Heavy flavour physics

Lecture 1

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Contents

Lecture 1

- What is flavour physics
- Brief historical reminder
- CKM mechanism
- SM flavour sector

What is flavour physics?

Fermions ("matter")	Bosons ("forces")
$\left\{\begin{array}{ccc} & \text{Quarks} & \\ uuu & ccc & ttt \\ ddd & sss & bbb \\ & & \\ & \text{Leptons} & \\ e & \mu & \tau \\ \nu_e & \nu_\mu & \nu_\tau \end{array}\right\} \times \left\{\begin{array}{c} \text{MATTER} \\ \text{ANTIMATTER} \end{array}\right\}$	$gggggggg$ γ W^+ W^- Z H

Flavour physics:

- transitions between different kinds of quarks
- its all about weak interactions...
- strong interactions as a "background"

Parameters of the Standard Model

- 3 gauge couplings: α_{EM}, α_{weak}, α_{strong}
- 2 Higgs parameters: v, m_H
- 6 quark masses:
- 3 quark mixing angles + 1 phase *(CKM matrix)*
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase) *(PMNS matrix)*

( ) = with Dirac neutrino masses

FLAVOUR PARAMETERS

Open questions in flavour physics

- Why are there so many different fermions?
- What is responsible for their organisation into generations / families?
- Why are there 3 generations / families each of quarks and leptons?
- Why are there flavour symmetries?
- What breaks the flavour symmetries?
- What causes matter–antimatter asymmetry?

Flavour physics issues

Families / generations

3 pairs of quarks (are we sure?)

3 pairs of leptons (are we sure?)

Hierarchies

$m(t) > m(c) > m(u)$ $m(b) > m(s) > m(d)$

$m(\tau) > m(\mu) > m(e)$ $m(\nu_\tau) > m(\nu_\mu) > m(\nu_e)$?

Mixings & couplings

hierarchy in quark mixings

what about lepton mixings?

Flavour physics issues

Mixings & couplings

universality

(no) flavour changing neutral currents (at tree level in the Standard Model)

Symmetry principles & their violation

P violation / C violation

CP violation / T violation

baryon asymmetry of the universe

lepton flavour violation

Unification

Why is heavy flavour physics interesting?

- Hope to learn something about the mysteries of the flavour structure of the Standard Model

- CP violation and its connection to the matter–antimatter asymmetry of the Universe

- Discovery potential far beyond the energy frontier via searches for rare or SM forbidden processes

What breaks the flavour symmetries?

- In the Standard Model, the vacuum expectation value of the Higgs field breaks the electroweak symmetry
- Fermion masses arise from the Yukawa couplings of the quarks and charged leptons to the Higgs field *(taking* $m_\nu = 0$*)*
- The CKM matrix arises from the difference between weak eigenstates and mass eigenstates
- Consequently, the only flavour-changing interactions are the charged current weak interactions

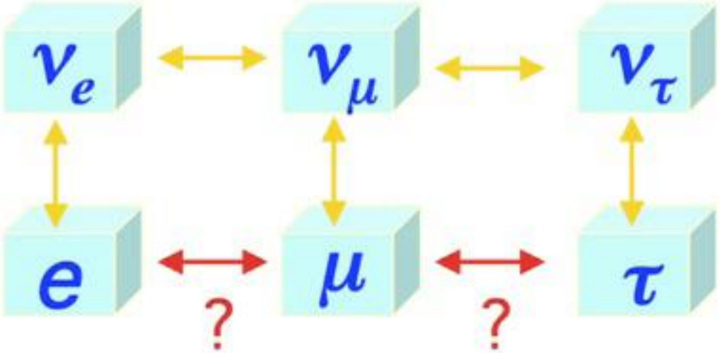
  → *no flavour-changing neutral currents (GIM mechanism)*

  → *not generically true in most extensions of the SM*

  → *flavour-changing processes provide sensitive tests*

Lepton flavour violation

- No right-handed neutrinos in the SM, implies they are massless
  - → neutrinos only left-handed (chirality)
  - → antineutrinos only right-handed (chirality)
- Neutrino oscillations show they have small but finite masses
  - → **where are the right-handed neutrinos?**
  - → charged lepton flavour violation
  - → physics beyond the Standard Model
- Why do we not observe the decay $\mu \rightarrow e\gamma$?
  - → exact (but accidental) lepton flavour conservation in the SM with $m_\nu=0$
  - → SM loop contributions suppressed by $(m_\nu / m_W)^4$
  - → LFV - a mechanism beyond the SM needed



What causes matter–antimatter asymmetry?

- The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks

$$V_{CKM} = U_u U_d^+$$

(U - diagonalisation of mass matrices)

- It is a 3x3 complex unitary matrix

  → described by 9 (real) parameters

  → 5 can be absorbed as phase differences between the quark fields

  → 3 can be expressed as (Euler) mixing angles

  → the fourth makes the CKM matrix complex (i.e. gives it a phase)

  - weak interaction couplings differ for quarks and antiquarks
  - CP violation

Symmetries

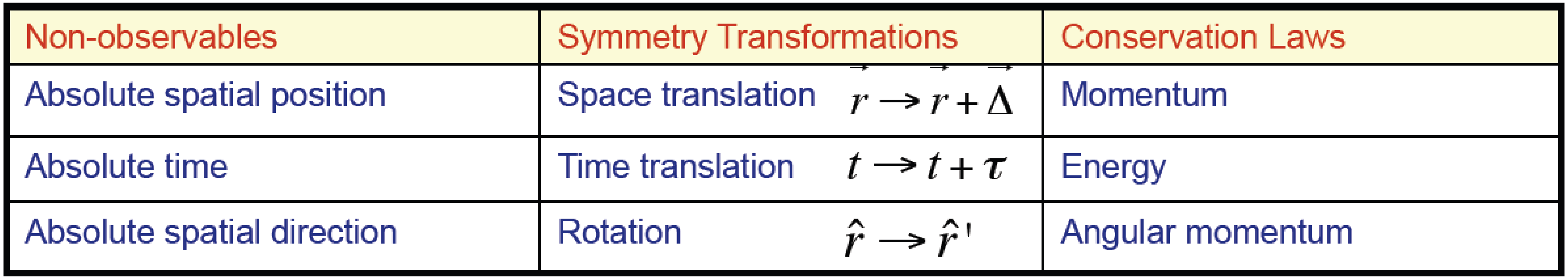
The (probably) most important concept in physics: concept of symmetry

> T. D. Lee:
>
> „The root to all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; the non-observables”

- if a quantity is fundamentally non-observable it is related to an exact symmetry
- if a quantity could in principle be observed by an improved measurement the symmetry is said to be broken

Noether theorem: symmetry ⇔ conservation law

Examples

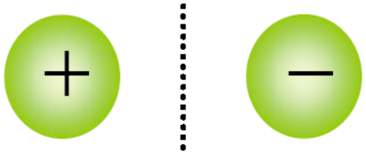


Non-observables	Symmetry Transformations	Conservation Laws
Absolute spatial position	Space translation $\vec{r} \rightarrow \vec{r} + \vec{\Delta}$	Momentum
Absolute time	Time translation $t \rightarrow t + \tau$	Energy
Absolute spatial direction	Rotation $\hat{r} \rightarrow \hat{r}'$	Angular momentum

Three discrete symmetries

Charge conjugation C

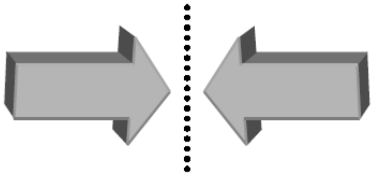
Particle ⇔ Anti-particle



$e^- \rightarrow e^+$

$\gamma \rightarrow \gamma$

Parity P

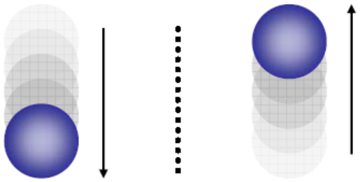


$\vec{r} \rightarrow -\vec{r}$

$\vec{p} \rightarrow -\vec{p}$

$\vec{L} \rightarrow \vec{L}$

Time inversion T



$t \rightarrow -t$

CPT theorem

- all interactions are invariant under combined *C*, *P* and *T*
- implies particle and anti-particle have equal masses and lifetimes
- one of the most important and generally valid theorems in *local* quantum field theory

Brief history

1932: Isospin

What is the difference between the proton (charge = +1) and the neutron (charge = 0)?

- masses almost identical
- coupling to the strong interaction identical

Heisenberg (1932) proposed (p,n) members of isospin doublet:

- p: $(I,I_z) = (1/2,+1/2)$ n: $(I,I_z) = (1/2,-1/2)$
- pions form an isospin triplet $\pi^{+,0,-}$: $(I,I_z) = (1, +1,0,-1)$
- *All particles in the same isospin representation are identical if EM is switched off*

Strong interaction same for proton & neutron

- Hamiltonian invariant under global SU(2) rotation
- pions thought to be Yukawa particles
- gauge bosons responsible for mediating strong force

Isospin is not an exact symmetry

- nonetheless, very useful concept
- successful because $m_u \sim m_d$ and $m_u,m_d < \Lambda_{QCD}$

1947: Strangeness

New particle observed

- produced in strong interaction
- long lifetime O(10^{-10} s) – “strange”
- decays only weakly

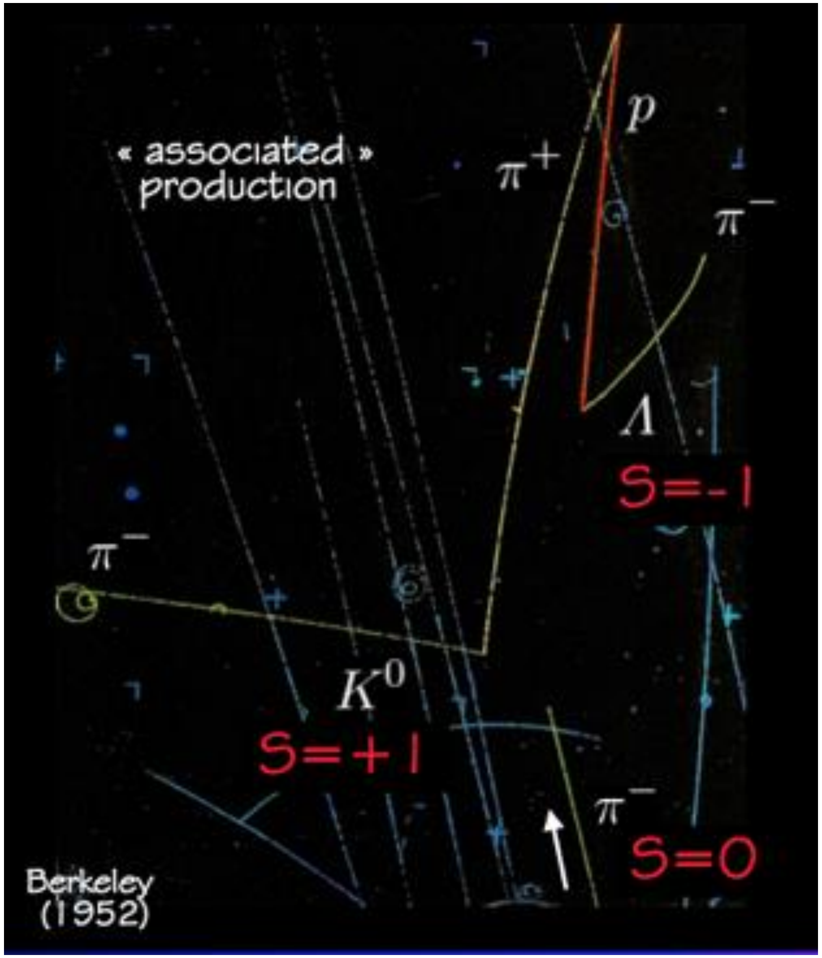
Observation in 1947 by G. D. Rochester and C. C. Butler

