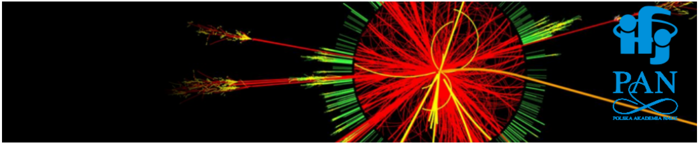


Particle Physics- Standard Model(3)

A. Bożek, IFJ PAN Kraków

Outline

This is an “introduction”

- Historical overview.
- the elementary particles
- the elementary forces
- Symmetries:
  - Gauge symmetry
  - Problem of mass
  - Spontaneous symmetry breaking

This lecture is not a complete course in particle physics and will only touch some most general problems.
Further reading:

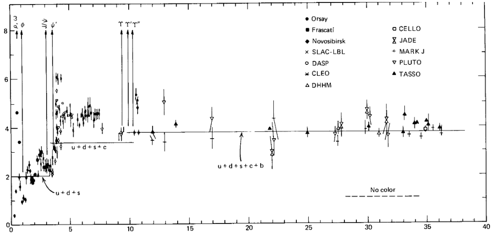
- D. H. Perkins, “Introduction to High Energy Physics”,
- F. Halzen, A. Martin: “Quarks and Leptons”.

Further lecture to watch listen on SM and BSM physics:

- Prof. Yuval Grossman (Cornell U.)
- Standard Model and Flavor - Lecture (`https://www.youtube.com/watch?v=GGzRdiBd8w8`)

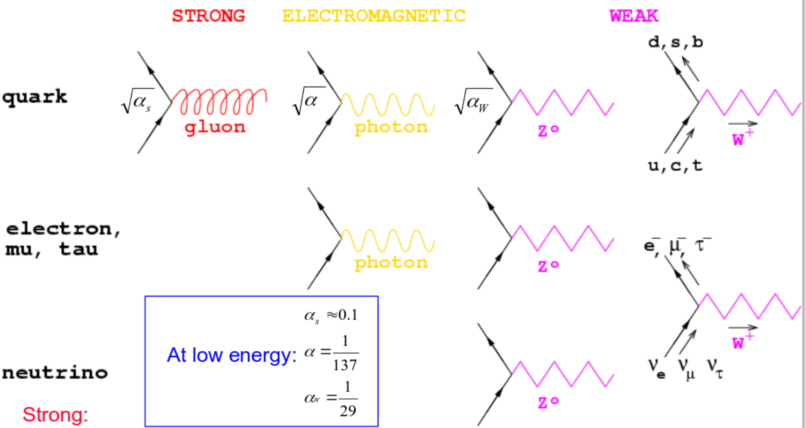
R ratio :Quarks electric charge, number of QCD charges (colors)

R is the ratio of the hadronic cross section to the muon cross section in electron–positron collisions:
$R = \frac{\sigma(e^+e^- \to \text{hadron})}{\sigma(e^+e^- \to \mu^+\mu^-)}$



R also provides experimental confirmation of the electric charge of quarks, in particular the charm quark and bottom quark, and the existence of three quark colors. A simplified calculation of R yields
$R = 3\sum e_q^2$,
where the sum is over all quark flavors with mass less than the beam energy. e_q is the electric charge of the quark, and the factor of 3 accounts for the three colors of the quarks. QCD corrections to this formula have been calculated.

The main standard model verticies



Strong:
All quarks (and anti-quarks)
No change of flavour

EM:
All charged particles
No change of flavour

Weak neutral current:
All particles
No change of flavour

Weak charged current:
All particles
Flavour changes

Feynman diagrams revisited

$\sigma \propto |T_{fi}|^2$

The Feynman diagrams give us the amplitude, c.f. ψ in QM whereas probability is $|\psi|^2$

(1) So, two electro-magnetic vertices:
e.g. $e^-e^+ \rightarrow \mu^-\mu^+$ amplitude gets factor from each vertex $\sqrt{\alpha}\sqrt{\alpha} = \alpha$

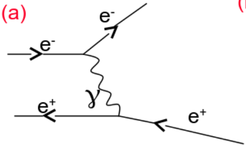
Crosssection gets amplitude squared $\propto \alpha^2$

for $e^-e^+ \rightarrow q\bar{q}$ with quarks of charge q (1/3 or 2/3) $\propto (q\sqrt{\alpha}\sqrt{\alpha})^2 = q^2\alpha^2$

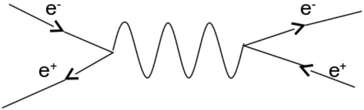
- Also remember : u,d,s,c,t,b quarks and they each come in 3 colours
- Scattering from a nucleus would have a Z term

(2) If we have several diagrams contributing to same process, we much consider *interference* between them e.g.

(a)



(b)



Same final state, get terms for $(a+b)^2 = a^2 + b^2 + ab + ba$

Quantum Chromodynamics (QCD)

QED – mediated by spin 1 bosons (photons) coupling to conserved electric charge
QCD – mediated by spin 1 bosons (gluons) coupling to conserved colour charge

u,d,c,s,t,b have same 3 colours (red,green,blue), so identical strong interactions [c.f. isospin symmetry for u,d], leptons are colourless so don't feel strong force

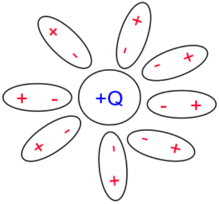
- Significant difference from QED:
  - photons have no electric charge
  - But gluons do have colour charge – eight different colour mixtures.

Hence, gluons interact with each other. Additional Feynman graph vertices:

Self-interaction

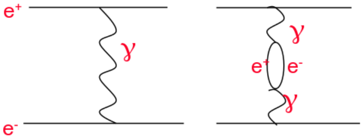
These diagrams and the difference in size of the coupling constants are responsible for the difference between EM and QCD

Running constant in QED



Charge +Q in dielectric medium
Molecules nearby screened,
At large distances don't see full charge
Only at small distances see +Q

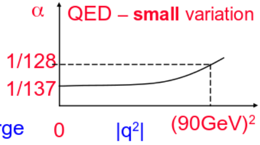
Also happens in vacuum – due to spontaneous production of virtual e^+e^- pairs



And diagrams with
two loops ,three loops….
each with smaller effect: α, α^2….

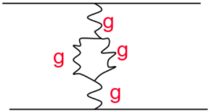
As a result coupling strength grows with $|q^2|$ of photon,

higher energy ⇒smaller wavelength gets closer to bare charge

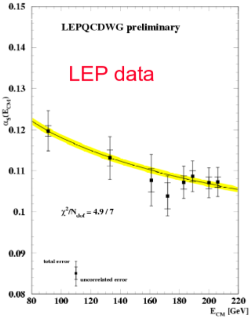


Running constant in QCD

- Exactly same replacing photons with gluons and electrons with quarks
- But also have gluon splitting diagrams



This gives anti-screening effect.
Coupling strength **falls** as $|q^2|$ increases

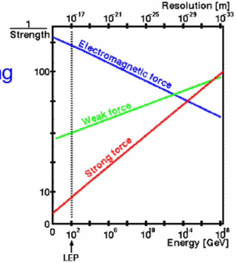


Strong variation in strong coupling
From $\alpha_s \approx 1$ at $|q^2|$ of 1 GeV^2
To α_s at $|q^2|$ of 10^4 GeV^2

Hence:
- Quarks scatter freely at high energy

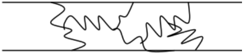
- Perturbation theory converges very Slowly as $\alpha_s \approx 0.1$ at current expts And lots of gluon self interaction diagrams

Grand Unification ?



