



Particle Physics for non-specialists

Part 5: E-p scattering, QCD, Weak
and EW Interactions

Anna Kaczmarska
IFJ PAN, Kraków

Some slides/ideas taken from wonderful
lectures of prof. Tadeusz Lesiak
and T. Potter, K. Grebieszko,
M.A. Thomson, A. Obłąkowska-Mucha,
F. Żarnecki, R. Kass

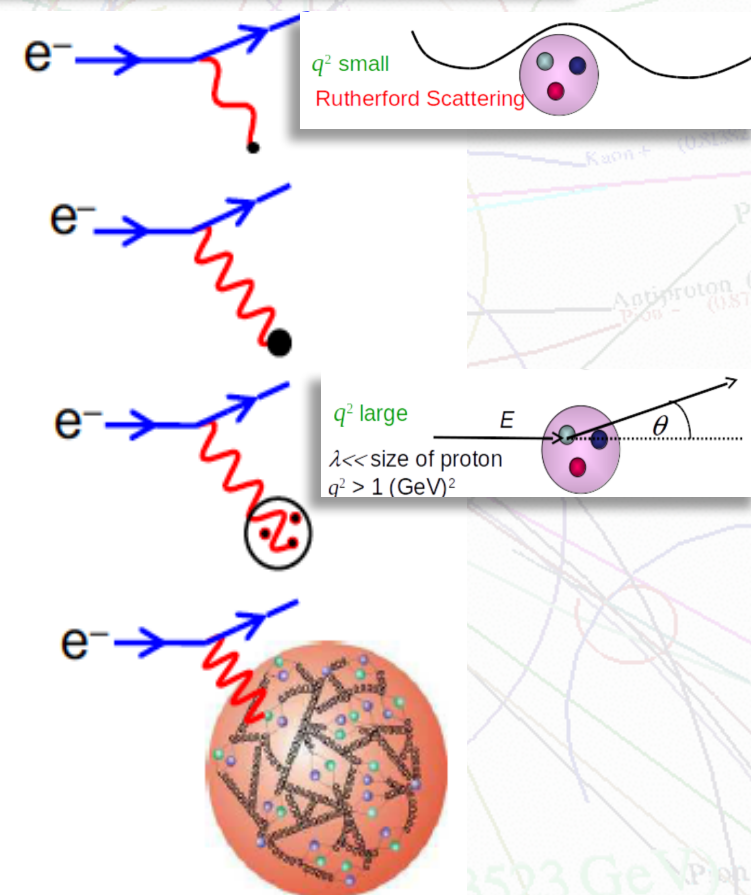
Content of the course

- **The Particle Physics for non-specialists course** consists of 6 lectures (2x45')
 - I. Concepts and history, basic terms
 - II. Accelerators
 - III. Detectors
 - IV. Symmetries, the quark model, Feynman diagrams, QED
 - V. e-p scattering, QCD, Weak and Electroweak Interactions**
 - VI. Higgs Boson, Beyond Standard Model, Neutrino Physics
- Slides will be available on indico
 - <https://indico.ifj.edu.pl/event/285/>
- **Literature**
 - Perkins *Introduction to High Energy Physics*
 - Griffiths *Introduction to Elementary Particles*
 - Martin, Shaw *Particle Physics*
 - Halzen & Martin: *Quarks & Leptons: an Introductory Course in Modern Particle Physics*
 - Particle Data Group: "Review of Particle Physics" [<http://pdg.lbl.gov>]
- **The (mini-)exam**
 - written form, short answers to question (from the list - to be provided earlier)

Electron-Proton (e-p) Scattering

- **Electron-proton scattering** (\rightarrow exchange of virtual photon) **as a probe of the structure of the proton**
- In e-p \rightarrow e-p scattering the nature of the interaction of the virtual photon with the proton **depends strongly on wavelength λ**

- At very low electron energies $\lambda \gg r_p$: scattering equivalent to that from a "point-like" spinless object (Rutherford scattering)
- At low electron energies $\lambda \sim r_p$: scattering is equivalent to that from a **extended charged object**
- At high electron energies $\lambda < r_p$: the wavelength is sufficiently short to **resolve sub-structure**. **Scattering from constituent quarks.**
- At very high electron energies $\lambda \ll r_p$: the proton appears to be a sea of quarks and gluons.

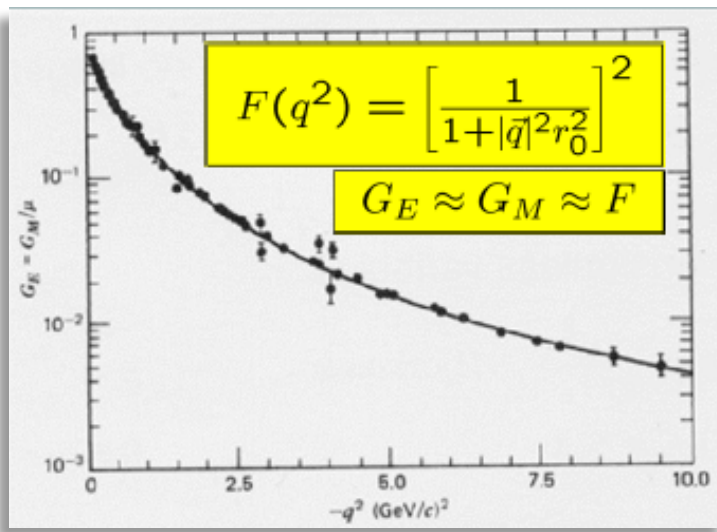


Elastic e-p Scattering

- An elastic e-p scattering with proton as an extended object can be described by

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega} \Big|_{\text{point}} |F(q^2)|^2 \quad \mathbf{q} \Rightarrow \text{4-momentum transfer between e and p}$$

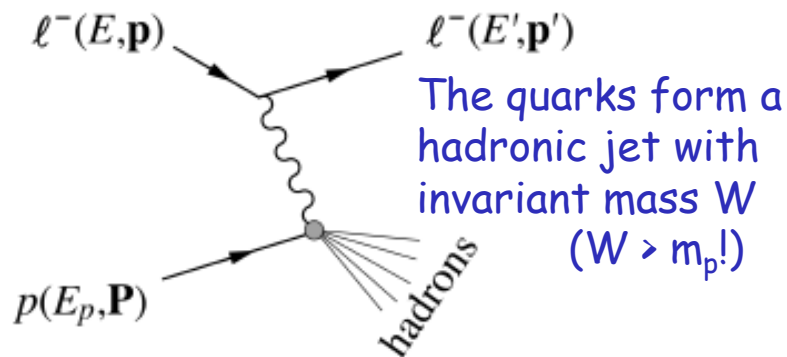
- $F(q^2)$ - form factor describing the charge distribution inside the proton
 - it is Fourier transform of density of charge distribution
- $F(q^2)$ measures probability that electron/photon will "see" proton as a whole
 - Low** q^2 probes distances larger than size of proton ($r \approx 1$ fm). No sensitivity to charge distribution $\Rightarrow F(0) = 1$, photon see whole charge of a target
 - Large** q^2 probes inside the proton and the form factor $F(q^2) < 1$
 - $F(q^2)$ decreases with q^2



- Form factors can be presented as electric G_E and magnetic G_M form factors
- They have to be measured experimentally
- From them one can get proton radius:
 - the proton is an extended object with a radius of 0.81 fm
 - although suggestive, does not imply proton is composite!

(Deep) Inelastic e-p Scattering

- During **Inelastic Scattering** the proton breaks up into its constituent quarks which then form a hadronic jet. For large Q^2 - **Deep Inelastic Scattering (DIS)**

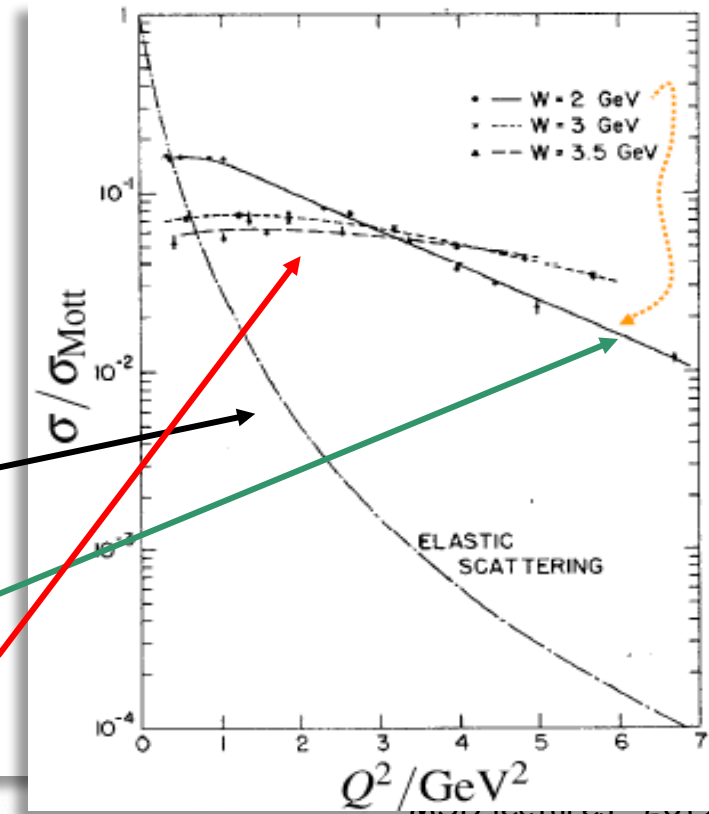


- Described by **two** variables (kinematics of e as we measure it), 4-momentum and energy transfer:

$Q^2 = 4EE' \sin^2 \frac{\theta}{2}$

$\nu = E - E'$
- W** measure "inelasticity" of scattering: large $W \rightarrow$ DIS, small W - inelastic

- Cross section of inelastic e-p scattering depends on **two structure functions** (corresponding to two form factors) $W_{1,2} = W_{1,2}(Q^2, \nu)$
- Expected to drop with increasing Q^2 as the form factors \Rightarrow experiment SLAC-MIT Stanford, USA, (1968), 7-18 GeV e^- on H_2 target
 - Elastic scattering falls off rapidly - proton is not point-like i.e. form factors (\Rightarrow previous slide)
 - Inelastic scattering only weakly dependent on Q^2
 - DIS (large W) almost independent of Q^2 (scaling)!**
 - e "sees" the same regardless on E increase!

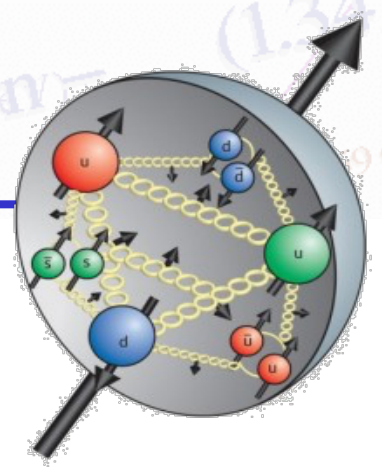


The Quark-Parton Model

- **Bjorken Scaling** (1968): for $Q^2 \rightarrow \infty$ and $\nu \rightarrow \infty$, such that $x = Q^2/2M\nu$ (Bjorken variable) is fixed
 - the structure functions depend only on x $[0,1]$ ($x=1$ for elastic scattering)
 - $MW_1(Q^2, \nu) \rightarrow F_1(x)$ and $\nu W_2(Q^2, \nu) \rightarrow 2xF_2(x)$
 - this behavior is called *scaling* - x is the *scaling variable*
 - if $W_{1,2}$ do not depend on Q^2 , but only on x , it hints at **scattering off point-like objects** (e.g. "form factor" $\rightarrow 1$)
- **Scaling found a natural explanation in the parton model proposed by Feynman (1969)**
 - before quark model (Gell-Mann and Zweig - 1964) was accepted
- **Partons** are constituents of the proton (hadrons)
 - they are point-like fermions like the leptons
 - but unlike the leptons they take part in strong interactions as well as electromagnetic and weak interactions
- Today the Feynman partons are understood to be identical with the quarks postulated by Gell-Mann and Zweig
- **The electron collides elastically with a parton that carries a fraction x of the proton momentum**
- At high momentum ("infinite" momentum) the partons are free
 - collision of one parton with the electron does not affect the other partons
 - this leads to scaling in x



Parton Distribution Functions (pdfs)



- The probability of a parton of type i having a fraction x of the proton energy is the **parton distribution function (pdf)** $f_i(x)$
- The proton structure functions are sums over the parton pdfs:

$$F_2^p(x) = \sum_i xQ_i^2 f_i(x) \quad 2xF_1^p(x) = F_2^p(x)$$

- To get the structure function of a proton we must add all the quark contributions
- From valence quarks:

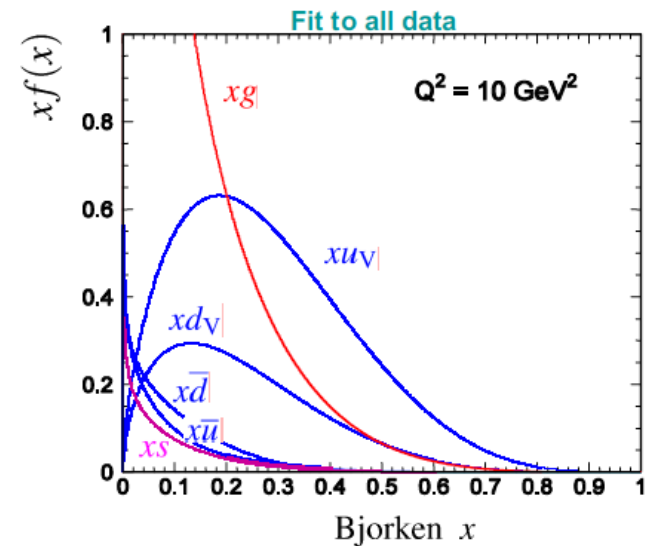
$$F_2^p(x) = \sum_i xQ_i^2 f_i(x) = \frac{4}{9}xu(x) + \frac{1}{9}xd(x)$$

$$F_2^n(x) = \sum_i xQ_i^2 f_i(x) = \frac{4}{9}xd(x) + \frac{1}{9}xu(x)$$

- Measurements of the structure functions enable us to determine the parton distribution functions

$$\int_0^1 F_2^p(x) dx = \frac{4}{9}f_u + \frac{1}{9}f_d = 0.18 \quad \int_0^1 F_2^n(x) dx = \frac{4}{9}f_d + \frac{1}{9}f_u = 0.12$$

where $f_u = \int_0^1 xu(x) dx = 0.36$ and $f_d = \int_0^1 xd(x) dx = 0.18$



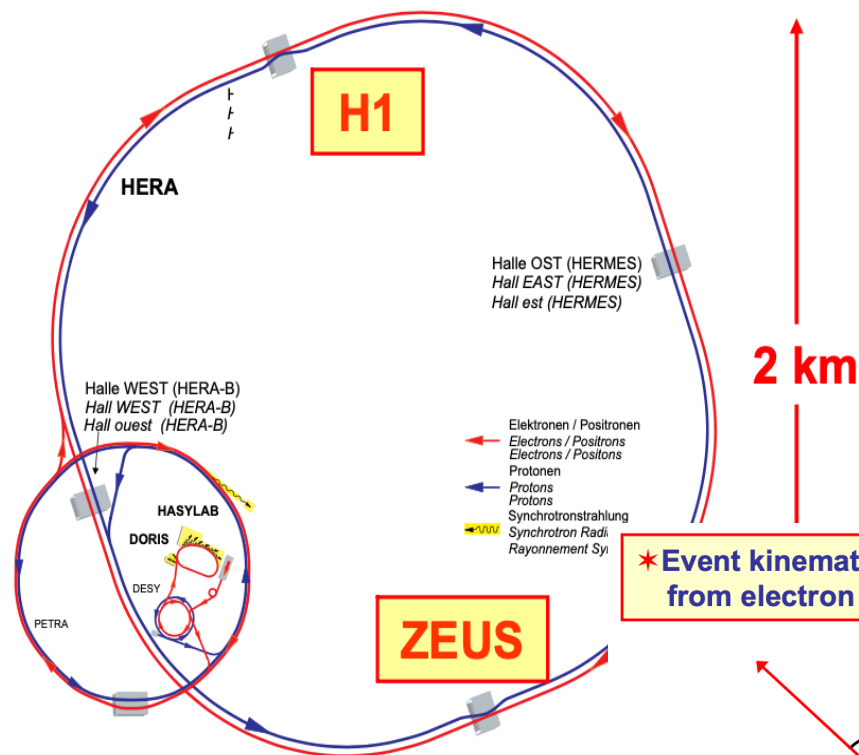
- Valence quarks carry only 54% of the proton momentum!**
- Something else is carrying the other half!
 - (Sea) gluons!**
- Ultimately the parton distribution functions are obtained from a fit to all experimental data including neutrino scattering

