Accelerating hydrodynamic description of pseudorapidity distributions and the initial energy density at RHIC and LHC

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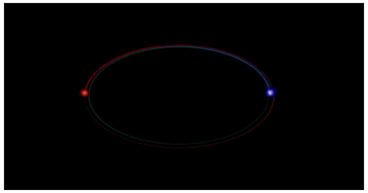
Many thanks to:
Xin-Nian Wang, T.S. Biró, M. I. Nagy.

Motivation



- *What is the initial temperature and thermal evolution of the produced matter?*

- *What is the viscosity of the produced matter? ...* http://www.bnl.gov/physics/rhiciiscience/

Outline

- Accelerating ideal solution of hydrodynamics and results (perfect flow for fitting p+p, Cu+Cu, Au+Au, Pb+Pb data).

  Csörgő, Nagy, Csanád (CNC) arXiv: 0605070, 0710.0327, 0805.1562, Csanád, et. arXiv:1609.07176. Z. F. Jiang, et. arXiv: 1711.10740.

- New perturbative solutions of viscous hydrodynamics.
- Summary and outlook.

Motivation

The next phase … will focus on detailed investigations of the QGP, "both to quantify its properties and to understand precisely how they emerge from the fundamental properties of QCD"

-- The frontiers of nuclear science, a long range plan

- *What is the initial temperature and thermal evolution of the produced matter?*

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CNC solutions of relativistic hydrodynamics

Hydrodynamics can be a universal tool to study the QGP. In Rindler coordinate, 5 different sets of the parameters *λ, κ, d* and K for 5 possible cases as follows (λ is the accelerate parameter.):

Csörgő, Nagy, Csanád(CNC)
arXiv: 0605070,
0710.0327, 0805.1562,....

Case	λ	d	κ	ϕ
a.)	2	**R**	d	0
b.)	1/2	**R**	1	(κ+1)/κ
c.)	3/2	**R**	(4d-1)/3	(κ+1)/κ
d.)	1	**R**	**R**	0
e.)	**R**	**R**	1	0

⇐ accelerating, *d* dimension (case a.)

⇐ *d* dimensional (T. S. Biró) (cases b.) and c.))

⇐ **Hwa-Bjorken, Buda-Lund type** (case d.))

⇐ Special EoS, but general velocity (case e.))

In all ideal cases, the velocity field and the pressure is expressed as :

$$v = \tanh \lambda\eta_s, \quad p = p_0\left(\frac{\tau_0}{\tau}\right)^{\lambda d \frac{(\kappa+1)}{\kappa}}\left(\cosh\frac{\eta_s}{2}\right)^{-(d-1)\phi}$$

The initial energy density estimation

Rapidity distribution:

$$\frac{dN}{dy} \approx \frac{dN}{dy}\bigg|_{y=0} \cosh^{\pm\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) e^{-\frac{m}{T_f}\left[\cosh^{\alpha}\left(\frac{y}{\alpha}\right)-1\right]}$$

⇔

Pseudorapidity distribution:

$$\frac{dN}{d\eta} \approx \frac{\overline{p}}{\overline{\overline{E}}} \frac{dn}{dy} = \frac{\overline{p_T} \cosh\eta}{\sqrt{m^2 + \overline{\overline{p_T^2}}}} \frac{dN}{dy}$$

$$\alpha = \frac{2\lambda - 1}{\lambda - 1}.$$

The initial energy density estimation

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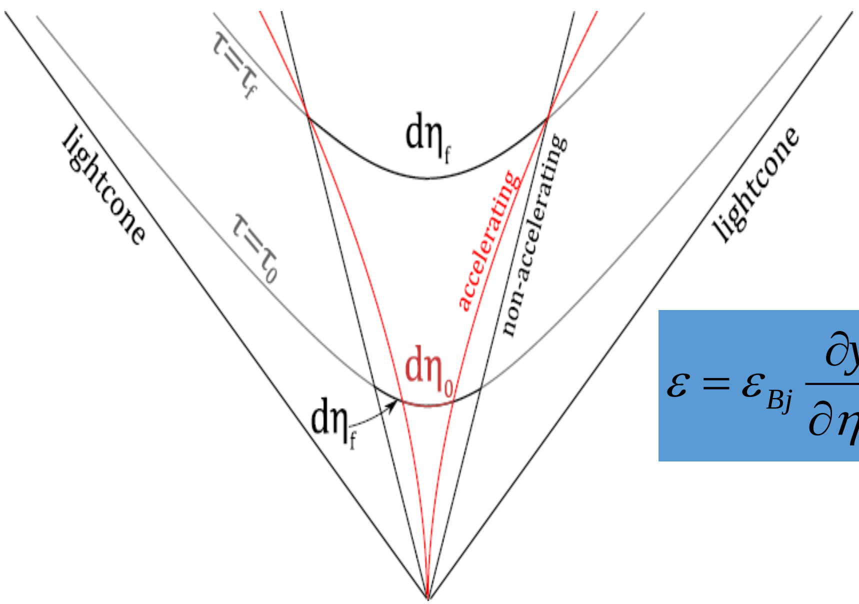


Flow element's motion path

$$\varepsilon_{Bj} = \frac{1}{R^2\pi\tau_0}\frac{dE}{d\eta} = \frac{\langle E\rangle}{R^2\pi\tau_0}\frac{dn}{d\eta}$$

J. D. Bjroken, Phys. Rev. D 27, 140 (1983)

$$\varepsilon = \varepsilon_{Bj}\frac{\partial y}{\partial\eta_f}\frac{\partial\eta_f}{\partial\eta_i}$$

For an accelerating flow,
Two main modifications for **Initial conditions:** $y \neq \eta$; $\eta_{final} \neq \eta_{initial}$.

