Proton intermittency analysis in NA61/SHINE

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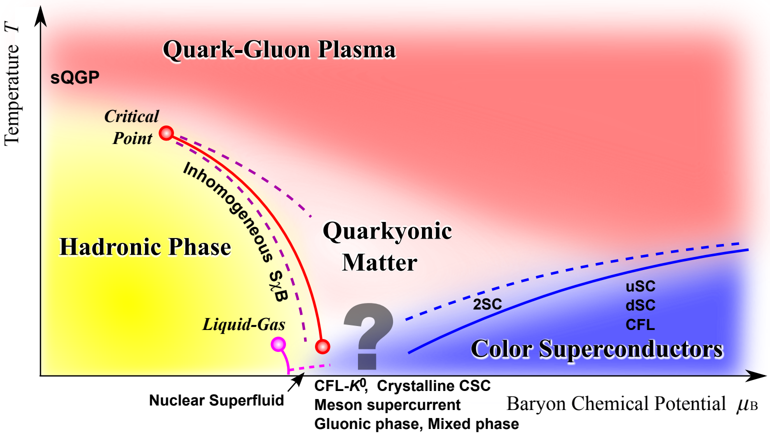
Phase diagram of QCD

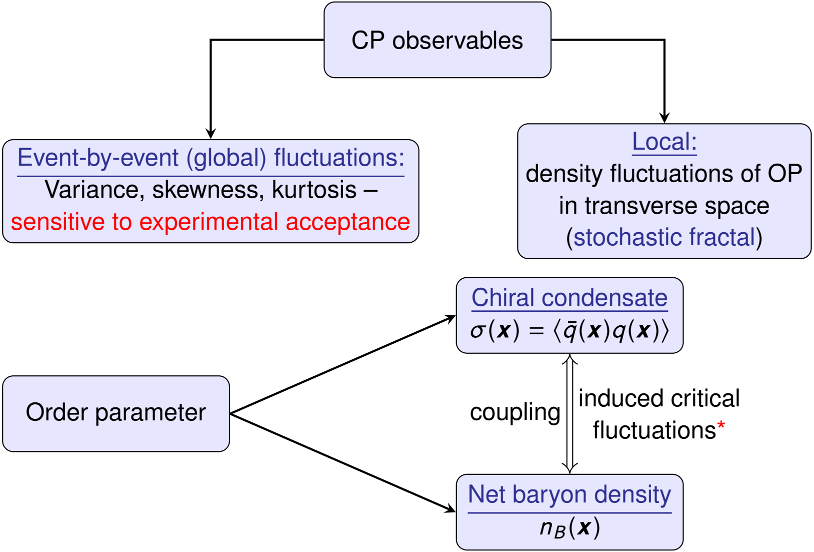
- Objective: Detection / existence of the QCD Critical Point (CP)



K. Fukushima, T. Hatsuda, Rept. Prog. Phys. 74:014001 (2011)

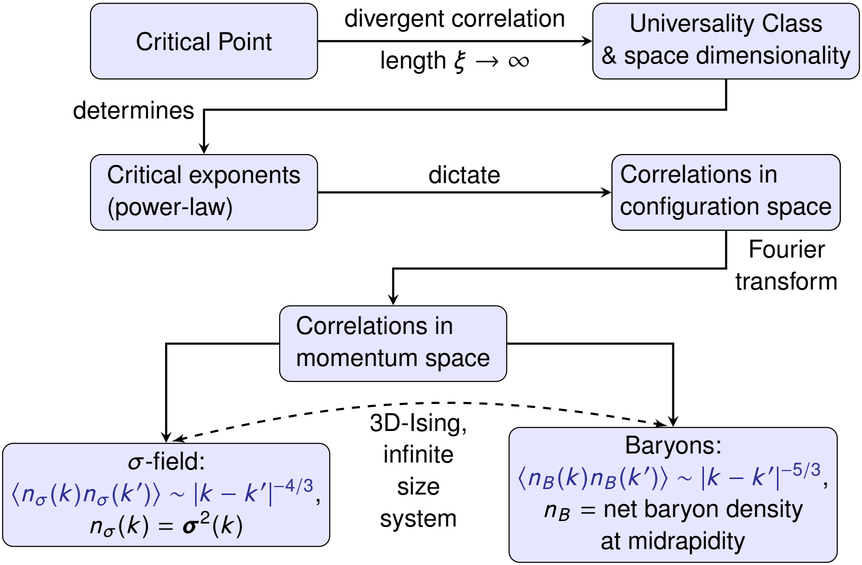
- Look for observables tailored for the CP; Scan phase diagram by varying energy and size of collision system.

Critical Observables; the Order Parameter (OP)



*[Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003)]

Self-similar density fluctuations near the CP

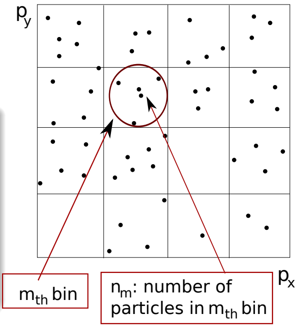


Observing power-law fluctuations: Factorial moments

Experimental observation of local, power-law distributed fluctuations ⇒ Intermittency[1-3] in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions[3])

- Net protons used as proxy for net baryons (same critical fluctuations[4]); finally, protons can be used (dominant contribution) & anti-protons dropped.
- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size ⇔ number of cells M:



$$F_2(M) \equiv \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2},$$

where $\langle \ldots \rangle$ denotes averaging over events.

[1] [J. Wosiek, *Acta Phys. Polon.* **B 19** (1988) 863-869]
[2] [A. Bialas and R. Hwa, *Phys. Lett.* **B 253** (1991) 436-438]
[3] [F.K. Diakonos, N.G. Antoniou and G. Mavromanolakis, PoS (CPOD2006) 010, Florence]
[4] [Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003)]

Subtracting the background from factorial moments

- Experimental data is noisy $\Rightarrow$ a background of non-critical pairs must be subtracted at the level of factorial moments.
- Intermittency will be revealed at the level of subtracted moments $\Delta F_2(M)$.

Partitioning of pairs into critical/background

$$\langle n(n-1)\rangle = \underbrace{\langle n_c(n_c-1)\rangle}_{\text{critical}} + \underbrace{\langle n_b(n_b-1)\rangle}_{\text{background}} + \underbrace{2\langle n_b n_c\rangle}_{\text{cross term}}$$

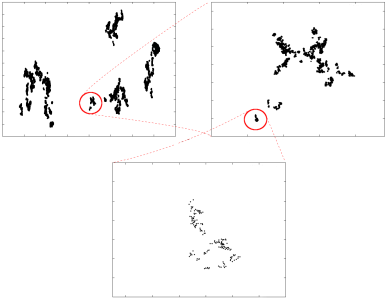
$$\underbrace{\Delta F_2(M)}_{\text{correlator}} = \underbrace{F_2^{(d)}(M)}_{\text{data}} - \lambda(M)^2 \cdot \underbrace{F_2^{(b)}(M)}_{\text{background}} - 2 \cdot \underbrace{\lambda(M)}_{\text{ratio } \frac{<n>_b}{<n>_d}} \cdot (1-\lambda(M))\, f_{bc}$$

- The cross term can be neglected under certain conditions (non-trivial! Justified by Critical Monte Carlo* simulations)

** [Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]*

Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
  - Only protons produced
  - One cluster per event, produced by random Lévy walk: $\tilde{d}_F^{(B,2)} = 1/3 \Rightarrow \phi_2 = 5/6$
  - Lower / upper bounds of Lévy walks $p_{\text{min,max}}$ plugged in.
  - Cluster center exponential in p_T, slope adjusted by T_c parameter.
  - Poissonian proton multiplicity distribution.



Input parameters

Parameter	p_{min} (MeV)	p_{max} (MeV)	λ_{Poisson}	T_c (MeV)
Value	$0.1 \rightarrow 1$	$800 \rightarrow 1200$	$\langle p \rangle_{\text{non-empty}}$	163

** [Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]*

Scaling of factorial moments – Subtracting mixed events

For $\lambda \lesssim 1$ (background domination), two approximations can be applied:

1. Cross term can be neglected
2. Non-critical background moments can be approximated by (uncorrelated) mixed event moments; then,

$$\Delta F_2(M) \simeq \Delta F_2^{(e)}(M) \equiv F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

For a critical system, ΔF_2 scales with cell size (number of cells, M) as:

$$\Delta F_2(M) \sim \left(M^2\right)^{\varphi_2}$$

where φ_2 is the intermittency index.

Theoretical prediction for φ_2

universality class, effective actions

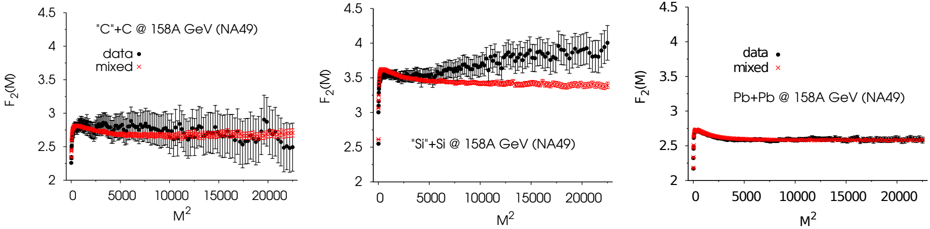
$$\varphi_{2,cr}^{(p)} = \frac{5}{6}\ (0.833\ldots)$$

net baryons (protons)

[N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis, K. S. Kousouris, Phys. Rev. Lett. **97**, 032002 (2006)]

NA49: “C”+C, “Si”+Si, Pb+Pb at 158A GeV/c

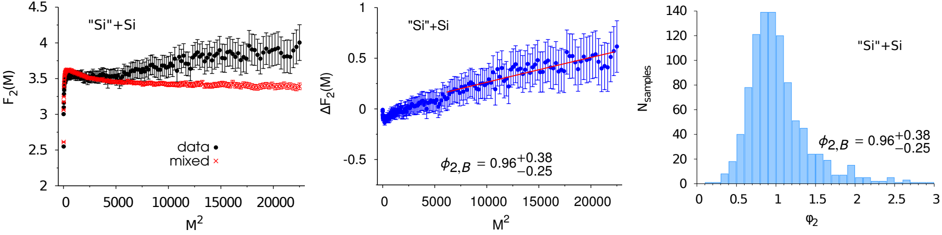
- 3 sets of NA49 collision systems were analysed[1], at 158A GeV/c: “C”+C, “Si”+Si, Pb+Pb (“C”=C,N ; “Si”=Si,Al,P)
- Factorial moments of proton transverse momenta analyzed at mid-rapidity
- Fit with $\Delta F_2^{(e)}(M\ ;\ \mathcal{C},\phi_2) = e^{\mathcal{C}} \cdot \left(M^2\right)^{\phi_2}$, for $M^2 \geq 6000$



- No intermittency detected in the “C”+C, Pb+Pb datasets.
- Evidence for intermittency in “Si”+Si – but large statistical errors.

[1] [T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]

NA49: "C"+C, "Si"+Si, Pb+Pb at 158A GeV/c

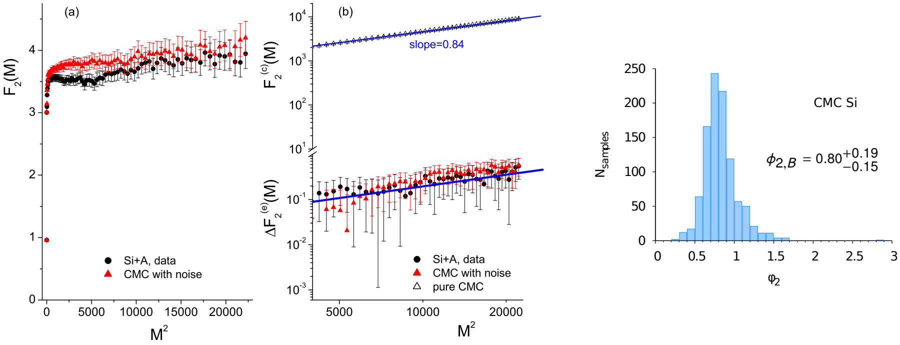


- Evidence for intermittency in "Si"+Si – but large statistical errors.
- Based on CMC simulation, we estimate a fraction of $\sim$ 1% critical protons are present in the sample.
- Estimated intermittency index[1]: $\phi_{2,B} = 0.96^{+0.38}_{-0.25}$(stat.) $\pm$ 0.16(syst.)

[1] [T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]

Noisy CMC (baryons) – estimating the level of background

- $F_2(M)$ of noisy CMC approximates "Si"+Si for $\lambda \approx 0.99$
- $\Delta F_2^{(e)}(M)$ reproduces critical behaviour of pure CMC, even though their moments differ by orders of magnitude!



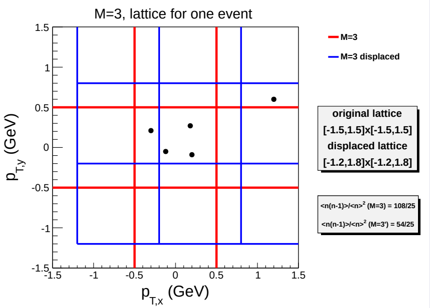
- Noisy CMC results show our approximation is reasonable for dominant background.

Improving calculation of $F_2(M)$ via lattice averaging

- Problem: With low statistics/multiplicity, lattice boundaries may split pairs of neighboring points, affecting $F_2(M)$ values (see example below).

- Solution: Calculate moments several times on different, slightly displaced lattices (see example)
- Average corresponding $F_2(M)$ over all lattices. Errors can be estimated by variance over lattice positions.
- Lattice displacement is larger than experimental resolution, yet maximum displacement must be of the order of the finer binnings, so as to stay in the correct p_T range.

Displaced lattice — a simple example



Improved confidence intervals for ϕ_2 via resampling

- In order to estimate the statistical errors of $\Delta F_2(M)$, we need to produce variations of the original event sample. This, we can achieve by using the statistical method of resampling (bootstrapping) $\Rightarrow$
  - Sample original events with replacement, producing new sets of the same statistics (# of events)
  - Calculate $\Delta F_2(M)$ for each bootstrap sample in the same manner as for the original.
  - The variance of sample values provides the statistical error of $\Delta F_2(M)$.

