



# 3 DIMENSIONAL LÉVY FEMTOSCOPY WITH PHENIX

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MÁTÉ CSANÁD FOR PHENIX @ WPCF 2018, MAY 24

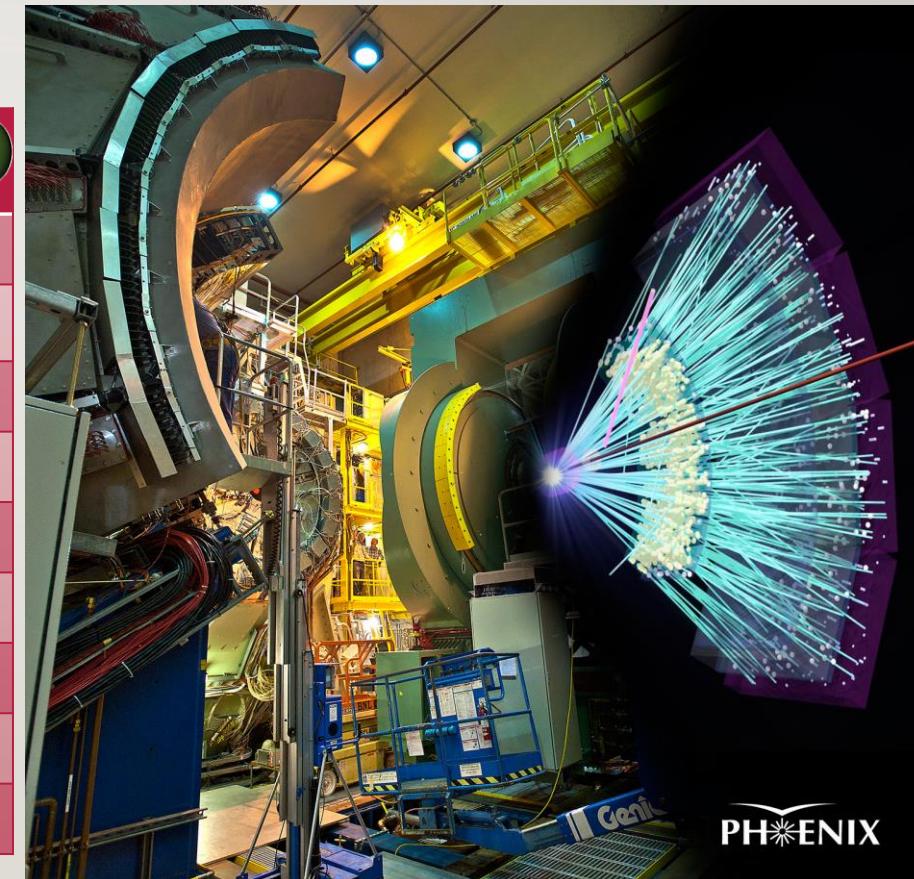
EÖTVÖS UNIVERSITY, BUDAPEST, HUNGARY



## 2<sub>/22</sub> THE PHENIX EXPERIMENT AND THE BES

- Collision energies: 7.7 to 200 GeV (20-400 MeV in  $\mu_B$ , 140-170 MeV in  $T$ )
- This talk: 200 GeV Au+Au

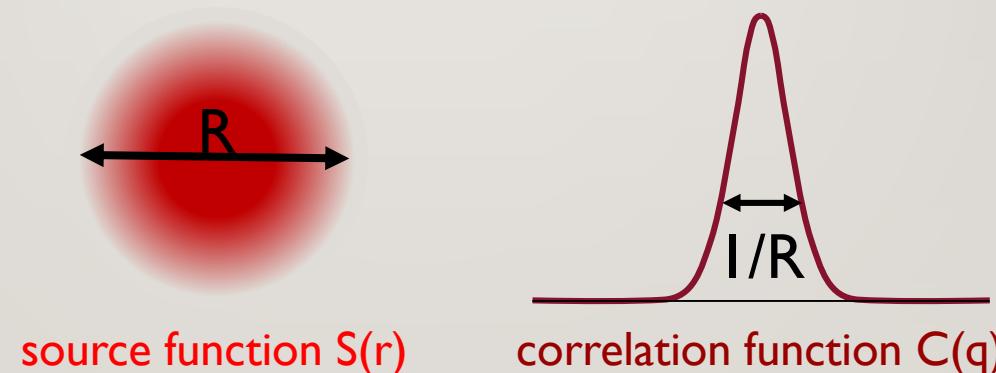
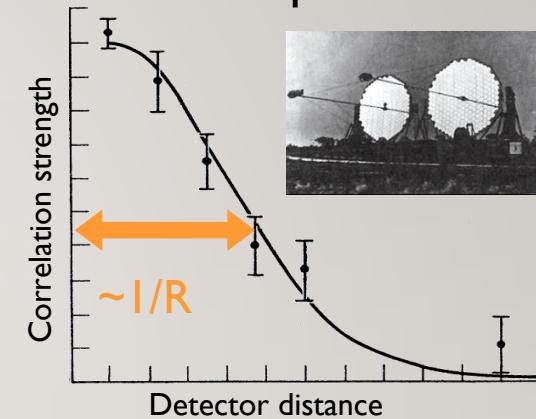
$\sqrt{s_{NN}}$ [GeV]	2	3	4	5	6	7	8	9	10
510	✓								
200	✓	✓	✓	✓	✓	✓	✓	✓	
130							✓		
62.4	✓			✓		✓		✓	
39			✓				✓		
27							✓		
20			✓		✓		✓		
14.5							✓		
7.7							✓		



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# FEMTOSCOPY: THE HBT EFFECT

- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance  $\Rightarrow$  source size
  - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
  - Momentum correlation  $C(q)$  related to source  $S(r)$
  - $C(q) \cong 1 + |\int S(r) e^{iqr} dr|^2$  (under some assumptions)



- Measure  $C(q)$ : map out source space-time geometry on femtometer scale!

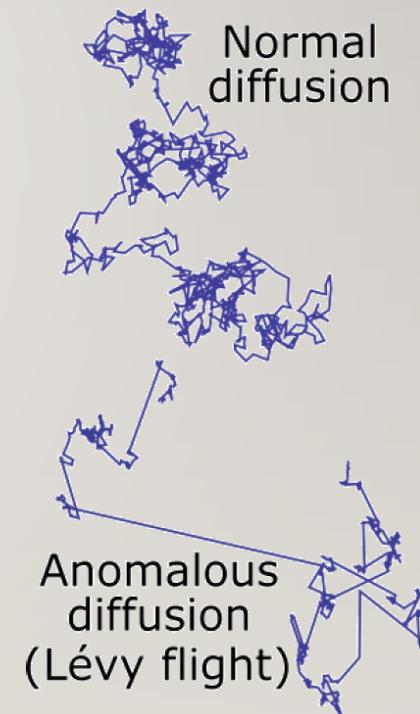
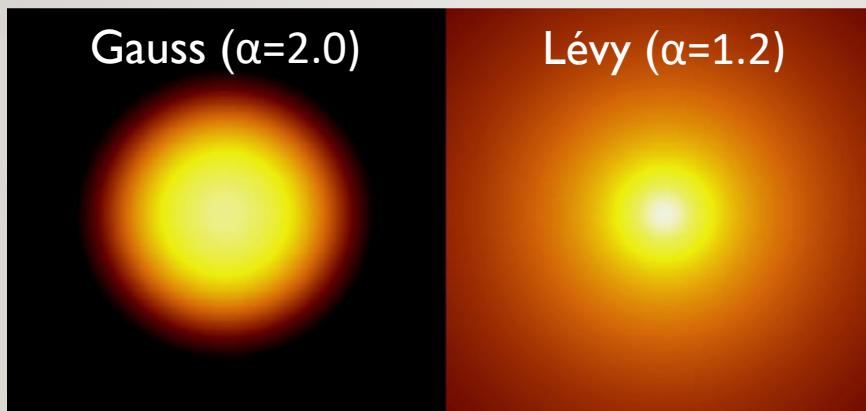
# LEVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Expanding medium, increasing mean free path: anomalous diffusion

