QCD phase-diagram and multiparticle proton correlations

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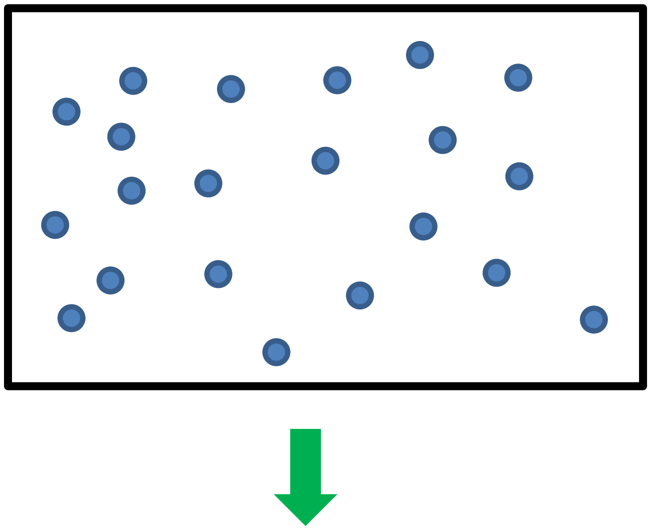
correlation, interaction
cumulants and STAR data
clusters
two event classes

AB, V.Koch, N.Strodthoff, PRC 95 (2017) 054906
AB, V.Koch, V.Skokov, EPJC 77 (2017) 288
AB, V.Koch, D.Oliinychenko, J.Steinheimer, 1804.04463

Poisson distribution



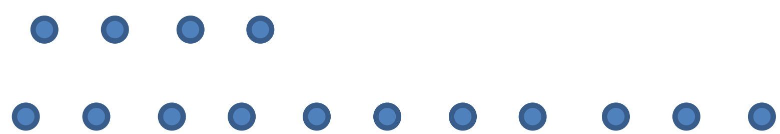
$$N = 10^{10}$$

$$p = 10^{-9}$$

$$\langle n \rangle = Np = 10$$

event # 1

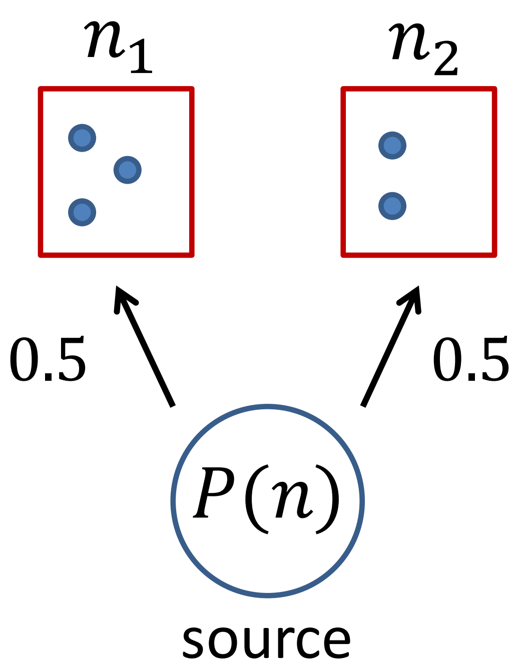
event # 2



$P(n) =$ Poisson if $N \to \infty,\ p \to 0,\ \ Np = \langle n \rangle$

Such source (multiplicity distribution) is characterized by **no correlations,** $\boldsymbol{C_n = 0}$, $n = 2{,}3, \dots$

In what sense "no correlations"



$$P(n_1, n_2) \stackrel{?}{=} P(n_1)P(n_2)$$

It is true for $P(n) =$ Poisson only

fixed N

finite N

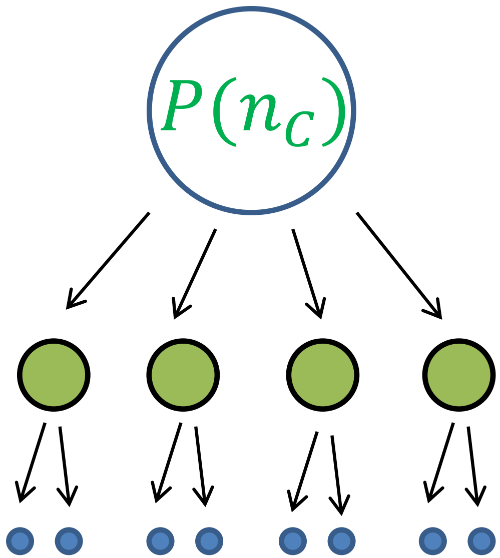
resonances

volume fluctuation

$$P(n_1, n_2) = P(n)\frac{n!}{n_1!\,n_2!}\left(\frac{1}{2}\right)^{n_1}\left(\frac{1}{2}\right)^{n_2}$$

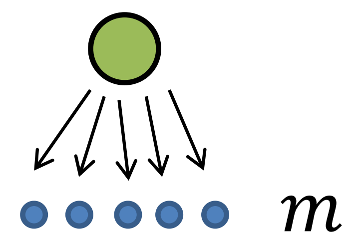
$$n = n_1 + n_2$$

Multi-particle correlations



Poisson

m particle cluster



$$\boldsymbol{C_2} \neq 0$$

$$\boldsymbol{C_k} = 0, k > 2$$

$$\boldsymbol{C_{2,3,\ldots,m}} \neq 0$$

$$\boldsymbol{C_k} = 0, k > m$$

factorial cumulants

$$\boldsymbol{C_k} = \frac{d^k}{dz^k} \ln\left(\sum_n P(n) z^n\right)\Big|_{z=1}$$

Interaction can change correlations

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2) dy_1 dy_2$$

factorial cumulant (integrated correlation function)

For Poisson $\boldsymbol{C_2} = 0$ but $\boldsymbol{C_2}(y_1, y_2)$ can have a non-trivial shape due to, e.g., interactions

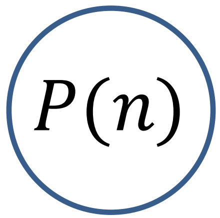
For example:

$$\boldsymbol{C_2}(\phi_1, \phi_2) \sim \cos(2\Delta\phi), \quad \Delta\phi = \phi_1 - \phi_2$$

- hydrodynamics
- initial stage (e.g., CGC)

