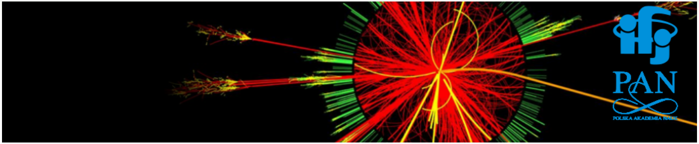
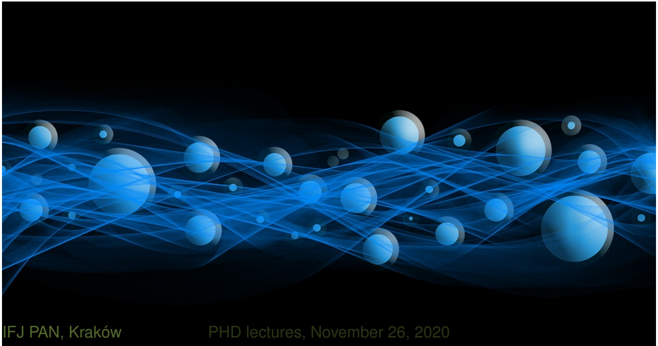


Particle Physics- Standard Model(4)

A. Bożek, IFJ PAN Kraków



Outline

This is an "introduction"

- Historical overview.
- the elementary particles
- the elementary forces
- Symmetries:
    - Gauge symmetry
    - Problem of mass
    - Spontaneous symmetry breaking

This lecture is not a complete course in particle physics and will only touch some most general problems.
Further reading:

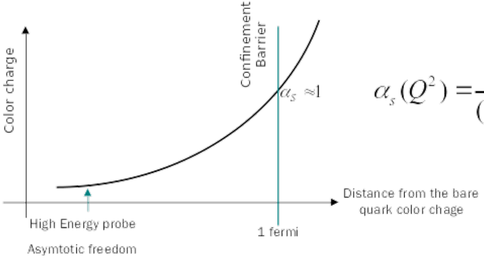
- D. H. Perkins, "Introduction to High Energy Physics",
- F. Halzen, A. Martin: "Quarks and Leptons".

Further lecture to watch listen on SM and BSM physics:

- Prof. Yuval Grossman (Cornell U.)
- Standard Model and Flavor - Lecture (https://www.youtube.com/watch?v=GGzRdiBd8w8)

Quark confinment

→ Quark confinement(Only colorless states are physically observable) is explained in QCD by infrared divergences due to the massless gluons



$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2_{QCD})}$$

Heavy quark Efective Theory HQET

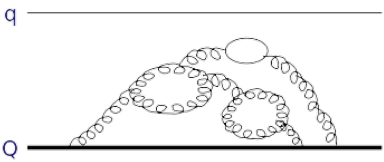
Heavy Quark : $m_Q > \Lambda_{QCD}$

Heavy Quark limit : $m_Q \to \infty$

Heavy Quark + light quark system

Comptom wavelength of Q : $\lambda_Q \sim \frac{1}{m_Q}$

To resolve the quantum number of Heavy quark, need a hard probe with $Q^2 \geq m_Q^2$



"Brown muck"

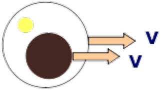
light quark q cannot see the quantum numbers of Heavy Quark

Heavy quark Efective Theory HQET

The configuration light Degree of freedoms with different heavy quark flavor, spin system of hadron does not change if the velocity of heave quark is same.

We can regard heavy quark velocity as conserved quantity

Heavy Quark velocity ≒ Meson velocity
Momentum transfer ~ Λ_{QCD} ⇒ velocity change ~ Λ_{QCD} /m_Q ~ 0



Therefore this picture gives spin – flavor symmetry in QCD under $m_Q \to \infty$ limit.
N_h heavy quark flavor → $SU(2N_h)$ spin-flavor symmetry group

It provide the relations between the properties of hadrons with different flavor and spin of heavy quark.
Such as B, D, B^*,D^*, Λ_b Λ_c

HQET spectroscopy

Strong Interaction dynamics is independent of the spin and mass of the heavy quark by heavy quark symmetry.

Therefore hadronic states can be classified by the quantum number of the light DOF such as flavor, spin, parity, etc.