Metzler, Klafter, Physics Reports 339 (2000) 1-77, Csanad, Csorg, Nagy, Braz.J.Phys. 37 (2007) 1002

- Levy-stable distribution:  $\mathcal{L}(\alpha, R; r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$

- From generalized central limit theorem, power-law tail  $\sim r^{-(1+\alpha)}$
- Special cases:  $\alpha = 2$  Gaussian,  $\alpha = 1$  Cauchy



- Shape of the correlation functions with Levy source:

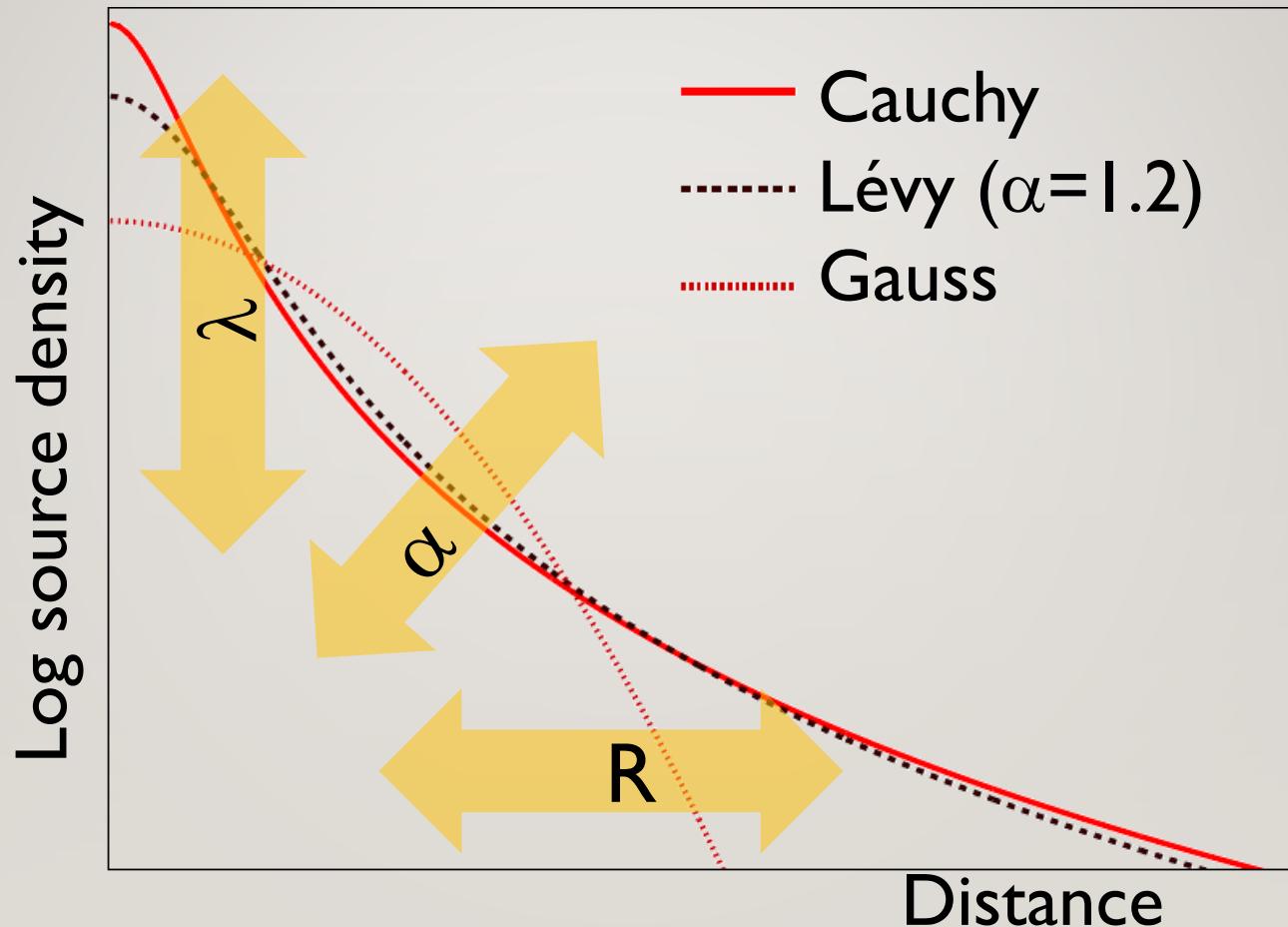
$$C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$$

$\alpha = 2$ : Gaussian  
 $\alpha = 1$ : Exponential

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# LVY VERSUS GAUSS VERSUS EXPONENTIAL

- No tail if  $\alpha = 2$ , power law if  $\alpha < 2$ ; correlation between  $\alpha$  and  $R, \lambda$



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# SPATIAL CORRELATION VERSUS DISTRIBUTION

- Assuming several things (thermal emission, no interactions, etc):

$$\begin{aligned}C_2(q, K) &= \int S\left(r_1, K + \frac{q}{2}\right) S\left(r_2, K - \frac{q}{2}\right) \left|\Psi_2^{(0)}(r_1, r_2)\right|^2 dr_1 dr_2 \\&\cong 1 + \left| \int S(r, K) e^{iqr} dr \right|^2\end{aligned}$$

- Let us introduce pair distribution (a.k.a. spatial correlation)

$$D(r, K) = \int S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right) d\rho$$

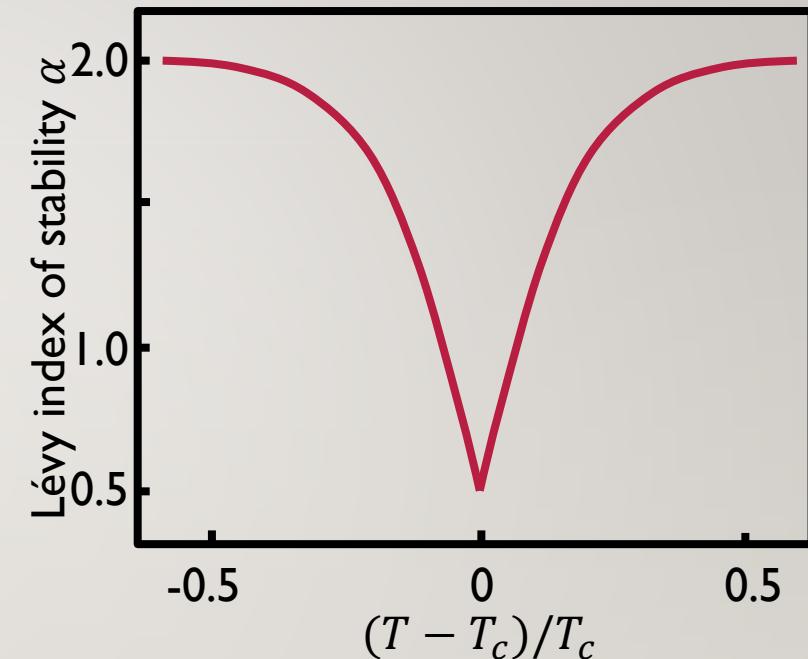
- With this, the correlation function becomes

$$C_2(q, K) \cong \int D(r, K) \left|\Psi_2^{(0)}(r)\right|^2 dr = 1 + \int D(r, K) e^{iqr} dr$$

- Correlation function measures spatial correlation function**

# Lvy index as a critical exponent?

- Critical spatial correlation:  $\sim r^{-(d-2+\eta)}$ ;  
Lvy source:  $\sim r^{-(1+\alpha)}$ ;  $\alpha \Leftrightarrow \eta$ ?  
Csrg, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67,
- QCD universality class  $\leftrightarrow$  3D Ising  
Halasz et al., Phys.Rev.D58 (1998) 096007  
Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising:  $\eta = 0.50 \pm 0.05$   
Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising:  $\eta = 0.03631(3)$   
El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lvy HBT!
- Change in  $\alpha_{\text{Lvy}}$  proximity of CEP?
- Modulo finite size/time and non-equilibrium effects  $\rightarrow$  what does power law exponent mean?



