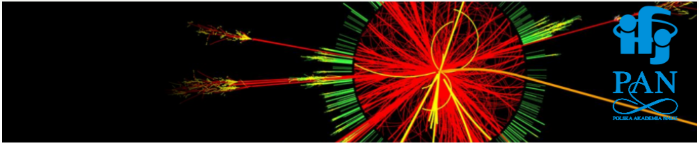

Particle Physics- Standard Model(2)

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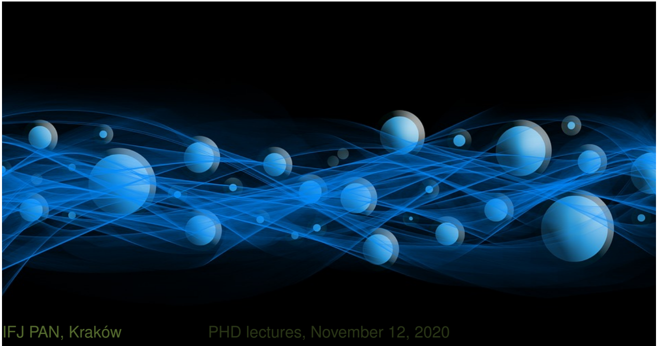


Outline

This is an “introduction”

- Historical overview.
- the elementary particles
- the elementary forces
- Symmetries:
  - Gauge symmetry
  - Problem of mass
  - Spontaneous symmetry breaking

This lecture is not a complete course in particle physics and will only touch some most general problems.
Further reading:

- D. H. Perkins, “Introduction to High Energy Physics”,
- F. Halzen, A. Martin: “Quarks and Leptons”.

Further lecture to watch listen on SM and BSM physics:

- Prof. Yuval Grossman (Cornell U.)
- Standard Model and Flavor - Lecture (`https://www.youtube.com/watch?v=GGzRdiBd8w8`)

GIM mechanism revisited

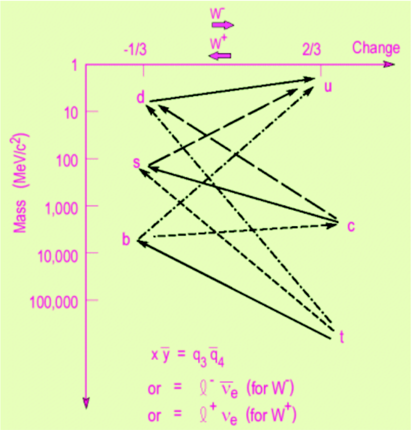
- Theory of weak interactions
  - Fermi's theory of weak interactions
  - V-A theory
  - Current - current theory, current algebra
  - W and Z bosons
  - Electroweak theory

Weak interaction revisited

- Weak interactions occur for all fundamental particles except gluons and photons. Weak interactions involve the exchange or production of W or Z bosons.
- Weak forces are very short-ranged. In ordinary matter, their effects are negligible except in cases where they allow an effect that is otherwise forbidden. There is a number of conservation laws that are valid for strong and electromagnetic interactions, but broken by weak processes. So, despite their slow rate and short range, weak interactions play a crucial role in the make-up of the world we observe.
- Any process where the number of particles minus the number of antiparticles of a given quark or lepton type changes, is weak decay process and involves a W-boson. Weak decays are thus responsible for the fact that ordinary stable matter contains only up and down type quarks and electrons. Matter containing any more massive quark or lepton types is unstable. If there were no weak interactions, then many more types of matter would be stable.

Quark decays revisited

- Quark flavour never changes except by weak interactions, like beta decay, that involve W bosons.
- Any quark type can convert to any other quark type with a different electric charge by emitting or absorbing a W boson. Different possible transitions are shown schematically in the diagram.



Weak interaction revisited

- Decay processes always proceed from a more massive quark to a less massive quark, because the reverse process would violate conservation of energy. Scattering processes can involve the reverse transitions, provided sufficient energy is available.

- Lepton number conservation rules
  Each charged lepton is converted only to its own neutrino type by emitting or absorbing a W boson. This leads to the three lepton number conservation laws.

Fermi's theory of weak intercation

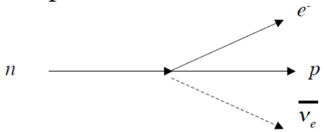
Beginning of the story:

- As an example Fermi considered β-decay of the neutron:

$$n \to p + e^- + \overline{\nu}_e$$

- Assumption: at a single point in the space-time, the wavefunction of the neutron is transformed into that of the proton, and the wavefunction of incoming neutrino (equivalent to outgoing antineutrino which we actually see) is transformed into that of electron.
- The schematic diagram for this process:

Such a process is called four-fermion interaction.