Spin-flavor symmetry in HQET predict some relations of properties of hadron states, typically mass spectrum of different Hadrons states

Meson	Constituent Quarks	J	P
D	c, (u or d)	0	-
D*	c, (u or d)	1	-
D_1	c, (u or d)	1	+
D_2^*	c, (u or d)	2	+
D_s	c, s	0	-
D_s^*	c,s	1	-

Meson	Constituent Quarks	J	P
B	b, (u or d)	0	-
B*	b, (u or d)	1	-
B_1	b, (u or d)	?	?
B_2^*	b, (u or d)	?	?
B_s	b, s	0	-
B_s^*	b,s	1	-

HQET spectroscopy

1. Ground state mesons

$$j_l = s_l = \frac{1}{2} \qquad J = j_l \pm \frac{1}{2} \qquad J = 0 \; or \; J = 1$$ degenerate states

Experimentally

$$m_{B^*} - m_B \approx 46\,\text{MeV}$$
$$m_{D^*} - m_D \approx 142\,\text{MeV}$$
$$m_{D_S^*} - m_{D_S} \approx 142\,\text{MeV}$$

Quite small as expected

⇒ Need a hyperfine correction of order $1/m_Q$

$$m_{M^*} - m_M \sim \frac{1}{m_Q}$$

So we can expect $m^2_{B^*} - m^2_B \approx m^2_{D^*} - m^2_D \approx const.$

$$m^2_{B^*} - m^2_B \approx 0.49\,\text{GeV}^2$$
$$m^2_{D^*} - m^2_D \approx 0.55\,\text{GeV}^2$$

HQET spectroscopy

2. Excited state mesons

$$s_l = \frac{1}{2},\ j_l = \frac{3}{2} \qquad \left[\begin{array}{l} J=1:\ D_1(2420) \\ J=2:\ D_2^*(2460) \end{array} \right.$$

degenerate states

$$m_{D_2^*} - m_{D_1} \approx 35\,\mathrm{MeV}$$

It is small mass splitting supporting our assertion

One can expect also

$$m^2_{B_2^*} - m^2_{B_1} \approx m^2_{D_2^*} - m^2_{D_1} \approx 0.17\ \mathrm{GeV}^2$$

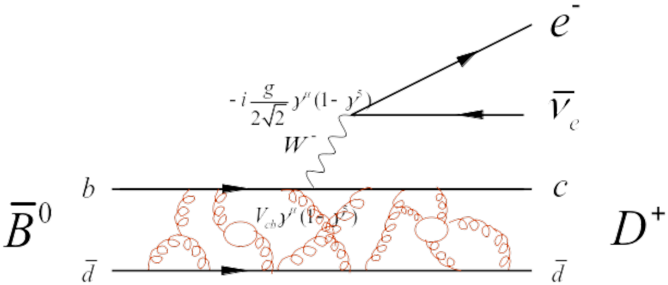
3. Excitation energy

$$m_{B_S} - m_B \approx m_{D_S} - m_D \approx 100\,\mathrm{MeV}$$

$$m_{B_1} - m_B \approx m_{D_1} - m_D \approx 557\,\mathrm{MeV}$$

Weak decay form factors

Physical picture of weak decay

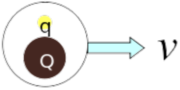


$$M = \frac{G_F}{\sqrt{2}} V_{cb} \left[\bar{u}(p_e)\gamma^\mu(1-\gamma_5)v(p_{\nu_e}) \right] \left\langle D^{(*)}(p) \,|\, \bar{c}\gamma_\mu(1-\gamma_5)b \,|\, B(p) \right\rangle$$

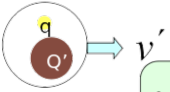
Hadronic matrix element $\left\langle D^{(*)}(p) \,|\, \bar{c}\gamma_\mu(1-\gamma_5)b \,|\, B(p) \right\rangle$
parameterized by several form factors.

