Recent lattice Results on the QCD phase diagram

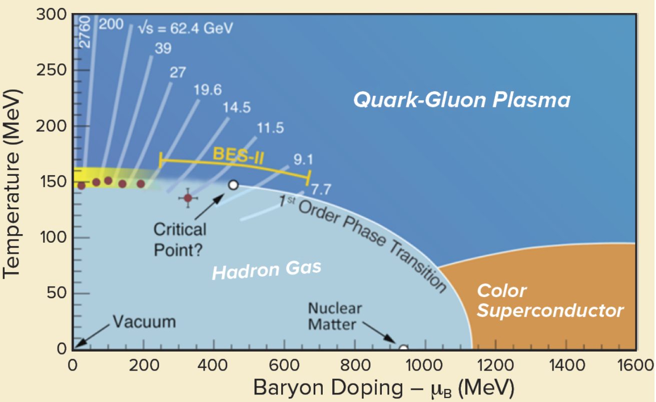
Jana N. Guenther

May 26th 2018

SFB TR55

WB collaboration

The (T, μ_B)-phase diagram of QCD

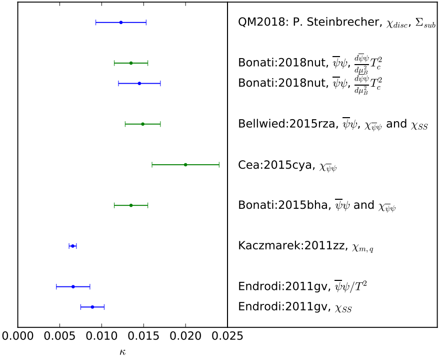


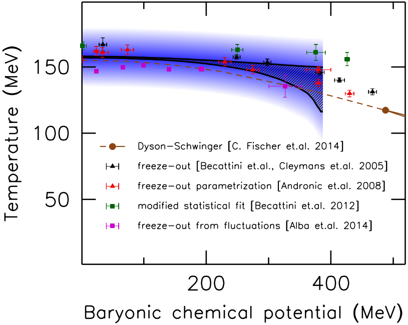
Our observables:
T_c, Equation of state, Fluctuations

Not covered in this talk

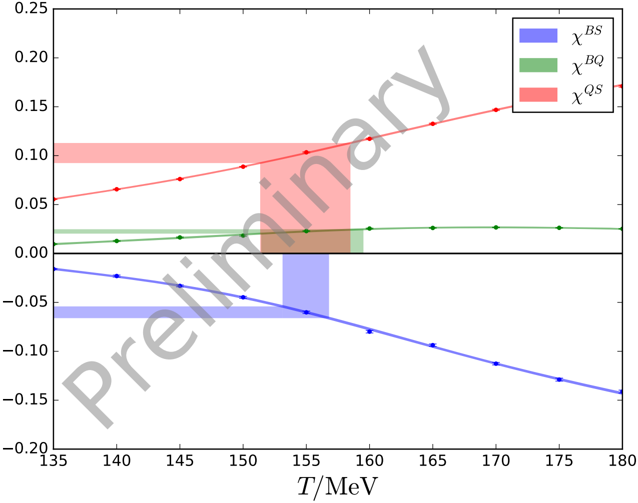
Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$





Also not in this talk



...nor anything about the Equation of States

Why LQCD?

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \overline{\psi}\left(\mathrm{i}\gamma_\mu D^\mu - m\right)\psi$$

- Because of the strong coupling and the self interaction of gluons perturbation theory is not feasible
- Path integral quantization:

$$\langle 0 | T \hat{\phi}_1 \ldots \hat{\phi}_n | 0 \rangle = \frac{\int D\phi\, \hat{\phi}_1 \ldots \hat{\phi}_n e^{\mathrm{i}\int \mathrm{d}x\, \mathcal{L}}}{\int D\phi\, e^{\mathrm{i}\int \mathrm{d}x\, \mathcal{L}}}$$

First problem: There are many points in space-time $D\phi = \prod_i \mathrm{d}\phi(x_i)$

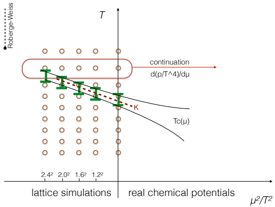
Solution: Replace continuous space by a discrete 4d lattice

Why aren't we finished yet?