Weak currents by Fermi

$$J_\mu^{EM} = -\overline{\psi}_e \gamma_\mu \psi_e$$

electromagnetic interaction from QED

$$H_{EM} = J_\mu^{EM} A^\mu = -e\overline{\psi}\gamma_\mu \psi A^\mu$$

Fermi (1934) introduced a weak hadronic current density

$V_\mu^{C+} = \overline{\psi}_P \gamma_\mu \psi_n$ (charged weak hadronic current)

in analogy to J_μ^{EM} and A^μ (photon)

V_μ^C weak field $W^\pm$ (W boson)

Fermi constructed further $l_\mu^C = \overline{\psi}_e \gamma_\mu \psi_\nu$

$$H_\beta = \frac{G_\beta}{\sqrt{2}}\left(V^{c\mu} l_\mu^{c+} + l^{c\mu} V_\mu^{c+}\right)$$

V-A theory

- The description of this process is given by the amplitude:

$$M = G_F(\overline{\psi}_p \Gamma \psi_n)(\overline{\psi}_e \Gamma \psi_\nu)$$

  where the factors Γ are responsible for particle transformation, and G_F is the very well known Fermi coupling constant which determines the strength of the weak interaction (the rate of the decay).
- In 1956 **Feynman and Gell-Mann** proposed that the interaction factors Γ are a mixture of vector and axial-vector quantities, to account for parity violation in weak interactions.
  - Vector - changes sign if rotated through 180°.
  - Axial vector - like vector under rotation, but opposite sign to vector under parity transformation.
- *V-A* theory was proven by many experiments at that time, in particular by the fact that neutrino is left-handed (helicity -1/2) - its spin is antiparallel to its momentum.

V-A theory

leptonic current density, which results in left-handed neutrinos:

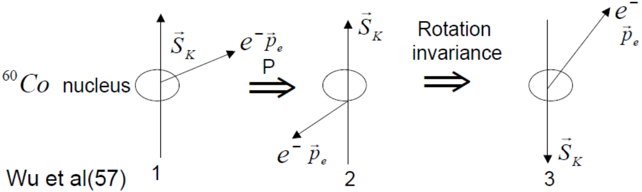
$$l_\mu^C = \overline{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu$$

hadronic current

$$h_\mu^{C+} = \overline{\psi}_p \gamma_\mu (1 - c_A \gamma_5) \psi_n \quad \text{(V-A structure)}$$

^{60}Co Parity-violation β decay

Pseudoscalar $\langle \vec{S}_K \cdot \vec{P}_e \rangle$ $\quad {}^{60}Co \to {}^{60}Ni + e^- + \overline{\nu}_e$



Vector and axial currents

$$j_\mu = \overline{\psi}\gamma_\mu\psi$$

$$j_\mu^5 = \overline{\psi}\gamma_\mu\gamma_5\psi$$

$$i\partial_\mu j^\mu = i\partial_\mu(\overline{\psi}\gamma^\mu\psi) = i(\partial_\mu\overline{\psi})\gamma^\mu\psi + i\overline{\psi}\gamma^\mu(\partial_\mu\psi)$$

$$= -m\overline{\psi}\psi + m\overline{\psi}\psi$$

$$i(\partial_\mu\overline{\psi})\gamma^\mu = -m\overline{\psi}$$

$$i\gamma^\mu(\partial_\mu\psi) = m\psi$$

$$\therefore\ \partial_\mu j^\mu = 0$$

Vector current is conserved

Vector and axial currents

$$i\partial_\mu j^5_\mu = i(\partial_\mu\overline{\psi})\gamma^\mu\gamma_5\psi + i\overline{\psi}\gamma^\mu\gamma_5(\partial_\mu\psi)$$
$$= i(\partial_\mu\overline{\psi})\gamma^\mu\gamma_5\psi - i\overline{\psi}\gamma_5\gamma^\mu(\partial_\mu\psi)$$
$$= -m\overline{\psi}\gamma_5\psi - m\overline{\psi}\gamma_5\psi$$
$$= -2m\overline{\psi}\gamma_5\psi$$

The axial vector current is not conserved for m≠0
m: current mass of the quark

Sugested reading `http://www.scholarpedia.org/article/Axial_anomaly`

Leptonic currents modern aproach

- We can also describe weak interactions in terms of the interactions of two "currents". In β-decay one current converts a neutron into a proton, and the other creates an electron and its antineutrino (absorbs or destroys neutrino).
- For simplicity consider *leptonic reactions* which involves only leptons. As a result of lepton number conservation law, the absorption of an electron neutrino is always accompanied by the creation of an electron and vice verse. Equivalently, the creation of a positron should be accompanied by the creation of an electron neutrino (or the absorption/destruction of electron antineutrino)

$$L^W = \overline{\psi}_e \Gamma \psi_{\nu_e} + \overline{\psi}_\mu \Gamma \psi_{\nu_\mu} + \overline{\psi}_\tau \Gamma \psi_{\nu_\tau} \qquad \overline{L}^W = \overline{\psi}_{\nu_e} \Gamma \psi_e + \overline{\psi}_{\nu_\mu} \Gamma \psi_\mu + \overline{\psi}_\tau \Gamma \psi_{\nu_\tau}$$

