

# Event by event fluctuations of the source shape: implications for the Levy shape and Event Shape Sorting

Boris Tomášik

Univerzita Mateja Bela, Banská Bystrica, Slovakia  
and FNSPE, České vysoké učení technické, Praha, Czech Republic

*[boris.tomasik@umb.sk](mailto:boris.tomasik@umb.sk)*

Results shown here are worked out by  
**Jakub Cimerman**, Renata Kopečná

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# Motivation

- Each collision event evolves differently and has different final state
  - Measured quantities are usually averaged over a large number of events
- 
- **Part 1: Influence on the shape of the correlation function**
  - **Part 2: Can we have a more exclusive selection of events?**

# Part 1: Correlation function from event averaging

see also:

A. Białas, Nucl. Phys. A 525 (1991) 345-360

C. Plumberg, U. Heinz, Phys. Rev. C 92 (2015) 044906 and add. 92 (2015) 049901

Non-Gaussian measured  $C(q)$  is fitted with Levy stable distribution

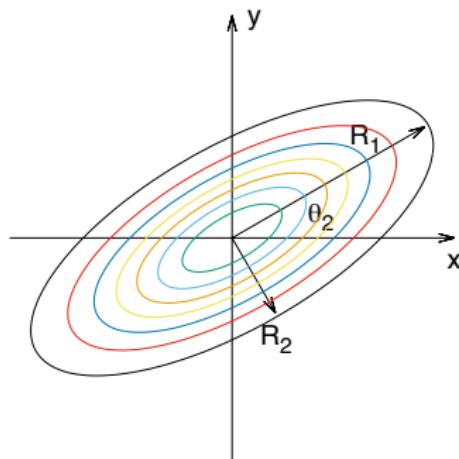
$$C(q) - 1 = \exp(-|Rq|^\alpha)$$

- (HBT) correlation function is characteristic for each source (event)
- the sizes of the sources and their orientation fluctuate
- the **measured** correlation function results from a weighted average of correlation functions from each event
- observed shape of the correlation function results from:
  - shapes of the individual sources
  - distribution of sizes and orientations (event plane angles)

Can we get Levy-shaped correlation function from averaging?

# Exercise 1: Levy-shaped $C(q)$ from Gaussian sources

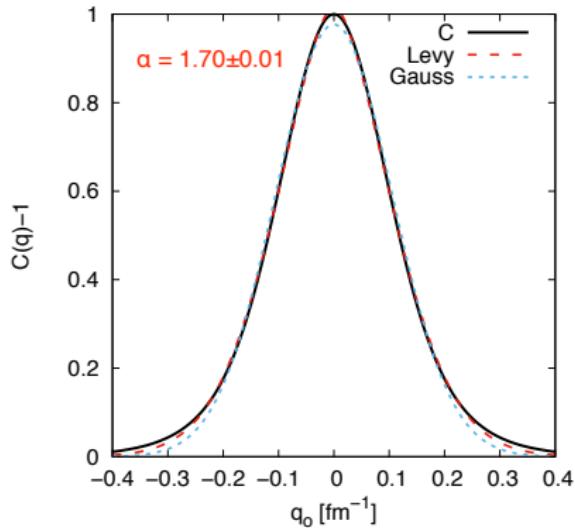
Toy model: elliptic Gaussian source



Averaging  $C(q)$ 's over:

- $R_1$  and  $R_2$  distributed like in optical Glauber model
- $\theta_2$  distributed uniformly

Averaged correlation function



# Extension of the blast-wave model to higher orders

The emission function:

$$S(x, p) d^4x = \frac{m_t \cosh(Y - \eta)}{(2\pi)^3} d\eta \, r dr \, d\varphi \frac{\tau \, d\tau}{\sqrt{2\pi \Delta \tau^2}} \\ \times \exp\left(-\frac{(\tau - \tau_0)^2}{2\Delta \tau^2}\right) \exp\left(-\frac{p^\mu u_\mu}{T}\right) \Theta(1 - \bar{r})$$

Transverse size

$$\bar{r} = \frac{r}{R(\varphi)} \quad R(\varphi) = R_0 \left( 1 - \sum_{n=2}^{\infty} \textcolor{red}{a_n} \cos(n(\varphi - \varphi_n)) \right)$$

Transverse expansion goes into  $u_\mu$ , parametrized by transverse rapidity

$$\rho(\bar{r}, \varphi) = \bar{r} \rho_0 \left( 1 + \sum_{n=2}^{\infty} 2\textcolor{red}{\rho_n} \cos(n(\varphi - \varphi_n)) \right)$$

## Exercise 2: Levy-shaped $C(q)$ from blast-wave sources

The model: blast-wave emission function  
with 2nd order anisotropy.