- Simulations take a lot of computer time
- Not everything can be calculated directly. For example:
  - Only observables that can be calculated in Euclidean space
  - Only thermal equilibrium
  - Only simulations at

$$\mu_B = 0 \Rightarrow \langle n_B \rangle = 0$$

heavy ion collision experiments



The sign problem

The QCD partition function:

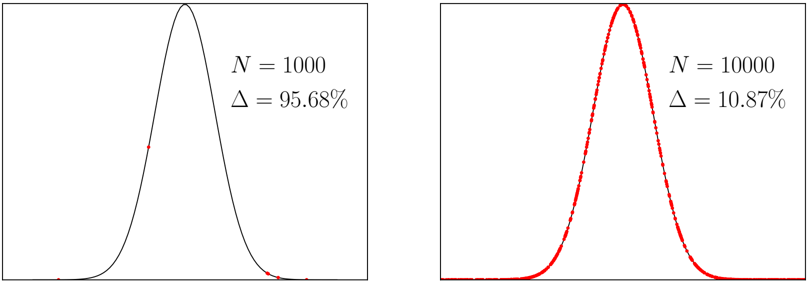
$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \det M(U) e^{-\beta S_G(U)}$$

- For Monte Carlo simulations $\det M(U)e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

The sign problem

$$\int_{-\infty}^{\infty}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \approx \int_{-100}^{100}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100-x_i^2)\frac{e^{-\frac{1}{2}x_i^2}}{\sqrt{2\pi}}\cdot\frac{200}{N}$$

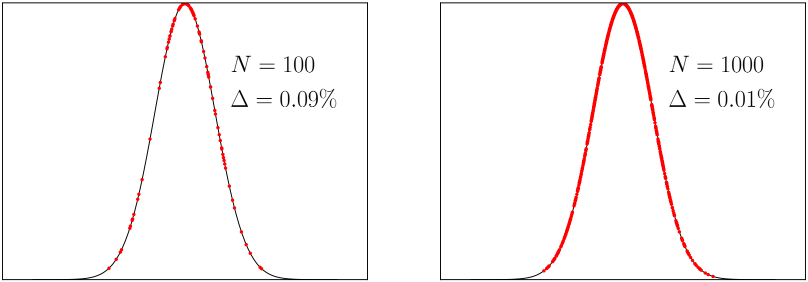
The x_i are drawn from a uniform distribution in the interval $[-100, 100]$



Importance sampling

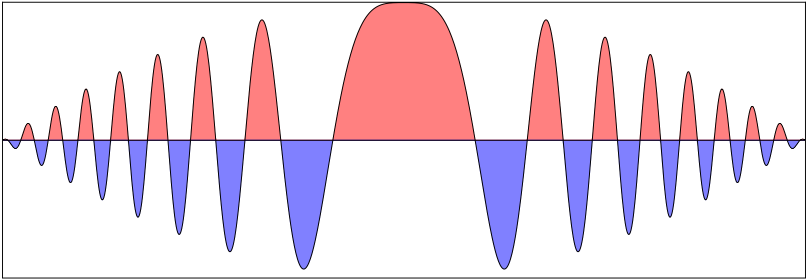
$$\int_{-\infty}^{\infty}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100-x_i^2)\cdot\frac{1}{N}$$

The x_i are drawn from a normal distribution



The sign problem

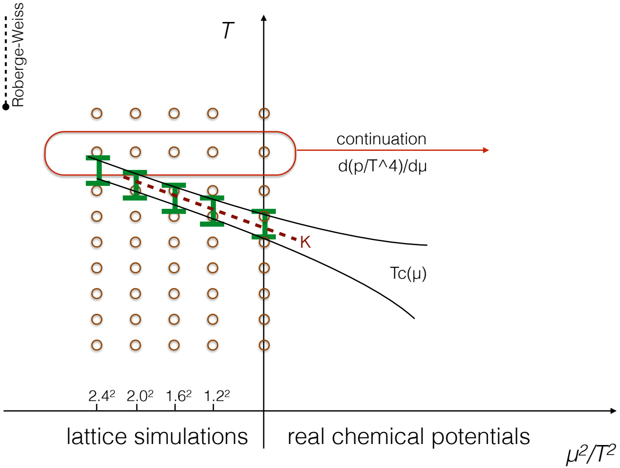
$$\int_{-\infty}^{\infty}(100 - x^2)\frac{e^{-\frac{i}{2}x^2}}{\sqrt{2\pi}}$$