Multi-particle correlations measure deviations from Poisson

Consider a source giving always **one particle**



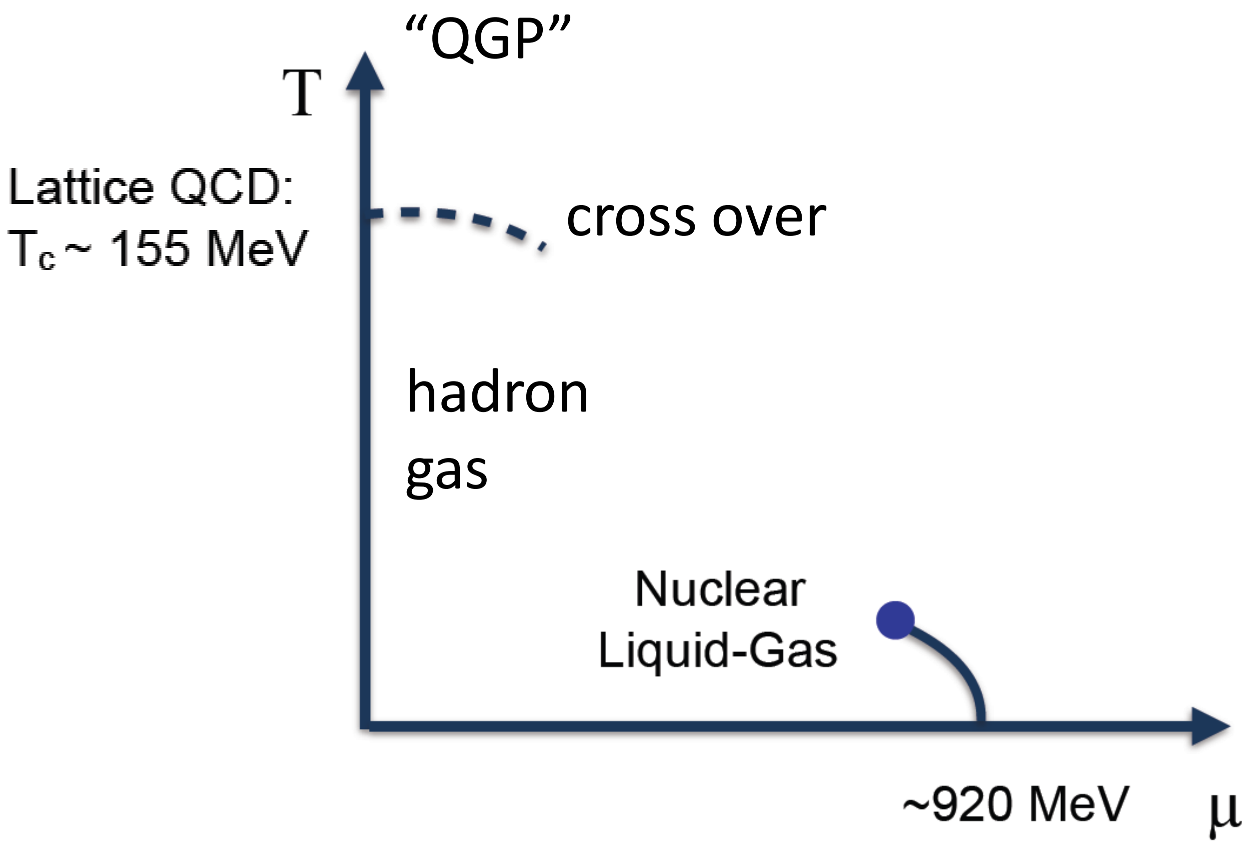
$$P(n) = 1 \quad \text{for} \quad n = 1$$
$$= 0 \quad \text{for} \quad n > 1$$

$$\boldsymbol{C_k} = \frac{d^k}{dz^k} \ln(z)\Big|_{z=1}$$

$$\boldsymbol{C_2} = -1,\ \ \boldsymbol{C_3} = 2,\ \ \boldsymbol{C_4} = -6, \dots,\ \ \boldsymbol{C_9} = 40320$$

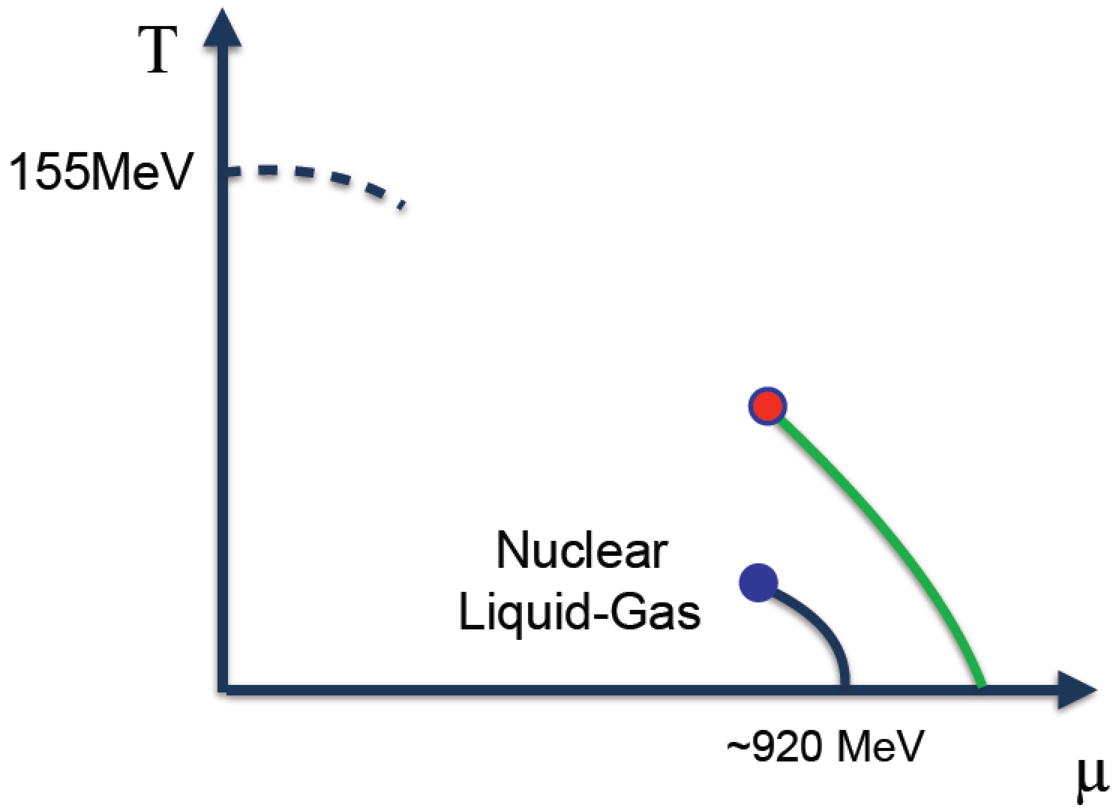
$$\boldsymbol{C_k} = (-1)^{k-1}(k-1)!$$

What we know about the QCD phase diagram



The rest is everybody's guess.

Usual expectation based on various effective models



On the experimental side all we can do is to measure various fluctuation observables and hope to see some nontrivial energy or/and system-size dependence

see, e.g.,
Stephanov, Rajagopal, Shuryak, PRL (1998)
Stephanov, PRL (2009)
Skokov, Friman, Redlich, PRC (2011)

There are some intriguing results:

STAR, HADES

Higher order cumulants

Proton v_1 (STAR)

HBT radii (STAR)

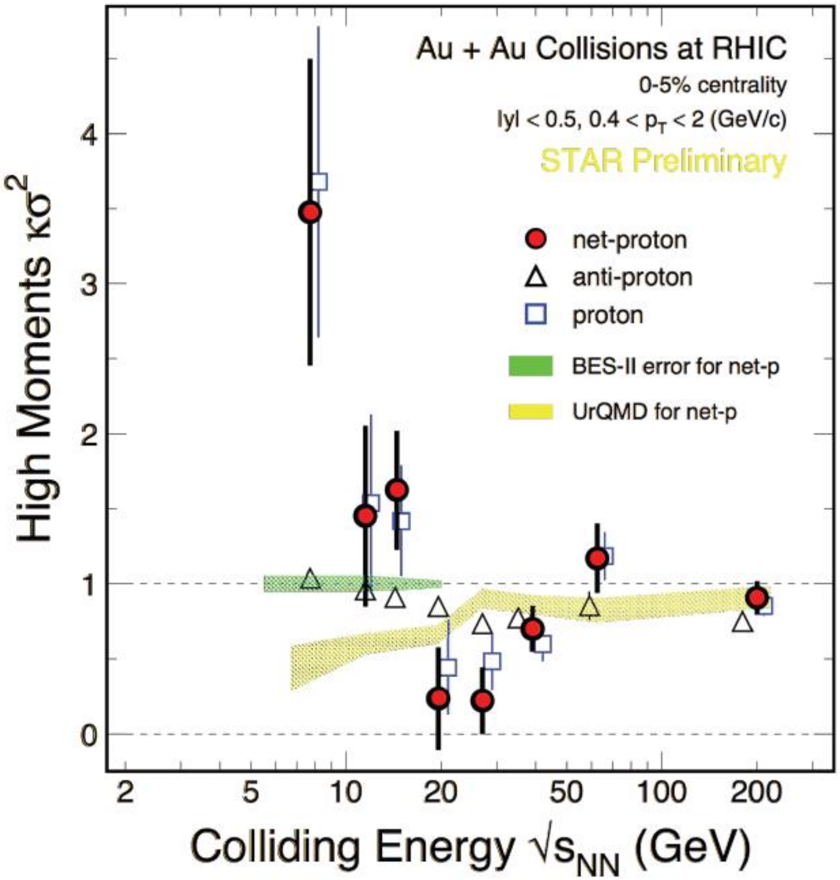
NA 49

Intermittency in the transverse momentum phase space

Strongly intensive variables

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



my notation

K_4/K_2

cumulants naturally appear in statistical physics

cumulants $$K_i = \frac{d^i}{dt^i} \ln\left(\sum_n P(n) e^{tn}\right)\Big|_{t=0}$$