Large x - valence quarks dominate
Small x - quarks from sea and gluons dominate
 Small component from $s(x)$

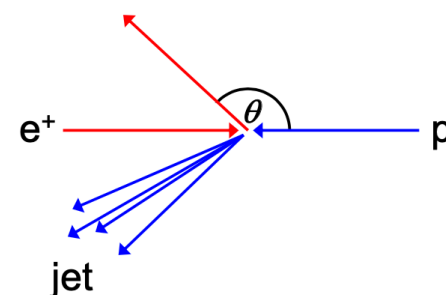
HERA $e^\pm p$ Collider : 1991-2007

★ DESY (Deutsches Elektronen-Synchrotron) Laboratory, Hamburg, Germany

e^\pm $\xrightarrow{27.5 \text{ GeV}}$ $\xleftarrow{820 \text{ GeV}}$ p $\sqrt{s} = 300 \text{ GeV}$



★ Event kinematics determined from electron angle and energy

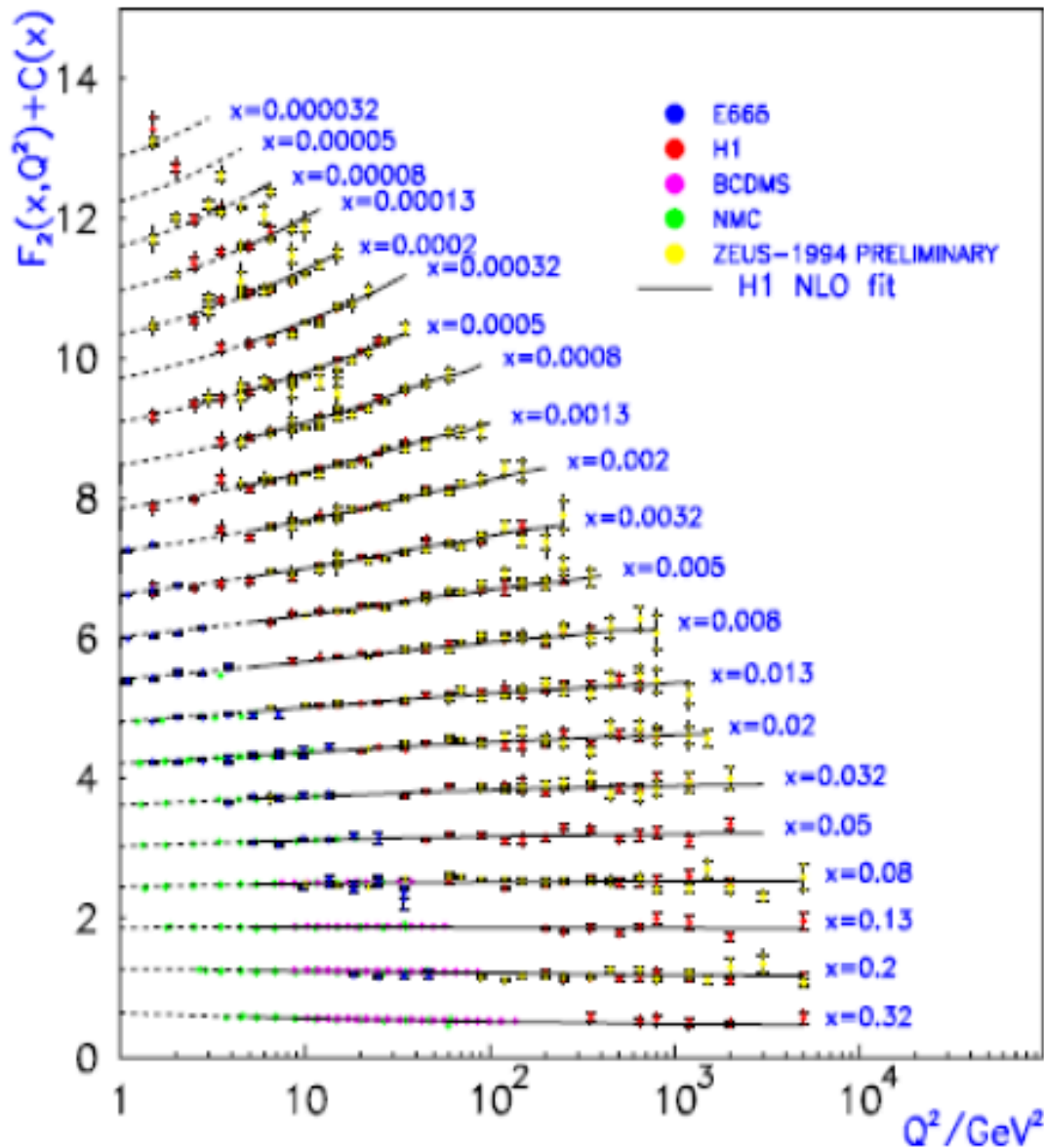


★ Also measure hadronic system (although not as precisely) – gives some redundancy

★ Two large experiments : H1 and ZEUS

★ Probe proton at very high Q^2 and very low x

Results on the proton structure function F_2 from experiments at CERN, Fermilab and DESY



Important Q^2 dependence of the data:

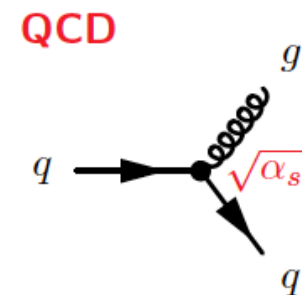
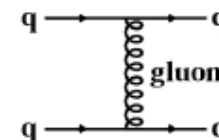
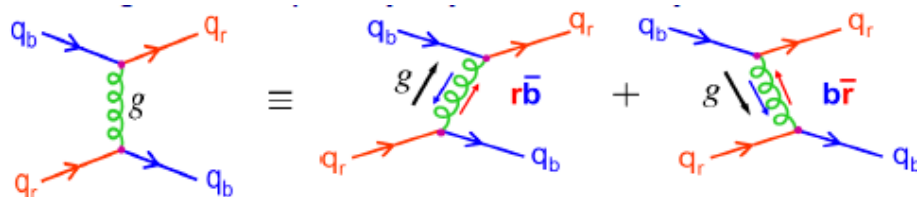
- At $x > 0.05$ the structure function F_2 independent of Q^2 , i.e. scaling
 - point-like partons $\rightarrow R(\text{quark}) < 10^{-18} \text{ m}$
- At smaller x violation of scaling!
- Scaling requires the partons to be like "free" particles, therefore scaling violation indicates the effect of binding on the partons - colour force transmitted by gluons
 - at small x gluon dominates
 - quarks can emit gluon
 - at smaller Q^2 we have smaller resolution and quark-gluon system is seen as one object
 - at higher Q^2 we can see them separately - thus dependence $F_2(x, Q^2)$
- Note: QCD foresees dependence $F_2(x, Q^2)$



**and
now
for something
completely different**

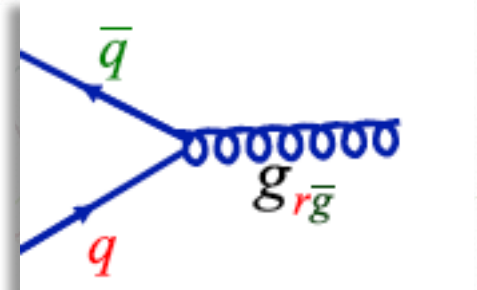
Quantum Chromodynamics (QCD)

- In the 70s, physicists (Gross, Politzel, Wilczek) developed a renormalizable quantum field theory of the strong interaction - **Quantum Chromodynamics (QCD)**
- QCD is a **non-abelian** gauge theory invariant under $SU(3)$ and as a result:
 - Describes the interaction of quarks with the particle responsible for the strong interaction: **massless spin 1 gluons**
 - The charge responsible for this interaction is called **colour**
 - Gluons couple only to objects that have "colour": quarks and gluons
 - There are three different charges ("colours"): **red, green, blue**
 - Note: in QED there is only one charge (electric)
 - Gluon can change the colour of a quark but not its flavour**. e.g. a red u-quark can become a blue u-quark via gluon exchange.
 - Both gluons and photons are massless** thus range of strong force should be ∞ as EM force
 - But it is ~ 1 fm!
- Difference in coupling constants**
 - $\alpha_{EM} \sim 1/137 \rightarrow$ good for perturbation approach
 - $\alpha_S \sim 0.1 - 1 \rightarrow$ perturbation approach does not work in many processes



Gluons

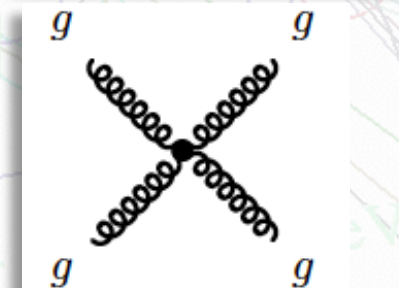
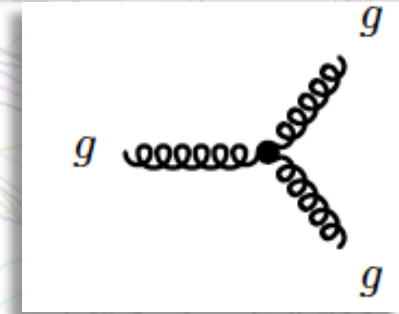
- Gluons are massless spin-1 bosons, which carry the colour quantum number
- Colour is exchanged via gluons, but always conserved (overall and at each vertex)
- Expect 9 gluons $SU(3)$: $3 \times \bar{3} = 8 \oplus 1$: $r\bar{b}$ $r\bar{g}$ $g\bar{r}$ $g\bar{b}$ $b\bar{g}$ $b\bar{r}$ $r\bar{r}$ $b\bar{b}$ $g\bar{g}$
- However: Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$ is colourless and does not participate in the strong interaction
 - \Rightarrow 8 coloured gluons
- QCD looks like a stronger version of QED. However, there is one big difference \Rightarrow **gluons carry colour charge**
 - Gluons can interact with other gluons
 - Note: In QED photon self-couplings are absent since the photon does not have an electric charge (technically - gluons self-interact because $SU(3)$ is non-abelian group)
- All particles (mesons and baryons) are colour singlets.
 - This "saves" the Pauli Principle
 - In the quark model the Ω^- consists of 3 s quarks in a totally symmetric state. Need something else to make the total wavefunction anti-symmetric \Rightarrow colour!



$$r\bar{g} \quad r\bar{b} \quad g\bar{b} \quad g\bar{r} \quad b\bar{r} \quad b\bar{g}$$

$$\sqrt{\frac{1}{2}}(r\bar{r} - g\bar{g})$$

$$\sqrt{\frac{1}{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$$



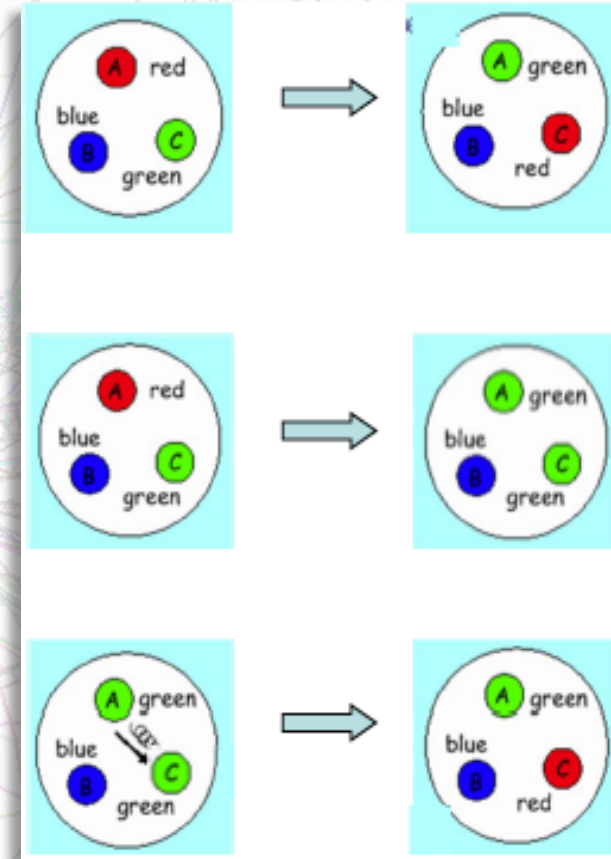
QCD - local gauge symmetry

- QED was invariant under gauge symmetry - U(1)

$$\psi'(\bar{x}, t) = e^{-ief(\bar{x}, t)} \psi(\bar{x}, t)$$
- The equivalent symmetry for QCD is invariance under

$$\psi'(\bar{x}, t) = e^{-ig \sum_{i=1}^8 \frac{\lambda_i \omega_i(\bar{x}, t)}{2}} \psi(\bar{x}, t)$$
 - SU(3) transformation (λ_i are eight 3x3 matrices)

- Global gauge symmetry:** e.g. change $r \leftrightarrow g$ everywhere
- Nothing changes: hadron remains "white"
- Local gauge symmetry:** e.g. change of one (A) quark $r \Rightarrow g$ everywhere
- Hadron is not "white" anymore!
- We can restore symmetry asking A quark to send to C a gluon with colors: r anti- g
- Quark C starts to be r !
- Hadron is white again - but asking for this we introduced new interaction
 - new gluon field transporting color charge



QCD vs QED

QED is an abelian gauge theory with U(1) symmetry:

$$\psi'(\bar{x}, t) = e^{-ief(\bar{x}, t)} \psi(\bar{x}, t)$$

QCD is a non-abelian gauge theory with SU(3) symmetry:

$$\psi'(\bar{x}, t) = e^{-ig \sum_{i=1}^8 \frac{\lambda_i \omega_i(\bar{x}, t)}{2}} \psi(\bar{x}, t)$$

Both are relativistic quantum field theories that can be described by Lagrangians:

QED:
$$L = \bar{\psi}(i\gamma^u \partial_u - m)\psi + e\bar{\psi}\gamma^u A_u \psi - \frac{1}{4} F^{uv} F_{uv}$$

m=electron mass
ψ=electron spinor

electron-γ
interaction

A_u =photon field
 $F_{uv}=\partial_u A_v - \partial_v A_u$

QCD:
$$L = \bar{q}_{jk}(i\gamma^u \partial_u - m)q_{jk} + g(\bar{q}_{jk}\gamma^u \lambda_a q_{jk})G_u^a - \frac{1}{4} G_{uv}^a G_a^{uv}$$

m=quark mass
j=color (1,2,3)
k=quark type (1-6)
q=quark spinor

quark-gluon
interaction

gluon-gluon
interaction
(3g and 4g)

G_u^a =gluon field (a=1-8)
 $G_{uv}^a = \partial_u G_v^a - \partial_v G_u^a - gf_{abc} G_u^b G_v^c$

$$[\lambda_a, \lambda_b] = if_{abc} \lambda_c$$

λ_a 's (a=1-8) are the generators of SU(3).
 λ_a 's are 3x3 traceless hermitian matrices.