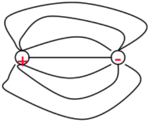
Range of strong forces

Gluons are massless, hence expect a QED like long range force
But potential is changed by gluon self coupling



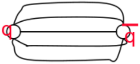
Qualitatively:

QED



Standard EM field

QCD

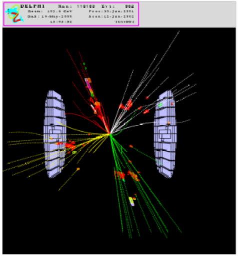


Field lines pulled into strings
By gluon self interaction

Form of QCD potential:

$$V_{QCD} = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

Coulomb like to start with,
but on ~1 fermi scale energy
sufficient for fragmentation



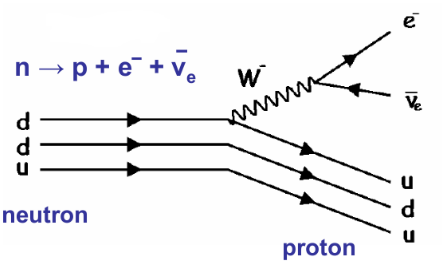
QCD – energy/unit length stored in field ~ constant.
Need infinite energy to separate qqbar pair.
Instead energy in colour field exceeds $2m_q$ and new
q qbar pair created in vacuum

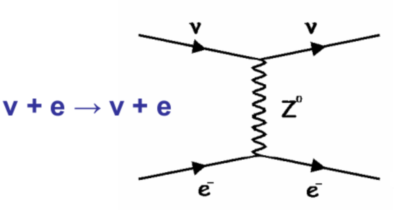
This explains absence of free quarks in nature.
Instead jets (fragmentation) of mesons/baryons

NB Hadrons are colourless, Force between
hadrons due to pion exchange. 140MeV→1.4fm

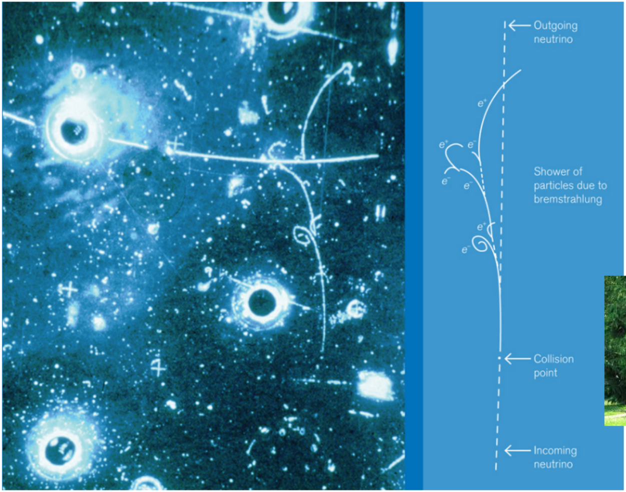
Weak interaction

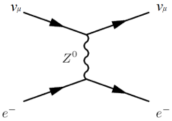
processes related to change flavor (quark decay)





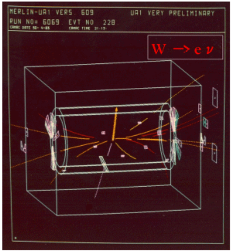
Weak neutral current discovery - indirect evidence of Z^0

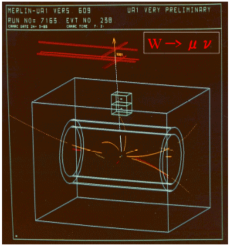


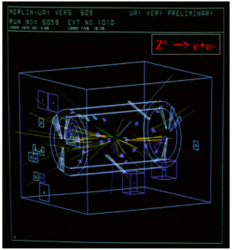


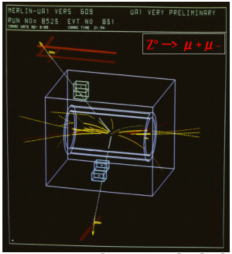
Gargamelle (Cern)

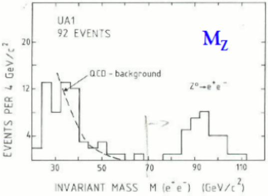
W and Z^0 discovery







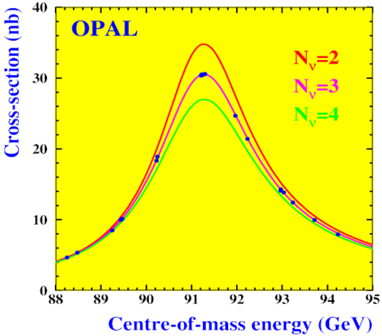




Rubbia and van der Meer were promptly awarded the 1984 Nobel Prize in Physics.

Number of generations?

- Determination of the Z^0 line-shape:
  - Reveals the number of 'light neutrinos'
  - Fantastic precision on Z^0 parameters
    - Corrections for phase of moon, water level in Lac du Geneve, passing trains,…



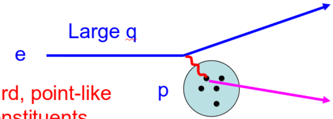
N_ν	2.984±0.0017
M_{Z0}	91.1852±0.0030 GeV
Γ_{Z0}	2.4948 ±0.0041 GeV

Existence of only 3 neutrinos

- Unless the undiscovered neutrinos have mass $m_{\nu}>M_Z/2$

Partons are reality !!!

New “Rutherford” like experiments, but with much higher energy.
probing structure of proton itself



Hard, point-like
Constituents,
behaving as if they are free particles

Partons are reality !!!

New “Rutherford” like experiments, but with much higher energy.
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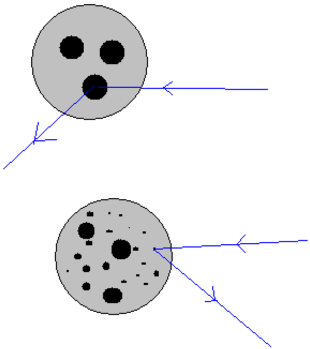
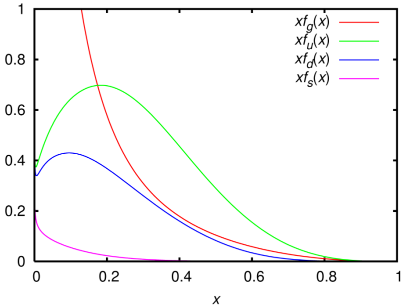


The scattering particle only sees the valence partons. At higher energies, the scattering particles also detects the sea partons.



The probability density for finding a particle with a certain longitudinal momentum fraction x at resolution scale q^2. inside proton.