- neutral particle (no track) → two charged pions
- charged particle (track) → charged pion + X

M. Gell-Mann & K. Nishijima (1953) introduce new quantum number

- **strangeness S**
- S conserved in strong interactions
- S not conserved in weak interactions

$$p + \pi^- \rightarrow \Lambda + K^0$$



Quark model (1960s)

Originally introduced as a model to explain the particle „zoo"

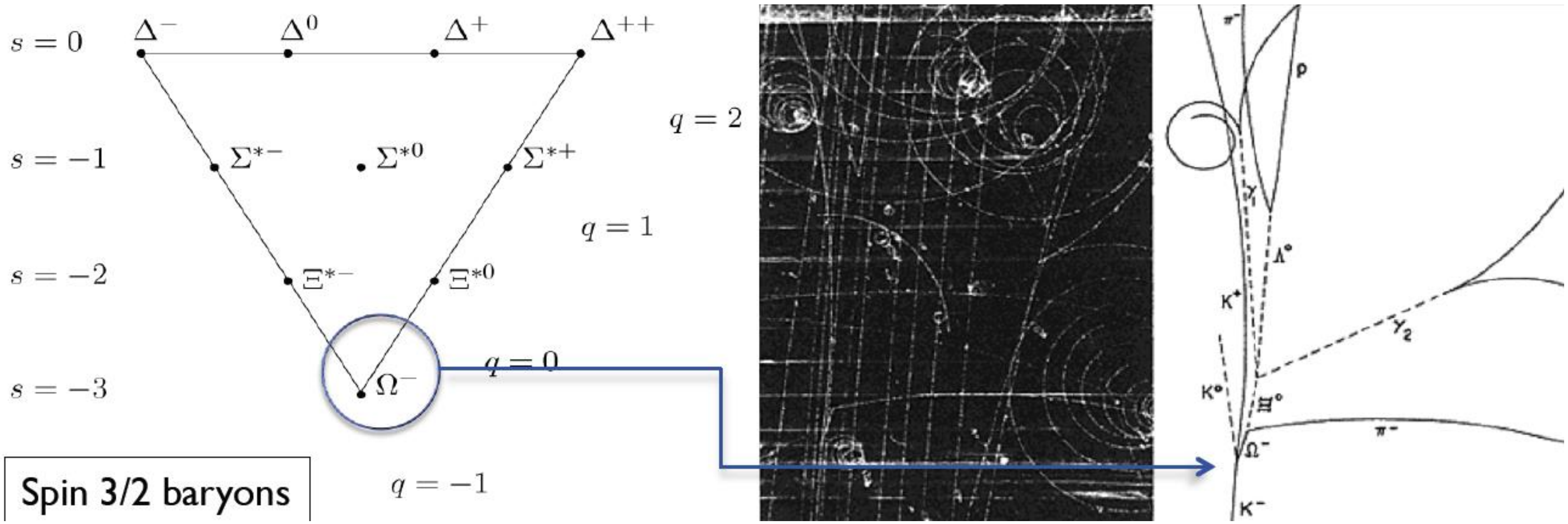
Gell-Mann/Nishijima law: $Q = e\,(I_z + Y/2)$

hipercharge Y = B + S (baryon number + strangeness)

Gell-Mann/Nishijima formula developed into **„eightfold way"** classification:

→ all known mesons and baryons could fit in **SU(3) representations**

Discovery of Ω^-(sss) [1964 at Brookhaven] in particular validated theory

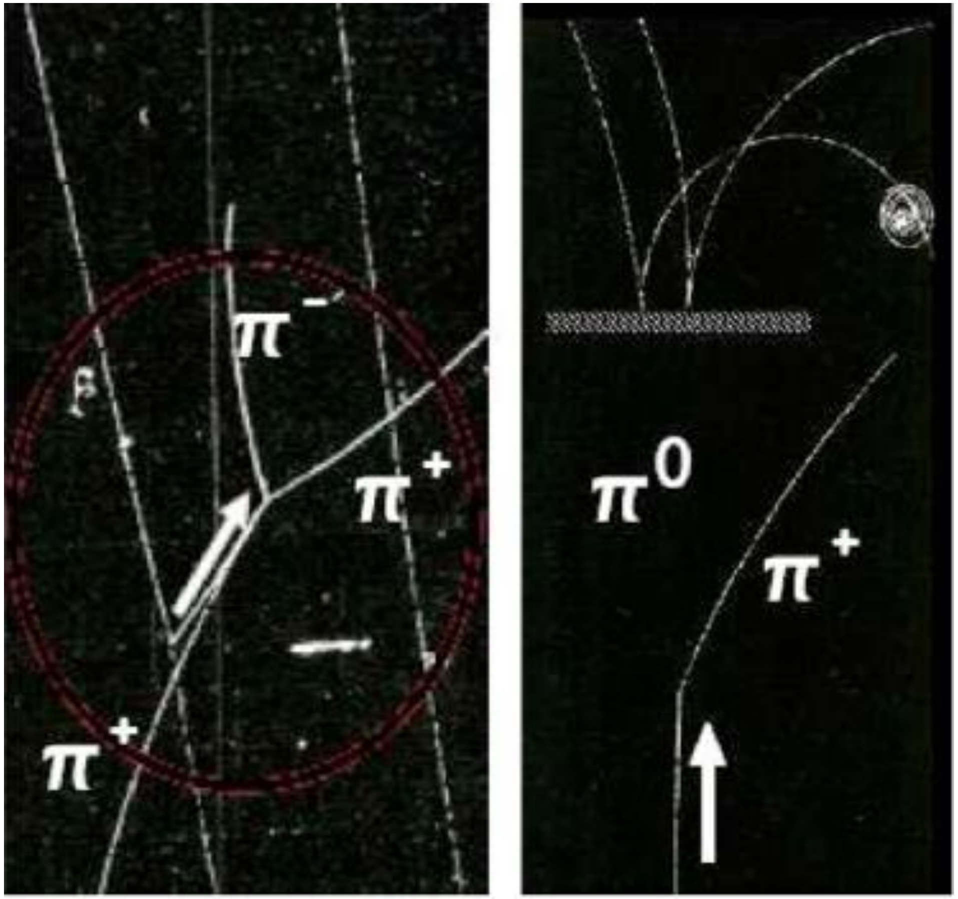


1950-56: The ϴ-τ puzzle

Observation of two strange mesons with:

- same mass
- same production rate
- same lifetime
- particle decaying to $\pi^+\pi^0$ was originally called the θ
- „another" particle (called τ) decaying to $\pi^+\pi^-\pi^+$ was also discovered

But: decay into final states with different parities



1956: Lee and Yang:

"Is parity violated in the weak interaction?"

$$\theta \to \pi^+\pi^0; \qquad P(\pi^+\pi^0) = +1$$

$$\tau \to \pi^+\pi^+\pi^-; P(\pi^+\pi^+\pi^-) = -1$$

1957: Parity violation

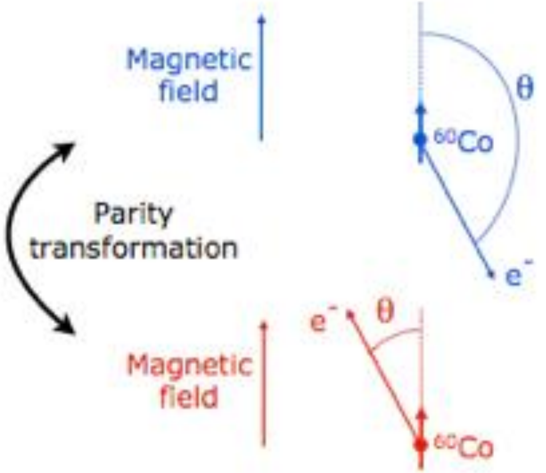
C. S. Wu

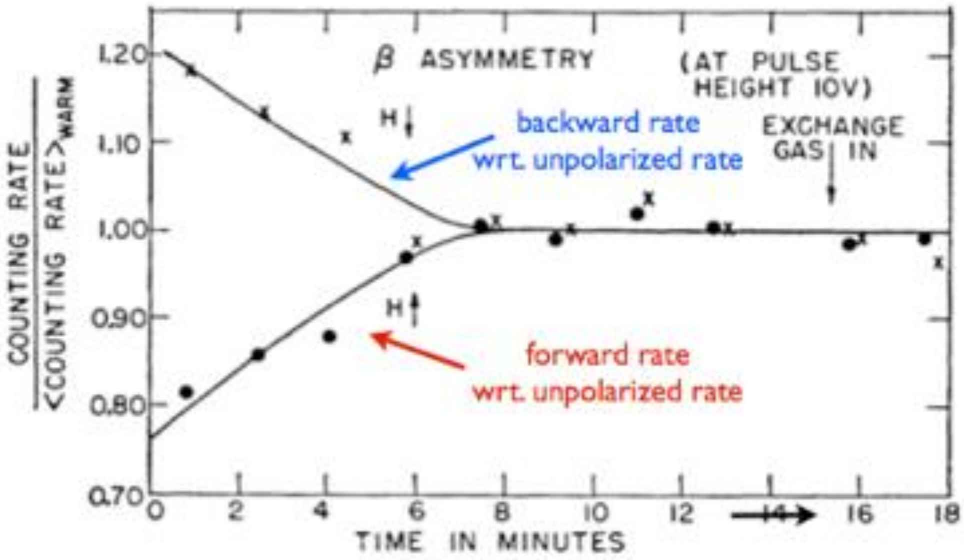
β rays(e-) from the ^{60}Co atoms emitted asymmetrically under parity inversion (by magnetic field).

Measure angular distribution of electrons from β decays of polarized ^{60}Co.