$f_{abc} \equiv$ structure constants of the group

How Strong is Strong ?

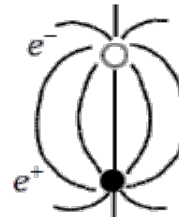
- QCD potential between quark and anti-quark has two components:
 - Short range, Coulomb-like term: $-\frac{4}{3} \frac{\alpha_s}{r}$
 - Long range, linear term: $+kr$, with $k \sim 1 \text{ GeV/fm}$

$$V_{\text{QCD}} = -\frac{4\alpha_s}{3r} + kr$$

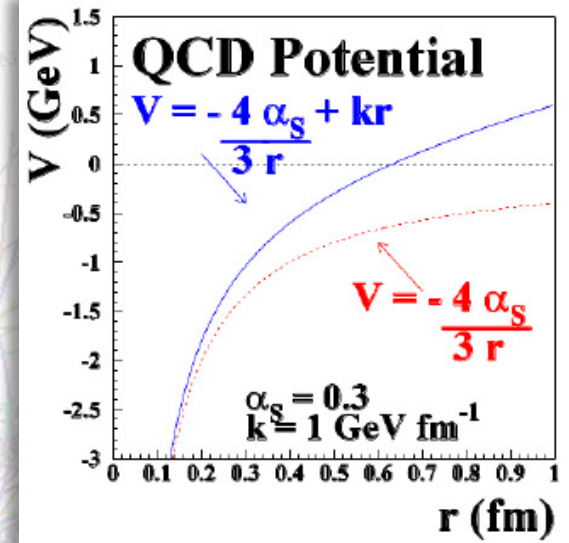
QCD
Colour field



QED
Electric field



- Self interactions of the gluons squeezes the lines of force into a narrow tube/string of approximately constant energy density ($\sim 1 \text{ GeV/fm}$)
- The string has a "tension" and as the quarks separate the string stores potential energy
- Energy required to separate two quarks is infinite
 - Quarks always come in combinations with zero net colour charge \Rightarrow **Confinement**



$$F = -\frac{dV}{dr} = \frac{4\alpha_s}{3r^2} + k$$

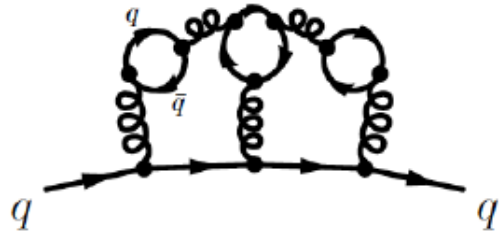
at large r

$$F = k \sim \frac{1.6 \times 10^{-10}}{10^{-15}} \text{ N} = 160,000 \text{ N}$$

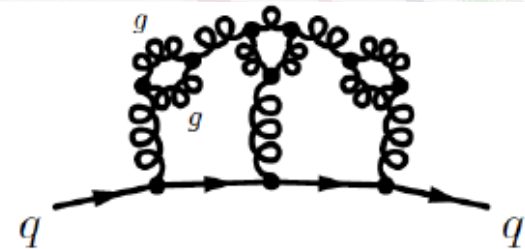
Equivalent to weight of ~ 150 people

Running α_s

- α_s specifies the strength of the strong interaction
- But, just as in QED, α_s is not a constant. It "runs" (i.e. depends on energy/distance)

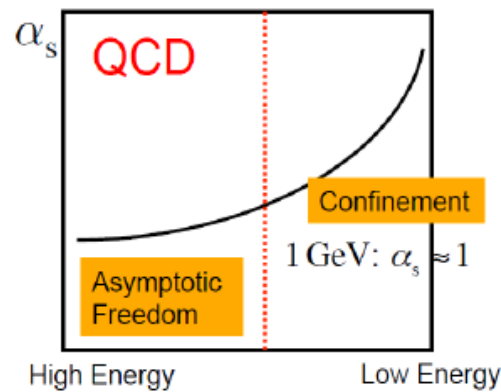
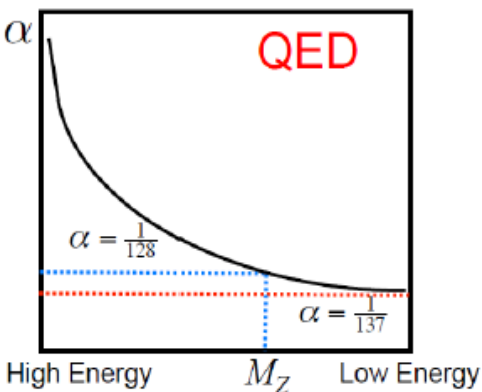


In QCD, quantum fluctuations lead to a cloud of virtual q - anti- q pairs \Rightarrow one of many (an infinite set) of such diagrams analogous to those for QED



In QCD, the gluon self-interactions also lead to a cloud of virtual gluons \Rightarrow one of many (an infinite set) of such diagrams. No analogy in QED, photons do not carry the charge of the interaction. This effect dominates! \rightarrow **Colour Anti-Screening**

- The cloud of virtual gluons carries colour charge and the effective colour charge decreases at smaller distances (high energy)!
- Hence, at low energies (<200 MeV, >1 fm) α_s is large ! Cannot use perturbation theory! **Confinement**
- But at high energies α_s is small. In this regime, can treat quarks as free particles and use perturbation theory ! **Asymptotic Freedom**



$$\sqrt{s} = 100 \text{ GeV}, \quad \alpha_s = 0.12$$

Hadronization and Jets

- What happens when we try to pull apart two quarks?
- Imagine q - anti- q produced at same point in space with very large momentum. They fly apart:



- The energy between the q - anti- q increases as they move apart $E \sim V(r) \sim kr$



- When $E > 2 m_q c^2 \dots$ (breaking of a "string")



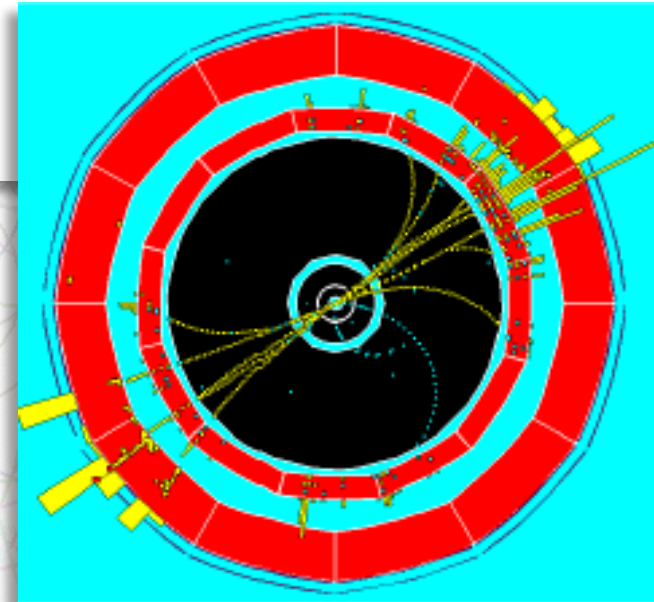
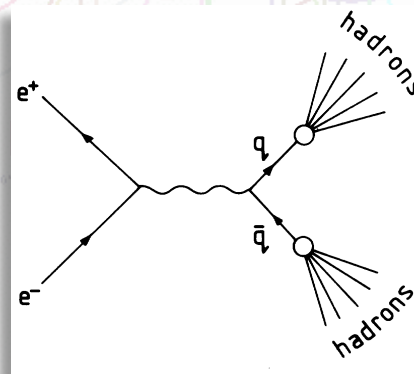
- As the kinetic energy decreases ... the hadrons freeze out



- This process is known as **hadronization**

- For quarks created with high energy start out with quarks and end up with narrowly collimated **jets of hadrons**

- This is how we see quarks and gluons in our detectors



What is the Evidence for Color?

- One of the most convincing arguments for color comes from a comparison of the cross sections for the two processes:

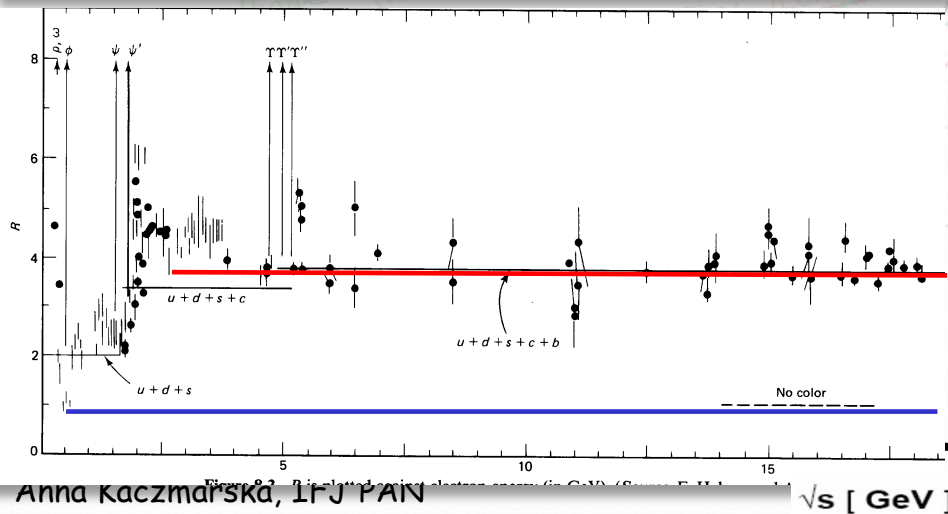
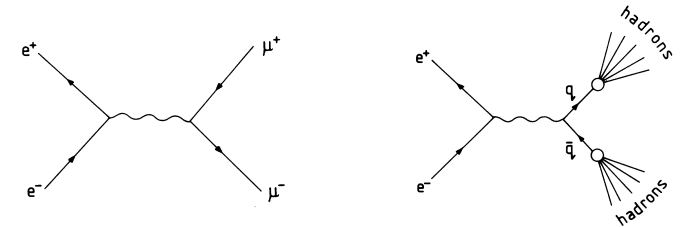
$$e^+e^- \rightarrow \mu^+\mu^- \quad \text{and} \quad e^+e^- \rightarrow q\bar{q}$$

- If color plays no role** in quark production then the ratio of cross sections should only depend on the charge (Q) of the quarks that are produced

$$R \equiv \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \sum_{i=1}^n Q_i^2$$

- However, if **color is important** for quark production then the above ratio should be multiplied by the number of colors (3)

$$R \equiv \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_{i=1}^n Q_i^2$$



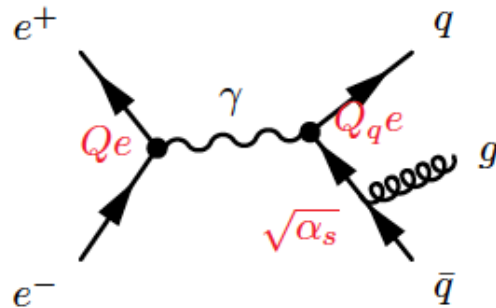
Agreement with e^+e^- colliders data under assumption of quark fractional charge and 3 colors

Energy	Ratio R
$\sqrt{s} > 2m_s \sim 1 \text{ GeV}$	$3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$ u,d,s
$\sqrt{s} > 2m_c \sim 4 \text{ GeV}$	$3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = 3\frac{1}{3}$ u,d,s,c
$\sqrt{s} > 2m_b \sim 10 \text{ GeV}$	$3\left(\dots + \frac{1}{9}\right) = 3\frac{2}{3}$ u,d,s,c,b
$\sqrt{s} > 2m_t \sim 350 \text{ GeV}$	$3\left(\dots + \frac{4}{9}\right) = 5$ u,d,s,c,b,t

What is the Evidence for Gluons?

- In QED, electrons can radiate photons. In QCD, quarks can radiate gluons

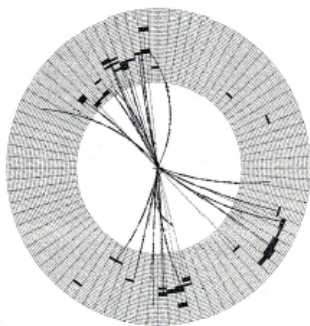
Example: $e^-e^+ \rightarrow q\bar{q}g$



$$M \sim \frac{Q_q}{q^2} \sqrt{\alpha} \sqrt{\alpha} \sqrt{\alpha_s}$$

- In QED we can detect the photons. In QCD, we never see free gluons due to confinement
- Experimentally, detect gluons as an additional jet: 3-jet events
 - Angular distribution of gluon jet depends on gluon spin

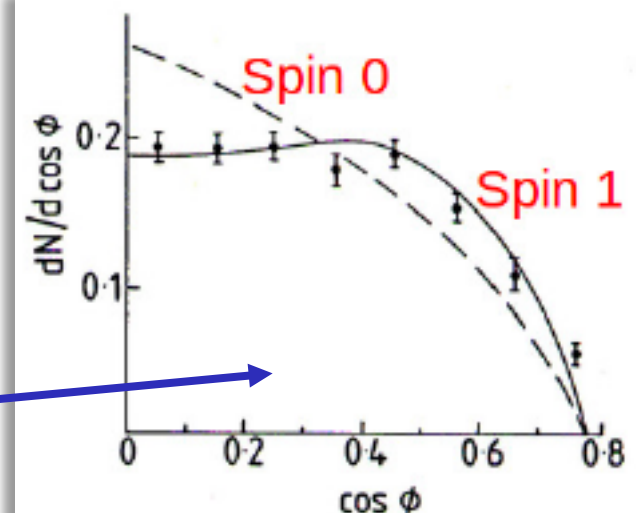
JADE event $\sqrt{s} = 31$ GeV
First direct evidence of gluons (1978)



Distribution of the angle, ϕ , between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cm frame).