$$\lambda = 1.18 \pm 0.01, \quad \tau_f/\tau_0 = 8 \pm 2,$$

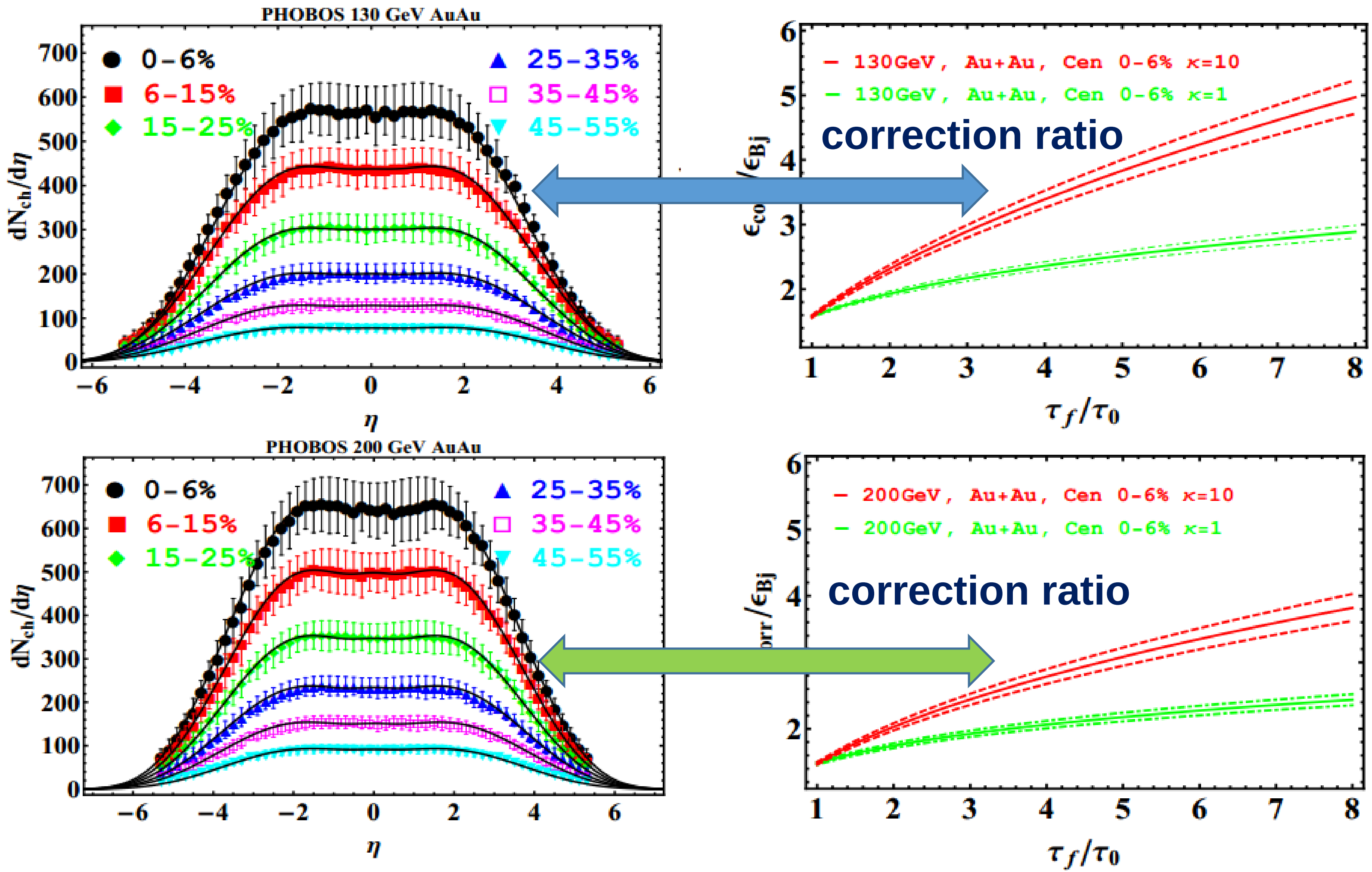
$$\varepsilon_{corr} = (2.0 \pm 0.1)\varepsilon_{Bj} = 10.0 \pm 0.5 \mathrm{GeV/fm}^3.$$

BRAHMS 200 GeV Au+Au data
arXiv: 0805.1562

For more recent, exact results that include the work done by pressure, see G.Kasza's talk at WPCF2018

The pseudorapidity distributions

Results for 130 GeV Au+Au and 200 GeV Au+Au collisions



(See Z.F. Jiang, C.B. Yang, M. Csanád, T. Csörgő, arXiv:1711.10740)

The initial energy density estimation

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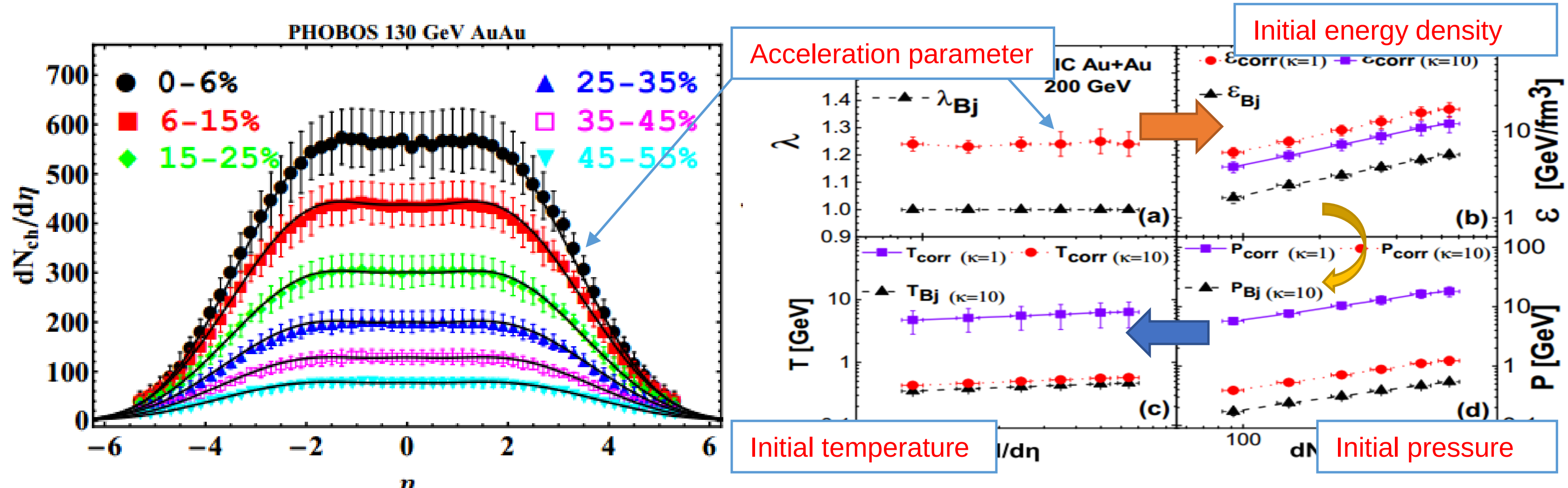