Weak decay form factors

Kinematical picture $M \to M' e\nu$



$t < t_0$



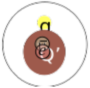
$t > t_0$

$$q^2 = (P' - P)^2 \approx (m_{Q'} v' - m_Q v)^2$$
$$= m_{Q'}^2 + m_Q^2 - 2 m_{Q'} m_Q v' \cdot v$$

$$w \equiv v \cdot v' \quad v \cdot v' = \frac{m_{M'}^2 + m_M^2 - q^2}{2 m_{M'} m_M} \qquad 0 \le q^2 \le (m_{M'} - m_M)^2$$

Maximum $q^2 = (m_{M'} - m_M)^2$; minimum $w = 1$ ⇨ Zero recoil

Minimum $q^2 = 0$; maximum w



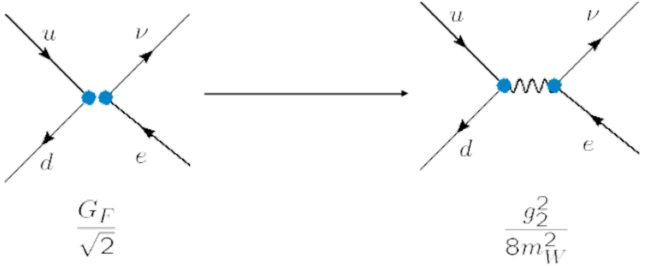
Short and long interactions

- There is a hierarchy between the W/Z/t scales and the energies/masses of the external particles we are interested in:

$$m_W, m_Z, m_t \longleftrightarrow m_b, m_c, m_s$$

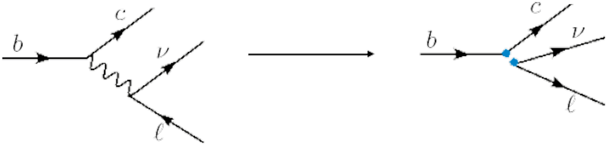
- This suggests that the physics at these two scales can be treated independently

- Historically this is what happened for the β-decay

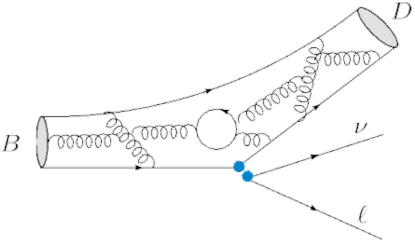


Example $B \to D\ell\nu$

- The first step is to identify the "core" short distance interaction:

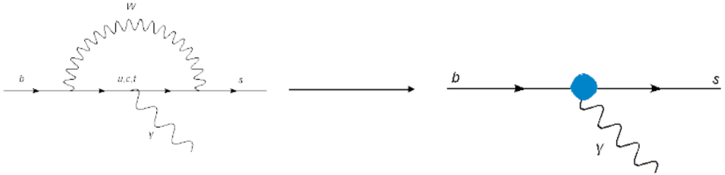


- The second step is to relate the *b* and *c* quarks to the particles we observe:



Example - inclusive decays $B \to X_S\gamma$

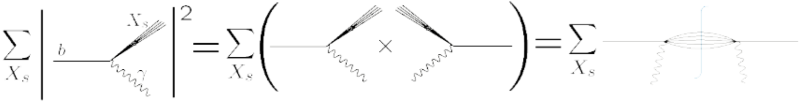
- The first step is to identify the "core" short distance interaction:



- Quark-hadron duality:

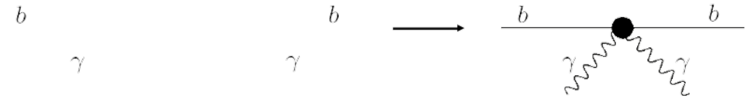
$$\sum_{X_s \in \text{hadrons}} \Gamma(b \to X_s\gamma) = \sum_{X_s \in \{\text{quarks,gluons}\}} \Gamma(b \to X_s\gamma)$$

- Optical theorem:

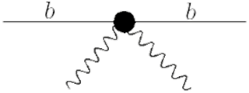


Example - inclusive decays $B \to X_S\gamma$

- This part of the calculation is perturbative (hopefully):



- The b quark has *non-perturbative* overlap with the B meson:

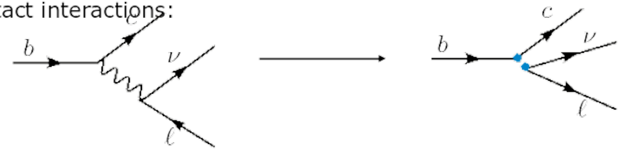
$\langle B|$  $|B\rangle = \Gamma(B \to X_s\gamma)$

Effective Hamiltonian

- The strategy described in the previous slides allows to decouple the problem of calculating short-distance effects from the complications of

  non-perturbative QCD

- Note that after “integrating out” the heavy field, we are left with new contact interactions:



$$\frac{g_2^2}{8}\frac{1}{m_W^2 - p_W^2}\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\nu}_L\gamma_\mu\ell_L\right) \qquad \frac{G_F}{\sqrt{2}}\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\nu}_L\gamma_\mu\ell_L\right) + O\left(\frac{p_W^2}{m_W^2}\right)$$

- We construct an effective Hamiltonian that does not contain the W, Z and t anymore, but has new operators.