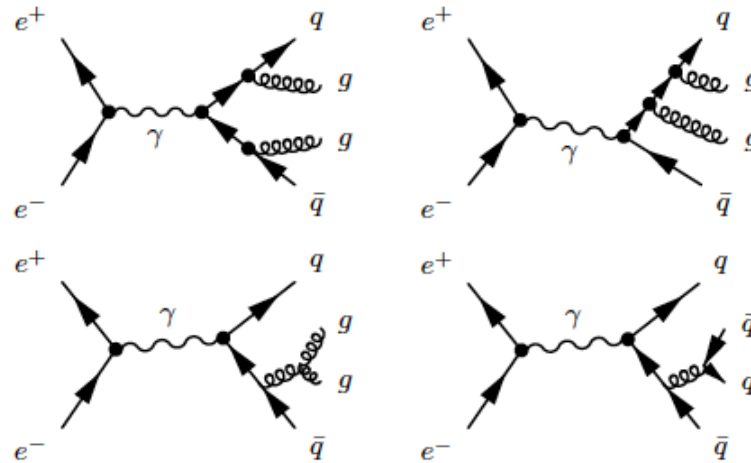
Distribution depends on the spin of the gluon.

=> Gluon is spin 1

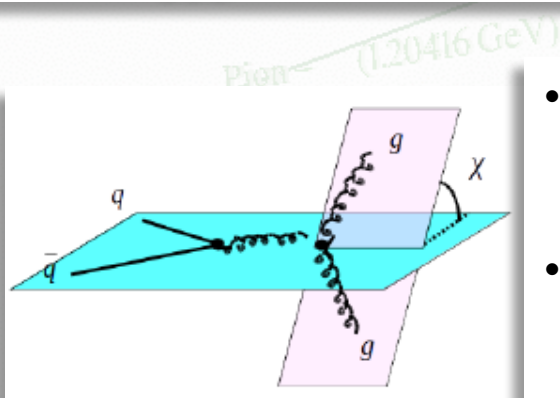


Evidence for Gluon Self-Interactions

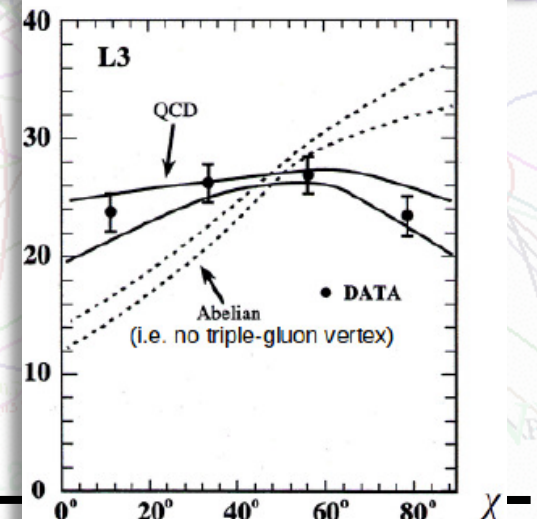
- Direct evidence for the existence of the gluon self-interactions comes from 4-jet events:



- The angular distribution of jets is sensitive to existence of triple gluon vertex
 - qqg vertex consists of two spin 1/2 quarks and one spin 1 gluon
 - ggg vertex consists of three spin-1 gluons => different angular distribution.



- Define the two lowest energy jets as the gluons (they are more likely to be lower energy than quark jets)
- Measure angle between the plane containing the "quark" jets and the plane containing the "gluon" jets.



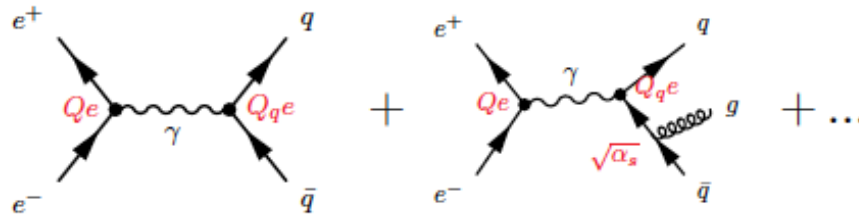
How to measure α_s

α_s can be measured in many ways and at different energies

Example 1: From the ratio

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

In practice, measure



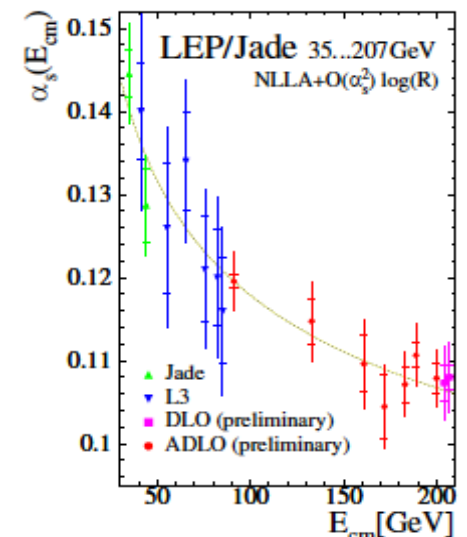
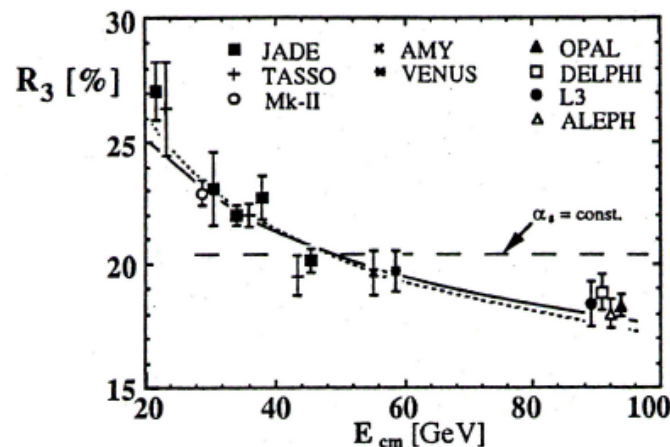
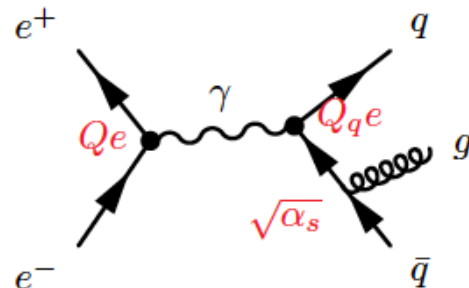
i.e. don't distinguish between 2 and 3 jets

When gluon radiation is included:

$$R = 3 \sum Q_q^2 \left(1 + \frac{\alpha_s}{\pi} \right)$$

Example 2: 3-jet rate $e^+e^- \rightarrow q \text{ anti-}q g$

$$R_3 = \frac{\sigma(e^+e^- \rightarrow 3 \text{ jets})}{\sigma(e^+e^- \rightarrow 2 \text{ jets})} \propto \alpha_s$$

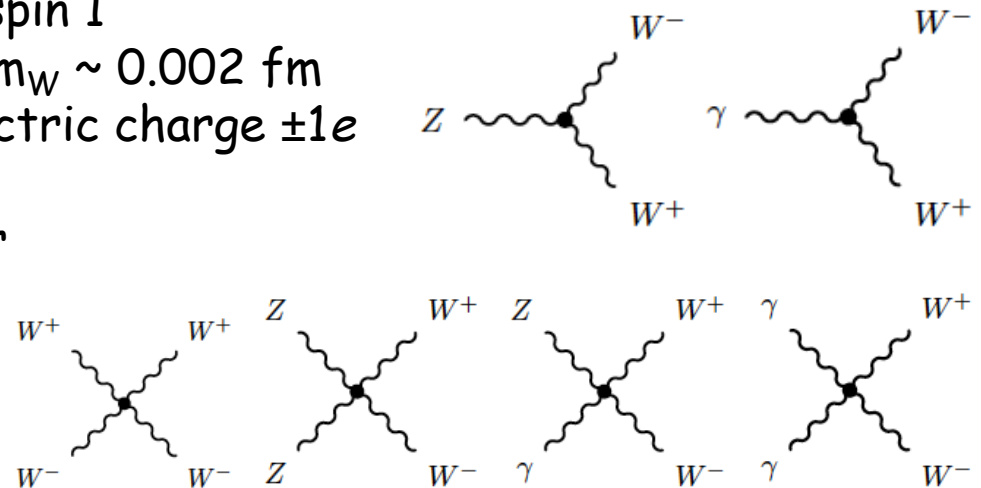




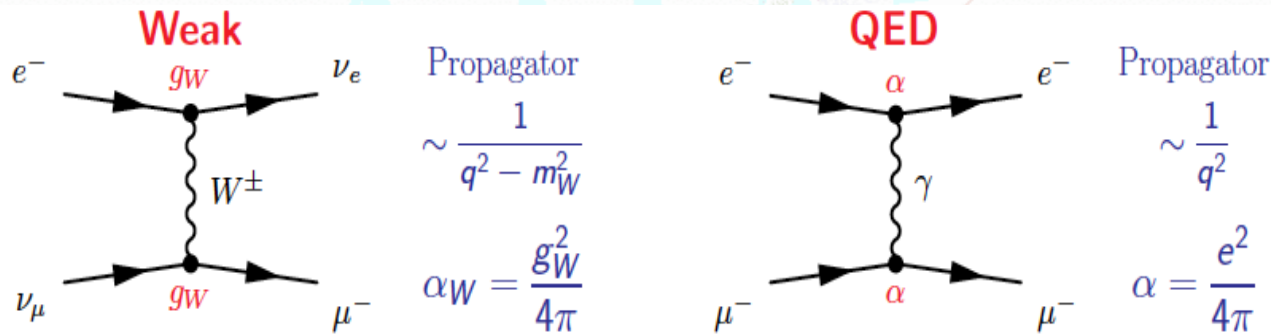
**and
now
for something
completely different**

The Weak Interaction

- The weak force is responsible for some of the most important phenomena:
 - Decays of the muon and tau leptons
 - Neutrino interactions
 - Decays of the lightest mesons and baryons
 - Radioactivity, nuclear fission and fusion
- Characteristics of Weak Processes:
 - Long lifetimes $10^{-13} - 10^3$ s
 - Small cross sections 10^{-13} mb
 - **neutrinos only interact weakly** \rightarrow have very small interaction cross-sections
 - one needs approximately 50 light-years of water to stop a 1 MeV neutrino!
- Weak Force is propagated by massive W^+ , W^- and Z^0 bosons
 - $M_Z = 91.2$ GeV; $M_W = 80.4$ GeV, both spin 1
 - Massive propagator \rightarrow short range $1/m_W \sim 0.002$ fm
 - Z^0 has no electric charge, W^\pm has electric charge $\pm 1e$
 - They carry weak charge
 - W and Z can interact with each other
 - W and γ interact (as W is charged)
- Two types of weak interaction
 - **Charged current (CC): W bosons**
 - **Neutral current (NC): Z bosons**



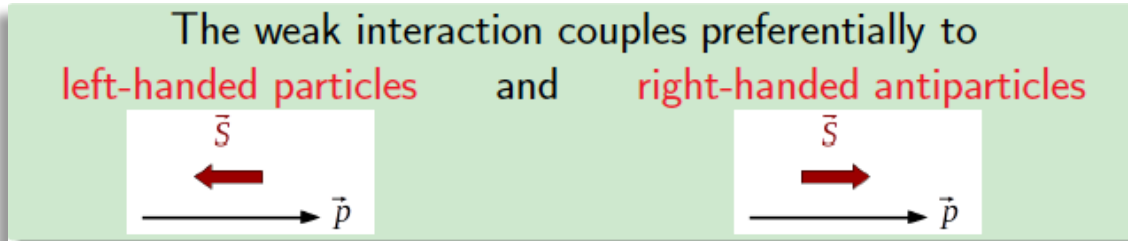
Charged Current Interactions



- Exchanged boson carries electromagnetic charge
- Flavour changing - only the CC weak interaction

$$g_W = 0.65 \quad \text{and} \quad \alpha_W = \frac{g_W^2}{4\pi} \sim \frac{1}{30}$$

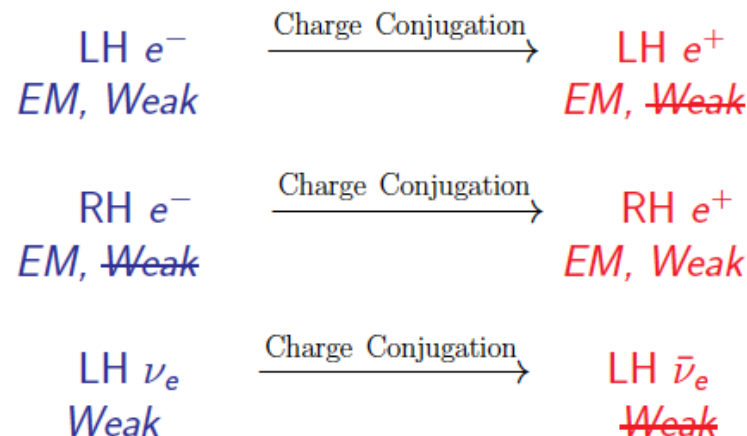
- Parity violating - only the CC weak interaction can violate parity conservation
 - The weak interaction treats LH and RH states differently and therefore can violate parity



\Rightarrow right-handed ν 's do not exist
left-handed $\bar{\nu}$'s do not exist

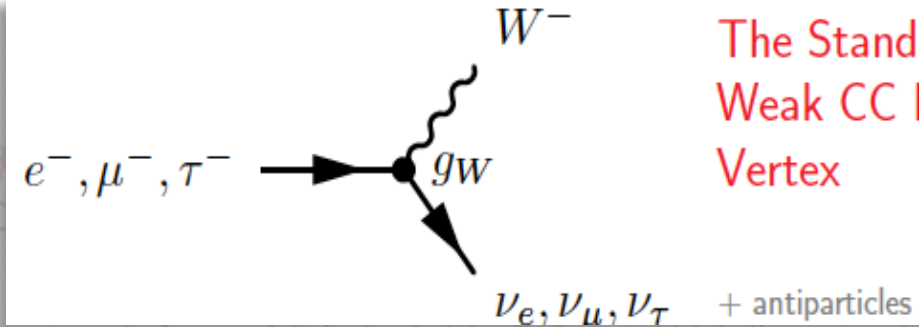
Even if they did exist, they would be unobservable.