$$R_0 = 7 \text{ fm}, T = 120 \text{ MeV}, \rho_0 = 0.8$$

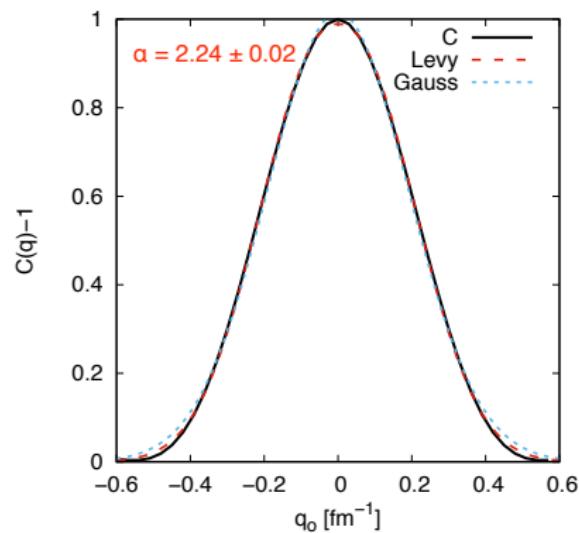
anisotropy parameters (flow and spatial)

$$\rho_2 = a_2 = 0.2$$

Averaging  $C(q)$ 's over:

- $\theta_2$  distributed uniformly
- $p_t$  from 0 to 1 GeV

Averaged correlation function



## Conclusion: Levy shapes

- The averaging of the correlation functions over many events may have an influence on the shape of the measured correlation function.
- The exact value of the Levy parameter depends on details of the source shape and the distribution of the source parameters.

## Part 2: Event Shape Sorting and femtoscopy

- try to avoid extensive averaging over very different events by identifying events with similar distribution of final state hadrons and taking averages over them only
- useful for data analyses of observables which depend on fireball shape
  - jet quenching
  - jet shapes
  - anisotropic flow of heavy flavors
  - azimuthal dependence of correlation radii
  - ...
- useful for the comparison with theoretical simulations
  - hydrodynamic simulations with specific geometry
  - identification of rare events

# Event Shape Engineering

- Two subevents
  - Subevent *a*: event selection
  - Subevent *b*: physical analysis
- Helps avoiding nonphysical biases (nonflow effects)
- Information loss
- Event selection according to the magnitude of the **reduced flow vector**  $q_n$

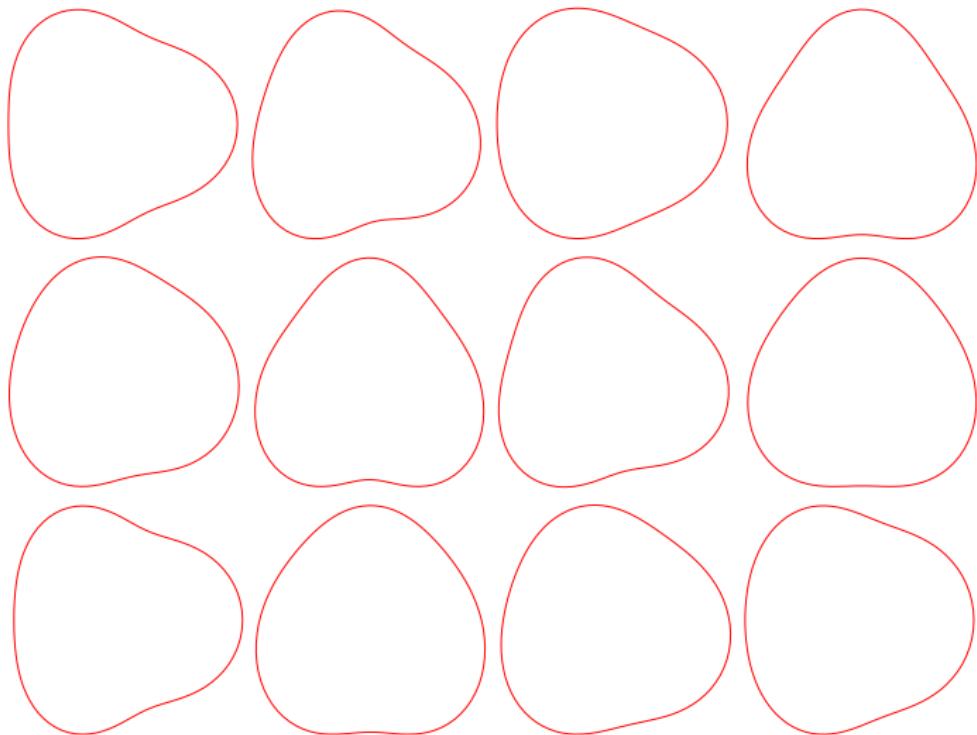
$$\vec{Q}_n = \left( \sum_{i=1}^M \cos(n\phi_i), \sum_{i=1}^M \sin(n\phi_i) \right),$$

$$q_n = |\vec{Q}_n|/\sqrt{M}.$$

[J. Schukraft, A. Timmins, S. A. Voloshin, Phys. Lett. B 719 (2013) 394-398]

# Event shapes

How to do Event Shape Engineering among these shapes...?



