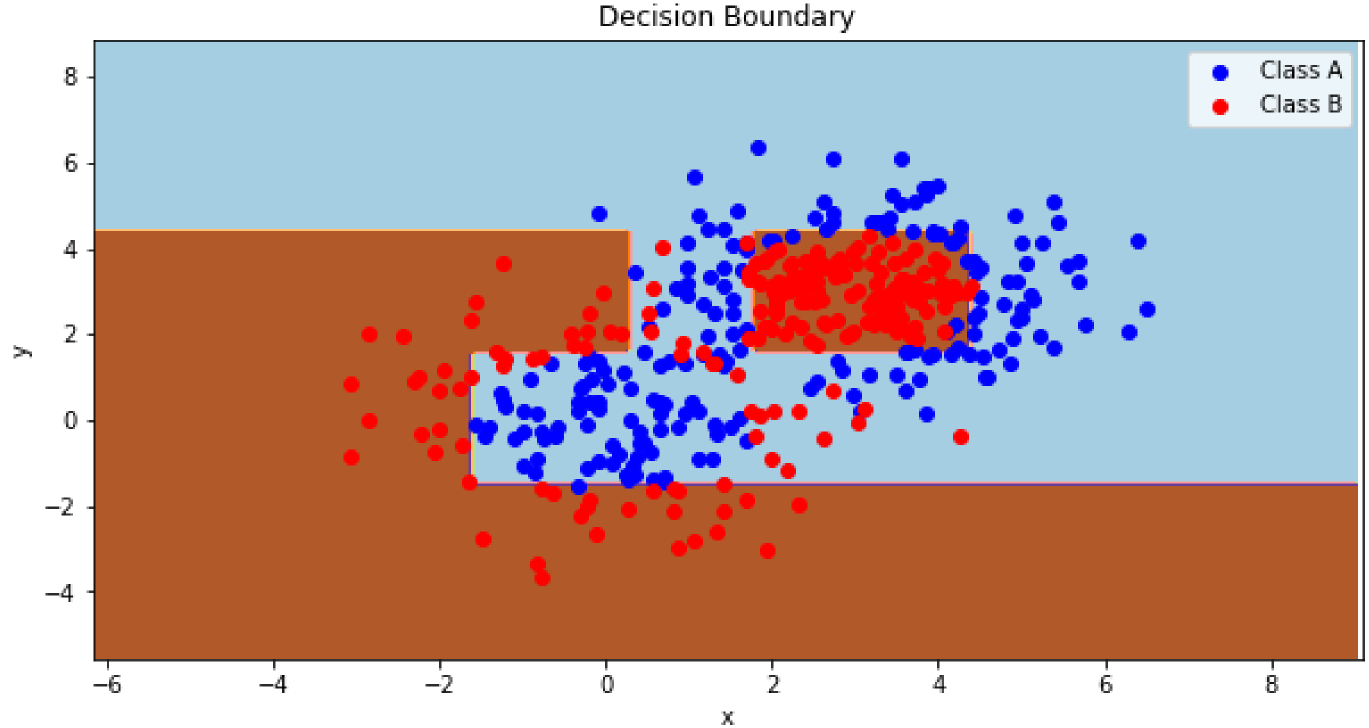


Ensemble Learning example

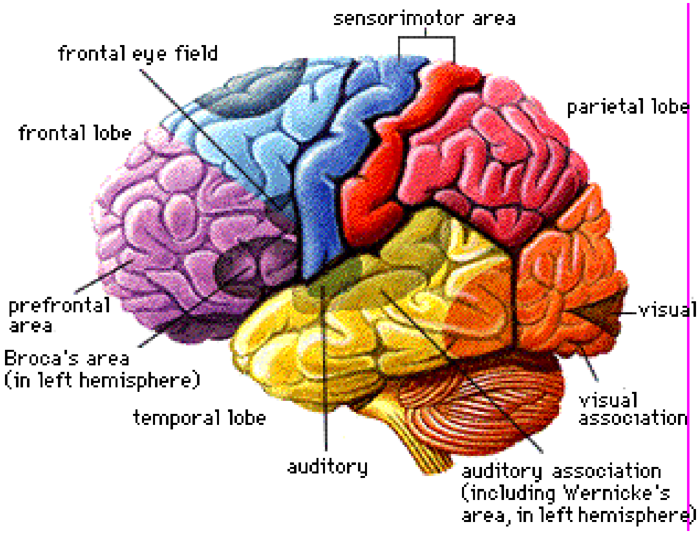
Any questions?

Example code to run various classification tasks, also with ensemble learning:

https://github.com/marcinwolter/MachineLearning2020/blob/main/EnsambleLearningExample.ipynb



Inspired by human brain



- **Human brain:**
  - 10^{14} neurons, frequency 100 Hz
  - Parallel processing of data (complex pattern recognition in 100 ms – 10 steps only!!!)
  - Learns on examples
  - Resistant for errors and damaged neurons
- **Neural Network:**
  - Just an algorithm, which might not reflect the way the brain is working.

History

1938 N. Rashevsky, neurodynamics – neural networks as dynamic systems, recurrent networks;

1943 W. McCulloch, W. Pitts, neural networks=logic systems;

1958 F. Rosenblatt, perceptron, network as a function;

1973 Chr. von der Malsburg, self-orgnization in the brain;

1982 Kohonen, Self-Organizing Maps

...