Cumulants are not optimal

$$K_2 = \langle(\delta N)^2\rangle \qquad \delta N = N - \langle N \rangle$$

N – number of protons

we neglect anti-protons,
good at low energies

$$K_3 = \langle(\delta N)^3\rangle$$

$$K_4 = \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2$$

$$K_n = \langle N \rangle + physics[2, \ldots, n]$$

$physics$ = two-, three-, n-particle factorial cumulants

for Poisson distribution $K_n = \langle N \rangle$, $(physics = 0)$

We have

$$K_2 = \langle N \rangle + \boldsymbol{C_2}$$

$$K_3 = \langle N \rangle + 3\boldsymbol{C_2} + \boldsymbol{C_3}$$

$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulants mix integ. correlation functions of different orders

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2) dy_1 dy_2$$

factorial cumulant

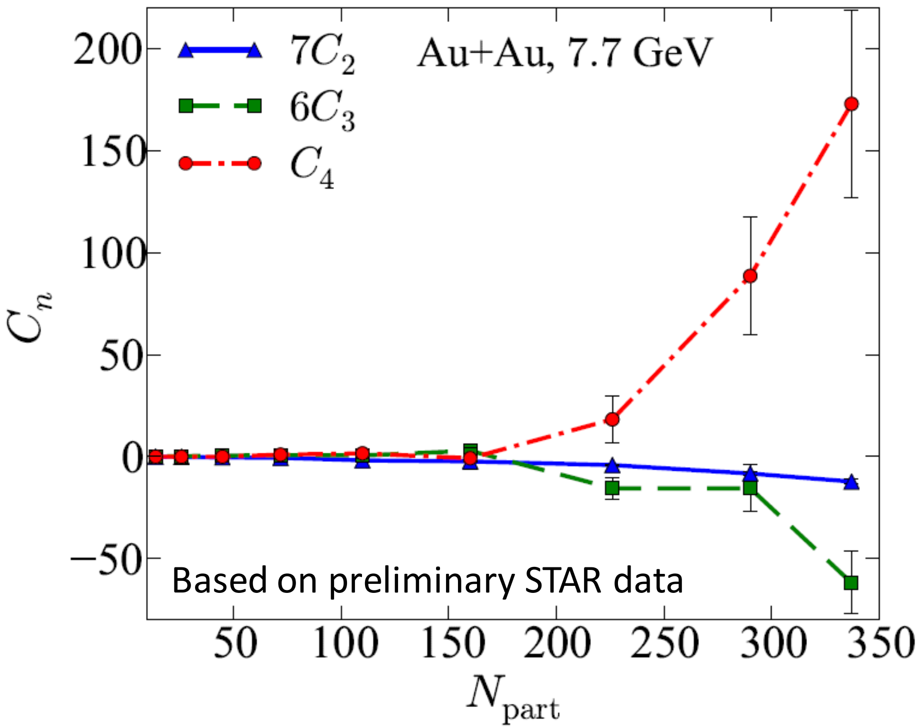
See, e.g.,

B. Ling, M. Stephanov, PRC 93 (2016) 034915

AB, V. Koch, N. Strodthoff , PRC 95 (2017) 054906

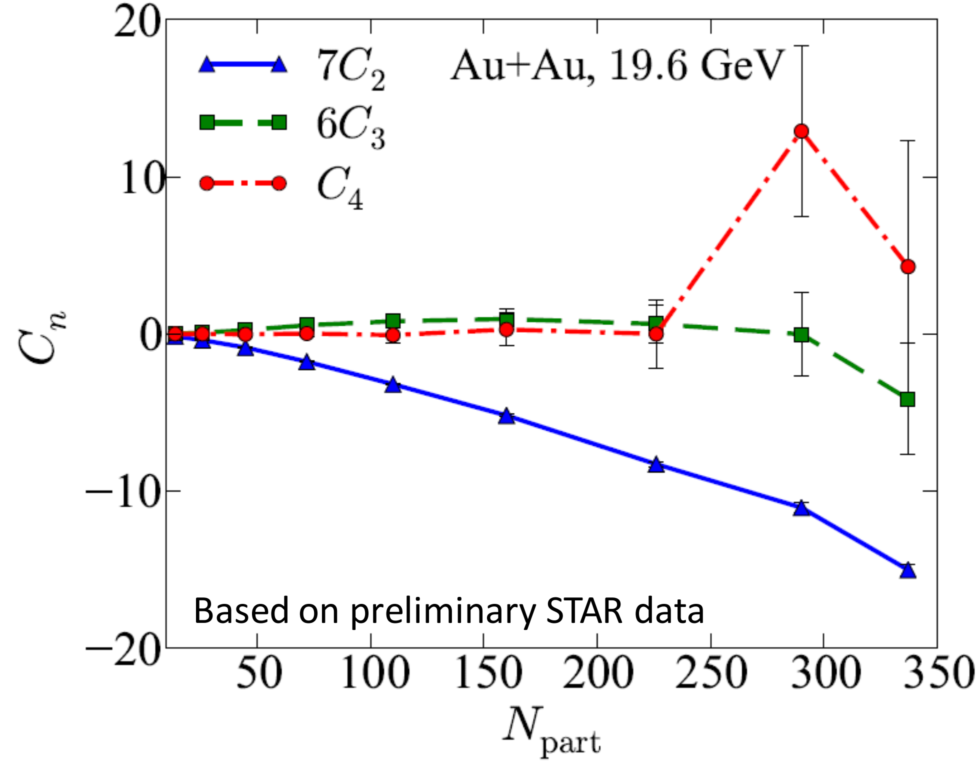
Using preliminary STAR data we obtain C_n

central signal at 7.7 GeV is driven by large 4-particle correlations



$C_4(7.7) \sim 170$

central signal at 19.6 GeV is driven by 2-particle correlations

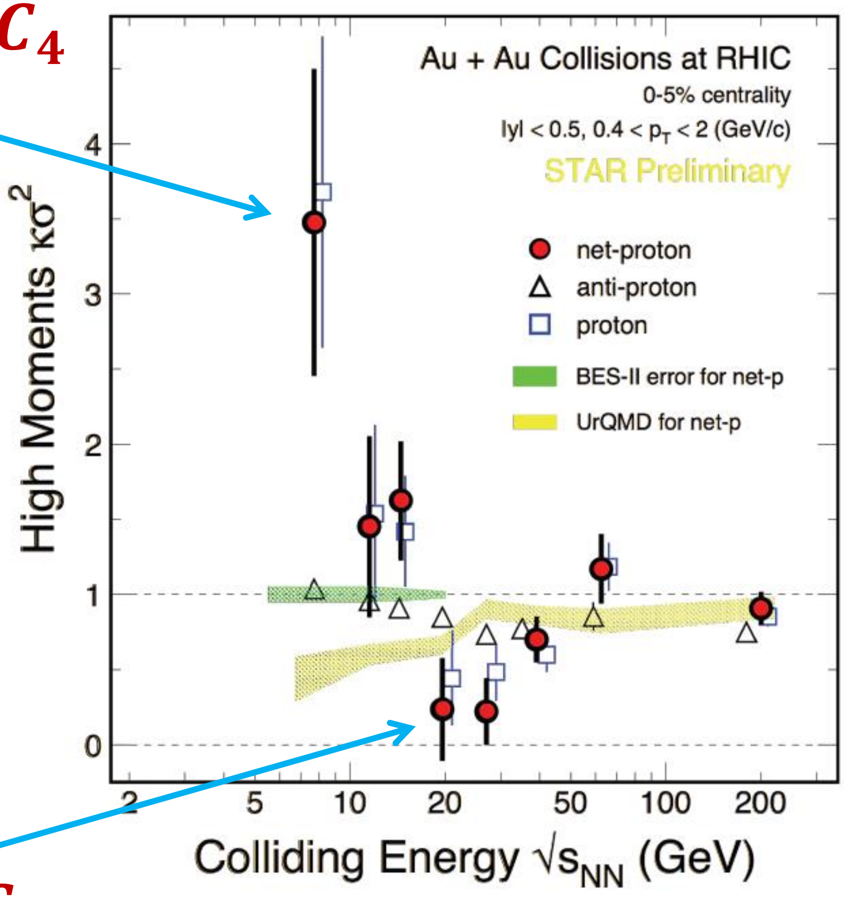


C_4 and $6C_3$ cancelation in most central coll.