Cabibo Kobayashi Maskawa matrix (CKM)

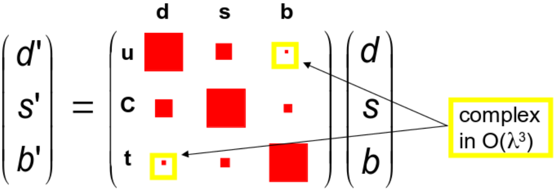
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates — **CKM matrix** — **mass eigenstates**

Unitarity

$$V_{CKM} V^{+}_{CKM} = 1$$

Magnitude of elements



Cabibo Kobayashi Maskawa matrix (CKM)

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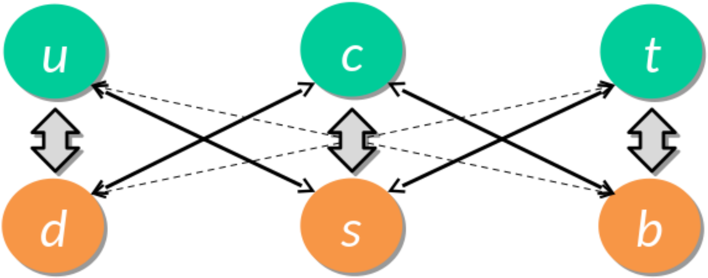
Unitarity

$$V_{CKM} V_{CKM}^{+} = 1$$

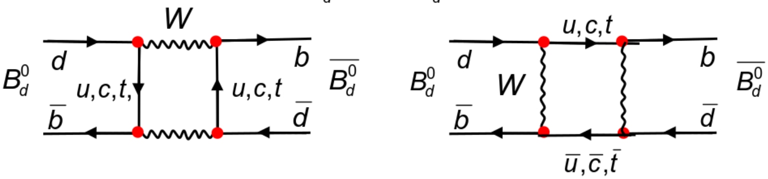
weak eigenstates **CKM matrix** **mass eigenstates**



Mixing of neutral mesons

As result of the quark mixing the Standard Model predicts oscillations of neutral mesons:

$$B_d^0 \Leftrightarrow \overline{B_d^0}$$



Similar graphs for other neutral mesons:

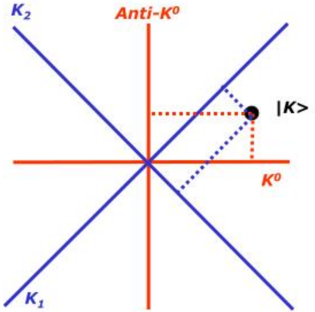
Neutral mesons: $\left|P^0\right\rangle$: $K^0 = \left|d\bar{s}\right\rangle$ $D^0 = \left|\bar{u}c\right\rangle$ $B_d^0 = \left|d\bar{b}\right\rangle$ $B_s^0 = \left|s\bar{b}\right\rangle$

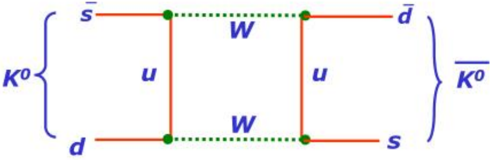
$\left|\overline{P^0}\right\rangle$: $\overline{K^0} = \left|\bar{d}s\right\rangle$ $\overline{D^0} = \left|\bar{u}c\right\rangle$ $\overline{B_d^0} = \left|d\bar{b}\right\rangle$ $\overline{B_s^0} = \left|s\bar{b}\right\rangle$

discovery of mixing 1960 2019 1987 2006

Mixing of neutral mesons

- $K^0 \to \overline{K^0}$ transition
  - Note 1: Two W bosons required ($\Delta S=2$ transition)
  - Note 2: many vertices, but still lowest order process...

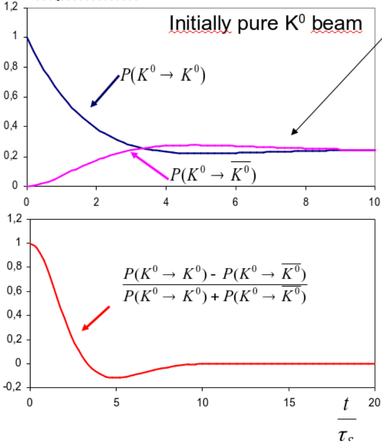




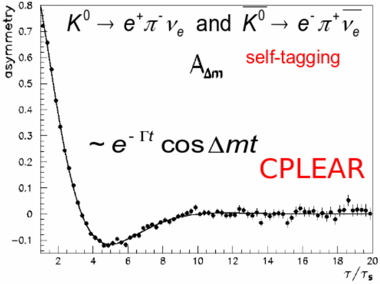
Neutral Kaons system

After the lifetime of the K_s the K^0 consists entirely out of K's, which are essentially an equal mixture of K^0 and K^0.

Expectation



Measurement



Discret Symmetries

- Three fundamental discrete Symmetries:
  - Parity (P) = Space inversion: $\vec{x} \rightarrow -\vec{x}$
  - Charge Conjugation (C) = particle $\rightarrow$ antiparticle
  - Time Reversal (T) = Time inversion: $x_0 \rightarrow -x_0$
- CPT Theorem:

> Assuming only local interactions, Lorentz invariance and Causality the product of the three symmetries C $\times$ P $\times$ T is always a symmetry.

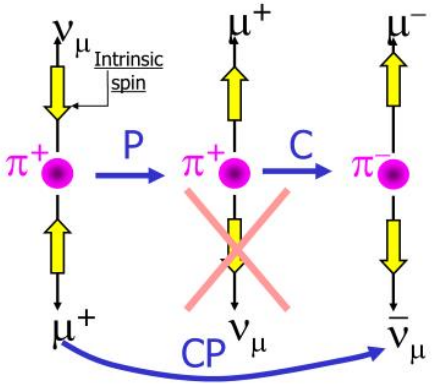
- ... this is always true for a Lagrangian field theory (with causal particle propagators)

The History of CP

- Historically it was believed that all three discrete symmetries hold separately
- First revolution:
  1956 Lee and Yang suggest P violation
  quickly experimentally confimed by Wu et al.
- ... CP was still believed to be conserved,
- until 1964:
- Second Revolution:
  Cronin and Fitch discover CP violation
  CPT theorem: CP Violation = T Violation

The Weak force and C,P parity violation

- What about C+P ≡ CP symmetry?
  - CP symmetry is parity conjugation (x,y,z → -x,-y,z) followed by charge conjugation ($X \to \overline{X}$)



100% P violation:
All ν's are lefthanded
All $\bar{\nu}$'s are righthanded

CP appears to be preserved in weak interaction!