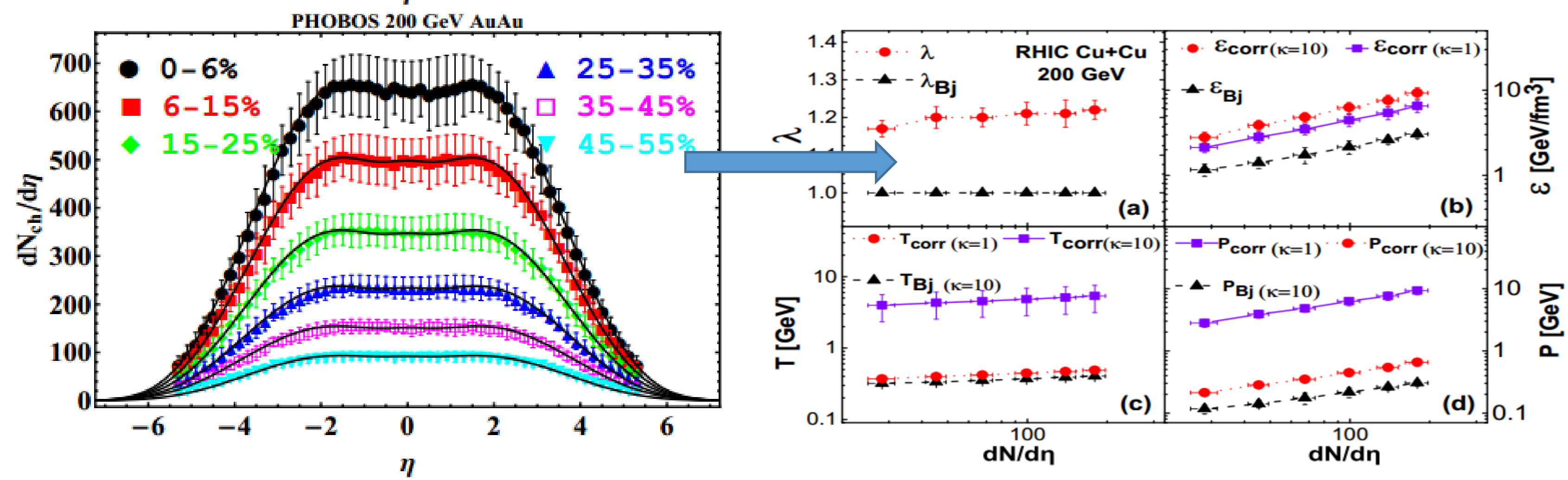

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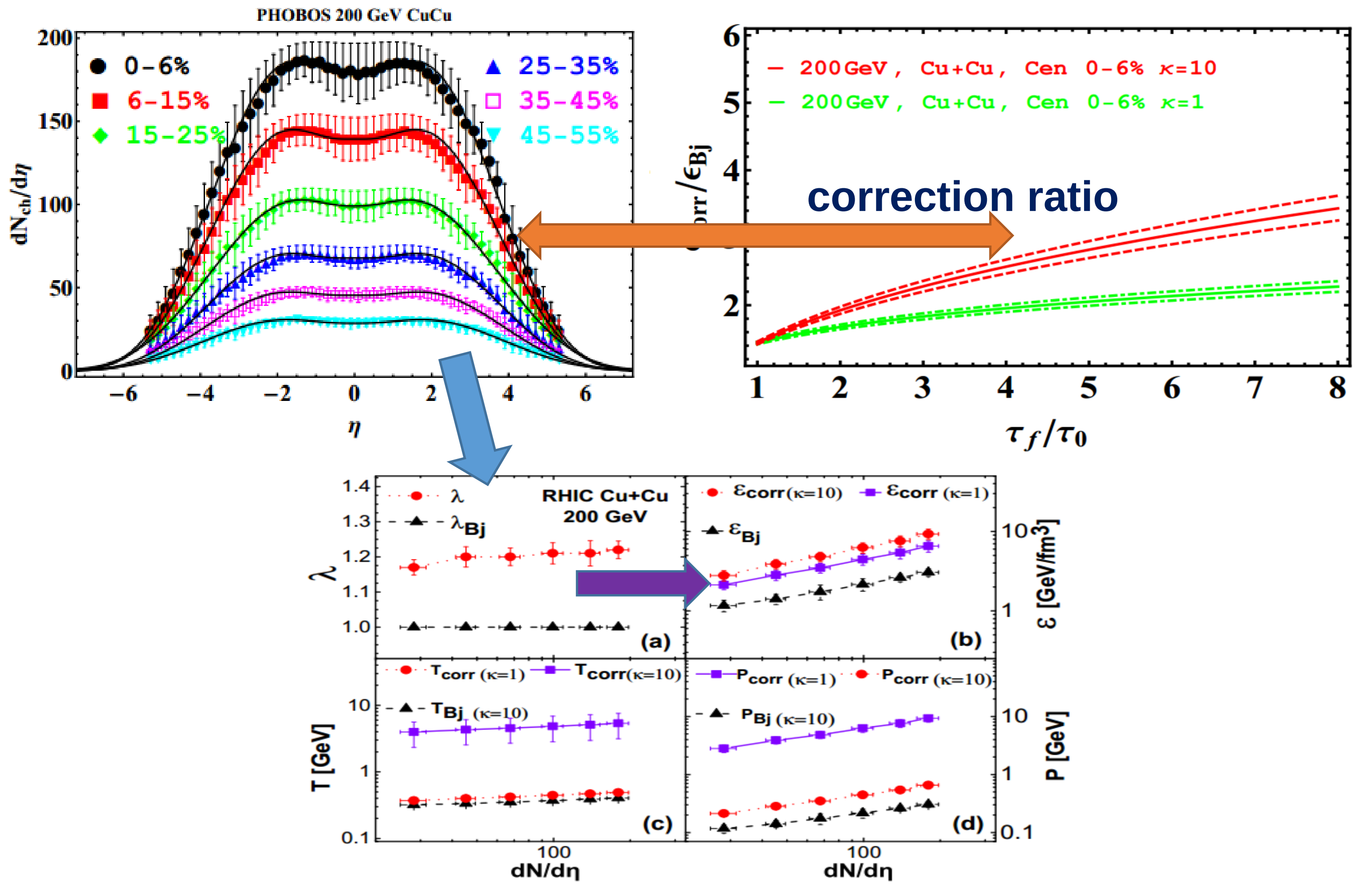
The initial energy density estimation