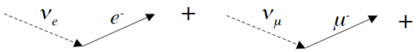
Leptonic currents modern aproach

the leptonic current is extended in an obvious way
to the muonic & tau-lepton sectors:

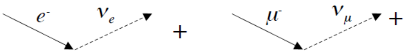
$$l_\mu^C = \overline{\psi}_e \gamma_\mu (1-\gamma_5)\psi_{\nu_e} + \overline{\psi}_\mu \gamma_\mu (1-\gamma_5)\psi_{\nu_\mu} + \overline{\psi}_\tau \gamma_\mu (1-\gamma_5)\psi_{\nu_\tau}$$

Leptonic currents

$L^W \equiv$

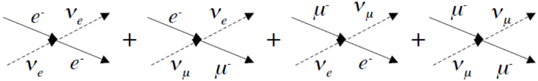


$\overline{L}^W \equiv$



- The *leptonic current* should be multiplied by its antiworld partner to generate all observed weak interactions of leptons.
- The first-order amplitude of the all leptonic processes can be written as:

$M = G_F \overline{L}^W L^W \equiv$



+.....

Hadronic current

- There are also *semileptonic reactions* which involve both leptons and hadrons, such as neutron β-decay, and *hadronic weak reactions* which involve only hadrons, such as the decays of neutral kaons into 2 or 3 pions.
- Hadronic weak current can be characterised in terms of its effect on the quantum numbers of hadrons.
- The total weak current is a sum of leptonic and hadronic components:

$$J^W = L^W + H^W$$

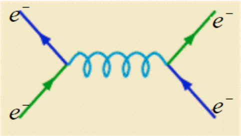
- Then, the amplitude of weak interactions can be presented as:

$$G_F \overline{J}^W J^W = G_F(\overline{L}^W L^W + \overline{L}^W H^W + \overline{H}^W L^W + \overline{H}^W H^W)$$

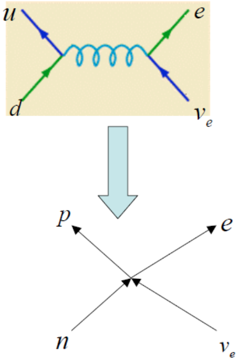
Problems with Fermi theory

- Low-energy weak interactions can be well explained in the framework of Fermi's theory (in the framework of the current-current theory in the V-A form).
- Problems begin when high-energy processes need to be described.
  - The theory predicts linear rise of the neutrino cross-sections with energy, which violates the unitarity principle - the probability for a particular process to occur should be less than or equal to unity. At the energy about 300 GeV the cross-section exceeds the maximal value allowed by the unitarity principle.
  - Very large cross-section at high energies contradict also cosmic-ray data. Neutrinos are the important part of secondary cosmic rays produced in the atmosphere but their interactions with matter were not frequent and put strong limit on their cross-section.

Problems with Fermi theory



$\Rightarrow$



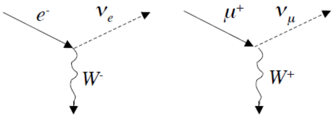
W GeV

bosons (82) range 1/1000 fermi

*: Fermi's theory as a low-energy limit of an interaction mediated by the exchange of bosons.

Problems with Fermi theory

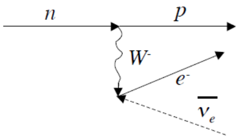
- Fermi's basic principle: analogy with electromagnetism.
- But electromagnetic processes are described as an exchange of photons.
- There should be vector particles which transmits the weak force.
- Weak interaction is of short range, hence the vector particle should be heavy.
- Since β-decay changes nuclear charge, the vector particles should carry the charge.
- Four-fermion point-like interactions were abandoned and replaced with a particle (*W*) exchange mechanism.



Examples of basic lepton processes with the creation or destruction of W-boson

Problems with *W*

- Neutron β-decay can now be shown as an exchange of W-boson.



- W-boson helped to solve some problems of Fermi's theory but produced others.
- W-boson has spin 1 and non-zero mass. As a consequence at high-momentum transfer its propagator is proportional to $p^2 / M^2 c^2$. The diagrams are diverging and the results of the theory are infinite.

Electroweak theory and Higgs mechanism

- First step: **1954 - C. N. Yang and R. Mills** developed a theory of interacting massless particles - gauge theory (the Lagrangian describing the interaction of particle wavefunctions remains invariant under certain symmetry transformations). The theory could accommodate particles like the photon and W-boson, but it required them to be massless. The infinities in the model could be reabsorbed (the model was "renormalisable").

Electroweak theory and Higgs mechanism

•1964 - **Peter Higgs** found a way to transform the Lagrangian describing two massless scalar particles and one massless vector gauge particle into the Lagrangian with one massive scalar particle and one massive vector gauge particle ("Higgs mechanism" for symmetry breaking - mechanism that can give masses to W-bosons).