Effective Hamiltonian

- $\mathcal{L}_{SM} \longrightarrow \mathcal{L}_{eff} \equiv [\mathcal{L}_{SM}]_{\text{no W,Z,t}} - \sum C_i O_i$

  Wilson coefficients

- For example:

$$b \rightarrow c\ell\nu \Rightarrow \begin{cases} C = \frac{G_F}{\sqrt{2}} \\ O = (\bar{c}_L \gamma^\mu b_L)\,(\bar{\nu}_L \gamma_\mu \ell_L) \end{cases}$$

$$b \rightarrow s\gamma \Rightarrow \begin{cases} C = 4\frac{G_F}{\sqrt{2}}\, V_{tb}^* V_{ts}\, f(\frac{m_t^2}{m_W^2}) \\ O = \frac{e m_b}{16\pi^2}\,(\bar{s}_L \sigma^{\mu\nu} b_R)\, F_{\mu\nu} \end{cases}$$

- One advantage is that a given operator contributes to many different processes

Convergence of perturbative SM theory

- Let us consider the perturbative expansion of a given amplitude:

  $$A(i \to f) \sim 1 + \alpha_s(1+L) + \alpha_s^2(1+L+L^2) + O(\alpha_s^3)$$

  where $L = \log \frac{m_W^2}{p_{\text{ext}}^2}$, $\alpha_s = g_s^2/(4\pi)$ and p_{ext} is a generic external momentum

- In general at order α_s^n we will find terms proportional to $\alpha_s^n L^n$, $\alpha_s^n L^{n-1}$, ...

- If $p_{\text{ext}}^2 \ll m_W^2$, $\alpha_s L \sim O(1)$ and the convergence of the perturbative expansion is spoiled

- We need to find a way to get rid of these logs

Convergence of perturbative SM theory

- The effective Hamiltonian approach is precisely the framework that will allow us to resum these logs at all orders in perturbation theory:

$$A(i \to f) \sim 1 + \alpha_s \log \frac{m_W^2}{p_{\text{ext}}^2} + \cdots$$
$$= 1 + \underbrace{\alpha_s \log \frac{m_W^2}{\mu^2}}_{\text{in the WC}} + \underbrace{\alpha_s \log \frac{\mu^2}{p_{\text{ext}}^2}}_{\text{in the matrix element of the eff. operator}} \cdots$$

$$A(i \to f) = C(\mu) \langle f | O(\mu) | i \rangle$$

- Unfortunately the problem is not solved yet: we can not choose a value of μ that eliminates at the same time the large logs in the Wilson coefficient and in

  the operator matrix element

Effective theories

- If a physical problem contains widely separated scales, it is almost always worthwhile to pursue an effective theory strategy

m_W

perturbative → integrate out

$$A(B \to X) = \sum C_i(\mu) \langle X | O_i(\mu) | B \rangle + O\left(\frac{m_b^2}{m_W^2}\right)$$

m_b

dependence on external states (possibly non-perturbative)

Effective theories

m_b

still perturbative → integrate out
but $p_{ext} \sim m_b$!!

$\sqrt{\Lambda_{QCD} m_b}$

$$\langle X \,|O_i(\mu)|\, B\rangle \;=\; C\; J * \Phi + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

Λ_{QCD}

non-perturbative

Effective theories Renormalized Group Equation

- A generic amplitude at the 1-loop level looks like:

  $$A(i \to f) \sim 1 + \alpha_s(1+L) + \alpha_s^2(1+L+L^2) + O(\alpha_s^3)$$

  where $L = \log m_W^2 / p_{\text{ext}}^2$

- We can write $\alpha_s \log \frac{m_W^2}{p_{\text{ext}}^2} = \underbrace{\alpha_s \log \frac{m_W^2}{\mu^2}}_{C(\mu)} + \underbrace{\alpha_s \log \frac{\mu^2}{p_{\text{ext}}^2}}_{\langle f|O(\mu)|i\rangle}$

