The universal structure of mountain ranges in the world

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Contributors:

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Praca Magisterska

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Kierunek studiów: **Informatyka Stosowana**

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Oprogramowanie do automatycznej detekcji przebiegu grani górskich na podstawie danych DEM

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Kraków, Sierpień 2016

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Universal features of mountain ridge networks on Earth

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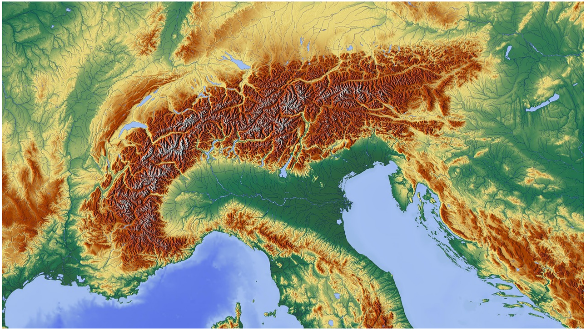
Analyzed data

- Shuttle Radar Topography Mission (2000)
  - spatial resolution: 3" (meridional: $\sim$ 90 m, zonal: $\sim$ 90 $*$ $\cos\phi$ m)
  - vertical resolution: 5-9 m, depending on a continent
- Selected mountain ranges:
  - Alps
  - Baetic Mountains
  - Pyrenees
  - Scandinavian Mountains
  - Atlas Mountains
  - Appalachian Mountains
  - Andes
  - Himalayas
  - Southern Alps

Range	Area (km^2)	Height (m)	Origin	No. data pts.
Alps	207,000	4,807	Orogenic	36,895,401
Baetic Mountains	100,000	3,478	Orogenic	11,255,768
Pyrenees	19,000	3,404	Orogenic	10,332,333
Scandinavian Mountains	243,000	2,469	Non-orogenic	138,497,654
Himalayas	594,400	8,848	Orogenic	40,295,462†
Southern Alps	36,700	3,724	Orogenic	23,619,193
Appalachian Mountains	531,000	2,037	Orogenic	208,854,395
Atlas Mountains	775,340	4,167	Orogenic	240,495,193
Andes	3,371,000	6,961	Orogenic	320,431,192†



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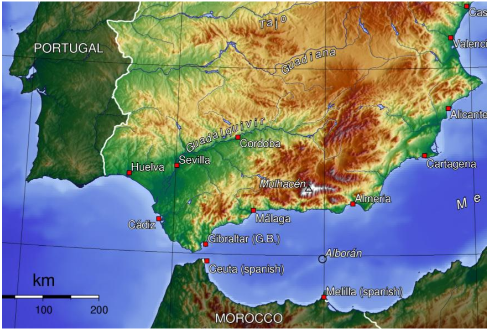

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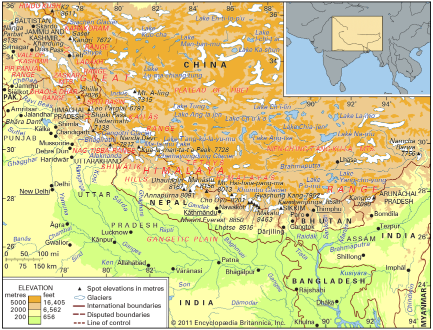

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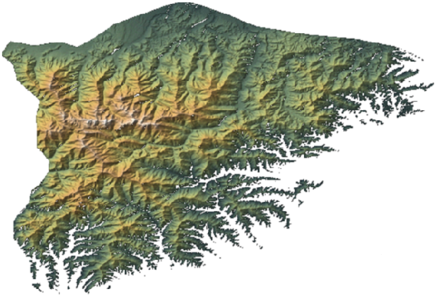
Ridge axis detection

Ridge axis detection:

- Profile recognition and polygon breaking algorithm
- MST-based optimization



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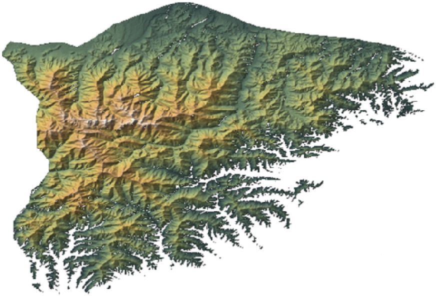
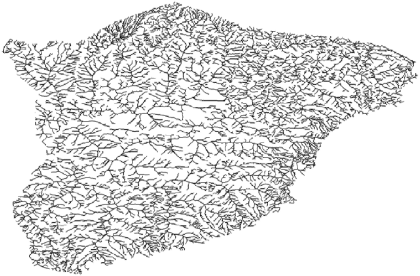