A first look at CP violation in Kaons

- There are two different neutral Kaons:

$$|K^0\rangle = |\bar{s}d\rangle \qquad \text{and} \qquad |\bar{K}^0\rangle = |\bar{d}s\rangle$$

- They are pseudoscalar particles:

$$\mathrm{P}|K^0\rangle = -|K^0\rangle \qquad \text{and} \qquad \mathrm{P}|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

- Charge conjugation is $q \leftrightarrow \bar{q}$, hence

$$\mathrm{C}|K^0\rangle = |\bar{K}^0\rangle \qquad \text{and} \qquad \mathrm{C}|\bar{K}^0\rangle = |K^0\rangle$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \quad \text{and} \quad |K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

A first look at CP violation in Kaons

- Kaons decay either into two or three pions (in an *S* wave state)
- CP Quantum numbers of the (neutral) final states

$$\mathrm{CP}|\pi\pi\rangle = |\pi\pi\rangle \quad \text{and} \quad \mathrm{CP}|\pi\pi\pi\rangle = -|\pi\pi\pi\rangle$$

- Assuming CP Conservation:

$$|K_S\rangle \to |\pi\pi\rangle \quad \text{and} \quad |K_S\rangle \not\to |\pi\pi\pi\rangle$$
$$|K_L\rangle \not\to |\pi\pi\rangle \quad \text{and} \quad |K_L\rangle \to |\pi\pi\pi\rangle$$

- Cronin and Fitch: $|K_L\rangle \to |\pi\pi\rangle$
- We are back to CPT as the only real symmetry

A first look at CP violation in Kaons

- The kaons are produced in mass eigenstates:
  - $|K^0>$: $s\bar{d}$
  - $|\bar{K}^0>$: $\bar{d}s$
- The CP eigenstates are:
  - CP=+1: $|K_1> = 1/\sqrt{2}\,(|K^0> - |\bar{K}^0>)$
  - CP= -1: $|K_2> = 1/\sqrt{2}\,(|K^0> + |\bar{K}^0>)$
- The kaons decay as short-lived or long-lived kaons:
  - $|K_S>$: predominantly CP=+1 $\quad |K_S\rangle = \frac{|K_1\rangle + \varepsilon|K_2\rangle}{\sqrt{1+|\varepsilon|^2}}$,
  - $|K_L>$: predominantly CP= -1 $\quad |K_L\rangle = \frac{|K_2\rangle + \varepsilon|K_1\rangle}{\sqrt{1+|\varepsilon|^2}}$.

- $\eta_{+-} \equiv \dfrac{\langle \pi^+\pi^- | H | K_L \rangle}{\langle \pi^+\pi^- | H | K_S \rangle}$
- $\eta_{+-} = (2.236 \pm 0.007) \times 10^{-3}$
- $|\varepsilon| = (2.232 \pm 0.007) \times 10^{-3}$

CP Violation in weak interactions

- Kaons decay either into two or three pions (in an S wave state)
- CP Quantum numbers of the (neutral) final states

$$\mathrm{CP}|\pi\pi\rangle = |\pi\pi\rangle \quad \text{and} \quad \mathrm{CP}|\pi\pi\pi\rangle = -|\pi\pi\pi\rangle$$

- Assuming CP Conservation:

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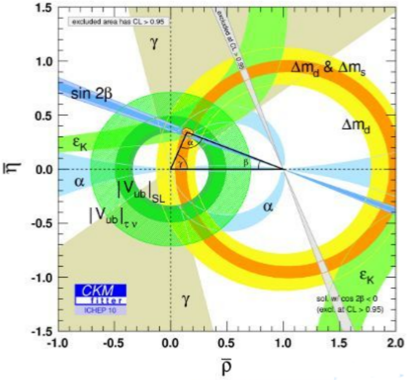
- Cronin and Fitch: $|K_L\rangle \to |\pi\pi\rangle$
- We are back to CPT as the only real symmetry

CP Violation in weak interactions B-factory

- In 2000: The *B* Factories go into operation:
- First observation of non-Kaon CP Violation
- CP Violation in the *B* system is in (almost too good) agreement with the predictions of KM:



Why we need CP violation

- CP is needed to generate the Baryon-Antibaryon Asymmetry of the Universe: $\Delta = n_{\mathrm{Bar}} - n_{\overline{\mathrm{Bar}}} \neq 0$
- The Sakharov Conditions: (Sakharov 1967)

1. Baryon number violation: $H_{\mathrm{eff}}(\Delta \neq 0) \neq 0$
2. CP violation: $\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$
3. Absence of thermal equilibrium: Time is irrelevant in equilibrium, hence CPT implies CP

- The fundamental theory has to have CP violation
- NB: The SM has $H_{\mathrm{eff}}(\Delta \neq 0) \neq 0$

P transformation

- Parity P: $\vec{x} \to -\vec{x}$
- There has to be an operator P in Hilbert Space
- If P is a symmetry: $\mathrm{P}|0\rangle = |0\rangle \quad [H\,,\,\mathrm{P}] = 0$
- Scalar Field:

$$\mathrm{P}\phi(x_0, \vec{x})\mathrm{P}^\dagger = \phi(x_0, -\vec{x})$$

- Vector Field:

$$\left.\begin{array}{l} \mathrm{P}A^0(x_0, \vec{x})\mathrm{P}^\dagger = A^0(x_0, -\vec{x}) \\ \mathrm{P}A^i(x_0, \vec{x})\mathrm{P}^\dagger = -A^i(x_0, -\vec{x}) \end{array}\right\} \mathrm{P}A^\mu(x_0, \vec{x})\mathrm{P}^\dagger = A_\mu(x_0, -\vec{x})$$

P transformation

- Spinor field:

$$\mathrm{P}\psi(x_0, \vec{x})\mathrm{P}^\dagger = \gamma_0 \psi(x_0, -\vec{x})$$
$$\mathrm{P}\bar{\psi}(x_0, \vec{x})\mathrm{P}^\dagger = \bar{\psi}(x_0, -\vec{x})\gamma_0$$

- This is designed such that $\bar{\psi}(x)\gamma_\mu\psi(x)$ behaves like a vector field. Homework: check this!
- P invariance means that the action is invariant:

$$\mathrm{P}\, S\, \mathrm{P}^\dagger = S$$

C transformation

- Charge Conjugation C: particle $\leftrightarrow$ antiparticle
- There has to be an operator C in Hilbert Space
- If C is a symmetry: $\mathrm{C}|0\rangle = |0\rangle \quad [H, \mathrm{P}] = 0$
- Scalar Field:

$$\mathrm{C}\phi(x)\mathrm{C}^{\dagger} = \phi(x)^{\dagger}$$

- Vector Field:

$$\mathrm{C}A^{\mu}(x)\mathrm{C}^{\dagger} = -A^{\mu}(x)$$

- Spinor Field:

$$\mathrm{C}\psi(x)\mathrm{C}^{\dagger} = \mathcal{C}(\bar{\psi}(x))^{T}$$

CP violation from complex couplings

- Assume that the Lagrange operator is

$$\mathcal{L}(x) = \sum_i a_i \mathcal{O}_i(x) + \text{h.c.} = \sum_i \left(a_i \mathcal{O}_i(x) + a_i^* \mathcal{O}_i^\dagger(x) \right)$$

- Assume: The $\mathcal{O}_i$ behave like complex scalar fields

$$\text{CP}\,\mathcal{L}(x)\,\text{CP}^\dagger = \sum_i \left(a_i \mathcal{O}_i^\dagger(\bar{x}) + a_i^* \mathcal{O}_i(\bar{x}) \right) \quad \bar{x} = (x_0, -\vec{x})$$

$$\text{CP}\,S\,\text{CP}^\dagger - S = -2i \int d^4x \sum_i \left(\text{Im}\,a_i\, \mathcal{O}_i(x) - \text{Im}\,a_i \mathcal{O}_i^\dagger(x) \right)$$

CP violation, if one of the couplings is complex !

