Analyses of multi-pion HBT correlations for the pion-emitting sources with Bose-Einstein condensation

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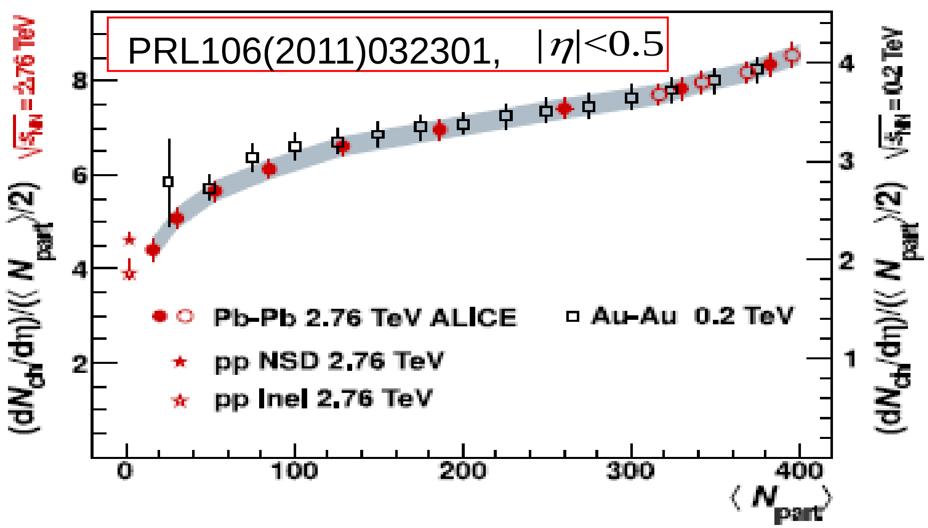
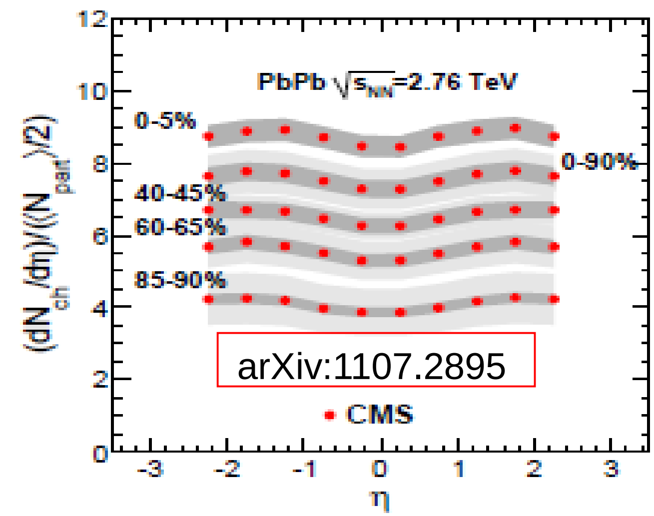
Outline

- Introduction
- B-E condensation of pion gas in harmonic oscillator potentials
- Density distribution and multi-pion HBT correlations
- Model calculations and results of correlation functions
- Comparison with experimental data
- Summary and outlook

Introduction

- HBT interferometry is a tool to study the space & time of particle-emitting sources.
- Because HBT correlation occurs only for chaotic sources. So, it can be used to probe the source chaotic degree.
- Pions are the most copiously produced particles in high-energy collisions. In the heavy-ion collisions at the RHIC and LHC, the detected identical pions are about hundreds and thousands.

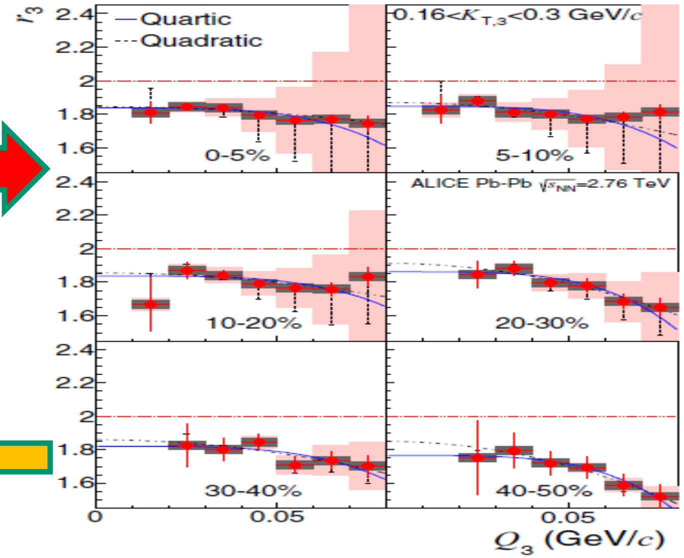




$$0-5\%,\ \langle N_{part}\rangle = 383,\ dN_{ch}/d\eta = 1601,\quad N_{ch} \approx 1601\times 4 = 6400$$

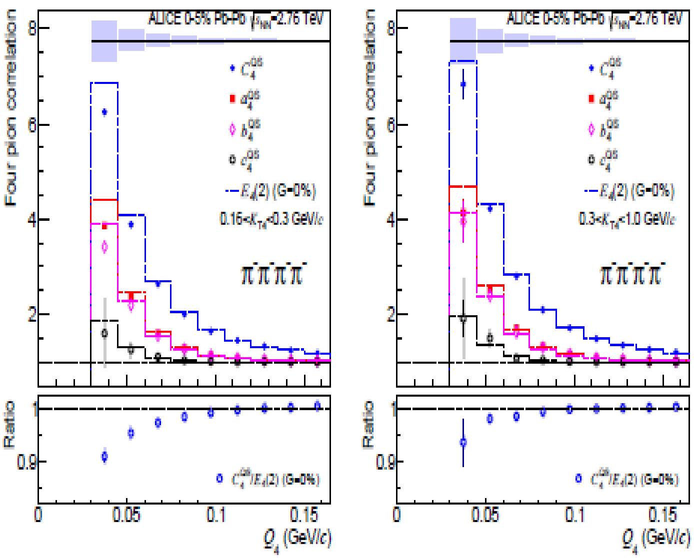
The high pion event multiplicity may possibly lead to occurrence of the pion condensate in ultra-relativistic heavy-ion collisions.

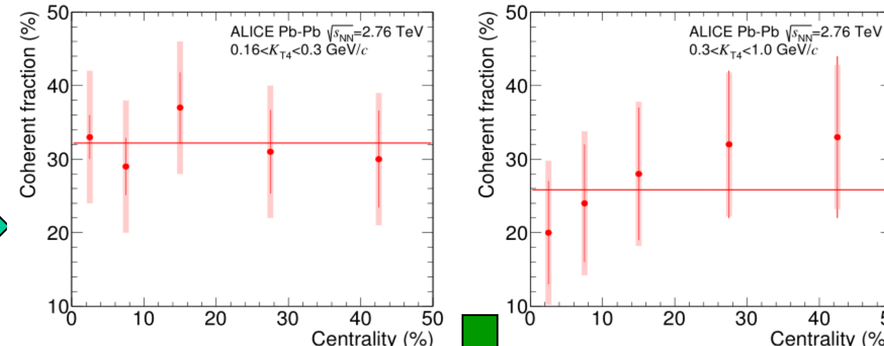
Three-pion HBT measurements for the $\sqrt{s_{NN}}$=2.76 TeV Pb-Pb at the LHC (PRC**89**,2014,024911) indicate that the pion sources are not chaotic completely and with the considerable degree of coherence



r_3 < 2 Partially coherent sources

(PRC**93**,054908,2016)





(a) $0.16 < K_{T4} < 0.3$ GeV/c (b) $0.3 < K_{T4} < 1.0$ GeV/c

20-40 % coherent fraction from the recent multi-pion measurement at ALICE

It is our motivation to study the possible pion Bose-Einstein condensation in ultra-relativistic heavy-ion collisions and investigate the effects of the condensation on pion HBT measurements.

BE condensation of pion gas in harmonic oscillator

- We consider an expanding pion source, the time-dependent harmonic oscillator potential is given by

$$V(r,t)=\frac{1}{2}m\omega^2(t)r^2=\frac{1}{2}\hbar\omega(t)\frac{r^2}{a^2(t)},$$

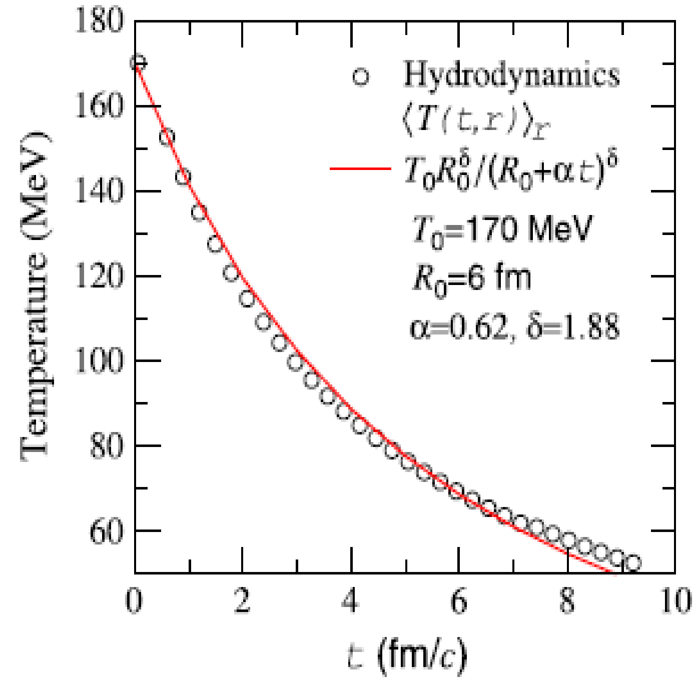
- Assuming the system relaxation time is smaller than source evolution time, we may deal the pion gas as a quasi-static adiabatic expansion, and have

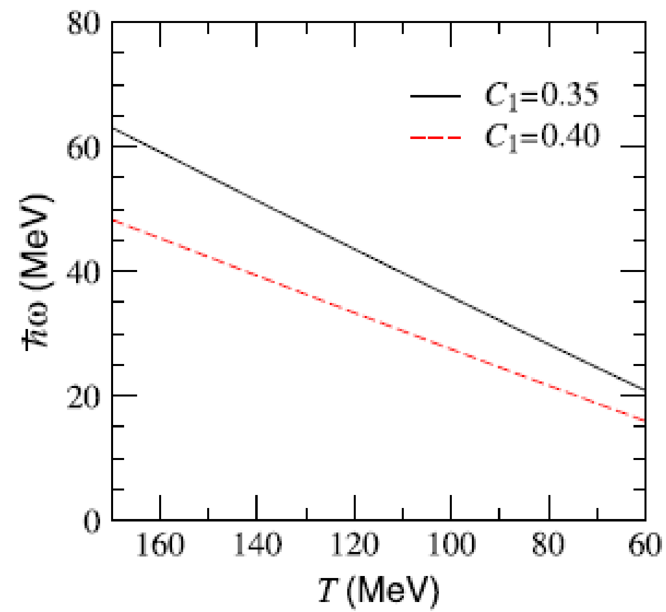
$$TV^{\gamma-1}=\text{const}$$

$$\Rightarrow \quad T=\frac{T_0R_0^{\delta}}{(R_0+\alpha t)^{\delta}},$$

$$a=C_1(R_0+\alpha t)$$

With the assumption of the quasi-static adiabatic expansion and the relation a(t), we can calculate the condensation for the expansion pion source at each evolving time as static sources.