[W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]

- Furthermore, we can obtain a distribution $P(\varphi_2)$ of φ_2 values. Each bootstrap sample of $\Delta F_2(M)$ is fit with a power-law:

$$\Delta F_2(M; \mathcal{C}, \varphi_2) = e^{\mathcal{C}} \cdot (M^2)^{\varphi_2}$$

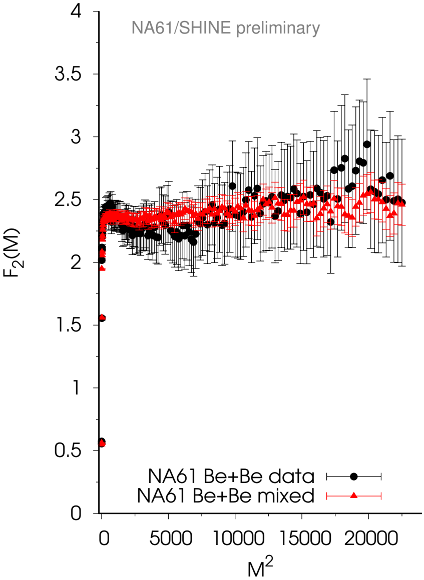
and we can extract a confidence interval for φ_2 from the distribution of values.

[B. Efron, *The Annals of Statistics* **7**,1 (1979)]

Systematic effect estimation

- Systematic uncertainties arise from:
  - Misidentification of protons & detector effects (e.g. acceptance)
  - The fact that $F_2(M)$ are correlated for different bin sizes M
  - Selection of M-range to fit for power-law
- Bin correlations are partially handled by the bootstrap φ_2 distribution, but that is insufficient! The effect of bin correlation has to be investigated through Critical and background Monte Carlo simulation; independent bins approach has also been attempted.
- Other systematic uncertainties are estimated by varying proton and M-range selection

NA61/SHINE: Be+Be at 150A GeV/c



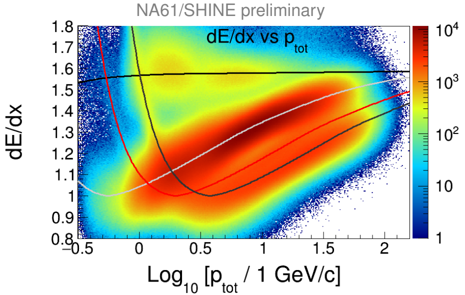
- $F_2(M)$ of data and mixed events overlap ⇒
- Subtracted moments $\Delta F_2(M)$ fluctuate around zero ⇒
- No intermittency effect is observed.
- Preliminary analysis with CMC simulation indicates an upper limit of ∼ 0.3% critical protons

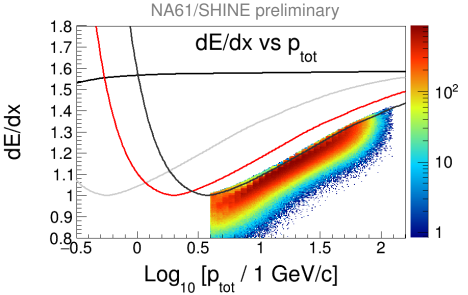
[PoS(CPOD2017) 054]

NA61/SHINE: ^{40}Ar + ^{45}Sc at 150A GeV/c

- First released results of preliminary analysis in Ar+Sc at 150A GeV/c – CPOD2018 Conference (Corfu, September 2018).
- NA61/SHINE CP task force created to verify and extend these results. Task force is spearheaded by IFJ Krakow group, with important contributions from Athens (NKUA), Warsaw (WUT, NCNR) and Frankfurt (FIAS).
- Intermittency analysis process:
    - Proton selection via particle energy loss dE/dx
    - Removal of split tracks – q_{inv} distribution & cut of proton pairs
    - Probe Δp_T distribution of proton pairs for power-law like behaviour in the limit of small p_T differences
    - Calculate factorial moments $F_2(M), \Delta F_2(M)$ for selected protons
    - Calculate intermittency index ϕ_2 (when possible) & estimate its statistical uncertainty
- Results were obtained for:
    - 0-5%, 5-10% and 10-15% centrality bins
    - 80%, 85% and 90% minimum proton purity selections

Proton selection

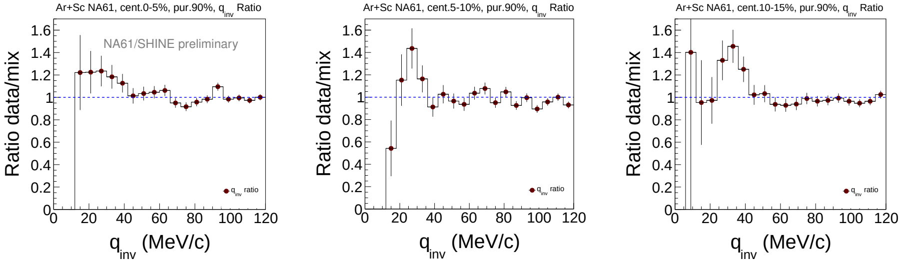




- Employ p_{tot} region where Bethe-Bloch bands do not overlap (3.98 GeV/c $\leq p_{\text{tot}} \leq 126$ GeV/c)
- Fit dE/dx distribution with 4-gaussian sum for $\alpha = \pi, K, p, e$ – Bins: p_{tot}, p_T
- 30 Bins in $\text{Log}_{10}(p_{\text{tot}})$: $10^{0.6} \rightarrow 10^{2.1}$ GeV/c
- 20 Bins in p_T: $0.0 \rightarrow 2.0$ GeV/c
- Proton purity: probability for a track to be a proton, $\mathcal{P}_p = p/(\pi + K + p + e)$
- Additional cut along Bethe-Blochs (avoid low-reliability region between p and K curves)

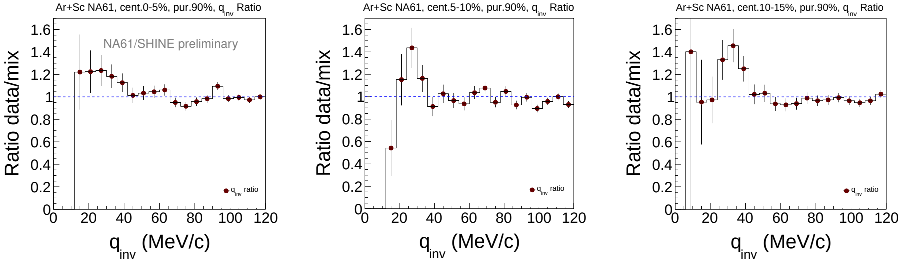
Split tracks & the q_{inv} cut

- Events may contain split tracks: sections of the same track erroneously identified as a pair of tracks that are close in momentum space.
- Three cuts to root them out:
  1. Ratio of points / potential points in a track (removes most)
  2. Minimum track distance in the detector (pair cut)
  3. q_{inv} cut (pair cut, physics-significant)
- q_{inv} distribution of track pairs probed in order to root the rest out:
  $q_{\mathrm{inv}}(p_i, p_j) \equiv \frac{1}{2}\sqrt{-(p_i - p_j)^2}$, p_i : 4-momentum of i^{th} track.
- We calculate the ratio of $q_{\mathrm{inv}}^{\mathrm{data}} / q_{\mathrm{inv}}^{\mathrm{mixed}}$.



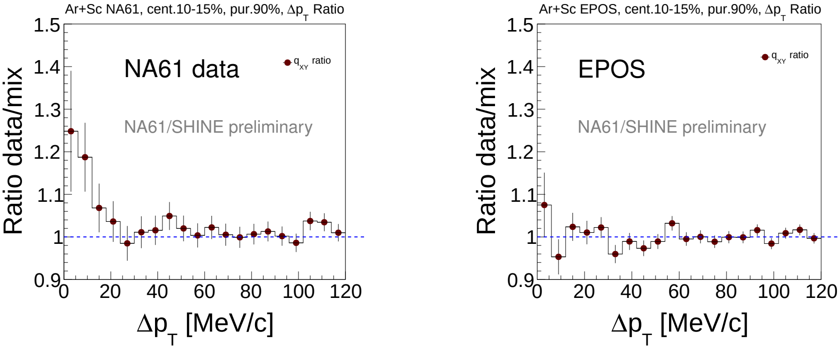
Split tracks & the q_{inv} cut

- A peak at low q_{inv} (below 20 MeV/c) indicates a possible split track contamination that must be removed.
- Anti-correlations due to F-D effects and Coulomb repulsion must be removed before intermittency analysis ⇒ “dip” in low q_{inv}, peak predicted around 20 MeV/c [Koonin, PLB 70, 43-47 (1977)]
- Universal cutoff of $q_{inv} > 7$ MeV/c applied to all sets before analysis.



Δp_T distributions: NA61 data vs EPOS*

- Ar+Sc at 150A GeV/c: $\Delta p_T = 1/2\sqrt{(p_{X_1} - p_{X_2})^2 + (p_{Y_1} - p_{Y_2})^2}$ distributions of protons selected for intermittency analysis



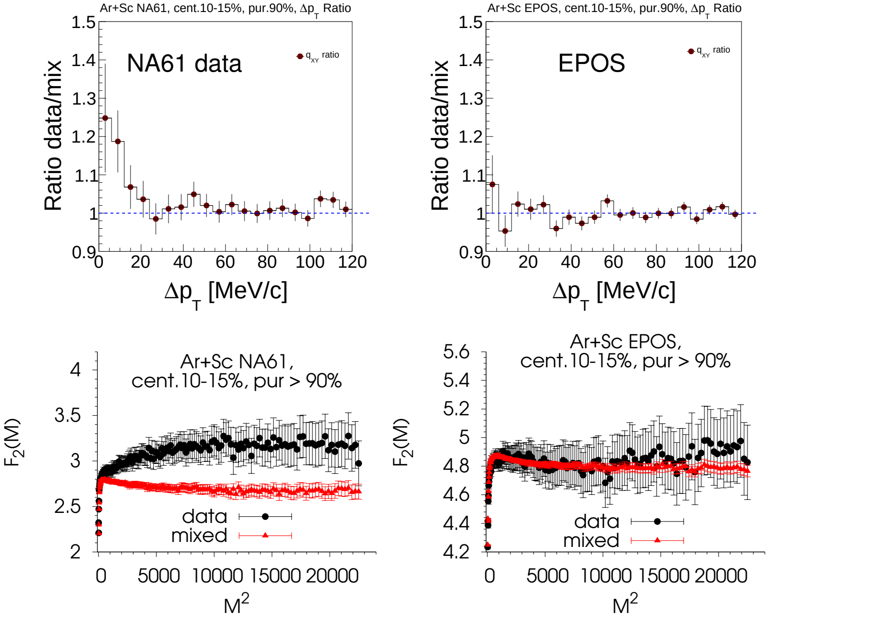
Significant peak at $\Delta p_T \to 0$

Flat distribution

- In NA61 data, we see strong correlations in $\Delta p_T \to 0 \Rightarrow$ indication of intermittent behaviour

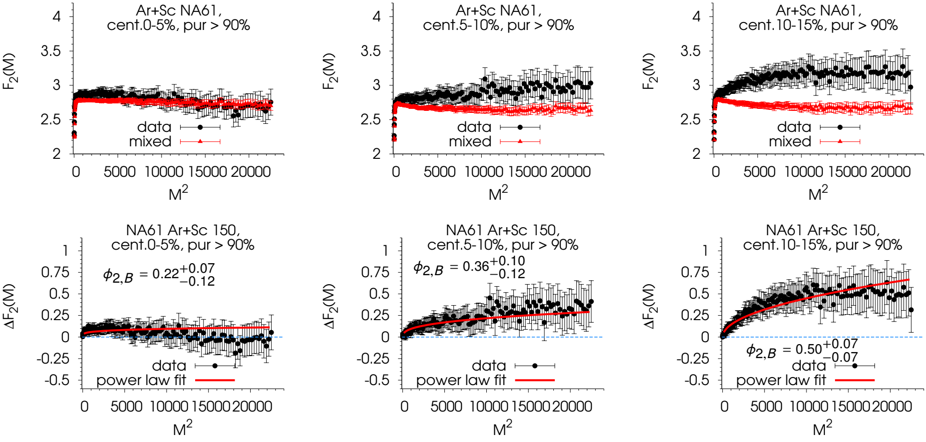
*[K. Werner, F. Liu, and T. Pierog, Phys. Rev. C 74, 044902 (2006)]

