QCD phase-diagram and multiparticle proton correlations

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correlation, interaction
cumulants and STAR data
clusters
two event classes

AB, V.Koch, N.Strodthoff, PRC 95 (2017) 054906
AB, V.Koch, V.Skokov, EPJC 77 (2017) 288
AB, V.Koch, D.Oliinychenko, J.Steinheimer, 1804.04463
Poisson distribution

\[ N = 10^{10} \]
\[ p = 10^{-9} \]
\[ \langle n \rangle = Np = 10 \]

\( P(n) = \text{Poisson if } N \to \infty, \ p \to 0, \ Np = \langle n \rangle \)

Such source (multiplicity distribution) is characterized by no correlations, \( C_n = 0, \ n = 2, 3, \ldots \)
In what sense “no correlations”

\[ P(n_1, n_2) = P(n_1)P(n_2) \]

It is true for \( P(n) = \text{Poisson only} \)

fixed N
finite N
resonances
volume fluctuation

\[ P(n_1, n_2) = P(n) \frac{n!}{n_1!n_2!} \left( \frac{1}{2} \right)^{n_1} \left( \frac{1}{2} \right)^{n_2} \]

\[ n = n_1 + n_2 \]
Multi-particle correlations

\[ C_2 \neq 0 \]
\[ C_k = 0, \; k > 2 \]

factorial cumulants
\[ C_k = \frac{d^k}{dz^k} \ln \left( \sum_n P(n)z^n \right) \bigg|_{z=1} \]
Interaction can change correlations

\[ \rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + C_2(y_1, y_2) \]

\[ C_2 = \int C_2(y_1, y_2)dy_1dy_2 \]

For Poisson \( C_2 = 0 \) but \( C_2(y_1, y_2) \) can have a non-trivial shape due to, e.g., interactions

For example:

\[ C_2(\phi_1, \phi_2) \sim \cos(2\Delta\phi), \quad \Delta\phi = \phi_1 - \phi_2 \]

- hydrodynamics
- initial stage (e.g., CGC)
Multi-particle correlations measure deviations from Poisson

Consider a source giving always one particle

\[ P(n) = 1 \text{ for } n = 1 \]
\[ = 0 \text{ for } n > 1 \]

\[ C_k = \left. \frac{d^k}{dz^k} \ln(z) \right|_{z=1} \]

\[ C_2 = -1, \quad C_3 = 2, \quad C_4 = -6, \ldots, \quad C_9 = 40320 \]

\[ C_k = (-1)^{k-1}(k - 1)! \]
What we know about the QCD phase diagram

Lattice QCD: $T_c \approx 155$ MeV

Cross over

hadron gas

“QGP”

Nuclear Liquid-Gas

$\approx 920$ MeV

The rest is everybody’s guess.
Usual expectation based on various effective models
On the experimental side all we can do is to measure various fluctuation observables and hope to see some nontrivial energy or/and system-size dependence

see, e.g.,
Stephanov, Rajagopal, Shuryak, PRL (1998)
Stephanov, PRL (2009)
Skokov, Friman, Redlich, PRC (2011)

There are some intriguing results:

**STAR, HADES**
- Higher order cumulants
- Proton $v_1$ (STAR)
- HBT radii (STAR)

**NA 49**
- Intermittency in the transverse momentum phase space
- Strongly intensive variables
Preliminary STAR data

\[ K_i = \frac{d^i}{dt^i} \ln \left( \sum_n P(n) e^{tn} \right) \bigg|_{t=0} \]

cumulants naturally appear in statistical physics
Cumulants are not optimal

\[ K_2 = \langle (\delta N)^2 \rangle \quad \delta N = N - \langle N \rangle \quad N - \text{number of protons} \]
we neglect anti-protons, good at low energies

\[ K_3 = \langle (\delta N)^3 \rangle \]

\[ K_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2 \]

\[ K_n = \langle N \rangle + \text{physics}[2, \ldots, n] \]

\(\text{physics} = \text{two-, three-, } n\text{-particle factorial cumulants}\)

for Poisson distribution \(K_n = \langle N \rangle, \ (\text{physics} = 0)\)
We have

\[ K_2 = \langle N \rangle + C_2 \]
\[ K_3 = \langle N \rangle + 3C_2 + C_3 \]
\[ K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4 \]

\[ \rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + C_2(y_1, y_2) \]

\[ C_2 = \int C_2(y_1, y_2) dy_1 dy_2 \]

See, e.g.,
B. Ling, M. Stephanov, PRC 93 (2016) 034915
AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906
Using preliminary STAR data we obtain $C_n$

Central signal at 7.7 GeV is driven by large 4-particle correlations

$C_n$ and $6C_3$ cancelation in most central coll.

$C_4(7.7) \sim 170$

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906
here we see $C_4$

and here $C_2$

e.g., baryon conservation
Let’s put the STAR numbers in perspective.