•1967 - 1968 - **Sheldon Glashow, Steven Weinberg and Abdus Salam** - development of the standard model of electroweak interactions, which unifies electromagnetic and weak interactions.

4 Fermi Theory with V-A

- OK as long as $q^2 << M_W$
- Matrix element

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J_1^\mu J_{2\ \mu}$$

where J_1,J_2 are the two currents and $G_F \sim 10^{-5}$ GeV^{-2}

- We know now that currents exchange a $W^\pm$
- Leptonic current:

$$J_{\ell\ \mu} = \overline{\psi}_{\nu_{ell}} (1 - \gamma_5) \psi_\ell$$

where ψ is a spinor

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

- Note: γ^5 is a pseudoscalar, so leptonic current is $V - A$
- If momentum transfer is large, replace 4-point interaction with W-propagator
- This is called a "charged current" interaction since a $W^\pm$ is exchanged.

4 Fermi Theory with V-A 2

- Before 1956 matrix element was defined

$$\mathcal{M}_{fi} = G_F g_{\mu\nu} \left[\overline{\psi}_3 \gamma^\mu \psi_1\right] \left[\overline{\psi}_4 \gamma^\nu \psi_2\right]$$

- Modification to add V-A interactions

$$\mathcal{M}_{fi} = \frac{1}{\sqrt{2}} G_F g_{\mu\nu} \left[\overline{\psi}_3 \gamma^\mu \left(1-\gamma^5\right) \psi_1\right] \left[\overline{\psi}_4 \gamma^\nu \left(1-\gamma^5\right) \psi_2\right]$$

where $1/\sqrt{2}$ is to keep value of G_F the same

- Replacing 4-Fermi interaction with propagator

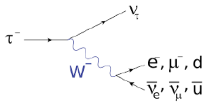
$$\mathcal{M}_{fi} = \left[\frac{g_W}{\sqrt{2}} \overline{\psi}_3 \gamma^\mu \left(1-\gamma^5\right) \psi_1\right] \left(\frac{-g^{\mu\nu} + q^\mu q^\nu / M_W^2}{q^2 - M_W^2}\right) \left[\frac{g_W}{\sqrt{2}} \overline{\psi}_4 \gamma^\nu \left(1-\gamma^5\right) \psi_2\right]$$

- In limit $q^2 << M_W$

$$\mathcal{M}_{fi} = \frac{g_W^2}{8 M_W^2} g_{\mu\nu} \left[\overline{\psi}_3 \gamma^\mu \psi_1\right] \left[\overline{\psi}_4 \gamma^\nu \psi_2\right]$$

$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 m_W^2}$$

τ decay



- $m_\tau = 1.777$ GeV
- Several possible decays:

$$\begin{aligned} \tau^- &\to e^- \overline{\nu}_e \nu_\tau \\ \tau^- &\to \mu^- \overline{\nu}_\mu \nu_\tau \\ \tau^- &\to d\overline{u}\ \nu_\tau \end{aligned}$$

In last case, the $d\overline{u}$ turns into hadrons with 100% probability

- All diagrams look like μ-decay
- If $G_F^\mu = G_F^e = G_F$, predict:

$$\begin{aligned} \Gamma_{\tau^- \to e^-} &= \Gamma_{\tau^- \to \mu^-} \\ &= (m_\tau/m_\mu)^2\, \Gamma(\mu) \end{aligned}$$

(difference in available phase space)

- Using the measured τ-lifetime and BR, check consistency of G_F

$$\begin{aligned} G_F^\tau/G_F^\mu &= 1.0023 \pm 0.0033 \\ G_F^e/G_F^\mu &= 1.000 \pm 0.004 \end{aligned}$$

Lepton universality for G_F

- For quark decays, need a factor of 3 for color. Predict

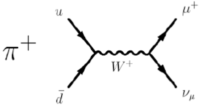
$$BR(\tau \to hadrons) = \frac{3}{3+1+1} = 60\%$$

- Experimental result:

$$BR(\tau \to hadrons) = (64.76 \pm 0.06)\%$$

Difference from 60% understood (QCD corrections; as for R)

π decay



$\tau_{\pi^+} = 2.6 \times 10^{-8}$ s

- $u\bar{d}$ annihilation into virtual W^+
- Depends on π^+ wave function at origin
  - Need phenomenological parameter that characterizes unknown wave function
- Write matrix element

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J_\pi^\mu \overline{u}(p)\gamma_\mu (1 - \gamma_5) v(k)$$

where $J_\pi^\mu = f(q^2)q^\mu$ since q^μ is the only available 4-vector

- But $q^2 = m_\pi^2$ so $J_\pi = f_\pi q^\mu$. f_π has units of mass (matrix element must be dimensionless)