Results for 200 GeV Cu+Cu collisions



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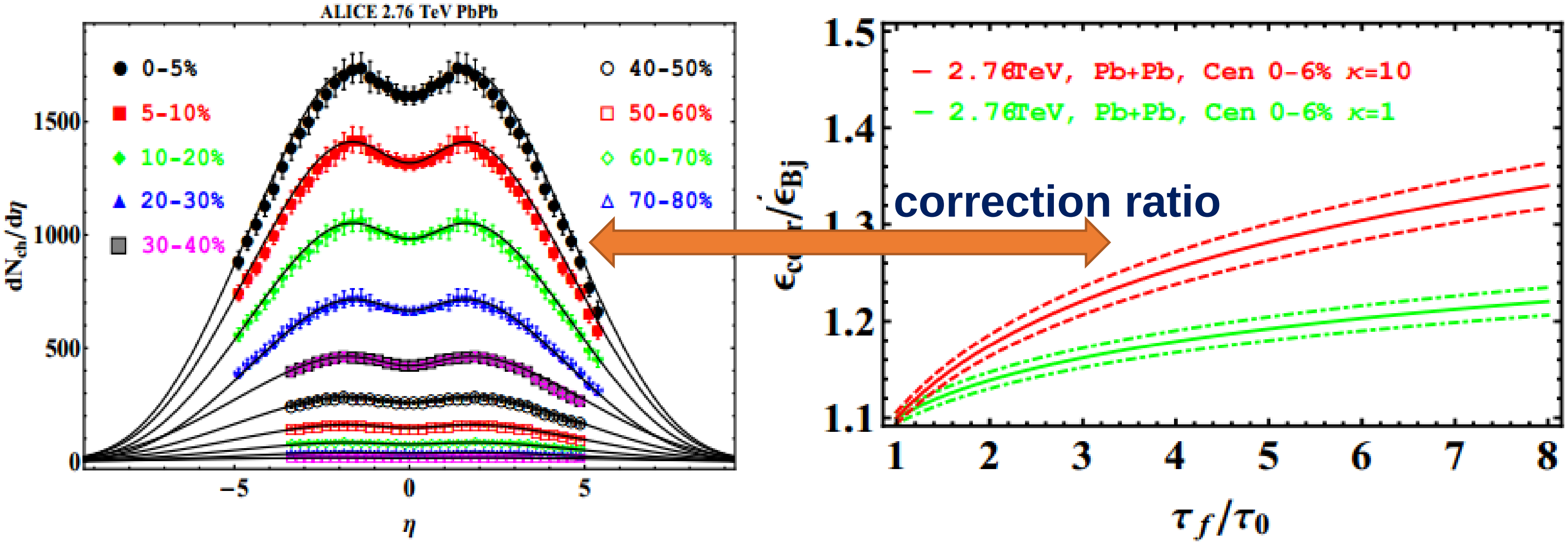
The initial energy density estimation

Results for 2.76 TeV Pb+Pb collisions



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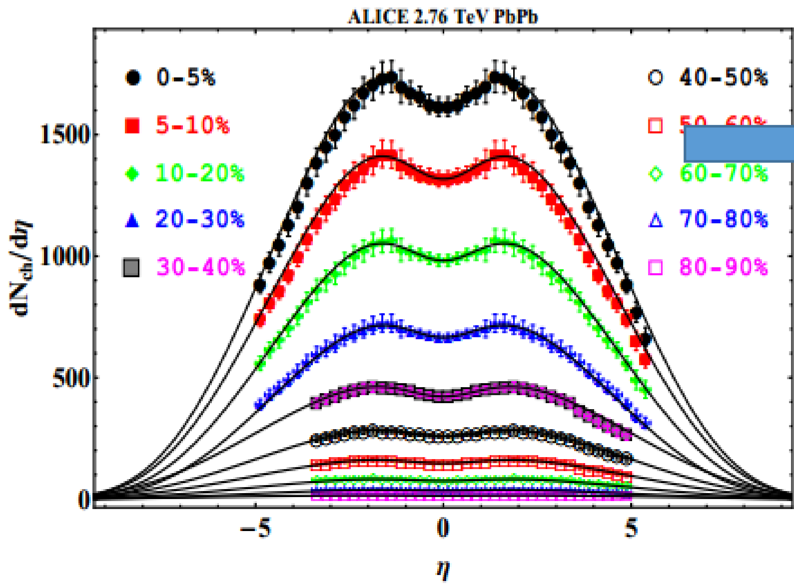

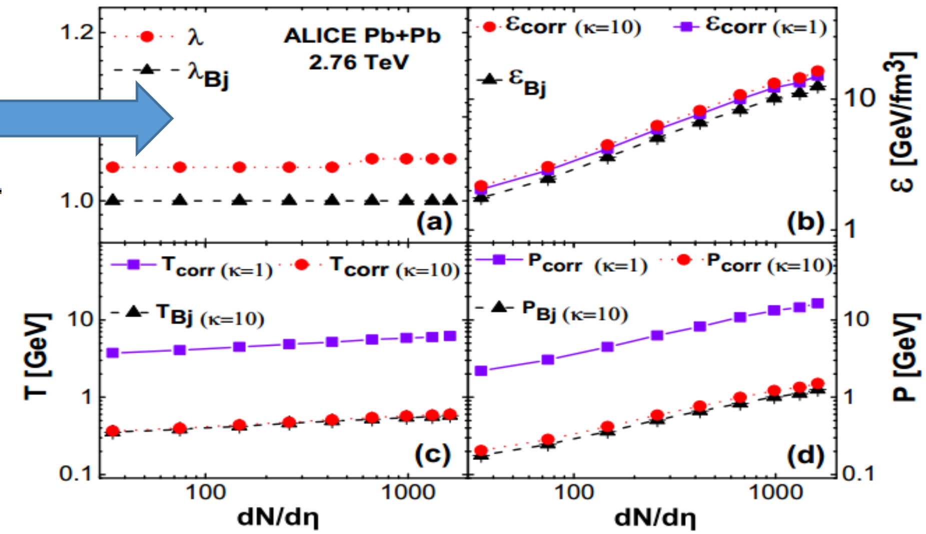

[(See Z.F. Jiang, C.B. Yang, M. Csanád, T. Csörgő, arXiv:1711.10740)](#)

- Hydro can be used to estimate the initial ε_0 . (J. D. Bjorken, Phys. Rev. D27 (1983) 140-151. Gyulassy M, Matsui T. 1984. Phys. Rev. D29: 419)
- The larger the center of mass energy, the smaller the acceleration parameter λ. The bigger $\langle N_{part} \rangle$, the bigger λ.
- From the fits, the initial energy density, initial temperature and initial pressure are reconstructed.

