

Let us consider a more general problem, viz, all spacetime foam models characterized by a (numerical) parameter α (which, for the holographic foam model, equals $2/3$):

$$\delta l \sim l(l_P/l)^\alpha \text{ and } \delta t \sim t(t_P/t)^\alpha,$$

where l_P and t_P are respectively the Planck length ($\sim 10^{-33}$ cm) and the Planck time. (Notations: “ \sim ” means equal, order-of-magnitudewise.) I will have to bring in the so-called correction factor (coming from the need for “proper averaging”) [Reference: arXiv:gr/qc/0305019, especially sections 4 & 5].

Consider two beams of light, with energy E_1 and E_2 respectively, emitted simultaneously from a source which is a distance L away from the detector. To the lowest order of approximation, the two beams travel with a speed c (which later I will take to be 1 for convenience, ditto for the Planck’s constant) so that the time it takes for the journey is $t = L/v$ where, again to the lowest order of approximation, is equal to L/c . The general expression for the fluctuation of t (valid separately for the two beams of light), δt (which will give rise to a spread in arrival times), is given by two terms; call them term a and term b.

$$\text{Term a: } \delta t_a = (\delta L)/v \sim (L^{1-\alpha}l_P^\alpha)/c = t^{1-\alpha}t_P^\alpha.$$

Term b: $\delta t_b = (L/v^2)\delta v \sim (L/c^2)c(E/E_P)^\alpha$. But for this term involving δv , we need a correction factor $(Et)^{-\alpha}$, yielding the corrected $\delta t_b \sim t^{1-\alpha}t_P^\alpha$.

Thus, order-of-magnitudewise, the two terms a and b are equal! Hence we get $\delta t \sim t^{1-\alpha}t_P^\alpha$ which is INdependent of energy! In other words, we have $\delta t_1 \sim \delta t_2 \sim t^{1-\alpha}t_P^\alpha$, where $t = L/c$. Therefore, the two beams of light with different energies arrive at the detector within $\sim (L/c)^{1/3}t_P^{2/3}$ of each other for $\alpha = 2/3$ (the holographic spacetime foam model) — which is very small indeed!