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

STAR Preliminary
X.Luo, N.Xu, 1701.02105

here we see C_4



and here C_2

e.g., baryon conservation

Let's put the STAR numbers in perspective.

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288

Suppose that we have **clusters** (distributed according to Poisson) decaying always to 4 protons

$$\boldsymbol{C_k} = \langle N_{\rm cl} \rangle \cdot 4!/(4-k)!$$

↑ mean number of clusters

$$\boldsymbol{C_4} = \langle N_{\rm cl} \rangle \cdot 24$$

for 5-proton clusters:

$$\boldsymbol{C_k} = \langle N_{\rm cl} \rangle \cdot 5!/(5-k)!$$

$$\boldsymbol{C_4} = \langle N_{\rm cl} \rangle \cdot 120$$

and $\langle N_{\rm cl} \rangle \sim 1$

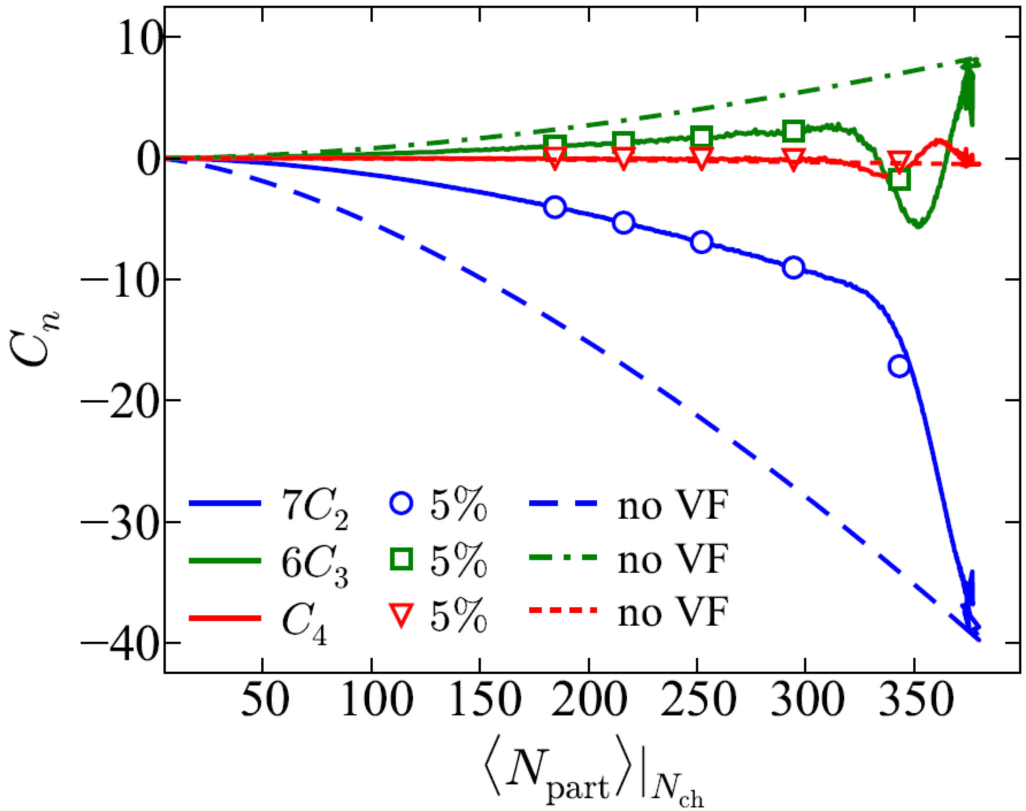
To obtain $\boldsymbol{C_4} \approx 170$ we need $\langle N_{\rm cl} \rangle \sim 7,$ it means 28 protons.
STAR sees on average 40 protons in central collisions.

In this model $C_2 > 0$ and $C_3 > 0$ contrary to the STAR data

Baryon conservation + volume fluctuation (minimal model)

- independent baryon stopping (baryon conservation by construction)
- N_{part} fluctuations (volume fluctuation - VF)

Au+Au, $\sqrt{s} = 7.7$ GeV



STAR

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$

we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288

See also, P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114

Can we describe the STAR data at 7.7 GeV with *ordinary* multiplicity distributions?

Model with **two event classes**

$$P(N) = (1-\alpha) P_{(a)}(N) + \alpha P_{(b)}(N)$$

Poisson, binomial, etc..

Poisson, binomial, etc.

That is, with probability $1-\alpha$ we have $P_{(a)}(N)$ and with probability α we have $P_{(b)}(N)$

AB, V. Koch, D. Oliinychenko, J. Steinheimer, 1804.04463

$$C_2 = \alpha(1-\alpha)\overline{N}^2 \approx \alpha\overline{N}^2,$$
$$C_3 = -\alpha(1-\alpha)(1-2\alpha)\overline{N}^3 \approx -\alpha\overline{N}^3,$$
$$C_4 = \alpha(1-\alpha)(1-6\alpha+6\alpha^2)\overline{N}^4 \approx \alpha\overline{N}^4,$$

$$\overline{N} = \langle N_{(a)}\rangle - \langle N_{(b)}\rangle$$

$$\frac{C_{n+1}}{C_n} \approx -\overline{N}$$

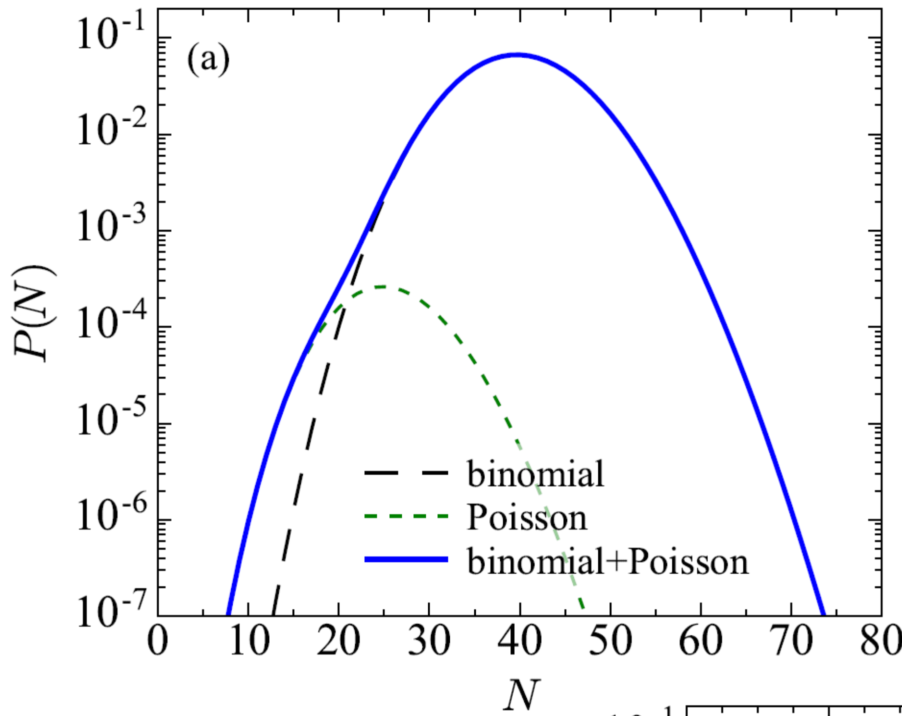
$$\frac{C_6}{C_5} \approx \frac{C_5}{C_4} \approx \frac{C_4}{C_3} = -17 \pm 6$$