- After spinor calculation, result for decay width:

$$\Gamma = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

- This came from:

$$|\mathcal{M}|^2 \sim G_F^2 m_\mu^2 \left(m_\pi^2 - m_\mu^2\right) f_\pi^2$$

$$\text{Phase Space} \sim \frac{|p|}{8\pi m_\pi^2}$$

π decay (2)

- From previous page

$$\Gamma = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

- Result for electron same with $m_\mu \to m_e$
- Thus

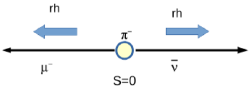
$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2}{m_\mu^2} \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2$$

- Since $m_e = 0.51$ MeV, $m_\mu = 105.65$ MeV and $m_{pi^+} = 139.57$ MeV

$$\frac{\Gamma_e}{\Gamma_\mu} \sim 1.2 \times 10^{-4}$$

This agrees with measurements

- Physically, result comes from helicity suppression

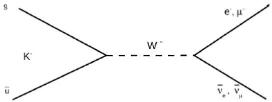


- Spin 0 pion, right-handed antineutrino forces μ^- to be right-handed
- But μ^- wants to be left-handed
  - rh component $\sim (v/c)^2 \sim m_\mu$
- The less relativistic the decay product is, the larger the decay rate

$K^{\pm}$ decay

- $K^{\pm}$ mass larger than $\pi^{\pm}$
- More options for decay

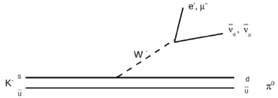
Leptonic



Same calculation as for $\pi^{\pm}$
Helicity suppression make decay rate to muons larger that to electrons

$$BR(K^- \to \mu^- \overline{\nu}_\mu) = (63.56 \pm 0.11) \times 10^{-2}$$
$$BR(K^- \to e^- \overline{\nu}_e) = (1.582 \pm 0.007) \times 10^{-5}$$

Semi-Leptonic

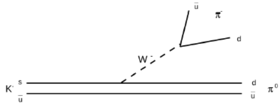


3-body decay: No helicity suppression

$$BR(K^- \to \pi^0 \mu^- \overline{\nu}_\mu) = (3.352 \pm 0.033)\ \%$$
$$BR(K^- \to \pi^0 e^- \overline{\nu}_e) = (5.07 \pm 0.04)\ \%$$

More phase space for decay to e

- Hadronic (several diagrams possible)



$$BR(K^- \to \pi^- \pi^0) = (20.67 \pm 0.08)\ \%$$
$$BR(K^- \to \pi^- \pi^0 \pi^0) = (1.760 \pm 0.023)\ \%$$
$$BR(K^- \to \pi^- \pi^- \pi^+) = (5.583 \pm 0.024)\ \%$$

G_F fenomenology

- Leptonic decays of μ and τ demonstrate that G_F is the same for all lepton species
- Leptoniv decay of charged pion and kaon tell us nothing about G_F since f_π and f_K (which depend on wave form ar origin) are unknown
- If we want to ask whether G_F is the same for hadronic currents as leptonic ones, we need to look at semileptonic decays
  - Analog of the β decay
- But, we have to make sure taht we are not affected by stron interactions corrections.

Is G_F really universal ?

- Muon decay rate is

$$\Gamma = \frac{G_F^2 {m_\mu}^5}{192\pi^3}$$

  in approximation where m_e ignored
- Same formula holds for nuclear β-decay
- A good choice of decay: $O^{14} \to N^{14*}\ e^+\nu_e\ (0^+ \to 0^+)$
- Correcting for available phase space we find

$$\begin{aligned} G_\mu &= 1.166 \times 10^{-5} \\ G_\beta &= 1.136 \times 10^{-5} \end{aligned}$$

Close but not the same!

- What's going on?

Is G_F really universal ? (2)

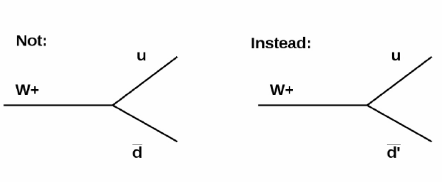
- Compare the following:

	Decay	Quark Level Decay
0^{14}:	$p \to n e^+ \nu_e$	$u \to d e^+ \nu_e$
	$\pi^- \to \pi^0 e^- \overline{\nu}_e$	$d \to u e^- \overline{\nu}_e$
	$K^- \to \pi^0 e^- \overline{\nu}_e$	$s \to u e^- \overline{\nu}_e$
	$\mu^+ \to \overline{\nu}_\mu e^+ \nu_e$	

- After correcting for phase space factors, G_F obtained from p and π^- agree with each other, but are slightly less than obtained from μ.
- G_F obtained from K^- decay appears much smaller
- Either G_F is not universal, or something else is going on!