1986 **backpropagation of errors, many application!**

....

2010 Deep Neural Networks – great progress in AI!!!!!!

Artificial Neural Network

Neural Network – a mathematical model which is composed out of many functions (typically nonlinear)

Tasks:

- **Event classification** – background vs signal classification
- **Regression** – approximation of a real function

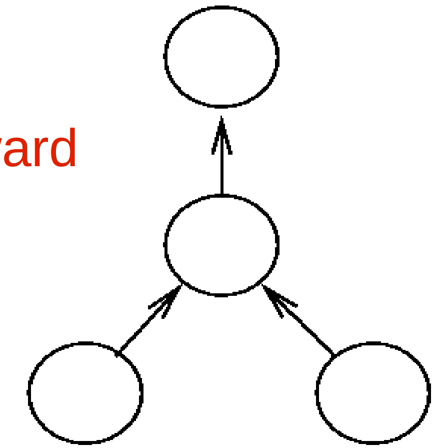
Two types of networks:

- **Feed forward** – information is sent from input layer to output without any loops
- **Recurrent** – recurrent loops.

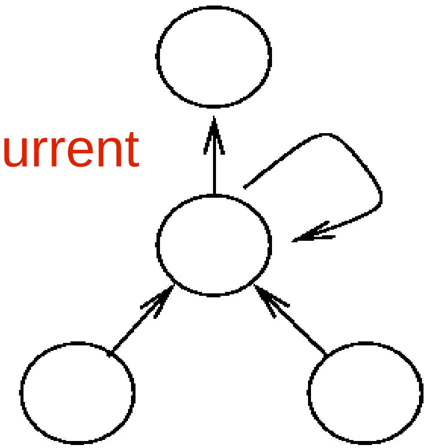
Learning:

- **supervised**
- **unsupervised**

Feed-forward



Recurrent

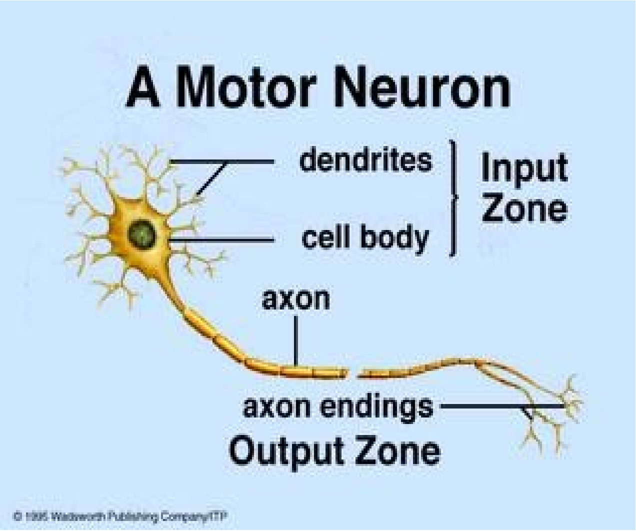


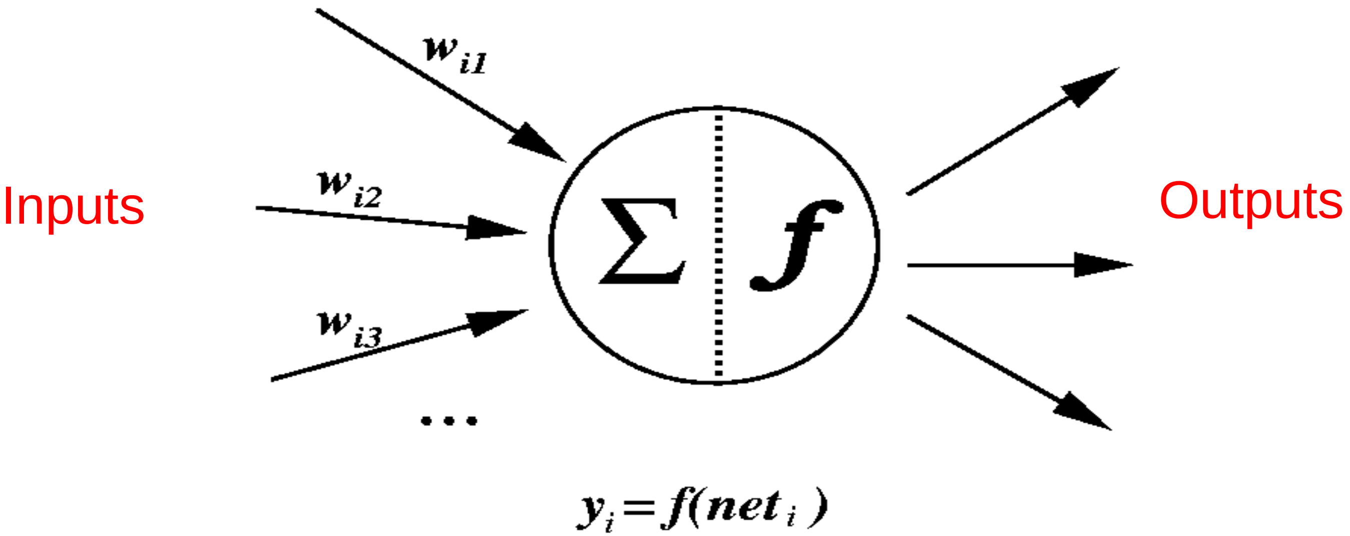
What are they used for?