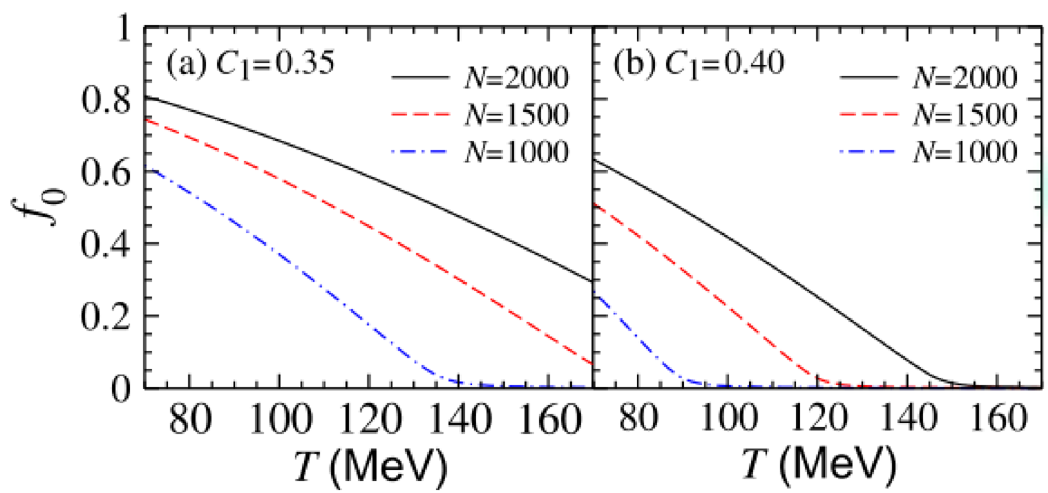
Condensation fraction

- In a canonical ensemble, the total number of bosons

$$\mathrm{N} = N_0 + N_T = \frac{Z}{1-Z} + \sum_{n>0}^{\infty} \frac{g_n\, Z\, e^{-\beta \widetilde{E_n}}}{1 - Z\, e^{-\beta \widetilde{E_n}}} \qquad , \widetilde{E_n} = E_n - E_0$$

Z — fugacity, which is related to E_0 and can be solved

Condensation fraction $f_0 = \frac{N_0}{N} = \frac{Z}{(1-Z)N}$



Condensation is significant for small characteristic length

For N=2000, the system has large condensation fraction, which may reach 0.7 at the temperatures 60-80 MeV.

However, for N=1000, the system has only small condensation fraction at low temperature for $C_1 = 0.40$ than for smaller source $C_1 = 0.35$.

Density distribution and pion HBT correlations

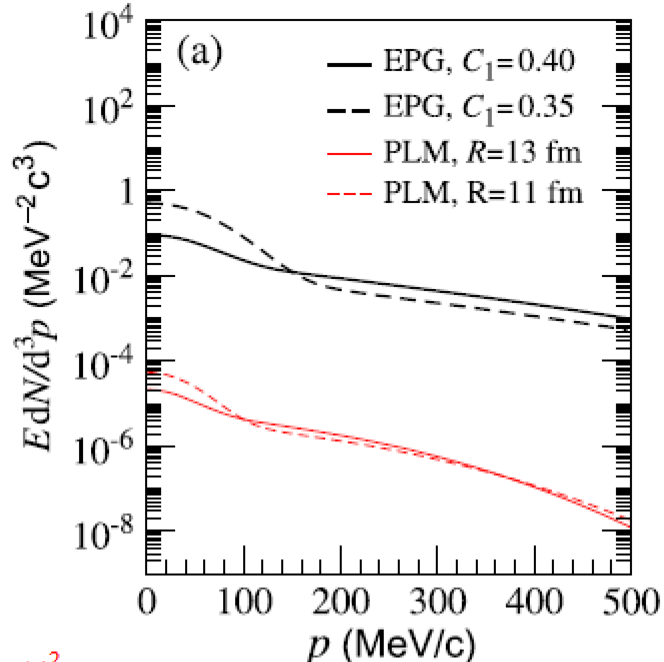
- In the EPG model, the one- and two-particle density matrices

$$G^{(1)}(p_1,p_2) = \sum_n u^*{}_n(p_1)u_n(p_2)\ \langle \hat{a}_n^+\ \hat{a}_n\rangle = \sum_n u^*{}_n(p_1)u_n(p_2)\ \frac{g_n\ Z\ e^{-\beta\widetilde{E_n}}}{1 - Z\ e^{-\beta\widetilde{E_n}}}$$

$$G^{(2)}(p_1,p_2;p_1,p_2) = \sum_{klmn} u_k^*(p_1)u_l^*(p_2)\ u_m(p_2)u_n(p_2)\langle \hat{a}_k^+\hat{a}_l^+\ \hat{a}_m\hat{a}_n\rangle$$

- The invariant single-pion momentum distribution is

$$\mathrm{E}\frac{dN}{dp} = \sqrt{p^2 + m_\pi^2}\ G^{(1)}(p,p)$$



- **Two-body density matrix in momentum space**

In the limit of a large number of particles, $N(N-1) \sim N^2(\gg N_T, N_0)$

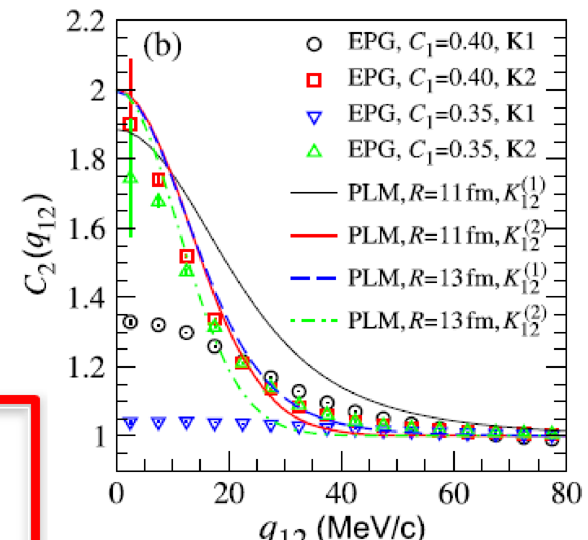
$$G^{(2)}(p_1,p_2;p_1,p_2) = G^{(1)}(p_1,p_1)G^{(1)}(p_2,p_2) + |G^{(1)}(p_1,p_2)|^2 - N_0{}^2|u_0(p_1)|^2|u_0(p_2)|^2$$

- **In BE correlation measurement**, we normalize the probability relative to the probability of detecting particle p_1 and p_2, and define the momentum correlation function C

$$\mathrm{C}(p_1,p_2) = \frac{G^{(2)}(p_1,p_2;p_1,p_2)}{G^{(1)}(p_1,p_1)G^{(1)}(p_2,p_2)}$$

$$\mathrm{C}(p,q) = \mathrm{C}(p_1,p_2) = 1 + \frac{|G^{(1)}(p_1,p_2)|^2 - N_0{}^2|u_0(p_1)|^2|u_0(p_2)|^2}{G^{(1)}(p_1,p_1)G^{(1)}(p_2,p_2)}$$

This is the general Bose-Einstein correlation for all situations



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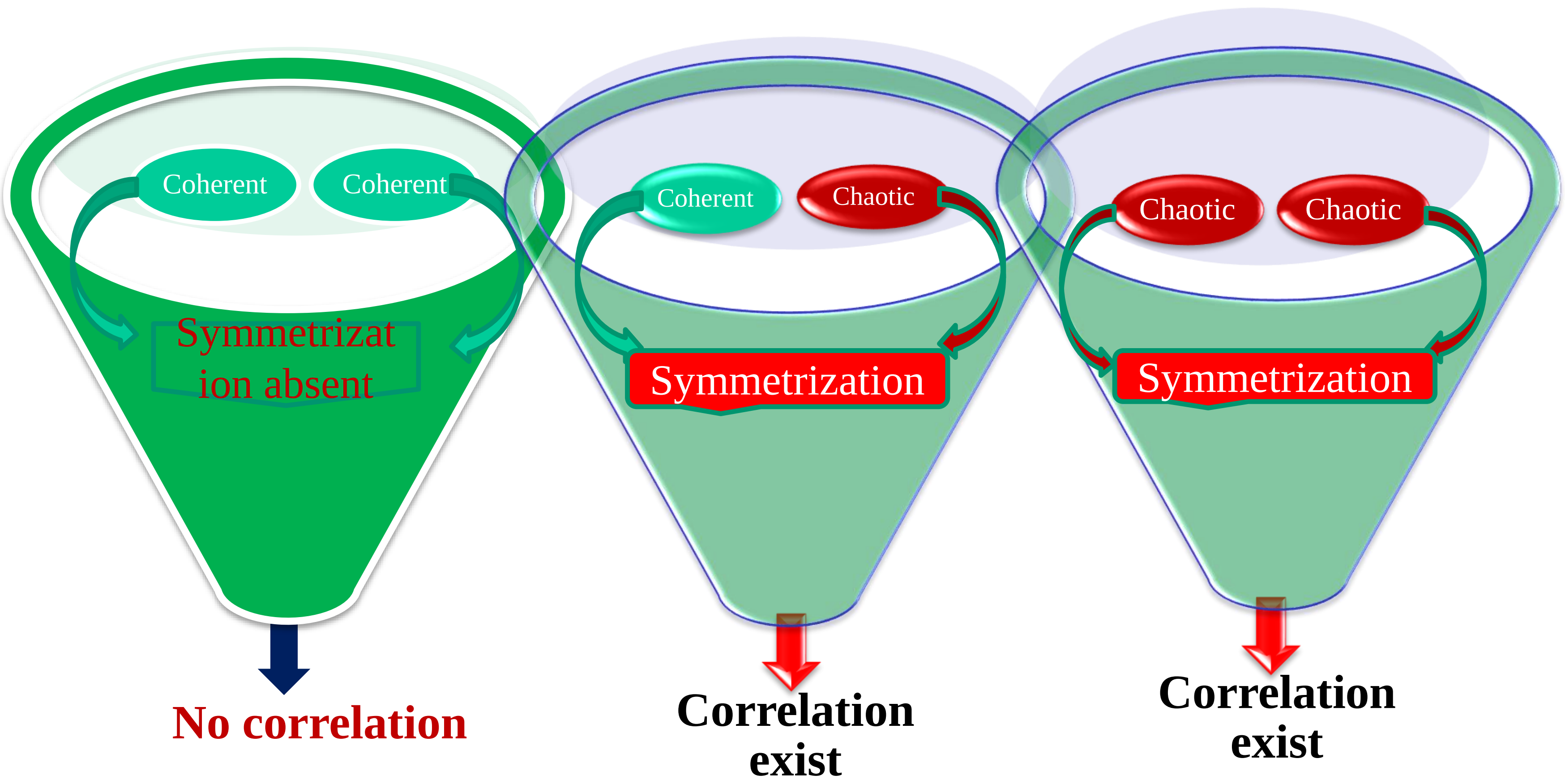
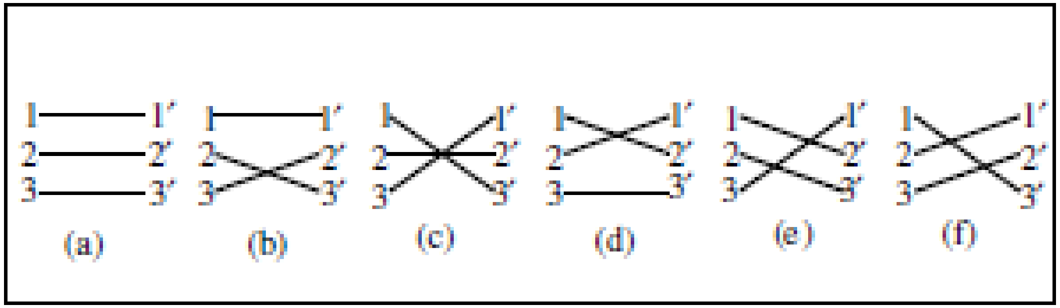