An Explanation: weak eigenstates

- Suppose strong and weak eigenstates of quarks not the same
- Weak coupling:



- Here d' is an admixture of down-type quarks
- Normalization of w.f. for quarks means if $d' = \alpha d + \beta s$, then $\sqrt{\alpha^2 + \beta^2} = 1$
- Can force this normalization by writing α and β in terms of an angle

$$d' = d \cos\theta_C + s \sin\theta_c$$

or

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_C \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

The Cabbibo angle

- Using

$$d' = d \cos\theta_C + s \sin\theta_C$$

we predict

$$p\&\pi \text{ decay} \approx G_F^2 \cos^2\theta_C$$

$$K \text{ decay} \approx G_F^2 \sin^2\theta_C$$

$$\mu \text{ decay} \approx G_F^2$$

- Using experimental measurements, we found

$$\cos\theta_C = 0.97420 \pm 0.00021$$

$$\sin\theta_C = 0.2243 \pm 0.0005$$

- However, in addition to the d' there should be an orthogonal down-type combination

$$s' = s \cos\theta_C - d \sin\theta_C$$

Does it interact weakly?

4th quark - early 70's discussion

- It's odd to have one charge 2/3 quark and two charge -1/3 quarks
- Suppose there is a heavy 4^{th} quark
- We could then have two families of quarks. In strong basis:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}$$

Call this new quark "charm"

- Then, the weak basis is

$$\begin{pmatrix} u \\ d' = d\cos\theta_C + s\sin\theta_c \end{pmatrix}, \quad \begin{pmatrix} c \\ s' = s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$

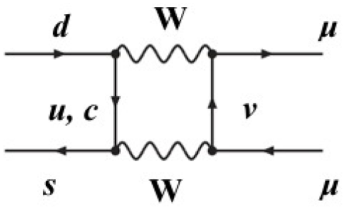
- There is a good argument for this charm quark in addition to G_F ...

GIM mechanism

- Glashow, Iliopoulous, Maiani (GIM) proposed existence of this 4^{th} quark (charm)
- Charm couples to the s' in same way u couples to the d'
- Reason for introducing charm: to explain why flavor changing neutral currents (FCNC) are highly suppressed
- Two examples of FCNC suppression:
  1. $BR(K_L^0 \to \mu^+\mu^-) = 6.84 \times 10^{-9}$
  2. $BR(K^+ \to \pi^+\nu\nu)/BR(K^+ \to \pi^0\mu\nu) < 10^{-7}$
- Why are these decay rates so small?
- It turns out that there is also a Z that couples to $f\overline{f}$ pairs, but it does not change flavor (same as γ)
- If only vector boson was the $W^{\pm}$, would require two bosons to be exchanged
  - Need second order charged weak interactions, but even this would give a bigger rate than seen unless there is a cancellation

GIM mechanism box diagrams

- Consider the "box" diagram



- $\mathcal{M}$ term with u quark $\propto \cos\theta_C \sin\theta_C$
- $\mathcal{M}$ term c quark $\propto -\cos\theta_C \sin\theta_C$
- Same final state, so we add $\mathcal{M}$'s
- Terms cancel in limit where we ignore quark masses

The cancelation is not accidental

- Matrix relating strong basis to weak basis is unitary

$$d'_i = \sum_j U_{ij} d_j$$

- Therefore is we sum over down-type quark pairs

$$\begin{aligned} \sum_i \overline{d}'_i d'_i &= \sim_{ijk} \overline{d}_j U^\dagger_{ji} U_{ik} d_k \\ &= \sum_j \overline{d}_j d_j \end{aligned}$$

- If an interaction is diagonal in the weak basis, it stays diagonal in the strong basis
- Independent of basis, there are no $d \longleftrightarrow s$ transitions

No flavor changing neutral current weak interactions
(up to terms that depend on the quark masses)

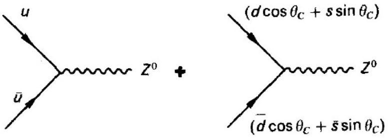
GIM mechanism historical view

- it was noted that all neutral-current transitions had ΔS=0
  - *strange* quark would transform into *up* but not into *down*
  - no "flavor-changing neutral currents"
    - … at tree level!
- Glashow, Iliopoulos and Maiani proposed a mechanism to explain this
  - now called the GIM mechanism

GIM mechanism historical view

$$\underbrace{u\bar{u} + (d\bar{d}\cos^2\theta_C + s\bar{s}\sin^2\theta_C)}_{\Delta S=0} + \underbrace{(s\bar{d} + \bar{s}d)\sin\theta_C\cos\theta_C}_{\Delta S=1},$$

$$\begin{pmatrix} u \\ d_C \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}$$



$$\underbrace{u\bar{u} + c\bar{c} + (d\bar{d} + s\bar{s})\cos^2\theta_C + (s\bar{s} + d\bar{d})\sin^2\theta_C}_{\Delta S=0}$$
$$+ \underbrace{(s\bar{d} + \bar{s}d - \bar{s}d - s\bar{d})\sin\theta_C\cos\theta_C}_{\Delta S=1}.$$