- C-symmetry is maximally violated in the weak interaction



Weak interactions of leptons

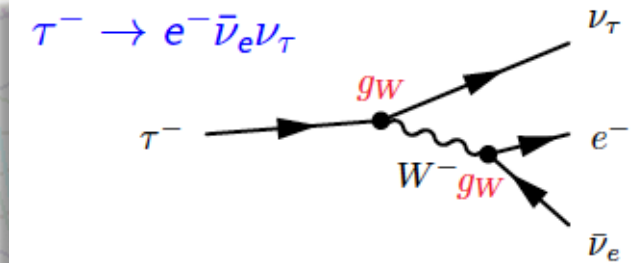
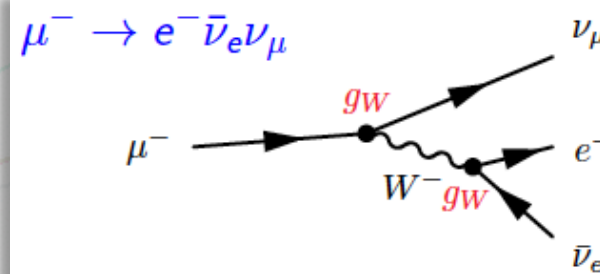
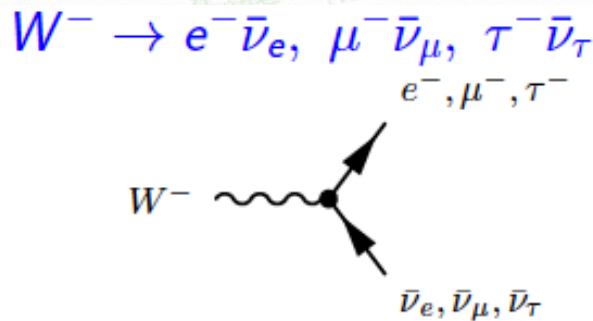
- All weak CC lepton interactions can be described by the W boson propagator and the weak vertex:



- W bosons only "couple" to the (left-handed) lepton and neutrino **within the same generation**
- Only the weak CC interaction changes lepton type, but **only within a generation**
 - Lepton number conservation for each lepton generation
- Coupling constant $\alpha_W = g_W^2/4\pi$

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

e.g. no $W^\pm e^- \nu_\mu$ coupling

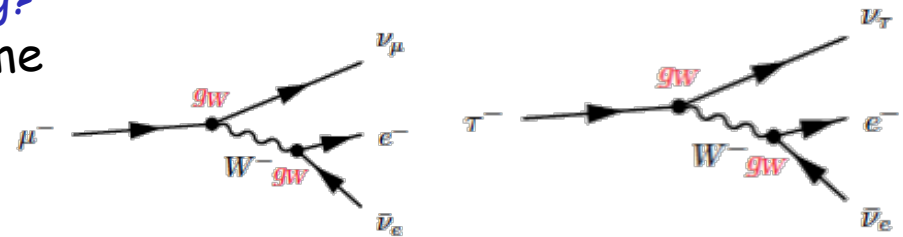


Universality of Weak Coupling

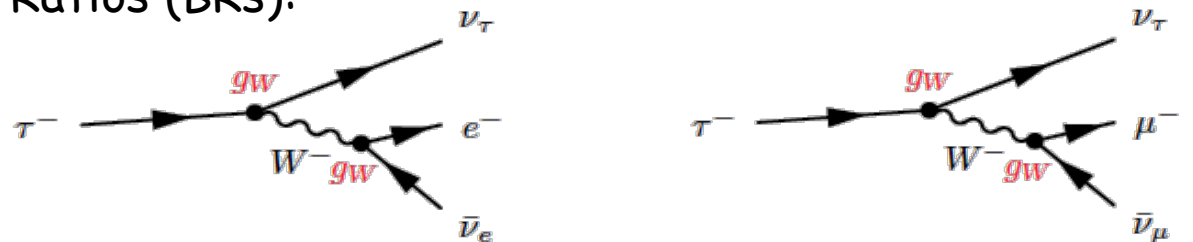
- Do all leptons have the same weak coupling?
 - Measurements of muon and tau lifetime

Prediction: $\frac{\tau_\tau}{\tau_\mu} = 0.1784 \frac{m_\mu^2}{m_\tau^5} = 1.33 \times 10^{-7}$

Experiment: $\frac{\tau_\tau}{\tau_\mu} = \frac{2.9 \times 10^{-13}}{2.2 \times 10^{-6}} = 1.32 \times 10^{-7}$

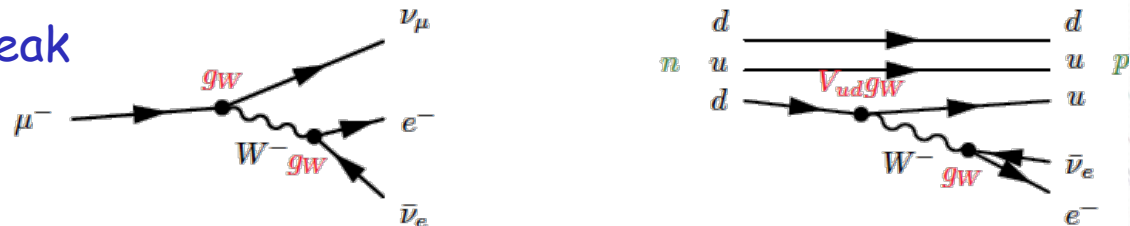


- The same! \Rightarrow The same weak CC coupling for μ and τ !
- Comparison of Branching Ratios (BRs):



- \Rightarrow The same weak CC coupling for e , μ and $\tau \Rightarrow$ Lepton Universality

- Compare coupling strength measured from μ decay with that from nuclear β decay
- Measured: $\beta/\mu = 0.974 \pm 0.003$
- Conclude that the strength of the weak interaction is almost the same for leptons as for quarks
- But the difference is significant, and has to be explained!



Weak Interactions of Quarks

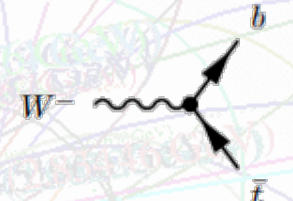
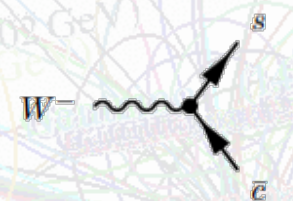
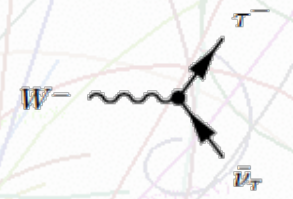
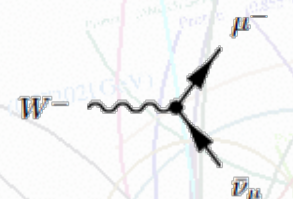
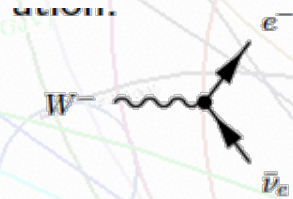
- Impose a symmetry between leptons and quarks, so weak CC couplings take place within one generation:

Leptons

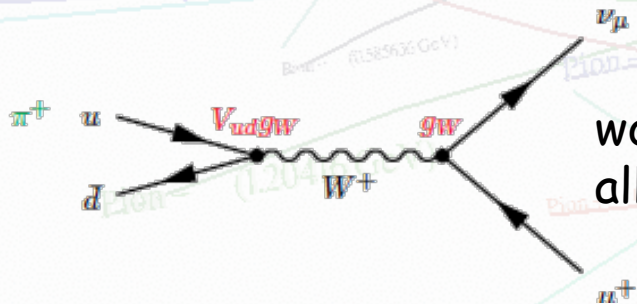
$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

Quarks

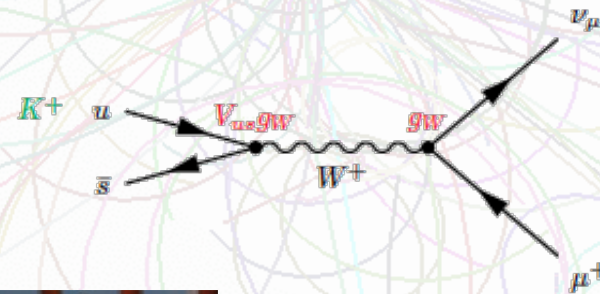
$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$



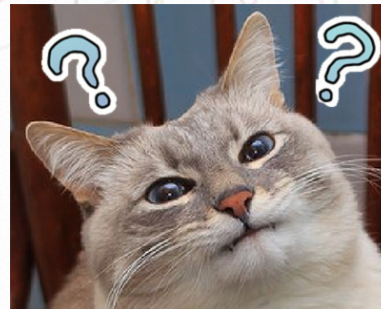
Thus



would be allowed



Would not! But we have observed this $K^+ \rightarrow \mu^+ \nu$ decay! (much smaller rate than π^+ decay)



Quark Mixing

- Instead, alter the lepton-quark symmetry to: (only considering 1st and 2nd gen. here)

$$\begin{array}{cc} \text{Leptons} & \text{Quarks} \\ \begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} & \begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \end{array} \quad \text{where } \begin{aligned} d' &= d \cos \theta_C + s \sin \theta_C \\ s' &= -d \sin \theta_C + s \cos \theta_C \end{aligned}$$

- Now, the down type quarks in the weak interaction are actually linear superpositions of the down type quarks i.e. weak eigenstates (d', s') are superpositions of the mass eigenstates (d, s)

$$\text{Weak Eigenstates} \quad \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad \text{Mass Eigenstates}$$

\Rightarrow Cabibbo angle $\theta_C \sim 13^\circ$ (from experiment)

- Now, the weak coupling to quarks is:

$$\begin{aligned} & \begin{array}{c} d \cos \theta_C + s \sin \theta_C \\ W^- \text{ vertex} \\ \bar{u} \end{array} = \begin{array}{c} d \\ W^- \text{ vertex} \\ \bar{u} \end{array} g_W \cos \theta_C + \begin{array}{c} s \\ W^- \text{ vertex} \\ \bar{u} \end{array} g_W \sin \theta_C \\ & \begin{array}{c} -d \sin \theta_C + s \cos \theta_C \\ W^- \text{ vertex} \\ \bar{c} \end{array} = \begin{array}{c} d \\ W^- \text{ vertex} \\ \bar{c} \end{array} -g_W \sin \theta_C + \begin{array}{c} s \\ W^- \text{ vertex} \\ \bar{c} \end{array} g_W \cos \theta_C \end{aligned}$$

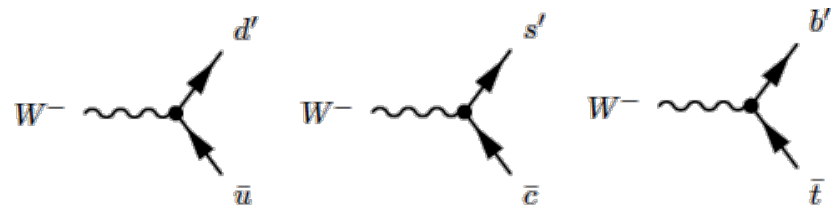
- Quark mixing explains the lower rate of $K^+ \rightarrow \mu^+ \nu$ compared to $\pi^+ \rightarrow \mu^+ \nu$ and the ratio of coupling strength

- $\beta/\mu = 0.974 \pm 0.003$
- $\beta/\mu = \cos \theta_C$

which holds for $\theta_C \sim 13^\circ$

Cabibbo-Kobayashi-Maskawa (CKM) Matrix

- Extend quark mixing to three generations



Weak Eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Mass Eigenstates

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} \cos \theta_C & \sin \theta_C & \sin^3 \theta_C \\ -\sin \theta_C & \cos \theta_C & \sin^2 \theta_C \\ \sin^3 \theta_C & -\sin^2 \theta_C & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0.975 & 0.220 & 0.01 \\ -0.220 & 0.975 & 0.05 \\ 0.01 & -0.05 & 1 \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

Cabibbo Allowed

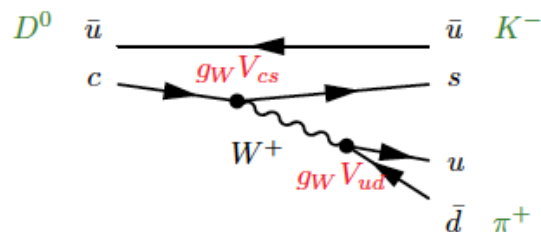
$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

Cabibbo Suppressed

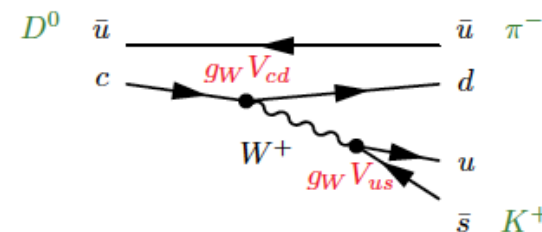
$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

Doubly Cabibbo Suppressed

$$D^0 \rightarrow K^- \pi^+$$



$$D^0 \rightarrow K^+ \pi^-$$



$$\frac{\Gamma(D^0 \rightarrow K^+ \pi^-)}{\Gamma(D^0 \rightarrow K^- \pi^+)} \sim \frac{(g_W^2 V_{cd} V_{us})^2}{(g_W^2 V_{cs} V_{ud})^2} = \frac{\sin^4 \theta_C}{\cos^4 \theta_C} \sim 0.0028$$

Measure 0.0038 ± 0.0008

Summary

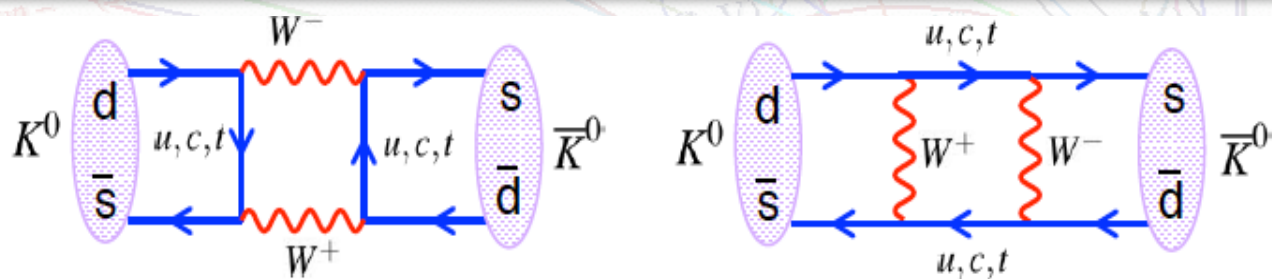
- W^\pm bosons always change quark flavour
- W^\pm prefers to couple to quarks in the same generation, but quark mixing means that cross-generation coupling can occur
- Crossing two generations is less probable than one

W-lepton coupling constant $\rightarrow g_W$

W-quark coupling constant $\rightarrow g_W V_{\text{CKM}}$

Neutral Kaons Mixing

- The neutral kaon is a bound state of a quark and an anti-quark: $k^0 = \bar{s}d$ $\bar{k}^0 = s\bar{d}$
- The k^0 and anti- k^0 are produced by the strong interaction and have definite strangeness \Rightarrow thus they cannot decay via the strong or electromagnetic interaction
- The neutral kaon decays via the weak interaction, which does not conserve strangeness
- The Weak Interaction also allows mixing of neutral kaons via "box diagrams"



$$|k_1\rangle = \frac{1}{\sqrt{2}}(|k^0\rangle + |\bar{k}^0\rangle)$$

$$|k_2\rangle = \frac{1}{\sqrt{2}}(|k^0\rangle - |\bar{k}^0\rangle)$$

$$CP|k_1\rangle = |k_1\rangle \quad \text{and} \quad CP|k_2\rangle = -|k_2\rangle$$

- K_1 ($CP=1$) and K_2 ($CP=-1$) are eigenstates of CP
- Experimentally two kinds of neutral kaons have been detected: K_S and K_L
 - $\tau(K_S) = 8.953 \pm 0.005 \times 10^{-11} \text{ s}$
 - $\tau(K_L) = 5.114 \pm 0.021 \times 10^{-8} \text{ s}$
 - These are eigenstates of weak interactions
- So... K_1 ($CP=1$) = K_S and K_2 ($CP=-1$) = K_L ?