Principle of symmetrization

Three-pion correlation functions



$$C_3(p_1, p_2, p_3) = \frac{G^{(3)}(p_1, p_2, p_3; p_1, p_2, p_3)}{G^{(1)}(p_1; p_1)G^{(1)}(p_2; p_2)G^{(1)}(p_3; p_3)},$$

$$\boldsymbol{C_3(p_1, p_2, p_3) = 1 + R(1,2) + R(2,3) + R(1,3) + R(1,2,3)}$$

Where

$$R(i, j) = \frac{|G^{(1)}(p_i, p_j)|^2 - N_0^2 |u_0(p_i)|^2 |u_0(p_j)|^2}{G^{(1)}(p_i, p_i)G^{(1)}(p_j, p_j)}$$

→ **Two-pion correlators**

And

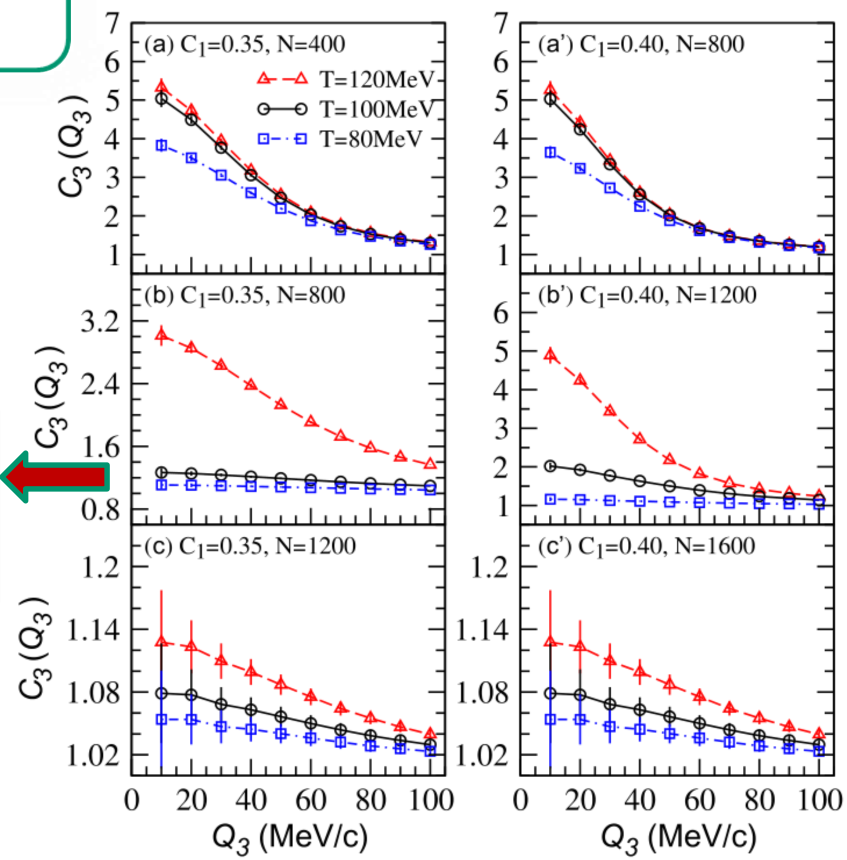
$$R(i, j, k) = 2\frac{\mathrm{Re}\,[G^{(1)}(p_i, p_j)G^{(1)}(p_j, p_k)G^{(1)}(p_k, p_i) - N_0^3 f_3(p_i, p_j, p_k)]}{G^{(1)}(p_i, p_i)G^{(1)}(p_j, p_j)G^{(1)}(p_k, p_k)}$$

→ **Three-pion correlators**

$$\begin{aligned} f_3(p_i, p_j, p_k) = {} & G^{(1)}(p_i, p_j)u_0(p_i)u_0^*(p_j)|u_0(p_k)|^2/N_0 \\ & + G^{(1)}(p_j, p_k)u_0(p_j)u_0^*(p_k)|u_0(p_i)|^2/N_0 \\ & + G^{(1)}(p_k, p_i)u_0(p_k)u_0^*(p_i)|u_0(p_j)|^2/N_0 \\ & - 2|u_0(p_i)|^2|u_0(p_j)|^2|u_0(p_k)|^2. \end{aligned}$$

Three-pion correlation functions

Three-pion correlations as a function of Q_3 for the EPG sources with the different temperatures and particle numbers

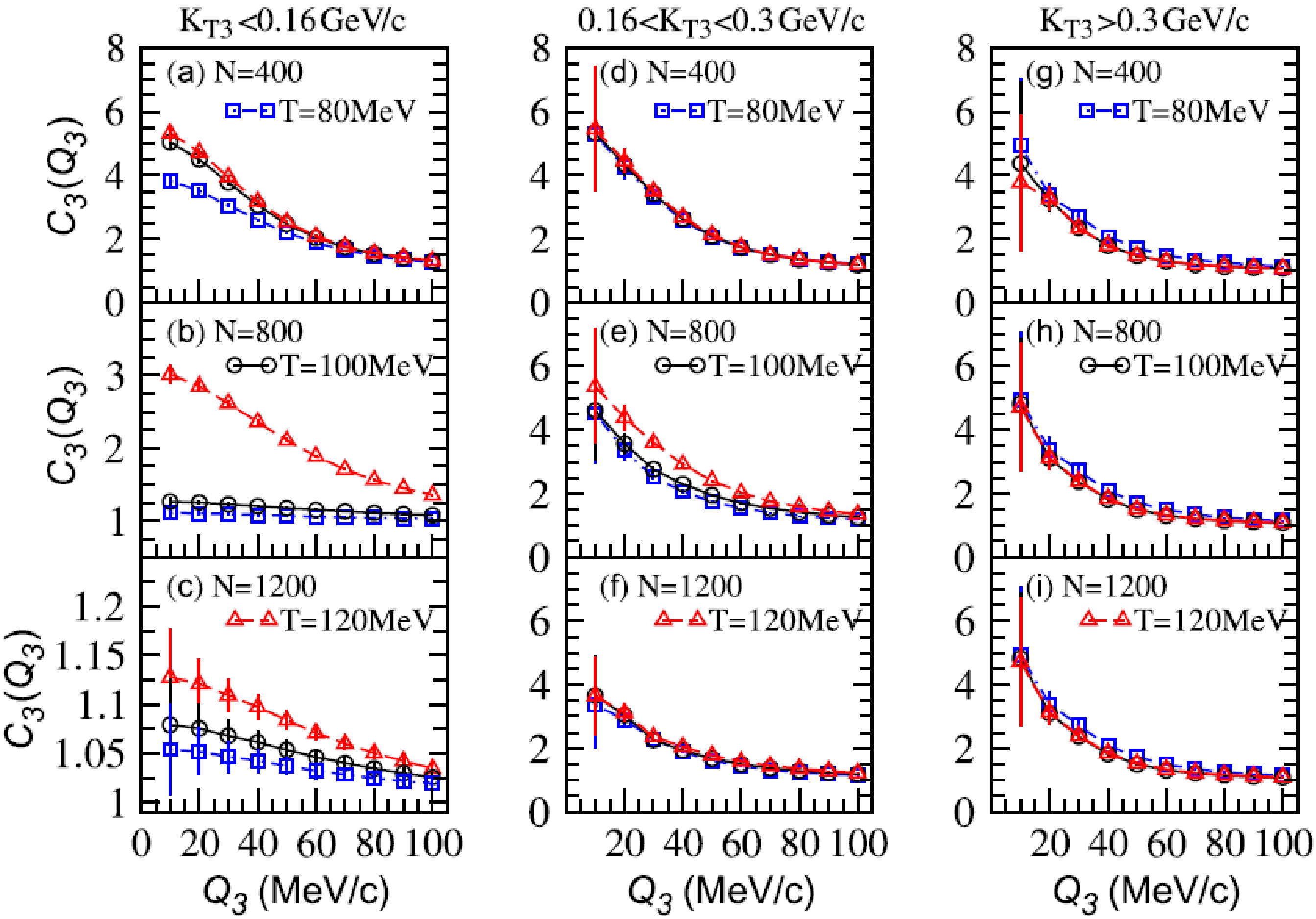


For the fixed N and T, the correlation functions for the sources with $C_1 = 0.35$ are lower than those for the sources with $C_1 = 0.40$

Because the source with a small C_1 has small characteristic length and high condensation fraction

$$Q_3 = \sqrt{Q_{12}^2 + Q_{23}^2 + Q_{13}^2}$$

Pion correlation functions with different K_{T3} intervals

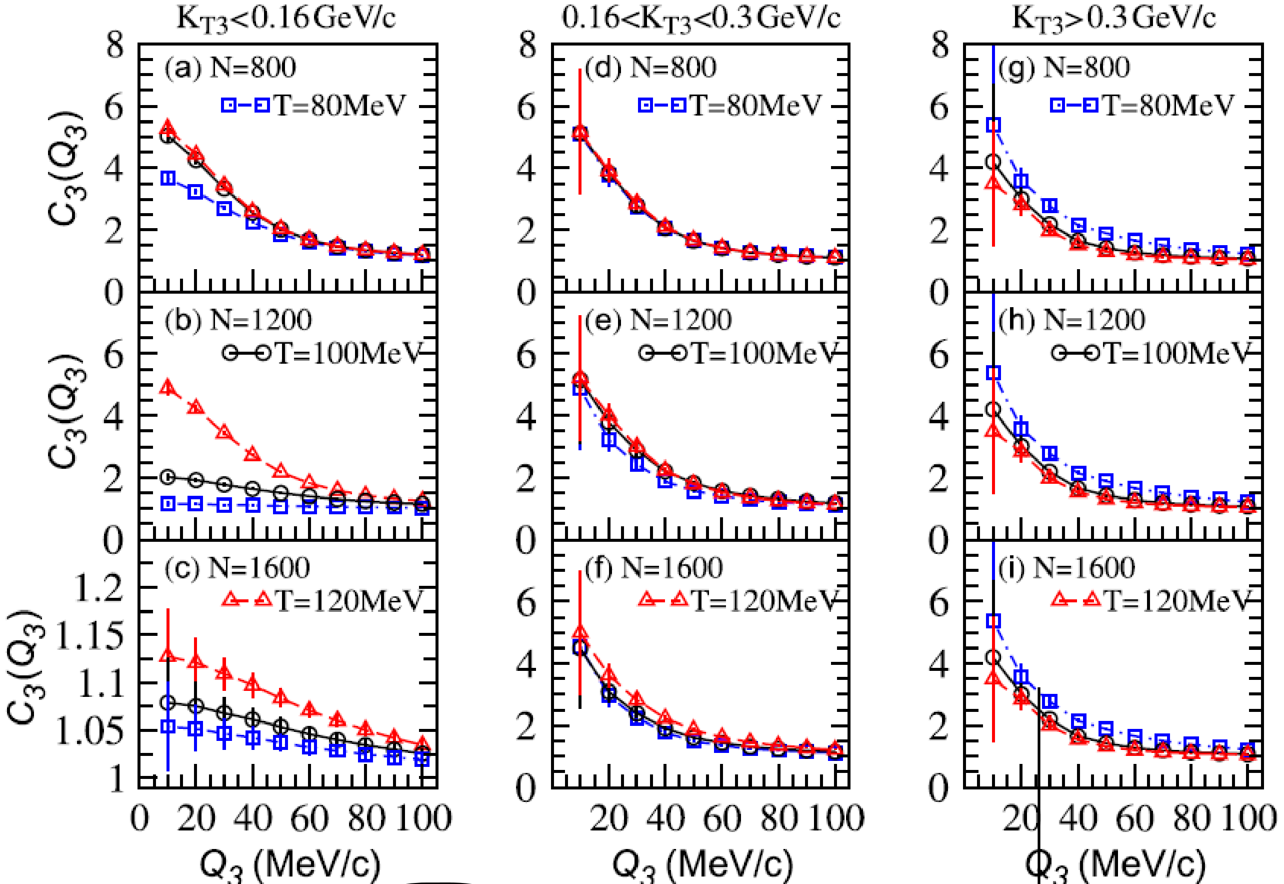


$$K_{T3} = \frac{|p_{T1} + p_{T2} + p_{T3}|}{3}$$

Three-pion correlation functions for the EPG sources with $C_1 = 0.35$

The widths of the correlation functions in the highest momentum interval are narrower than those in the lowest momentum intervals because the source has a wider spatial distribution for the pions emitted from excited states than that from ground state