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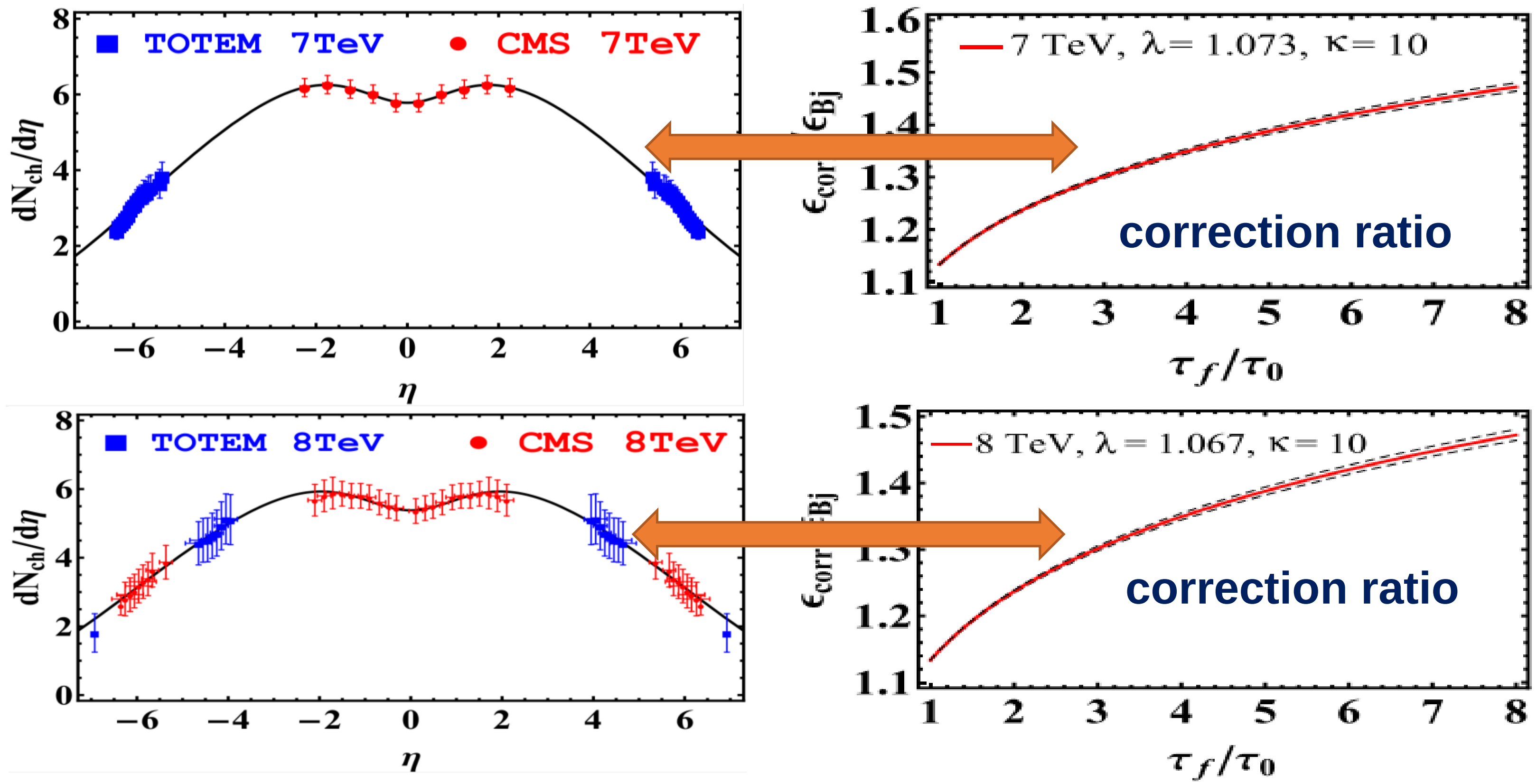
The pseudorapidity distributions

Results for 7 TeV & 8 TeV p+p collisios:



(M. Csanád, T. Csörgő, Z.F. Jiang. C.B. Yang, Universe. vol 3(2017). pp 1-9.)

$\sqrt{s}$	ε_{Bj}	f_c	ε_{corr}	λ	c_s^2	$dN/d\eta\|_{\eta=0}$
7 TeV	0.507	1.262	0.640	1.073	0.10	5.895(NSD)
8 TeV	0.500	1.240	0.644	1.067	0.10	5.38(Inelastic)

The initial energy density estimation

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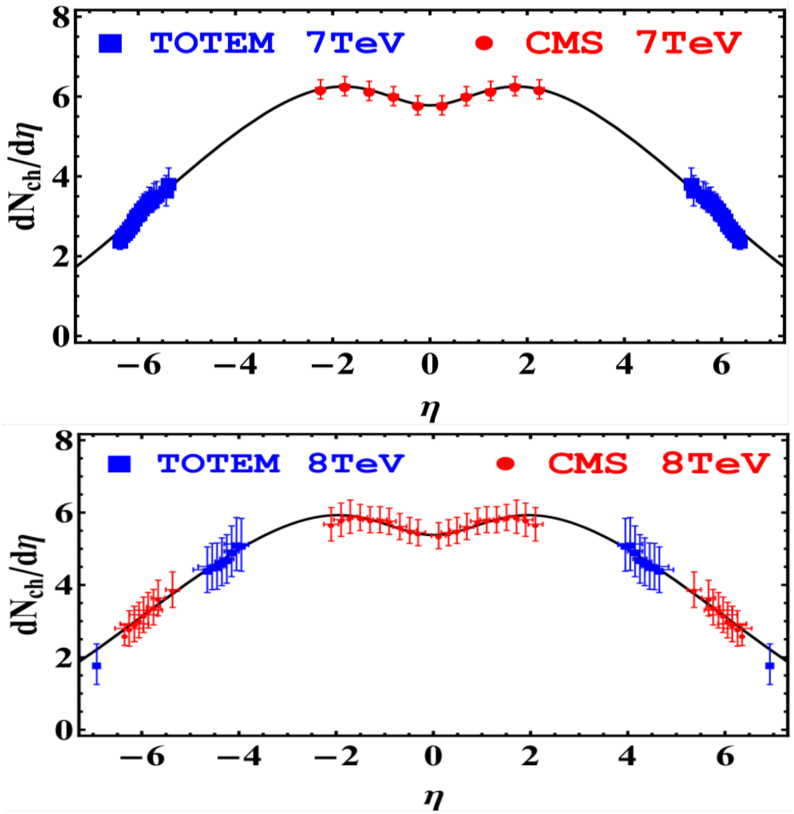