# THE COULOMB-EFFECT

- Plane-wave result, based on  $\left|\Psi_2^{(0)}(r)\right|^2 = 1 + e^{iqr}$ :

$$C_2(q, K) \cong \int D(r, K) \left|\Psi_2^{(0)}(r)\right|^2 dr = 1 + \int D(r, K) e^{iqr} dr$$

- If interaction:

$$\Psi_2^{(0)}(r) \rightarrow \Psi_2^{(\text{int})}(r_1, r_2)$$

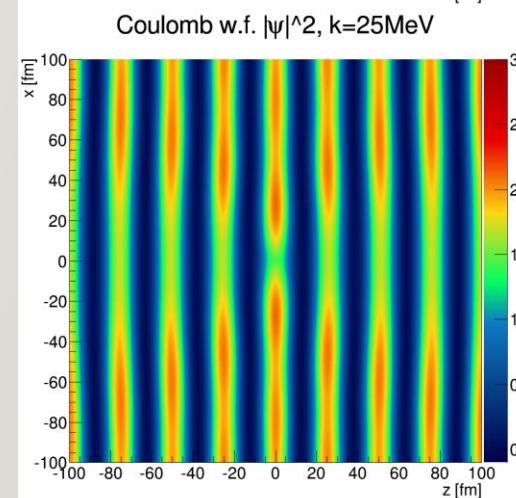
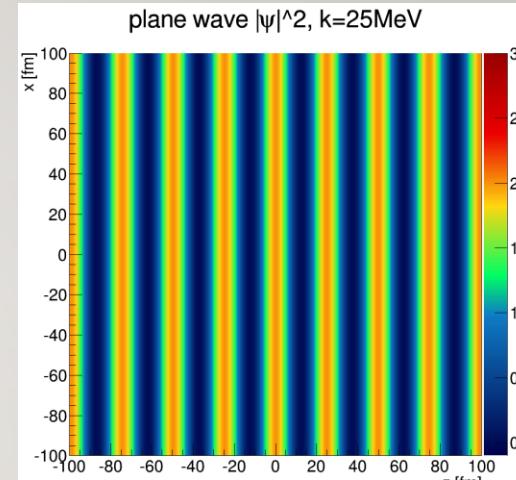
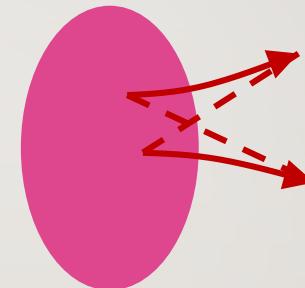
- For Coulomb:

$$\left|\Psi_2^{(C)}(r)\right|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1} \cdot (\text{complicated hypergeometric expression})$$

- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\text{Bose-Einstein}}(q)K(q), \text{ where } K(q) = \frac{\int D(r, K) \left|\Psi_2^{(C)}(r)\right|^2 dr}{\int D(r, K) \left|\Psi_2^{(0)}(r)\right|^2 dr}$$