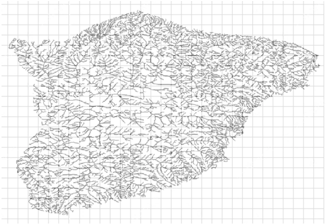


Fractal dimension of the ridge maps

Box-counting fractal dimension:

- cover a ridge map with boxes of size $;
- count the number n of boxes that contain a piece of a ridge line;
- repeatedly change the box size and calculate $n(l)$;
- plot $\ln n(l)$ vs. $\ln l$.

A ridge map is fractal if the following relation holds: $n(l) \sim l^{-D}$, where D is the box-counting dimension.



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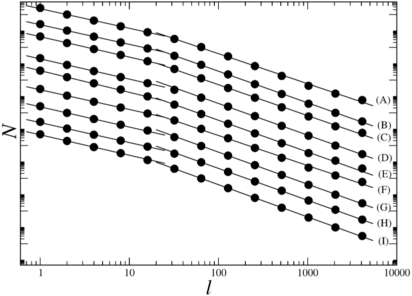


- two scaling regimes: $l < 20$ and $l > 20$;
- the cross-over scale of $l \approx 20$ equals to $\approx$ 2 km;
- a fractal ridge-line structure with $1.2 \leq D \leq 1.3$ below 2 km;
- a space-filling structure with $1.8 \leq D \leq 2.0$ above 2 km;
- similar results were reported for the drainage networks.

Range	D_{small}	D_{large}	d_f
Alps	1.20	1.84	1.65 ± 0.11
Baetic Mountains	1.20	1.93	1.70 ± 0.13
Pyrenees	1.25	1.89	1.61 ± 0.14
Scandinavian Mountains	1.24	1.94	1.62 ± 0.12
Himalayas	1.32	1.94	1.68 ± 0.12
Southern Alps	1.21	1.84	1.68 ± 0.11
Appalachian Mountains	1.24	1.96	1.65 ± 0.10
Atlas Mountains	1.25	1.97	1.65 ± 0.10
Andes	1.22	1.95	1.66 ± 0.11

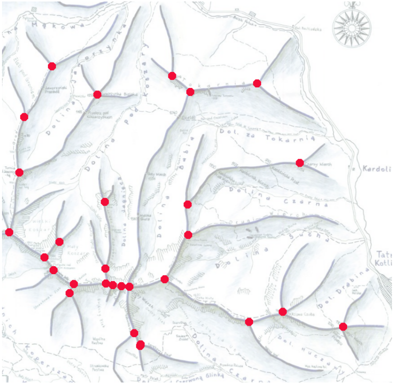
Topographic networks

Topographic networks

Topographic networks

Definition:

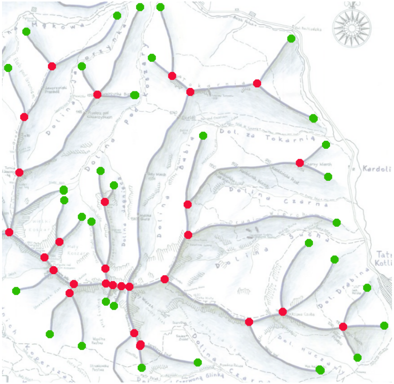
- Nodes T_1: ridge bifurcation points



Topographic networks

Definition:

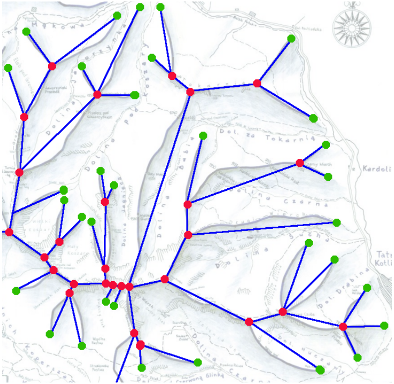
- Nodes T_1: ridge bifurcation points
- Nodes T_2: ridge ends