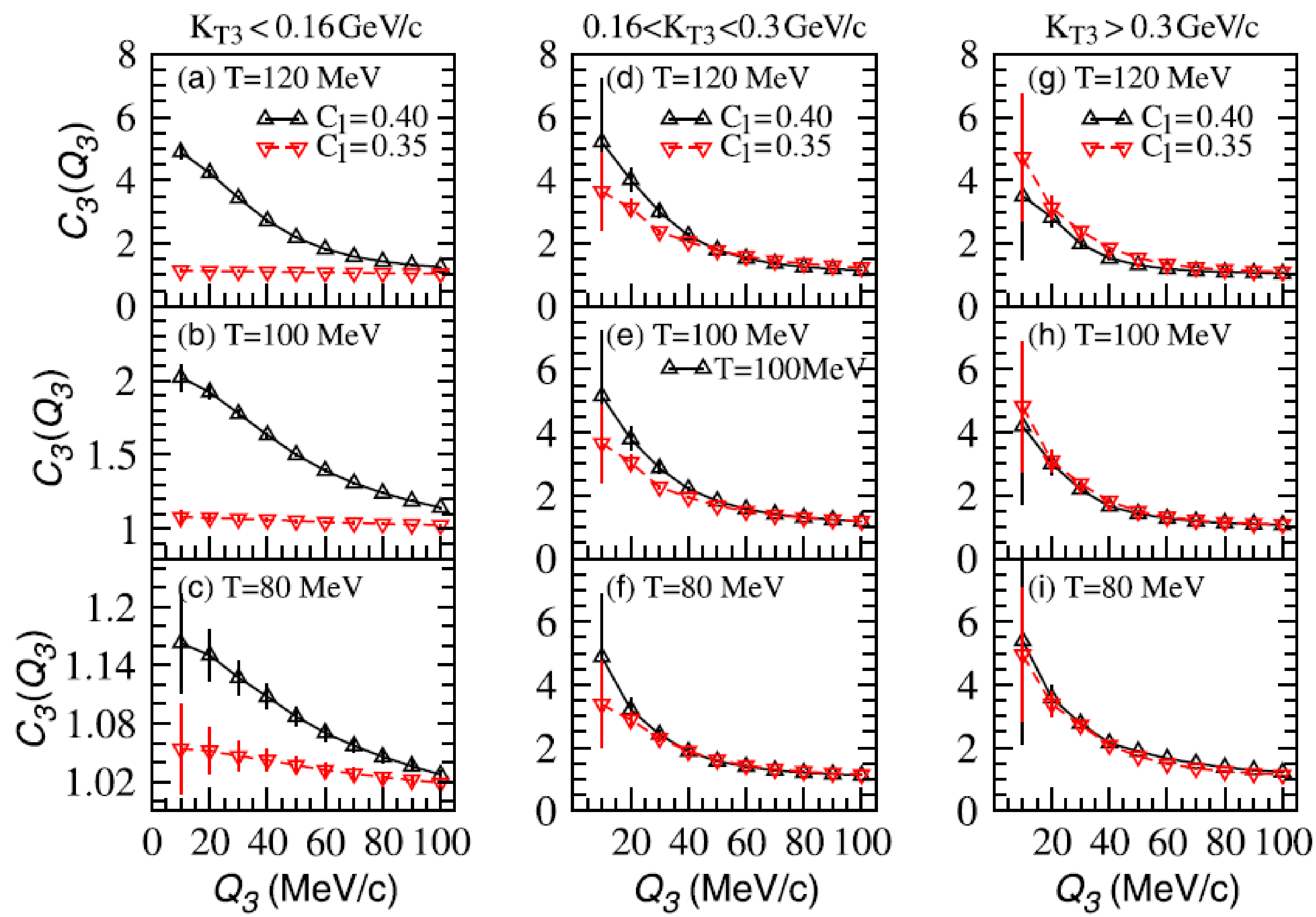
Three-pion correlation functions with $C_1 = 0.40$



Because the source spatial distribution is narrow at low temperature for the chaotic emission from excited states

The correlation functions for the sources with T=80 MeV are slightly higher than those for the sources with the higher temperatures in the highest momentum interval

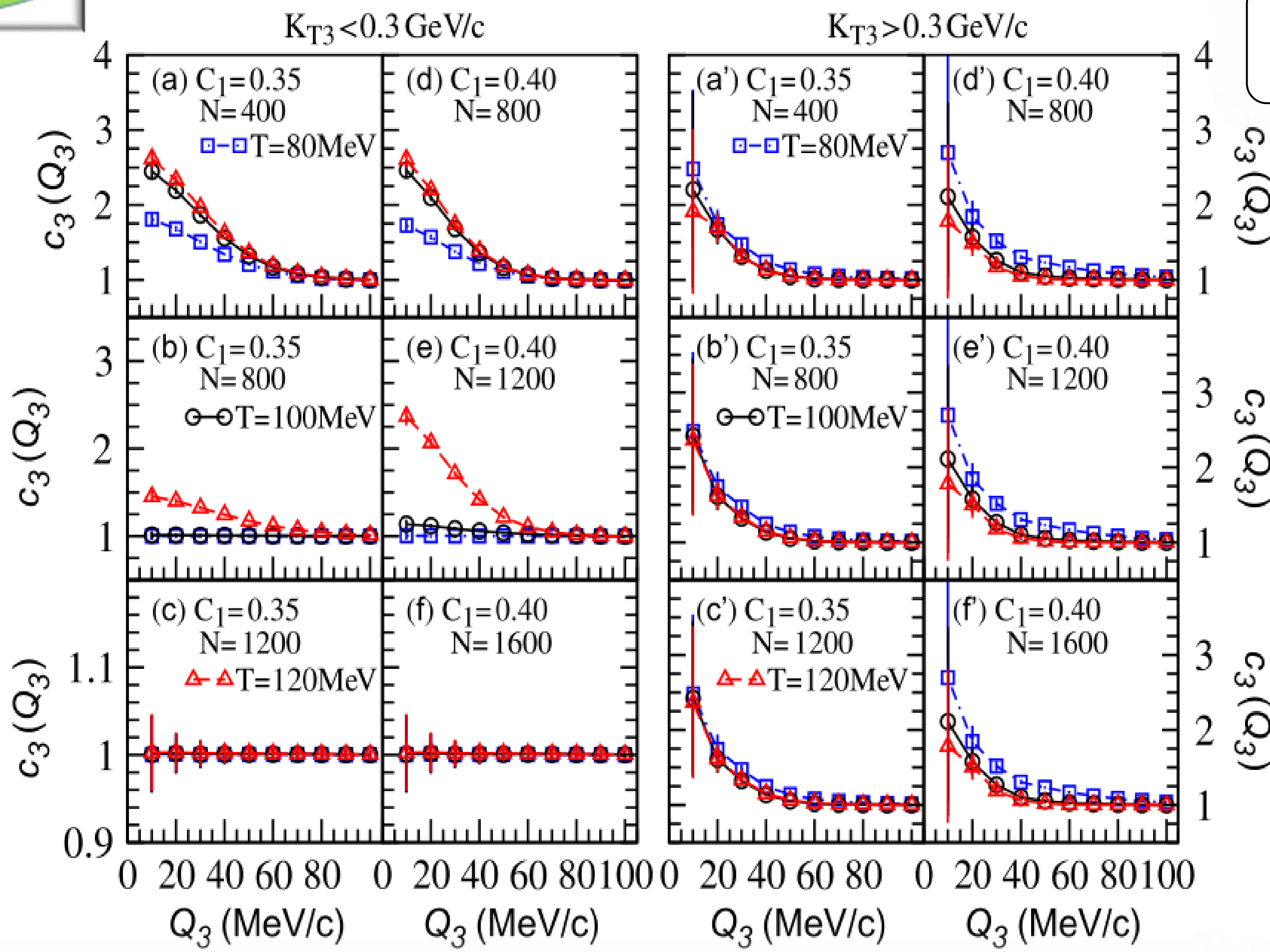
Comparison of C_3 with particle number N=1200



This indicates that the condensation effect on the correlation functions decreases with the increasing average transverse-momentum KT3 because the pions emitted chaotically from excited states have high average momentum