- In this analysis: assuming spherical source



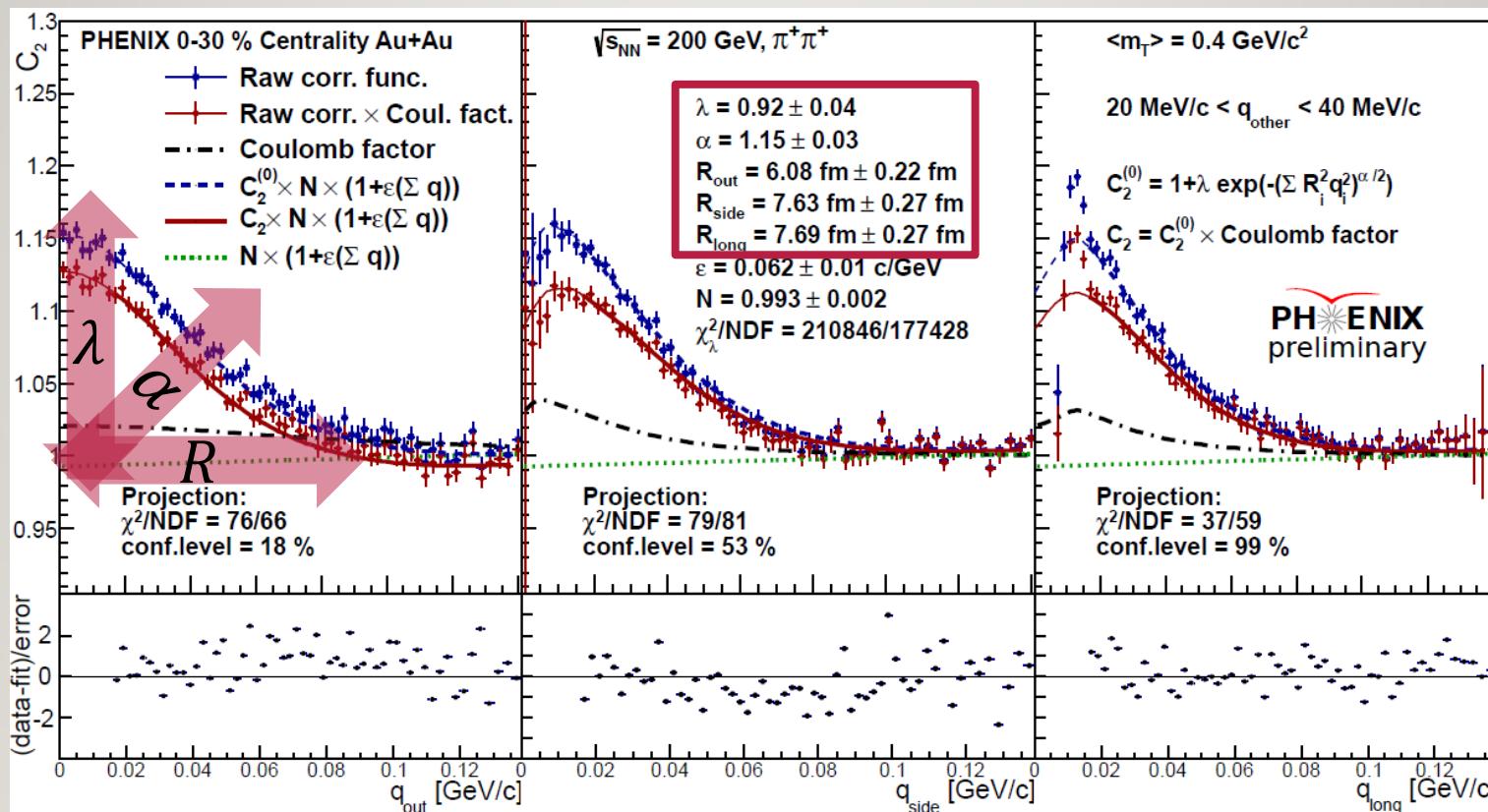
# LEVY HBT ANALYSIS

- Dataset used for the analysis:
  - Events: Run-10, Au+Au,  $\sqrt{s_{NN}} = 200 \text{ GeV}$ , 0-30% centrality:  $\sim 2$  billion events
  - Particle identification:
    - time-of-flight data from PbSc East/West, TOF East/West, momentum, flight length
    - $2\sigma$  cuts on  $m^2$  distribution
  - Single track cuts:  $2\sigma$  matching cuts in TOF & PbSc for pions
  - Pair-cuts:
    - A random member of pairs assoc. with hits on same tower were removed
    - customary shaped cuts in  $\Delta\phi - \Delta z$  plane for Drift Chamber, PbSc East/West, TOF East/West
- 3D corr. func. as a function of  $\vec{q}_{LCMS}$  in various  $m_T$  bins
  - $\vec{q}_{LCMS}$  is momentum difference longitudinal co-moving frame
  - Using Bertsch-Pratt frame:  $\vec{q}_{LCMS} = (q_{out}, q_{side}, q_{long})_{LCMS}$
  - Levy fits for 31  $m_T$  bins ( $0.228 < m_T < 0.871 \text{ GeV/c}$ ) with Coulomb effect

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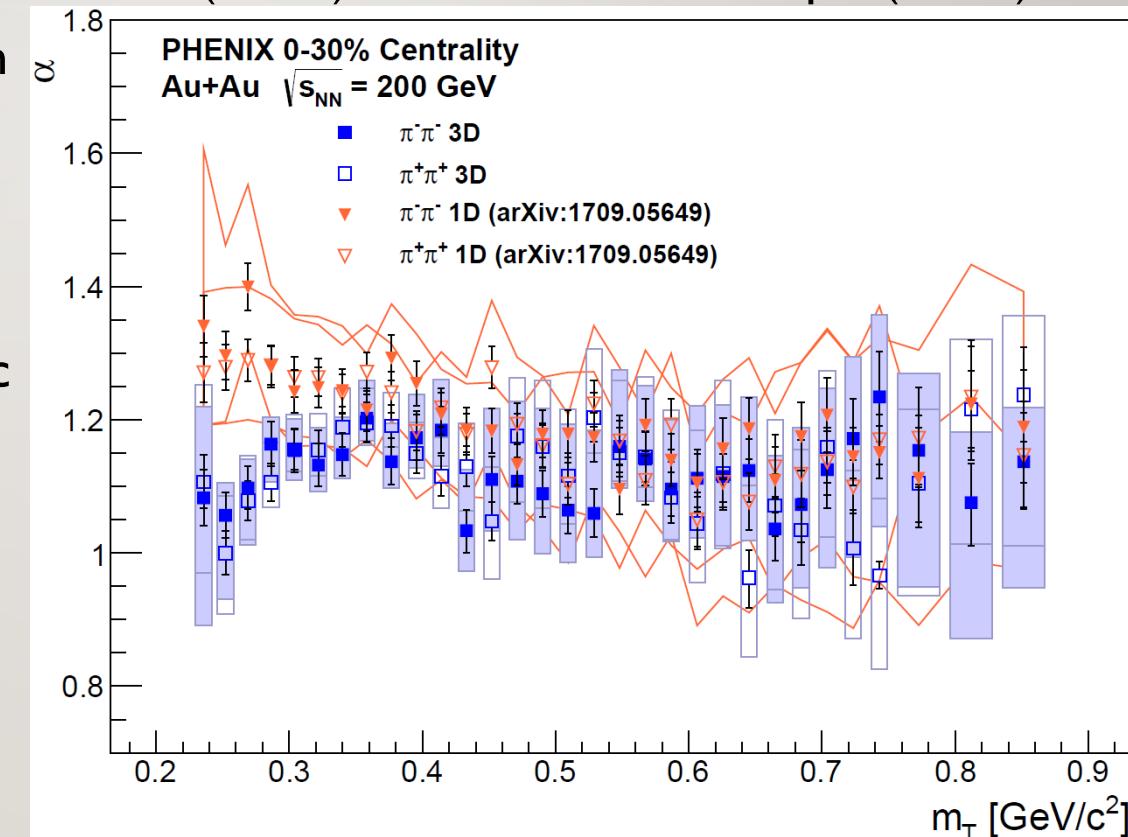
# EXAMPLE $C_2(q_{LCMS})$ CORRELATION FUNCTION

- Fitted 31  $m_T$  bins, modified log-likelihood fit, Coulomb-incorporated function  
E802: Phys.Rev. C66 (2002) 054906 [nucl-ex/0204001]; PHENIX: Phys.Rev.Lett. 93 (2004) 152302 [nucl-ex/0401003]
- Physical parameters:  $R$ ,  $\lambda$ ,  $\alpha$  measured versus pair  $m_T$



# LEVY EXPONENT (SHAPE PARAMETER) $\alpha$

- Compatible with 1D ( $Q_{LCMS}$ ) measurement of arXiv:1709.05649
- Measured value far from Gaussian ( $\alpha = 2$ ), inconsistent with expo. ( $\alpha = 1$ )
- Also far from the random field 3D Ising value at CEP ( $\alpha = 0.5$ )
- More or less constant (at least within systematic uncertainties)
- What do models and calculations say?



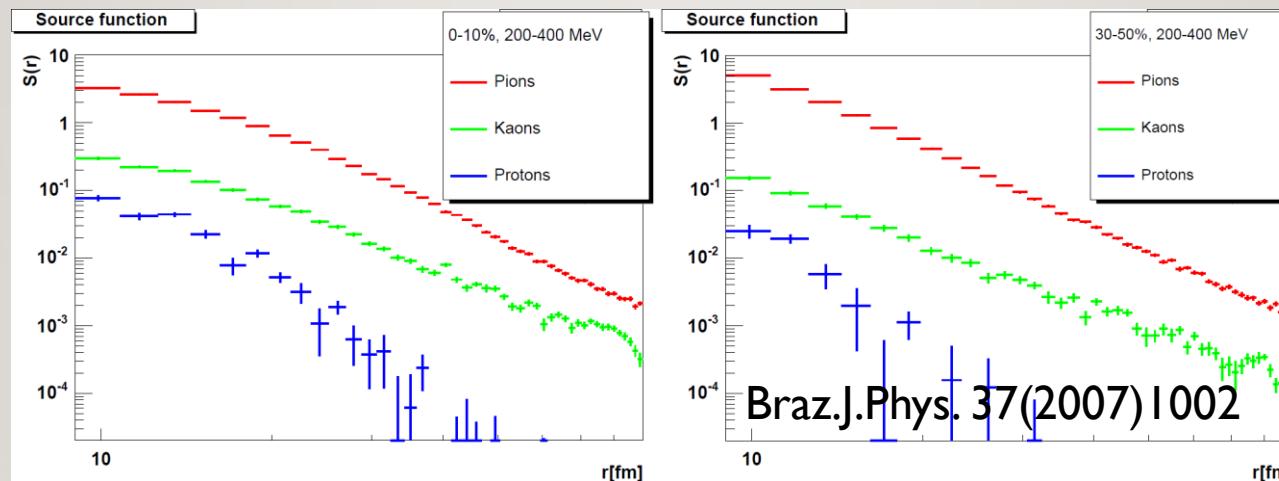
12<sub>/22</sub>

# THE IMPORTANCE OF A KAON ANALYSIS

- Kaons: smaller cross-section, larger mean free path
- Heavier power-law tail?
- Prediction for  $\pi, K, p$  based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering

Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]

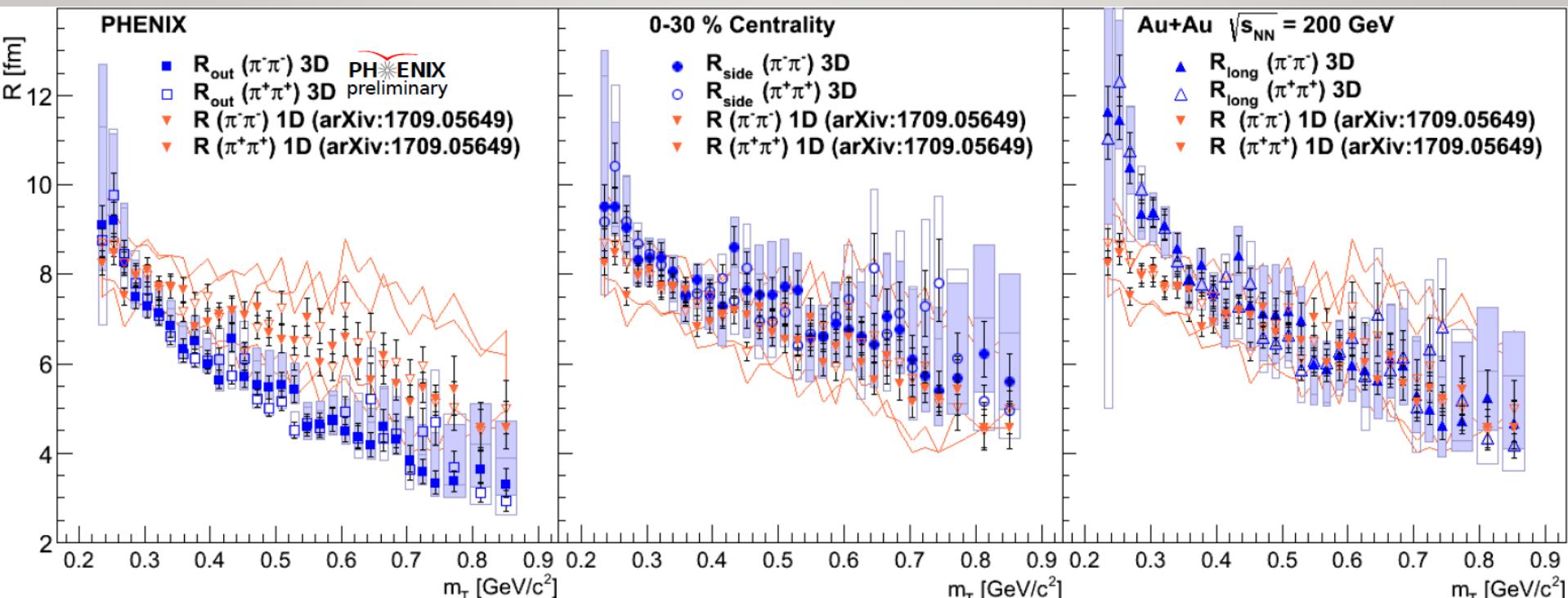
Csanad, Csorg, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]



- $R_{\text{HBT}}(\text{Kaon})$  mT-scaling or its violation for Lvy scale R?

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# LEVY SCALE PARAMETER R



- Compatibility with 1D Levy analysis
- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS radius!
  - There is no 2<sup>nd</sup> moment (variance or root mean square) for Levy distributions with  $\alpha < 2$ !
- Asymmetric source for small  $m_T$ , validity of Coulomb-approximation?

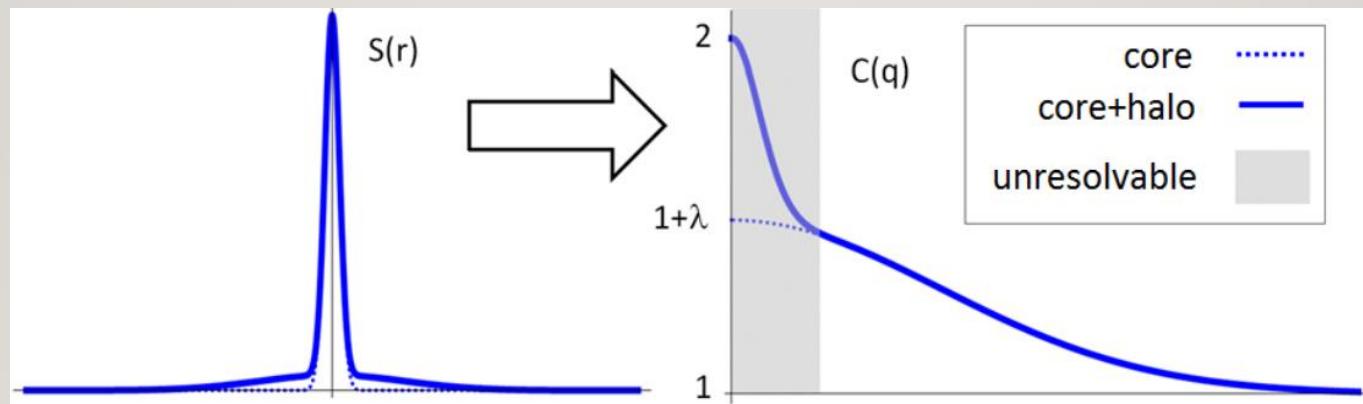
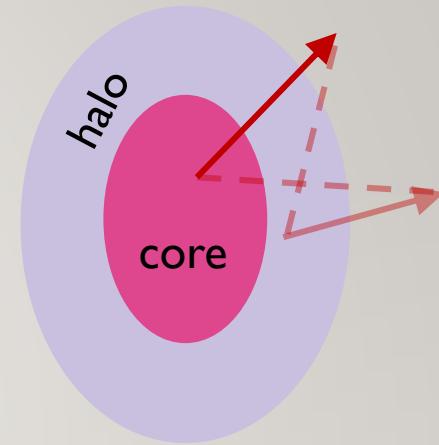
14<sub>/22</sub>

# CORRELATION STRENGTH $\lambda$ : CORE FRACTION

- Two-component source
  - Core: hydrodynamically expanding, thermal medium
  - Halo: long lived resonances ( $\gtrsim 10$  fm/c,  $\omega, \eta, \eta', K_0^S, \dots$ ), unresolvable experimentally
  - Define  $f_C = N_{\text{core}}/N_{\text{total}}$
- True  $q \rightarrow 0$  limit:  $C(0) = 2$
- Apparently  $C(q \rightarrow 0) \rightarrow 1 + \lambda$
- $\lambda(m_T) = f_C^2(m_T)$