... ordered

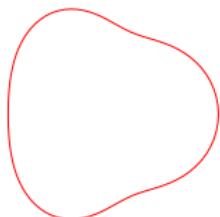
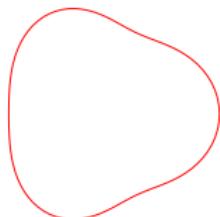
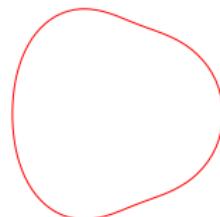
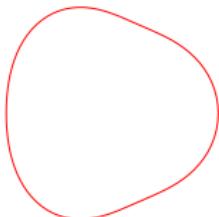
$$\Psi_{23} = 0$$

$$\begin{array}{c} v_2 = 0.04 \\ v_3 = 0.04 \end{array}$$

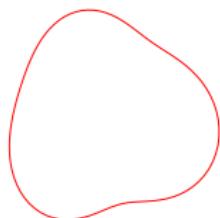
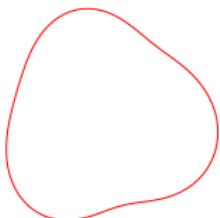
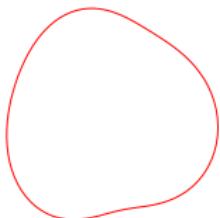
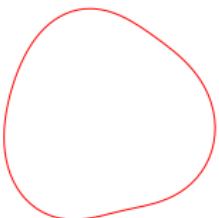
$$\begin{array}{c} v_2 = 0.06 \\ v_3 = 0.04 \end{array}$$

$$\begin{array}{c} v_2 = 0.04 \\ v_3 = 0.06 \end{array}$$

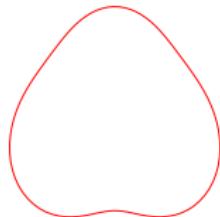
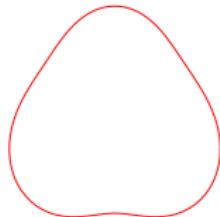
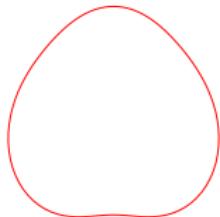
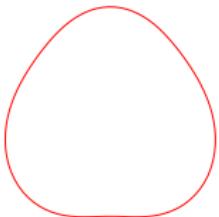
$$\begin{array}{c} v_2 = 0.06 \\ v_3 = 0.06 \end{array}$$



$$\Psi_{23} = 0.7$$



$$\Psi_{23} = 1.57$$



# Event Shape Sorting

- iteratively sorts events in such a way, that events with similar histograms (in azimuthal angle, e.g.) end up close to each other
- divides the totality of events into (customarily 10) event bins
- no need to specify a sorting variable, unlike Event Shape Engineering [J. Schukraft, A. Timmins, S. A. Voloshin, Phys. Lett. B 719 (2013) 394-398]

Algorithm based on:

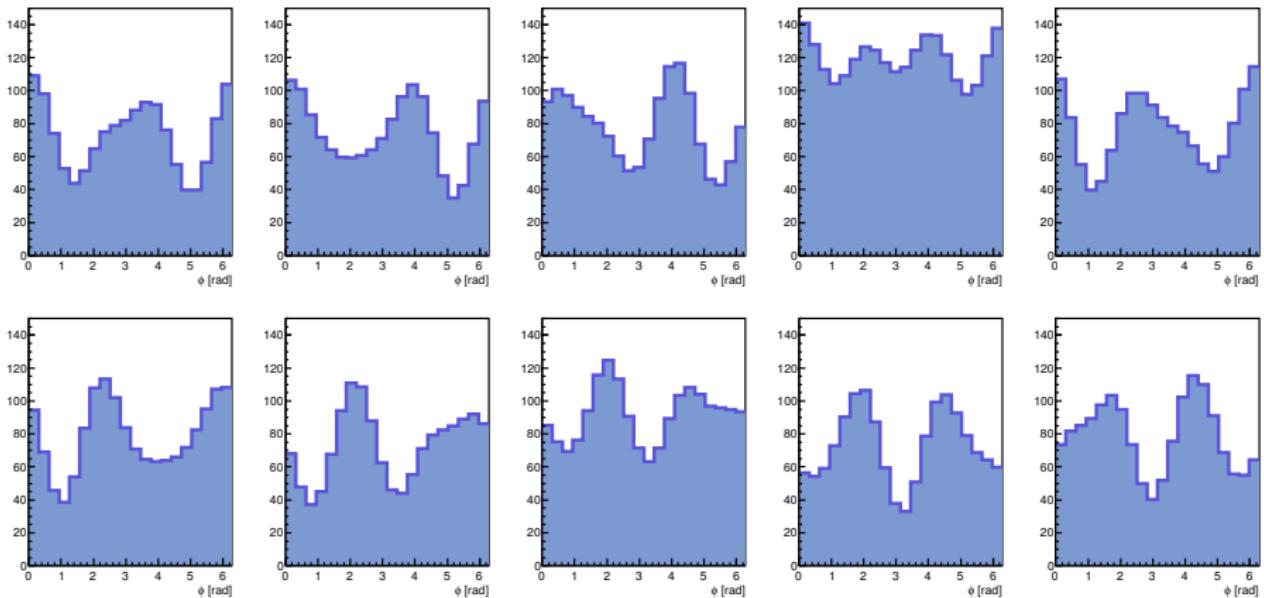
S. Lehmann, A.D. Jackson, B. Lautrup, arXiv:physics/0512238