Topographic networks

Definition:

- Nodes T_1: ridge bifurcation points
- Nodes T_2: ridge ends
- Edges: connect two neighbour nodes along a ridge



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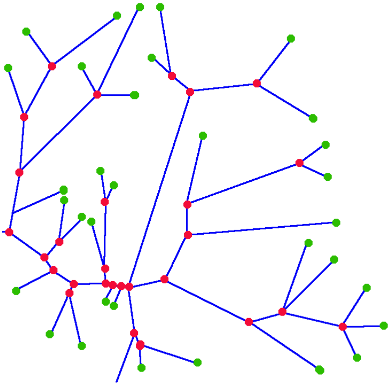

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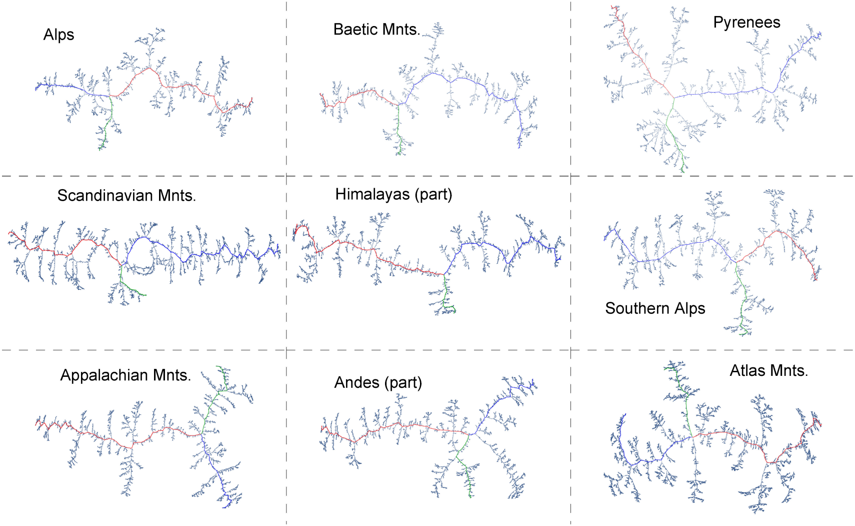


Topographic networks

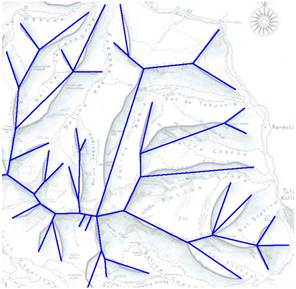
Topological properties:

- Acyclic and connected
- Number of nodes ($T_1 + T_2$): $95,356 \leq N_{\mathrm{T}} \leq 2,550,922$
- Maximum node degree: $6 \leq k_{\mathrm{T}} \leq 8$
- Network diameter: $1,148 \leq D_{\mathrm{T}} \leq 6,574$
- Average path length: $461 \leq L_{\mathrm{T}} \leq 2,434 \quad (11.5 \leq \ln L_{\mathrm{T}} \leq 13.8)$
- Decentralized (no hubs)
- Without the small-world property

Ridge networks

Definition:

- Nodes: ridges
- Edges: connect two ridges with a common junction point



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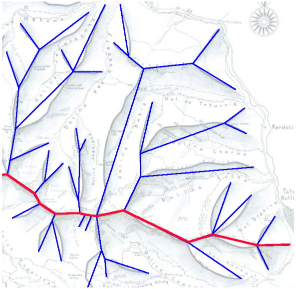

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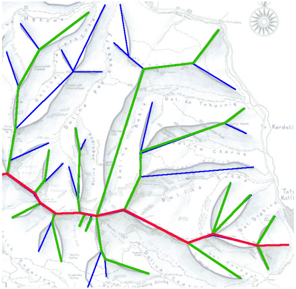

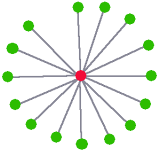

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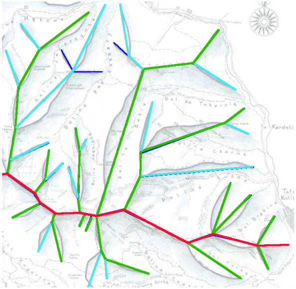