Most of the electrons are measured in the opposite direction to the spin of the ^{60}Co

→ **parity is maximally violated!**





^{60}Co polarization decreases as a function of time as the temperature increases

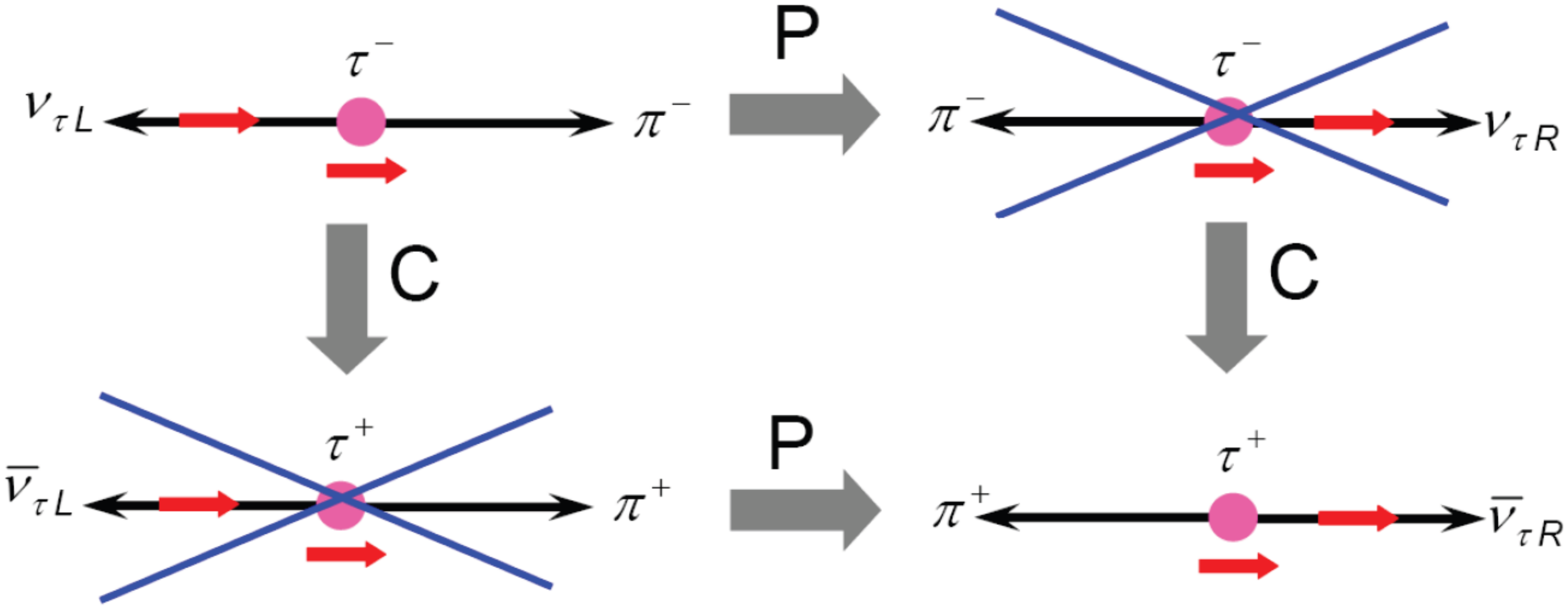
P, C and CP symmetries in weak interactions

Now understood that P is maximally violated in β decays

- no right-handed neutrinos

However, C is also maximally violated

- no left-handed antineutrinos



But CP was thought to be a good symmetry, until 1964

1964: CP violation - Cronin-Fitch experiment

Both $K^0 \rightarrow \pi\pi$ and anti-$K^0 \rightarrow \pi\pi$ occur

- K^0 may turn into its antiparticle, so are not mass eigenstates

The mass eigenstates are:

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

CP operator gives:

$$\mathbf{CP}|K^0\rangle = |\bar{K}^0\rangle, \mathbf{CP}|K_S\rangle = +|\bar{K}_S\rangle, \mathbf{CP}|K_L\rangle = -|\bar{K}_L\rangle$$

Thus:

$$\text{only } K_S \rightarrow \pi\pi, \text{ but } K_L \rightarrow 3\pi$$

Under CP symmetry:

K_S(CP=+1): can only decay (hadronically) to 2π's (CP=+1)

K_L(CP=-1): can only decay (hadronically) to 3π's (CP=-1)

If CP conserved, should *not* see the decay $K_L \rightarrow 2\pi$'s

1964: CP violation - Cronin-Fitch experiment

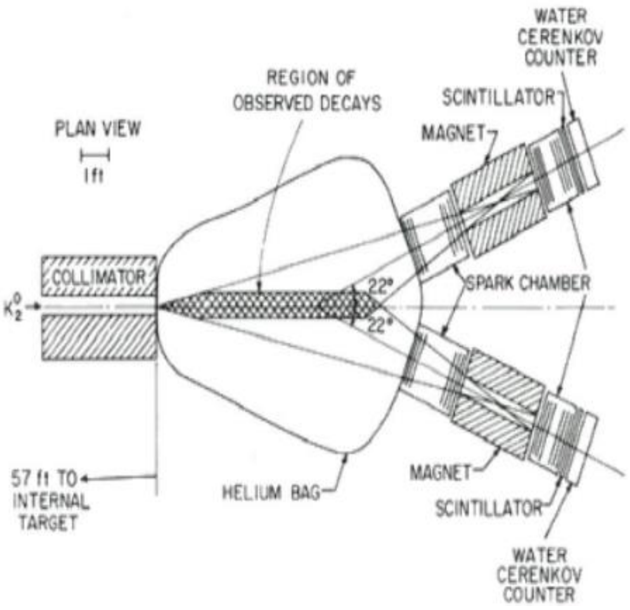
Observation of $K_2 \rightarrow \pi^+\pi^-$ → Christenson, Cronin, Fitch, Turlay (1964)

The experiment shoot protons on a target to produce K^0, after a long enough trip in a vacuum pipe, they achieved a pure K_2 beam.

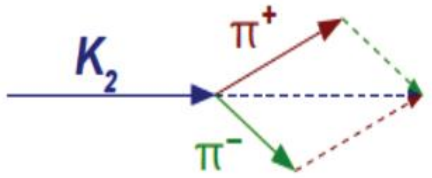
Experimentally use invariant mass (energy conservation) and angle between K_2 and $\pi^+\pi^-$ (momentum conservation).

Find excess of ~56 events in the signal region: **$BF(K_2 \rightarrow \pi^+\pi^-) \sim 2 \times 10^{-3}$**

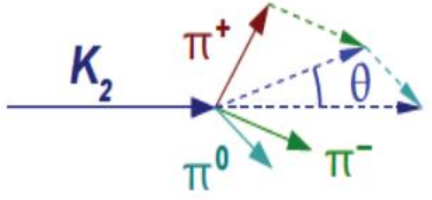
→ **CP violation!**

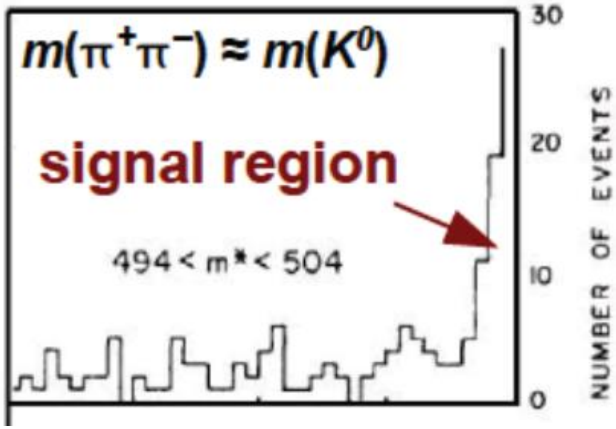


2-body decay (signal):



3-body decay (background):





1963: Cabibbo mixing

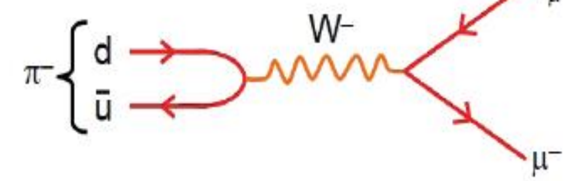
The weak coupling did not look to be universal:

s → u e.g. $K^+ \to \mu^+ \nu_\mu$

d → u e.g. $\pi^+ \to \mu^+ \nu_\mu$

s → u transitions suppressed by a factor ~20

Pion decay



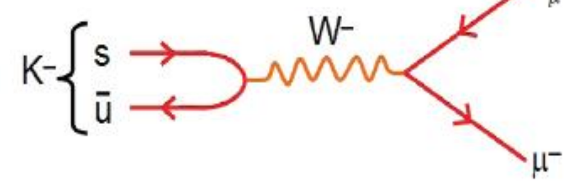
$\pi^-(d\bar{u}) \to \mu^- + \bar{\nu}_\mu$

Cabibbo (1963): **weak interactions couples to a linear combination:**

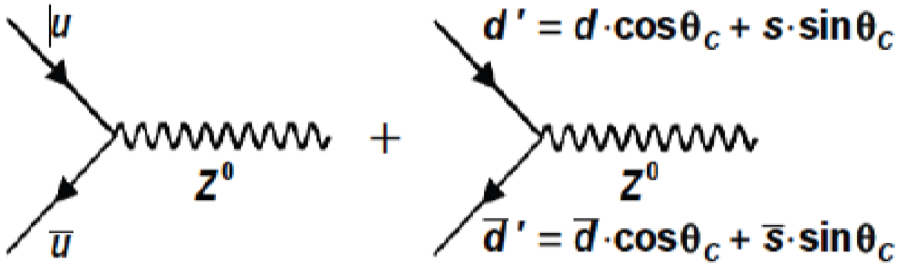
$$\boxed{d' = d \cdot \cos\theta_c + s \cdot \sin\theta_c}$$

$\sin\theta_c = 0.22$ (empirically)

Kaon decay



$K^-(s\bar{u}) \to \mu^- + \bar{\nu}_\mu$



$d' = d \cdot \cos\theta_C + s \cdot \sin\theta_C$

$\bar{d}' = \bar{d} \cdot \cos\theta_C + \bar{s} \cdot \sin\theta_C$

But, if the neutral weak currents also couples to *d′* expect large FCNC

Experimentally, however, $BR(K \to \mu\mu) \sim 7\times10^{-9}$

1970: GIM mechanism

$K^+ \to \mu^+\nu_\mu$ so why not $K^0 \to \mu^+\mu^-$?

$K^+ \to \pi^0\mu^+\nu_\mu$ so why not $K^0 \to \pi^0\mu^+\mu^-$?