- Using the fact that A(i→f) is μ independent, we can write a RGE for C(μ), whose solution resums all these logarithms:

  $$\frac{\mathrm{d}C(\mu)}{\mathrm{d}\log\mu} = \gamma\, C(\mu) \longrightarrow C(\mu) = C(\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{\gamma_0}{2\beta_0}}$$

- Now we can choose $\mu_0 \sim O(m_W)$ in order to minimize the logs in $C(\mu_0)$ and $\mu_b \sim O(m_b)$ to minimize the logs in $\langle f|O(\mu_b)|i\rangle$:

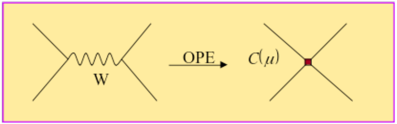
  $$A(i \to f) = C(\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu_b)}\right)^{\frac{\gamma_0}{2\beta_0}} \langle f|O(\mu_b)|i\rangle$$

Operators Products Expansion (OPE)

OPE allows to disentangle SD and LD effects by "integrating out" the W boson and other fields with mass larger than a certain factorization scale.

$$A = \langle H_{eff} \rangle = \sum_i C_i(\mu) \langle Q_i(\mu) \rangle$$

Wilson coefficients, determined by matching

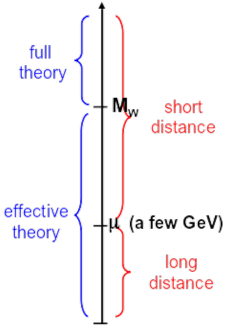


Due to asymptotic freedom of QCD, the strong interaction effects at short distances are calculable in perturbation theory.

However, as a result of matching procedure at the scale M_W and RG equations:

$$C_i(\mu) \quad \text{depend on} \quad \alpha_S(\mu) \log \frac{M_W}{\mu}$$

LARGE!
spoils the validity of the usual perturbation theory



Operators Products Expansion (OPE)

Basic structure of decay amplitudes:

$$A(M \to F) = \sum_i B_i V^i_{CKM} \eta^i_{QCD} F_i(x_t)$$

$\langle K^{(*)} | O_i | B \rangle$
(nonperturbative)

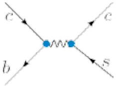
QCD RG factors
(RG improved
perturbation theory)

Inami-Lim fuctions
(perturbation theory)

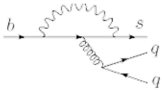
Effective theories: OPE - $b \to s$

$$H_{\rm eff} = -4\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^* \left[\sum_{i=1}^{10} C_i O_i + \sum_{i=3}^{6} C_{iQ} O_{iQ}\right]$$

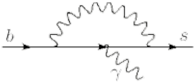
$$\left.\begin{aligned} O_1 &= (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L) \\ O_2 &= (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L) \end{aligned}\right\} \text{ current-current:}$$



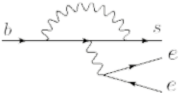
$$\left.\begin{aligned} O_3 &= (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q}\gamma^\mu q) \\ O_4 &= (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q}\gamma^\mu T^a q) \\ O_5 &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L) \sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q) \\ O_6 &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a b_L) \sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q) \end{aligned}\right\} \text{ QCD penguin:}$$



$$\left.\begin{aligned} O_7 &= \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \\ O_8 &= \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu} \end{aligned}\right\} \text{ magnetic moment:}$$



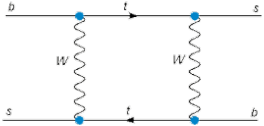
$$\left.\begin{aligned} O_9 &= (\bar{s}_L \gamma_\mu b_L) \sum (\bar{\ell}\gamma^\mu \ell) \\ O_{10} &= (\bar{s}_L \gamma_\mu b_L) \sum (\bar{\ell}\gamma^\mu \gamma_5 \ell) \end{aligned}\right\} \text{ semileptonic:}$$