Dealing with the sign problem

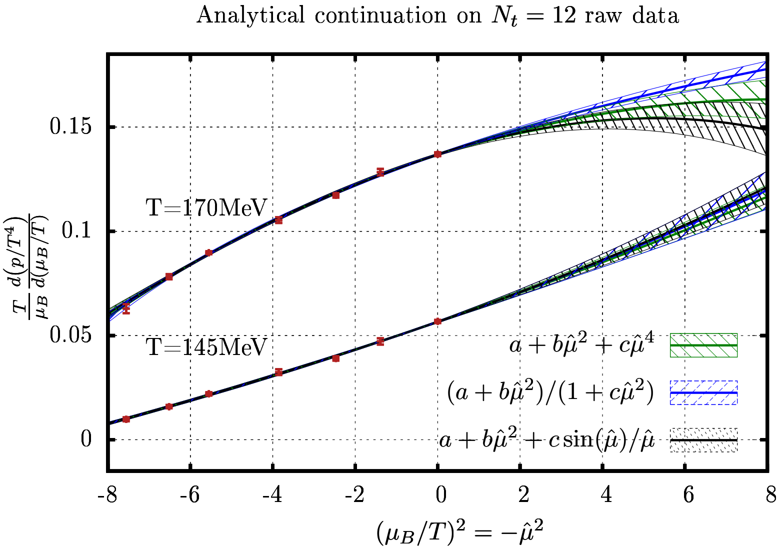
- Reweighting techniques
- Canonical ensemble
- Complex Langevin
- Lefshetz Thimble
- Density of state methods
- Dual variables
- Taylor expansion $\longrightarrow$ [Bazavov et al., Bazavov:2017dus], [Bonati et al., Bonati:2018nut]
- Imaginary μ
- ...

Analytic continuation



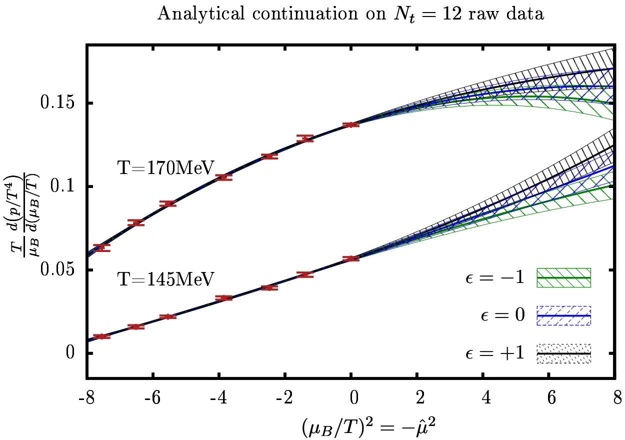
Common technique: [de Forcrand, Philipsen, deForcrand:2002hgr], [Bonati et al., Bonati:2015bha], [Cea et al., Cea:2015cya], [D'Elia et al., DElia:2016jqh], [Bonati et al., Bonati:2018nut] . . .

Different functions

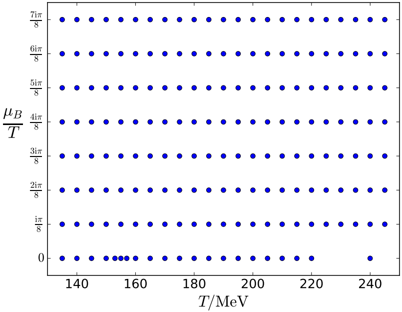


Different functions

Condition: $\chi_8 \lesssim \chi_4 \longrightarrow f(\hat{\mu}_B) = a + b\hat{\mu}_B^2 + c\hat{\mu}_B^4 + \frac{b\epsilon}{840}\hat{\mu}_B^6$



Simulation details



- Borsanyi et al., Borsanyi:2018grb, arXiv:1805.04445
- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavor, on LCP with pion and kaon mass
- Simulation at $\mu_S = \mu_Q = 0$
- Lattice size: $48^3 \times 12$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 1, 2, 3, 4, 5, 6$ and 7

The fit function

$$\chi_0^B(\hat{\mu}_B) = \frac{p}{T^4} = c_0 + c_2\hat{\mu}_B^2 + c_4\hat{\mu}_B^4 + c_6\hat{\mu}_B^6 + \frac{4!}{8!}c_4\epsilon_1\hat{\mu}_B^8 + \frac{4!}{10!}c_4\epsilon_2\hat{\mu}_B^{10}.$$

where ϵ_1 and ϵ_2 are drawn randomly from a normal with $\mu = -1.25$ and $\sigma = 2.75$ distribution.