Three-pion cumulant correlation functions

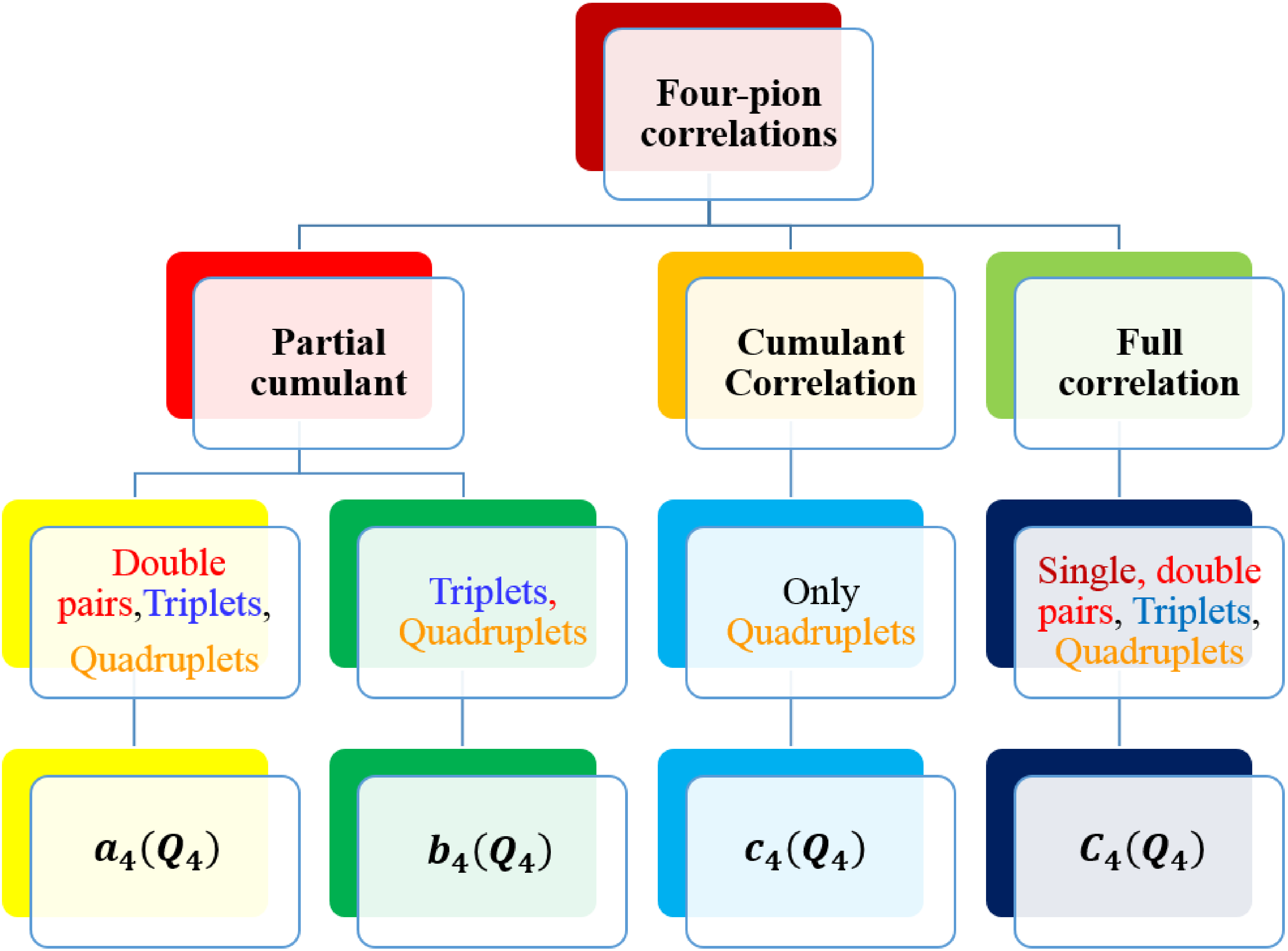
$$c_3(Q_3) = 1 + R(1,2,3)$$



Almost independent of N

The correlation from the pure pion-triplet interference, R(1, 2, 3), approaches zero when any pion pair among the three pions is uncorrelated

Classifications of Four-pion correlations



Mathematical expressions

1:

$$
\begin{aligned}
&C_4(p_1, p_2, p_3, p_4) \\
&= 1 + R(1,2) + R(1,3) + R(1,4) + R(2,3) + R(2,4) \\
&+ R(3,4) + R(1,2)R(3,4) + R(1,3)R(2,4) \\
&+ R(1,4)R(2,3) + R(1,2,3) + R(1,2,4) + R(1,3,4) \\
&+ R(2,3,4) + R(1,2,3,4) + R(1,2,4,3) + R(1,3,2,4)
\end{aligned}
$$

$$
R(i, j, k, l) = 2\frac{\mathrm{Re}\,[G^{(1)}(p_i, p_j)G^{(1)}(p_j, p_k)G^{(1)}(p_k, p_l)G^{(1)}(p_l, p_i) - N_0^4 f_4(p_i, p_j, p_k, p_l)]}{G^{(1)}(p_i, p_i)G^{(1)}(p_j, p_j)G^{(1)}(p_k, p_k)G^{(1)}(p_l, p_l)}
$$

$$
\begin{aligned}
f_4(p_i, p_j, p_k, p_l) = {} & G^{(1)}(p_i, p_j)G^{(1)}(p_j, p_k)u_0(p_i)u_0^*(p_k)|u_0(p_l)|^2/N_0^2 \\
& + G^{(1)}(p_i, p_j)G^{(1)}(p_l, p_i)u_0^*(p_j)u_0(p_l)|u_0(p_k)|^2/N_0^2 \\
& + G^{(1)}(p_j, p_k)G^{(1)}(p_k, p_l)u_0(p_j)u_0^*(p_l)|u_0(p_i)|^2/N_0^2 \\
& + G^{(1)}(p_l, p_i)G^{(1)}(p_k, p_l)u_0^*(p_i)u_0(p_k)|u_0(p_j)|^2/N_0^2 \\
& + G^{(1)}(p_l, p_i)G^{(1)}(p_j, p_k)u_0^*(p_i)u_0(p_j)u_0^*(p_k)u_0(p_l)/N_0^2 \\
& + G^{(1)}(p_i, p_j)G^{(1)}(p_k, p_l)u_0(p_i)u_0^*(p_j)u_0(p_k)u_0^*(p_l)/N_0^2 \\
& - 2G^{(1)}(p_i, p_j)u_0(p_i)u_0^*(p_j)|u_0(p_k)u_0(p_l)|^2/N_0 \\
& - 2G^{(1)}(p_j, p_k)u_0(p_j)u_0^*(p_k)|u_0(p_i)u_0(p_l)|^2/N_0 \\
& - 2G^{(1)}(p_k, p_l)u_0(p_k)u_0^*(p_l)|u_0(p_i)u_0(p_j)|^2/N_0 \\
& - 2G^{(1)}(p_l, p_i)u_0(p_l)u_0^*(p_i)|u_0(p_j)u_0(p_k)|^2/N_0 \\
& + 3|u_0(p_i)|^2|u_0(p_j)|^2|u_0(p_k)|^2|u_0(p_l)|^2.
\end{aligned}
$$

2:

$$
\begin{aligned}
&a_4(p_1, p_2, p_3, p_4) \\
&= 1 + R(1,2)R(3,4) + R(1,3)R(2,4) + R(1,4)R(2,3) \\
&+ R(1,2,4) + R(1,3,4) + R(1,2,3) + R(2,3,4) \\
&+ R(1,2,3,4) + R(1,2,4,3) + R(1,3,2,4)
\end{aligned}
$$

3:

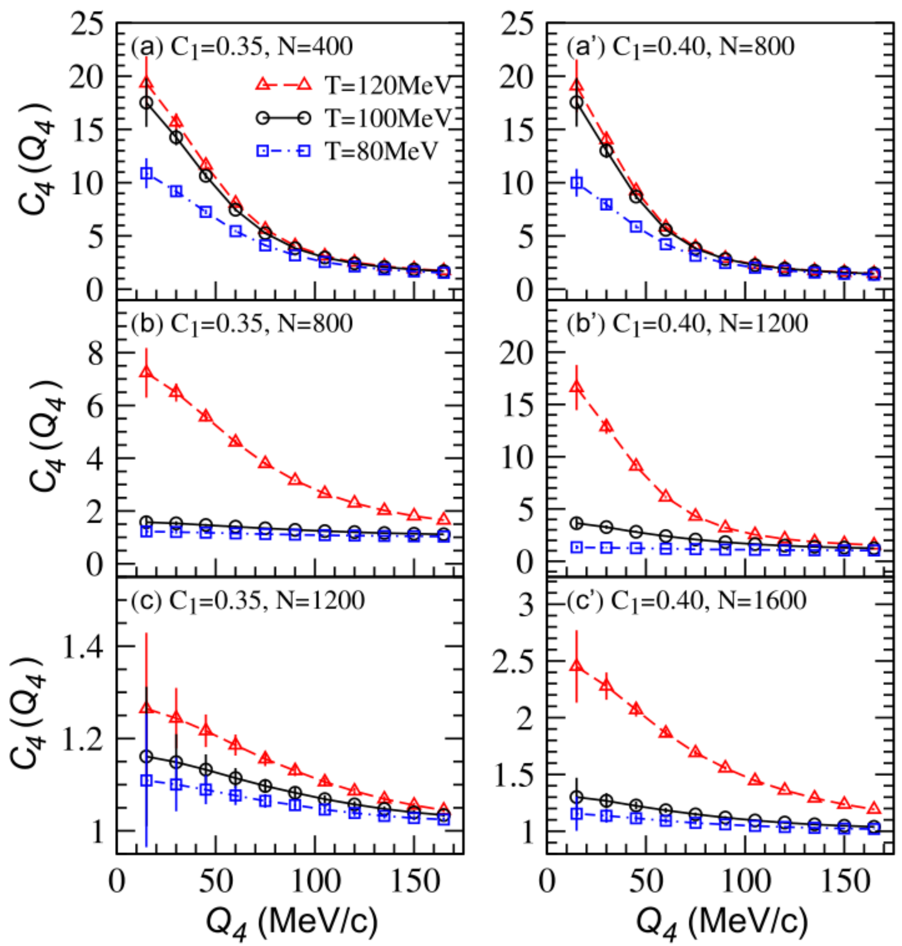
$$
\begin{aligned}
&b_4(p_1, p_2, p_3, p_4) \\
&= 1 + R(1,2,3) + R(1,2,4) + R(1,3,4) + R(2,3,4) \\
&+ R(1,2,3,4) + R(1,2,4,3) + R(1,3,2,4)
\end{aligned}
$$

4:

$$
c_4(p_1, p_2, p_3, p_4) = 1 + R(1,2,3,4) + R(1,2,4,3) + R(1,3,2,4)
$$

Four-pion correlation functions

Higher order QS correlations contain additional information about the source which cannot be learned from 2- and 3- particle correlations alone



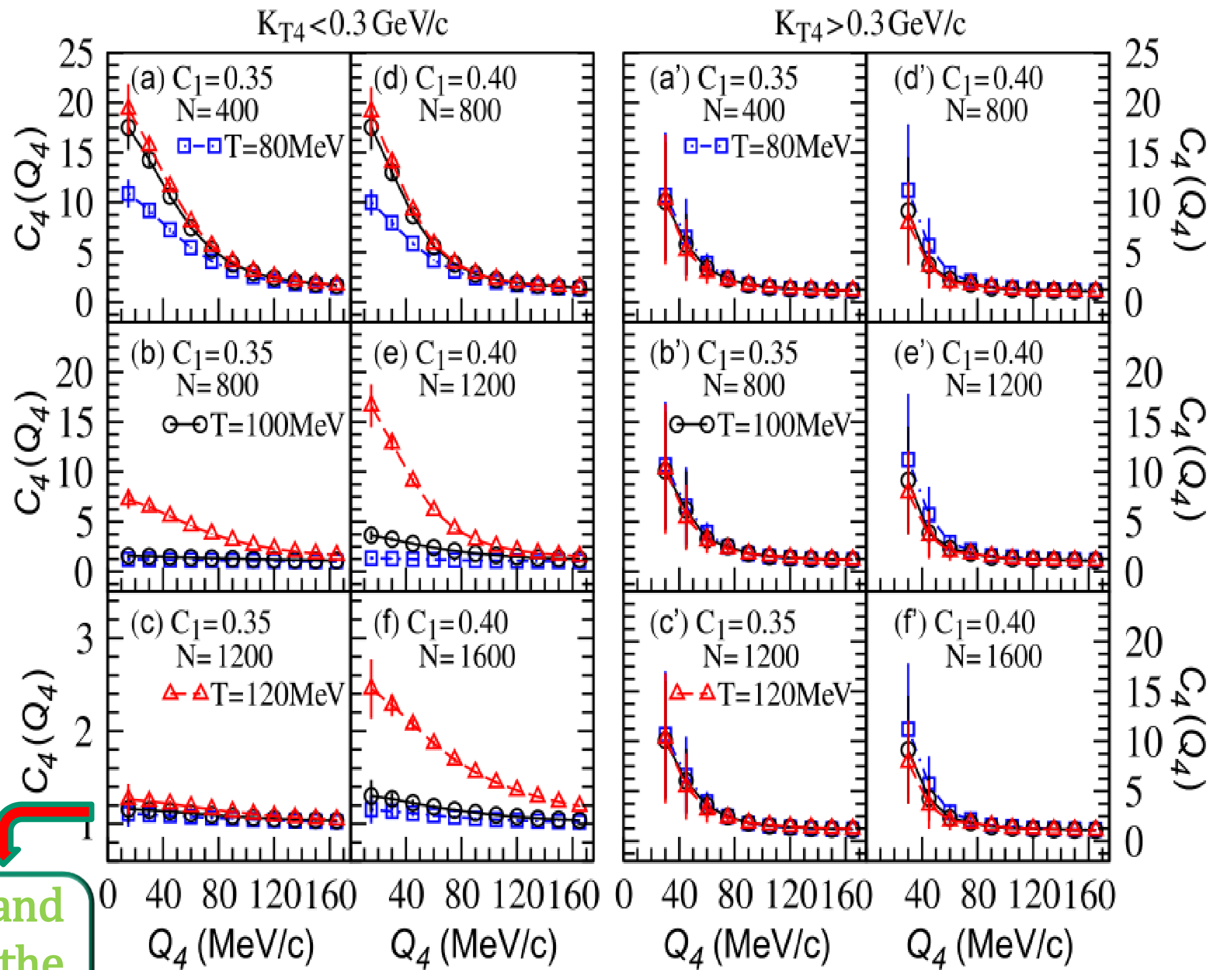
Four-pion correlations are higher than three-pion correlation functions