Δp_T distributions & $F_2(M)$: NA61 data vs EPOS

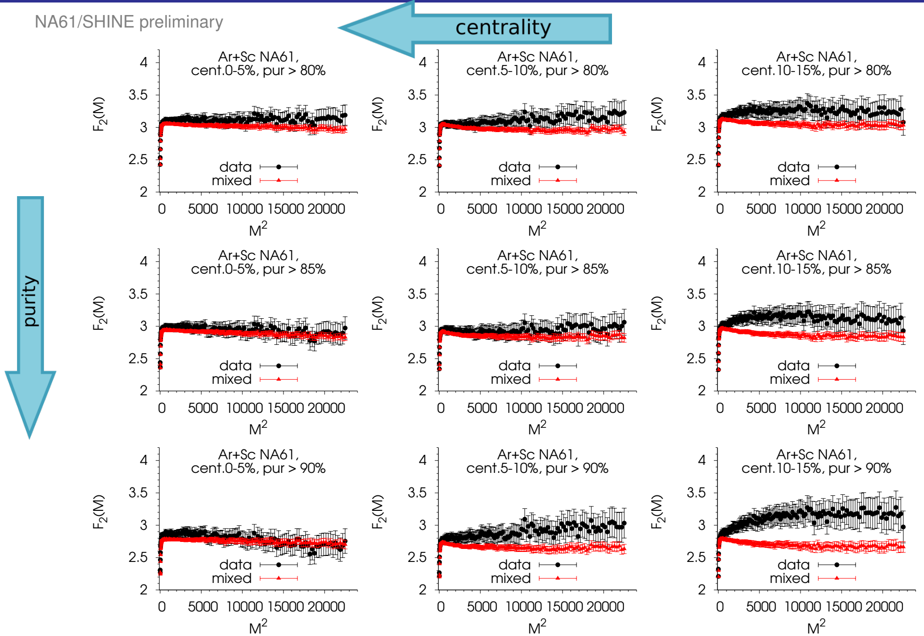


NA61/SHINE preliminary

NA61/SHINE: Ar+Sc at 150A GeV/c: $F_2(M)$, $\Delta F_2(M)$

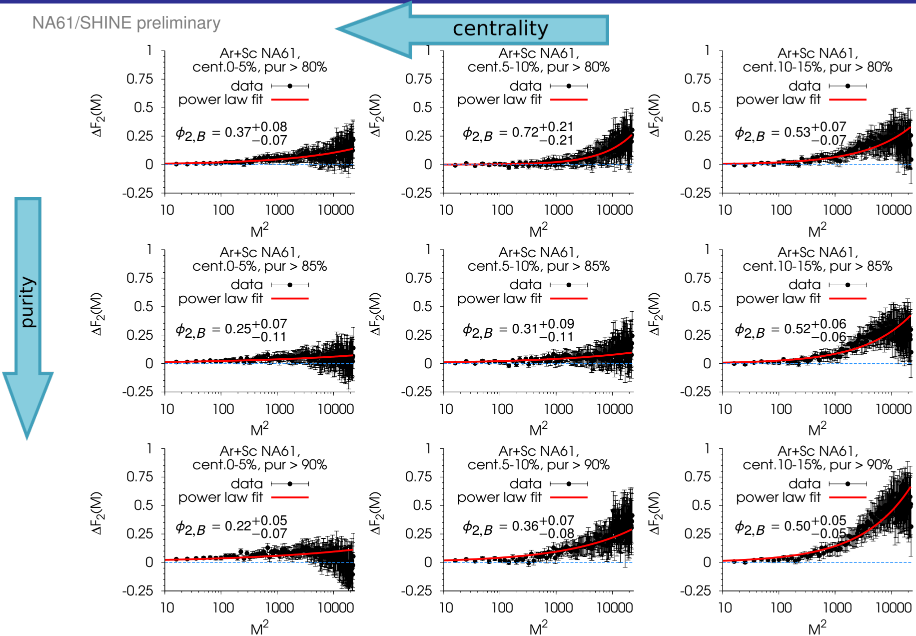


NA61/SHINE: Ar+Sc at 150A GeV/c: $F_2(M)$



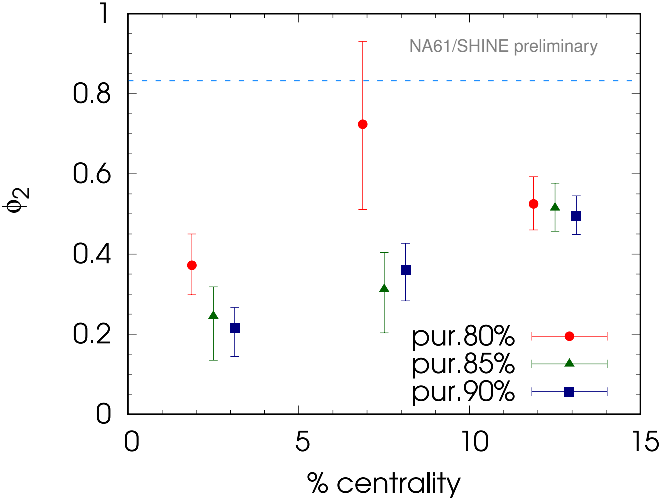
NA61/SHINE: Ar+Sc at 150A GeV/c: $\Delta F_2(M)$

NA61/SHINE preliminary



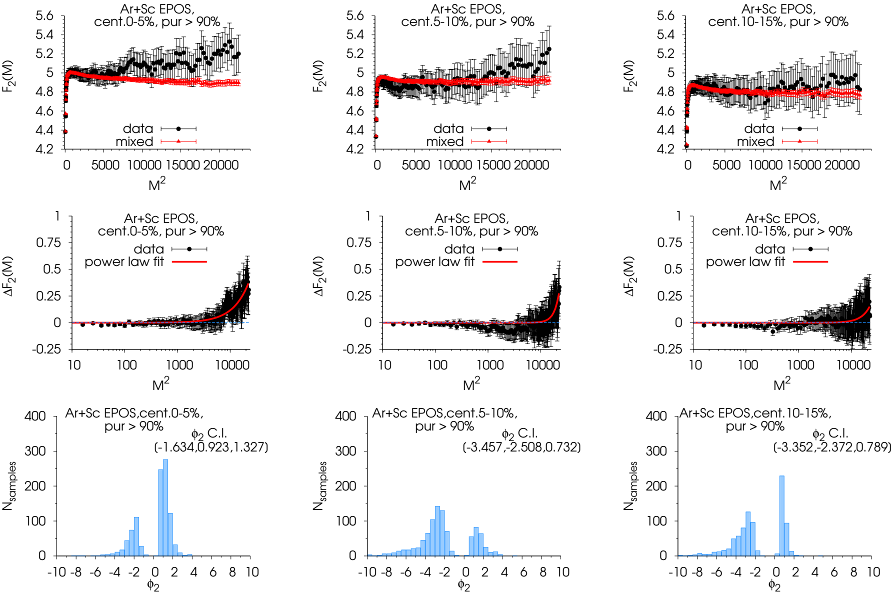
NA61/SHINE: Ar+Sc at 150A GeV/c: Summary

- Based on CMC simulation*, we estimate a fraction of 0.7% critical protons are present in the sample.



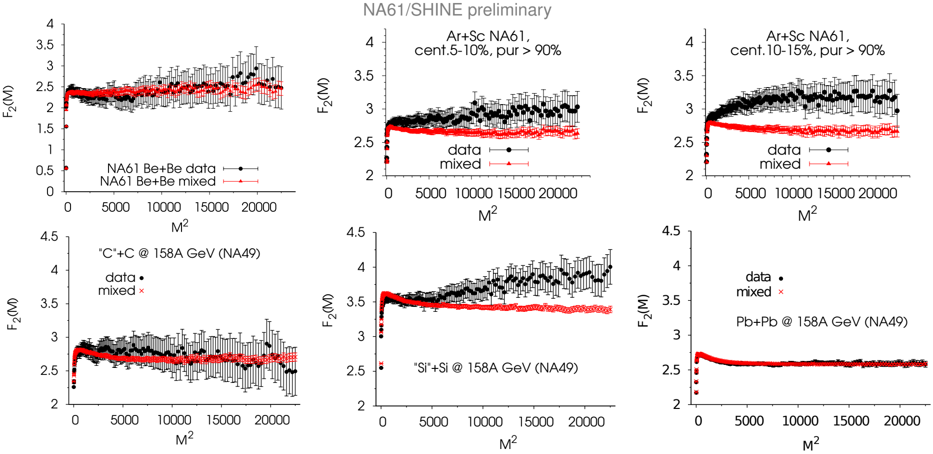
[Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]

Ar+Sc EPOS: $F_2(M)$, $\Delta F_2(M)$, ϕ_2 bootstrap distribution



NA61/SHINE preliminary

Intermittency analysis at 150/158A GeV/c: Summary

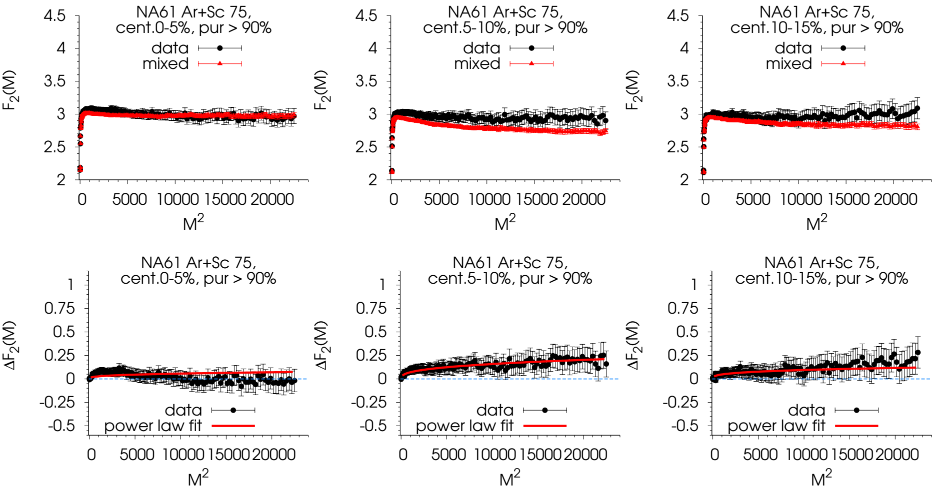


- Indication of intermittency effect in middle-central NA61/SHINE Ar+Sc collisions
- First possible evidence of CP signal in NA61/SHINE
- Effect quality increases with increased proton purity selection, up to 90% proton purity; EPOS does not reproduce observed effect.

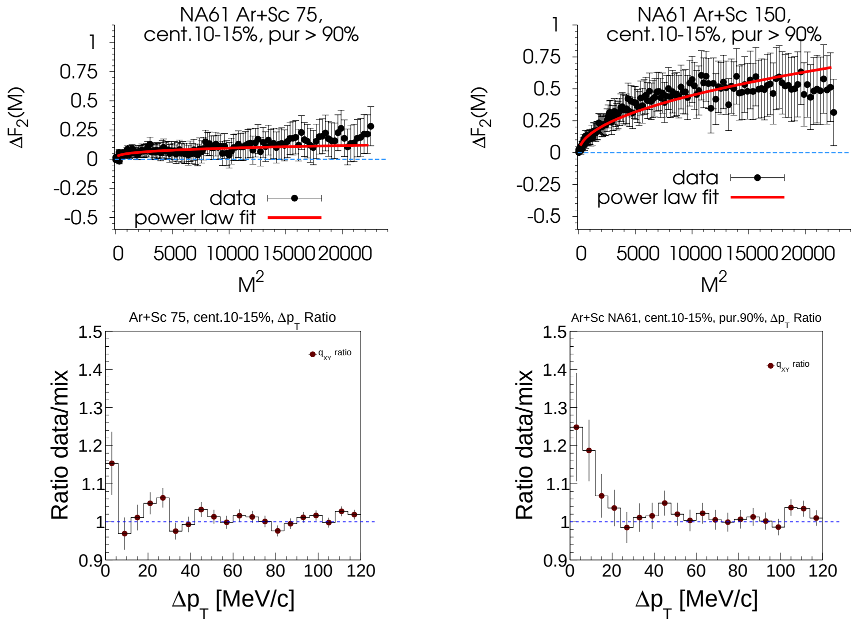
^{40}Ar + ^{45}Sc at 75A GeV/c – Cuts investigation – control subset

- We need an independent data set in order to investigate/optimize cut selection
- We randomly partition original set of events:
  - 30% of events ⇒ control subset
  - 70% of events ⇒ analysis subset
- Event statistics in control subset:
  - 0- 5% most central ⇒ 166K events
  - 5-10% most central ⇒ 160K events
  - 10-15% most central ⇒ 157K events
- Event statistics in analysis subset:
  - 0- 5% most central ⇒ 387K events
  - 5-10% most central ⇒ 375K events
  - 10-15% most central ⇒ 367K events
- In what follows, we present intermittency analysis results on the control & analysis subsets

$F_2(M)$, $\Delta F_2(M)$ – Ar+Sc 75 NA61 (analysis, 90% purity)

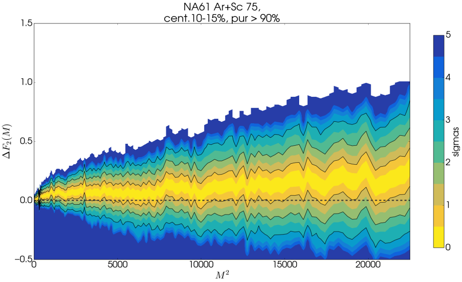


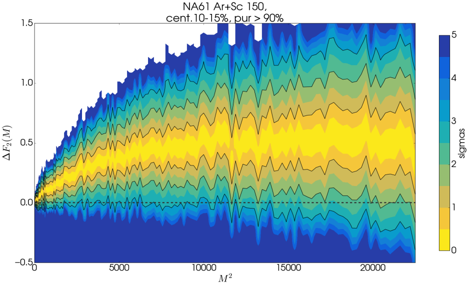
$\Delta F_2(M)$, Δp_T – Ar+Sc 75 , 150 comparison (analysis)



$\Delta F_2(M)$ – Ar+Sc 75 , 150 comparison (analysis)

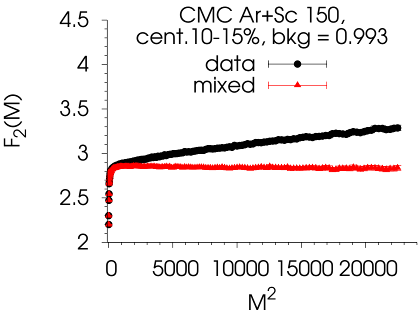
- $\Delta F_2(M)$ bootstrap distributions – contour map of sigmas from the median

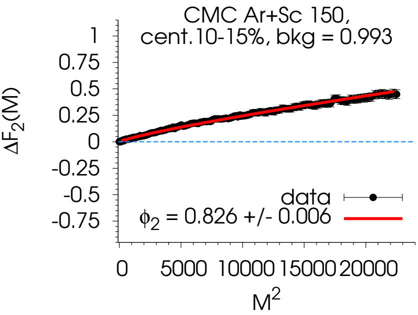




- $\sim 1\sigma$ separation of $\Delta F_2(M)$ from zero in Ar+Sc 75
- $\sim 2 - 3\sigma$ separation of $\Delta F_2(M)$ from zero in Ar+Sc 150

$F_2/\Delta F_2(M)$ – CMC ArSc150 + 99.3% noise (10M run)



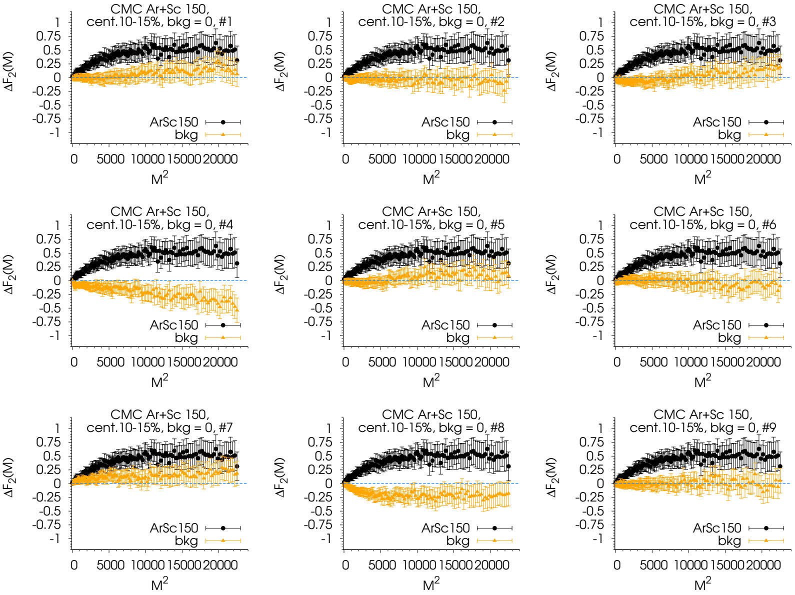


- $F_2(M)$ and $\Delta F_2(M)$ values have converged to almost their "true" (expected) values
- intermittency index ϕ_2 is very close to the theoretically expected value of a pure critical system.
- We can now use these settled $F_2(M)$ values to check the convergence of various sub-sampling schemes – starting with non-overlapping sub-samples.