CP in the Standard Model

There are two sources of CP violation in the SM:

1. CKM CP violation: CP Violation encoded in the quark (and lepton) mass matrices
2. Strong CP violation: CP violation through the vacuum structure of QCD

- (1) is phenomenologically confirmed
- (2) remains an open question

Structure of the Standard Model

- SM is a chiral gauge theory: Left and right handed components of fermions are in different multiplets
- $\rightarrow$ Implementation of Parity Violation
- $\rightarrow$ Fermion mass terms require symmetry breaking!

$$\mathcal{L}_{\text{mass}} = m\bar{\psi}_L\psi_R + \text{h.c.}$$

- There are three quarks with electric charge $+2/3e$: Up-type quarks
- There are three quarks with electric charge $-1/3e$: Down-type quarks

Structure of the Standard Model

- All quarks are known to be massive
  $\rightarrow$ we need both left and right handed components

$$\mathcal{U}_L = \begin{bmatrix} u_L \\ c_L \\ t_L \end{bmatrix} \quad \mathcal{U}_R = \begin{bmatrix} u_R \\ c_R \\ t_R \end{bmatrix} \quad \mathcal{D}_L = \begin{bmatrix} d_L \\ s_L \\ b_L \end{bmatrix} \quad \mathcal{D}_R = \begin{bmatrix} d_R \\ s_R \\ b_R \end{bmatrix}$$

- Mass terms: Two 3×3 mass matrices:

$$\mathcal{L}_{\text{mass}} = \bar{\mathcal{U}}_L \cdot M_u \cdot \mathcal{U}_R + \bar{\mathcal{D}}_L \cdot M_d \cdot \mathcal{D}_R$$

- M_u and M_d originate from spontaneous symmetry breaking:

$$M_u = Y_u \langle v \rangle \quad M_d = Y_d \langle v \rangle$$

Structure of the Standard Model complex phases

- Origin of CKM-like CP violation:
  Quark Mass Matrices = Quark Yukawa Couplings
- The two mass matrices do not commute:

$$[M_u\,,\ M_d] \neq 0$$

- Relative rotation of the Eigenbases of M_u vs. M_d:
  CKM matrix V_{CKM}

$$M_u^{\mathrm{diag}} = V_{\mathrm{CKM}}^{\dagger} \cdot M_d^{\mathrm{diag}} \cdot V_{\mathrm{CKM}}$$

- The CKM matrix is unitary:

$$V_{\mathrm{CKM}}^{\dagger} \cdot V_{\mathrm{CKM}} = 1 = V_{\mathrm{CKM}} \cdot V_{\mathrm{CKM}}^{\dagger}$$

Structure of the Standard Model CKM matrix

- Express everything in terms of mass eigenstates: Redefinition of the fields

$$\mathcal{D}' = V_{\rm CKM} \cdot \mathcal{D}$$

- The CKM matrix reappears ONLY in the charged current interaction

$$\mathcal{L}_{\rm CC} = \bar{U}_L(\gamma^\mu W^{\pm}_\mu) \cdot V_{\rm CKM} \cdot \mathcal{D}_L + \text{h.c.}$$

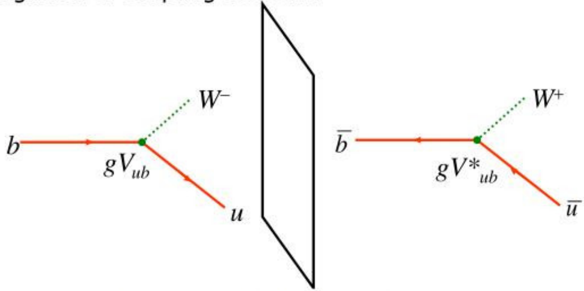
- Usual definition

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM complex phases and CP violation

Why complex phases matter

- CP conjugation of a W boson vertex involves complex conjugation of coupling constant



Above process violates CP if $V_{ub} \neq V_{ub}^*$

- With 2 generations V_{ij} is always real and $V_{ij} \equiv V_{ij}^*$
- *With 3 generations V_{ij} can be complex* → ***CP violation built into weak decay mechanism!***

CKM interpretation

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Off diagonal zeros of $V^\dagger_{CKM} V_{CKM} = 1 = V_{CKM} V^\dagger_{CKM}$
- $V^\dagger_{CKM} V_{CKM} = 1 : \begin{cases} V_{ub} V^*_{ud} + V_{cb} V^*_{cd} + V_{tb} V^*_{td} = 0 \\ V_{ub} V^*_{us} + V_{cb} V^*_{cs} + V_{tb} V^*_{ts} = 0 \\ V_{us} V^*_{ud} + V_{cs} V^*_{cd} + V_{ts} V^*_{td} = 0 \end{cases}$
- $V_{CKM} V^\dagger_{CKM} = 1 : \begin{cases} V_{ud} V^*_{td} + V_{us} V^*_{ts} + V_{ub} V^*_{tb} = 0 \\ V_{ud} V^*_{cd} + V_{us} V^*_{cs} + V_{ub} V^*_{cb} = 0 \\ V_{cd} V^*_{td} + V_{cs} V^*_{ts} + V_{cb} V^*_{tb} = 0 \end{cases}$

Wolfenstein Parametrisation of CKM

- Diagonal CKM matrix elements are almost unity
- CKM matrix elements decrease as we move off the diagonal
- Wolfenstein Parametrization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix}$$

- Expansion in $\lambda \approx 0.22$ up to λ^3
- A, ρ, η of order unity

CKM triangle interpretation

Deriving the triangle interpretation

- Starting point: the 9 unitarity constraints on the CKM matrix

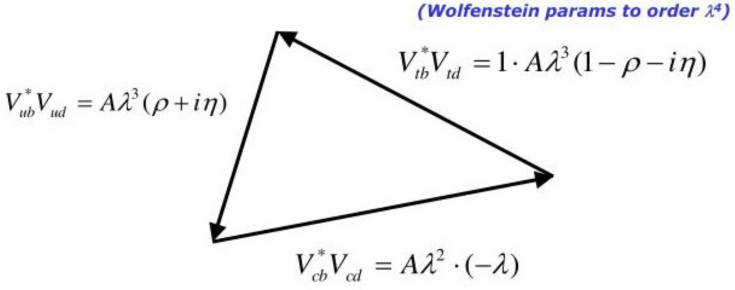
$$V^{+}V = \begin{pmatrix} V^{*}_{ud} & V^{*}_{cd} & V^{*}_{td} \\ V^{*}_{us} & V^{*}_{cs} & V^{*}_{ts} \\ V^{*}_{ub} & V^{*}_{cb} & V^{*}_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Pick (arbitrarily) orthogonality condition with (i,j)=(3,1)

$$V^{*}_{ub}V_{ud} + V^{*}_{cb}V_{cd} + V^{*}_{tb}V_{td} = 0$$

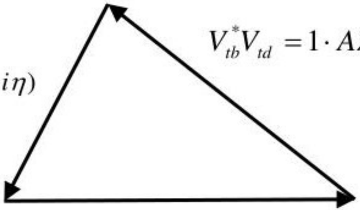
CKM triangle interpretation -visualisation