Because there are more contributions of the correlations of single pion pair, double pion pair, pure pion triplet and quadruplet

$$Q_4 = \sqrt{q_{12}^2 + q_{13}^2 + q_{14}^2 + q_{23}^2 + q_{24}^2 + q_{34}^2}$$

Four-pion correlations with different K_{T4} intervals

Follow experimental momentum cuts

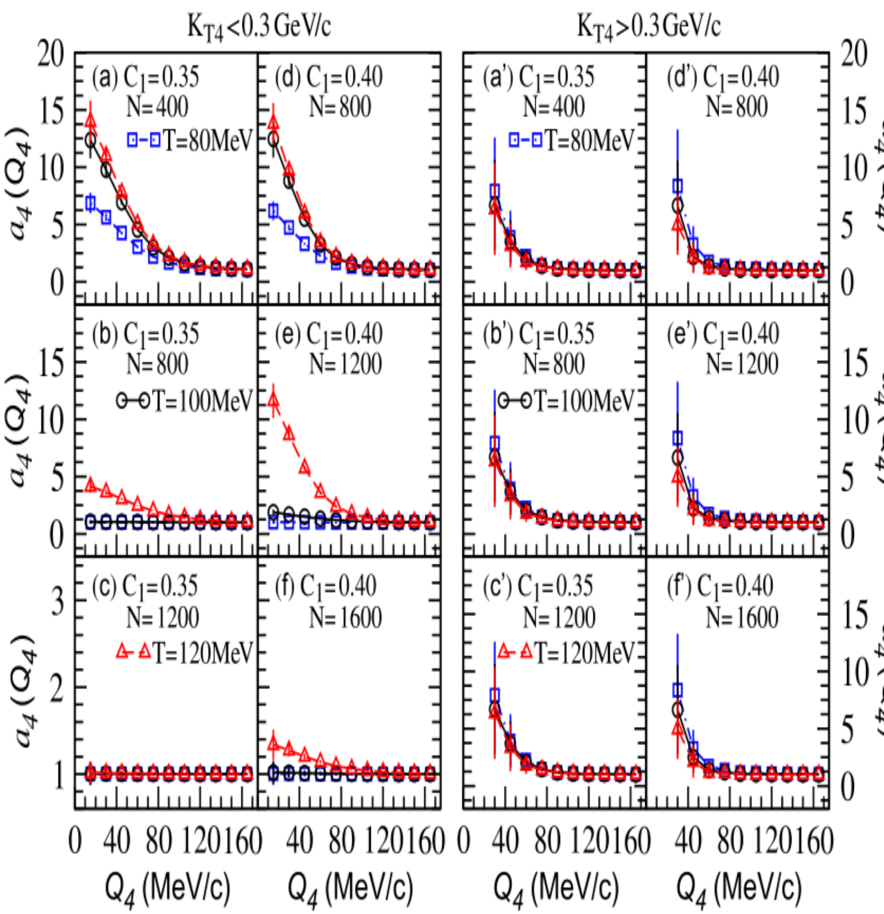


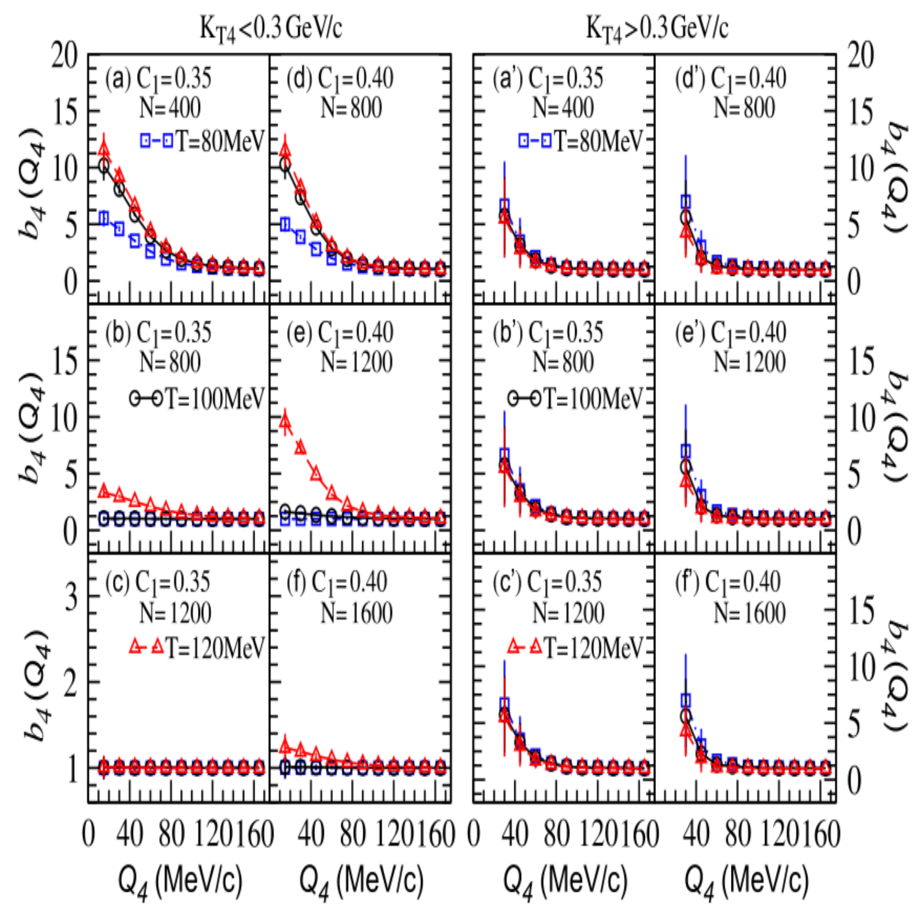
Large number of particles and low temperature welcome to the coherence in case of small source

$$K_{T4} = \frac{|p_{T1} + p_{T2} + p_{T3} + p_{T4}|}{4}$$

This finite condensation may decrease the correlation functions at low pion transverse momenta, but influence only slightly at high pion transverse momentum

