Cumulants of net-proton number fluctuations from ALICE at the LHC

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Motivation: To map the QCD phase diagram

- At LHC energies, $\mu_B \approx 0$.
- Lattice QCD: $T_c \approx 154 \pm 9$ MeV

Estimation from models with ALICE data:
- $T_{\text{freeze-out}} \approx 156 \pm 3$ MeV

- Chemical freeze-out line is close to the crossover line!

Precise determination of freeze-out parameters (T and $\mu_B$) at the LHC can help to locate the phase boundary at $\mu_B \approx 0$.

Motivation: Connecting experiment with lattice QCD

In lattice QCD, the cumulants ($C_n$) of the distributions of conserved charges (net-charge, net-baryon, net-strangeness) are related to the generalized quark-number susceptibilities ($\chi_n^q$)

$$\chi_n^q = \frac{\partial^n \left( \frac{p}{T^4} \right)}{\partial \left( \frac{\mu_B}{T} \right)^n}$$

We measure $C_n = VT^3 \chi_n^q$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Lattice QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_4 / C_2 = \frac{\chi_4^q}{\chi_2^q}$</td>
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</table>

A direct comparison of experimental results with lattice QCD!
Motivation: Connecting experiment with lattice QCD

Freeze-out temperature from the ratio of cumulants of net-baryons

- From lattice QCD, freeze-out parameters can be extracted from experimental results at $\mu_B = 0$.
- Net-proton fluctuations are a good proxy for net-baryon number fluctuations.

Experimental measurements of the ratio of cumulants of net-protons at the LHC will help to constrain the lattice QCD predictions in a model independent way.

[Y. Hatta, M.A. Stephanov, PRL 91 102003 (2003)]

WPCF 2018, Krakow, 23rd May 2018

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Experimental setup and dataset

ALICE detector

- **Inner Tracking System (ITS):** Vertex, tracking, PID
- **Time Projection Chamber (TPC):** Tracking, PID

Minimum-bias Pb-Pb collision data

<table>
<thead>
<tr>
<th>Collision energy</th>
<th>Pb-Pb 2.76 TeV (Run1)</th>
<th>Pb-Pb 5.02 TeV (Run2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of events</td>
<td>$14 \times 10^6$</td>
<td>$59 \times 10^6$</td>
</tr>
</tbody>
</table>

- **Kinematic cuts:** $0.4 < p_T < 1.0$ GeV/c, $|\eta| < 0.8$
- **Proton identification:** TPC (next slide)

V0 detector:
- Trigger, centrality estimation
- V0A: $2.8 < \eta < 5.1$, V0C: $-3.7 < \eta < -1.7$

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Proton Identification

Energy loss ($dE/dx$) of particles in TPC gas

- (Anti-)Proton identified using the nσ cut around the expected value of energy loss of particles in TPC detector (this presentation).

Generalized Gaussian fit to TPC $dE/dx$

- Proton identification using Identity method: Assign a Bayesian probability for each particle species.
- Get the true numbers using response matrix built from the parameterization of TPC $dE/dx$.

$C_2$ results from both the methods agree with each other

M. Gazdzicki et al., PRC 83, 054907 (2011)
M. I. Gorenstein, PRC 84, 024902 (2011)

M. I. Gorenstein, PRC 84, 024902 (2011)
Higher moments and cumulants

- Measure net-proton numbers on event-by-event basis: $\Delta p = p - \bar{p}$
- The $n^{th}$ moments: $m'_n = <(\Delta p)^n>$

<table>
<thead>
<tr>
<th>Cumulants from moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1 = m'_1$</td>
</tr>
<tr>
<td>$C_2 = m'_2 - m'_1^2$</td>
</tr>
<tr>
<td>$C_3 = m'_3 - 3m'_1m'_2 + 2m'_1^3$</td>
</tr>
<tr>
<td>$C_4 = m'_4 - 4m'_1m'_3 - 3m'_2^2 + 12m'_1^2m'_2 - 6m'_1^4$</td>
</tr>
</tbody>
</table>
Analysis methodology

1) Detector efficiency correction: method 1

- Assumption: detector efficiency follows Binomial distribution.