$BR(K_L \to \mu^+\mu^-) \sim 7 \cdot 10^{-9}$ $BR(K_L \to e^+e^-) \sim 10^{-11}$

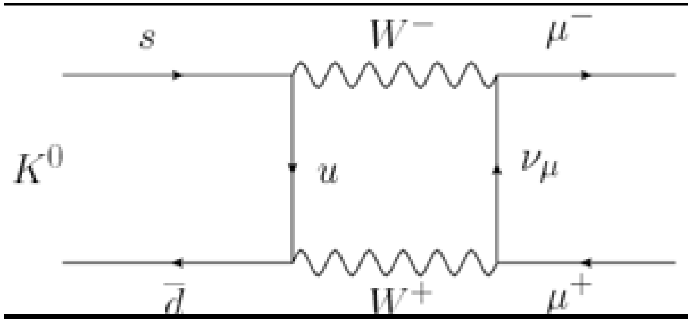
$BR(K^0 \to \pi^0\mu^+\mu^-) <\sim 10^{-10}$

GIM (Glashow, Iliopoulos, Maiani) mechanism (1970)

assume a **new (not yet observed) quark**
in SU(2) quark doublets

no tree level flavour changing neutral currents

suppression of FCNC via loops

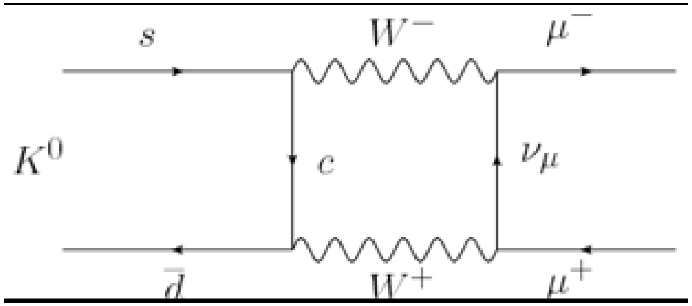


Requires that quarks come in pairs (doublets)

prediction of a **2nd** up-type quark

additional Feynman graph cancels the „*u*-box graph”

prediction of m(c) ≈ 1.5 GeV
→ *Gaillard and Lee (1974)*



CKM mechanism

1973: The CKM mechanism

1973: **Kobayashi & Maskawa** demonstrate that CP violation arises naturally from quark mixing if there are **3 generations of quarks**

CKM Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

3x3 matrix of complex numbers

- ⇒ 18 parameters
- unitary ⇒ 9 parameters
- quark fields absorb unobservable phases ⇒ 4 parameters

3 mixing angles and 1 phase allowing for CP violation

1974: Discovery of the charm quark

Observation of a **narrow resonance** at a mass of **3.1 GeV**

in **proton-Be collisions** at BNL

($p+Be \to e^+e^-+X$, Ting et al.)

in **e+e- → e+e-,μ+μ-, hadrons** at SLAC

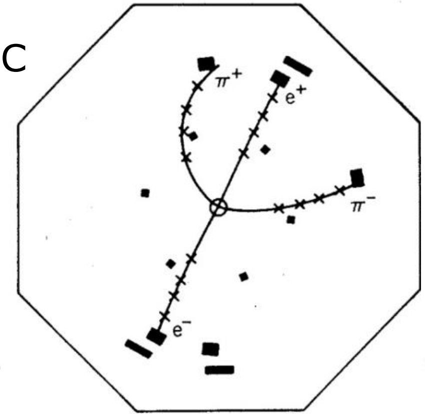
(Richter et al.)

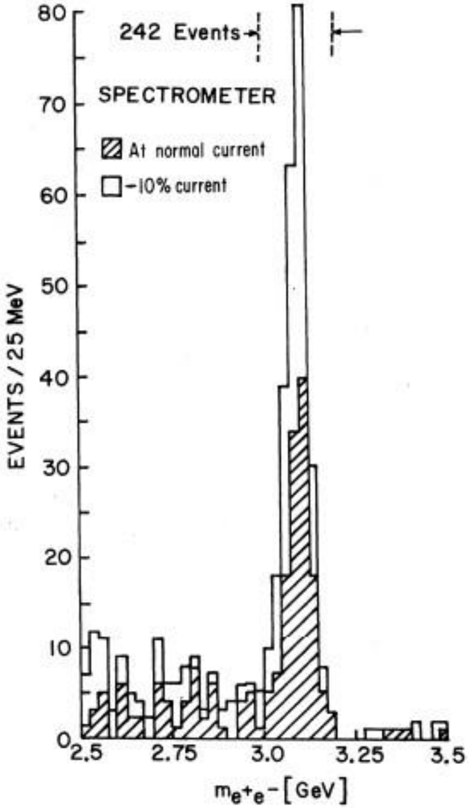
New resonance J/ψ

narrow width, therefore a **long lifetime**

(excluding interpretations as a uds state)

most plausible explanation was a bound state
of a **new quark (charm)** with mass **~1.5 GeV!**





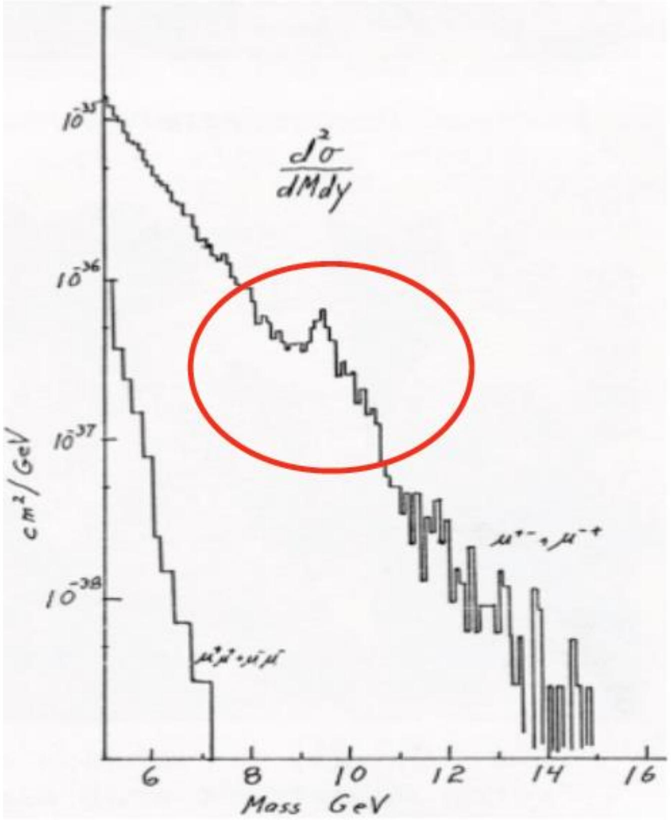
Soon after confirmed by the observation of new cc states and of open charm *(D mesons)*

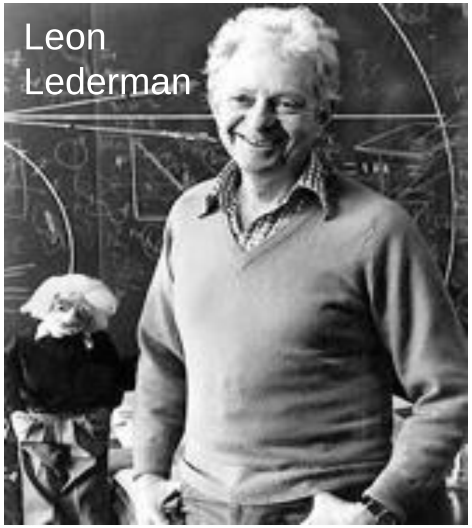
1977: Discovery of the bottom quark

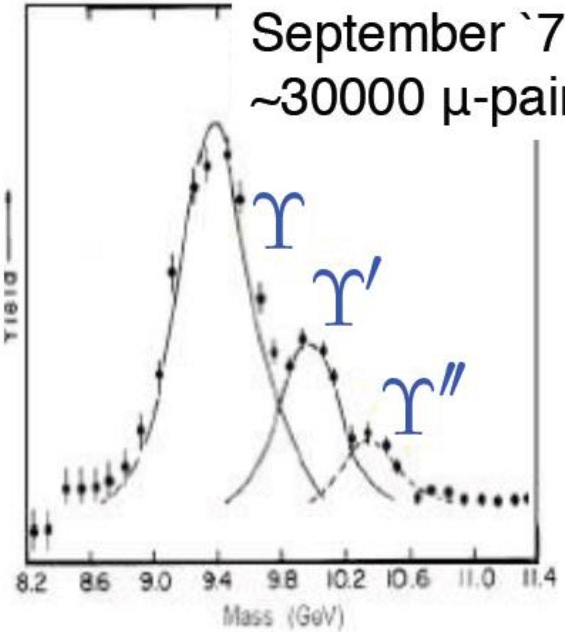
Are there really 3 generations?

Fermilab E288 Experiment observed excess of di-muon events at a mass of around 9-10 GeV *(3 resonances)*

$$p + \mathrm{Cu} \rightarrow \mu^+\mu^- + X$$



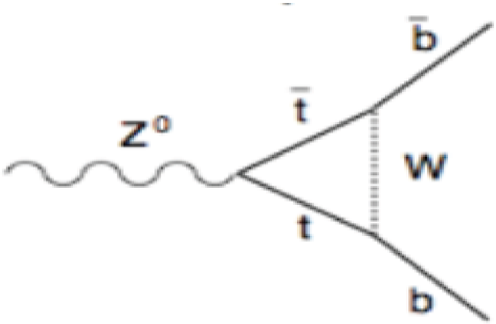

Leon Lederman


September `77:
~30000 μ-pairs

Discovery of bottomonium!

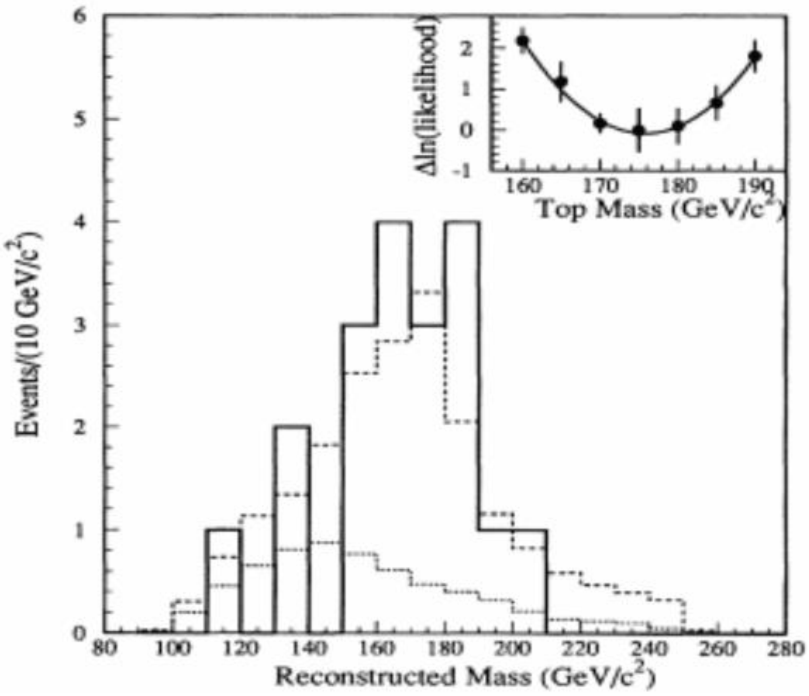
1995: Discovery of the top quark

- **t-quark** mass was predicted to be large **(>50 GeV)** from **B-mixing measurements (ARGUS, 1987)**
- between **150 and 200 GeV from LEP** precision EW measurements **in the 90s**



CDF/D0 (1995): Observation of tt(bar) production in pp(bar) collisions **at the Tevatron**

CDF: 175±8±10 GeV
D0: 199^{+19}_{-21}±22 GeV



Leptons

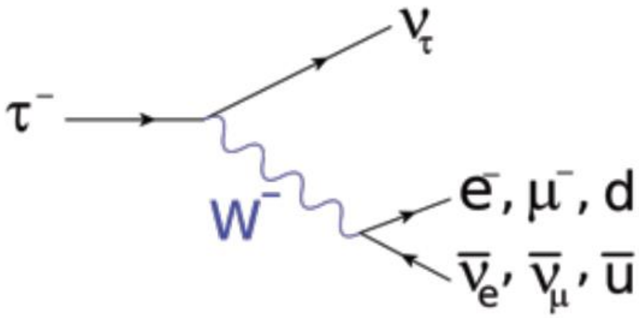
- Andesron, Neddemeyer discovered the μ with cosmic rays at Caltech in 1936. But because its mass was so close to the Yukawa pion, it was not recognized as a heavy electron until 1947 → Who ordered that?”