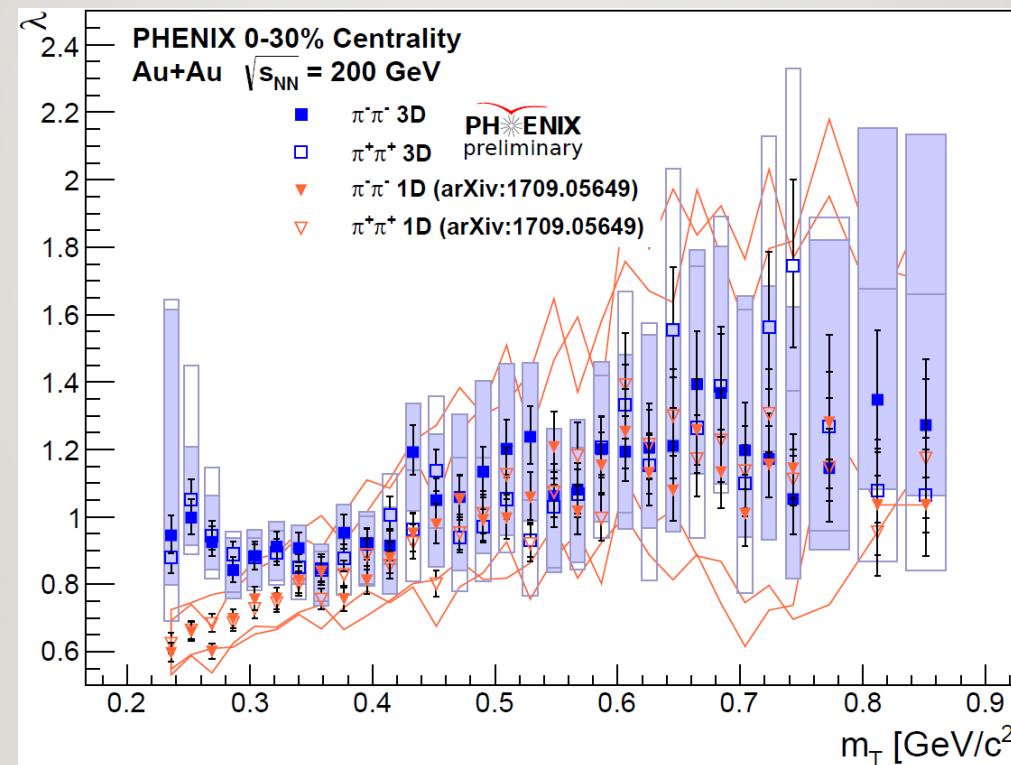
Bolz et al, Phys.Rev. D47 (1993) 3860-3870

Cs rg , L rstad, Zim nyi, Z.Phys. C71 (1996) 491-497



# 15<sub>/22</sub> LVY CORRELATION STRENGTH $\lambda$

- Compatibility of 1D and 3D results: low-mT decrease
- Small discrepancy at small mT: due to large Rlong at small mT?



# 16<sub>/22</sub> REMINDER: RELATION TO IN-MEDIUM $\eta'$ MASS

- Connection to chiral restoration

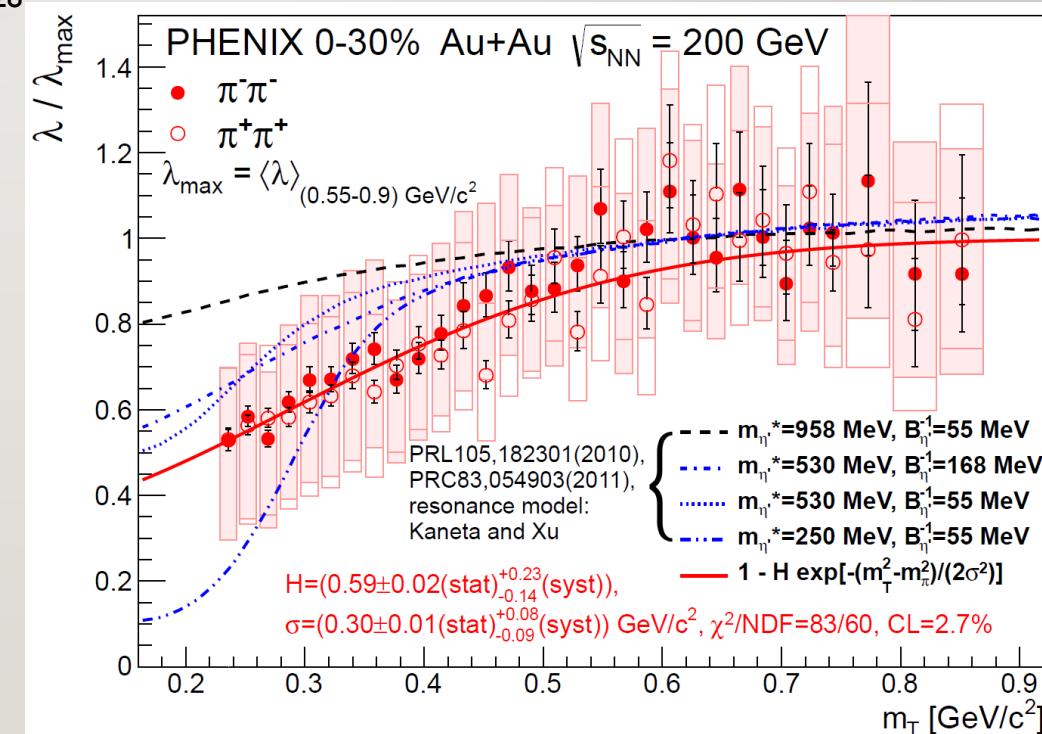
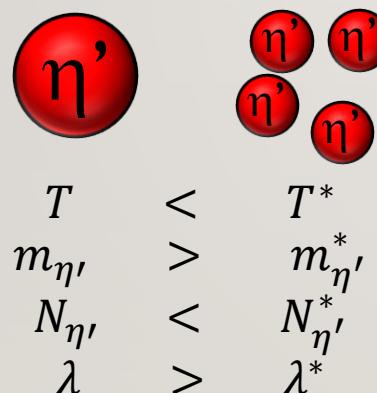
- Decreased  $\eta'$  mass  $\rightarrow \eta'$  enhancement  $\rightarrow$  halo enhancement
- Kinematics:  $\eta' \rightarrow \pi\pi\pi\pi$  with low  $m_T$   $\rightarrow$  decreased  $\lambda(m_T)$  at low  $m_T$
- Dependence on in-medium  $\eta'$  mass?

Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028

Vance, Cs rg , Kharzeev, PRL 81 (1998) 2205

Cs rg , V rtesi, Sziklai, PRL105 (2010) 182301

- 3D results compatible with 1D



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# SIDE-NOTE: THREE-PION LVY HBT

- Recall: two particle correlation strength  $\lambda = f_C^2$  where  $f_C = N_{\text{core}}/N_{\text{total}}$
- Generalization for higher order correlations:  $\lambda_2 = f_C^2$ ,  $\lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence ( $p_C$ ):