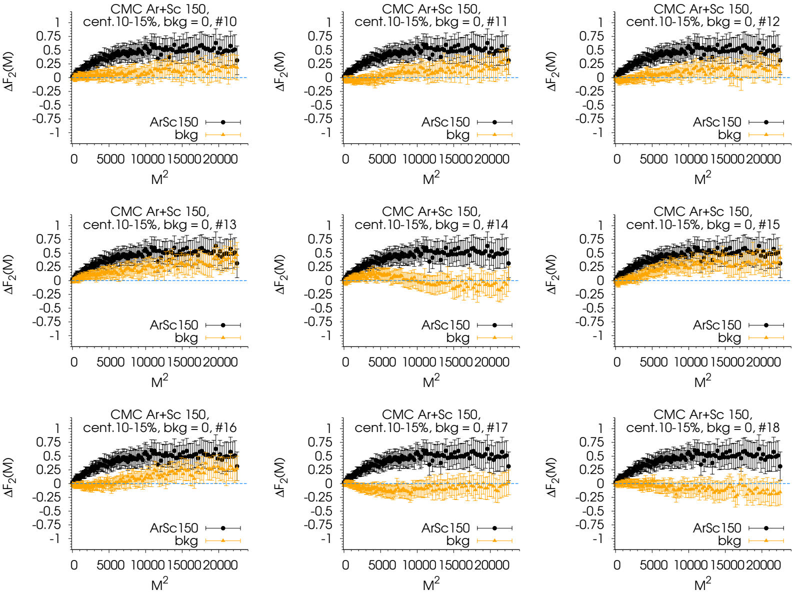
Critical Monte Carlo – spurious signal test

- We try to estimate how likely it is for a spurious signal to appear in non-critical events for low statistics.
- Comparison of $F_2(M)$ of data & CMC is risky due to CMC not including any pair cuts (TTD, q_{inv}).
- In contrast, $\Delta F_2(M)$ is relatively safer – pair cuts are applied to both original data and mixed events.
- For a statistics of 134×150K events, we calculate $\Delta F_2(M)$ for:
    1. CMC with 100% noise (pure background)
- We compare resulting $\Delta F_2(M)$ to $\Delta F_2(M)$ of NA61 ArSc150 data (10-15% centrality)

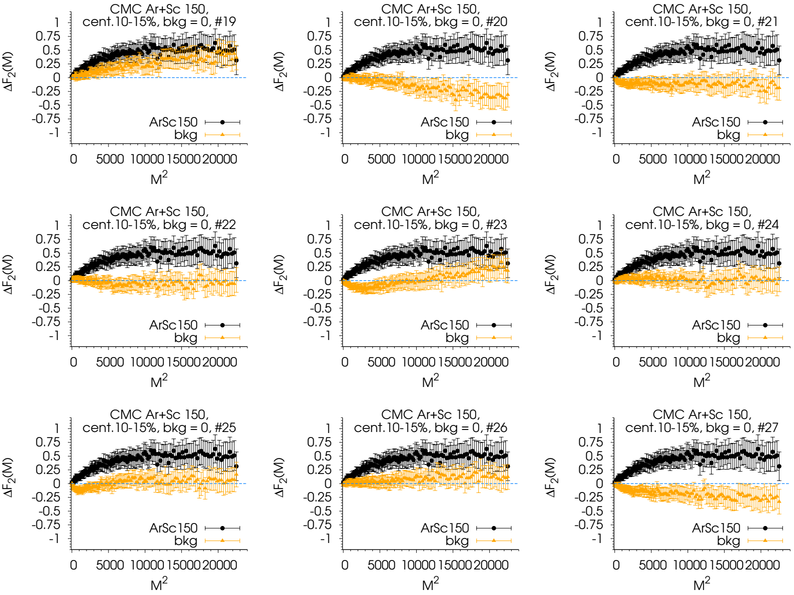
Spurious signal test, background – $\Delta F_2(M)$



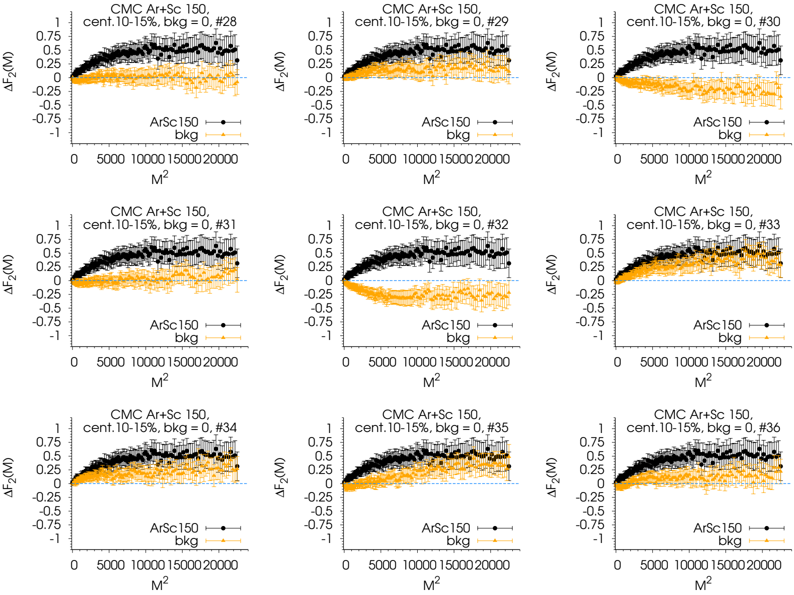
Spurious signal test, background – $\Delta F_2(M)$



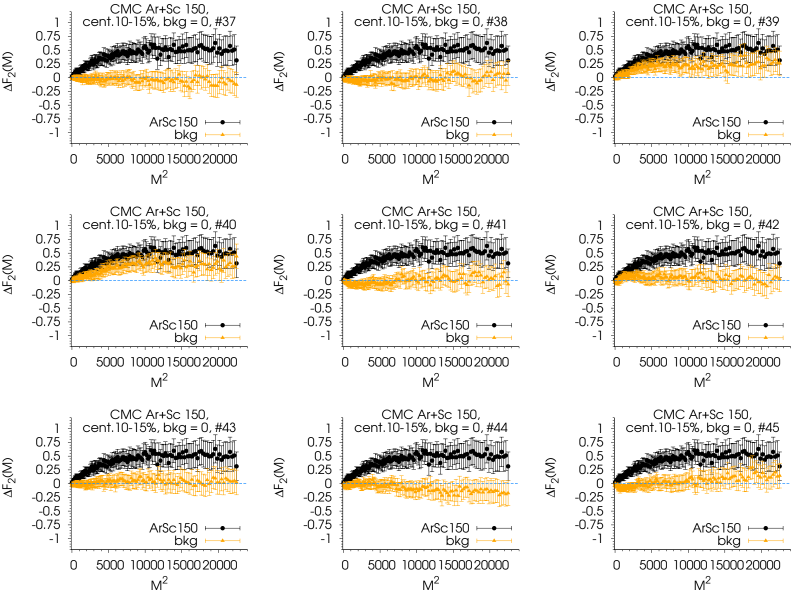
Spurious signal test, background – $\Delta F_2(M)$



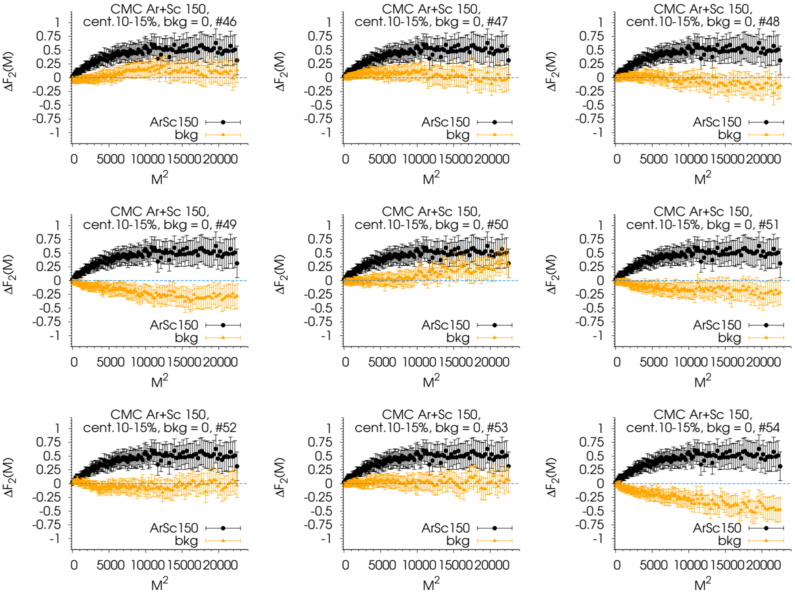
Spurious signal test, background – $\Delta F_2(M)$



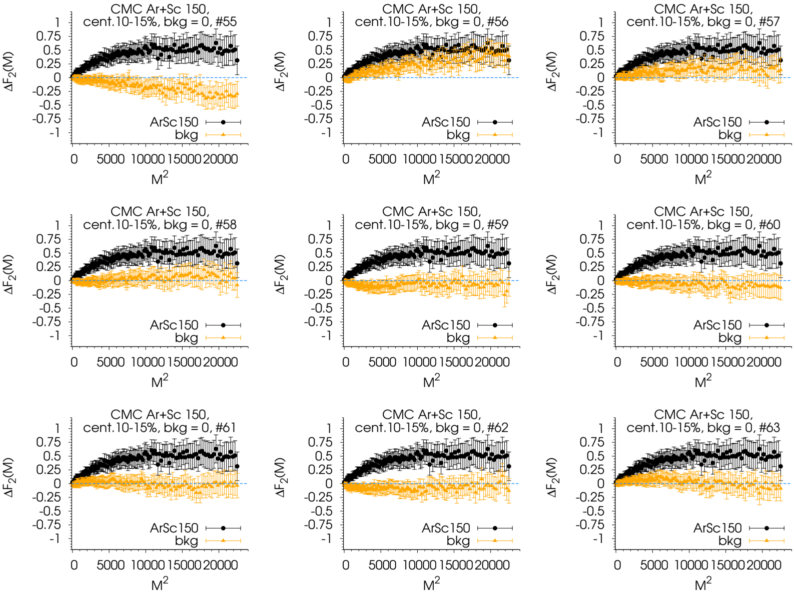
Spurious signal test, background – $\Delta F_2(M)$



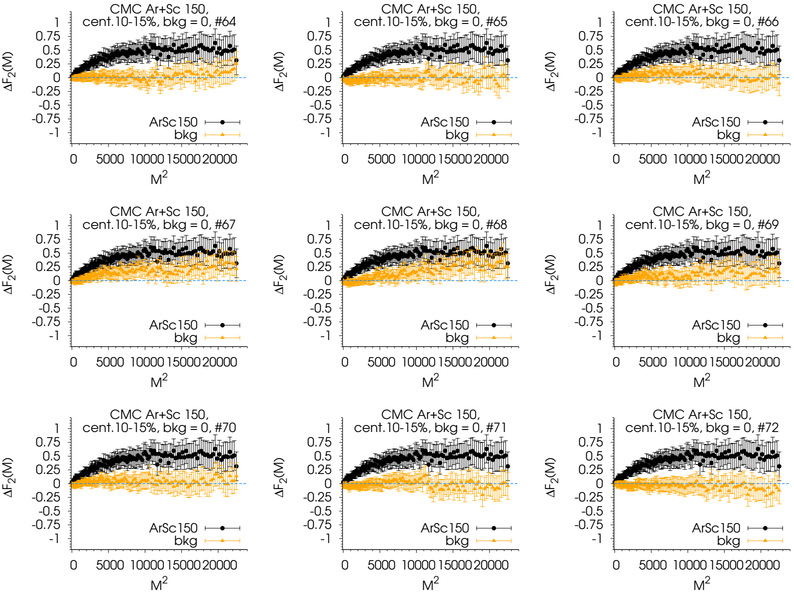
Spurious signal test, background – $\Delta F_2(M)$