- Sum of three complex vectors is zero →
  Form triangle when put head to tail



CKM triangle interpretation -visualisation

- Phase of 'base' is zero → Aligns with 'real' axis,



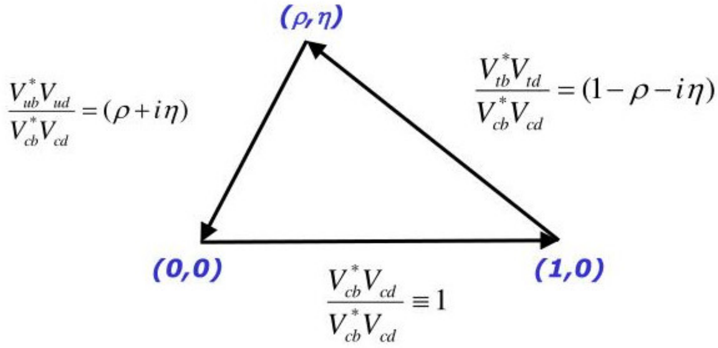
$$V_{tb}^* V_{td} = 1 \cdot A\lambda^3 (1 - \rho - i\eta)$$

$$V_{ub}^* V_{ud} = A\lambda^3 (\rho + i\eta)$$

$$V_{cb}^* V_{cd} = A\lambda^2 \cdot (-\lambda)$$

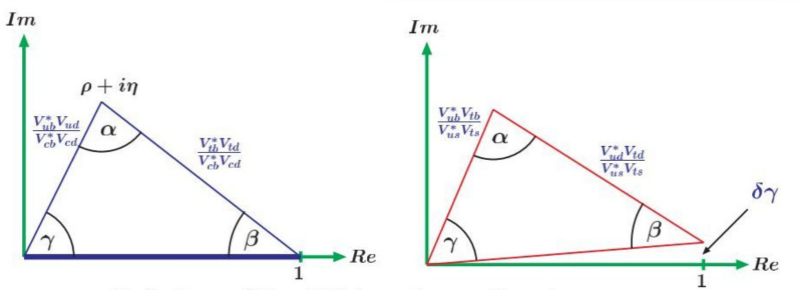
CKM triangle interpretation -visualisation

- Divide all sides by length of base



- Constructed a triangle with apex (ρ,η)

CKM triangle interpretation -visualisation



- Definition of the CKM angles α, β and γ
- To leading order Wolfenstein:

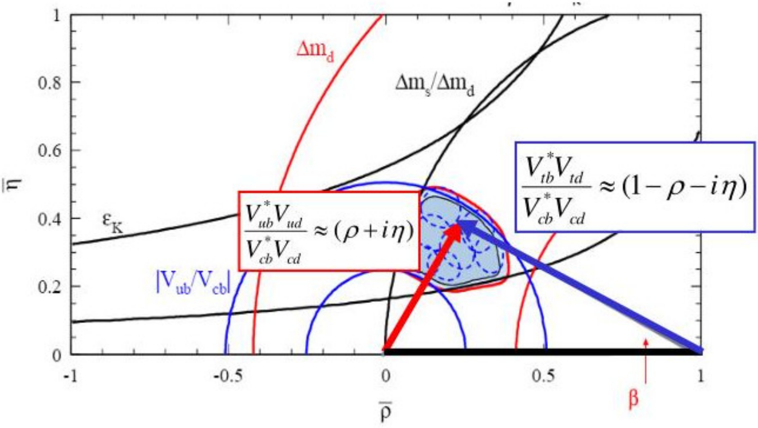
$$V_{ub} = |V_{ub}|e^{-i\gamma} \quad V_{tb} = |V_{tb}|e^{-i\beta}$$

all other CKM matrix elements are real.

- $\delta\gamma$ is order λ^5

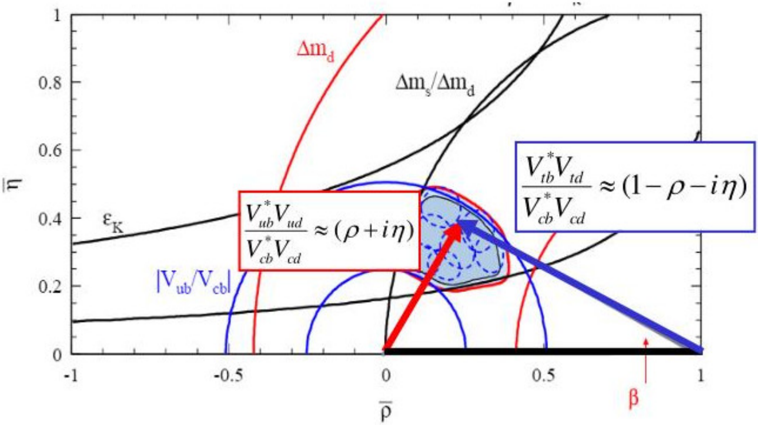
CKM triangle interpretation -visualisation

- We can now put this triangle in the (ρ,η) plane



CKM triangle interpretation -visualisation

- We can now put this triangle in the (ρ,η) plane



The Standard Model Lagrangian

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

- $L_{Kinetic}$: • Introduce the massless fermion fields
  • Require local gauge invariance ➔ gives rise to existence of gauge bosons

- L_{Higgs} : • Introduce Higgs potential with $<\phi> \neq 0$
  • Spontaneous symmetry breaking

  } $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$
  The W^+, W^-, Z^0 bosons acquire a mass

- L_{Yukawa} : • Ad hoc interactions between Higgs field & fermions

The Standard Model Lagrangian field notation

$Q = T_3 + Y$

Fermions: $\psi_L = \left(\frac{1-\gamma_5}{2}\right)\psi \; ; \; \psi_R = \left(\frac{1+\gamma_5}{2}\right)\psi$ with $\psi = Q_L, u_R, d_R, L_L, l_R, \nu_R$

Quarks:

Under SU2:
Left handed doublets
Right hander singlets

- $\begin{pmatrix} u^I (3,2,1/6) \\ d^I (3,2,1/6) \end{pmatrix}_{Li} \equiv Q^I_{Li} \, (3,2,1/6)$

Interaction rep. (I); Left-handed (L); generation index (i); $SU(3)_C$; $SU(2)_L$; Hypercharge Y (=avg el.charge in multiplet)

- $u^I_{Ri} \, (3,1,2/3)$
- $d^I_{Ri} \, (3,1,-1/3)$

Leptons:

- $\begin{pmatrix} \nu^I (1,2,-1/2) \\ l^I (1,2,-1/2) \end{pmatrix}_{Li} \equiv L^I_{Li} \, (1,2,-1/2)$
- $l^I_{Ri} \, (1,1,-1)$
- $\left(\nu^I_{Ri}\right)$

Scalar field:

- $\phi \, (1,2,1/2) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$

Note:
Interaction representation: standard model interaction is independent of generation number