From this we can calculate the derivatives that we can measure on the lattice:

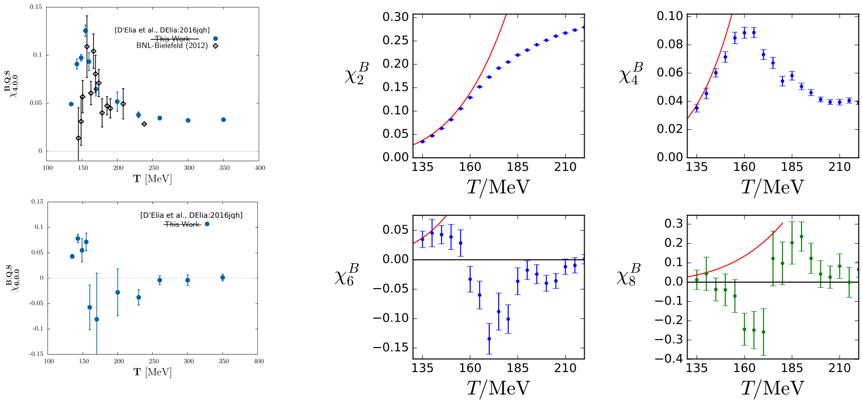
$$\chi_1^B(\hat{\mu}_B) = 2c_2\hat{\mu}_B + 4c_4\hat{\mu}_B^3 + 6c_6\hat{\mu}_B^5 + \frac{4!}{7!}c_4\epsilon_1\hat{\mu}_B^7 + \frac{4!}{9!}c_4\epsilon_2\hat{\mu}_B^9$$

$$\chi_2^B(\hat{\mu}_B) = 2c_2 + 12c_4\hat{\mu}_B^2 + 30c_6\hat{\mu}_B^4 + \frac{4!}{6!}c_4\epsilon_1\hat{\mu}_B^6 + \frac{4!}{8!}c_4\epsilon_2\hat{\mu}_B^8$$

$$\chi_3^B(\hat{\mu}_B) = 24c_4\hat{\mu}_B + 120c_6\hat{\mu}_B^3 + \frac{4!}{5!}c_4\epsilon_1\hat{\mu}_B^5 + \frac{4!}{7!}c_4\epsilon_2\hat{\mu}_B^7$$

$$\chi_4^B(\hat{\mu}_B) = 24c_4 + 360c_6\hat{\mu}_B^2 + c_4\epsilon_1\hat{\mu}_B^4 + \frac{4!}{6!}c_4\epsilon_2\hat{\mu}_B^6.$$

χ_2^B, χ_4^B, χ_6^B and χ_8^B

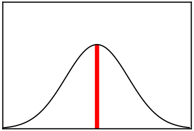


[D'Elia et al., DElia:2016jqh], see also [Bazavov et al., Bazavov:2017dus]

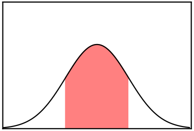
Observables

Cumulants of the net baryon number distributions:

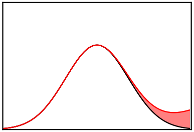
- mean M_B



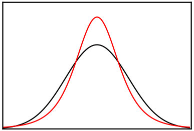
- variance σ_B^2



- skewness S_B: asymmetry of the distribution

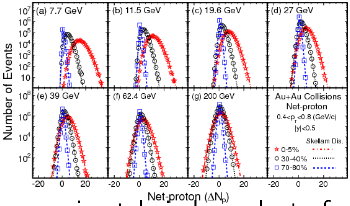


- kurtosis κ_B: "tailedness" of the distribution



Calculating observables

We have derivatives with respect to $\hat{\mu}_B$, $\hat{\mu}_Q$ and $\hat{\mu}_S$ of the pressure at $\mu_S = \mu_Q = 0$. Notation:



$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k},$$

with $\hat{\mu}_i = \mu/T$.

We want ratios of the cumulants that are approximately independent of the volume at $\mu_B > 0$, $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$:

$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = \hat{\mu}_B r_{12}^{B,1} + \hat{\mu}_B^3 r_{12}^{B,3} + \ldots$$

$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\hat{\mu}_B)}{\chi_1^B(T,\hat{\mu}_B)} = r_{31}^{B,0} + \hat{\mu}_B^2 r_{31}^{B,2} + \ldots$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \ldots$$

[Bazavov et al., Bazavov:2017dus], [Karsch, Karsch:2017zzw]
plot: [STAR, Adamczyk:2013dal]

Measured observables

On each ensemble we measure the $\chi^{BQS}_{i,j,k}$ up to the fourth derivative:

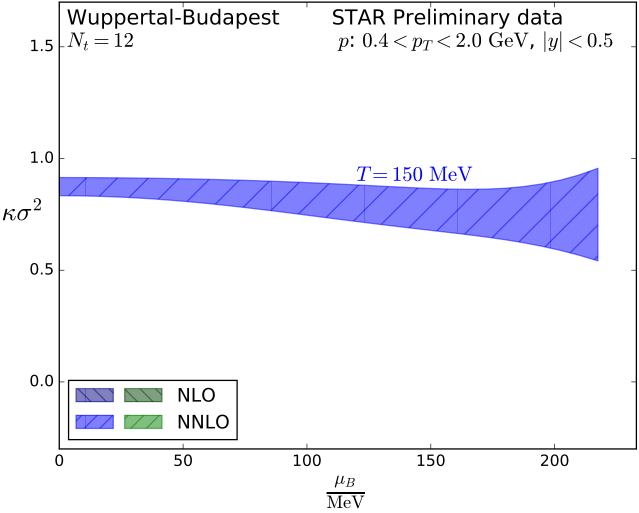
- χ^{B}_{1}
- χ^{B}_{2}
- χ^{B}_{3}
- χ^{B}_{4}
- χ^{Q}_{1}
- $\chi^{BQ}_{1,1}$
- $\chi^{BQ}_{1,2}$
- $\chi^{BQ}_{1,3}$
- χ^{S}_{1}
- $\chi^{BS}_{1,1}$
- $\chi^{BS}_{1,2}$
- $\chi^{BS}_{1,3}$

- χ^{Q}_{2}
- $\chi^{BQ}_{2,1}$
- $\chi^{BQ}_{2,2}$
- χ^{S}_{2}
- $\chi^{BS}_{2,1}$
- $\chi^{BS}_{2,2}$
- $\chi^{QS}_{1,1}$
- $\chi^{BQS}_{1,1,1}$
- $\chi^{BQS}_{2,1,1}$

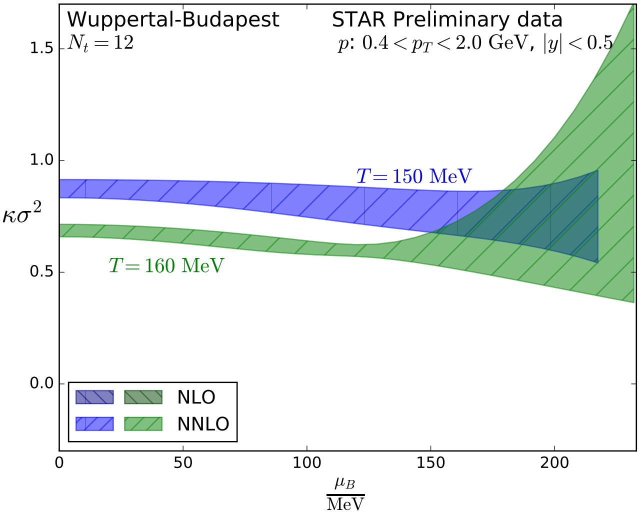
- χ^{Q}_{3}
- $\chi^{BQ}_{3,1}$
- χ^{S}_{3}
- $\chi^{BS}_{3,1}$
- $\chi^{QS}_{2,1}$
- $\chi^{BQS}_{1,2,1}$
- $\chi^{QS}_{1,2}$
- $\chi^{BQS}_{1,1,2}$

- χ^{Q}_{4}
- χ^{S}_{4}
- $\chi^{QS}_{3,1}$
- $\chi^{QS}_{2,2}$
- $\chi^{QS}_{1,3}$