Factorial moments

\[ F_{i,k} = \sum_{x_1, \ldots, x_i} \sum_{\bar{x}_1, \ldots, \bar{x}_k} \frac{a_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k)}{\epsilon(x_1) \ldots \epsilon(x_i) \epsilon(\bar{x}_1) \ldots \epsilon(\bar{x}_k)}. \]

where

\[ a_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k) = \langle n(x_1)[n(x_2) - \delta_{x_1, x_2}] \cdots [n(x_i) - \delta_{x_1, x_i} - \cdots - \delta_{x_{i-1}, x_i}] \rangle \tilde{n}(\bar{x}_1)[\tilde{n}(\bar{x}_2) - \delta_{\bar{x}_1, \bar{x}_2}] \cdots [\tilde{n}(\bar{x}_k) - \delta_{\bar{x}_1, \bar{x}_k} - \cdots - \delta_{\bar{x}_{k-1}, \bar{x}_k}] \rangle. \]

Cumulants from factorial moments

\[ K_1 = \langle N_1 \rangle - \langle N_2 \rangle, \]
\[ K_2 = N - K_1^2 + F_{02} - 2F_{11} + F_{20}, \]
\[ K_3 = K_1 + 2K_1^3 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30} - 3K_1(N + F_{02} - 2F_{11} + F_{20}), \]
\[ K_4 = N - 6K_1^4 + F_{04} + 6F_{03} + 7F_{02} - 2F_{11} - 6F_{12} - 4F_{13} + 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40} + 12K_1^2(N + F_{02} - 2F_{11} + F_{20}) - 3(N + F_{02} - 2F_{11} + F_{20})^2 - 4K_1(K_1 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30}), \]

where \[ N = \langle N_1 \rangle + \langle N_2 \rangle = F_{00} + F_{01}. \]
Analysis methodology

1) Detector efficiency correction: method 2

- Recent method: T Nonaka et al., PRC 95, 064912, (2017)
- Assumption: detector efficiency follows Binomial distribution.

Recent method:

\[
\begin{align*}
\langle Q \rangle_c &= \langle q_{(1,1)} \rangle_c, \\
\langle Q^2 \rangle_c &= \langle q_{(1,1)}^2 \rangle_c + \langle q_{(2,1)} \rangle_c - \langle Q \rangle_c, \\
\langle Q^3 \rangle_c &= \langle q_{(1,1)}^3 \rangle_c + 3\langle q_{(1,1)}q_{(2,1)} \rangle_c - 3\langle q_{(1,1)}q_{(2,1)} \rangle_c + \langle q_{(3,1)} \rangle_c - 3\langle q_{(3,2)} \rangle_c + 2\langle q_{(3,3)} \rangle_c, \\
\langle Q^4 \rangle_c &= \langle q_{(1,1)}^4 \rangle_c + 6\langle q_{(1,1)}q_{(2,1)} \rangle_c - 6\langle q_{(1,1)}q_{(2,1)} \rangle_c + 4\langle q_{(1,1)}q_{(3,1)} \rangle_c + 3\langle q_{(2,1)}^2 \rangle_c + 3\langle q_{(2,2)}^2 \rangle_c - 12\langle q_{(1,1)}q_{(3,2)} \rangle_c \\
&\quad + 8\langle q_{(1,1)}q_{(3,3)} \rangle_c - 6\langle q_{(2,1)}q_{(2,2)} \rangle_c + \langle q_{(4,1)} \rangle_c - 7\langle q_{(4,2)} \rangle_c + 12\langle q_{(4,3)} \rangle_c - 6\langle q_{(4,4)} \rangle_c.
\end{align*}
\]

Moments

\[
q_{(r,s)} = q(a_r/p_r) = \sum_{i=1}^{M} (a_i^r/p_i^s)n_i.
\]

- Simpler and faster than method 1.
- Method 1 and 2 give the same results!

\[a_i = 1 \text{ for proton, -1 for anti-proton} \]

\[p_i = \text{Efficiency in } i^{th} \ p_T \text{ bin.} \]

\[n_i = \text{Number of observed (anti-)} \ p_T \text{ proton} \]

\[M = \text{Total Number of } p_T \text{ bins} \]
Analysis methodology

2) Centrality bin-width correction (CBWC):

- To minimize the impact parameter (or volume) variations due to the finite centrality bin. [X. Luo et al J. Phys. G 40, 105104 (2013)]

$$C_n = \frac{\sum_{i=1}^{k} m_i C_{n,i}}{\sum_{i=1}^{k} m_i}$$

- With or without CBWC, the results are consistent for ALICE measurements.
- Cumulants up to 3rd order are not sensitive to volume fluctuations!