- Expert systems
- Pattern recognition
- Predictions (meteorology, stock market...)

Neuron – the basic element

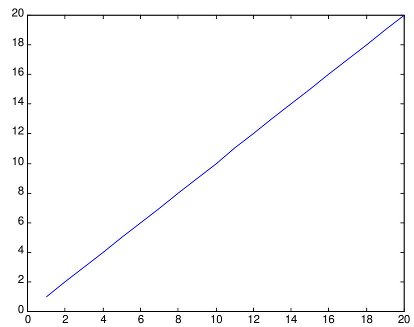
- Function of a weighted average of inputs $y_i = f(\sum_j w_{ij} y_j)$





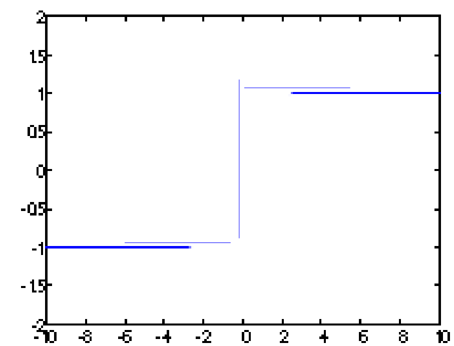
- Function *f* is called the **activation function**

Typical activation functions

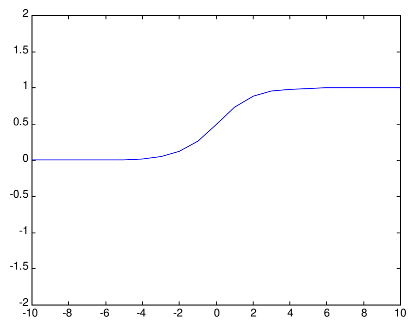


Linear (the whole network is called “linear”)

$$y = x$$

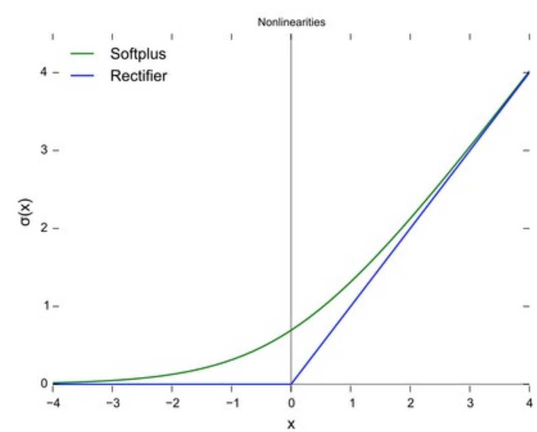


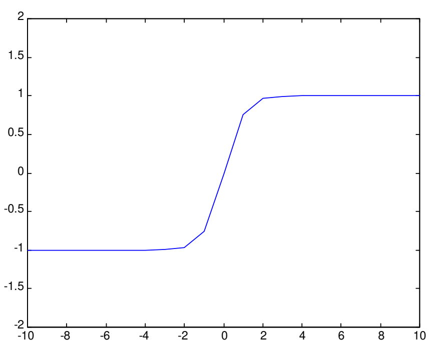
Step



[Logistic](#)

$$y = \frac{1}{1 + \exp(-x)}$$





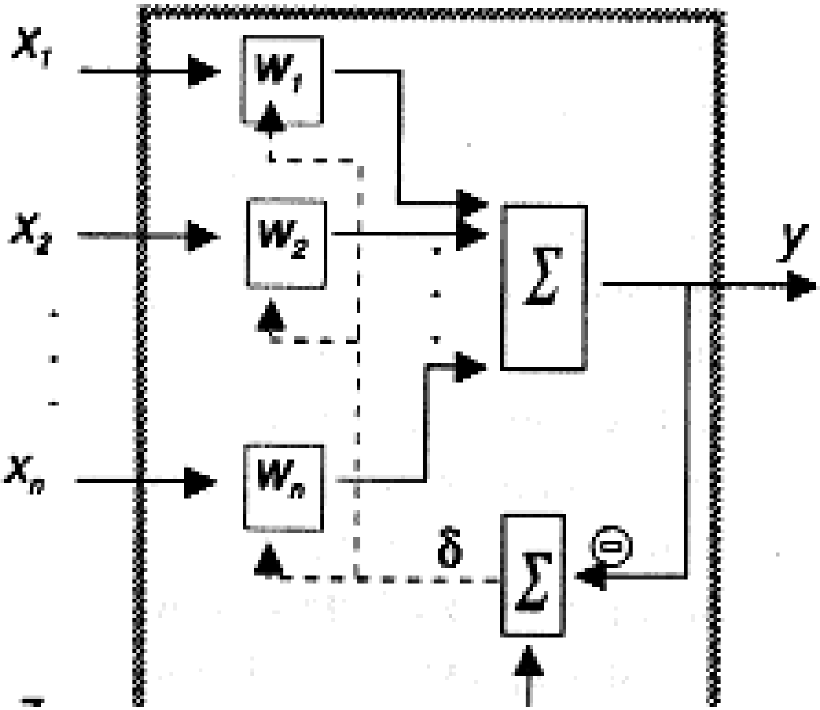
Hyperbolical tangent

$$y = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}$$

Rectified Linear (Relu), popular in **Deep Neural Networks**

Training a single neuron

Neuron is trained on examples
Supervised learning – the proper answers are known



ADALINE

(Adaptive Linear Network)

- X_i – input data
- Y - output value
- Z - the true output value (supervised training!)