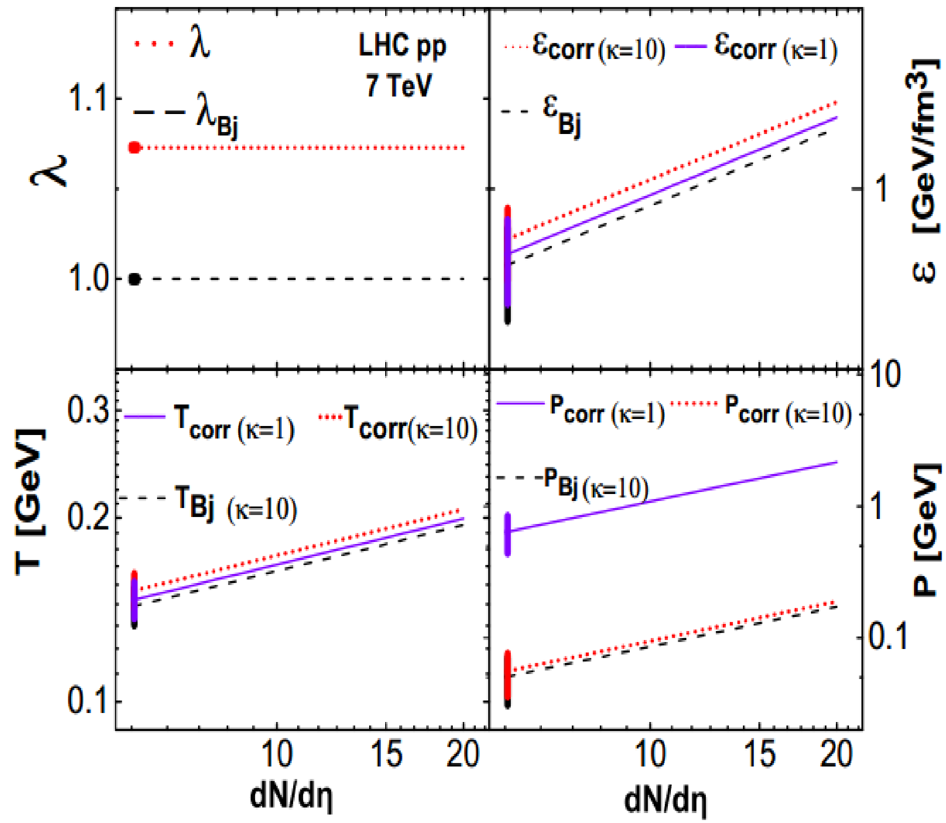

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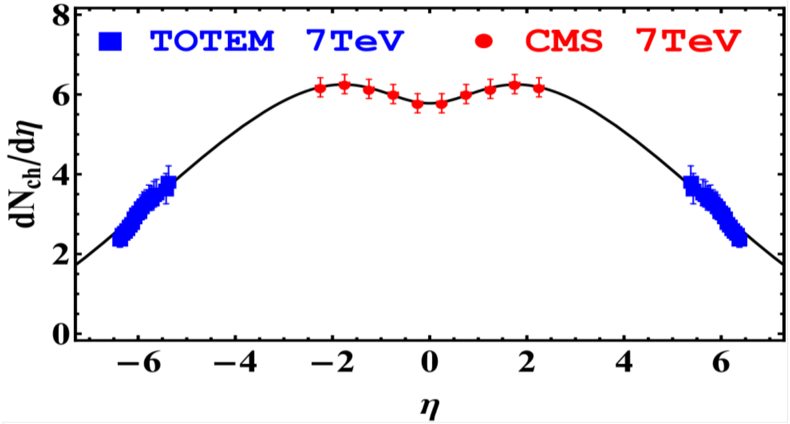

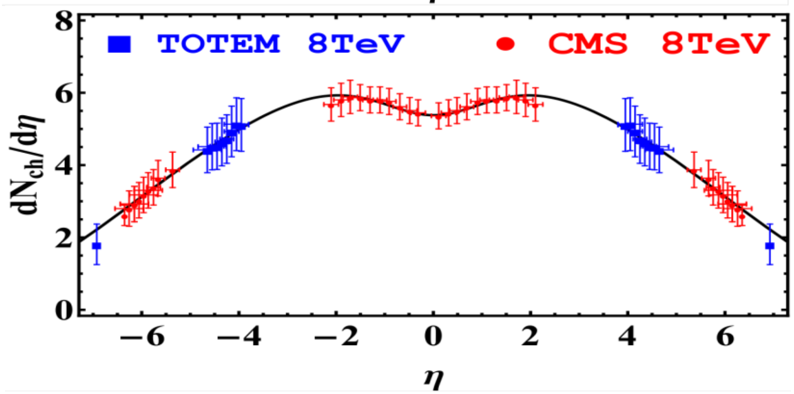

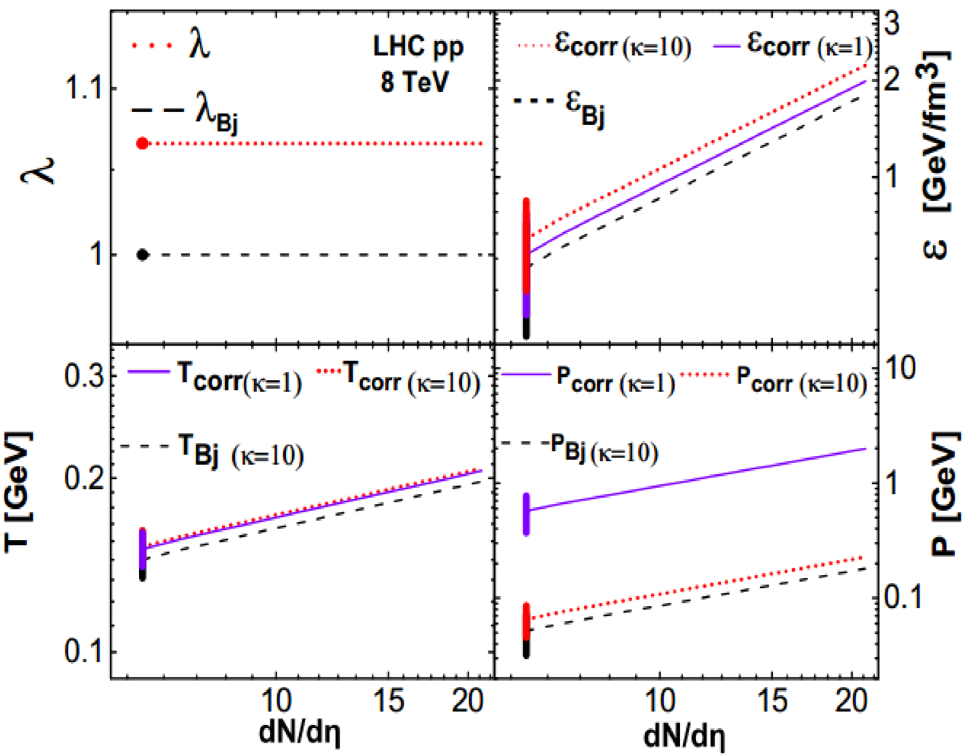

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The CKCJ solutions

The new CKCJ family of exact hydro solutions

$$
\begin{aligned}
\eta_x(H) &= \Omega(H) - H, \\
\Omega(H) &= \frac{\lambda}{\sqrt{\lambda-1}\sqrt{\kappa-\lambda}} \arctan\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}} \tanh(H)\right), \\
\sigma(\tau,H) &= \sigma_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda} \mathcal{V}_\sigma(s) \left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\frac{\lambda}{2}}, \\
T(\tau,H) &= T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{\kappa}} \mathcal{T}(s) \left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\frac{\lambda}{2\kappa}}, \\
\mathcal{T}(s) &= \frac{1}{\mathcal{V}_\sigma(s)}, \\
s(\tau,H) &= \left(\frac{\tau_0}{\tau}\right)^{\lambda-1} \sinh(H) \left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\lambda/2}.
\end{aligned}
$$

T. Csörgő, G. Kasza, M. Csanád, Z.F. Jiang, CKCJ solutions: arXiv: **1805.01427**

For more details, see T. Csörgő's talk at WPCF2018 on a new family of exact solutions of relativistic hydrodynamics.