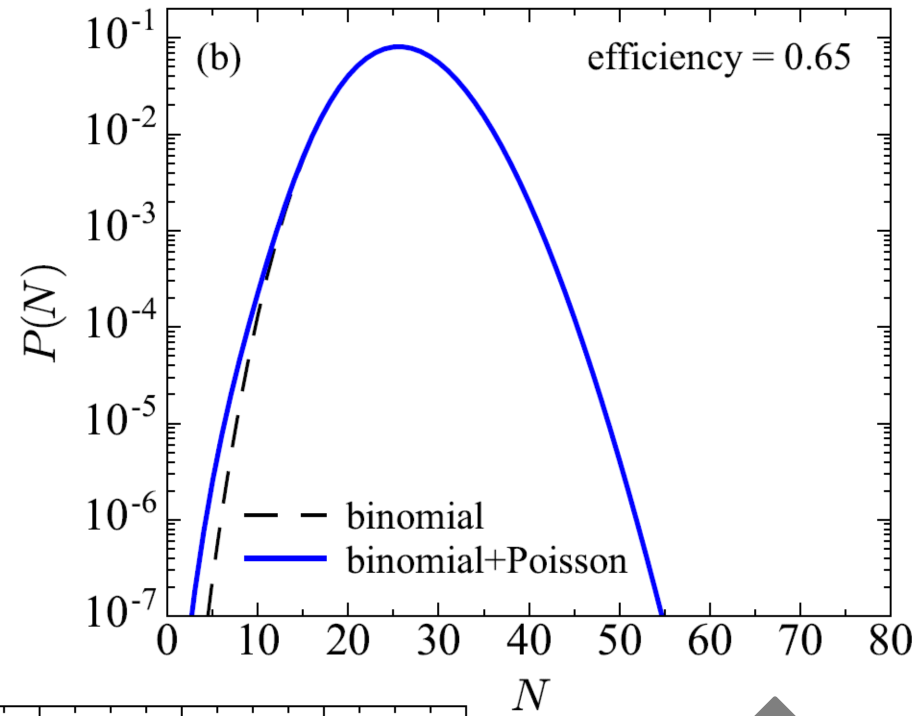
parameter-free prediction

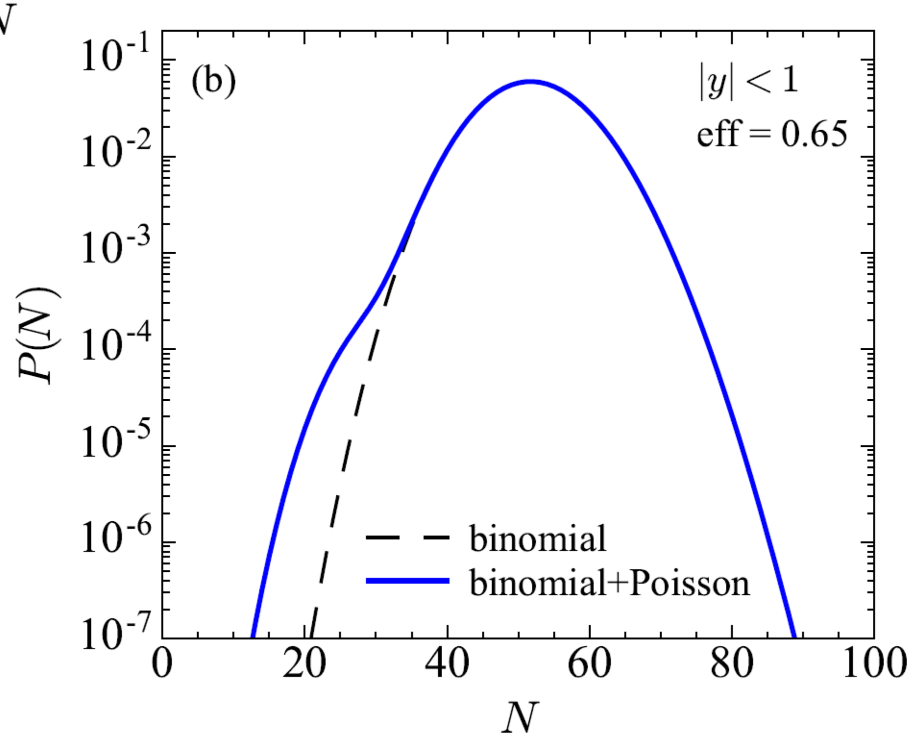
We can describe the data with $\alpha \approx 0.0033$