- TASK – minimize the loss function:

  Minimize: $\chi^2 = \sum (z^{(j)} - y^{(j)})^2$

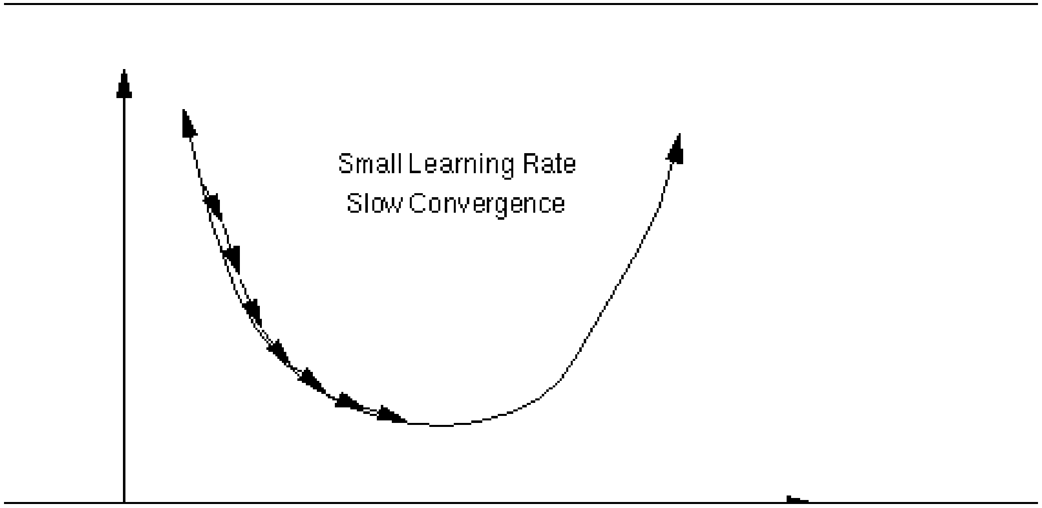
- New set of weights:

$$\delta = z - y$$

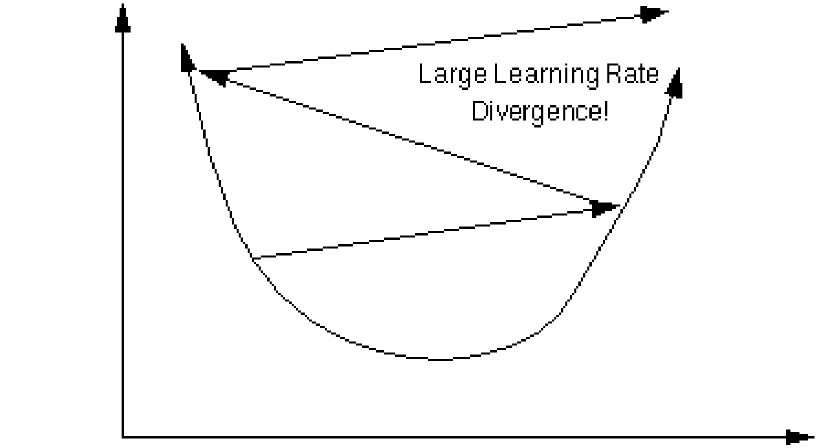
- η - learning speed

$$W' = W + \eta \cdot \delta \cdot X$$

Speed of learning



- To small

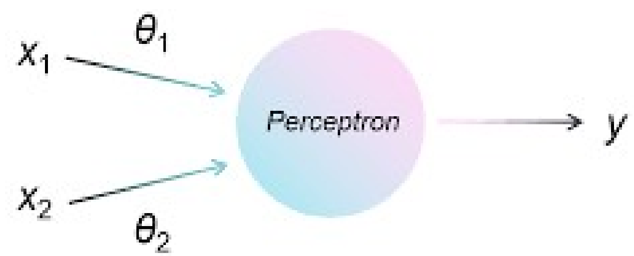


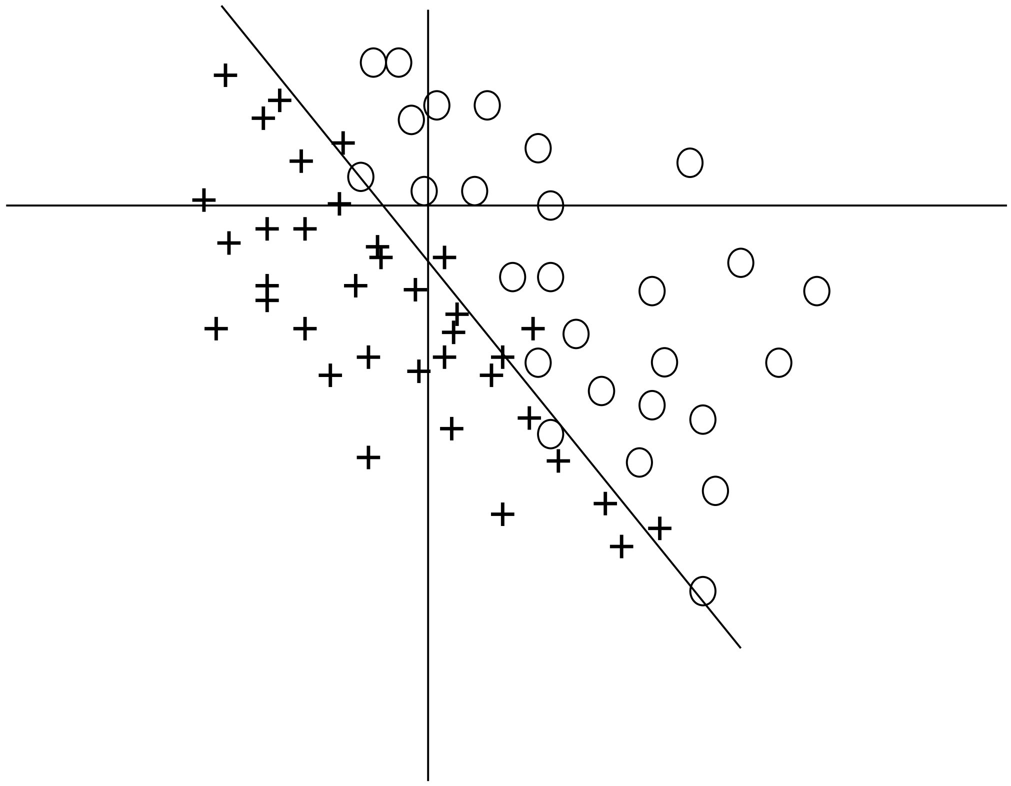
- To big

What can a single neuron (perceptron) do?

- Perceptron (with a step activation function) can divide a plane by a straight line (in general: division by a hyperplane in the n-dimensional space).

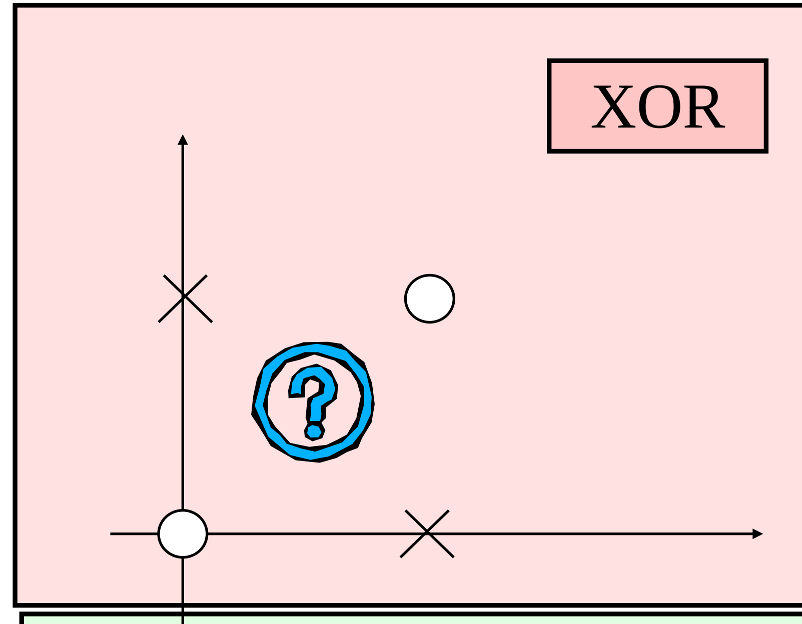
- Points above the line are classified as “1” (signal) and below as “0” - background.