The CKCJ solutions

Final state observables:

$$\frac{dn}{dy} \approx \frac{dn}{dy}\bigg|_{y=0} \cosh^{-\frac{1}{2}\alpha(\kappa)-1}\left(\frac{y}{\alpha(1)}\right) \exp\left(-\frac{m}{T_f}\left[\cosh^{\alpha(\kappa)}\left(\frac{y}{\alpha(1)}\right)-1\right]\right),$$

$$\frac{dn}{d\eta_p} \approx \frac{dn}{dy}\bigg|_{y=0} \frac{\langle p_T(y)\rangle \cosh(\eta_p)}{\sqrt{m^2+\langle p_T(y)\rangle^2 \cosh(\eta_p)}} \cosh^{-\frac{1}{2}\alpha(\kappa)-1}\left(\frac{y}{\alpha(1)}\right) \exp\left(-\frac{m}{T_f}\left[\cosh^{\alpha(\kappa)}\left(\frac{y}{\alpha(1)}\right)-1\right]\right),$$

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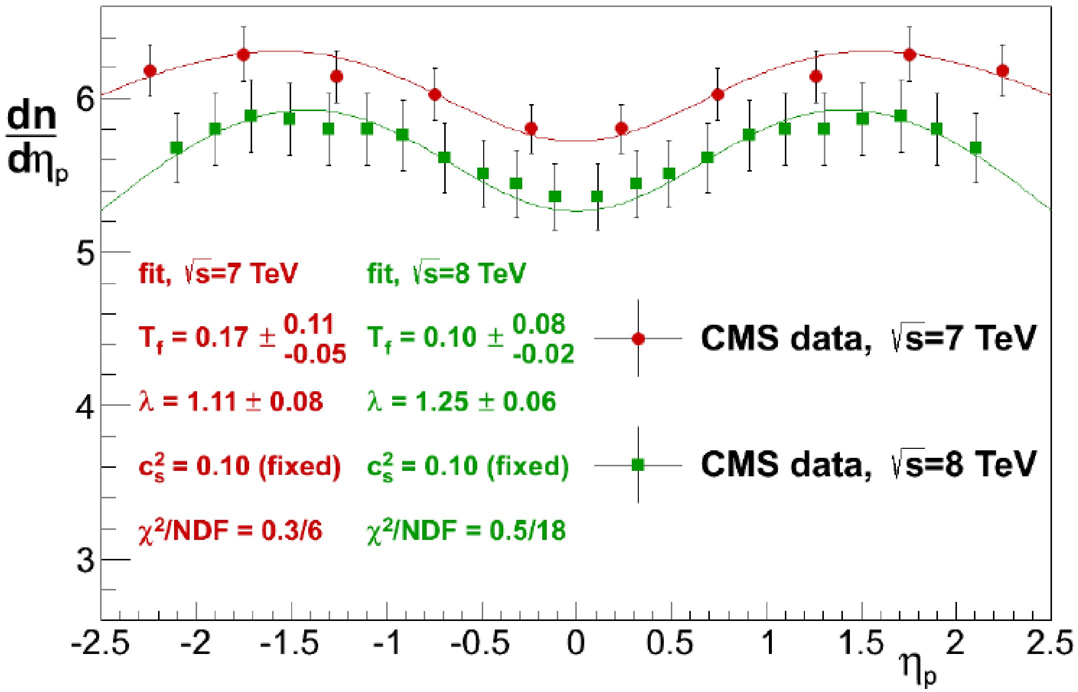
The CKCJ solutions

Results for 7 TeV & 8 TeV p+p collisios at CMS:



For a new method to estimate the initial energy density from the CKCJ solution: see G.Kasza's talk at WPCF 2018.

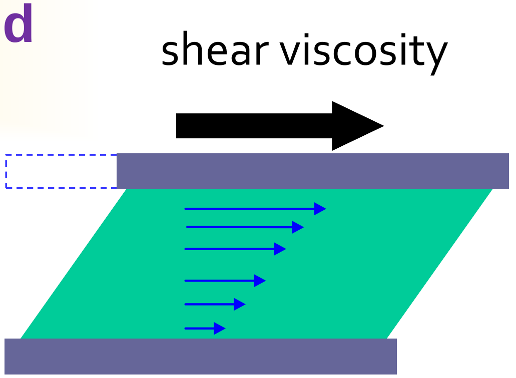
See May 22th, **G.Kasza's talk** talk: Initial energy density from new, exact solutions of relativistic hydrodynamics.

Relativistic accelerating viscous hydrodynamics

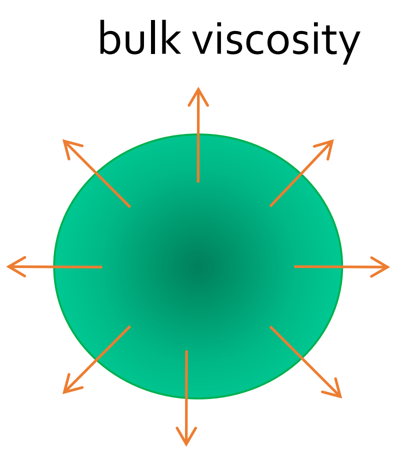
Longitudinal acceleration effect makes the fluid cooling faster,
The viscosity will creating heat and makes the fluid cooling slower.

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$u^{\mu} = (\cosh\Omega, 0, 0, \sinh\Omega) \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu}$$



Shear viscosity tensor: $\pi^{\mu\nu}$ **Bulk viscosity:** Π.