Theory	Experiments
$K_1 \rightarrow \pi^+ + \pi^-$	$K_S \rightarrow \pi^+ + \pi^-$
$K_1 \rightarrow 2\pi^0$	$K_S \rightarrow 2\pi^0$
$K_2 \rightarrow \pi^+ + e^- + \bar{\nu}_e$	$K_L \rightarrow \pi^+ + e^- + \bar{\nu}_e$
$K_2 \rightarrow \pi^- + e^+ + \nu_e$	$K_L \rightarrow \pi^- + e^+ + \nu_e$
$K_2 \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$	$K_L \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$
$K_2 \rightarrow \pi^- + \mu^+ + \nu_\mu$	$K_L \rightarrow \pi^- + \mu^+ + \nu_\mu$
$K_2 \rightarrow 3\pi^0$	$K_L \rightarrow 3\pi^0$
$K_2 \rightarrow \pi^+ + \pi^- + \pi^0$	$K_L \rightarrow \pi^+ + \pi^- + \pi^0$

Neutral Kaons and CP violation

- James Cronin and Val Fitch 1964: CP violation in weak decays!

$$K_L \rightarrow \pi^+ + \pi^-$$

- Rare ~0.2% of events
- Physical states K_S and K_L are mixture of K_1 and K_2 states

$$K_L \sim K_2 + \epsilon K_1$$

$$K_S \sim K_1 - \epsilon K_2$$

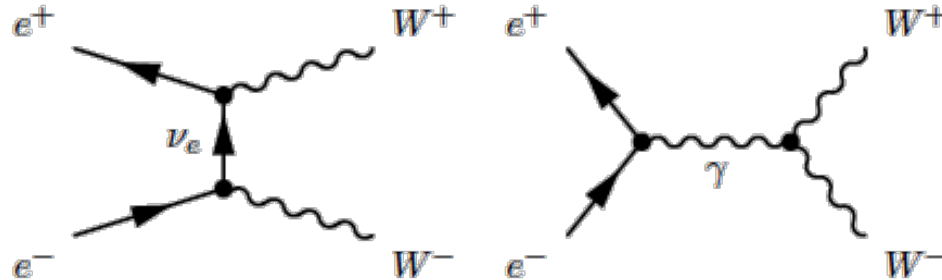
- ϵ is a (small) complex number that allows for CP violation through mixing
- If CP conserved $\epsilon = 0$ but it is not: $\epsilon \sim 2.3 \times 10^{-3}$



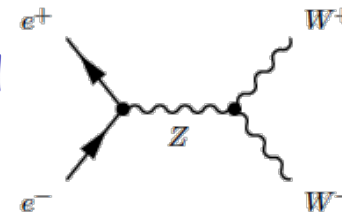
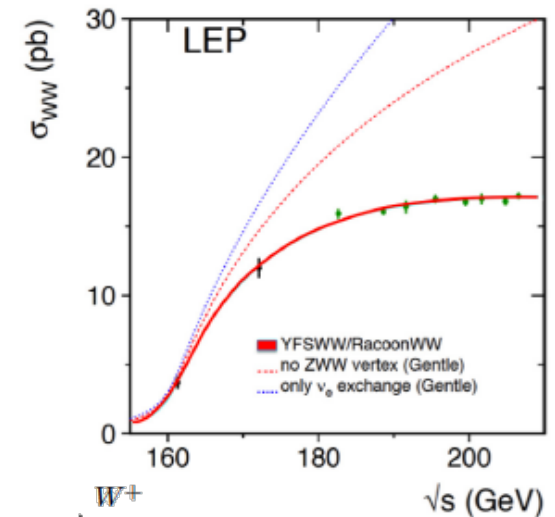
- There can be two types of CP violation in K_L decay:
 - indirect ("mixing"): $K_L \rightarrow \pi\pi$ because of its K_1 component
 - direct: $K_L \rightarrow \pi\pi$ because the amplitude for K_2 allows $K_2 \rightarrow \pi\pi$
 - It turns out that both types of CP violation are present and indirect \gg direct!
- CP violation discovered also in B meson systems (1999)
- The origins of CP violation are still unknown
- CP violation is of interest in Cosmology to explain why we live in a matter Universe rather than equal amounts of matter and antimatter
- The amount of CP Violation seen in K^0 and B^0 decays is not enough to explain this!

Electroweak Unification

- Consider $e^-e^+ \rightarrow W^+W^-$: 2 diagrams (+interference)



- Cross-section diverges at high energy
- Divergence cured by introducing Z boson
- Extra diagram for $e^-e^+ \rightarrow W^+W^-$
- Idea only works if γ , W , Z couplings are related
 - \Rightarrow **Electroweak Unification**



- Unify QED and the weak force \Rightarrow electroweak model
- Invariance under $SU(2) \times U(1)$ transformations
 - four **massless** gauge bosons W^+ , W^- , W_3 , B
 - The two neutral bosons W_3 and B then mix to produce the physical bosons Z^0 and γ
 - Photon properties must be the same as QED
 - predictions of the couplings of the Z^0 in terms of those of the W and γ

The GWS Model

- The Glashow, Weinberg and Salam model treats EM and weak interactions as different manifestations of a single unified electroweak force (Nobel Prize 1979)
- Start with 4 massless bosons W^+ , W_3 , W^- and B . The neutral bosons mix to give physical bosons.
- Physical fields: W^+ , Z , W^- and A (photon).



$$Z = W_3 \cos \theta_W - B \sin \theta_W$$

$$A = W_3 \sin \theta_W + B \cos \theta_W \quad \theta_W \text{ Weak Mixing Angle}$$

- W , Z "acquire" mass via the Higgs mechanism \Rightarrow next lecture
- The beauty of the GWS model is that it makes exact predictions of the W and Z masses and of their couplings with only 3 free parameters:

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} = \frac{e^2}{8m_W^2 \sin^2 \theta_W}$$

$$m_{W^\pm} = \left(\frac{\sqrt{2}e^2}{8G_F \sin^2 \theta_W} \right)^{1/2} \quad m_Z = \frac{m_W}{\cos \theta_W}$$

$$\alpha_{EM} = \frac{e^2}{4\pi} \quad g \sim e$$

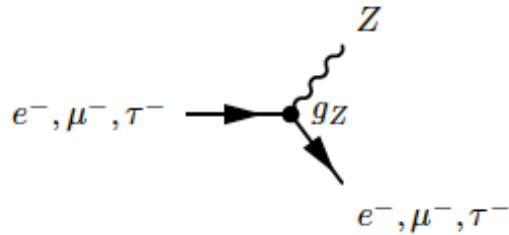
$$g_W = \frac{e}{\sin \theta_W}$$

$$g_Z = \frac{e}{\sin \theta_W \cos \theta_W} = \frac{g_W}{\cos \theta_W}$$

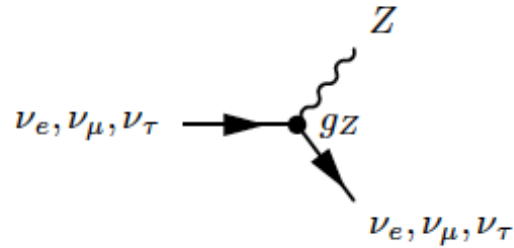
If we know α_{EM} , G_F , $\sin \theta_W$ (from experiment), everything else is defined.

The Weak NC Vertex

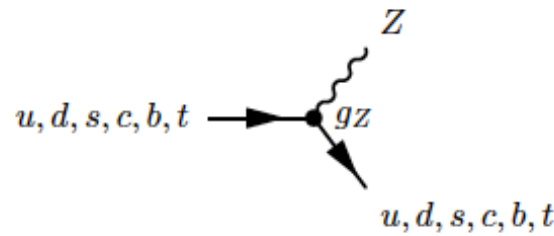
- All weak neutral current interactions can be described by the Z boson propagator and the weak vertices:



The Standard Model
Weak NC Lepton
Vertex



+ antiparticles

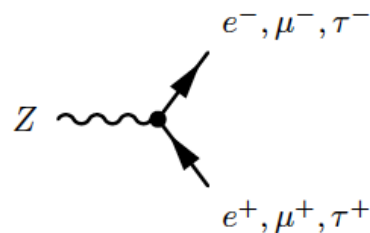


The Standard Model
Weak NC Quark Vertex

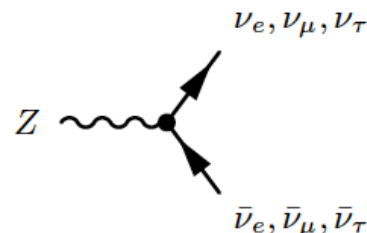
+ antiparticles

- Z never changes type of particle
 - Z never changes quark or lepton
- Z couplings are a mixture of EM and weak couplings, and therefore depend on $\sin^2\theta_W$

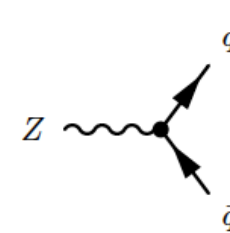
$$Z \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$$



$$Z \rightarrow \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$$

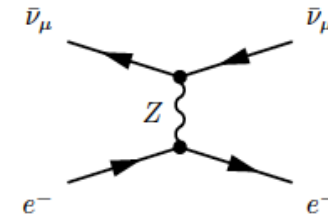


$$Z \rightarrow q\bar{q}$$



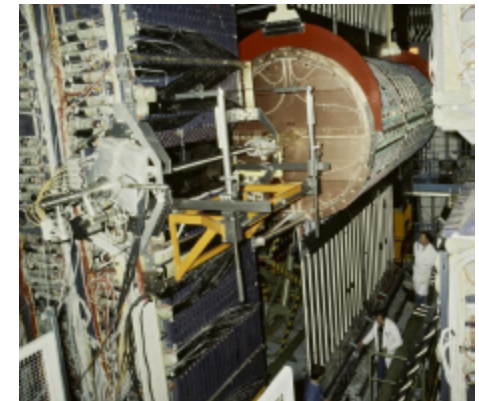
Evidence for GWS Model

- **Discovery of Neutral Currents (1973, Gargamelle)**
 - The process $e\nu \rightarrow e\nu$ was observed
 - Only possible Feynman diagram (no W diagram)
 - Indirect evidence for Z



- Direct Observation of W and Z (1983, UA1 at CERN)
 - First direct observation in p-anti-p collisions at $\sqrt{s} = 540 \text{ GeV}$ via decays into leptons

$$\begin{aligned}
 p\bar{p} &\rightarrow W^\pm + X & p\bar{p} &\rightarrow Z + X \\
 &\hookrightarrow e^\pm \nu_e, \mu^\pm \nu_\mu & &\hookrightarrow e^+ e^-, \mu^+ \mu^-
 \end{aligned}$$



- Precision Measurements of the Standard Model (1989-2000)
- LEP e^+e^- collider provided many precision measurements of the Standard Model
- Designed as a Z and W boson factory
- Precise measurements of the properties of Z and W bosons provide the most stringent test of our current understanding of particle physics
- LEP is the highest energy e^+e^- collider ever built $\sqrt{s} = 90 - 209 \text{ GeV}$
- 4 experiments combined saw 1.6×10^7 Z events, 3×10^4 W events



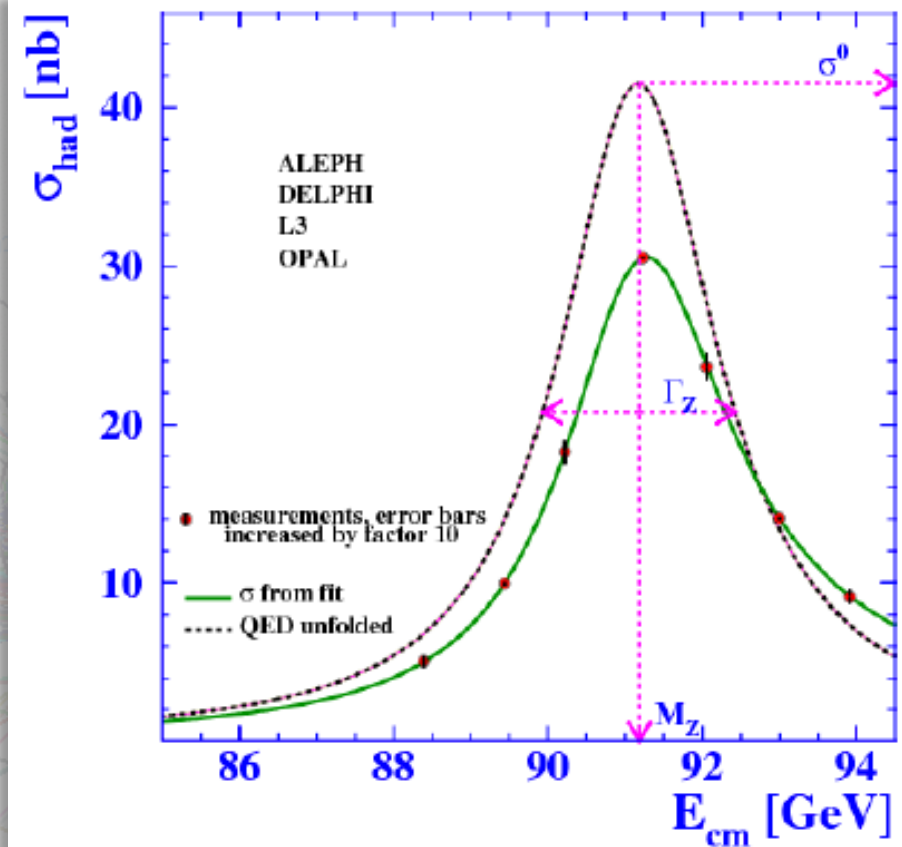
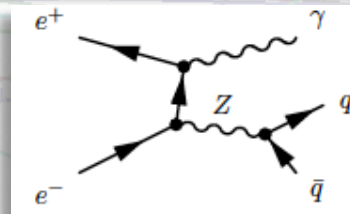
The Z Resonance

- Run LEP at various centre-of-mass energies (\sqrt{s}) close to the peak of the Z resonance and measure $\sigma(e^+e^- \rightarrow q \text{ anti-}q)$
- Determine the parameters of the resonance:
 - Mass of the Z, m_Z
 - Total decay width, Γ_Z
 - Peak cross-section, σ_0
- One subtle feature: need to correct measurements for QED effects due to radiation from the e^+e^- beams

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

$$\sigma_{q\bar{q}}^0 = 41.450 \pm 0.037 \text{ nb}$$



- **For such precision \Rightarrow detailed understanding of the accelerator and astrophysics!** E.g.
 - tidal distortions of the Earth by the Moon cause the rock surrounding LEP to be distorted - changing the radius by 0.15 mm (total 4.3 km). This is enough to change the centre-of-mass energy.
 - Also need a train timetable. Leakage currents from the TGV rail via Lake Geneva follow the path of least resistance... using LEP as a conductor.