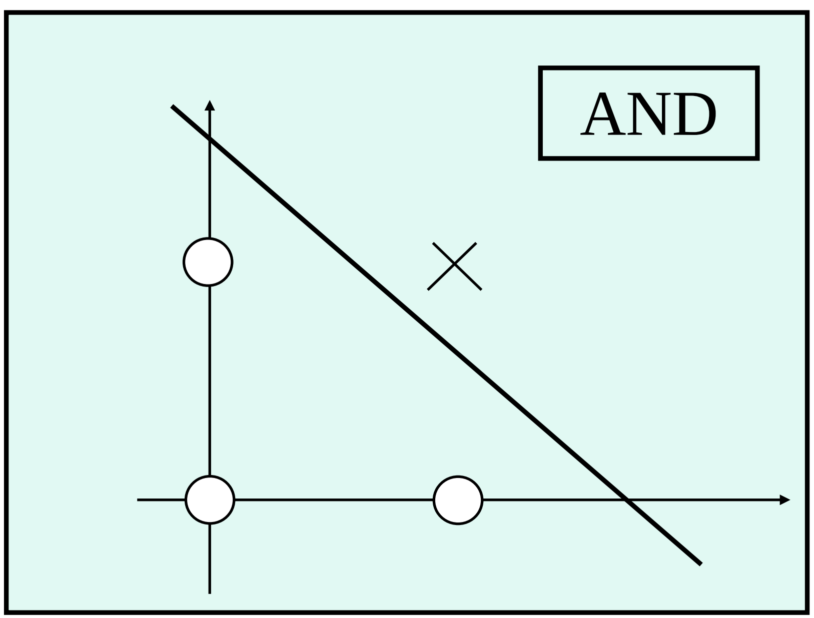


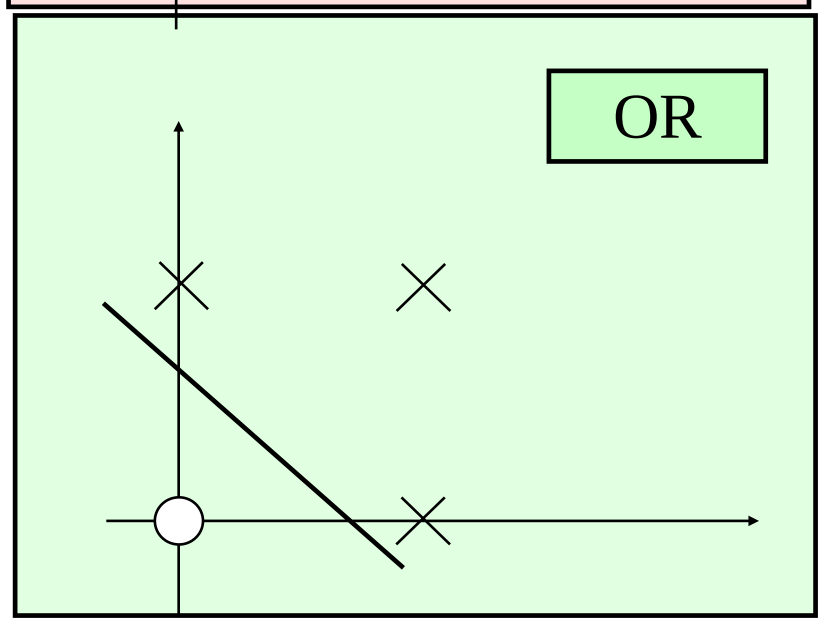


What the perceptron can not do?

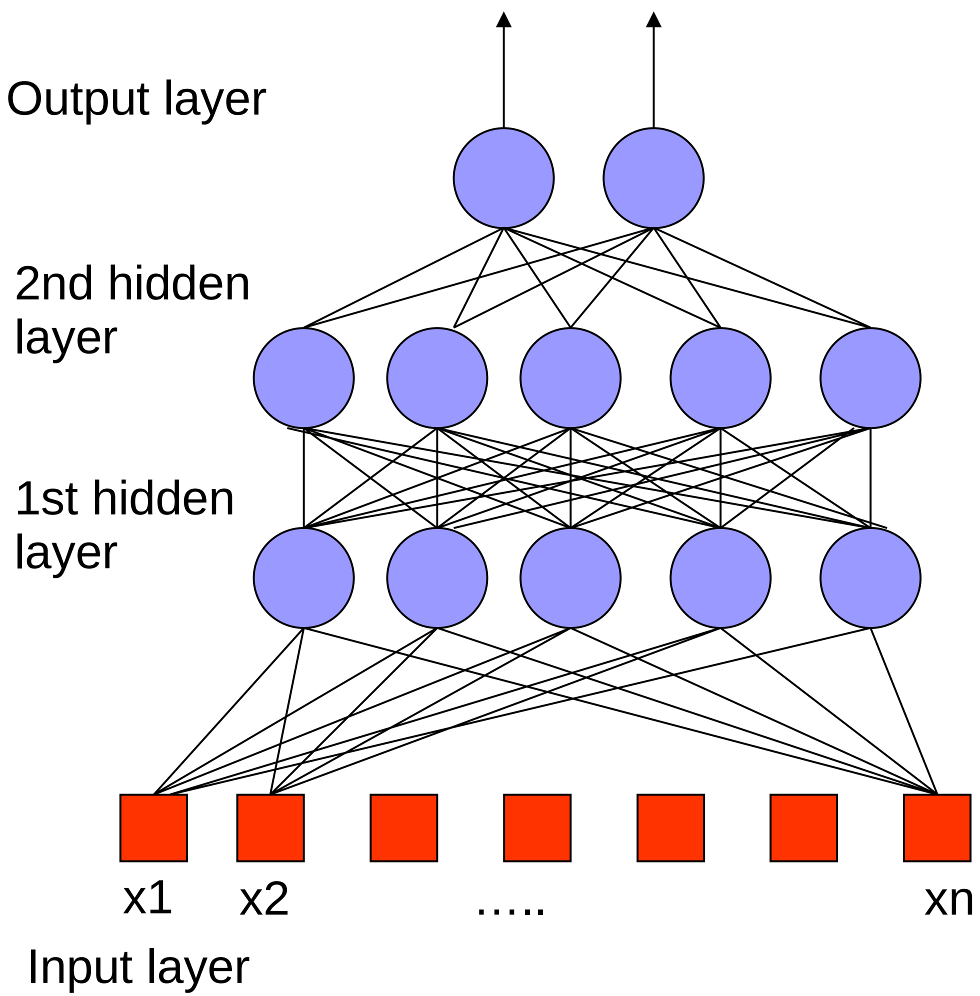
- A single perceptron can't separate the linearly not separable classes, for example the XOR function.
- The discovery of these limitations (1969) stopped the development of Neural Networks for some time.