Shear tensor:

$$\sigma^{\mu\nu} \equiv \partial^{\langle\mu}u^{\nu\rangle} \equiv (\frac{1}{2}(\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\mu}_{\beta}\Delta^{\nu}_{\alpha}) - \frac{1}{d}\Delta^{\mu\nu}\Delta_{\alpha\beta})\partial^{\alpha}u^{\beta}.$$

The fundamental equations of viscous fluid:

$$e = \kappa p, \; \partial_{\nu}T^{\mu\nu} = \mathbf{0}.$$

Assume: $n \approx 0, \quad \partial(nu^{\mu}) = \mathbf{0}.$

Equations of viscosity hydrodynamic

The second law of thermodynamics: $\partial_\mu S^\mu \geq 0$

$$\tau_\pi \Delta^{\alpha\mu}\Delta^{\beta\nu}\dot{\pi}_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta(\partial \cdot u) - \frac{1}{2}\Pi\frac{\zeta T}{\tau_\Pi}\partial_\lambda\left(\frac{\tau_\Pi}{\zeta T}u^\lambda\right)$$

Israel-Stewart equations.

viscous hydro: near-equilibrium system

Equations of viscosity hydrodynamic

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Israel-Stewart equations.

viscous hydro: near-equilibrium system

The Navier-Stokes approximation,

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} \qquad \Pi = -\zeta\left(\partial_\rho \mu^\rho\right)$$

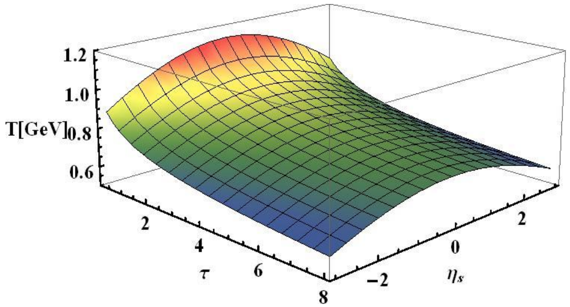
The shear viscosity and bulk viscosity,

Strongly coupled AdS/CFT prediction: $\eta / s \geq 1/4\pi \approx 0.08$ D.T. Son, et,al. 05

Via lattice calculation: $\zeta / s \leq 0.015$ H.B. Meyer, et,al. 07 10.3717

Solutions form viscous hydrodynamics

The temperature profile:

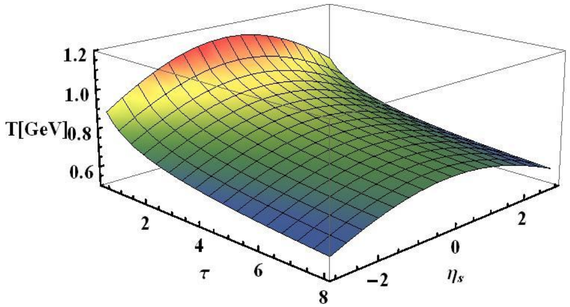


$$T(\tau,\eta_s) = T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{1+\epsilon}{\kappa}}$$

$$\times\left[\exp[-\frac{1}{2}\epsilon(1-\frac{1}{\kappa})\eta_s^2] + \frac{R_0^{-1}}{\kappa-1}\left(2\epsilon + \exp[-\frac{1}{2}\epsilon(1-\frac{1}{\kappa})\eta_s^2] - (2\epsilon+1)\left(\frac{\tau_0}{\tau}\right)^{\frac{\kappa-\epsilon-1}{\kappa}}\right)\right]$$

Solutions form hydrodynamic equations

The temperature profile:



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Contribution from ideal terms.
One special case of our recent work.

Contribution from viscous effect

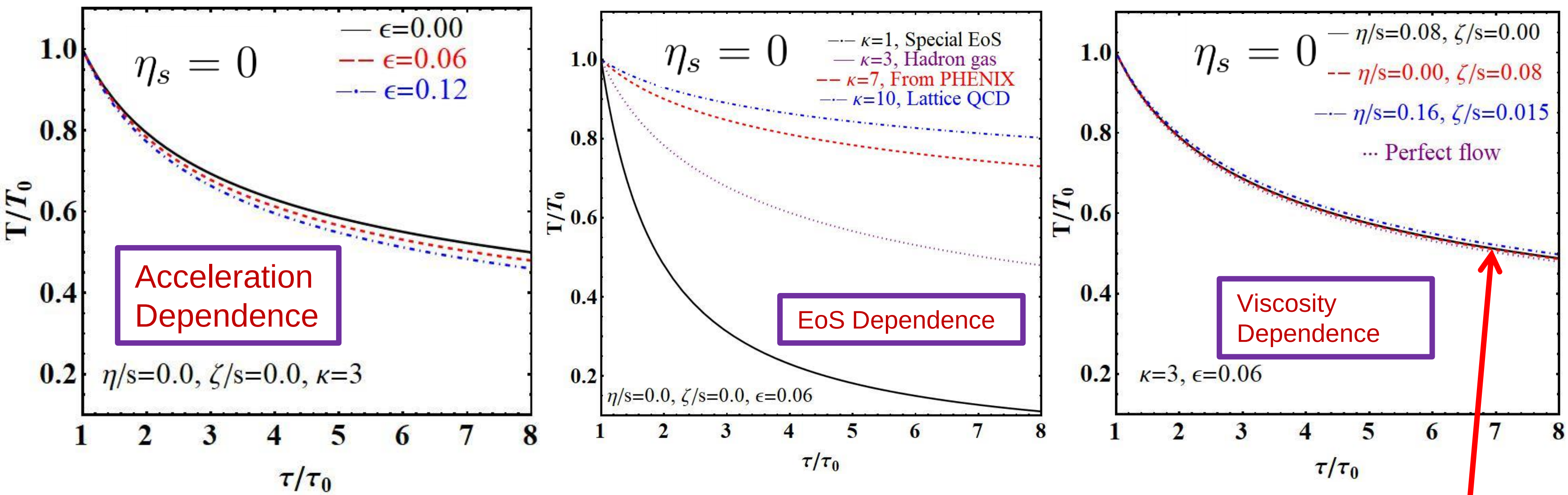
$$R_0^{-1} = \frac{\Pi_d}{T_0\tau_0}$$

Reynolds number
[A. Muronga, arXiv: 0309055]

- A non-zero Reyonlds numbers R_0^{-1} makes cooling rate smaller,
- A non-vanishing acceleration ϵ makes the colling rate is larger.

Open question: setting viscosity as the perturbative term.

Temperature evolution



- Acceleration effect comes from the pressure gradient, makes the cooling ratio **larger** than non-acceleration flow.
  [M. Nagy, T. Csorgo, M. Csanád: arXiv:0709.3677v1]
- EoS is an important modified factor.
  κ=1 a very special case, CNC solution.
  κ=7 comes from **[PHENIX, arXiv:nucl-ex/0608033v1]** .
- Viscosity effect make the cooling rate samller. [H. Song, S. Bass, U. Heinz. et, PRL2011]

The acceleration effect is almost fully **compensated** by the viscous contribution.

A brief summary and outlook

Summary:
1. By using exact accelerating solutions of perfect flow, the initial thermodynamics quantities are estimated.
2. The perturbative solution with viscous correction are obtained.

Outlook:
1. 2rd I-S problem, rotation, CLVisc 3+1D code;
2. Jet/heavy quarkonium disturbance evolution on accelerating medium background...

Thank you for your attention

[arXiv: 1609.07176, 1711.10740, 1805.01427…]()