- In 1930 Pauli proposed the existence of the neutrino to explain β decay. In 1956 Reines and Cowan using neutrinos from nuclear reactors, demonstrated their existence using the inverse β decay reaction:

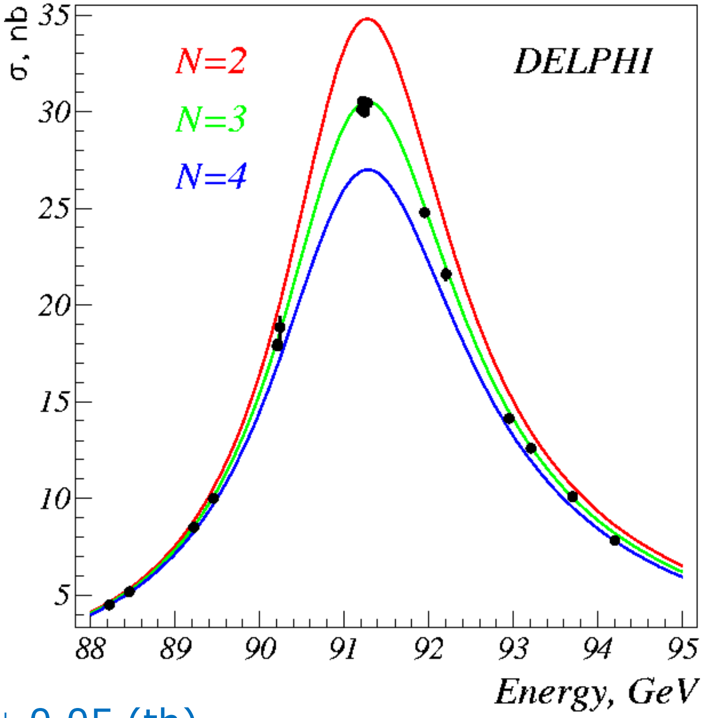
$$\text{anti-}\nu \; p \rightarrow n \; e^+$$

- In 1962 Lederman, Schwartz and Steinberger discovered that there were at least two kind of neutrinos with different properties. They used $\pi \rightarrow \mu \; \nu$ decays

- The τ lepton was observed in a series of experiments between 1974-77 by Perl et al. at SLAC. They found a number of unexplained events of the type $e^+e^- \rightarrow e\mu^+ \geq 2$ undetected. The interpretation was $e+e \rightarrow \tau+\tau \rightarrow e\mu + 4\nu$ with $m_\tau \sim 1.6\text{-}2$ GeV



Three families in lepton sector



- When LEP started producing e^+e^- collisions around the mass of the Z boson, there were already indications that the number of light neutrinos was three from previous experiments as well as from astrophysical arguments

- In 1989 after few months since the first collisions, the LEP experiments were able to measure precisely the total width of the Z boson, related to the N_ν

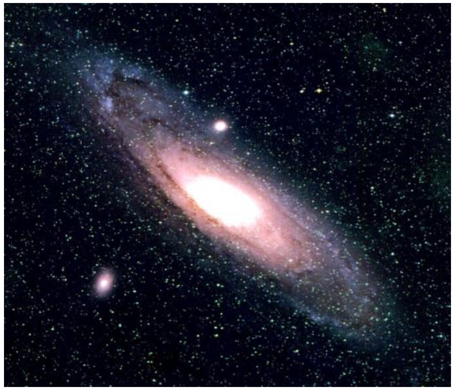
- For instance, ALEPH measured:

$$N_\nu = 3.27 \pm 0.24(\text{stat}) \pm 0.16(\text{syst}) \pm 0.05\ (\text{th})$$

- LEP measurements became very precise with more statistics, and the final number, $N_\nu = 2.9840 \pm 0.0082$, leaves no doubt that there are not more than three light neutrinos

- The third neutrino (ν_τ) was observed in 2000 by the DONUT Collaboration at Fermilab.

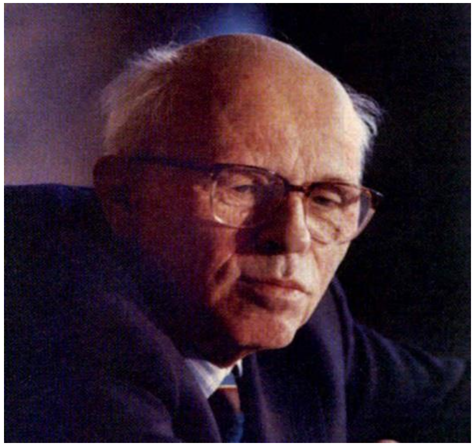
Matter-antimatter asymmetry



- We know that the matter – anti-matter symmetry in the Universe is broken: the Universe consists of matter.
- But, shortly after the Big Bang, there should have been equal amounts of matter and anti-matter

  → how did the Universe develop a preference of matter?

In 1966, Andrei Sakharov showed that necessary for evolution of matter dominated universe, from symmetric initial state, are:



(1) baryon number violation

(2) C & CP violation

(3) thermal inequilibrium

- No significant amounts of antimatter observed!
- (N(baryon) – N(antibaryon)) / $N_\gamma \sim 10^{-10}$

Standard Model *CPV* cannot explain matter asymmetry in the universe
→ the *only* CP violating phase in SM leads to $10^{-17}\ \Delta N_B/N_\gamma$

SM flavour sector

Flavour in Standard Model

- Higgs field was introduced to give masses to W^+, W^- and Z^0 bosons (after SBB)
- Since we have a Higgs field we can add (ad-hoc) interactions between the Higgs field ϕ and the fermions in a gauge invariant way (Yukawa couplings):

$$-L_{Yukawa} = Y_{ij}\left(\overset{\text{doublets}}{\overline{\psi}_{Li}\,\phi}\right)\overset{\text{singlet}}{\psi_{Rj}} + h.c.$$

- The quark flavour structure within the SM is described by 6 couplings and 4 CKM params
- It is convenient to move the CKM matrix from Yukawa sector to the weak current sector
- We can diagonalize the Y_{ij} matrices, such that we arrive in the „mass basis”

However, then the Lagrangian of the charged weak current should also be rewritten:

$$-L_{W^+} = \frac{g}{\sqrt{2}}\left(\overline{u},\overline{c},\overline{t}\right)_L \underbrace{\left(V_{CKM}\right)}_{\text{CKM matrix (rotation matrix)}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \gamma^\mu W_\mu^+$$

V_{CKM} originates from the diagonalization of the Yukawa couplings

Weak interactions in the SM

- After SSB, the charged current of a W⁻ exchange can be written as:

$$J^{\mu -} = (\overline{u}_L, \overline{c}_L, \overline{t}_L)\gamma^\mu V_{\mathrm{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

- Weak interaction only couples to left-handed field

  → left-handed quarks or right-handed anti-quarks

  → manifestly violates parity

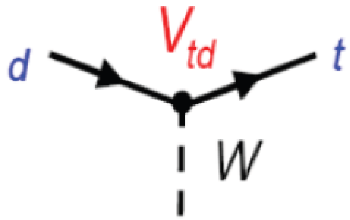
The weak eigenstates are related to the mass eigenstates by the CKM matrix:

$$\underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{Weak eigenstates}} = V_{\mathrm{CKM}} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{Mass eigenstates}}$$

CP transformation & the weak interaction

Quarks

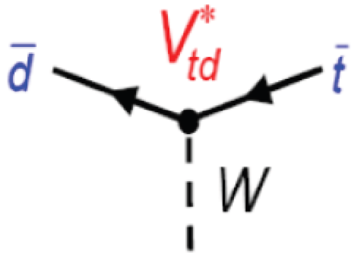
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



- - - - - CP - - - - -

Anti-quarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V^*_{ud} & V^*_{us} & V^*_{ub} \\ V^*_{cd} & V^*_{cs} & V^*_{cb} \\ V^*_{td} & V^*_{ts} & V^*_{tb} \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$



CP violation requires complex matrix elements

Relative phases

Q: ***How many parameters does the CKM matrix have?***

18 parameters (9 complex numbers):

9 unitary conditions: $V_{CKM} V_{CKM}^{\dagger} = 1$
5 relative phases of the quark fields
4 parameters (*)

(*) 3 (real) **Euler angles** and 1 **phase** (single source of CP violation in the SM)

- with 2 generations there is only one real (Euler) angle: the Cabbibo angle
- CP violation requires 3 generations!

When I do a phase transformation of the (left-handed) quark fields:

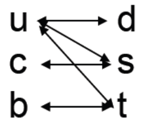
$$u_{Lj} \to e^{i\phi_j} u_{Lj} \qquad d_{Lk} \to e^{i\phi_k} d_{Lk}$$

And a simultaneous transformation of the CKM matrix:

$$V \to \begin{pmatrix} e^{-i\phi_u} & & \\ & e^{-i\phi_c} & \\ & & e^{-i\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi_d} & & \\ & e^{-i\phi_s} & \\ & & e^{-i\phi_b} \end{pmatrix}$$

or $V_{jk} \to \exp\left(-i\left(\phi_j + \phi_k\right)\right) V_{jk}$

There are only 5 relative phases (+ one overall phase)

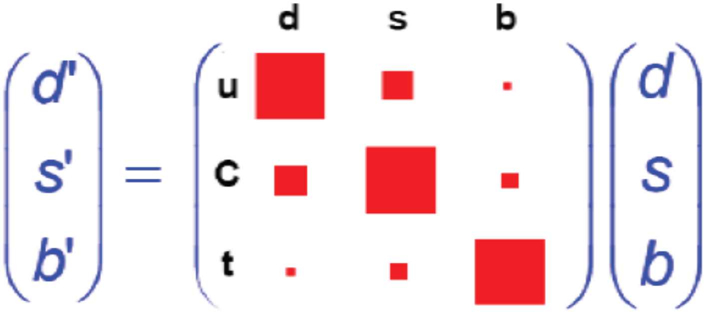


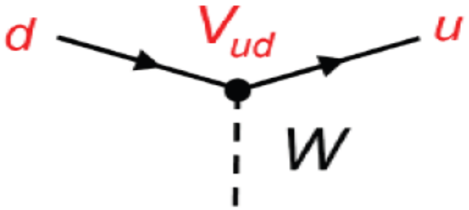
The charged current (i.e. the physics) remains invariant:

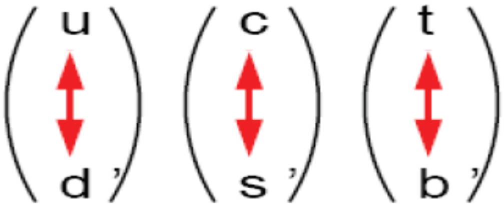
$$J^{\mu}_{CC} = \overline{u_{Li}} \gamma^{\mu} V_{ij} d_{Lj}$$

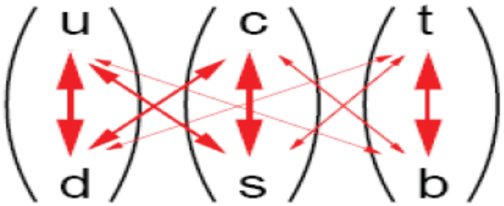
In other words, I can always absorb the 5 relative phases by redefining the quark fields
→ **these 5 phases are unobservable**

Hierarchy in quark mixing









- Diagonal elements of CKM matrix are close to one
- Only small off-diagonal contributions
- Mixing between quark families is „CKM suppressed”

Wolfenstein parametrization

Makes use of the fact that the off-diagonal elements are small compared to the diagonal elements