So, maybe a network of perceptrons?

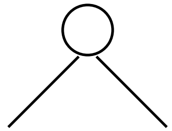
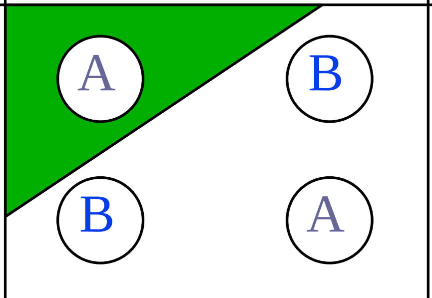
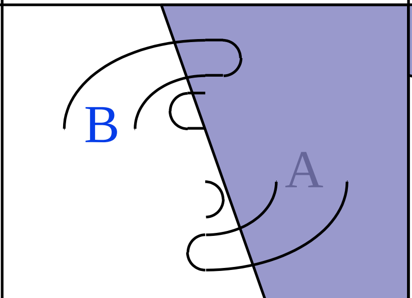
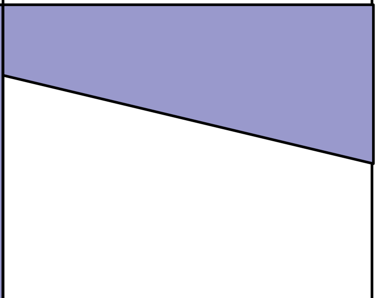
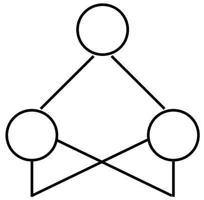
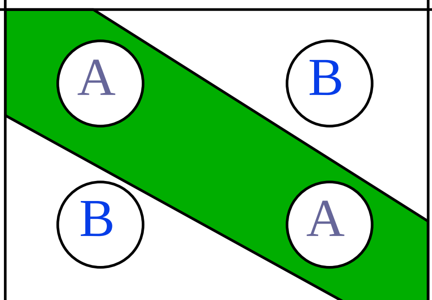
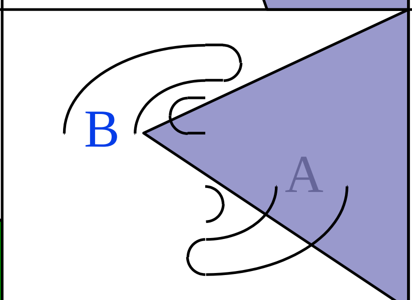
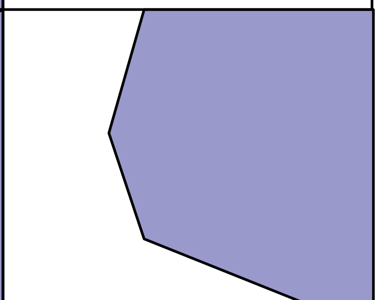
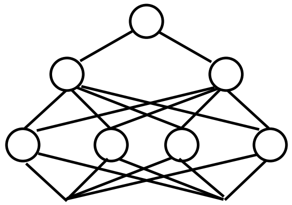
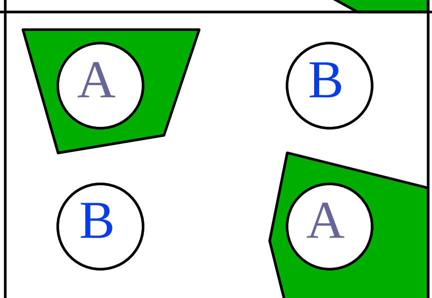
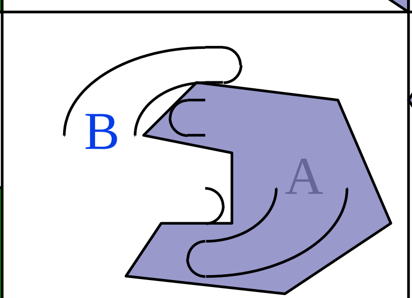
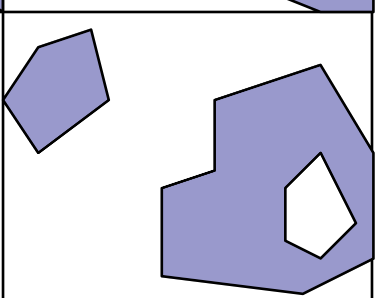


- Feed forward network – the information propagates from input to output.
- The net is the sum of many activation functions (in general non-linear)
- A network complicated enough can reproduce any function.

What a network can do?

applet general1

(step activation function)

Structure	Types of Decision Regions	Exclusive-OR Problem	Classes with Meshed regions	Most General Region Shapes
Single-Layer 	*Half Plane Bounded By Hyperplane*			
Two-Layer 	*Convex Open Or Closed Regions*			
Three-Layer 	Arbitrary (Complexity Limited by No. of Nodes)			

How to train a multilayer network?