The Standard Model Lagrangian field notation

$Q = T_3 + Y$

Explicitly:

- The left handed quark doublet :

$$Q^I_{Li}(3,2,1/6) \;=\; \begin{pmatrix} u^I_r, u^I_g, u^I_b \\ d^I_r, d^I_g, d^I_b \end{pmatrix}_L, \begin{pmatrix} c^I_r, c^I_g, c^I_b \\ s^I_r, s^I_g, s^I_b \end{pmatrix}_L, \begin{pmatrix} t^I_r, t^I_g, t^I_b \\ b^I_r, b^I_g, b^I_b \end{pmatrix}_L \qquad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = 1/6)$$

- Similarly for the quark singlets:

$$u^I_{Ri}(3,1,\ 2/3) \;=\; \left(u^I_r, u^I_r, u^I_r\right)_R, \left(c^I_r, c^I_r, c^I_r\right)_R, \left(t^I_r, t^I_r, t^I_r\right)_R \qquad (Y = 2/3)$$

$$d^I_{Ri}(3,1,-1/3) \;=\; \left(d^I_r, d^I_r, d^I_r\right)_R, \left(s^I_r, s^I_r, s^I_r\right)_R, \left(b^I_r, b^I_r, b^I_r\right)_R \qquad (Y = -1/3)$$

- The left handed leptons: $L^I_{Li}(1,2,-1/2) = \begin{pmatrix} \nu^I_e \\ e^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\mu \\ \mu^I \end{pmatrix}_L, \begin{pmatrix} \nu^I_\tau \\ \tau^I \end{pmatrix}_L \qquad \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix} \quad (Y = -1/2)$

- And similarly the (charged) singlets: $l^I_{Ri}(1,1,-1) = e^I_R, \mu^I_R, \tau^I_R \qquad (Y = -1)$

Niels Tuning (16)

The Standard Model Lagrangian kinetic term

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

$L_{Kinetic}$: Fermions + gauge bosons + interactions

Procedure:
Introduce the Fermion fields and demand that the theory is local gauge invariant under $SU(3)_C x SU(2)_L x U(1)_Y$ transformations.

Start with the Dirac Lagrangian: $L = i\bar{\psi}(\partial^\mu \gamma_\mu)\psi$

Replace: $\partial^\mu \rightarrow D^\mu \equiv \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y$

Fields:
- G_a^μ : 8 gluons
- W_b^μ : weak bosons: W_1, W_2, W_3
- B^μ : hypercharge boson

Generators:

L_a	: Gell-Mann matrices:	$\frac{1}{2}\lambda_a$	(3x3)	$SU(3)_C$
T_b	: Pauli Matrices:	$\frac{1}{2}\tau_b$	(2x2)	$SU(2)_L$
Y	: Hypercharge:			$U(1)_Y$

For the remainder we only consider Electroweak: $SU(2)_L$ x $U(1)_Y$

Niels Tuning

The Standard Model Lagrangian The Higgs potential

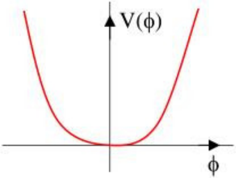
$$\mathsf{L}_{SM} = \mathsf{L}_{Kinetic} + \mathsf{L}_{Higgs} + \mathsf{L}_{Yukawa}$$

$$\mathsf{L}_{Higgs} = D_\mu \phi^\dagger D^\mu \phi - V_{Higgs} \qquad V_{Higgs} = \frac{1}{2}\mu^2\left(\phi^\dagger\phi\right) + |\lambda|\left(\phi^\dagger\phi\right)^2$$

Symmetry

$\mu^2 > 0$:

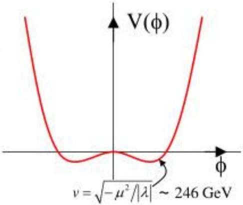
$<\varphi> = 0$



Broken Symmetry

$\mu^2 < 0$:

$<\varphi> = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$



Spontaneous Symmetry Breaking: The Higgs field adopts a non-zero vacuum expectation value

Procedure: $\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \Re e\,\varphi^+ + i\Im m\,\phi^+ \\ \Re e\,\varphi^0 + i\Im m\,\phi^0 \end{pmatrix}$

Substitute: $\Re e\,\varphi^0 = \frac{v + H^0}{\sqrt{2}}$

(The other 3 Higgs fields are "eaten" by the W, Z bosons)

1. $G_{SM} : \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right) \rightarrow \left(SU(3)_C \times U(1)_{EM}\right)$
2. The W^+, W^-, Z^0 bosons acquire mass
3. The Higgs boson H appears

The Standard Model Lagrangian

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

- $L_{Kinetic}$: •Introduce the massless fermion fields
  •Require local gauge invariance → gives rise to existence of gauge bosons
  ➔ CP Conserving
- L_{Higgs} : •Introduce Higgs potential with $<\phi> \neq 0$
  •Spontaneous symmetry breaking
  ➔ CP Conserving

  } $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$
  The W^+, W^-, Z^0 bosons acquire a mass
- L_{Yukawa} : •Ad hoc interactions between Higgs field & fermions
  ➔ CP violating with a single phase

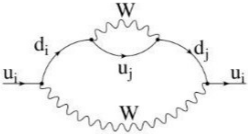
- $L_{Yukawa} \rightarrow L_{mass}$:
  - fermion weak eigenstates:
    - mass matrix is (3x3) non-diagonal

    } ➔ CP-violating
  - fermion mass eigenstates:
    - mass matrix is (3x3) diagonal

    } ➔ CP-conserving!
- $L_{Kinetic}$ in mass eigenstates: CKM – matrix ➔ CP violating with a single phase

Flavor diagonal CP violation

- Flavour diagonal CP Violation is well hidden: Check this!
  e.g electric dipole moment of the neutron:
  At least three loops (Shabalin)



$$d_e \sim e\frac{\alpha_s}{\pi}\frac{G_F^2}{(16\pi^2)^2}\frac{m_t^2}{M_W^2}\mathrm{Im}\Delta\,\mu^3$$
$$\sim 10^{-32} e\,cm \quad \text{with } \mu \sim 0.3\,\mathrm{GeV}$$
$$d_{\mathrm{exp}} \leq 3.0\times 10^{-26} e\,cm$$

Strong CP violation

$M_u^{\rm diag}$ and $M_d^{\rm diag}$ do not necessarily have real eigenvalues

- Using mass eigenstates

$$\mathcal{L}_{\rm mass} = \bar{\mathcal{U}}_L \cdot M_u^{\rm diag} \cdot \mathcal{U}_R + \bar{\mathcal{U}}_R \cdot M_u^{\rm diag\,\dagger} \cdot \mathcal{U}_L + \mathcal{U} \leftrightarrow \mathcal{D}$$
$$= \bar{\mathcal{U}} \cdot \left(M_u^{\rm diag} + M_u^{\rm diag\,\dagger} \right) \cdot \mathcal{U} + \bar{\mathcal{U}} \gamma_5 \cdot \left(M_u^{\rm diag} - M_u^{\rm diag\,\dagger} \right) \cdot \mathcal{U}$$