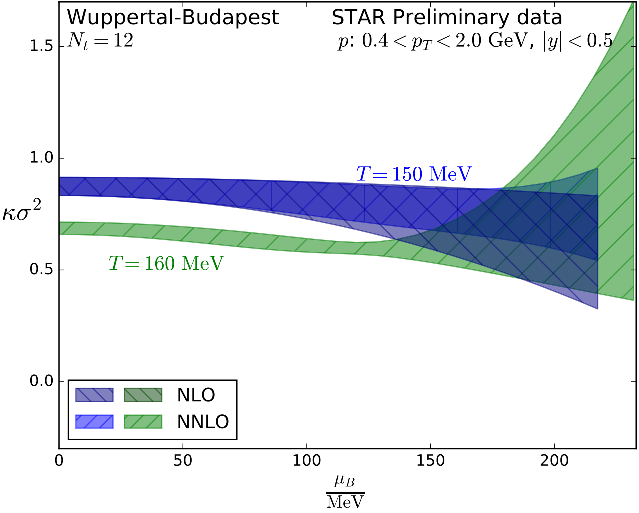
Extrapolation $\kappa\sigma^2$



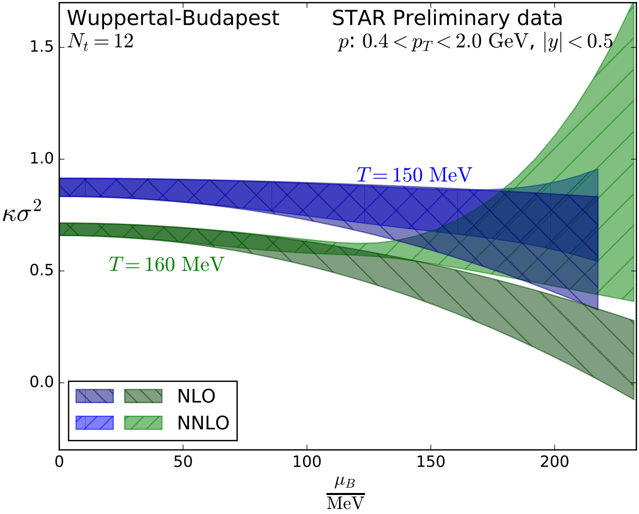
Extrapolation $\kappa\sigma^2$



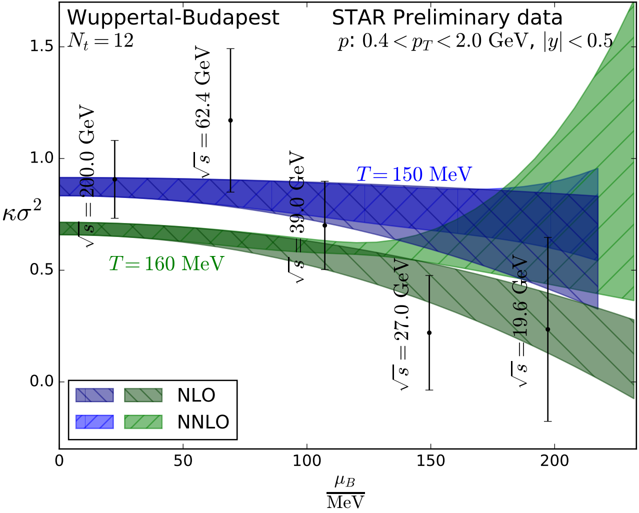
Extrapolation $\kappa\sigma^2$



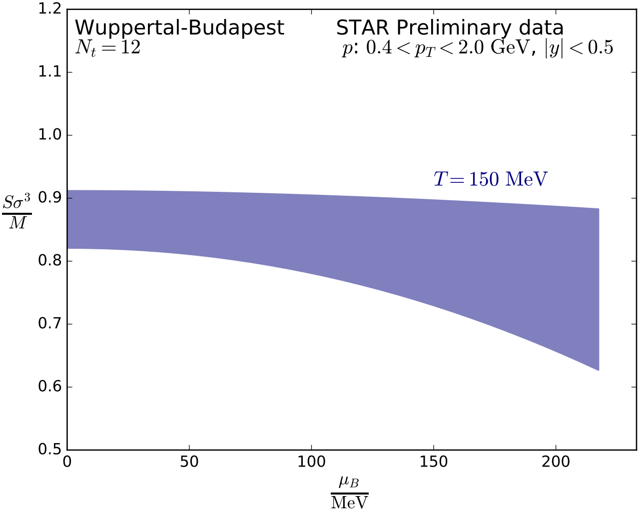
Extrapolation $\kappa\sigma^2$



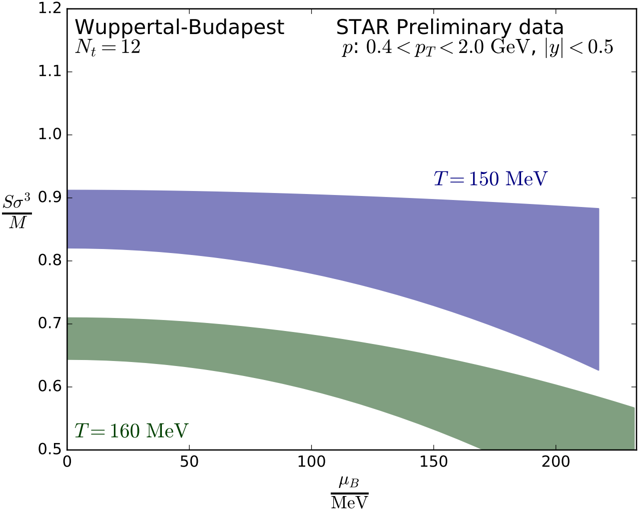
Extrapolation $\kappa\sigma^2$



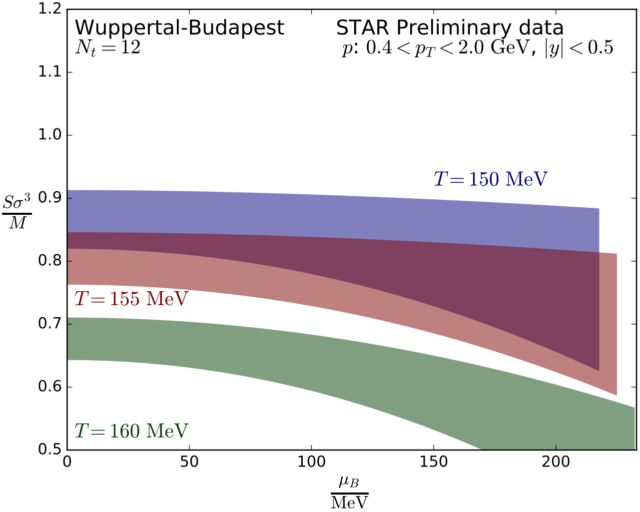
Extrapolation $\kappa\sigma^2$



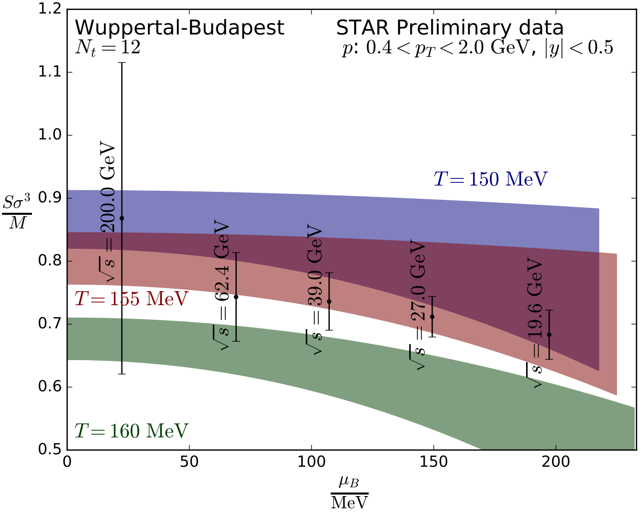
Extrapolation $\kappa\sigma^2$



Extrapolation $\kappa\sigma^2$



Extrapolation $\kappa\sigma^2$



Convergence radius estimators