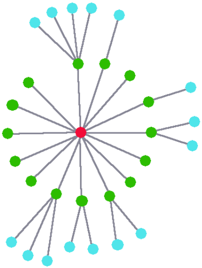

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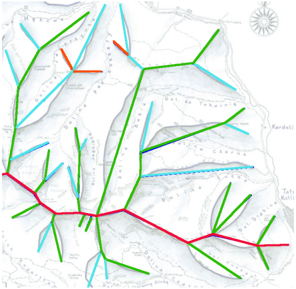

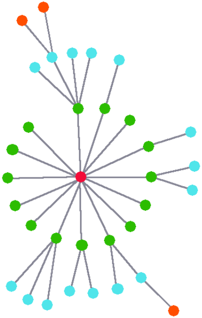

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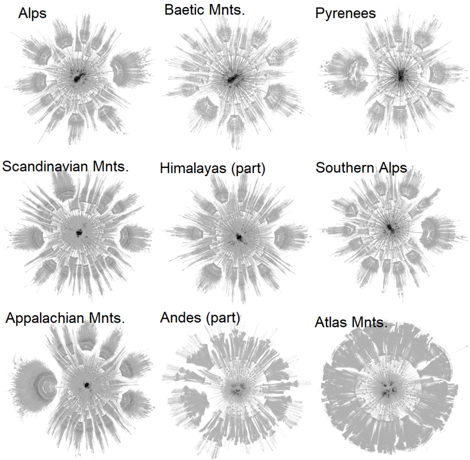

Ridge networks

Topological properties:

- Acyclic and connected
- Number of nodes: $50,027 \leq N_{\mathrm{R}} \leq 1,686,481$
- Maximum node degree: $852 \leq k_{\mathrm{R}} \leq 5,330$
- Network diameter: $21 \leq D_{\mathrm{R}} \leq 26$
- Average path length: $8.7 \leq L_{\mathrm{R}} \leq 9.6$
  $(10.8 \leq \ln L_{\mathrm{R}} \leq 13.2)$
- Highly centralized
- Small-world networks

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- Scale-free node degree distributions $P(X > k_{\mathrm{R}}) \sim k_{\mathrm{R}}^{-\beta}$
  with $1.6 \leq \beta \leq 1.7$

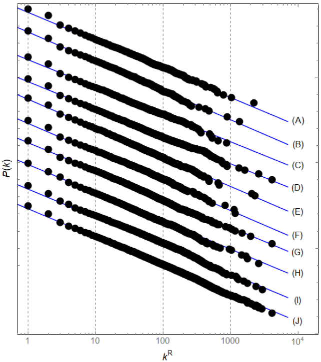


Figure: (A) Alps, (B) Baetic Mnts., (C) Pyrenees, (D) Scandinavian Mnts., (E) Himalayas, (F) Southerns Alps, (G) Appalachians, (H) Atlas, (I) Andes, (J) all mountains.

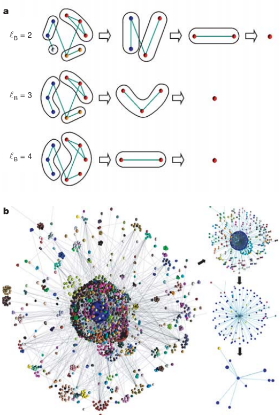
Self-similarity

Box-covering algorithm for identifying network self-similarity:

[C. Song *et al.*, Nature 433, 392-395 (2005)]

- define a length parameter l;
- select a random seed node;
- split the network into clusters such that a minimum path between any two nodes is $d \leq l - 1$;
- calculate the number of the clusters: N_c;
- choose different seed nodes and calculate $\langle N_c \rangle$;
- replace each cluster with a single node (renormalization) and repeat the previous steps;
- calculate $\langle N_c(l) \rangle$;

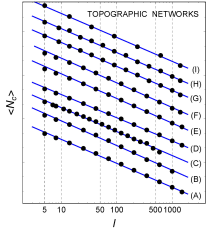
A network is self-similar if $\langle N_c(l) \rangle / N \sim l^{-d_f}$.