S. Lehmann, A. D. Jackson and B. E. Lautrup, Scientometrics **76** (2008) 369  
[physics/0701311 [physics.soc-ph]]

Published in

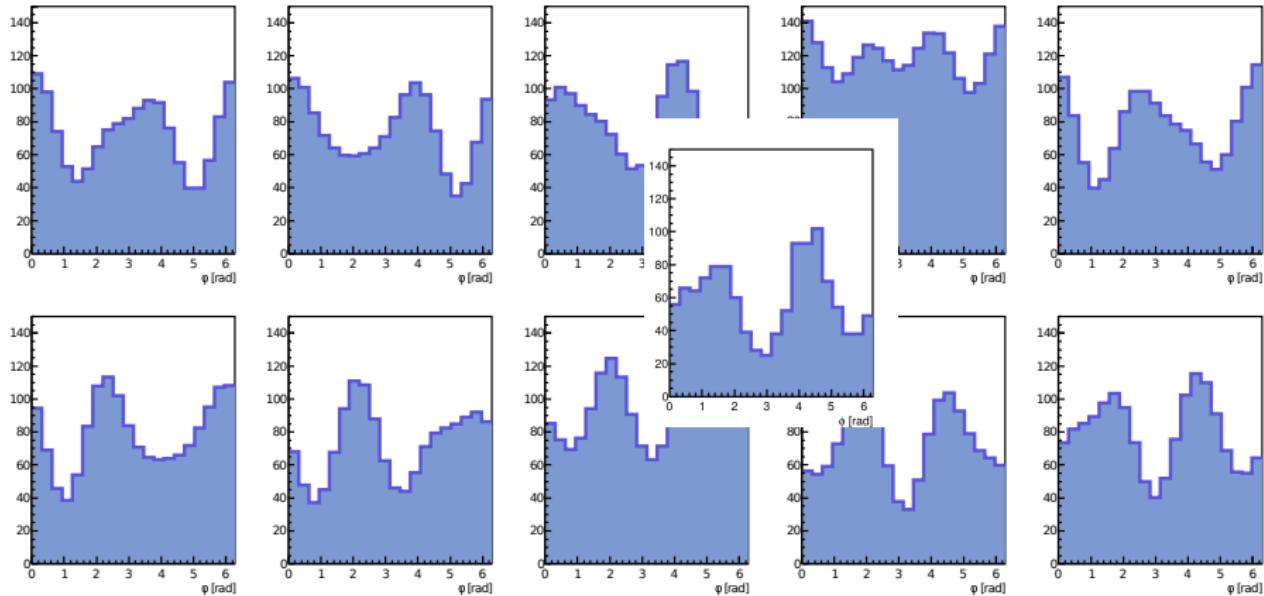
R. Kopečná, B. Tomášik: Eur. Phys. J. A **52** (2016) 115.

# Assigning event to event bin



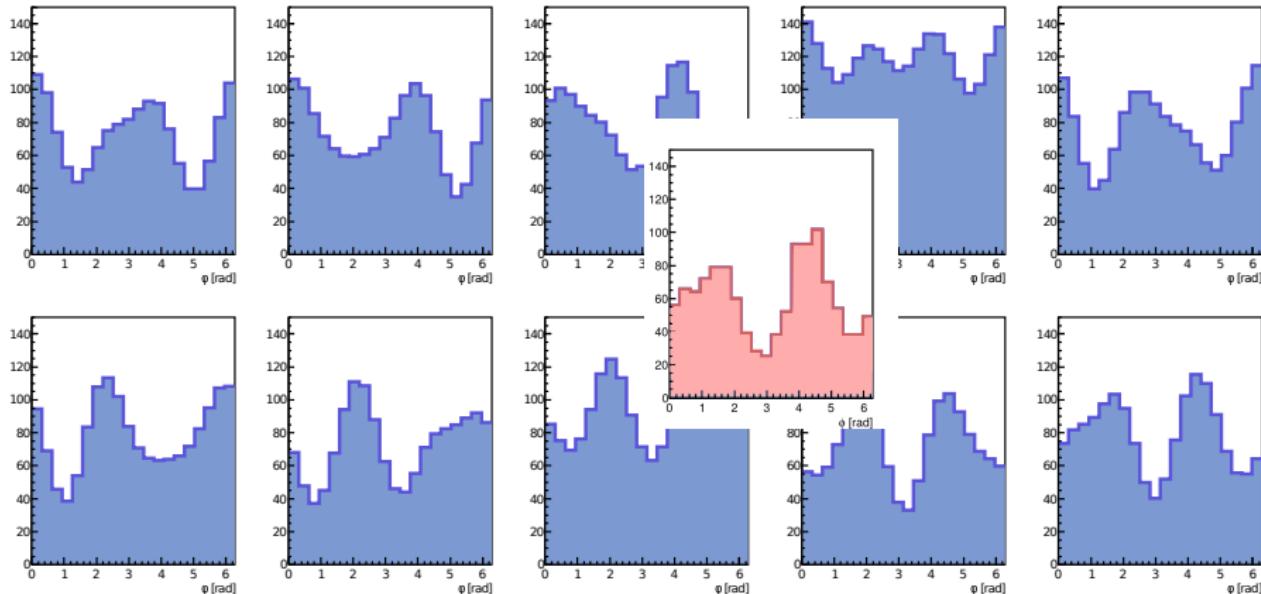
# Assigning event to event bin

To which event bin is this event similar?