1. Ratio test for the pressure:

$$p(\mu) = p_0 + p_2\hat{\mu}^2 + p_4\hat{\mu}^4 + p_6\hat{\mu}^6 + \dots$$

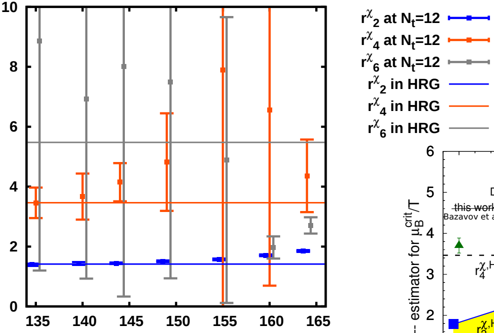
   Ratio test $\rightarrow r^p_{2n} = \sqrt{\frac{p_{2n}}{p_{2n+2}}}$

2. Ratio test for the susceptibility:

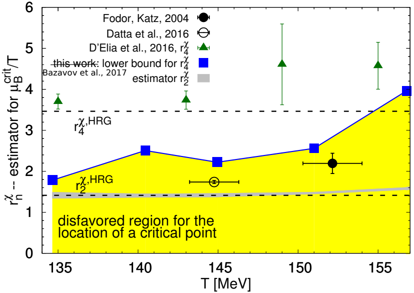
$$\chi_2(\mu) = 2p_2 + 12p_4\hat{\mu}^2 + 30p_6\hat{\mu}^4 + \dots$$

   Ratio test $\rightarrow r^\chi_{2n} = \sqrt{\frac{2n(2n-1)}{(2n+1)(2n+2)}} r^p_{2n}$

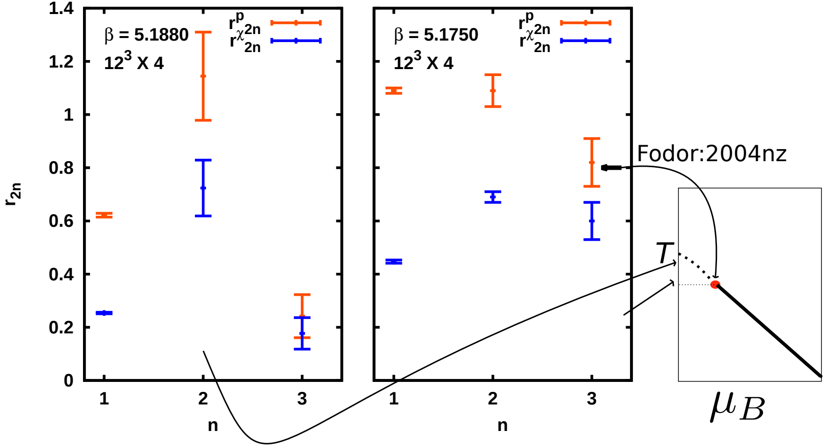
Ratios for the radius of convergence



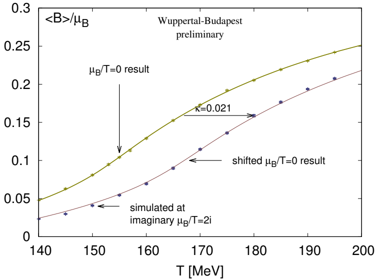
[Bazavov et al., Bazavov:2017dus]