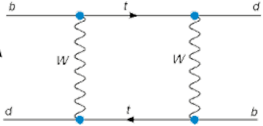


Effective theories: OPE - *B* mixing

$$H_{\text{eff}} = \frac{G_F^2 m_W^2}{16\pi^2}(V_{tb}^* V_{tq})^2 \sum_{i=1}^{8} C_i O_i$$

$$\begin{aligned}
O^{VLL} &= (\bar{b}_L \gamma_\mu q_L)(\bar{b}_L \gamma^\mu q_L) \\
O^{VRR} &= (\bar{b}_R \gamma_\mu q_R)(\bar{b}_R \gamma^\mu q_R) \\
O_1^{LR} &= (\bar{b}_L \gamma_\mu q_L)(\bar{b}_R \gamma^\mu q_R) \\
O_2^{LR} &= (\bar{b}_R q_L)(\bar{b}_L q_R) \\
O_1^{SLL} &= (\bar{b}_R q_L)(\bar{b}_R q_L) \\
O_1^{SRR} &= (\bar{b}_L q_R)(\bar{b}_L q_R) \\
O_2^{SLL} &= (\bar{b}_R \sigma_{\mu\nu} q_L)(\bar{b}_R \sigma^{\mu\nu} q_L) \\
O_2^{SRR} &= (\bar{b}_L \sigma_{\mu\nu} q_R)(\bar{b}_L \sigma^{\mu\nu} q_R)
\end{aligned}$$





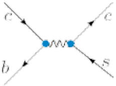
Effective operators products expansion - $b \to s$

$$H_{\rm eff} = -4\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1}^{10} C_i O_i + \sum_{i=3}^{6} C_{iQ} O_{iQ} \right]$$

$$O_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L)$$

$$O_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

current-current:



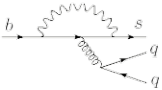
$$O_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q)$$

$$O_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q)$$

$$O_5 = (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q)$$

$$O_6 = (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q)$$

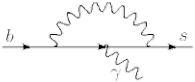
QCD penguin:



$$O_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$$O_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

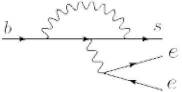
magnetic moment:



$$O_9 = (\bar{s}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \ell)$$

$$O_{10} = (\bar{s}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

semileptonic:



The Standard Model lagrangian

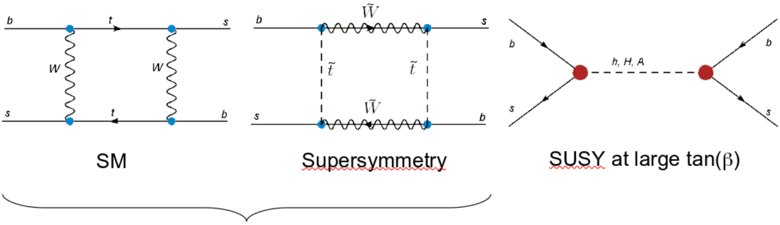
$$
\begin{aligned}
\mathcal{L}_{SM} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F^{i}_{\mu\nu}F^{i,\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu} + \theta_{\mathrm{QCD}}\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^{a}_{\mu\nu}G^{a}_{\rho\sigma} \\
& + \bar{L}_L\, i\not{D}\, L_L + \bar{Q}_L\, i\not{D}\, Q_L + \bar{E}_R\, i\not{D}\, E_R + \bar{D}_R\, i\not{D}\, D_R + \bar{U}_R\, i\not{D}\, U_R \\
& + |D_\mu\phi|^2 - V(\phi) \\
& - \left[\bar{L}_L\, \phi\, \hat{y}^e\, E_R + \bar{Q}_L\, \phi\, \hat{y}^d\, D_R + \bar{Q}_L\, \tilde{\phi}\, \hat{y}^u\, U_R + \text{h.c.}\right]
\end{aligned}
$$

$g_1,\ g_2,\ g_3,\ \theta_{\mathrm{QCD}}$		4	
$\lambda,\ v$		2	19 parameters
$\hat{y}^e,\ \hat{y}^d,\ \hat{y}^u \ \rightarrow$	$m_e,\ m_\mu,\ m_\tau,$ $m_d,\ m_s,\ m_b,$ $m_u,\ m_c,\ m_t,$ $V_{\mathrm{CKM}}\ (\lambda, A, \rho, \eta)$	13	

The effective hamiltonian and physics beyong SM

- A great advantage of the effective Hamiltonian approach is the extreme transparency to new physics

- Since the matrix elements of the various operators are dominated by large distance physics, new physics can enter only by:
  1. modifying the Wilson coefficients
  2. inducing new operators



$$O^{VLL} = (\bar{b}_L \gamma_\mu q_L)(\bar{b}_L \gamma^\mu q_L)$$

$$O_2^{LR} = (\bar{b}_R q_L)(\bar{b}_L q_R)$$
$$O_1^{SLL} = (\bar{b}_R q_L)(\bar{b}_R q_L)$$