More than two generation

- Generalize to N families of quark ($N = 3$ as far as we know)
- U is a unitary $N \times N$ matrix and d'_i is an N-column vector

$$d'_i = \sum_{j=1}^{N} Y_{ij} d_j$$

- How many independent parameters do we need to describe U?
  - $N \times N$ matrix: N^2 elements
  - But each quark has an unphysical phase: can remove $2N - 1$ phases (leaving one for the overall phase of U)
  - So, U has $N^2 - (2N - 1)$ independent elements
- However, an orthogonal $N \times N$ matrix has $\frac{1}{2}N(N-1)$ real parameters
  - So U has $\frac{1}{2}N(N-1)$ real parameters
  - $N^2 - (2N-1) - \frac{1}{2}N(N-1)$ imaginary phases $(= \frac{1}{2}(N-1)(N-2))$
- $N = 2$ 1 real parameter, 0 imaginary
- $N = 3$ 3 real parameters, 1 imaginary
- Three generations requires an imaginary phase: CP Violation inherent

CKM matrix

- Write hadronic current

$$J^{\mu} = -\frac{g}{\sqrt{2}} \left(\overline{u}\ \overline{c}\ \overline{t} \right) \gamma_{\mu} \frac{(1-\gamma_5)}{2} V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- V_{CKM} gives mixing between strong (mass) and (charged) weak basis
- Often write as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Wolfenstein parameterization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Here λ is the $\approx \sin\theta_C$.

CKM matrix (2)

- From previous page

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Impose Unitary and use all experimental measurements

$$\lambda = 0.22453 \pm 0.00044 \qquad A = 0.836 \pm 0.015$$
$$\rho = 0.122^{+0.018}_{-0.17} \qquad \eta = 0.355^{+0.12}_{-0.11}$$

- Result for the magnitudes of the elements is:

$$\begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359 \pm 0.00011 & 0.04214 \pm 0.00076 \\ 0.00896 \pm 00024 & 0.04133 \pm 0.00074 & 0.999105 \pm 000032 \end{pmatrix}$$

b quark third family

Two families were known in 1977. In the weak interactions, the two families appear rotated (Cabibbo angle)

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix};$$

Third family

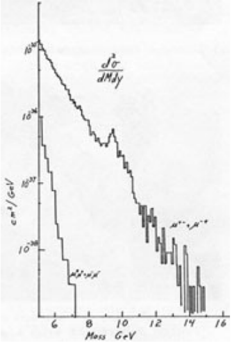
$$\begin{pmatrix} d_C \\ s_C \end{pmatrix} = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

The mixing in fact, involves all three families (CKM matrix)

Upsilon meson discovery : b quark

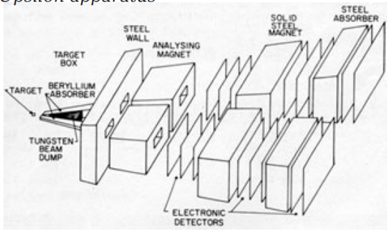
''Observation of a Dimuon Resonance at 9.5 GeV in 400 GeV Proton-Nucleus Collisions''

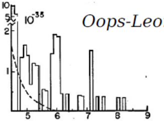
Summer of 1977, a team of physicists, led by Leon M. Lederman, working on experiment 288 in the proton center beam line of the Fermilab fixed target areas discovered the Upsilon Υ



1970 proposal: study the rare events that occur when a pair of muons or electrons is produced in a collision of the proton beam from the acccelerator on a platinum target Only one Upsilon is produced for every 100 billion protons which strike the target

The Upsilon apparatus

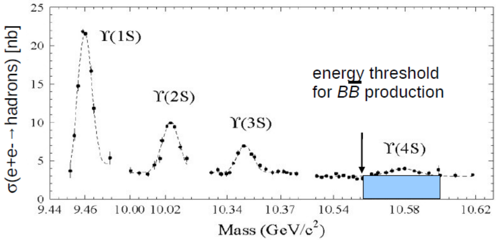


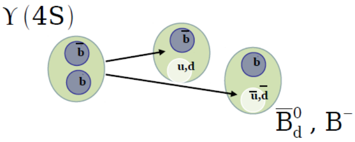


Oops-Leon !!

''The Upsilon fits very nicely into the picture of a super-atom consisting of the bound state of a bottom quark and antiquark.''