$N_t = 4$ Toy model with critical endpoint



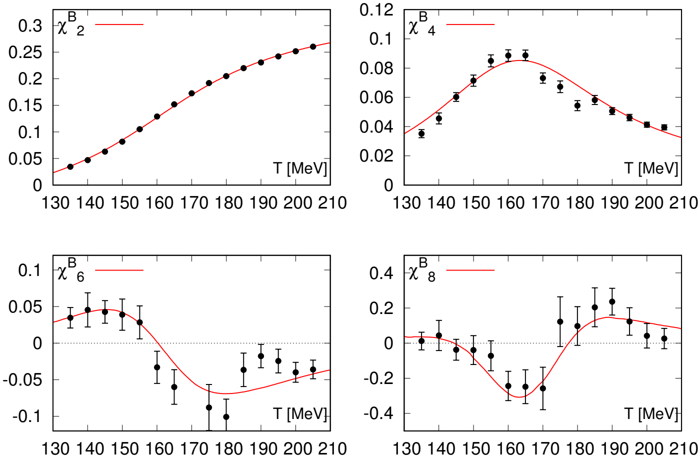
Toy model without critical endpoint



- Start with some parametrization of the curve χ_1^B/μ_B at $\mu = 0$
- Assume that the only difference in the physics at finite μ is a shift in this curve
- The inflection point of this curve is one possible definition of T_c, so shift the curve by using the κ values found in the literature
- You now have a model prediction of χ_1^B for any finite μ, differentiate it a few times at $\mu = 0$ to get estimates of χ_4^B, χ_6^B and χ_8^B

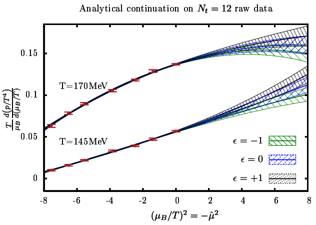
NOTE: The model assumes no criticality

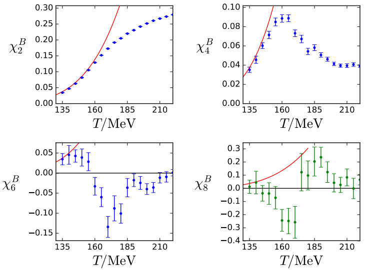
Fluctuations in the toy model

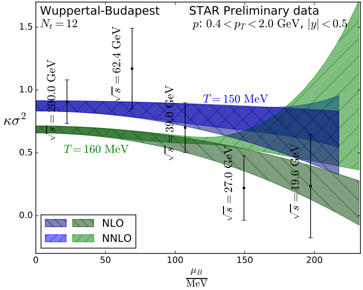


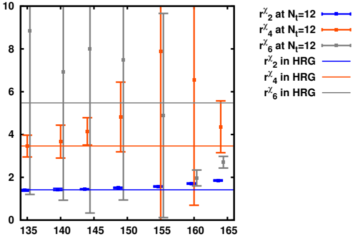
RED CURVE: The simple model described in the previous slide without criticality

Summary


Analytical continuation on $N_t = 12$ raw data
T=170MeV
T=145MeV
$\epsilon = -1$
$\epsilon = 0$
$\epsilon = +1$
$\frac{T}{\mu_B}\frac{d(p/T^4)}{d(\mu_B/T)}$
$(\mu_B/T)^2 = -\hat{\mu}^2$


χ_2^B
χ_4^B
χ_6^B
χ_8^B
T/MeV


Wuppertal-Budapest
$N_t = 12$
STAR Preliminary data
p: $0.4 < p_T < 2.0$ GeV, $|y| < 0.5$
$\sqrt{s} = 200.0$ GeV
$\sqrt{s} = 62.4$ GeV
$\sqrt{s} = 39.0$ GeV
$\sqrt{s} = 27.0$ GeV
$\sqrt{s} = 19.6$ GeV
$T = 150$ MeV
$T = 160$ MeV
$\kappa\sigma^2$
NLO
NNLO
$\frac{\mu_B}{\text{MeV}}$


r^{χ}_2 at N_t=12
r^{χ}_4 at N_t=12
r^{χ}_6 at N_t=12
r^{χ}_2 in HRG
r^{χ}_4 in HRG
r^{χ}_6 in HRG