Analysis methodology

3) Statistical error estimation:

- Subgroup method: Divide the data sample into ‘k’ subsamples randomly.

\[ C_n = \frac{\sum_{i}^{k} C_n^i}{k} \]

Statistical error

\[ \delta C_n = \frac{\sigma}{\sqrt{k}} \]

where

\[ \sigma = \sqrt{\frac{\sum_{i=1}^{k} (C_n^i - <C_n>)^2}{k - 1}} \]

- Number of subsamples used: 30
- Statistical errors estimated from Bootstrap and Subgroup method agree with each other.
Results: proton and anti-proton $C_1$

- For both energies, proton and anti-proton numbers are similar within systematic errors.
Results: Cumulants of net-proton distributions

- Cumulants of net-proton distributions for both energies are the same within statistical and systematic errors.

- $C_2$ results are consistent with Identity method.  
  
Baseline estimation

- Assume proton and anti-proton distributions are independent Poissonian distributions => net-proton distribution is a Skellam distribution.

- $C_n(\text{Skellam}) = C_1(p) + (-1)^n C_1(\bar{p})$

- $C_1(\Delta p) = C_3(\Delta p) = C_1(p) - C_1(\bar{p})$

- $C_2(\Delta p) = C_4(\Delta p) = C_1(p) + C_1(\bar{p}) \Rightarrow \frac{C_4}{C_2} = 1.$

- Used in Hadron Resonance Gas (HRG) model calculations.
  
Results: Ratio of cumulants of net-proton distributions

- $C_3/C_2$ and $C_4/C_2$ results for 2.76 TeV and 5.02 TeV are the same within the statistical error bars.
- $C_3/C_2$ and $C_4/C_2$ of central events agree with Skellam expectations within statistical errors.
Results: RHIC to LHC

- $C_3/C_2$ follows a decreasing trend as a function of collision energy.
- Within experimental uncertainties $C_4/C_2$ at the LHC agrees with the Skellam baseline.
Freeze-out temperature at the LHC

Implied freeze-out temperature from $C_4/C_2$ consistent with thermal fits to particle yields. (assuming net-protons are good proxy to net-baryons on the lattice)

- Need more precise data to further constrain the freeze-out temperature.

S. Borsányi et al PRL 111, 062005 (2013)
Summary

- The net-proton higher order cumulants up to 4\textsuperscript{th} order and their ratios in Pb-Pb collisions at 2.76 and 5.02 TeV are presented as a function of centrality.

- $C_3$, $C_4$ and their ratios with respect to $C_2$ generally agree with Skellam expectations within the uncertainties.

- From RHIC to LHC the ratios of cumulants approach the Skellam baseline.

- The results may be used in conjunction with lattice calculations to estimate the freeze-out temperature.

- News: Second cumulants of net-\Lambda fluctuations are measured in ALICE (see A. Ohlson talk in QM 2018)
Outlook

- The analysis will be extended for higher $p_T$ (up to 2 GeV/c) and for other particle species in different pseudo-rapidity windows.

- Measurement of net-proton higher order cumulants with Identity method.

- Off-diagonal cumulants of net-particles.

- The upcoming dedicated Pb-Pb run at 5.02 TeV will increase the statistics to further constrain the freeze-out parameters.

THANK YOU
Backup slides
Efficiency correction method

\[ N_{\text{rec.}} = N_{\text{rec.primary}} + N_{\text{cont.}} \]

Now \( N_{\text{rec.primary}} = N_{\text{rec}} - N_{\text{cont.}} \)

\[ \Rightarrow N_{\text{gen.primary}} = N_{\text{rec}} \frac{(1 - \delta)}{\varepsilon} \]

\[ \Rightarrow N_{\text{gen.primary}} = \frac{N_{\text{rec.}}}{\omega} \]

Correction factor \( \omega = \frac{\varepsilon}{(1 - \delta)} \)

where \( N_{\text{cont.}} = N_{\text{material}} + N_{\text{secondary}} + N_{\text{mis.identified}} \)

where contamination factor (\( \delta \)) = \( \frac{N_{\text{cont.}}}{N_{\text{rec.}}} \)

Efficiency (\( \varepsilon \)) = \( \frac{N_{\text{rec.primary}}}{N_{\text{gen.primary}}} \)