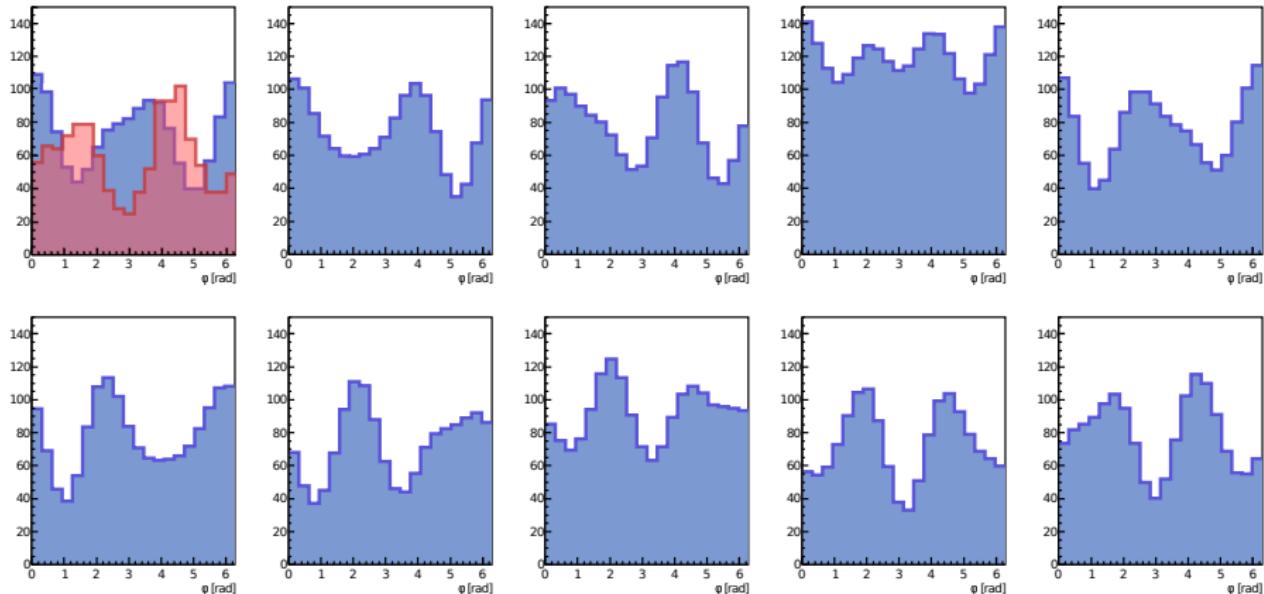
# Assigning event to event bin

Calculate Bayesian probability that the event belong to each event bin



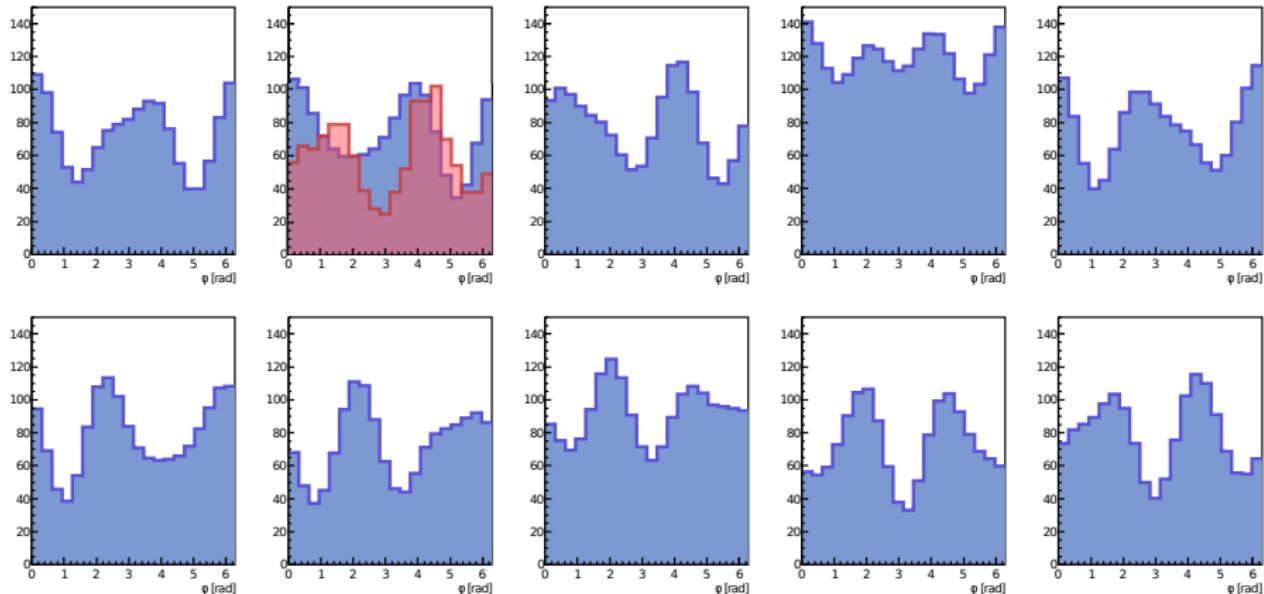
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 1