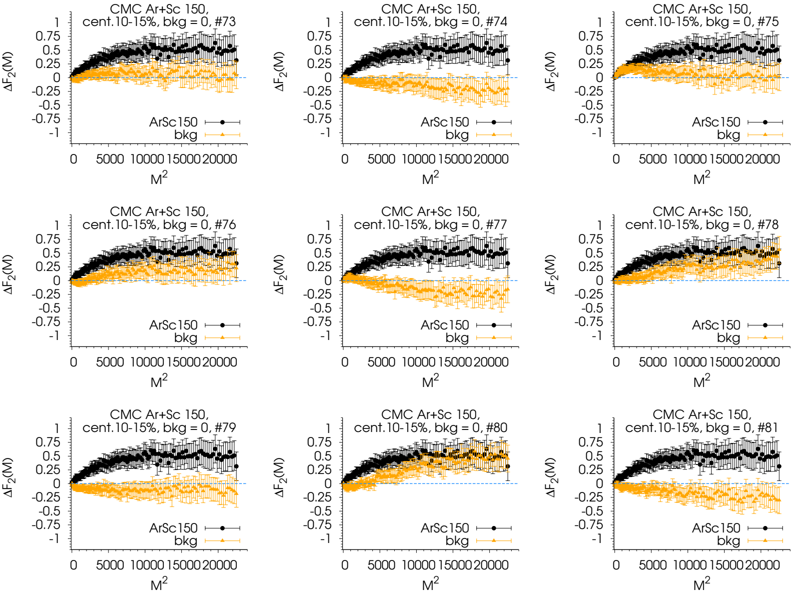
Spurious signal test, background – $\Delta F_2(M)$



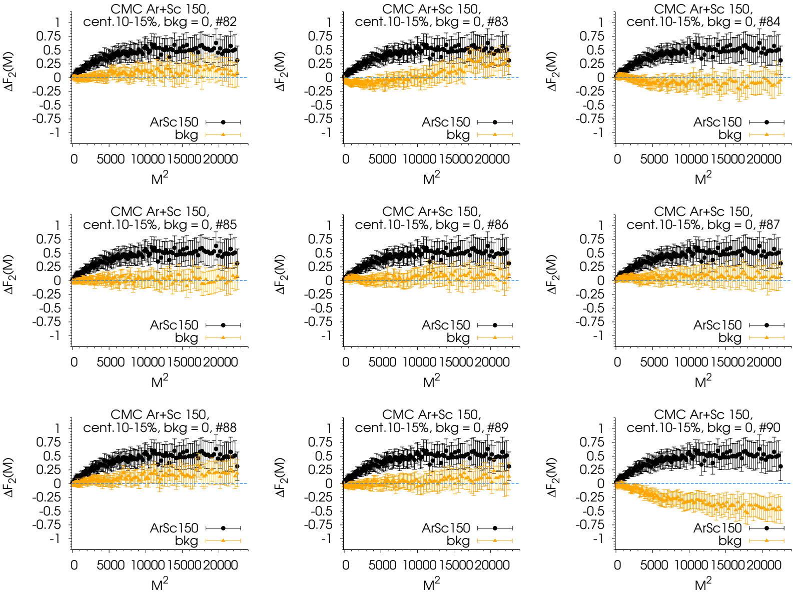
Spurious signal test, background – $\Delta F_2(M)$



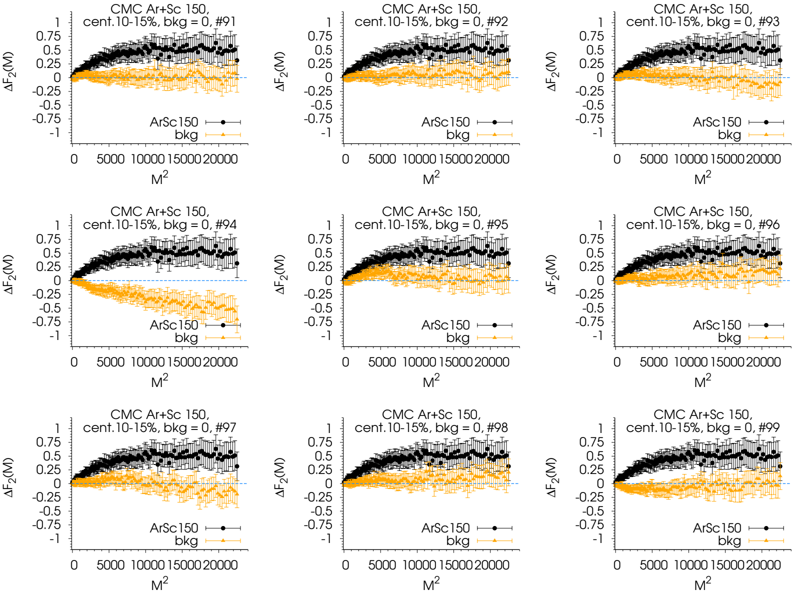
Spurious signal test, background – $\Delta F_2(M)$



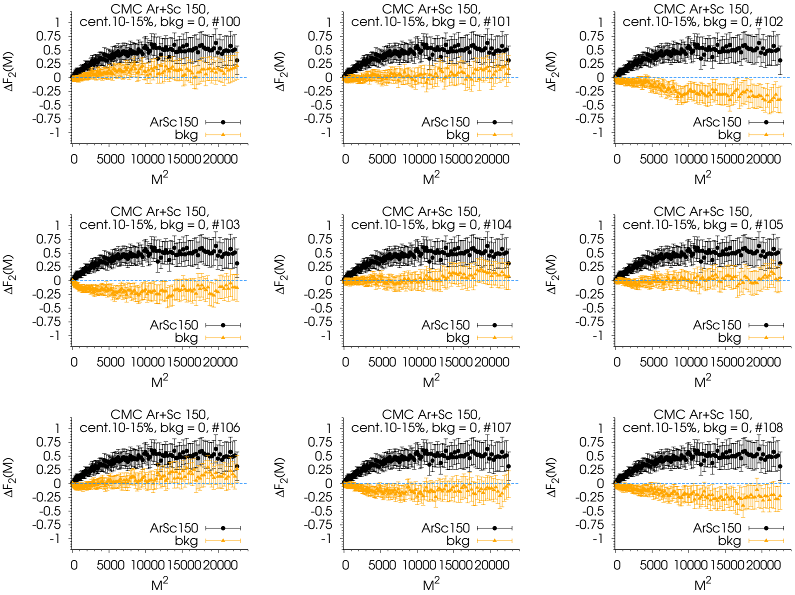
Spurious signal test, background – $\Delta F_2(M)$



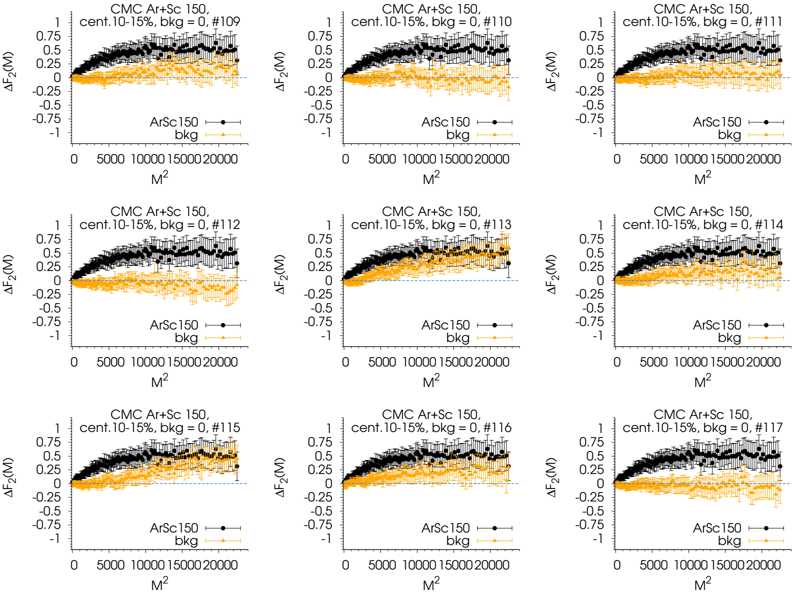
Spurious signal test, background – $\Delta F_2(M)$



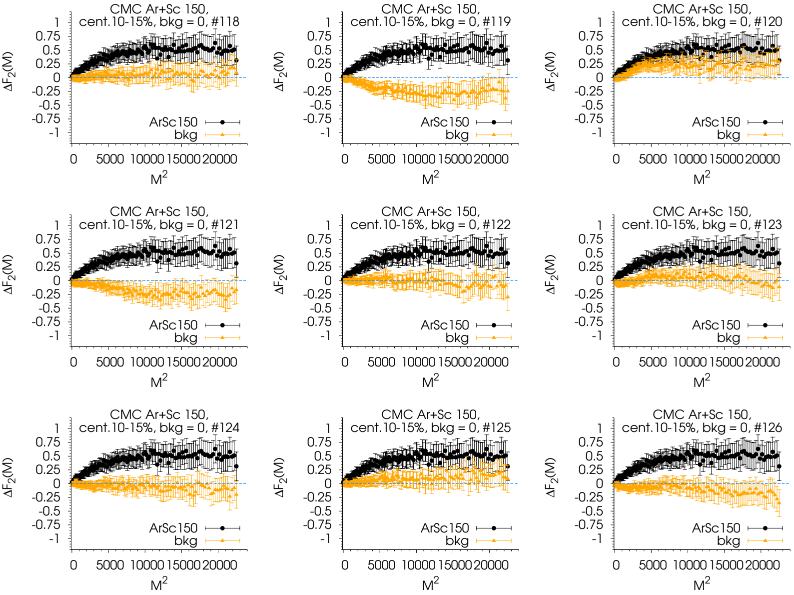
Spurious signal test, background – $\Delta F_2(M)$



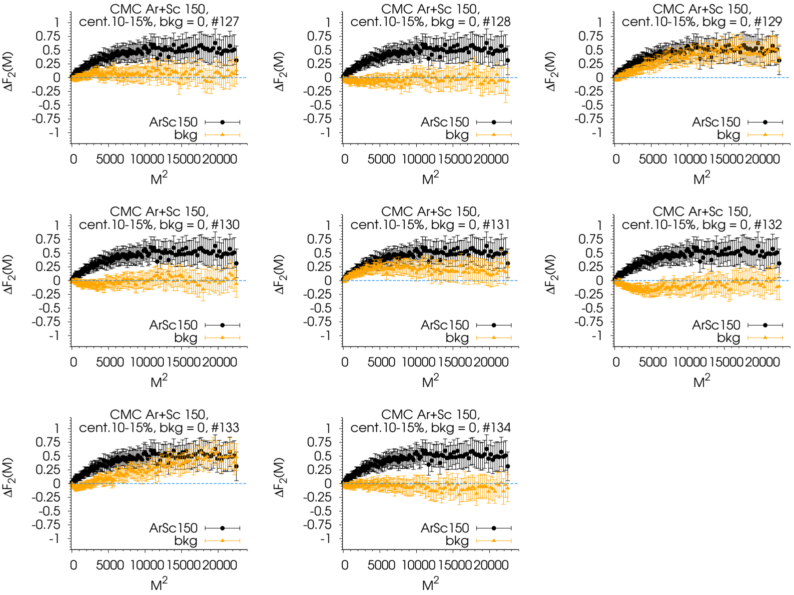
Spurious signal test, background – $\Delta F_2(M)$



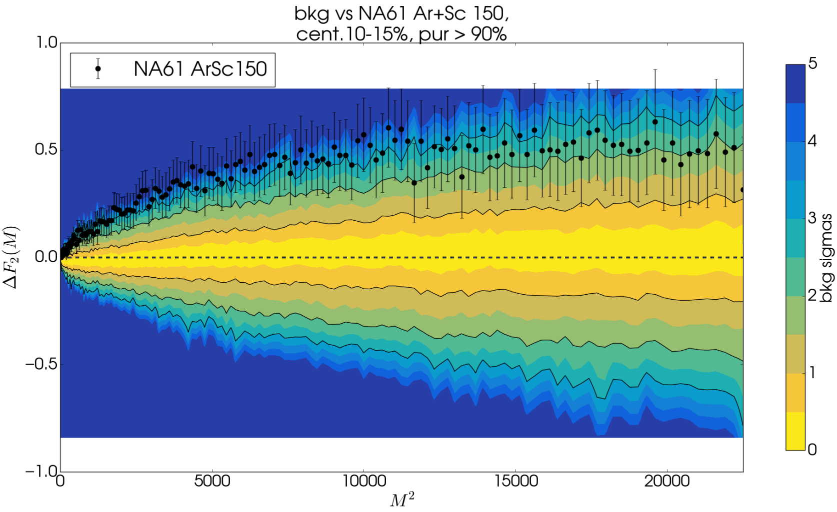
Spurious signal test, background – $\Delta F_2(M)$



Spurious signal test, background – $\Delta F_2(M)$



Spurious signal test, background – $\Delta F_2(M)$ C.I.



- Average ArSc150 $\Delta F_2(M) \sim 2 - 3\sigma$ away from random background $\Delta F_2(M)$

Summary and outlook

- We performed intermittency analysis on a variety of medium sized systems, on central to middle-central collisions: NA49 Si+Si, C+C & Pb+Pb at 158A GeV/c, as well as NA61/SHINE Ar+Sc at 150A GeV/c & 75A GeV/c;
- We find an indication of intermittency effect in middle-central NA61/SHINE Ar+Sc collisions at 150A GeV/c, consistently with our previously published analysis of intermittency in NA49 Si+Si at 158A GeV/c;
- In our estimation of a power-law intermittency index ϕ_2, statistical and systematic errors are significant;
- In the case of Ar+Sc at 150A GeV/c, 10-15% centrality, a non-zero $\Delta F_2(M)$ signal can be established at $\sim 2\sigma$ confidence level;
- Establishing a power-law scaling is, however, still challenging.
- ***"First possible indication for critical point from NA61/SHINE"***
  – Larry McLerran, *Theoretical Summary* talk at CPOD2018.

Summary and outlook

- CMC simulation shows Ar+Sc at 150A GeV/c data to be compatible with a 0.5-0.7% critical component of protons;
- Expanding the analysis to other NA61/SHINE systems (Xe+La, Pb+Pb) and SPS energies (Ar+Sc) will hopefully lead to a **more reliable interpretation** of the observed intermittency signal in terms of the **critical point**
- The NA61/SHINE CP task force, led by the IFJ Krakow group, is working on extending the Ar+Sc scan to lower energies, as well as scrutinizing intermittency methodology in order to reduce detector dependence and improve result robustness.