$$\langle N_{(a)}\rangle \approx 40,\ \langle N_{(b)}\rangle \approx 25$$

Now we can plot $P(N)$


(a)
$P(N)$
N
binomial
Poisson
binomial+Poisson

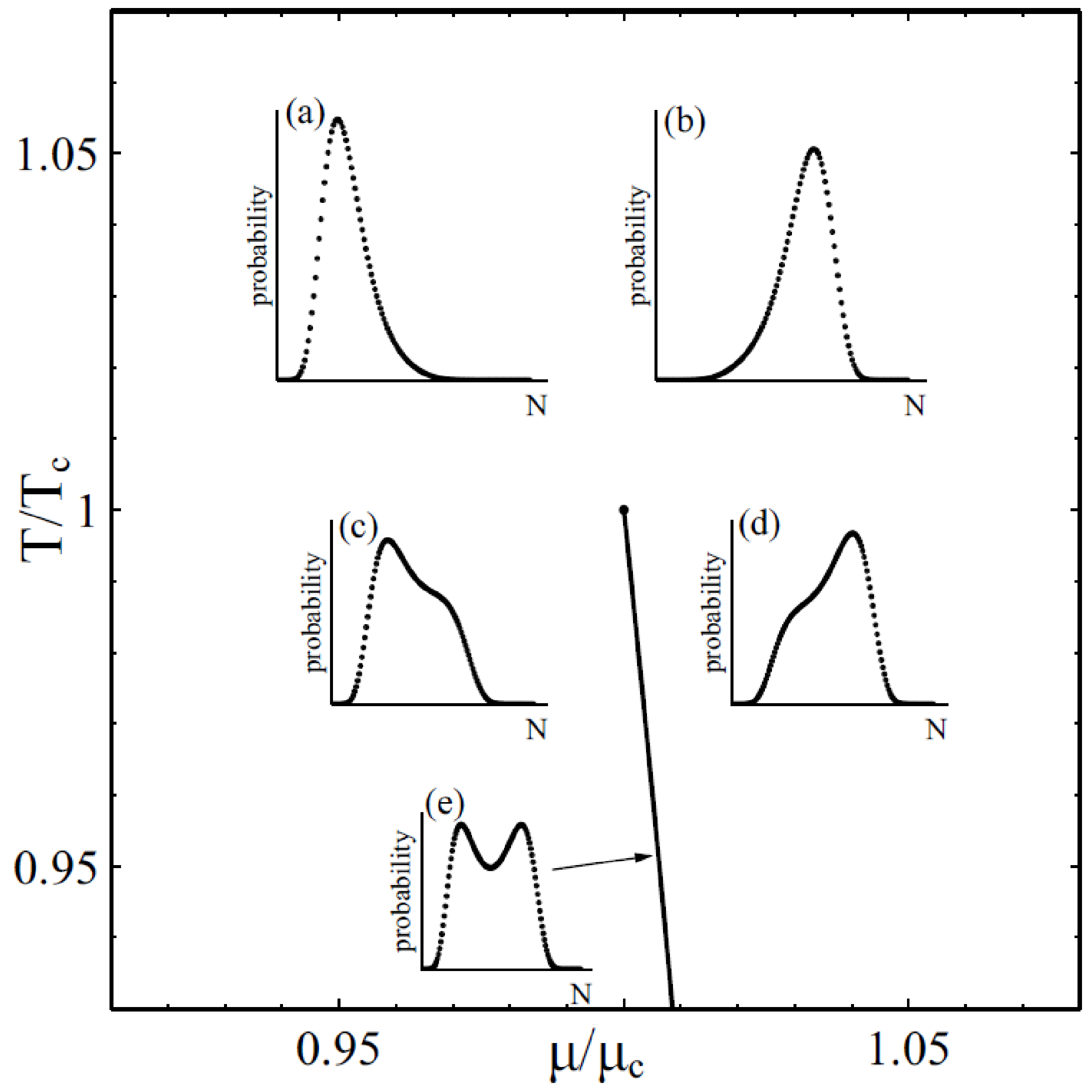

(b)
efficiency = 0.65
$P(N)$
N
binomial
binomial+Poisson

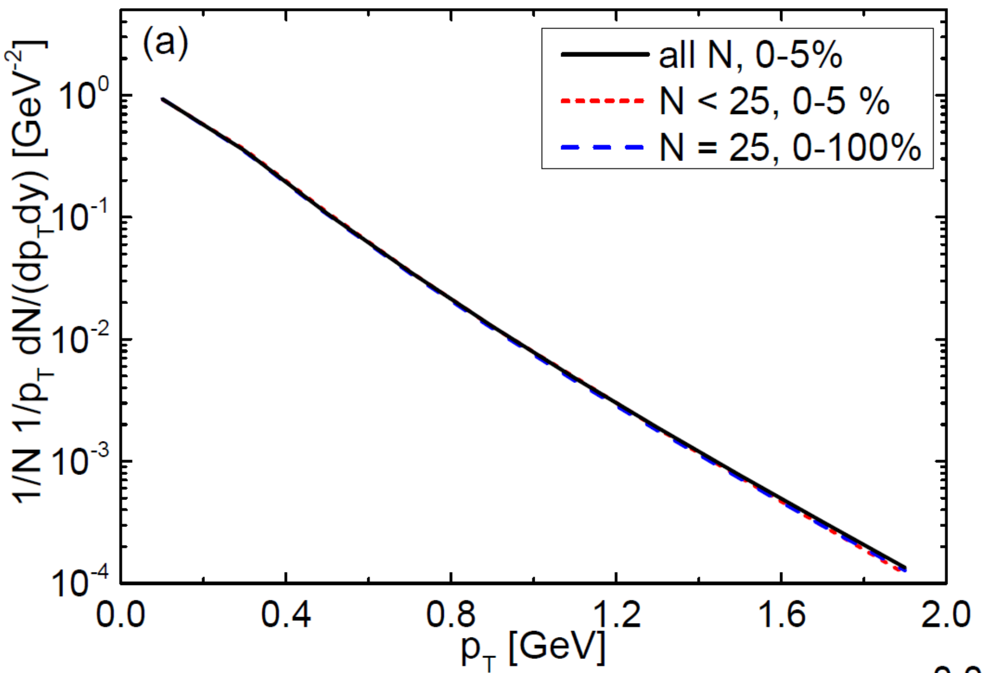

(b)
$|y| < 1$
eff = 0.65
$P(N)$
N
binomial
binomial+Poisson

$|y| < 0.5$

$|y| < 1$

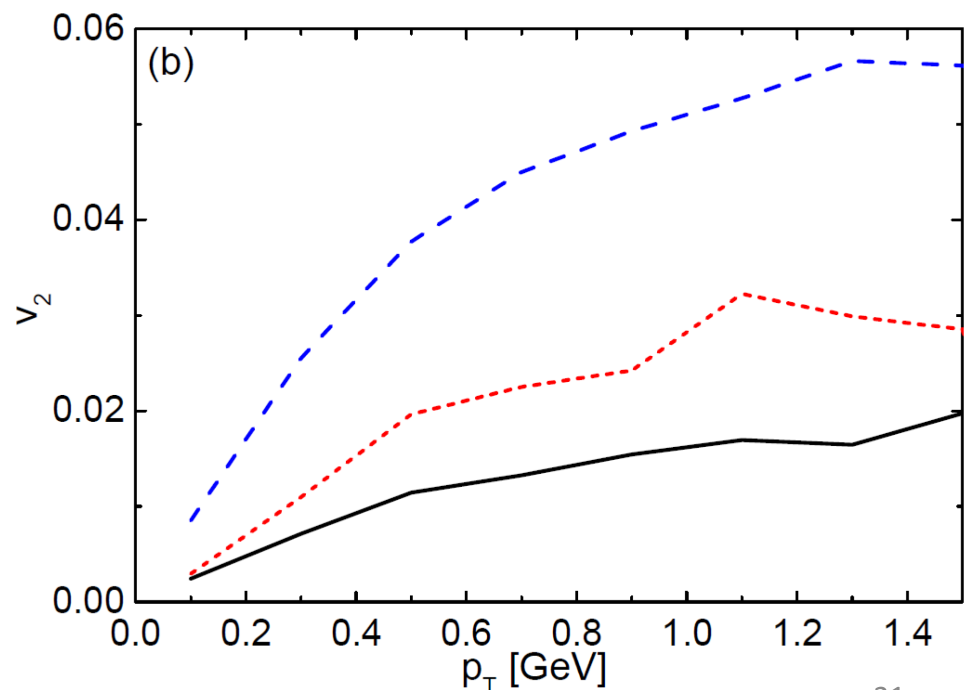
A finite volume van der Walls model





Cuts in the number of protons for central collisions

UrQMD



AB, V. Koch, D. Oliinychenko, J. Steinheimer, 1804.04463

Conclusions

Four-proton factorial cumulant (int. correlation function) at 7.7 GeV is surprisingly large.

Proton clusters?

Two event classes? Parameter-free predictions.

Volume fluctuation and baryon conservation seem to be irrelevant for C_3 and C_4. C_2 is likely dominated by background.

Backup

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + \boldsymbol{c_2}(y_1, y_2)]$$

reduced correlation function

e.g., does not depend on binomial efficiency

integrated reduced correlation function "coupling"

$$\boldsymbol{c_2} = \frac{\int \rho(y_1)\rho(y_2)\boldsymbol{c_2}(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2} = \frac{\boldsymbol{C_2}}{\langle N\rangle^2}$$