Number of Generations

- Currently know of three generations of fermions
- The Z boson couples to all fermions, including neutrinos. Therefore, the total decay width, Γ_Z , has contributions from all fermions with $m_f > m_Z/2$

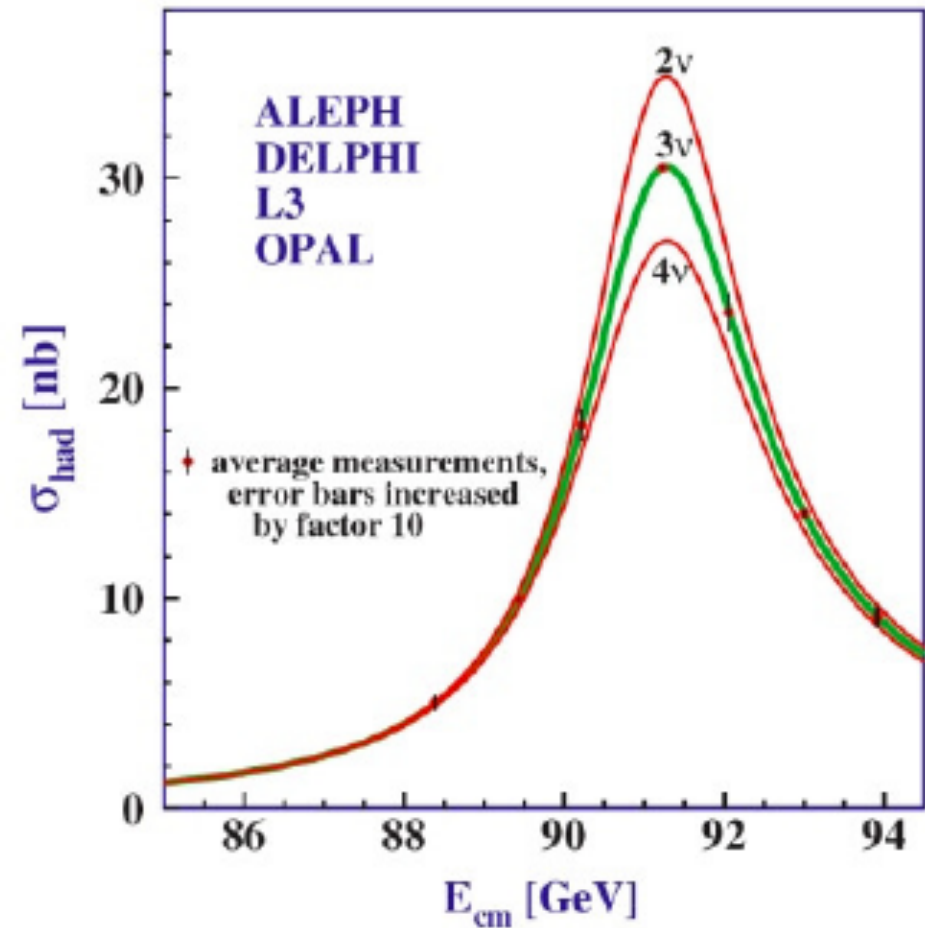
$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}$$

with $\Gamma_{\nu\bar{\nu}} = \Gamma_{\nu_e\bar{\nu}_e} + \Gamma_{\nu_\mu\bar{\nu}_\mu} + \Gamma_{\nu_\tau\bar{\nu}_\tau}$

- If there were fourth generation, and its neutrino would be light, it would be produced at LEP
- The neutrinos cannot be observed directly, but measured Z width depends on the number of neutrinos flavours the Z decays into
- Consistent with just three neutrinos!
 - Only three generations of matter ??

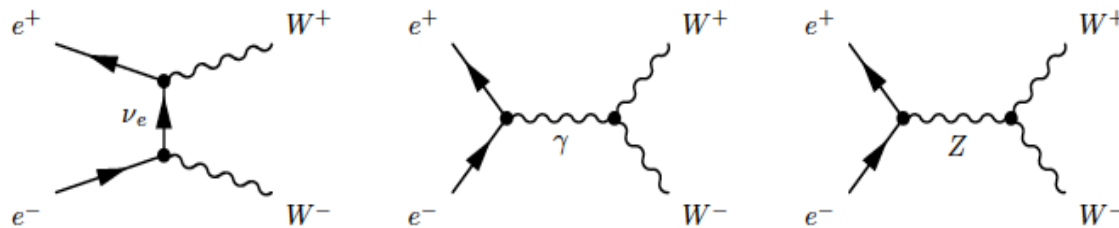
In addition:

- $\Gamma_{ee}, \Gamma_{\mu\mu}, \Gamma_{\tau\tau}$ are consistent \Rightarrow universality of the lepton couplings to the Z boson
- Γ_{qq} consistent with the expected value which assumes 3 colours

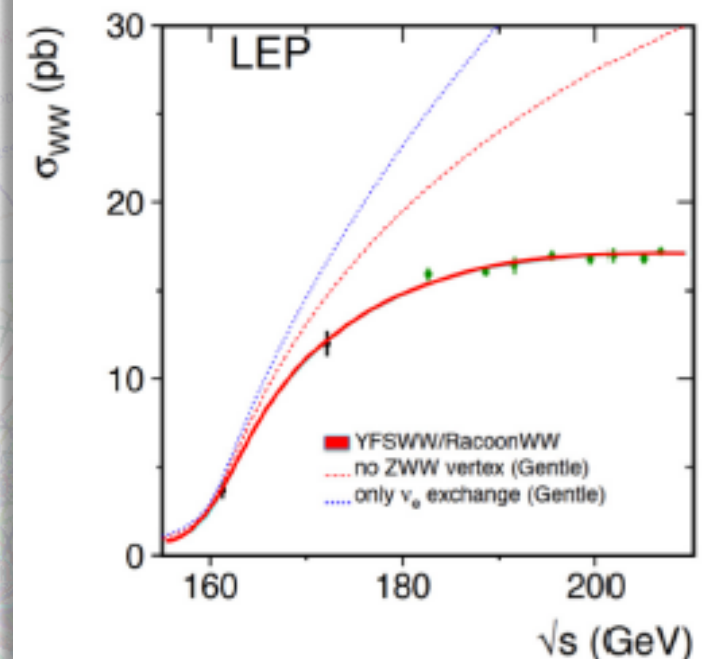


W⁺W⁻ at LEP

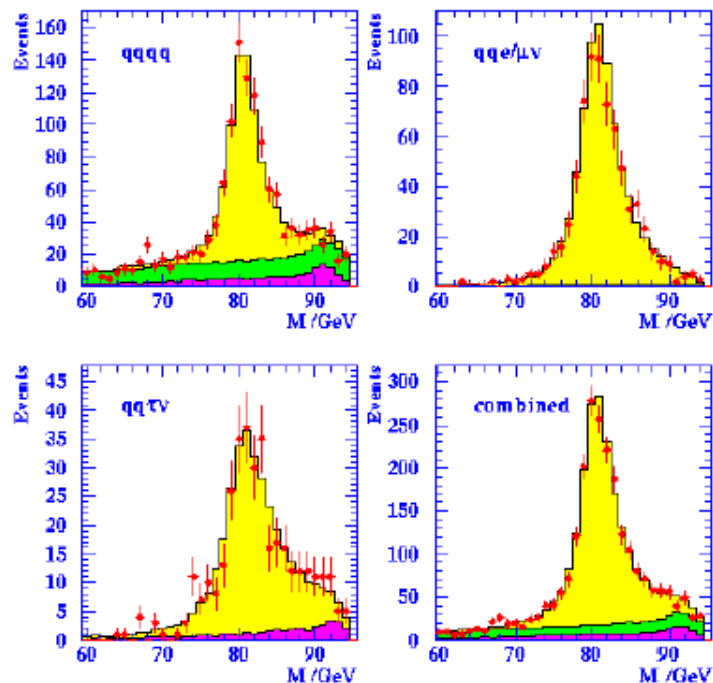
- In e⁺e⁻ collisions W bosons are produced in pairs
- Standard Model: 3 possible diagrams (destructive interference):



- Cross-section sensitive to the presence of the Triple Gauge Boson vertex



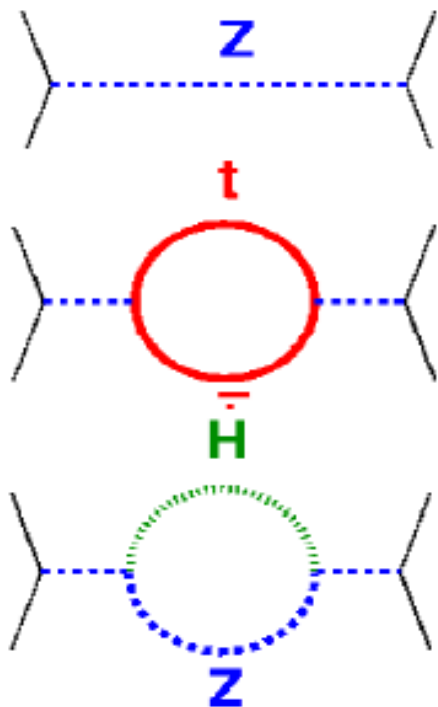
OPAL 189 GeV (prelim)



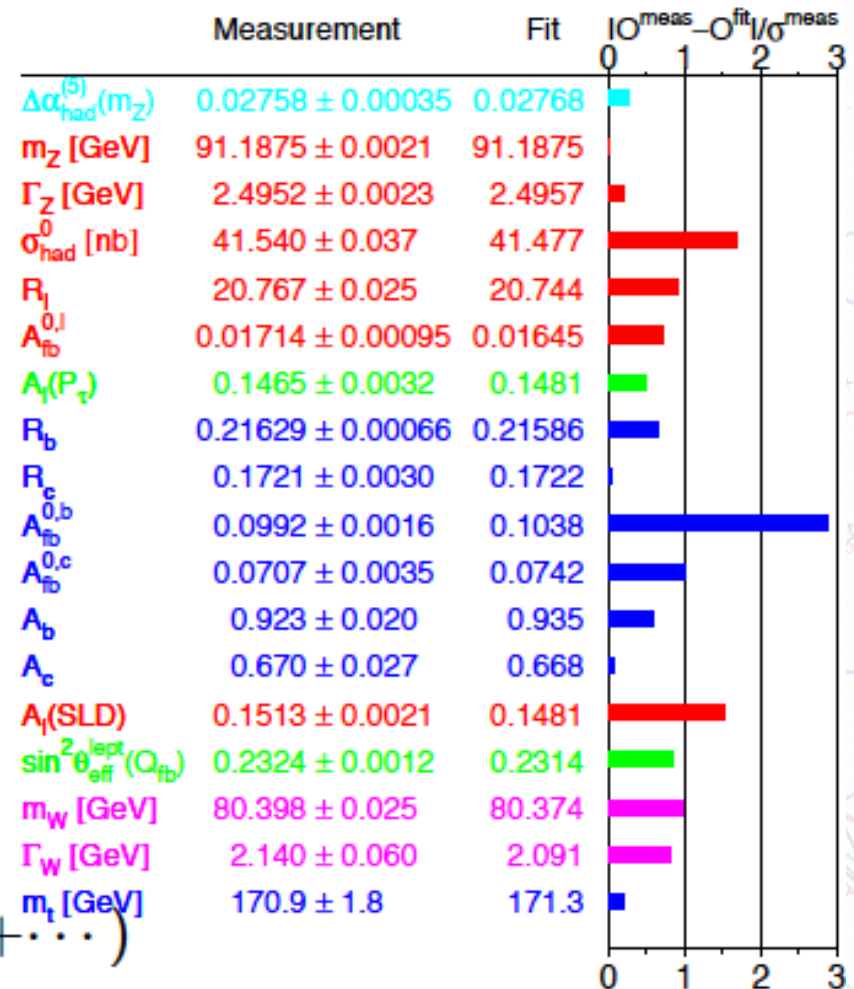
- Also precise measurement of m_W
- Unlike e⁺e⁻ → Z, W boson production at LEP was not a resonant process
- m_W was measured by measuring the invariant mass in each event
 - $m_W = 80.423 \pm 0.038 \text{ GeV}$
 - $\Gamma_W = 2.12 \pm 0.11 \text{ GeV}$

LEP: Precision Tests of Loop Corrections

e^+e^- machines can see effects of virtual particles



$$M_Z^2 = M_Z^{2\text{0th order}} (1 + \mathcal{O}(m_t^2) + \mathcal{O}(\ln m_h^2) + \dots)$$