Thank you!

Acknowledgements

This work was supported by the National Science Centre, Poland under grant no. 2014/14/E/ST2/00018.

Back Up Slides

Observing power-law fluctuations

Experimental observation of local, power-law distributed fluctuations

$\Downarrow$

Intermittency in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions*)

- Net proton density carries the same critical fluctuations as the net baryon density, and can be substituted for it.

  [Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003)]
- Furthermore, antiprotons can be dropped to the extent that their multiplicity is much lower than of protons, and proton density analyzed.

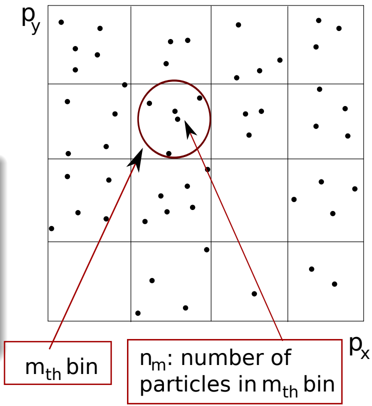
[J. Wosiek, *Acta Phys. Polon.* **B 19** (1988) 863-869]
[A. Bialas and R. Hwa, *Phys. Lett.* **B 253** (1991) 436-438]
*[F.K. Diakonos, N.G. Antoniou and G. Mavromanolakis, PoS (CPOD2006) 010, Florence]

Observing power-law fluctuations: Factorial moments

- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells M:

$$F_2(M) \equiv \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^2},$$

where $\langle \ldots \rangle$ denotes averaging over events.



Scaling of factorial moments – Subtracting mixed events

For $\lambda \lesssim 1$ (background domination), $\Delta F_2(M)$ can be approximated by:

$$\Delta F_2^{(e)}(M) = F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

For a critical system, ΔF_2 scales with cell size (number of cells, M) as:

$$\Delta F_2(M) \sim \left(M^2\right)^{\varphi_2}$$

where φ_2 is the intermittency index.

Theoretical predictions for φ_2

universality class, effective actions

$\varphi_{2,cr}^{(\sigma)} = \frac{2}{3}\ (0.66\ldots)$

sigmas (neutral isoscalar dipions)

[N. G. Antoniou **et al**, Nucl. Phys. A **693**, 799 (2001)]

$\varphi_{2,cr}^{(p)} = \frac{5}{6}\ (0.833\ldots)$

net baryons (protons)

[N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis, K. S. Kousouris, Phys. Rev. Lett. **97**, 032002 (2006)]

Subtracting the background from factorial moments

- Experimental data is noisy $\Rightarrow$ a background of non-critical pairs must be subtracted at the level of factorial moments.
- Intermittency will be revealed at the level of subtracted moments $\Delta F_2(M)$.

Partitioning of pairs into critical/background

$$\langle n(n-1)\rangle = \underbrace{\langle n_c(n_c-1)\rangle}_{\text{critical}} + \underbrace{\langle n_b(n_b-1)\rangle}_{\text{background}} + \underbrace{2\langle n_b n_c\rangle}_{\text{cross term}}$$

$$\underbrace{\Delta F_2(M)}_{\text{correlator}} = \underbrace{F_2^{(d)}(M)}_{\text{data}} - \lambda(M)^2 \cdot \underbrace{F_2^{(b)}(M)}_{\text{background}} - 2 \cdot \underbrace{\lambda(M)}_{\text{ratio } \frac{<n>_b}{<n>_d}} \cdot (1-\lambda(M))\, f_{bc}$$

- The cross term can be neglected under certain conditions (non-trivial! Justified by Critical Monte Carlo* simulations)

** [Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]*

Scaling of factorial moments – Subtracting mixed events

For $\lambda \lesssim 1$ (background domination), two approximations can be applied:

1. Cross term can be neglected
2. Non-critical background moments can be approximated by (uncorrelated) mixed event moments; then,

$$\Delta F_2(M) \simeq \Delta F_2^{(e)}(M) \equiv F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

For a critical system, ΔF_2 scales with cell size (number of cells, M) as:

$$\Delta F_2(M) \sim \left(M^2\right)^{\varphi_2}$$

where φ_2 is the intermittency index.

Theoretical prediction for φ_2

universality class, effective actions

$$\varphi_{2,cr}^{(p)} = \frac{5}{6}\ (0.833\ldots)$$

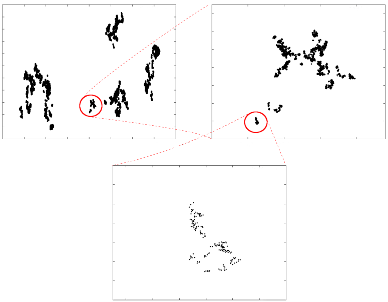
net baryons (protons)

[N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis, K. S. Kousouris, Phys. Rev. Lett. **97**, 032002 (2006)]

Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
  - Only protons produced
  - One cluster per event, produced by random Lévy walk: $\tilde{d}_F^{(B,2)} = 1/3 \Rightarrow \phi_2 = 5/6$
  - Lower / upper bounds of Lévy walks $p_{min,max}$ plugged in.
  - Cluster center exponential in p_T, slope adjusted by T_c parameter.
  - Poissonian proton multiplicity distribution.

Lévy walk example



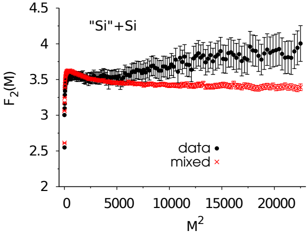
Input parameters

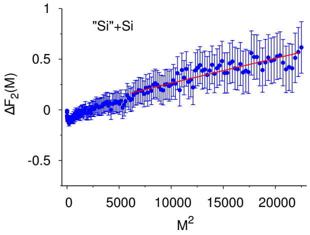
Parameter	p_{min} (MeV)	p_{max} (MeV)	λ_{Poisson}	T_c (MeV)
Value	$0.1 \rightarrow 1$	$800 \rightarrow 1200$	$\langle p \rangle_{\text{non-empty}}$	163

* [Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]

NA49: “C”+C, “Si”+Si, Pb+Pb at 158A GeV/c

- 3 sets of NA49 collision systems were analysed, at 158A GeV/c
  [T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]
- Factorial moments of proton transverse momenta analyzed at mid-rapidity
- Fragmentation beams used for C and Si (“C”=C,N ; “Si”=Si,Al,P) – components were merged to enhance statistics

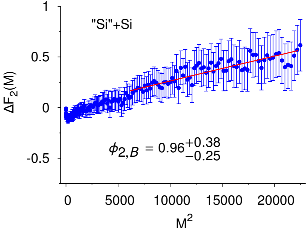


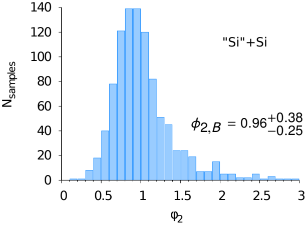


- Fit with $\Delta F_2^{(e)}(M\ ;\ C, \phi_2) = e^C \cdot \left(M^2\right)^{\phi_2}$, for $M^2 \geq 6000$
- No intermittency detected in the “C”+C, Pb+Pb datasets.

NA49: "C"+C, "Si"+Si, Pb+Pb at 158A GeV/c

- Evidence for intermittency in "Si"+Si – but large statistical errors.





- Bootstrap distribution of ϕ_2 values is highly asymmetric due to closeness of $F_2^{(d)}(M)$ to $F_2^{(m)}(M)$.
- Based on CMC simulation, we estimate a fraction of $\sim$ 1% critical protons are present in the sample.
- Estimated intermittency index: $\phi_{2,B} = 0.96^{+0.38}_{-0.25}(\text{stat.}) \pm 0.16(\text{syst.})$

[T. Anticic *et al.*, Eur. Phys. J. C 75:587 (2015), arXiv:1208.5292v5]

Ar+Sc at 150A GeV/c: NA61 data vs EPOS

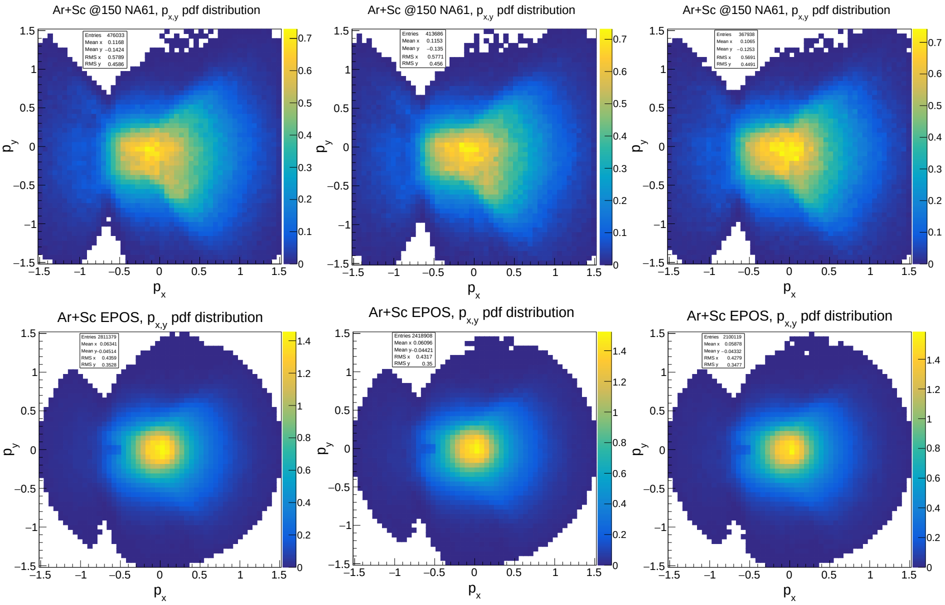
EPOS – proton p_T statistics

Centrality	#events	$\langle p \rangle_{\lvert p_T \rvert \leq 1.5\ GeV, \lvert y_{CM} \rvert \leq 0.75}$ Non-empty	With empty	$\Delta p_{x,y}$
0- 5%	293,412	3.06 ± 1.60	2.89 ± 1.70	0.35 - 0.43
5-10%	252,362	2.72 ± 1.45	2.49 ± 1.58	0.35 - 0.43
10-15%	274,072	2.45 ± 1.33	2.16 ± 1.48	0.35 - 0.43

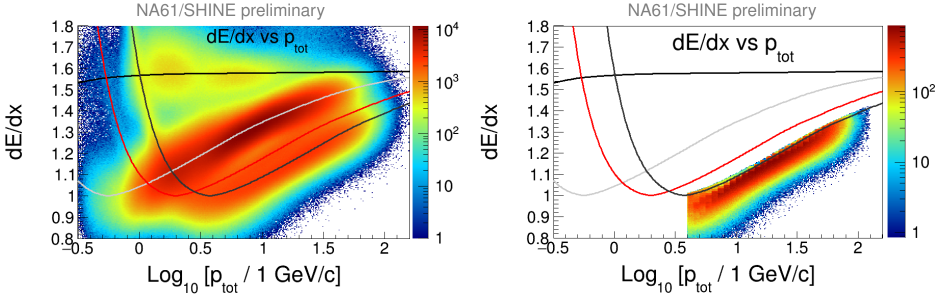
$^{40}Ar + ^{45}Sc$ NA61 data – proton p_T statistics

Centrality	#events	$\langle p \rangle_{\lvert p_T \rvert \leq 1.5\ GeV, \lvert y_{CM} \rvert \leq 0.75}$ Non-empty	With empty	$\Delta p_{x,y}$
0- 5%	144,362	3.44 ± 1.79	3.30 ± 1.89	0.46 - 0.58
5-10%	148,199	3.00 ± 1.61	2.79 ± 1.73	0.46 - 0.58
10-15%	142,900	2.81 ± 1.53	2.58 ± 1.66	0.45 - 0.57

$p_{X,Y}$ spectra comparison – NA61 vs EPOS (0 – 15%)

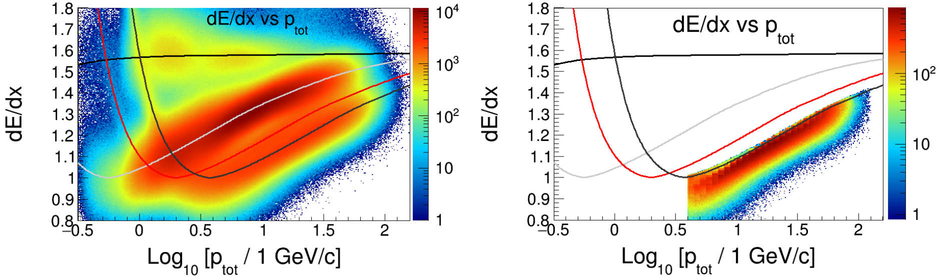


Proton selection



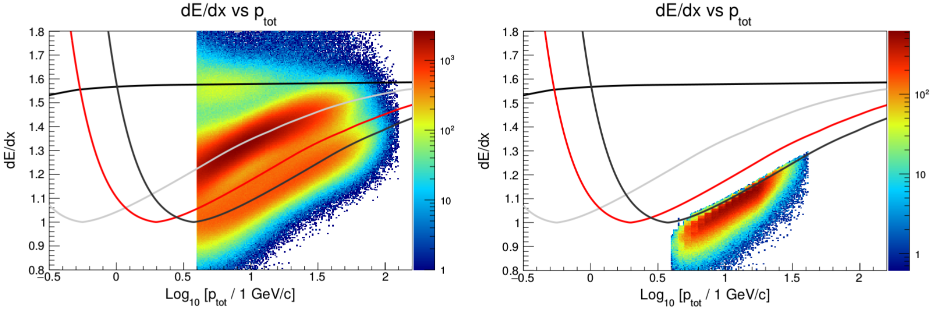
- Employ p_{tot} region where Bethe-Bloch bands do not overlap (3.98 GeV/c $\leq p_{tot} \leq$ 126 GeV/c)
- Fit dE/dx distribution with 4-gaussian sum for $\alpha = \pi, K, p, e$ – Bins: p_{tot}, p_T
- 30 Bins in $Log_{10}(p_{tot})$: $10^{0.6} \rightarrow 10^{2.1}$ GeV/c
- 20 Bins in p_T: 0.0 $\rightarrow$ 2.0 GeV/c
- Proton purity: probability for a track to be a proton, $\mathcal{P}_p = p/(\pi + K + p + e)$
- Additional cut along Bethe-Blochs (avoid low-reliability region between p and K curves)

dE/dx vs p_{tot} (proton ID)



- Avoid p_{tot} region where Bethe-Bloch curves overlap (3.98 GeV/c $\leq p_{tot} \leq$ 126 GeV/c)
- Using Hans Dembinski/Raul R Prado's dE/dx fitting software – Bins: p_{tot}, p_T
- Presented in Moscow meeting by Prado, Herve & Unger
- 30 Bins in $Log_{10}(p_{tot})$: $10^{0.6} \rightarrow 10^{2.1}$ GeV/c
- 20 Bins in p_T: 0.0 $\rightarrow$ 2.0 GeV/c
- Preliminary p selection: 90% purity removing deuterons from the model
- Cut along Bethe-Blochs: $BB_p + 0.15(BB_K - BB_p)$

dE/dx simulation & proton purity assignment in EPOS



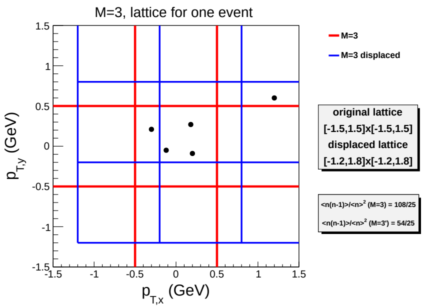
- Used dE/dx spectra from Ar+Sc @150 data in the 6% - 18% centrality interval
- For each track, assign a dE/dx value based on particle species and phase space bin
- Apply dE/dx & purity cuts identical to NA61/SHINE data

Improving calculation of $F_2(M)$ via lattice averaging

- Problem: With low statistics/multiplicity, lattice boundaries may split pairs of neighboring points, affecting $F_2(M)$ values (see example below).