Suppose that we have clusters (distributed according to Poisson) decaying always to 4 protons

\[
C_k = \langle N_{cl} \rangle \cdot 4!/(4 - k)!
\]

\[
C_4 = \langle N_{cl} \rangle \cdot 24
\]

For 5-proton clusters:

\[
C_k = \langle N_{cl} \rangle \cdot 5!/(5 - k)!
\]

\[
C_4 = \langle N_{cl} \rangle \cdot 120
\]

And \( \langle N_{cl} \rangle \sim 1 \)

To obtain \( C_4 \approx 170 \) we need \( \langle N_{cl} \rangle \sim 7 \), it means 28 protons. STAR sees on average 40 protons in central collisions.

In this model \( C_2 > 0 \) and \( C_3 > 0 \) contrary to the STAR data.
Baryon conservation + volume fluctuation (minimal model)
- independent baryon stopping (baryon conservation by construction)
- $N_{\text{part}}$ fluctuations (volume fluctuation - VF)

Au+Au, $\sqrt{s} = 7.7$ GeV

STAR

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$

we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288
See also, P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114
Can we describe the STAR data at 7.7 GeV with *ordinary* multiplicity distributions?

Model with **two event classes**

\[ P(N) = (1 - \alpha)P_a(N) + \alpha P_b(N) \]

That is, with probability \(1 - \alpha\) we have \(P_a(N)\) and with probability \(\alpha\) we have \(P_b(N)\)

AB, V. Koch, D. Oliinychenko, J. Steinheimer, 1804.04463
We can describe the data with \( \alpha \approx 0.0033 \).

\[
C_2 = \alpha (1 - \alpha) \bar{N}^2 \approx \alpha \bar{N}^2,
C_3 = -\alpha (1 - \alpha)(1 - 2\alpha) \bar{N}^3 \approx -\alpha \bar{N}^3,
C_4 = \alpha (1 - \alpha)(1 - 6\alpha + 6\alpha^2) \bar{N}^4 \approx \alpha \bar{N}^4,
\]

\[
\bar{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle
\]

\[
\frac{C_{n+1}}{C_n} \approx -\bar{N}
\]

\[
\frac{C_6}{C_5} \approx \frac{C_5}{C_4} \approx \frac{C_4}{C_3} = -17 \pm 6
\]

parameter-free prediction

We can describe the data with \( \alpha \approx 0.0033 \)

\[
\langle N_{(a)} \rangle \approx 40, \langle N_{(b)} \rangle \approx 25
\]

Now we can plot \( P(N) \)
\[ |y| < 1 \]  
\[ \text{eff} = 0.65 \]  
\[ |y| < 0.5 \]  
\[ |y| < 1 \]
A finite volume van der Walls model
Cuts in the number of protons for central collisions

UrQMD
Conclusions

Four-proton factorial cumulant (int. correlation function) at 7.7 GeV is surprisingly large.

Proton clusters?

Two event classes? Parameter-free predictions.

Volume fluctuation and baryon conservation seem to be irrelevant for $C_3$ and $C_4$. $C_2$ is likely dominated by background.
Backup
\[ \rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + c_2(y_1, y_2) \]

reduced correlation function

\[ \rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + c_2(y_1, y_2)] \]

e.g., does not depend on binomial efficiency

integrated reduced correlation function “coupling”

\[ c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2) dy_1 dy_2}{\int \rho(y_1)\rho(y_2) dy_1 dy_2} = \frac{C_2}{\langle N \rangle^2} \]

and the second order cumulant

\[ K_2 = \langle N \rangle + \langle N \rangle^2 c_2 \]

\[ c_2 \]
Constant correlation

\[ c_n(y_1, p_{t1}, \ldots, y_n, p_{tn}) = c_n^0 = \text{const} \quad \rightarrow \quad c_n = c_n^0 \]

\[ c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2} \]

\[ s = 7.7 \text{ GeV} \]

AB, V. Koch, PRC 96, 054905 (2017)
Comparison of 7.7, 11.5 and 19.6 GeV

Based on preliminary STAR data
Couplings’ point of view and **global baryon conservation**

Based on preliminary STAR data

Global baryon conservation

\[-c_2 = 1/B \approx 2 \cdot 10^{-3}\]

\[-c_3 = -2/B^2 \approx -10^{-5}\]

\[c_4 = -6/B^3 \approx -10^{-7}\]

\[B \approx 400\]
Critical point: everybody’s guess

**Figure 4:** Comparison of predictions for the location of the QCD critical point on the phase diagram. **Black points are model predictions:** NJLa89, NJlb89 – [12], CO94 – [13, 14], INJL98 – [15], RM98 – [16], LSM01, NJL01 – [17], HB02 – [18], CJT02 – [19], 3NJL05 – [20], PNJL06 – [21]. **Green points are lattice predictions:** LR01, LR04 – [22], LTE03 – [23], LTE04 – [24]. The two dashed lines are parabolas with slopes corresponding to lattice predictions of the slope $dT/d\mu_B^2$ of the transition line at $\mu_B = 0$ [23, 25]. The red circles are locations of the freezeout points for heavy ion collisions at corresponding center of mass energies per nucleon (indicated by labels in GeV) – Section 5.