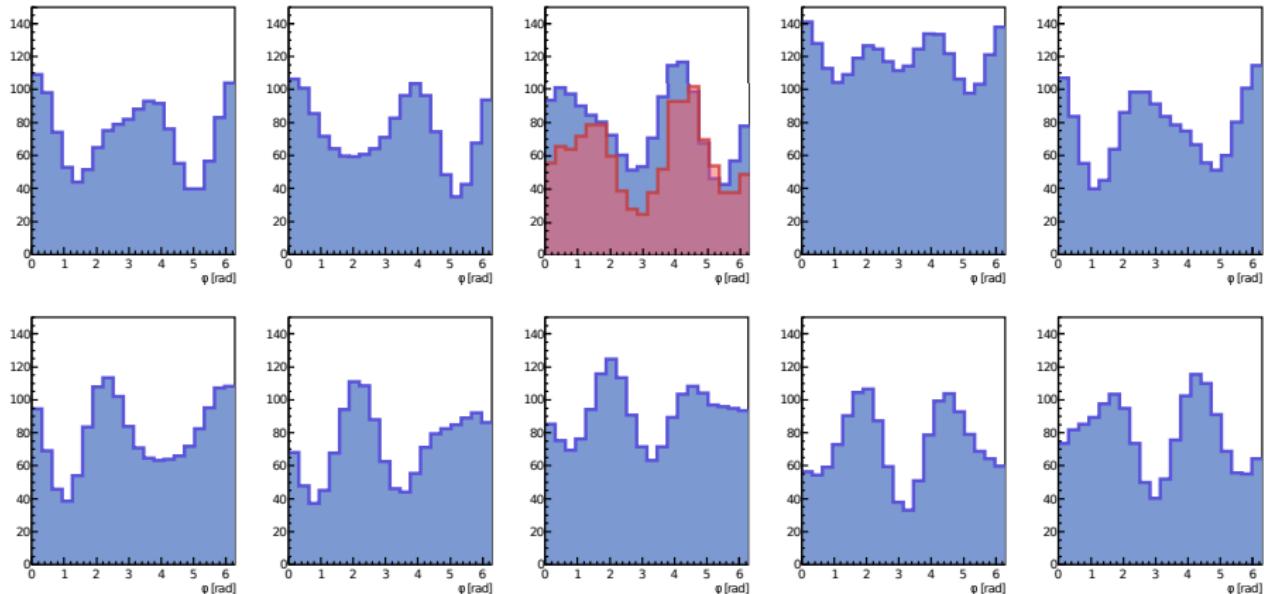
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 2



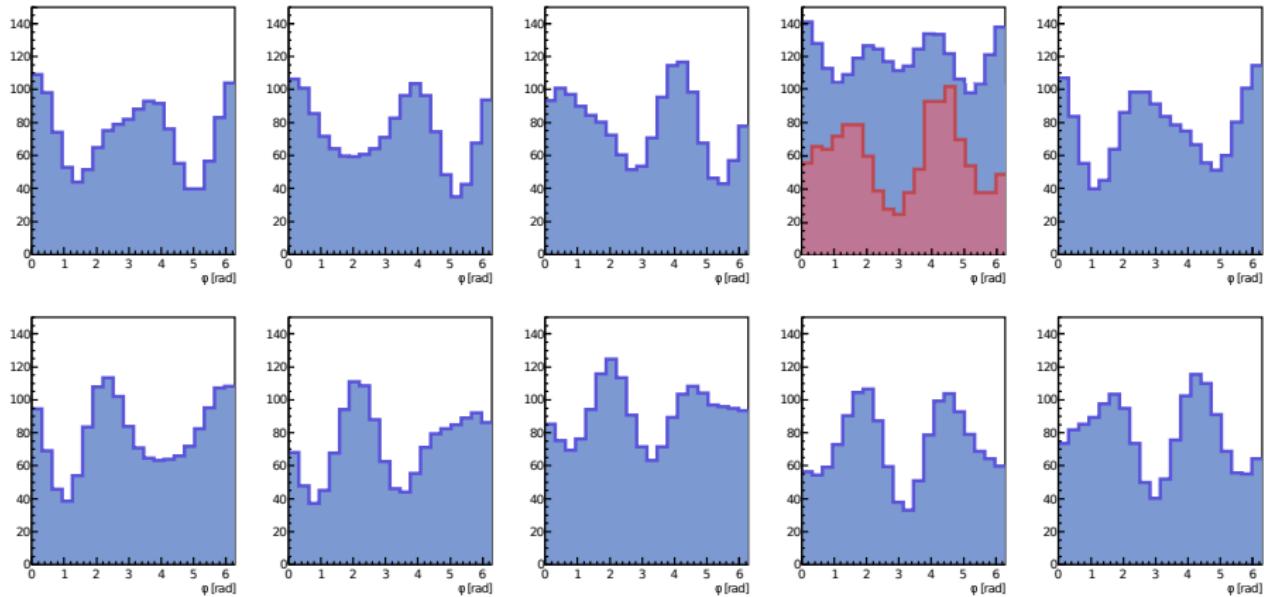
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 3