Partial cumulant correlation functions



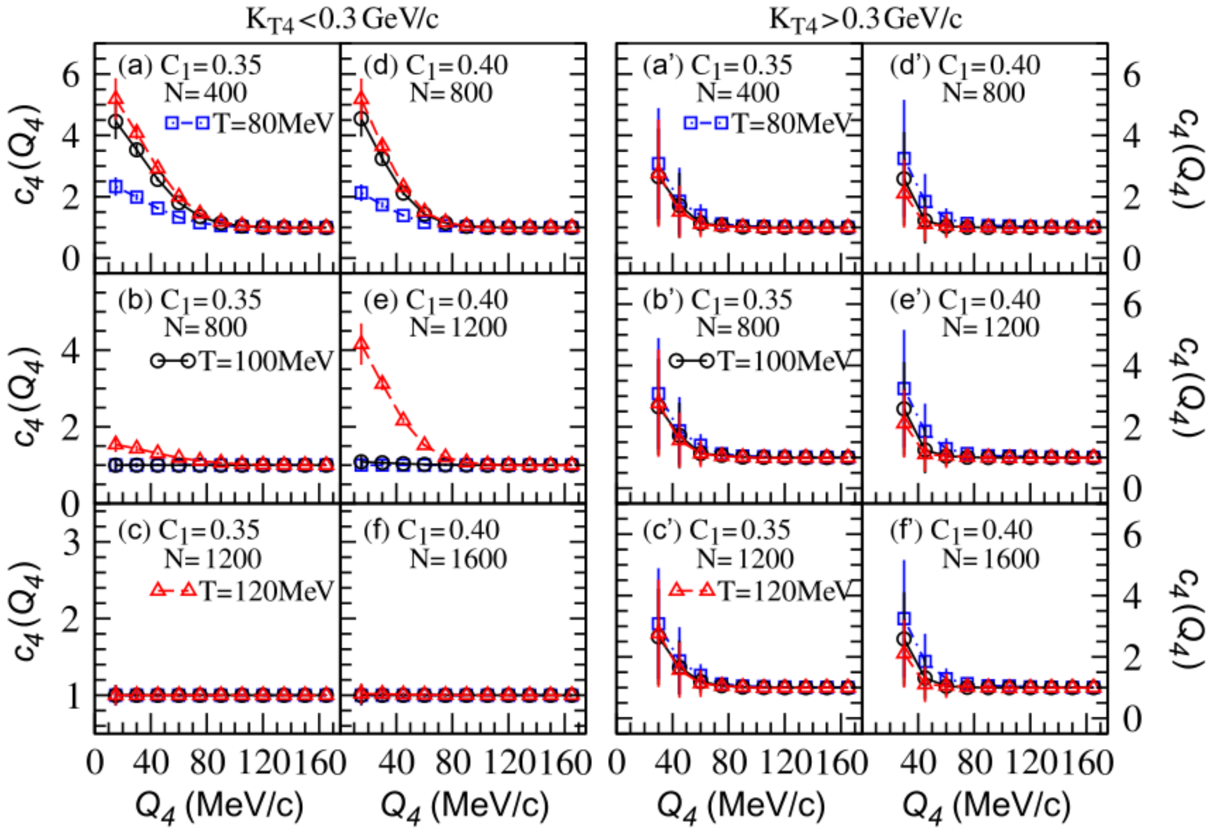


Correlations of single pair are removed

Correlations of single and double pair are removed

Cumulant correlation functions

$$c_4(p_1, p_2, p_3, p_4) = 1 + R(1,2,3,4) + R(1,2,4,3) + R(1,3,2,4)$$

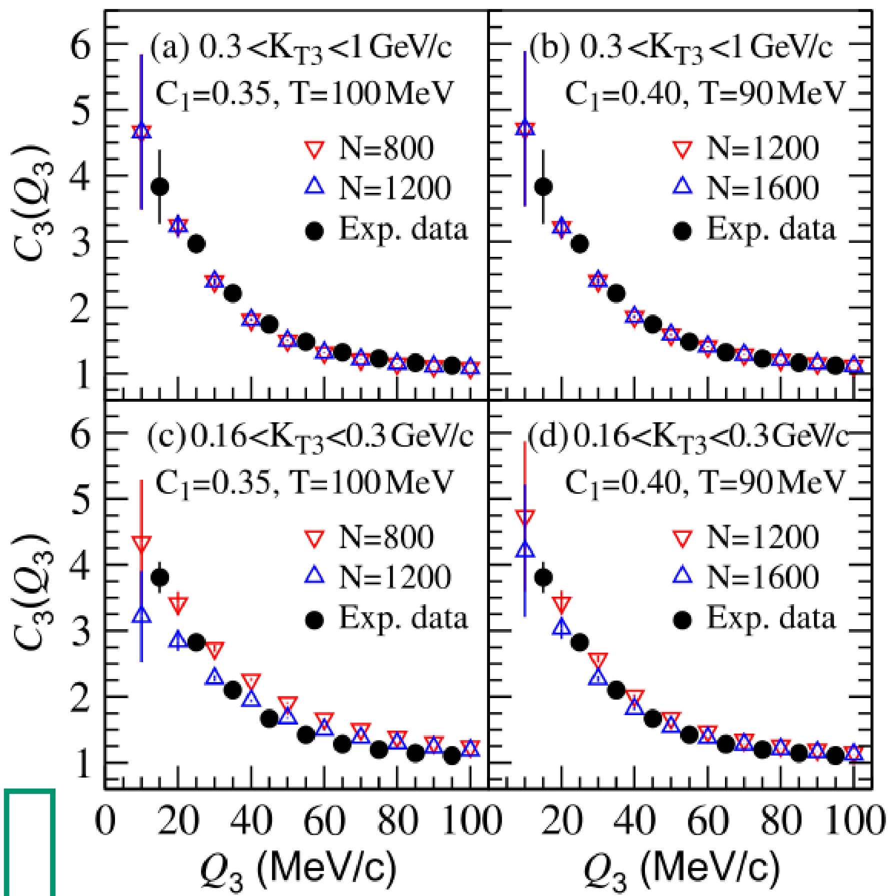


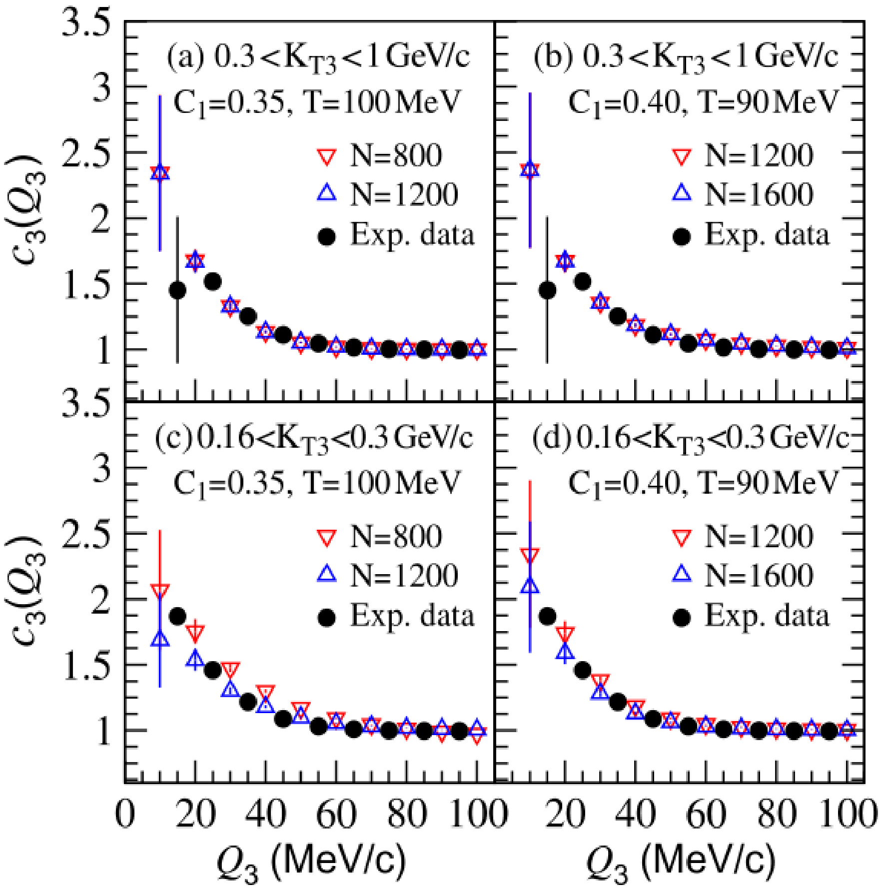
Contains only the correlations of pure quadruplet interference

c4(Q4) is more sensitive to source condensation compared to a4(Q4) and b4(Q4)

Model results with experimental data

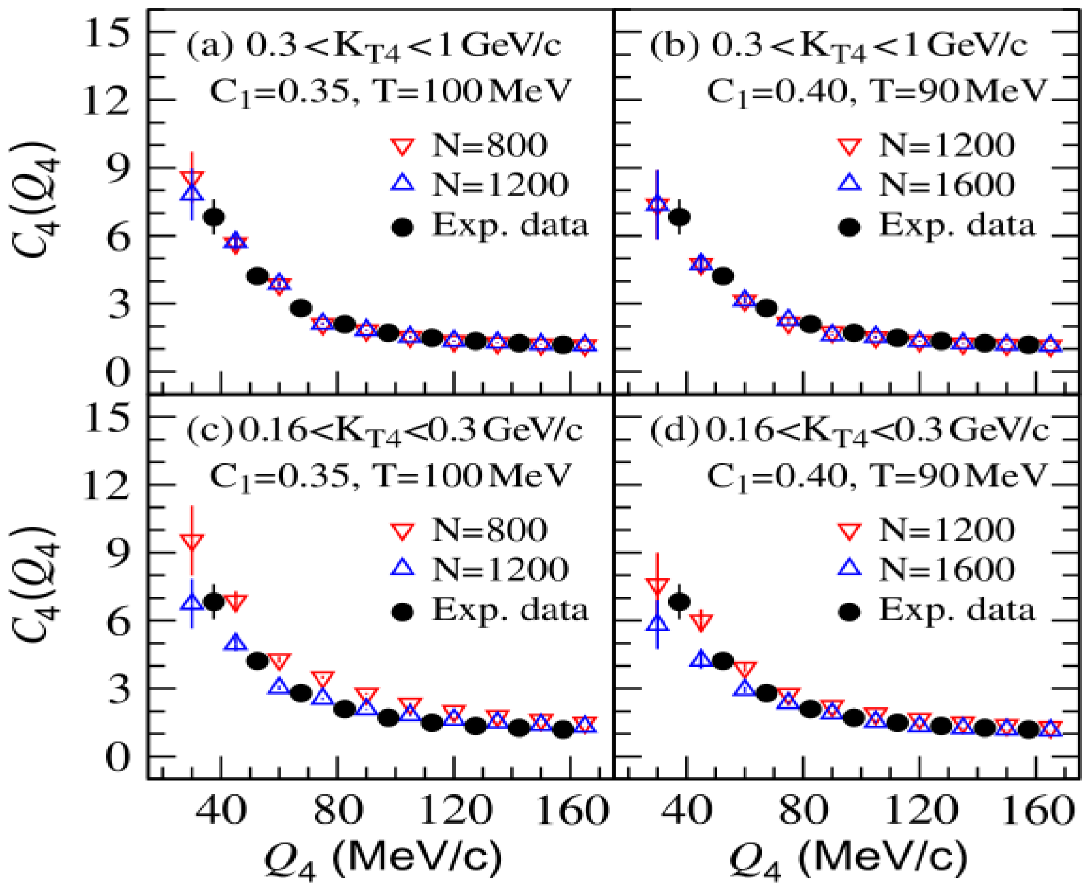
Model results and experimental data





This may be because the average longitudinal momentum of the three pions in the spherical EPG model is smaller than that in the experiment in low transverse-momentum intervals