$$\lambda_2 = f_C^2[(1 - p_C)^2 + 2p_C(1 - p_C)]$$

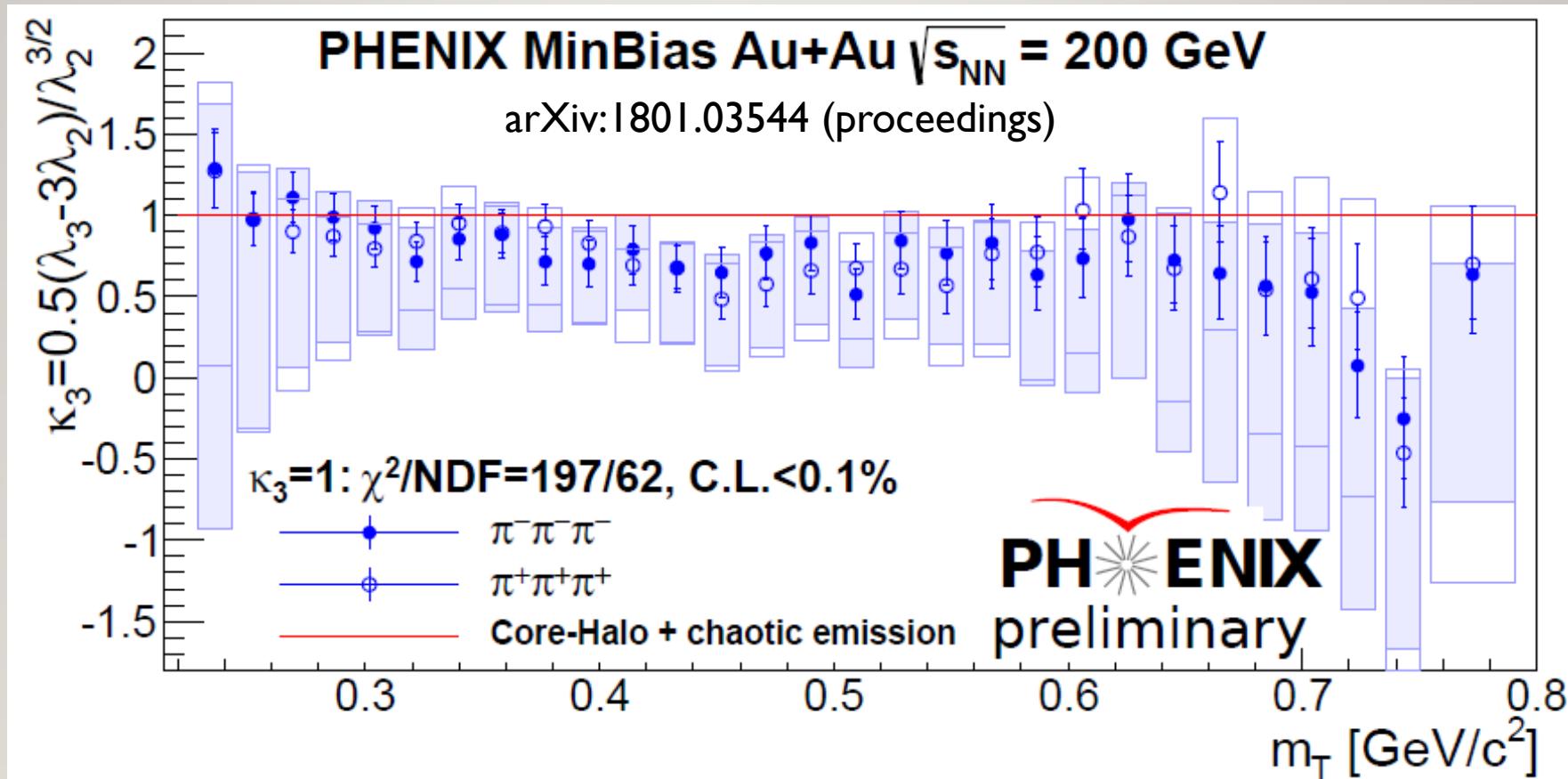
$$\lambda_3 = 2f_C^3[(1 - p_C)^3 + 3p_C(1 - p_C)^2] + 3f_C^2[(1 - p_C)^2 + 2p_C(1 - p_C)]$$

- Introduce core-halo independent parameter  $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2^3}}$ 
  - does not depend on  $f_C$
  - $\kappa_3 = 1$  if no coherence
- Finite meson sizes?  
Gavrilik, SIGMA 2 (2006) 074 [[hep-ph/0512357](#)]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
  - Random fields create random phase shift, on average distorts Bose-Einstein correlations

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# TEST OF CORE-HALO MODEL / COHERENCE

- Recall:  $\kappa_3 = 1$  in pure core-halo model,  $\kappa_3 \neq 1$  if coherence



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# PHENIX LVY HBT STATUS

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- Bose-Einstein correlations measured from 15 to 200 GeV
- Levy fits yield statistically acceptable description
- Levy parameters  $R, \lambda, \alpha$ : 3D measurement confirms 1D results
  - Stability parameter  $\alpha < 2 \leftrightarrow$  anomalous diffusion?
  - Linear scaling of  $I/R^2$  vs  $m_T \leftrightarrow$  hydro (but non-Gaussian source!)
  - Low- $m_T$  decrease in  $\lambda(m_T) \leftrightarrow$  core-halo model, in-medium  $\eta'$  mass?
- Three-particle analysis: chaotic or coherent emission?

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# OPEN QUESTIONS

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- Collision energy and centrality dependence?
  - Non-monotonicity in  $\alpha(\sqrt{s_{NN}})$  or  $\alpha(\text{centrality})$ ?
  - Hole in  $\lambda(m_T)$  at low  $\sqrt{s_{NN}}$ ? Really due to  $\eta'$ ?
  - Lower energies and centrality dependence: see the talk of D. Kincses
- What is the reason for the appearance of Lvy distributions for pions?
  - What is the Lvy exponent for kaons?
  - Kaons have smaller total cross-section thus larger mean free path, heavier tail?
  - Does  $m_T$  scaling hold for Lvy scale  $R$ ?
- Correlation strength versus core-halo picture: are there other effects?
  - Three-particle correlations may show if coherence or other effects play a role
  - Other effects may also play a role (finite meson sizes, random field phase shift, etc)

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# LEVY HBT WITH PHENIX

This work  
preliminary

$\pi$   
pairs  
200 GeV  
0-30%  
3D

stay tuned

$\pi$   
pairs  
200 GeV  
0-30%  
ID

Final data, PRC accepted  
arXiv:1709.05649  
WPCF18: Sndor Lks

Preliminary  
arXiv:1801.08827  
WPCF18:  
Daniel Kincses

$\pi$   
pairs  
200 GeV  
cent. dep.  
ID

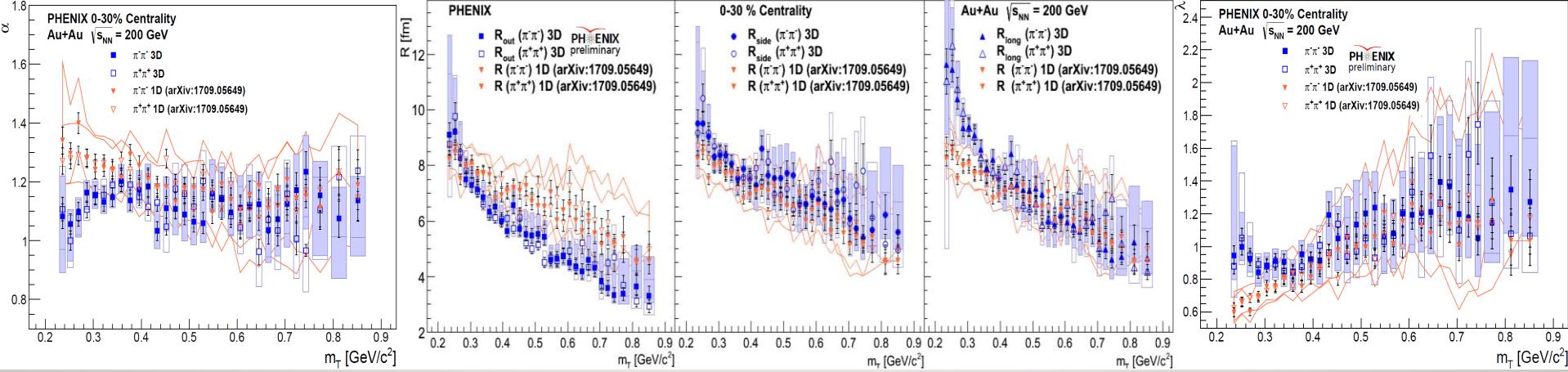
Preliminary  
arXiv:1801.03544  
WPCF17: A. Bagoly