Self-similarity

Topographic networks:

- self-similar with universal $d_f \approx 1.7$ for $l > 5$
- no other model approximates better the data



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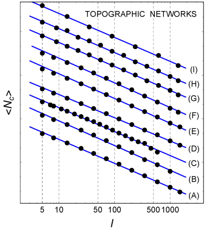
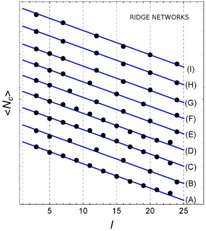


Ridge networks:

- exponential relation: $\langle N_c(l) \rangle \sim e^{-l/2}$ for $l \leq 25$



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Multiscale self-similarity

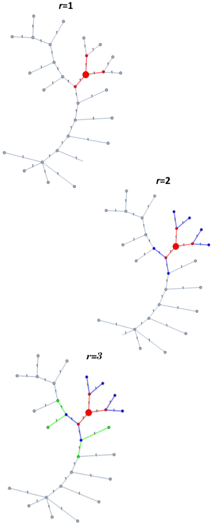
Sandbox algorithm for identifying network multifractality:

[J.-L. Liu *et al.*, Chaos 25, 023103 (2015)]

- define a radius parameter r;
- calculate a distance matrix describing the path lengths for all the node pairs;
- select a random seed node;
- count the number of nodes that fall inside a circle of radius r centered at the seed node;
- repeat the node counting for different values of r and calculate $n(r)$;
- choose different seed nodes and calculate the average $\langle n(r) \rangle$;
- calculate the generalized fractal dimensions D_q for some range of q values by using the formula:

$$D_q = \lim_{r \to \infty} \frac{\ln \langle [n(r)/N]^{q-1} \rangle}{(q-1)\ln(r/d)}.$$

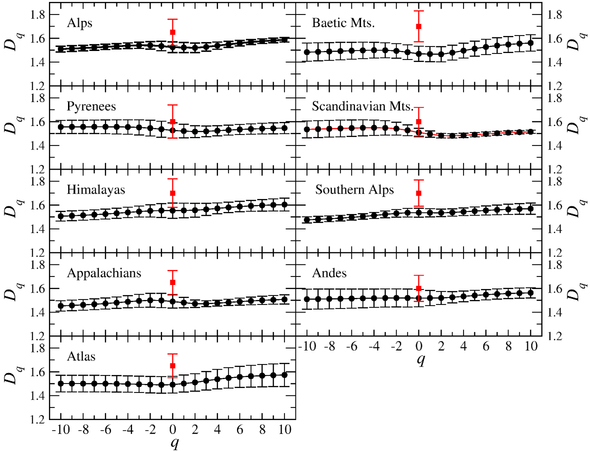
A network is multifractal if D_q depends on q.



Multiscale self-similarity

Topographic networks:

- $1.45 \leq D_q \leq 1.60$
- $1.50 \leq D_0 \leq 1.55$
- $D_q \approx$ const



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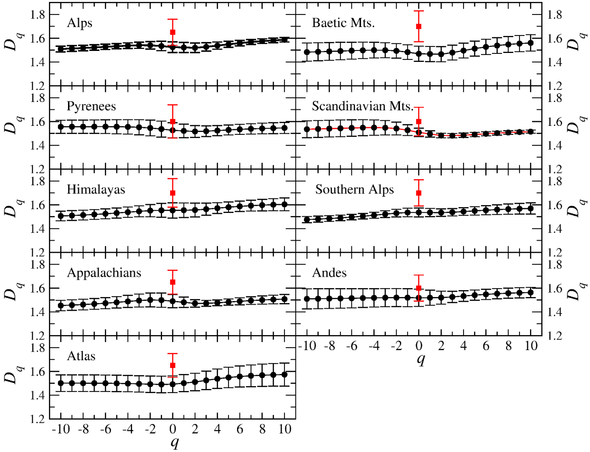
- $1.50 \leq D_0 \leq 1.55$
- $D_q \approx \mathrm{const}$
- monofractal structure



Universality

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[B. Klinkenberg, K.C. Clarke, in: Automated Pattern Analysis in Petroleum Exploration, pp. 201-212 (Springer, 1992)].