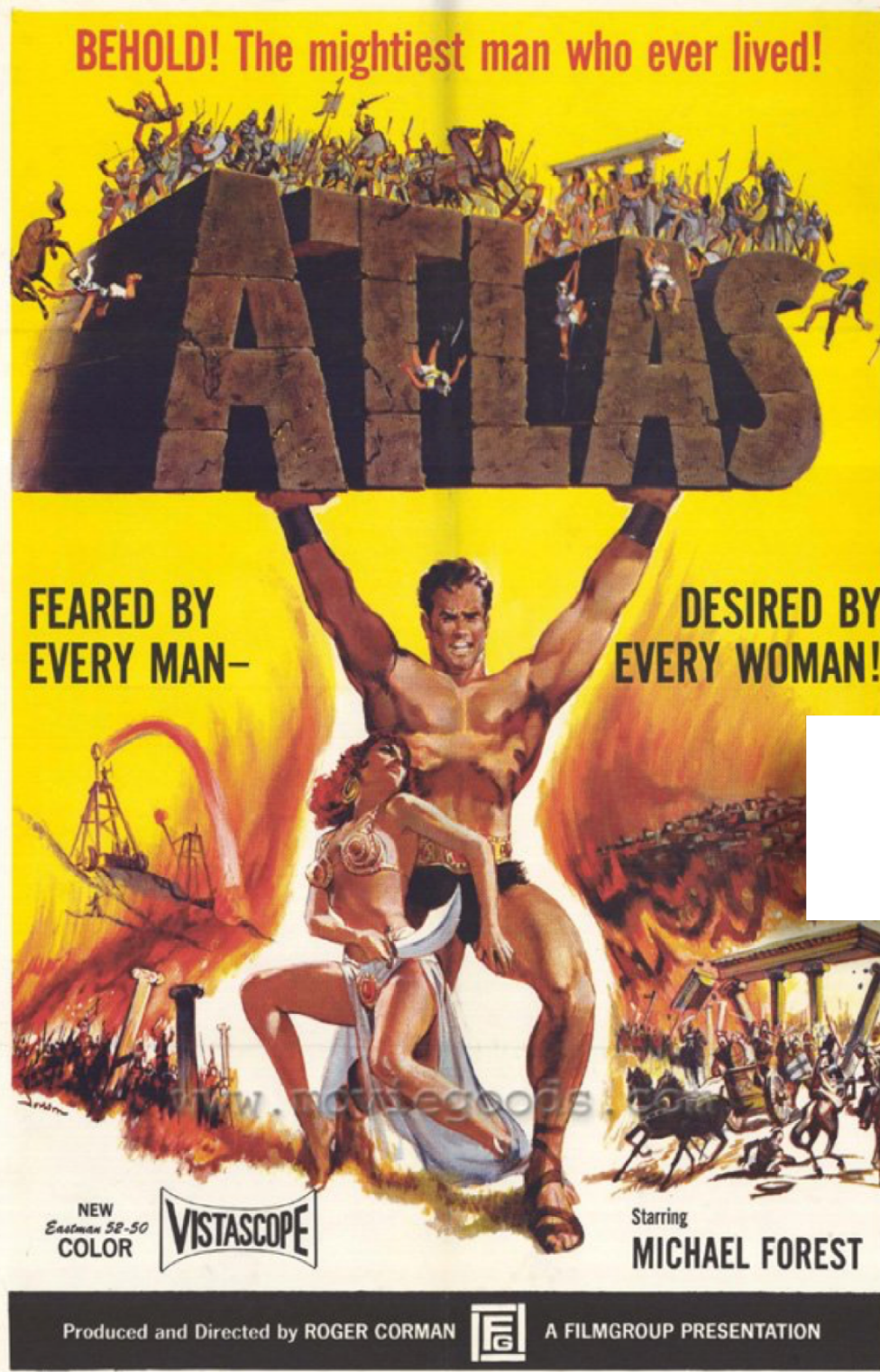
Sensitivity to New Physics!

THE END

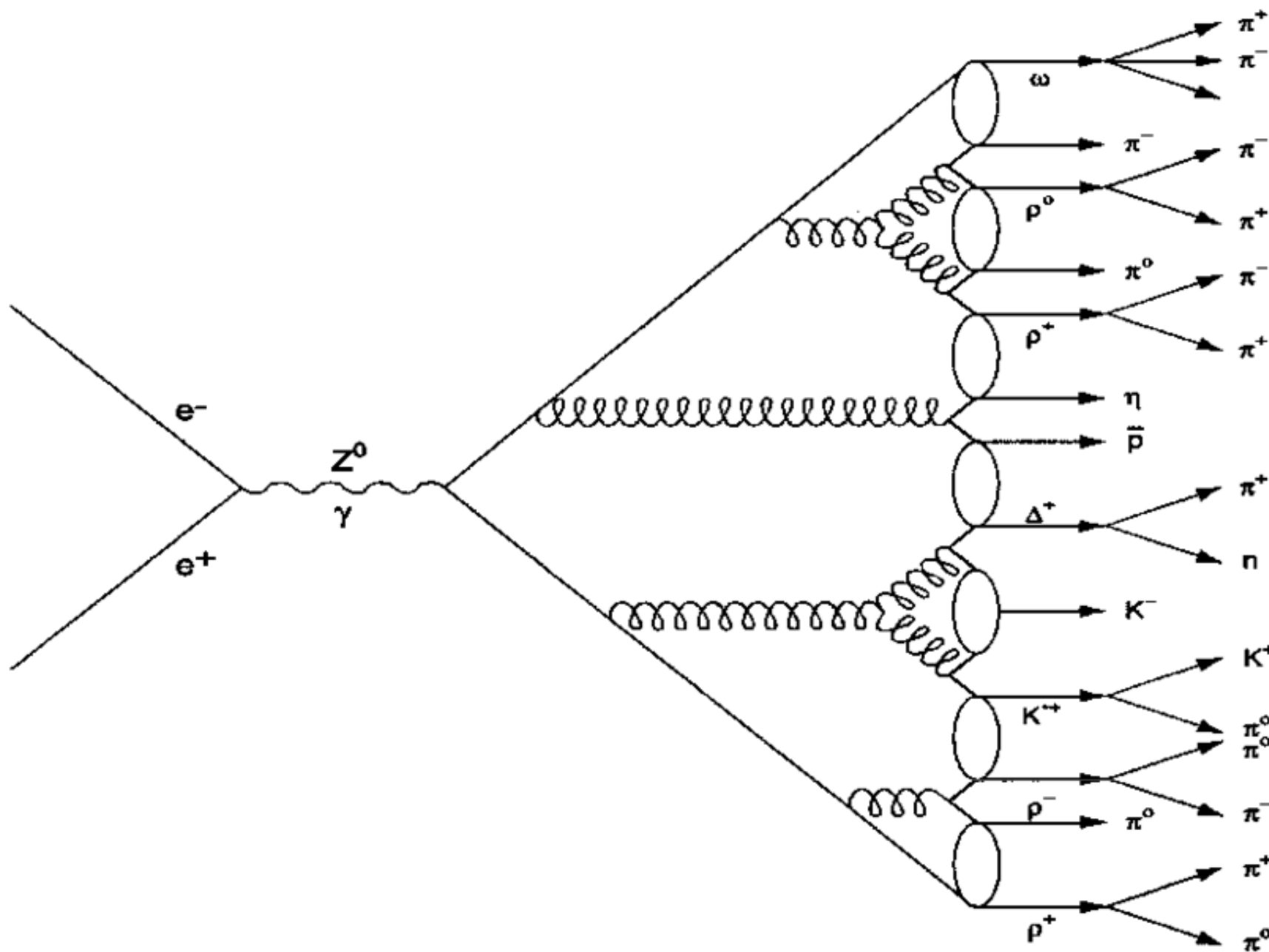


A
WARNER BROS. —
FIRST NATIONAL PICTURE



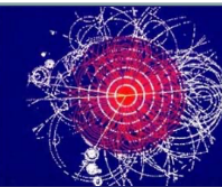


Dodatkowe slajdy
dla ciekawskich



03-87
B290A1B

Ogólny test teorii elektrosłabej



➤ Tzw. „pull”

$$P = \frac{O^{meas} - O^{fit}}{\sigma^{meas}}$$

➤ Model Standardowy (EW) przeszedł globalny test wspaniale.

➤ Można było oczekiwać, że teoria EW przestanie się zgadzać z danymi przy precyzji pomiarów rzędu procenta.

➤ Tymczasem zgodność dane-przewidywania nawet na poziomie promila.

➤ Zależność obserwacji od poprawek radiacyjnych daje ważne wyniki (przykłady):

1. Masa kwarka t została obliczona z globalnych fitów jako $m_t \approx 170 \pm 20$ GeV jeszcze przed jego odkryciem $O \sim m_t^2$

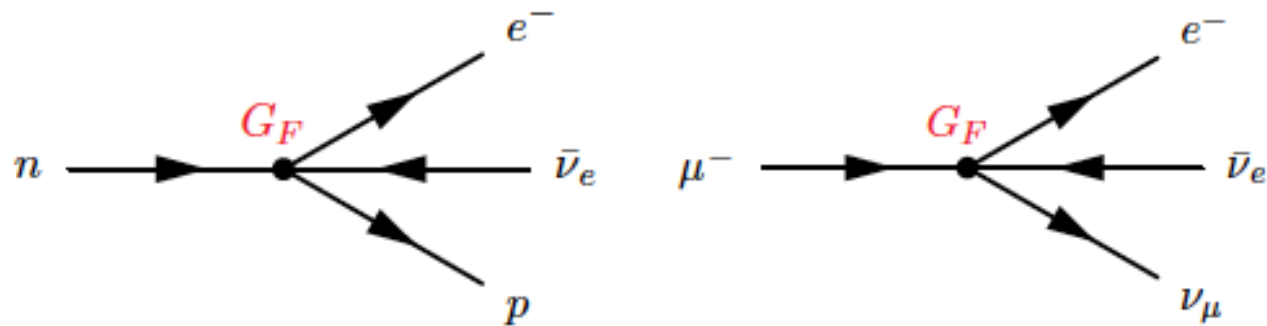
2. Dzięki globalnym fitom mamy ograniczenie na masę cząstki Higgsa $(100-300)$ GeV; $O \approx \log m_H^2$

Fermi Theory *The old ("imperfect") idea*

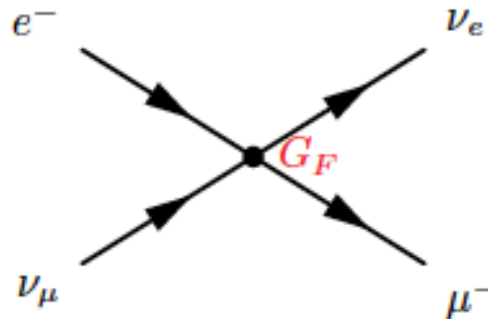
Weak interaction taken to be a "4-fermion contact interaction"

- No propagator
- Coupling strength given by the Fermi constant G_F
- $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$

β -decay in Fermi Theory



Neutrino scattering in Fermi Theory

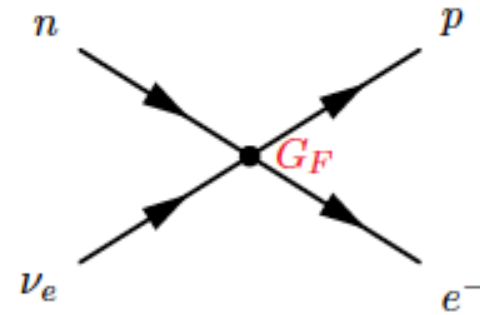


Why must Fermi Theory be “Wrong”?

$$\nu_e + n \rightarrow p + e^-$$

$$d\sigma = 2\pi |M_{fi}|^2 \frac{dN}{dE} = 2\pi 4G_F^2 \frac{E_e^2}{(2\pi)^3} d\Omega$$

$$\sigma = \frac{G_F^2 s}{\pi} \quad \text{See Appendix F}$$



where E_e is the energy of the e^- in the centre-of-mass system and \sqrt{s} is the energy in the centre-of-mass system.

In the laboratory frame: $s = 2E_\nu m_n$ (fixed target collision, see Chapter 3)

$$\Rightarrow \sigma \sim (E_\nu / \text{MeV}) \times 10^{-43} \text{ cm}^2$$

- ν 's only interact **weakly** \therefore have very small interaction cross-sections.
- Here **weak** implies that you need approximately 50 light-years of water to stop a 1 MeV neutrino!

However, as $E_\nu \rightarrow \infty$ the cross-section can become very large. Violates maximum value allowed by conservation of probability at $\sqrt{s} \sim 1 \text{ TeV}$ (“unitarity limit”). This is a big problem.

\Rightarrow Fermi theory breaks down at high energies.

Mieszanie kwarków

Parametryzacja

Parametryzacja Wolfenstein'a macierzy CKM:

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad \begin{aligned} \lambda &\approx \sin \theta_C \\ A, \rho, \eta &\sim 1 \end{aligned}$$

Elementy V_{td} i V_{ub} mogą być zespolone !

⇒ bezpośrednie łamanie CP w oddziaływaniach słabych

(w odróżnieniu od łamania pośredniego, poprzez mieszanie stanów o różnej symetrii)

Bardzo subtelny efekt...

Bezpośrednie łamanie CP zaobserwowano jedynie w rozpadach mezonów K^0 i B^0

Postać teoretyczna macierzy CKM

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$s_{ij} = \sin(\theta_{ij})$ $c_{ij} = \cos(\theta_{ij})$

Faza jest odpowiedzialna za łamanie CP

Tylko te człony są praktycznie zespolone

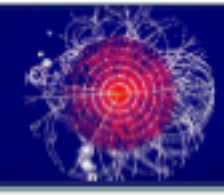
Czynnik fazowy zawsze mnożony jest przez najmniejszy kąt mieszania s_{13}



Efekty łamania CP są bardzo małe, z wyjątkiem szczególnych przypadków



Unifikacja elektroslaba



- Siłę elektroslabą opisuje lokalna teoria cechowania (gauge) oparta na iloczynie prostym dwóch grup symetrii $SU(2)_T \times U(1)_Y$
- $SU(2)_T$ - zachowana liczba kwantowa - słaby izospin (T); bozony: W_1, W_2, W_3 .
- $U(1)_Y$ - zachowana liczba kwantowa - hipertładunek (Y); bozon: B.
- Model elektroslaby już na pierwszym etapie swojej konstrukcji łamie parzystość: stany lewoskrętne tworzą dublety słabego izospinu, stany prawoskrętne - singlety.

Gdzie tu
 W^+, W^- i Z^0 ?

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$$

$$\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$$

Fermion Type			T_3	Y	Q
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1/2	-1/2	0
$\nu_{e,R}$	$\nu_{\mu,R}$	$\nu_{\tau,R}$	-1/2	-1/2	-1
e_R	μ_R	τ_R	0	0	0
			0	-1	-1
$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	1/2	1/6	2/3
u_R	c_R	t_R	-1/2	1/6	-1/3
d_R	s_R	b_R	0	2/3	2/3
			0	-1/3	-1/3

Model Weinberga-Salama

Pola materii:

leptonowe dublety lewoskrętne

$$I_3^W = +\frac{1}{2}$$

$$I_3^W = -\frac{1}{2}$$

$$\begin{pmatrix} \nu_{eL} \\ e_L^- \end{pmatrix}, \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L^- \end{pmatrix}, \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L^- \end{pmatrix},$$

leptonowe singlety prawoskrętne

$$e_R^-, \quad \mu_R^-, \quad \tau_R^-$$

kwarkowe dublety lewoskrętne

$$I_3^W = +\frac{1}{2}$$

$$I_3^W = -\frac{1}{2}$$

$$\begin{pmatrix} u_L \\ d_L' \end{pmatrix}, \quad \begin{pmatrix} c_L \\ s_L' \end{pmatrix}, \quad \begin{pmatrix} t_L \\ b_L' \end{pmatrix},$$

kwarkowe singlety prawoskrętne

$$d_R, \quad u_R, \quad s_R, \quad c_R, \quad b_R, \quad t_R$$

gdzie:

$$\psi_L(x) = \frac{1}{2}(1 + \gamma_5)\psi(x) \quad \psi_R(x) = \frac{1}{2}(1 - \gamma_5)\psi(x)$$