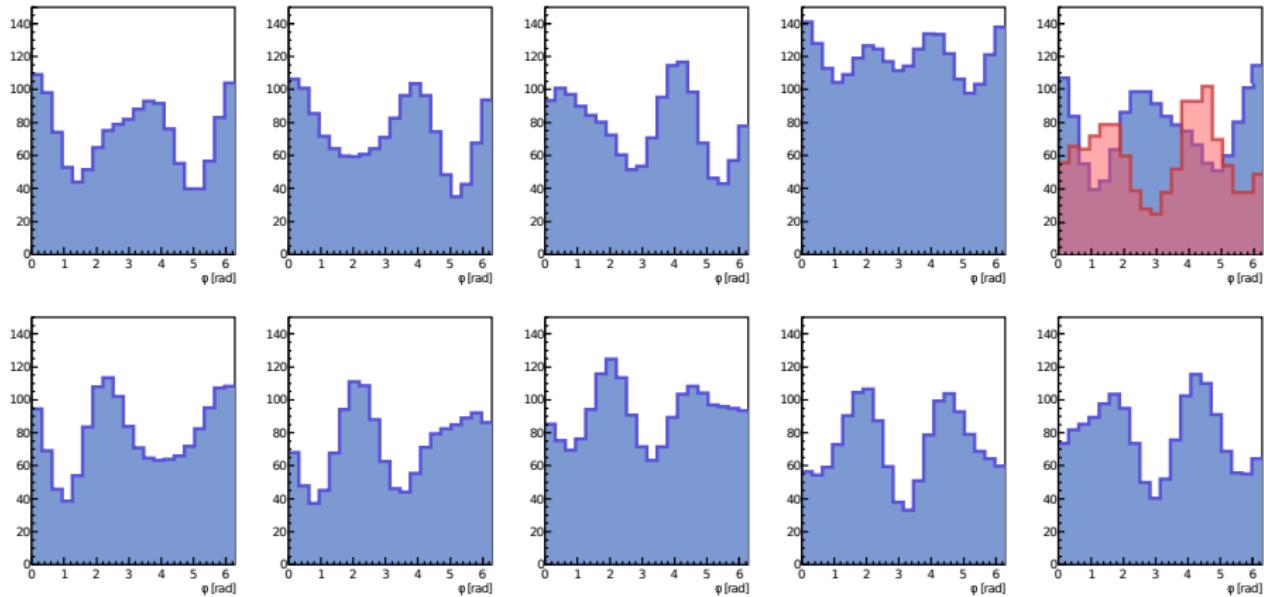
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 4



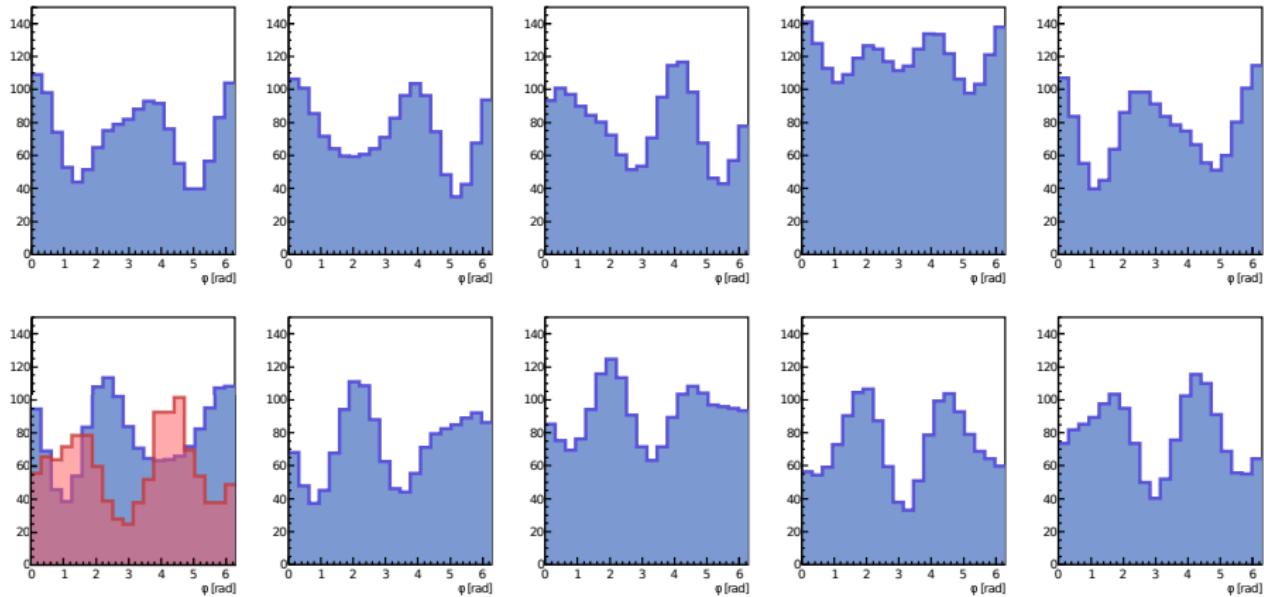
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 5