- Solution: Calculate moments several times on different, slightly displaced lattices (see example)
- Average corresponding $F_2(M)$ over all lattices. Errors can be estimated by variance over lattice positions.
- Lattice displacement is larger than experimental resolution, yet maximum displacement must be of the order of the finer binnings, so as to stay in the correct p_T range.

Displaced lattice — a simple example



Improved confidence intervals for ϕ_2 via resampling

- In order to estimate the statistical errors of $\Delta F_2(M)$, we need to produce variations of the original event sample. This, we can achieve by using the statistical method of resampling (bootstrapping) $\Rightarrow$
  - Sample original events with replacement, producing new sets of the same statistics (# of events)
  - Calculate $\Delta F_2(M)$ for each bootstrap sample in the same manner as for the original.
  - The variance of sample values provides the statistical error of $\Delta F_2(M)$.

[W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]

- Furthermore, we can obtain a distribution $P(\varphi_2)$ of φ_2 values. Each bootstrap sample of $\Delta F_2(M)$ is fit with a power-law:

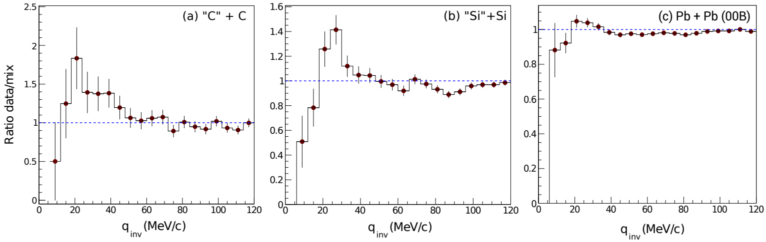
$$\Delta F_2(M; \mathcal{C}, \varphi_2) = e^{\mathcal{C}} \cdot (M^2)^{\varphi_2}$$

and we can extract a confidence interval for φ_2 from the distribution of values.

[B. Efron, *The Annals of Statistics* **7**,1 (1979)]

Split tracks; the q_{inv} cut in analysed datasets

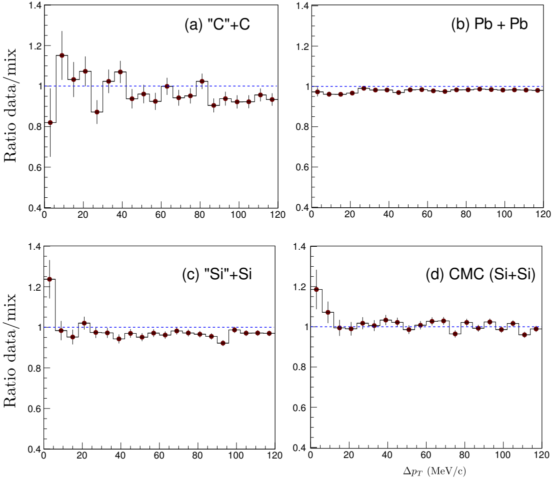
- Split tracks can create false positive for intermittency $\Rightarrow$ must be reduced or removed.
- q_{inv}-test – distribution of track pairs: $q_{inv}(p_i, p_j) \equiv \frac{1}{2}\sqrt{-(p_i - p_j)^2}$, p_i : 4-momentum of i^{th} track.
- Calculate ratio $q_{inv}^{data}/q_{inv}^{mixed}$ $\Rightarrow$ peak at low q_{inv} (below 20 MeV/c): possible split track contamination.



- Anti-correlations due to F-D effects and Coulomb repulsion must be removed before intermittency analysis $\Rightarrow$ "dip" in low q_{inv}, peak predicted around 20 MeV/c [Koonin, PLB 70, 43-47 (1977)]
- Universal cutoff of $q_{inv} > 25$ MeV/c applied to all sets before analysis.

NA49 analysis – Δp_T distributions

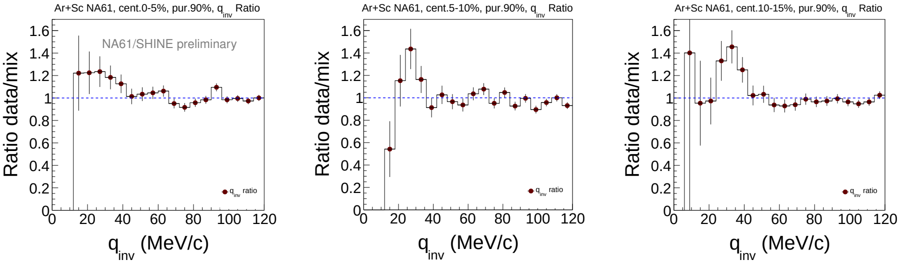
- We measure correlations in relative p_T of protons via
$$\Delta p_T = 1/2\sqrt{(p_{X_1} - p_{X_2})^2 + (p_{Y_1} - p_{Y_2})^2}$$



- Strong correlations for $\Delta p_T \to 0$ indicate power-law scaling of the density-density correlation function $\Rightarrow$ intermittency presence
- We find a strong peak in the "Si"+Si dataset
- A similar peak is seen in the Δp_T profile of simulated CMC protons with the characteristics of "Si"+Si.

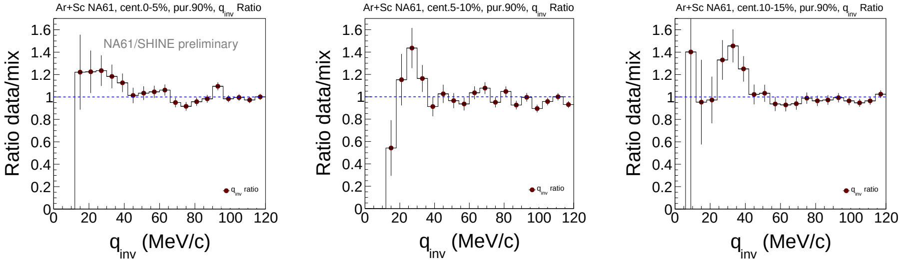
Split tracks & the q_{inv} cut

- Events may contain split tracks: sections of the same track erroneously identified as a pair of tracks that are close in momentum space.
- Three cuts to root them out:
  1. Ratio of points / potential points in a track (removes most)
  2. Minimum track distance in the detector (pair cut)
  3. q_{inv} cut (pair cut, physics-significant)
- q_{inv} distribution of track pairs probed in order to root the rest out:
  $q_{inv}(p_i, p_j) \equiv \frac{1}{2}\sqrt{-(p_i - p_j)^2}$, p_i : 4-momentum of i^{th} track.
- We calculate the ratio of $q_{inv}^{data} / q_{inv}^{mixed}$.



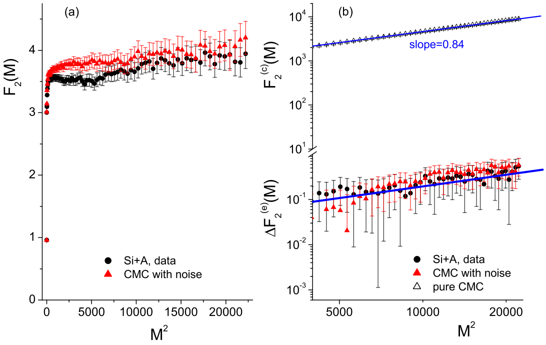
Split tracks & the q_{inv} cut

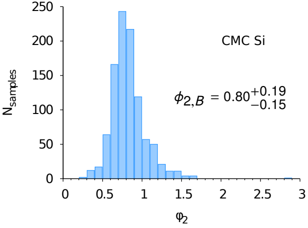
- A peak at low q_{inv} (below 20 MeV/c) indicates a possible split track contamination that must be removed.
- Anti-correlations due to F-D effects and Coulomb repulsion must be removed before intermittency analysis ⇒ "dip" in low q_{inv}, peak predicted around 20 MeV/c [Koonin, PLB 70, 43-47 (1977)]
- Universal cutoff of $q_{inv} > 7$ MeV/c applied to all sets before analysis.



Noisy CMC (baryons) – estimating the level of background

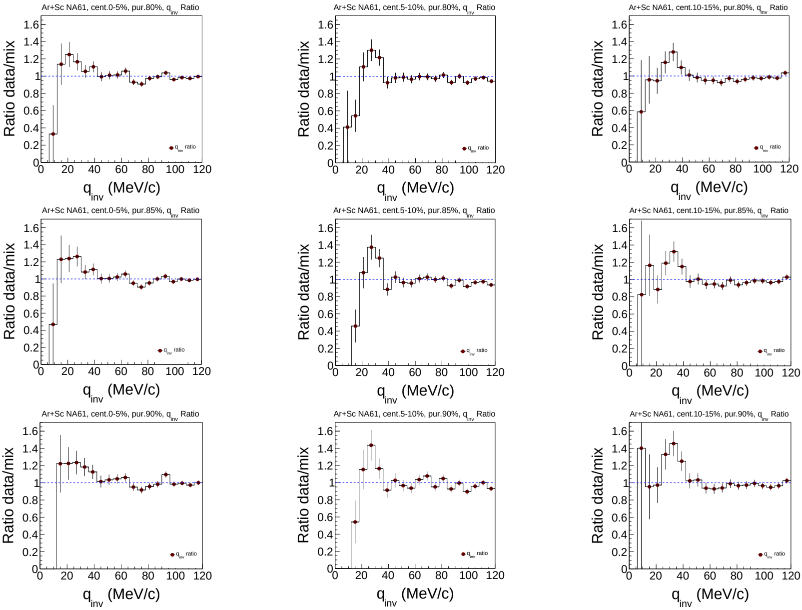
- $F_2(M)$ of noisy CMC approximates “Si”+Si for $\lambda \approx 0.99$
- $\Delta F_2^{(e)}(M)$ reproduces critical behaviour of pure CMC, even though their moments differ by orders of magnitude!



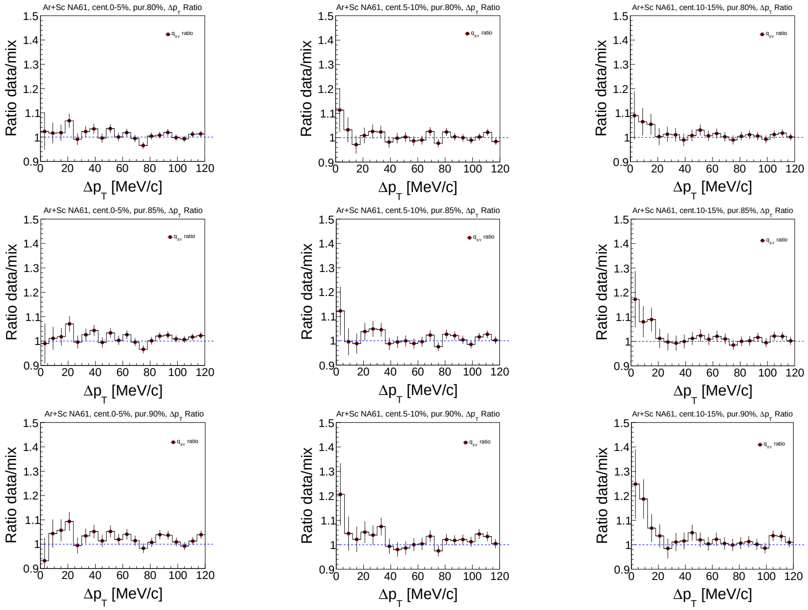


- Noisy CMC results show our approximation is reasonable for dominant background.