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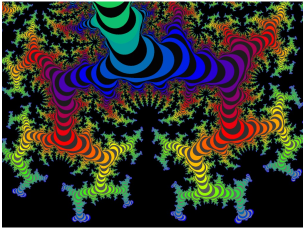
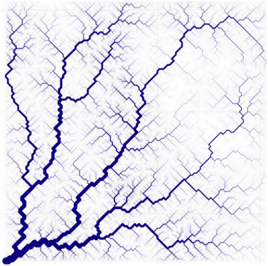
- Optically, the ridge structure of a typical mountain range looks self-similar and resembles a mathematical fractal.

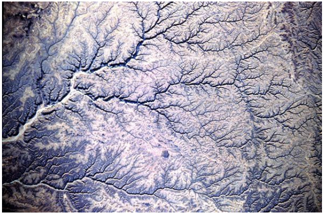


Universality

Why are the ridge-valley systems fractal?

- The drainage systems are typically dendritic and fractal.
- The dendritic, self-similar drainage systems form minimum spanning trees minimizing the energy dissipation.
- The drainage channels can form optimal trees if the terrain structure is locally easily erodable.





Universality

Drainage system formation:

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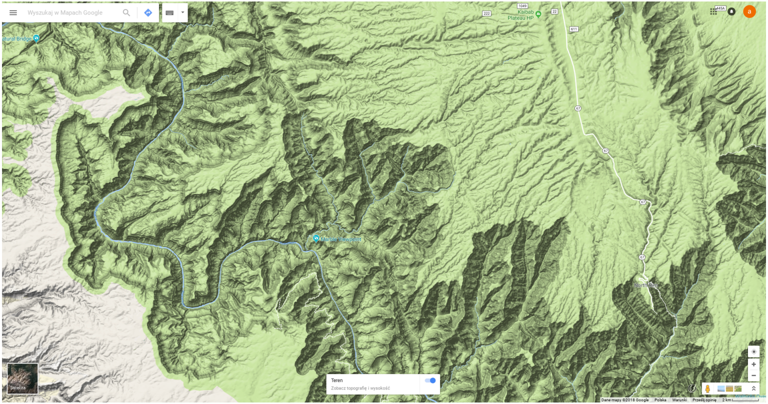
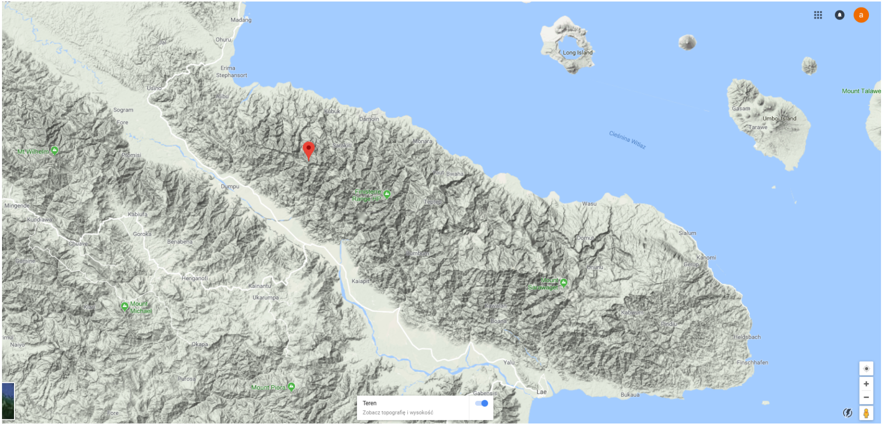


Universality

Drainage system formation:



Conclusions

Conclusions:

- The ridge maps are self-similar with $1.2 \leq D \leq 1.3$ on short spatial scales below 2 km and almost space-filling with $1.8 \leq D \leq 2.0$ on long spatial scales above 2 km.
- The topographic networks are self-similar with a common scaling exponent $d_f \approx 1.7$.
- The ridge networks are small-world and scale-free with a common scaling exponent $1.6 \leq \beta \leq 1.7$.
- No trace of multiscaling was identified on the binary network level; on the other hand, by allowing for the weighted edges, multifractality can be observed.
- These values are roughly invariant under changing the mountain range irrespective of the range's height, area, drainage patterns, and origin.

Future directions:

- Searching for a relation between the ridge structure and the scaling exponents: β and d_f.
- Studying weighted network representations of the mountain ridges for .
- Developing a network growth model that can reproduce the ridge and valley system formation.