- The term $\bar{\mathcal{U}} \gamma_5 \mathcal{U}$ should not be there!
- Can be removed by a chiral transformation:

$$\mathcal{U} \to \exp\left(-i\theta\gamma_5\right)\mathcal{U} \qquad \mathcal{U} \leftrightarrow \mathcal{D}$$

if

$$\theta = \mathrm{Arg\,Det} M \quad \text{with} \quad M = \begin{pmatrix} M_u^{\rm diag} & 0 \\ 0 & M_d^{\rm diag} \end{pmatrix}$$

Strong CP violation

if the chiral transformations were a symmetry

- This is only classically true
- QFT: The Chiral symmetry is anomalous!

$$\partial_\mu \left(\mathcal{U}\gamma_\mu\gamma_5\mathcal{U}\right) = \mathcal{U}\gamma_5 \cdot M_u^{\mathrm{diag}} \cdot \mathcal{U} + \frac{\alpha_s}{4\pi} G^{\mu\nu,a}\tilde{G}^a_{\mu\nu}$$

and the same for $\mathcal{D}$.

- Hence: Removing the γ_5 term generates a new term in the SM action:

$$S \rightarrow S - i(\mathrm{Arg\,Det}M) \int d^4x \, \frac{\alpha_s}{8\pi} G^{\mu\nu,a}\tilde{G}^a_{\mu\nu}$$

Strong CP violation

Quantum Theory modifies QCD

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \theta \, \frac{\alpha_s}{8\pi} G^{\mu\nu,a} \tilde{G}^a_{\mu\nu}$$

- Hence the dynamics depend only on

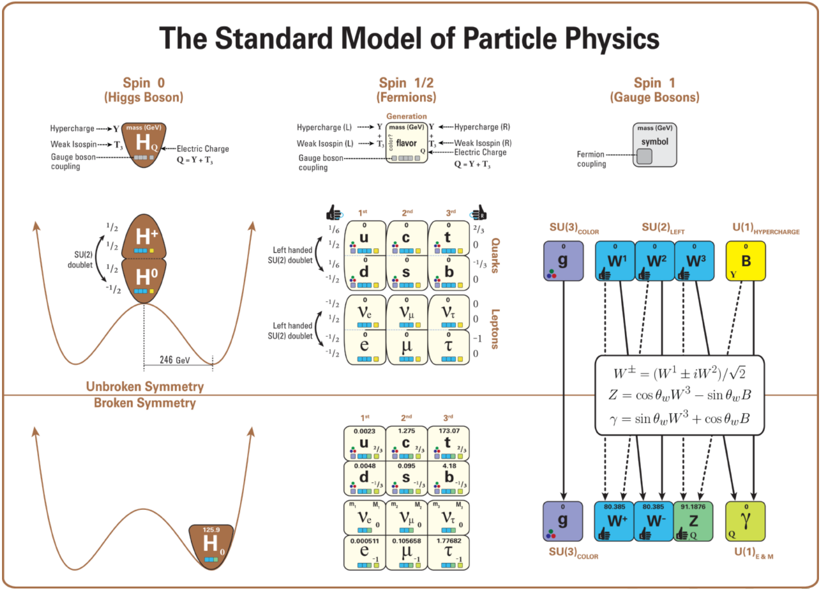
$$\bar{\theta} = \theta - \mathrm{Arg}\,\mathrm{Det} M$$

- This solves certain problems, but it creates new ones
- $G^{\mu\nu,a}\tilde{G}^a_{\mu\nu}$ breaks P as well as CP!
- It generates a neutron electric dipole moment

$$d_N^{TH} \sim 10^{-16}\bar{\theta}\,\mathrm{e\,cm} \quad d_N^{exp} \leq 1.1 \times 10^{-25}\,\mathrm{e\,cm}$$

- Strong CP Problem: Why is $\bar{\theta} \leq 10^{-9\pm 1}$ so small?

The Standard Model



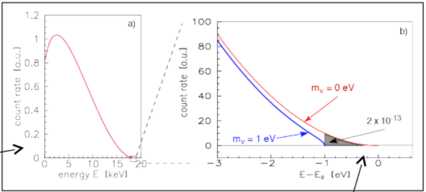
Neutrinos: beta decays

Understanding beta decays (energy, angular momentum)

$$A(Z,N) \rightarrow A(Z-1,N+1) + e^- + \bar{\nu}_e$$

Example: $^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \beta^- + \bar{\nu}$

The spectrum of the recoiling electron (non monoenergetic) was indicating the presence of invisible energy



Neutrino mass effects on the spectrum endpoint

Pauli hypotesis (1932): the presence of a new particle could save the energy conservation of:

- Energy
- Momentum
- Angular momentum

Neutrino hypotesis!

Experimental confirmation in 1956 (Reines & Cowan experiment)

Neutrinos: discovery

In a nuclear power reactor, antineutrinos come from β decay of radioactive nuclei produced by ^{235}U and ^{238}U fission. And their flux is very high.

1. The antineutrino reacts with a proton and forms n and e^+

$$\bar{\nu}_e \, p \rightarrow n \, e^+$$

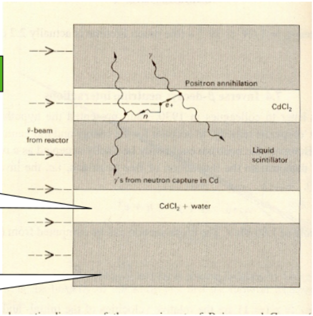
Inverse Beta Decay

2. The e^+ annihilates immediately in gammas

3. The n gets slowed down and captured by a Cd nucleus with the emission of gammas (after several microseconds delay)

Water and cadmium

Liquid scintillator



4. Gammas are detected by the scintillator: the signature of the event is the delayed gamma signal

$$\sigma\left(\bar{\nu}_e p \rightarrow n e^+\right) \approx 10^{-43} \; cm^2$$

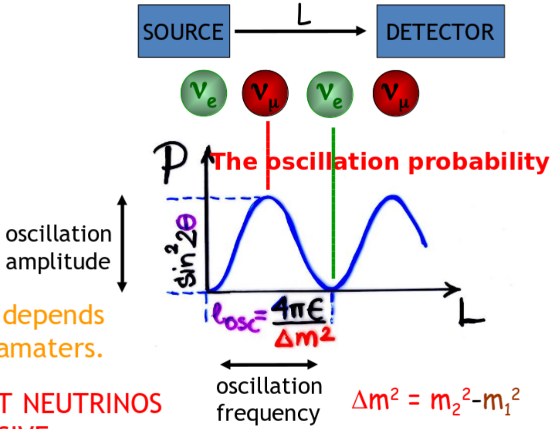
1956: Reines and Cowan at the Savannah nuclear power reactor

Neutrinos: mixing

Bruno Pontecorvo
(1913 – 1993)

Pontecorvo, 1957

Neutrinos can modify their flavor while travelling.
This is the neutrino oscillation phenomenon.



$\Delta m^2 = m_2^2 - m_1^2$

The phenomenon depends on oscillation paramaters.

IT REQUIRES THAT NEUTRINOS ARE MASSIVE.

Recapitulation