q_{inv} proton distributions – NA61/SHINE

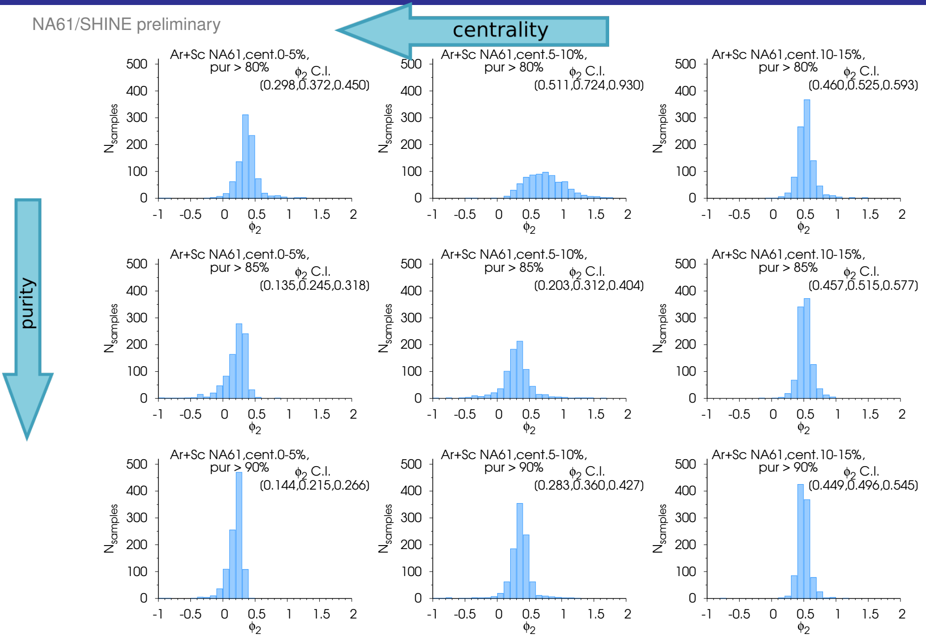


Δp_T proton distributions – NA61/SHINE

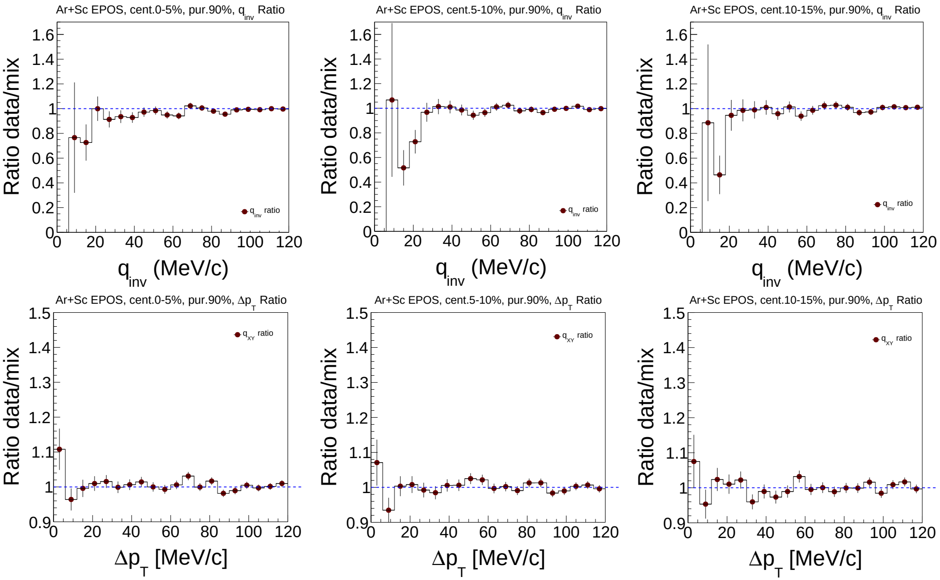


NA61/SHINE: Ar+Sc at 150A GeV/c: ϕ_2 bootstrap dist.

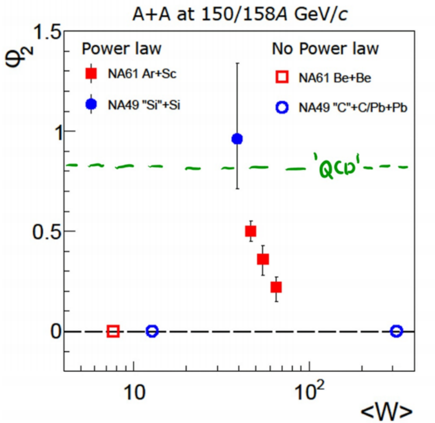
NA61/SHINE preliminary



q_{inv} & Δp_T distributions – EPOS

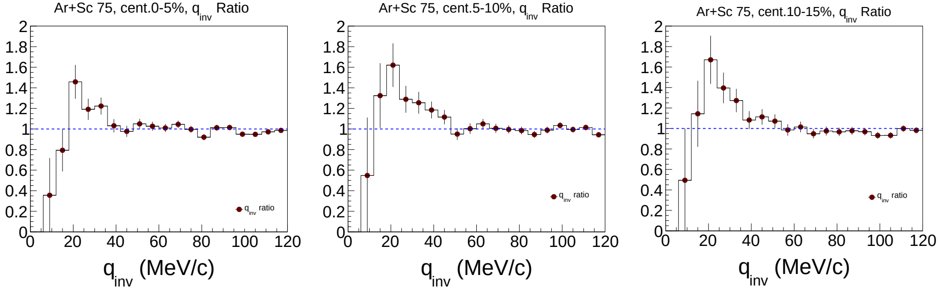


Intermittency analysis at 150/158A GeV/c: Summary



- Indication of intermittency effect in middle-central NA61/SHINE Ar+Sc collisions
- First possible evidence of CP signal in NA61/SHINE
- Effect quality increases with increased proton purity selection, up to 90% proton purity; EPOS does not reproduce observed effect.

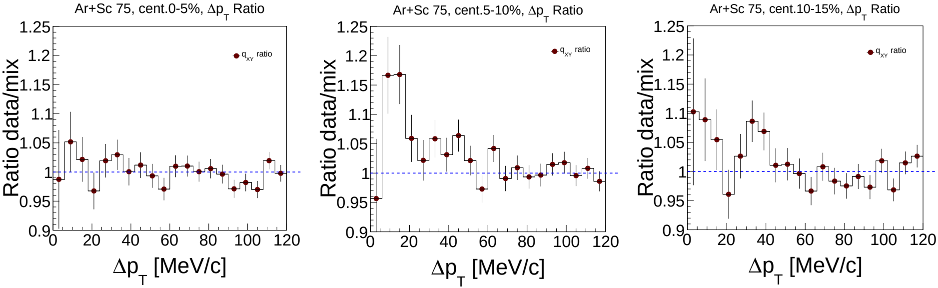
q_{inv} distributions – NPratio & TTD cuts (control subset)



- q_{inv} distribution seems to improve with TTD cut, apart for large fluctuations in the 1st bin
- Removing 1st bin: ⇒ Cut: $q_{inv} > 6$ MeV/c

Δp_T distributions – NPratio & TTD cuts (control subset)

- Applied cuts: $TTD > 2$cm, $q_{inv} > 6$ MeV/c, PP

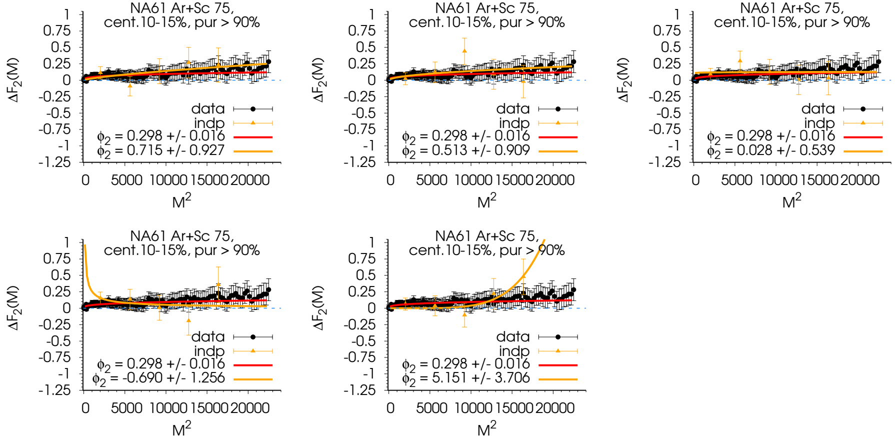


- No enhanced correlations for $\Delta p_T \to 0$ in the 1st and 2nd centrality bins.
- An enhancement in the 2nd bin for intermediate $\Delta p_T \Rightarrow$ 1st order region?
- "Sort of" an enhancement in the 3rd bin for $\Delta p_T \to 0$

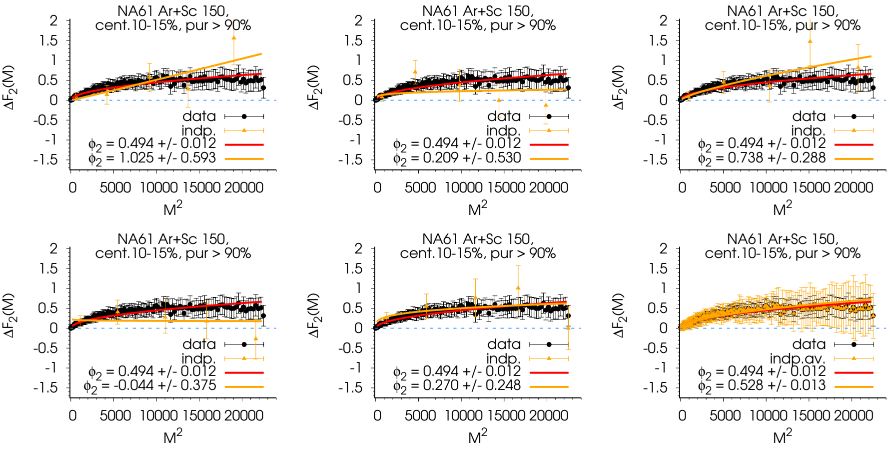
Independent bins test

- We attempt to mitigate the correlated bin effect by using non-overlapping event subsets for different M values
- As a first test, we use the full control + analysis statistics in the 3rd centrality bin
- Initial test for Ar+Sc 75:
  - statistics in the 3rd centrality bin $\Rightarrow \sim$ 520K events
  - Random partition of events $\Rightarrow$ 5 sets $\times \sim$ 100K events
- Process repeated for Ar+Sc 150:
  - statistics in the 3rd centrality bin $\Rightarrow \sim$ 150K events
  - Random partition of events $\Rightarrow$ 5 sets $\times \sim$ 30K events
- Independent bins test applied on a **MC with a critical component (CMC)**, in order to examine the efficacy of $F_2(M)$ on a system with a **known proton-proton correlation function**

$\Delta F_2(M)$ – Ar+Sc 75 NA61 (independent samples)

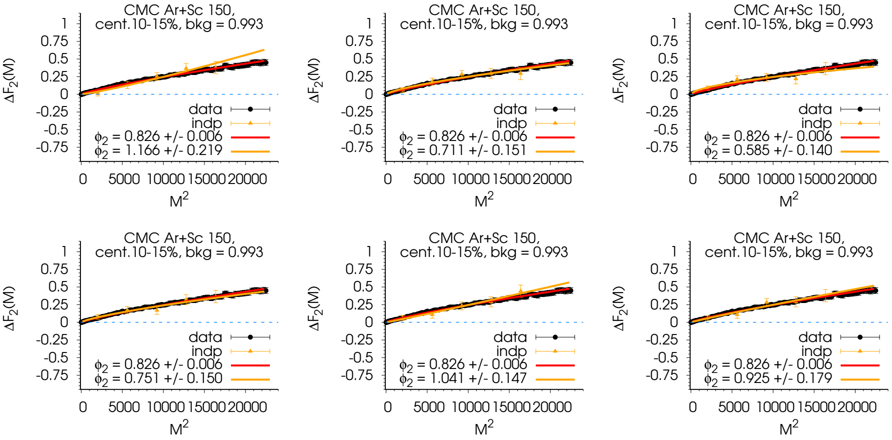


$\Delta F_2(M)$ – Ar+Sc 150 NA61 (independent samples)



$\Delta F_2(M)$ – CMC Ar+Sc 150 10M & indep. bins (2M)

- The original ϕ_2 fit for the 10M set is shown for reference (red line).

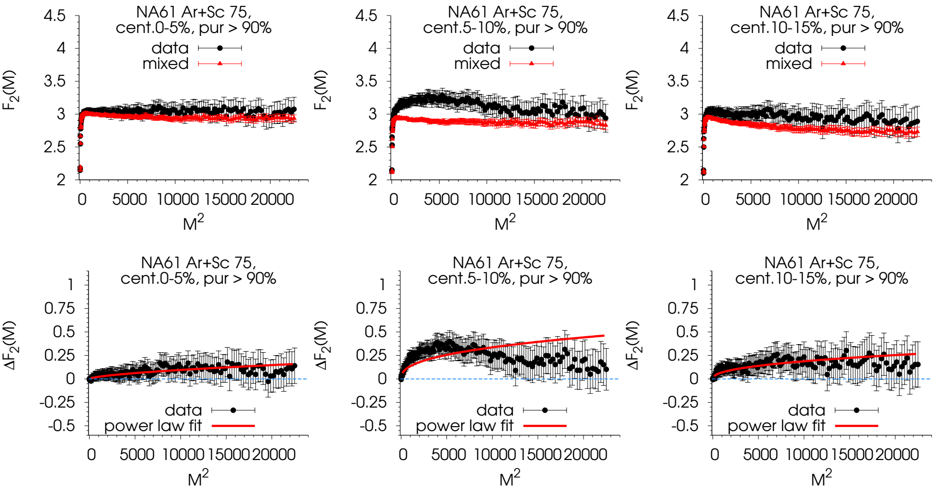


- $\Delta F_2(M)$ of independent bins almost converged to the correct trend
- Their ϕ_2 (orange line) seems very sensitive to slight displacement (too few points!).

Spurious Signal Test – Conclusions

- No observed cases of background $\Delta F_2(M)$ above NA61 ArSc150 data.
- $\sim$ 6% of background samples within 1 sigma of NA61 ArSc150 data $\Delta F_2(M)$ (from below).
- It is still not clear what possible distortions are introduced by pair cuts in NA61 data; it may be possible to simulate q_{inv} cut in CMC, but this will require an extension of CMC to 3D ($p_T \times y_{CM}$).

$F_2(M)$, $\Delta F_2(M)$ – Ar+Sc 75 NA61 (control, 90% purity)



$\Delta F_2(M)$, Δp_T – Ar+Sc 75 , 150 comparison (control)