→ expansion in $\lambda \approx V_{us}$, $A \approx V_{cb} / \lambda^2$ and ρ, η

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$\lambda \sim 0.22$ (=$\sin\theta_C$, sine of Cabibbo angle)
$A \sim 1$ (actually 0.80)
$\rho \sim 0.14$
$\eta \sim 0.34$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad A^2\lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

$$\rho + i\eta = \frac{\sqrt{1 - A^2\lambda^4}(\bar{\rho} + i\bar{\eta})}{\sqrt{1-\lambda^2}\,[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}$$

CKM angles and unitarity triangle

Writing the complex elements explicitly:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \sim \lambda^3 e^{-i\gamma} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \sim \lambda^3 e^{-i\beta} & \sim -\lambda^2 e^{-i\beta_s} & 1 \end{pmatrix} + O(\lambda^4)$$

Definition of the angles:

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

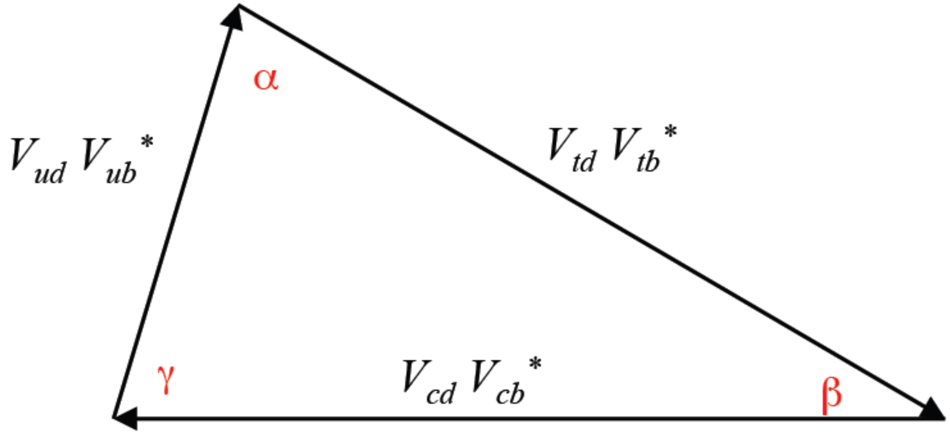
$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{tb}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

Using one of the 9 unitarity relations: $V_{\mathrm{CKM}} V^\dagger_{\mathrm{CKM}} = 1$

Multiply first „d" column with last „b" column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



CKM angles and unitarity triangle

Writing the complex elements explicitly:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \sim\lambda^3 e^{-i\gamma} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \sim\lambda^3 e^{-i\beta} & \sim -\lambda^2 e^{-i\beta_s} & 1 \end{pmatrix} + O(\lambda^4)$$

Definition of the angles:

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

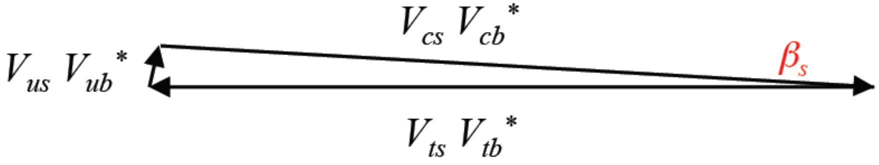
$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{tb}^*}{V_{cd}V_{cb}^*}\right)$$

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

Using another unitarity relation: $V_{\mathrm{CKM}} V_{\mathrm{CKM}}^{\dagger} = 1$

Multiply second „s” column with last „b” column:

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$



„Squashed unitarity triangle”

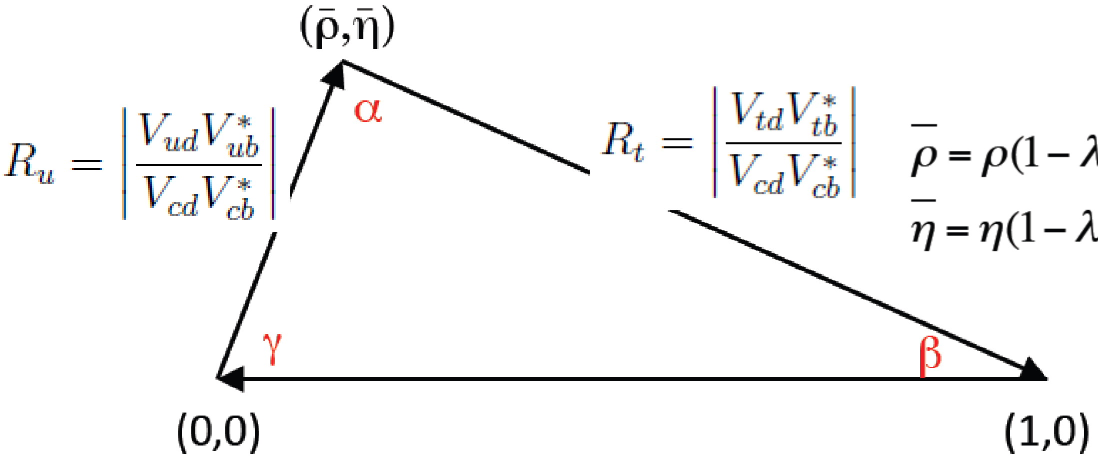
CKM angles and unitarity triangle

- Imposing unitarity to the CKM matrix results in six equations that can be seen as the sum of three complex numbers closing a triangle in the complex plane
- Two of these triangles are relevant for study of CP-violation in B-physics and define the angles

Current knowledge of UT
(from CKMFitter)

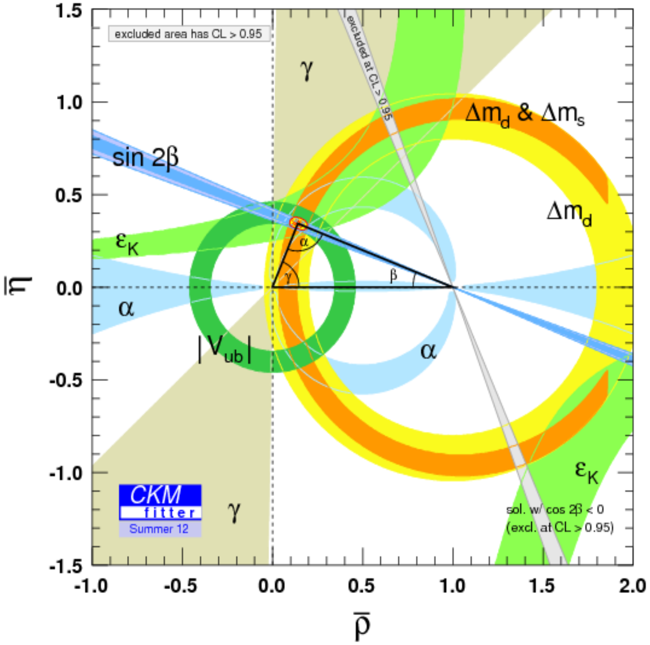
Normalized CKM triangle:

→ divide each side by $V_{cd}\,V^*_{cb}$



$$R_u = \left|\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right| \qquad R_t = \left|\frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}}\right|$$

$$\bar\rho = \rho(1-\lambda^2/2)$$
$$\bar\eta = \eta(1-\lambda^2/2)$$

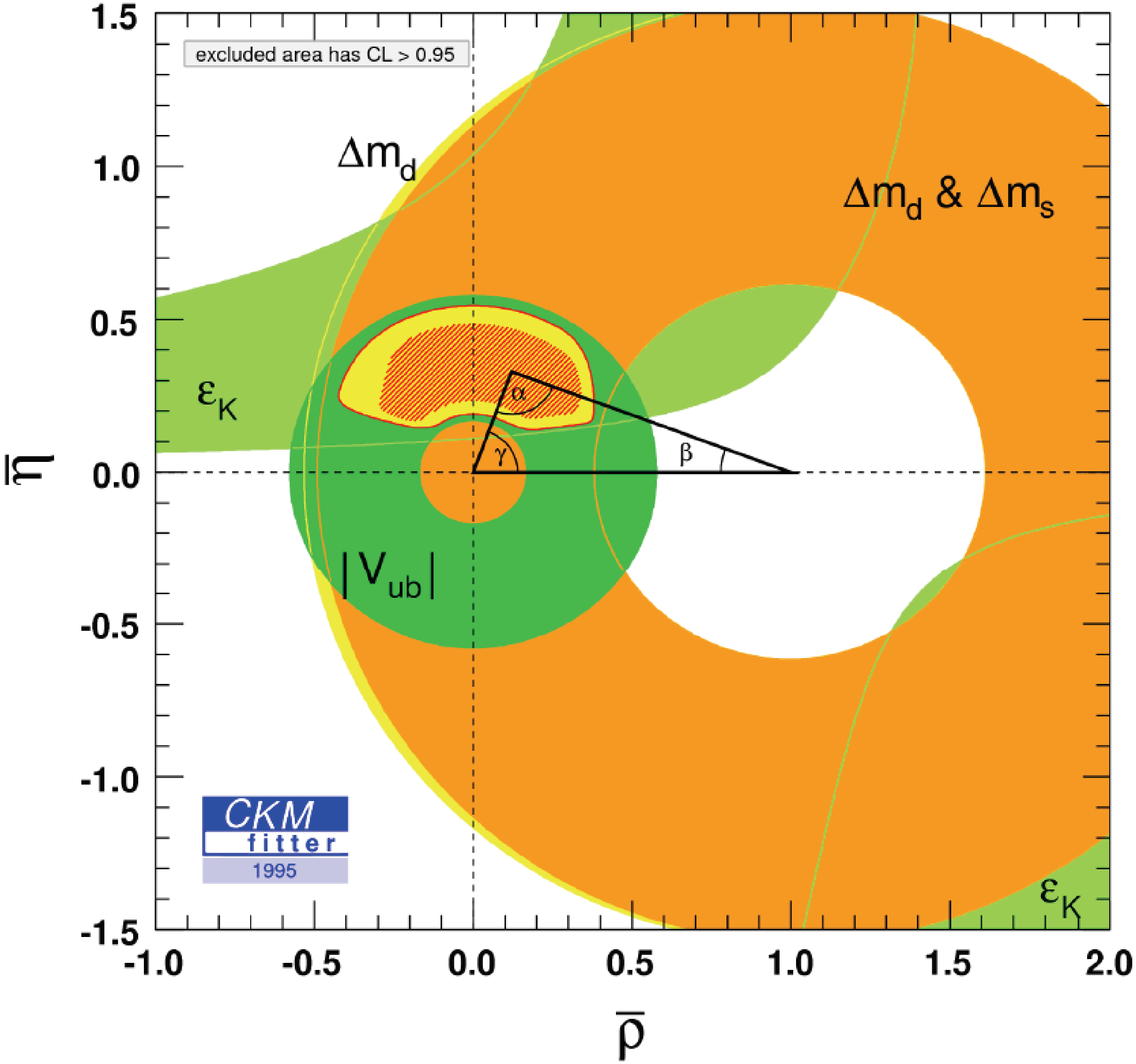


The unitarity triangle:

- Shows the size of the CP violation (no CPV means no triangle!)
- Presents our knowledge of CKM parameters
- Shows how consistent the measurements are!