Upsilon meson discovery : b quark



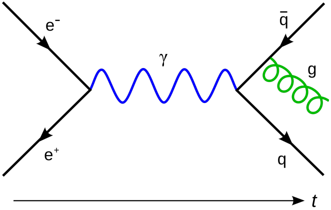


- 2 B's and nothing else !
- 2 B mesons are created simultaneously in a L=1 coherent state
  ⇒ before first decay, the final states contains a B and a $\overline{B}$

R ratio :Quarks electric charge, number of QCD charges (colors)

R is the ratio of the hadronic cross section to the muon cross section in electron–positron collisions:

$R = \frac{\sigma(e^+e^-)\rightarrow \text{hadron}}{\sigma(e^+e^-)\rightarrow \mu^+\mu^-}$



R also provides experimental confirmation of the electric charge of quarks, in particular the charm quark and bottom quark, and the existence of three quark colors. A simplified calculation of R yields

$R = 3\sum e_q^2$,

where the sum is over all quark flavors with mass less than the beam energy. e_q is the electric charge of the quark, and the factor of 3 accounts for the three colors of the quarks. QCD corrections to this formula have been calculated.

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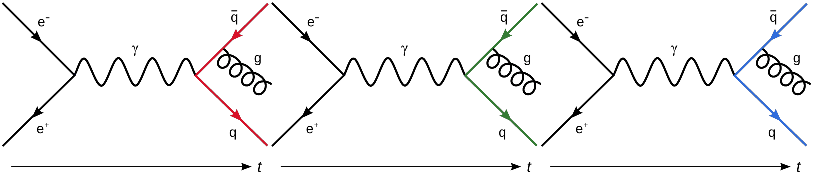

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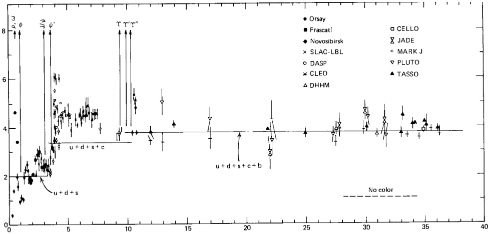

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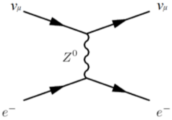
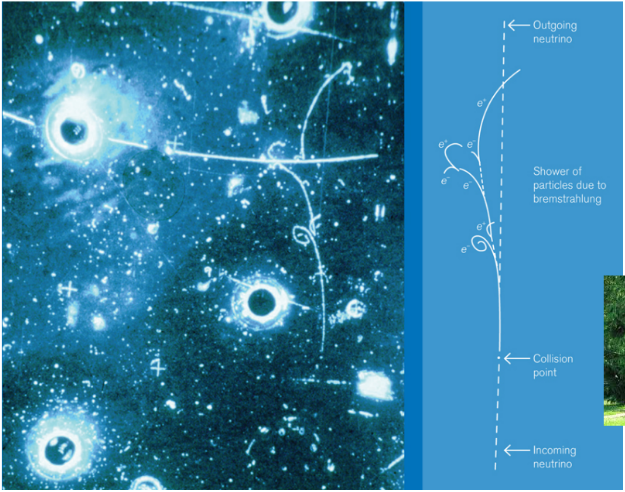
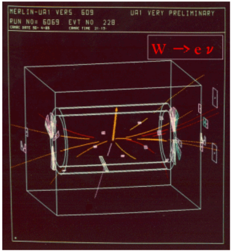
Weak neutral current discovery - indirect evidence of Z^0

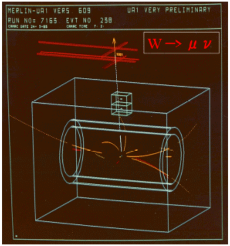


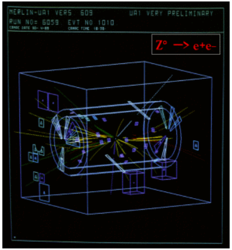


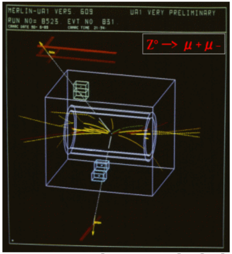
Gargamelle (Cern)

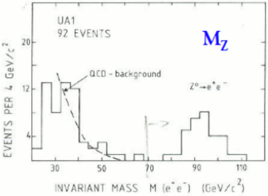
W and Z^0 discovery







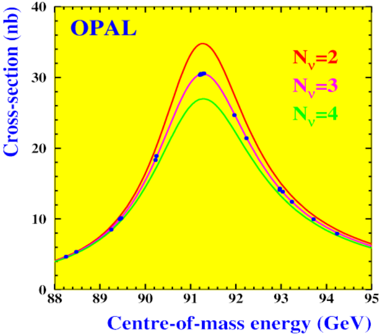




Rubbia and van der Meer were promptly awarded the 1984 Nobel Prize in Physics.

Number of generations?

- Determination of the Z^0 line-shape:
  - Reveals the number of 'light neutrinos'
  - Fantastic precision on Z^0 parameters
    - Corrections for phase of moon, water level in Lac du Geneve, passing trains,…



N_ν	2.984±0.0017
M_{Z0}	91.1852±0.0030 GeV
Γ_{Z0}	2.4948 ±0.0041 GeV

Existence of only 3 neutrinos

- Unless the undiscovered neutrinos have mass $m_\nu > M_Z/2$

Recapitulation