$\pi$   
pairs  
15-62 GeV  
0-30%  
ID

Preliminary  
arXiv:1711.06891  
WPCF18: Daniel Kincses

K  
pairs  
200 GeV  
0-30%  
ID

$\pi$   
triplets  
200 GeV  
0-30%  
ID



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# THANK YOU FOR YOUR ATTENTION

If you are interested in these subjects, come to:



## ZIMÁNYI SCHOOL'18

18. Zimányi

WINTER SCHOOL ON  
HEAVY ION PHYSICS

Dec. 3. - Dec. 7.,  
Budapest, Hungary

Janos Kass: Falanszter (Phalansterie)

József Zimányi (1931 - 2006)

<http://zimanyischool.kfki.hu/18>



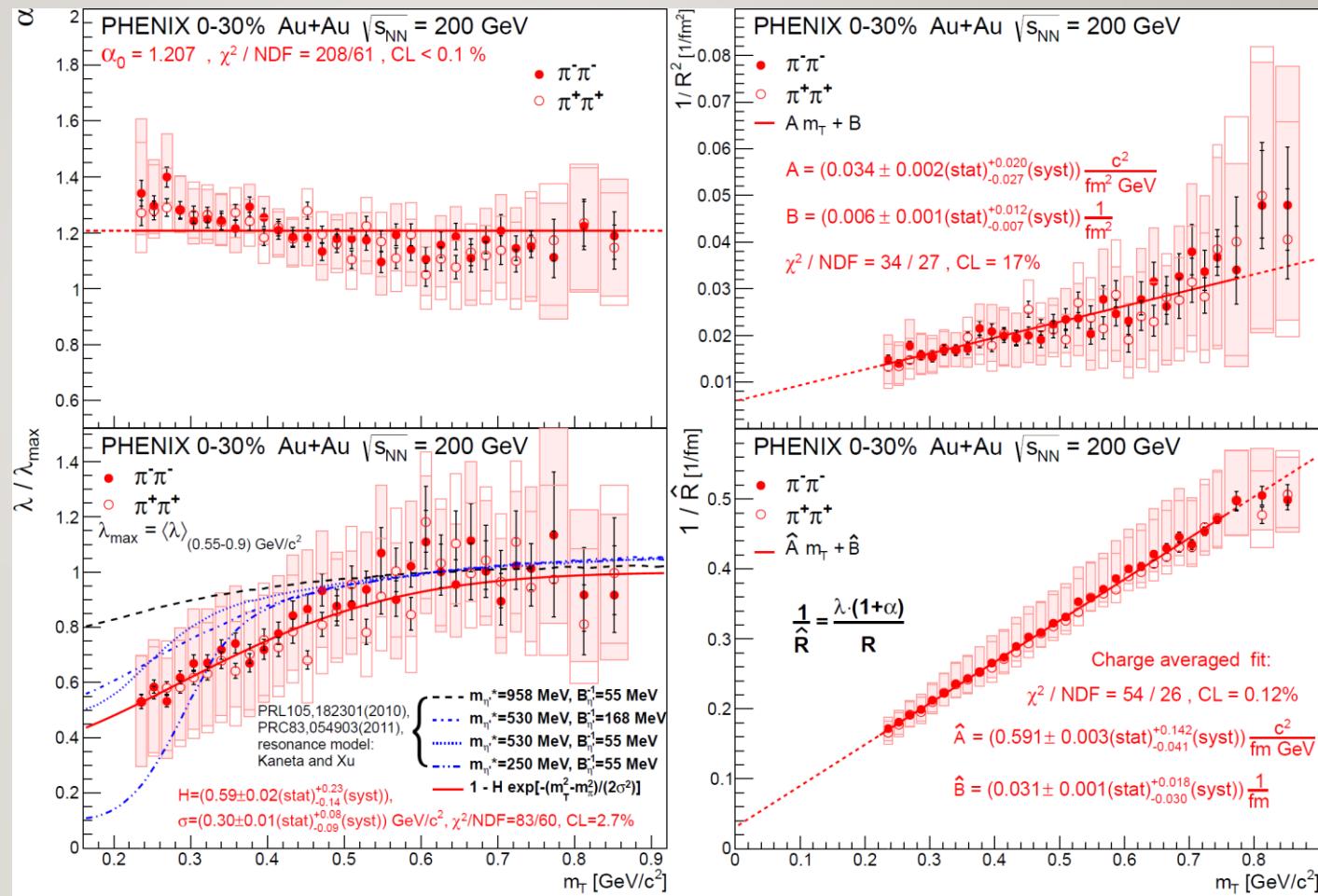


## 23 BACKUP

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# 24/22 200 GEV 1D ANALYSIS RESULTS RESULTS

- $\alpha$ : not 0.5 and not 2.0
- $R$ : hydro scaling
- $\lambda$ : „hole”, compatible with mass modification
- $\hat{R}$ : new scaling variable



# 25<sub>/22</sub> PRELIMINARY SYSTEMATIC ANALYSIS

- Uncertainty sources for each parameter:
  - PID, matching cuts, pair cuts, fit range, Coulomb-correction, q-binning
- Individual sources: very asymmetric
- Different for small and large  $m_T$
- Largest uncertainty:  $\lambda$

	$m_T < 500 \text{ MeV}/c^2$										$m_T > 500 \text{ MeV}/c^2$									
	$\lambda [\%]$		$R_{\text{out}} [\%]$		$R_{\text{side}} [\%]$		$R_{\text{long}} [\%]$		$\alpha [\%]$		$\lambda [\%]$		$R_{\text{out}} [\%]$		$R_{\text{side}} [\%]$		$R_{\text{long}} [\%]$		$\alpha [\%]$	
source	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
PID	2.7	8.4	2.1	2.3	1.9	3.8	4.5	2.7	2.1	1.5	2.9	5.1	1.7	2.6	1.7	2.8	1.8	3.1	1.6	1.1
ID match.	4.1	7.7	2.2	4.8	2.3	4.8	2.2	4.8	2.8	1.9	9.3	18.6	4.9	9.6	4.3	9.6	4.9	10.9	4.8	2.9
paircut	3.3	7.0	2.3	5.0	2.4	5.1	2.1	5.1	3.1	1.7	6.3	14.6	4.5	9.5	4.6	10.2	3.6	9.1	4.6	2.8
fitlim max	0.3	0.1	0.2	0.0	0.2	0.0	0.2	0.0	0.0	0.2	0.2	0.2	0.15	0.1	0.2	0.1	0.2	0.1	0.1	0.1
fitlim min	0.6	0.9	0.4	0.6	0.5	0.7	0.5	0.6	0.5	0.4	1.5	1.5	1.0	1.0	1.2	1.2	1.2	1.2	0.8	0.8
Coul. corr.	0.5	4.1	0.4	2.7	0.4	2.8	0.4	2.7	1.8	0.2	0.5	16.9	0.2	13.3	0.3	13.0	0.4	11.3	6.5	0.2
q-binning	10.5	0.4	0.0	7.2	7.8	0.3	7.4	0.3	0.2	4.1	15.9	0.0	0.3	3.3	12.3	0.0	12.8	0.0	0.0	9.0