Four-pion correlations with experimental data

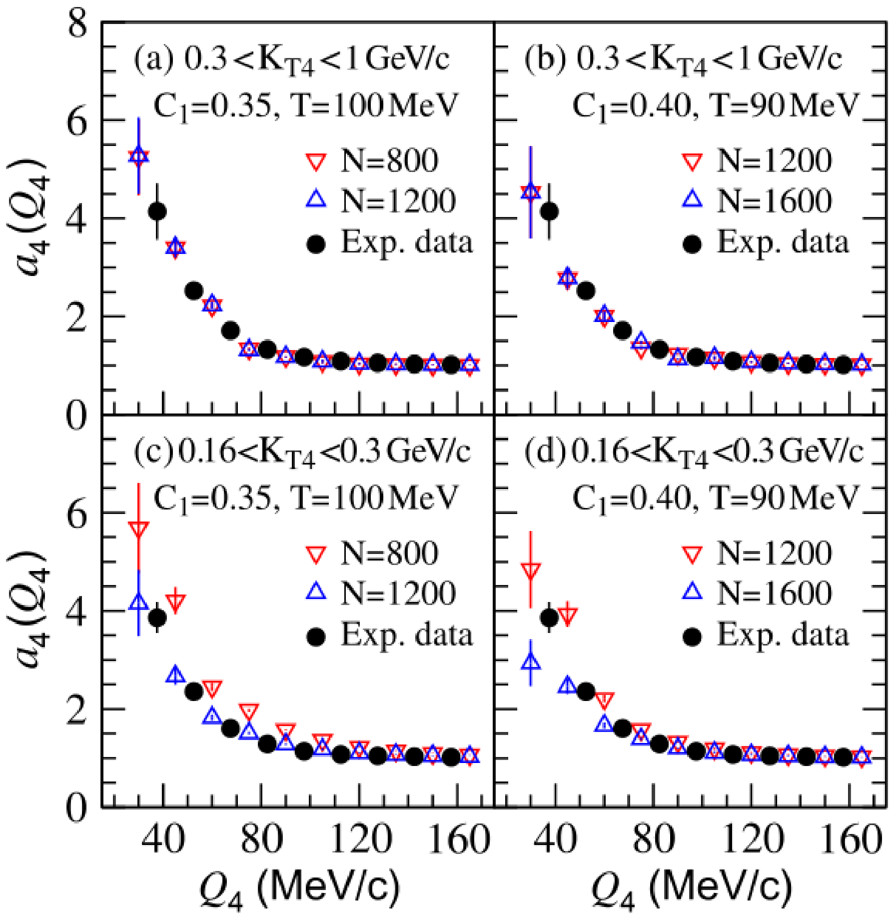


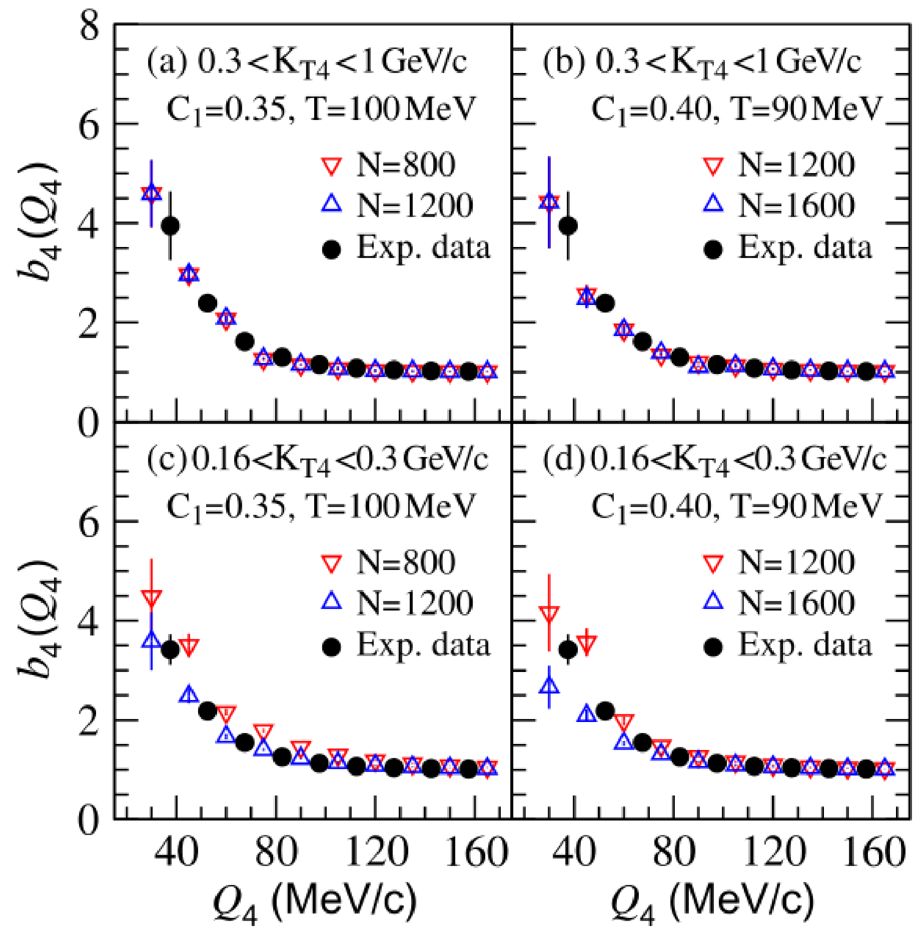
Why

In the small Q4 region the experimental results are between the model results for the sources with the small and large N

This may be because the average longitudinal momentum of the four pions in the spherical EPG model is smaller than that in the experiment in low transverse-momentum intervals

Partial cumulant correlations with experimental data



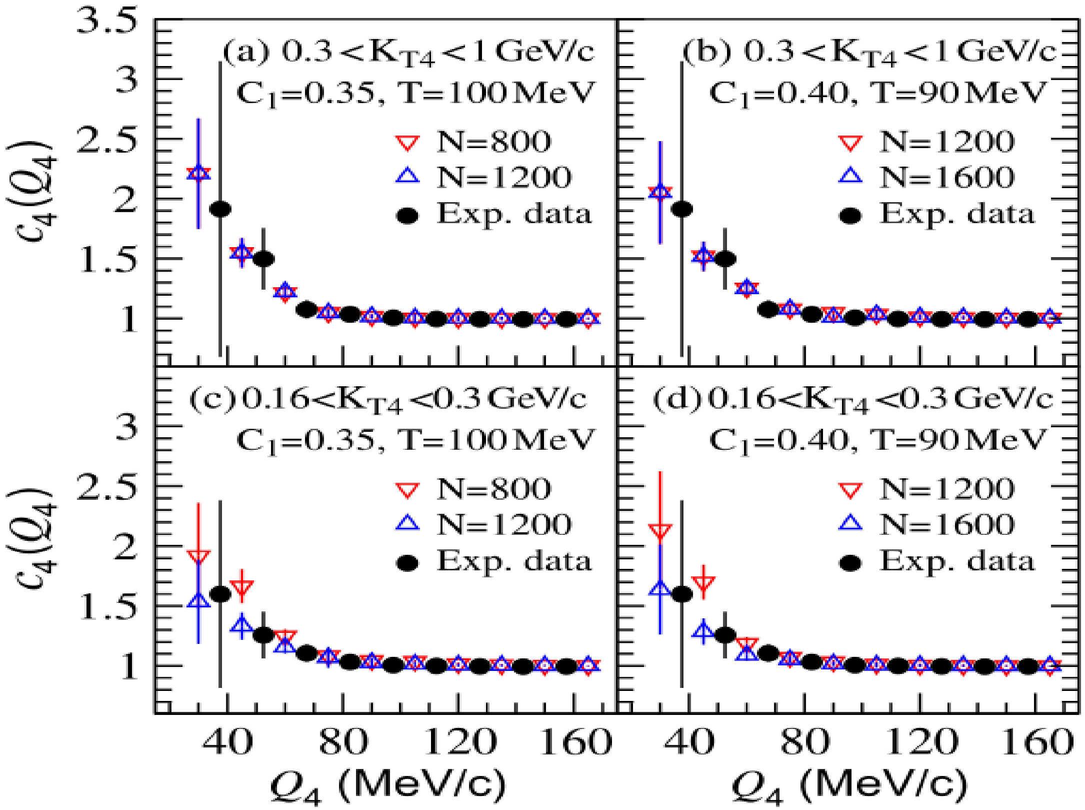


The model results in the high transverse momentum are almost independent of the source particle number N. They are constant with the experimental data in the high momentum interval

The experimental data are almost between the model results for N=1200,1600 for the sources with C1=0.40 and more consistent with the model results for N=1200 for the source with C1=0.35

Pb-Pb @ ALICE

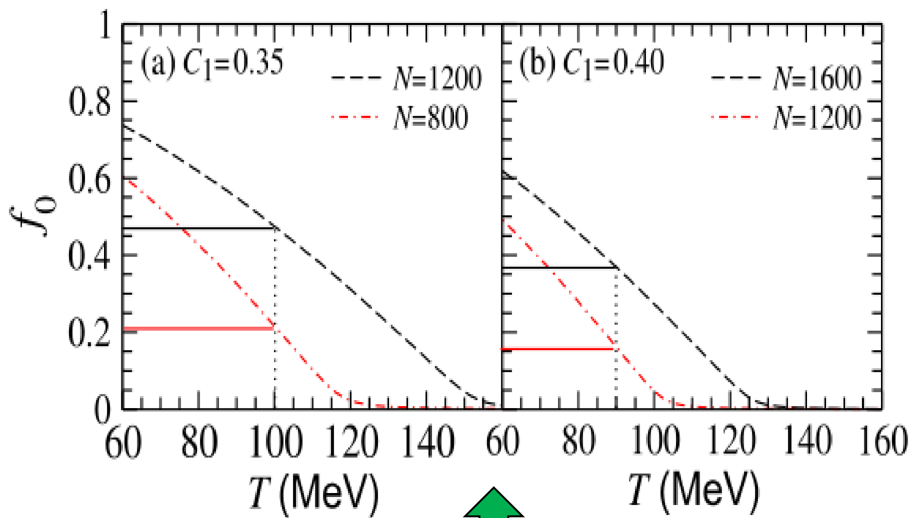
Cumulant correlations with experimental data



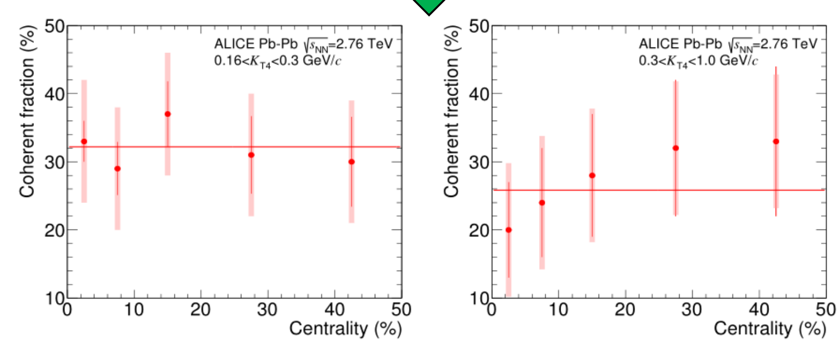
The model results are independent of the particle number of the source and consistant with the experimental data in high transverse momentum

Extracted condensation fractions

The values of condensation fraction are determined by the comparisons of the model results and experimental data of three- and four – pion correlations



They are consistent with the value of coherent fraction extracted by the ALICE collaboration



(a) $0.16 < K_{T4} < 0.3$ GeV/c (b) $0.3 < K_{T4} < 1.0$ GeV/c

In the EPG model the source size for small C1 parameter is smaller than that for a larger C1 parameter

Considering the source size is larger for central collisions than that for peripheral collisions in the experiments

Summary and outlook

We have calculated three and four-pion correlation functions and found that they are sensitive to the condensation fraction of EPG sources at low transverse momentum which is higher at large particle number and lower temperature

According to the EPG model, the condensation not only depends on the particle number which is smaller in the peripheral collisions than in central collisions, but also depends on the source size which is also smaller in peripheral collisions than in the central collisions

So the comprehensive effect of particle number and source size may lead to the result that the condensation fraction or coherent fraction is independent of collision centrality

So our model calculations explain the experimental data of both the three- and four-pion correlations

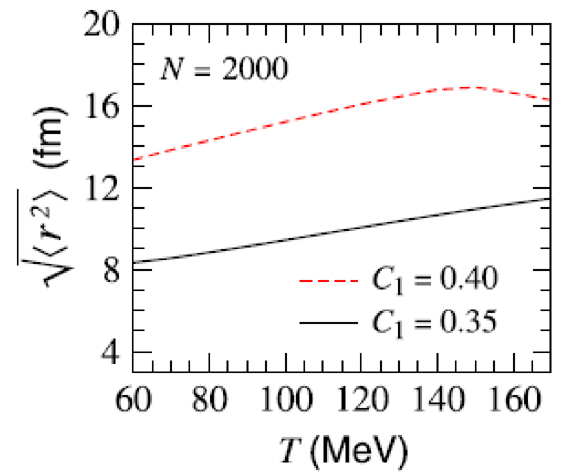
The source condensation fraction determine by the comparison is between 16% and 47%

Normalized four-pion correlation function r_4 cancel the effects of long-lived resonances, particle misidentification and experimental binning effect which we will study in future

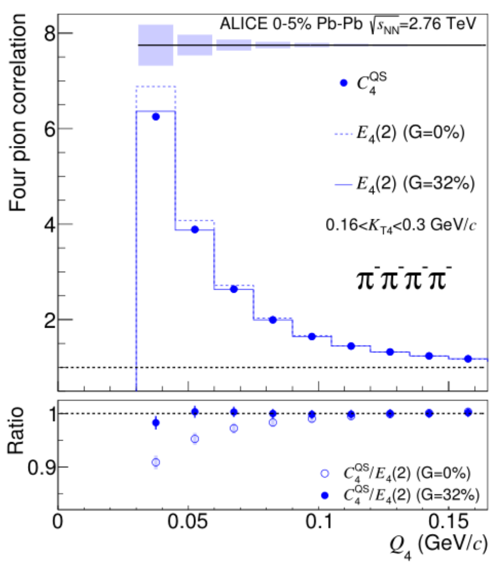
Thanks for your attention!

Backup

The RMSR of the sources for N=2000. Considering that the RMSR in Pb-Pb collisions at LHC to be on the order 10 fm, the value of parameter C1 being between 0.35 and 0.40 appear reasonable.



JPG**41(2014)**,125101



(PRC**93**,054908,2016)