Progress in UT

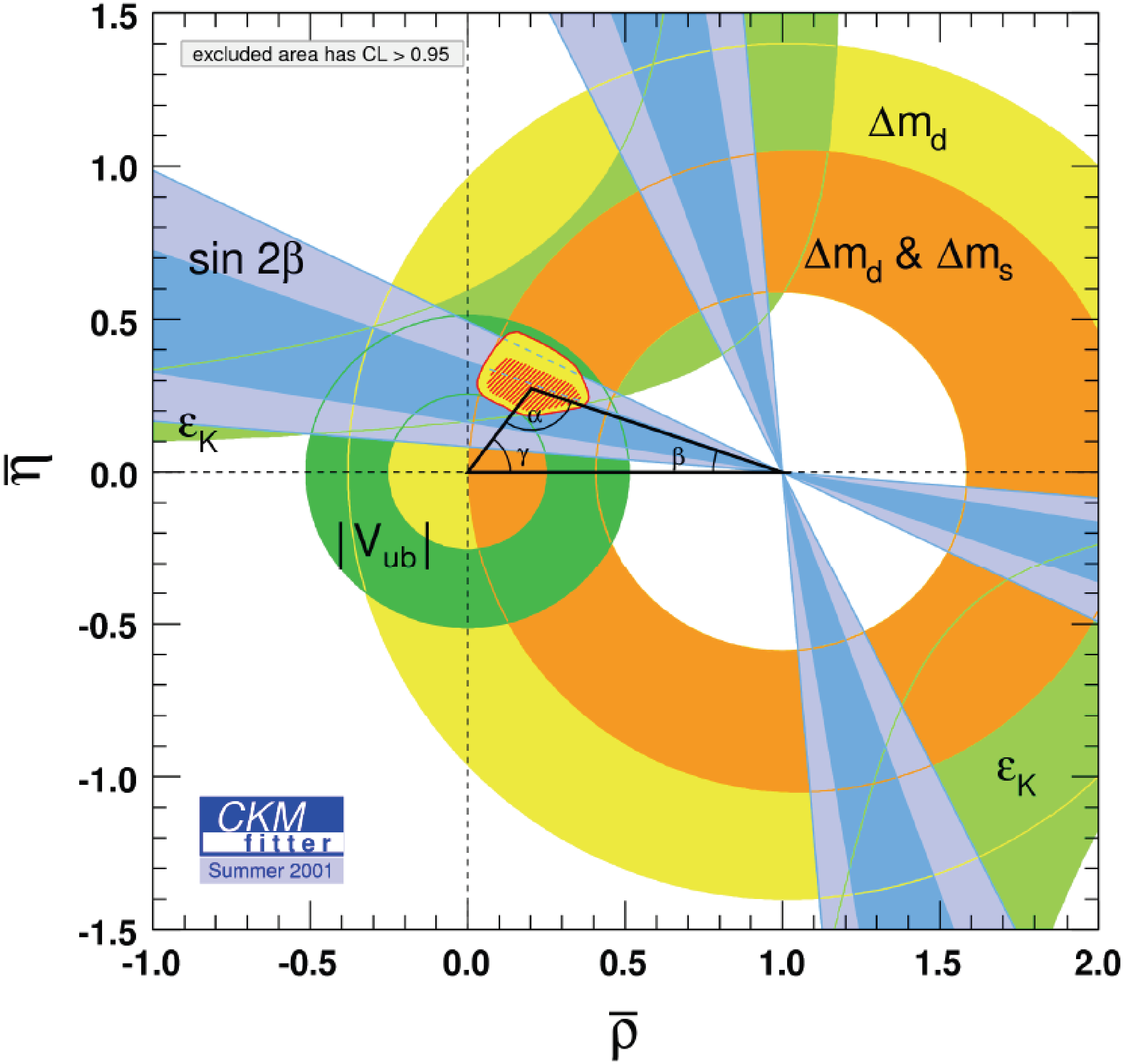
1995



Progress in UT

2001

first observation of non-kaon CPV



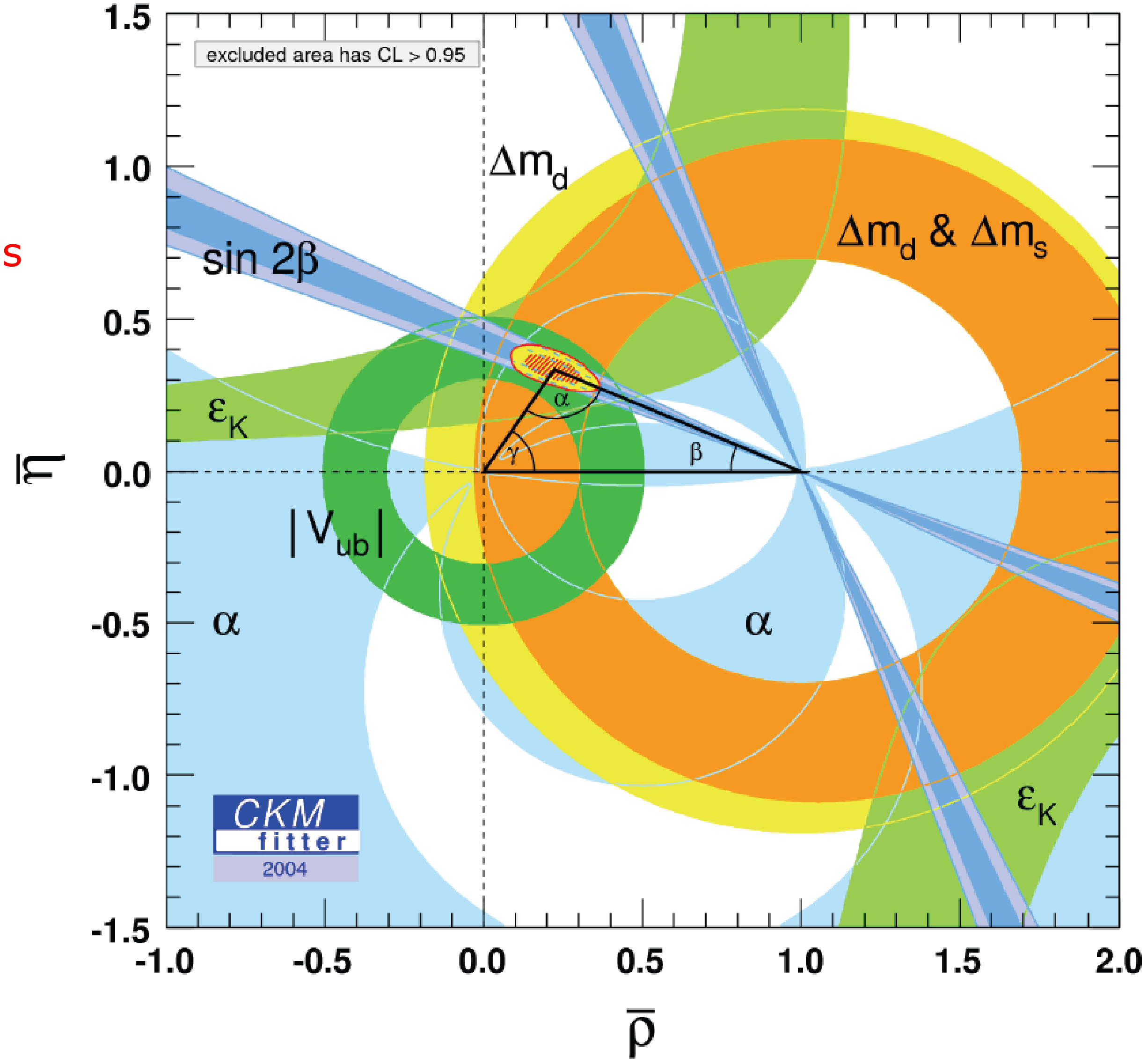
Progress in UT

2004

improvement in lattice calculations

more data on $A_{CP}(B \to J/\psi\ K_s)$

first constraints on the angle α from $B \to \rho\rho$



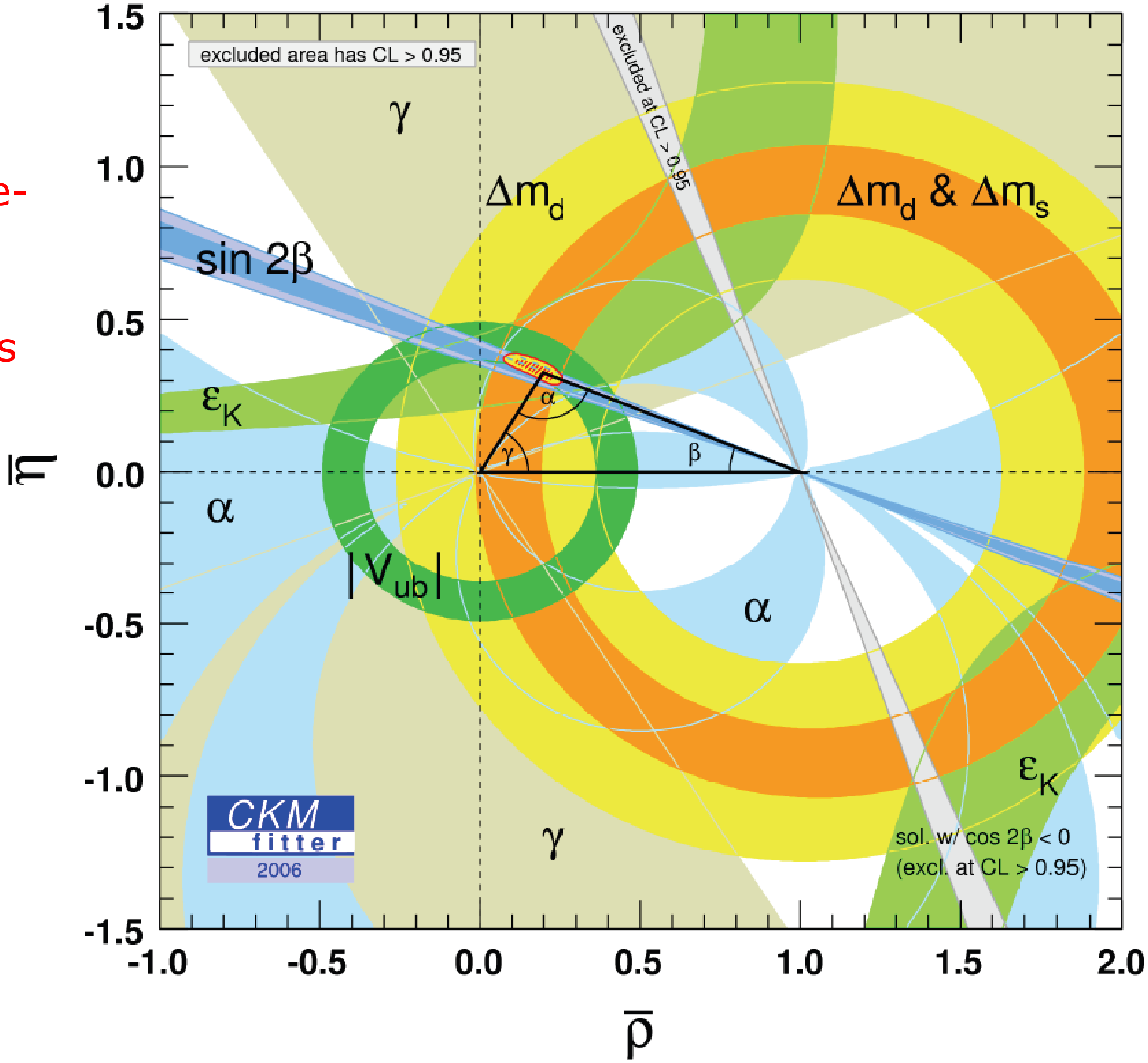
Progress in UT

2006

Tevatron measurement of Δm_s

tighter constraints on α

first constraints on γ from CPV in $B \rightarrow K \pi$

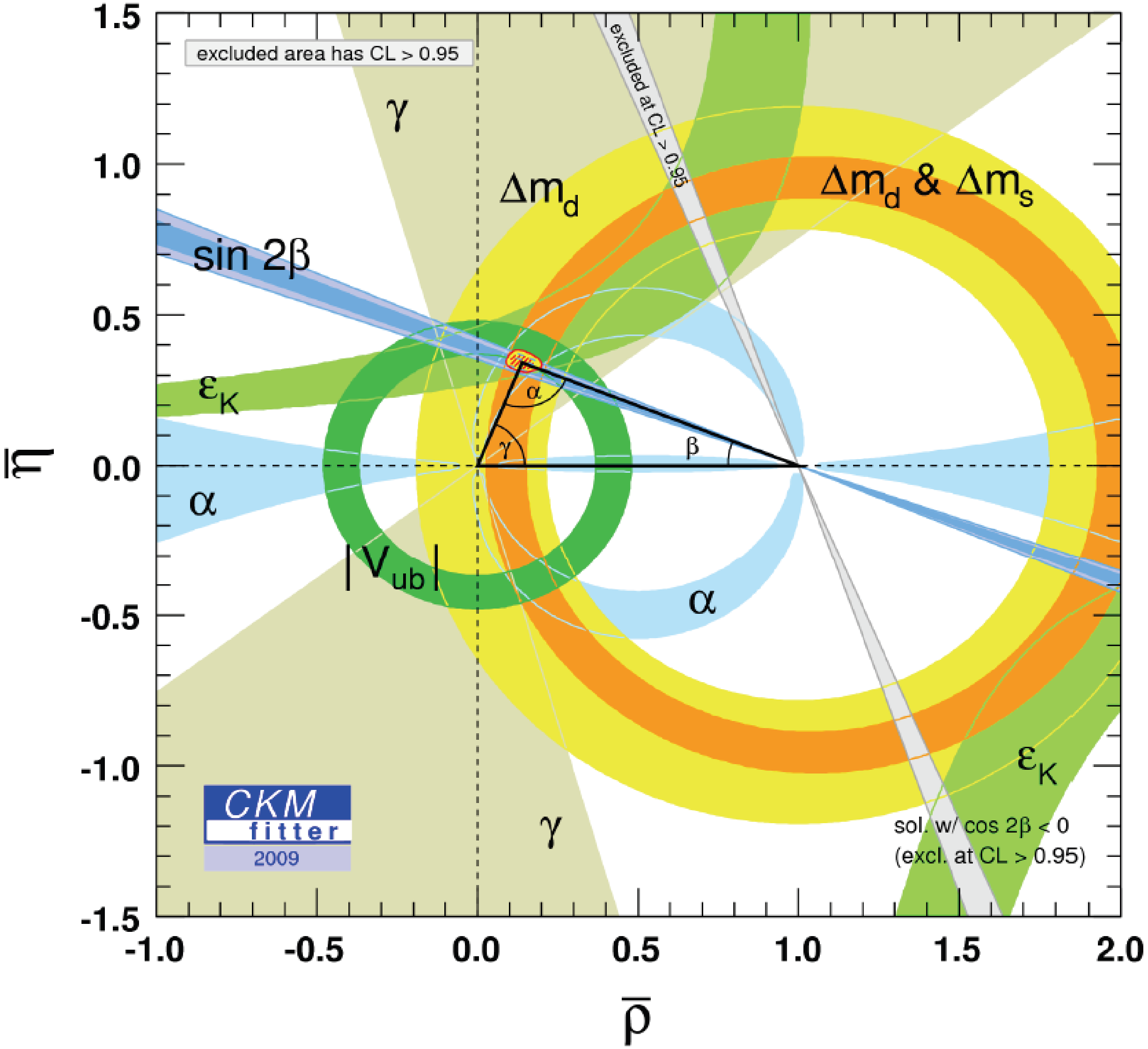


Progress in UT

2009

more contraints from B mixing

constraints from BaBar on β from CPV in $B \rightarrow \chi_{0c} K_s$

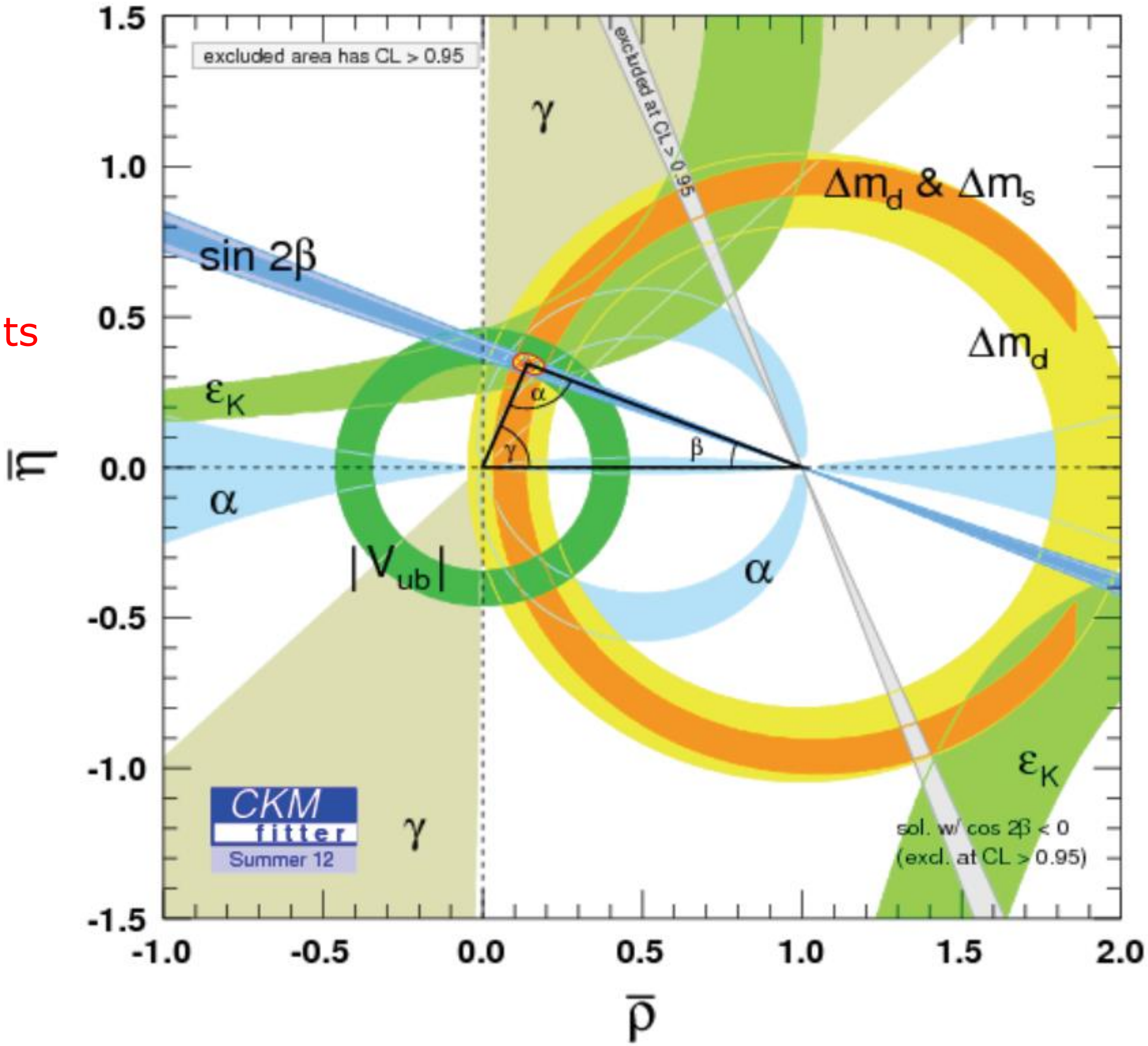


Progress in UT

2012

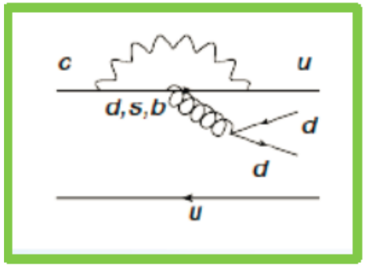