- Minimize the loss function by choosing a set of weights ω:

$$R(\omega) = \frac{1}{N} \sum_i [t_i - n(x_i, \omega)]^2$$

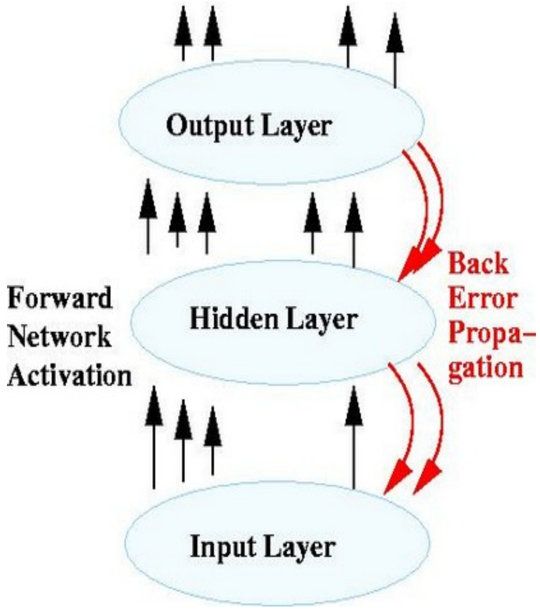
- Problem – how to correct the weights in the deeper layer, while comparing only outputs on the last layer?
- This problem stopped the development of Neural Networks for 30 years, until 80-ties.
- Solution - the **backpropagation** method. Errors $\delta = t - n(x, \omega)$ are propagated backward through the net using the actual weights.

Typical training procedure

- Two data samples: for training and for tests.
- $\chi^2=\Sigma(z-y)^2$ is calculated for both samples and compared to avoid **overtraining.**
- ***Backpropagation:*** difference between the expected and calculated value on output *y-f(x,w)* is propagated backward through the net using the actual weights:

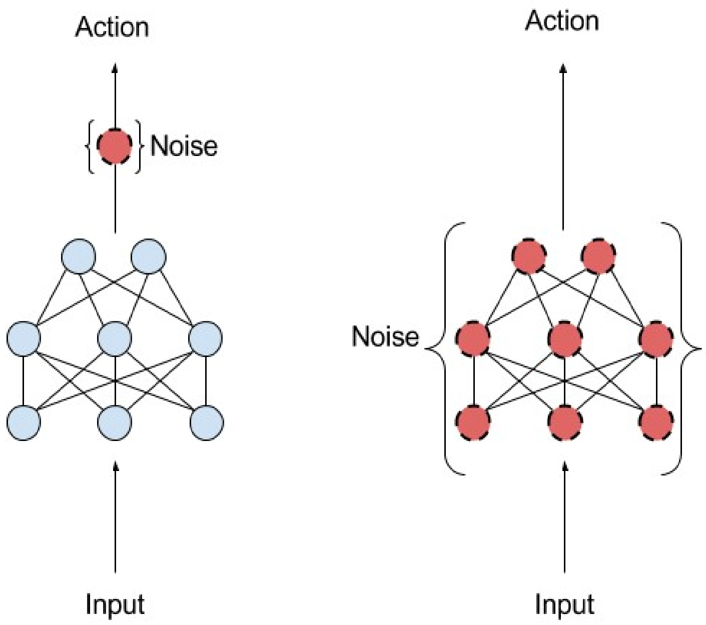
$$dw_{ij} = \rho x_i(t_j - y_j),$$

where ρ is a speed of learning, t_j the true value on the output *j*, y_j calculated by the net, and x_i is an actual value on the neuron *i* in the layer preceding the output layer.



Finding the minimum

- We never know, whether the global or a local minimum of the loss function $\chi^2=\Sigma(z-y)^2$ was found.
- Mechanisms preventing stopping in a local minimum:
  - Using random initial weights, repetition of training,
  - Addition of noise, so the minimizing algorithm can jump out of a local minimum (jittering).
- Regularization - both MLPRegressor and MLPClassifier use parameter alpha for regularization (L2 regularization) term which helps in avoiding overfitting by penalizing weights with large magnitudes.



Action-Space-Noise (left) and Parameter-Space-Noise (right)

https://matthiasplappert.com/publications/2017_Plappert_Master-thesis.pdf

Minimization algorithms in sklearn

- Stochastic Gradient Descent (SGD) - updates parameters using the gradient of the loss function with respect to a parameter that needs adaptation, i.e.
$$w \leftarrow w - \eta(\alpha \frac{\partial R(w)}{\partial w} + \frac{\partial Loss}{\partial w})$$
where η is the learning rate which controls the step-size in the parameter space search.
- Adam - similar to SGD, but it can automatically adjust the amount to update parameters based on adaptive estimates of lower-order moments.
- L-BFGS - approximates the Hessian matrix which represents the second-order partial derivative of a function. Further it approximates the inverse of the Hessian matrix to perform parameter updates.

Neural network examples

- Simple comparison of many classifiers

  https://github.com/marcinwolter/MachineLearning2020/blob/master/plot_mnist_filters.ipynb

- Neural network for hand-written digits classification

  https://github.com/marcinwolter/MachineLearning2020/blob/master/plot_digits_classif_mlp.ipynb

- Visualization of MLP weights

  https://github.com/marcinwolter/MachineLearning2020/blob/master/plot_mnist_filters.ipynb

https://playground.tensorflow.org
Nice NN demonstrator