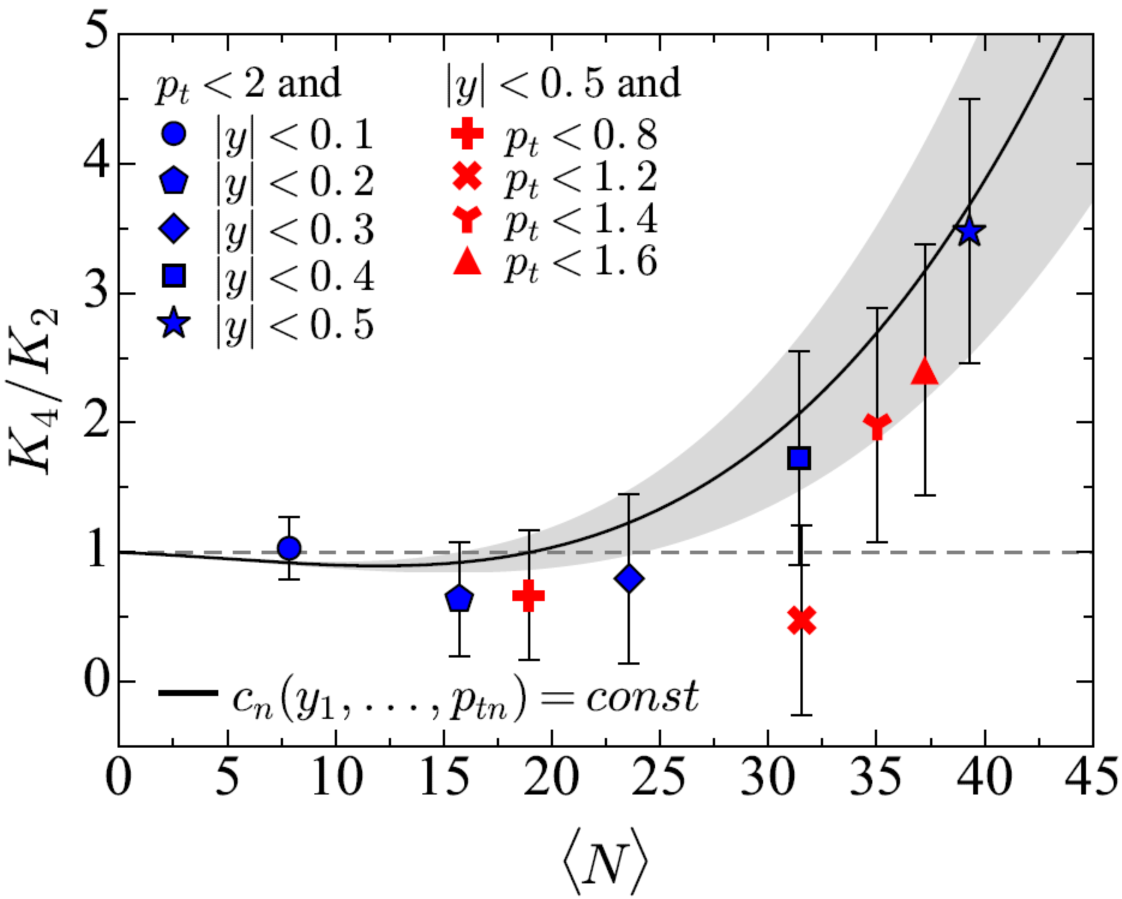
and the second order cumulant

$$K_2 = \langle N\rangle + \underbrace{\langle N\rangle^2 \boldsymbol{c_2}}_{\boldsymbol{C_2}}$$

Constant correlation

$$c_2 = \frac{\int \rho(y_1)\rho(y_2) c_2(y_1, y_2) dy_1 dy_2}{\int \rho(y_1)\rho(y_2) dy_1 dy_2}$$

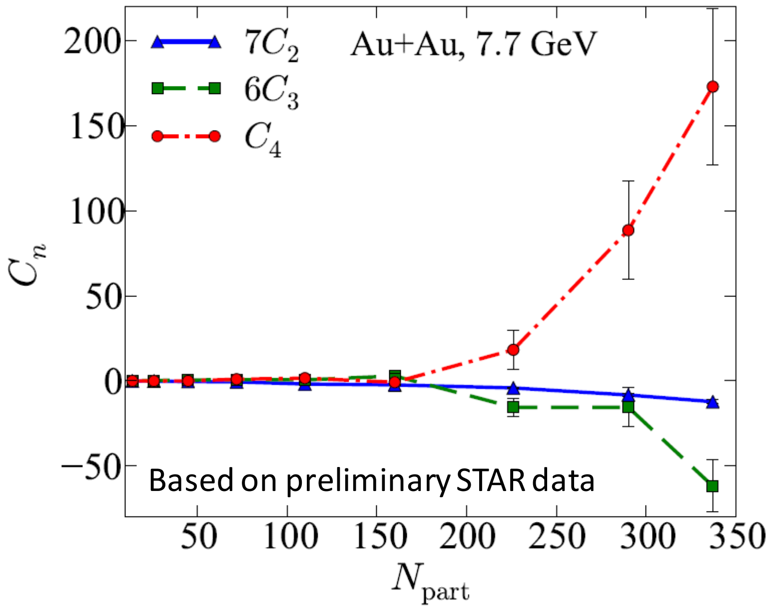
$$c_n(y_1, p_{t1}, \ldots, y_n, p_{tn}) = c_n^0 = const \quad \rightarrow \quad c_n = c_n^0$$

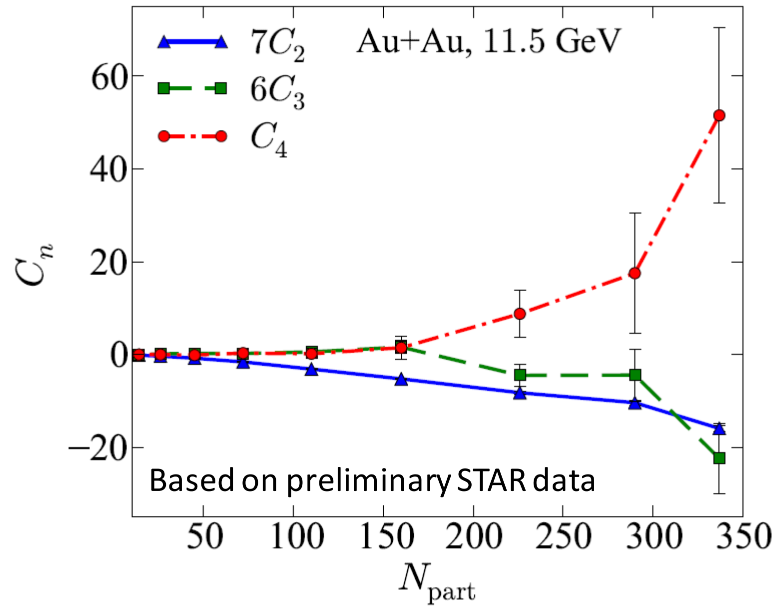


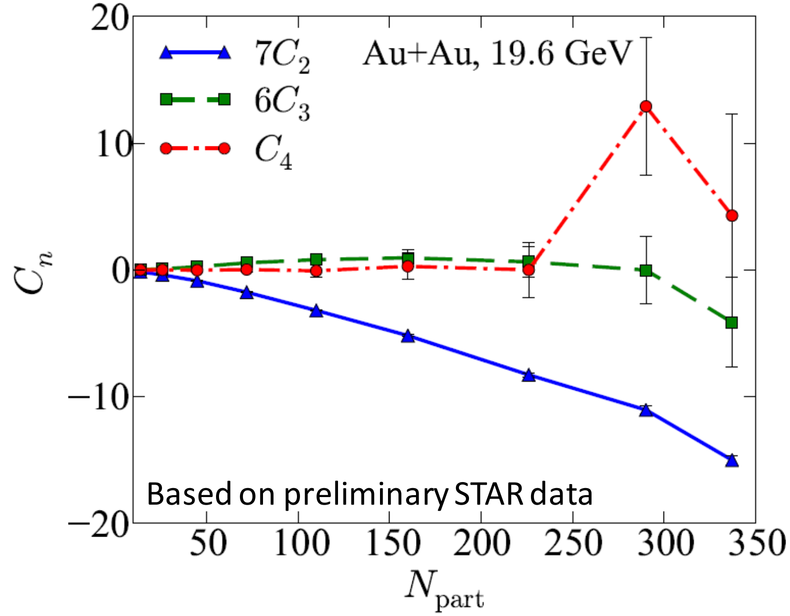
$\sqrt{s} = 7.7$ GeV

AB, V. Koch, PRC 96, 054905 (2017)