LHCb measurement of Δm_s

tighter constraints on γ

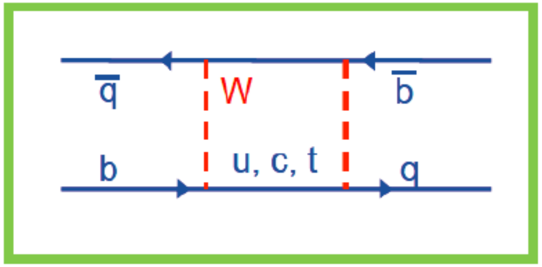


FCNC loops in the SM

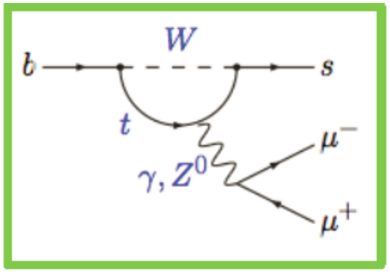
Map of flavour transitions and type of loop processes



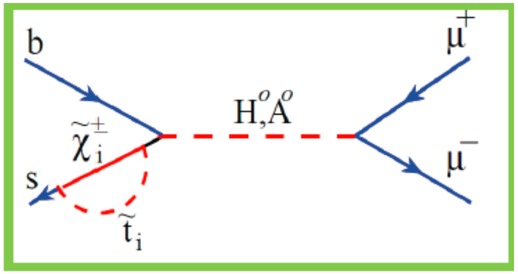
QCD penguin



ΔF=2 box



EW penguin



Higgs penguin

	b→s	b→d	c→u	s→d
QCD penguin	$A_{CP}(B_s \to hhh)$	$A_{CP}(B^0 \to hhh)$	**$\Delta a_{CP}(D \to hh)$**	$K \to \pi^0 ll$ ε'/ε
ΔF=2 box	**ΔM_{Bs}** **$A_{CP}(B_s \to J/\psi\varphi)$**	ΔM_{Bd} $A_{CP}(B^0 \to J/\psi K_s)$	x,y, q/p	ΔM_K ε_K
EW penguin	**$B \to K^{(*)}\mu\mu$** $B \to X_s\gamma$	$B \to \pi\mu\mu$ $B \to X\gamma$	$D \to X_u l\, l$	$K \to \pi^0 ll$ $K \to \pi^\pm \nu\nu$
Higgs penguin	**$B_s \to \mu\mu$**	**$B^0 \to \mu\mu$**	$D \to \mu\mu$	$K^0 \to \mu\mu$