Comparison of 7.7, 11.5 and 19.6 GeV

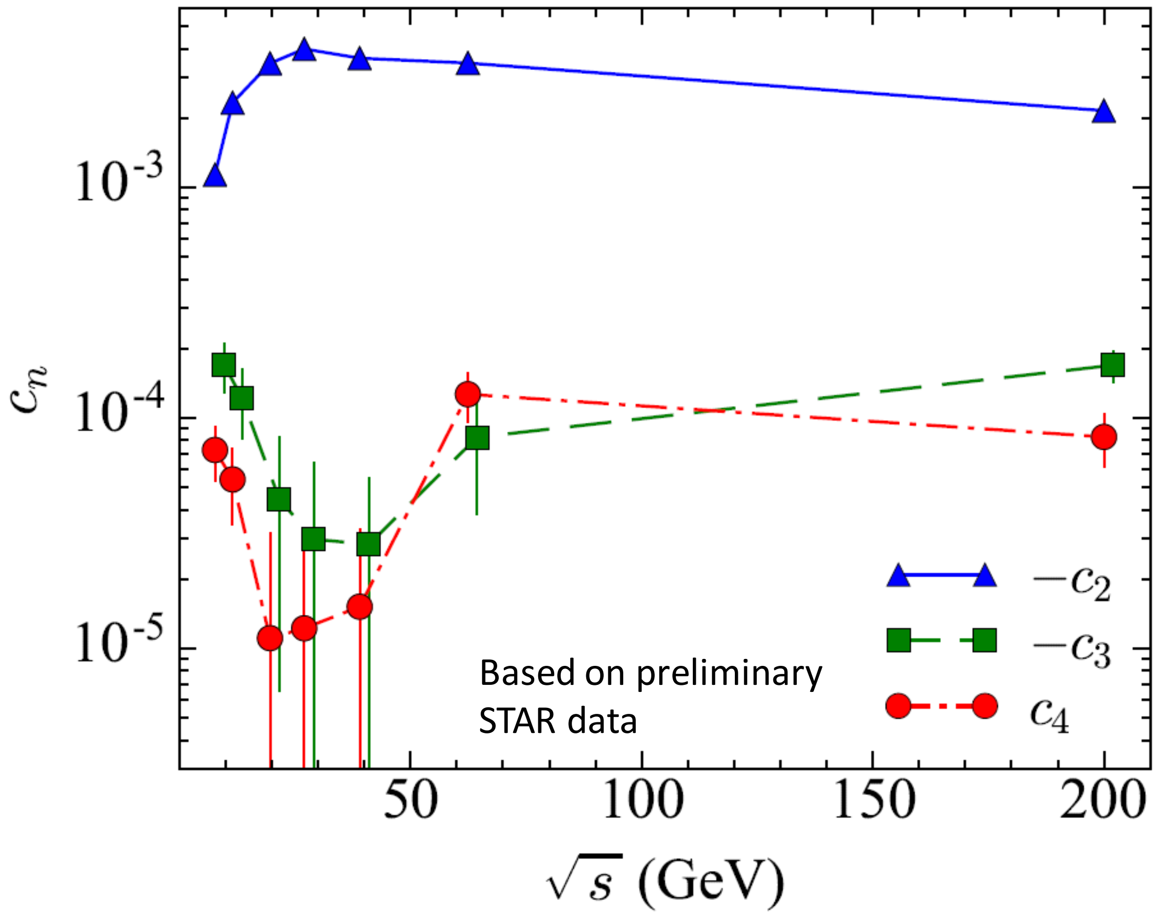






Couplings' point of view and **global baryon conservation**

AB, VK, preliminary



Global baryon conservation

$$-c_2 = 1/B \approx 2 \cdot 10^{-3}$$

$$-c_3 = -2/B^2 \approx -10^{-5}$$

$$c_4 = -6/B^3 \approx \underline{-10^{-7}}$$

$$B \approx 400$$

Critical point: everybody's guess

M. Stephanov, hep-lat/0701002

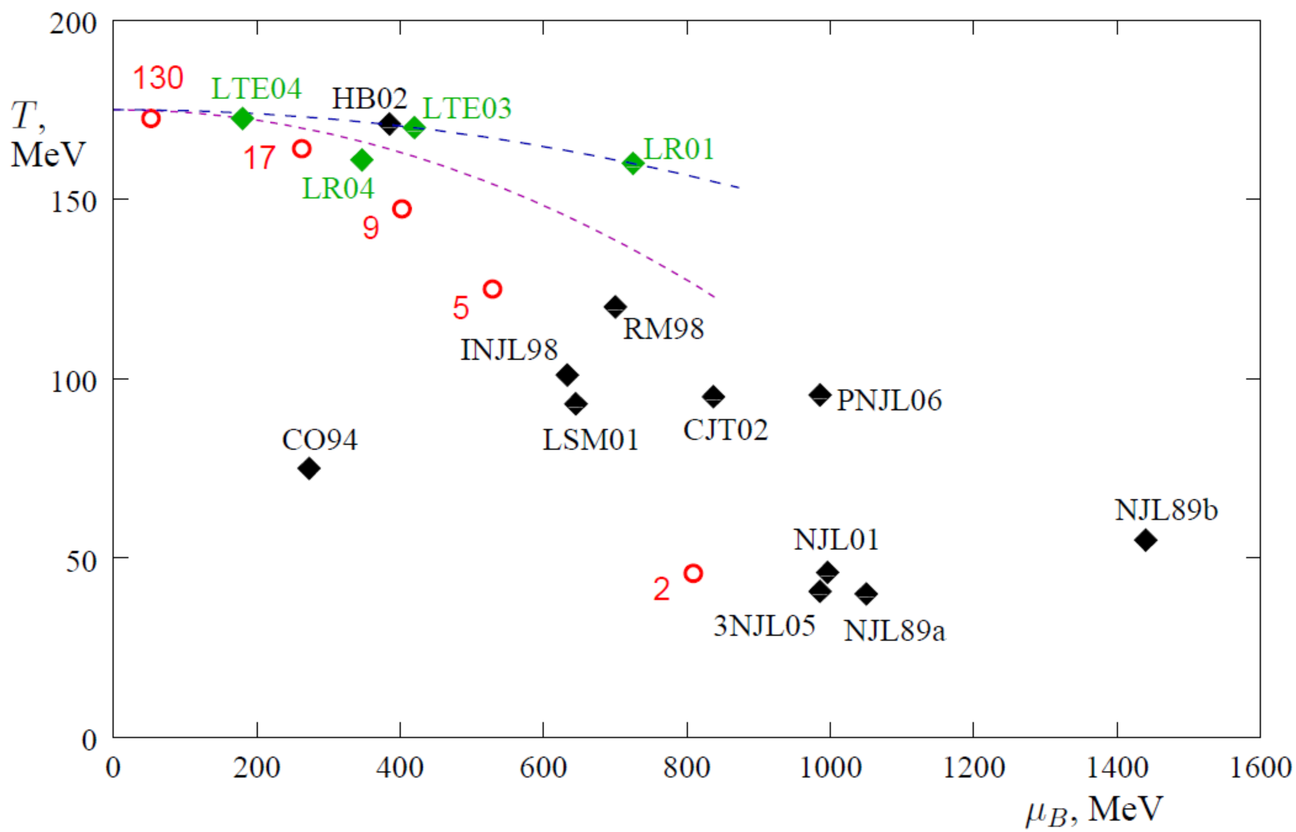


Figure 4: Comparison of predictions for the location of the QCD critical point on the phase diagram. Black points are model predictions: NJLa89, NJLb89 – [12], CO94 – [13, 14], INJL98 – [15], RM98 – [16], LSM01, NJL01 – [17], HB02 – [18], CJT02 – [19], 3NJL05 – [20], PNJL06 – [21]. Green points are lattice predictions: LR01, LR04 – [22], LTE03 – [23], LTE04 – [24]. The two dashed lines are parabolas with slopes corresponding to lattice predictions of the slope $dT/d\mu_B^2$ of the transition line at $\mu_B = 0$ [23, 25]. The red circles are locations of the freezeout points for heavy ion collisions at corresponding center of mass energies per nucleon (indicated by labels in GeV) – Section 5.