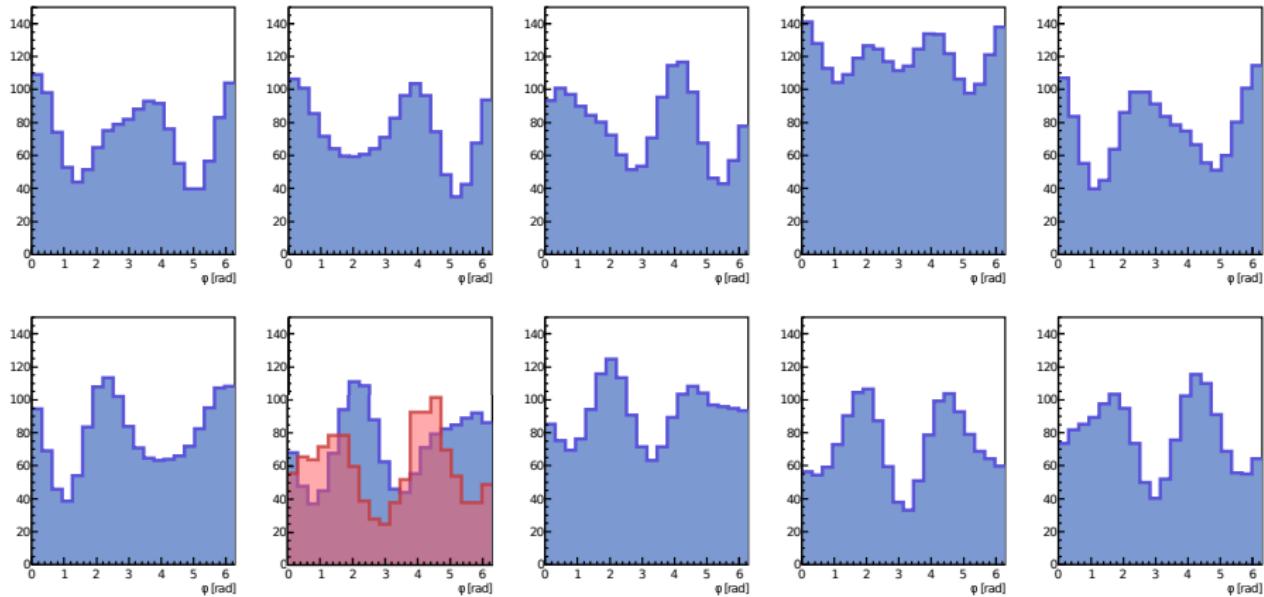
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 6



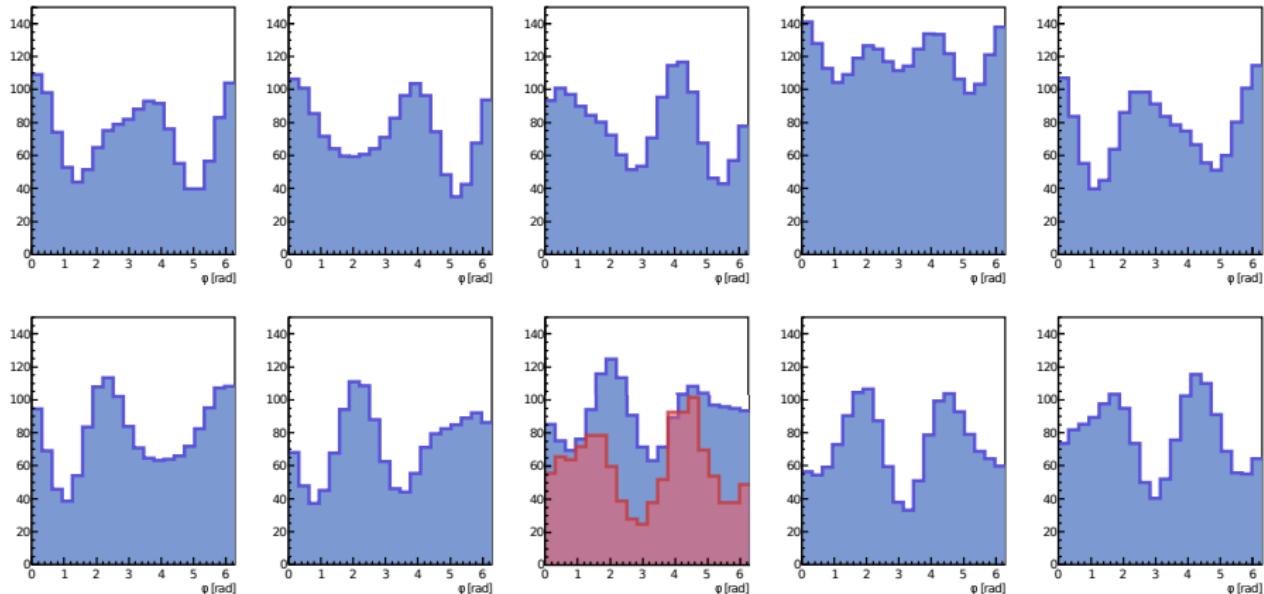
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 7



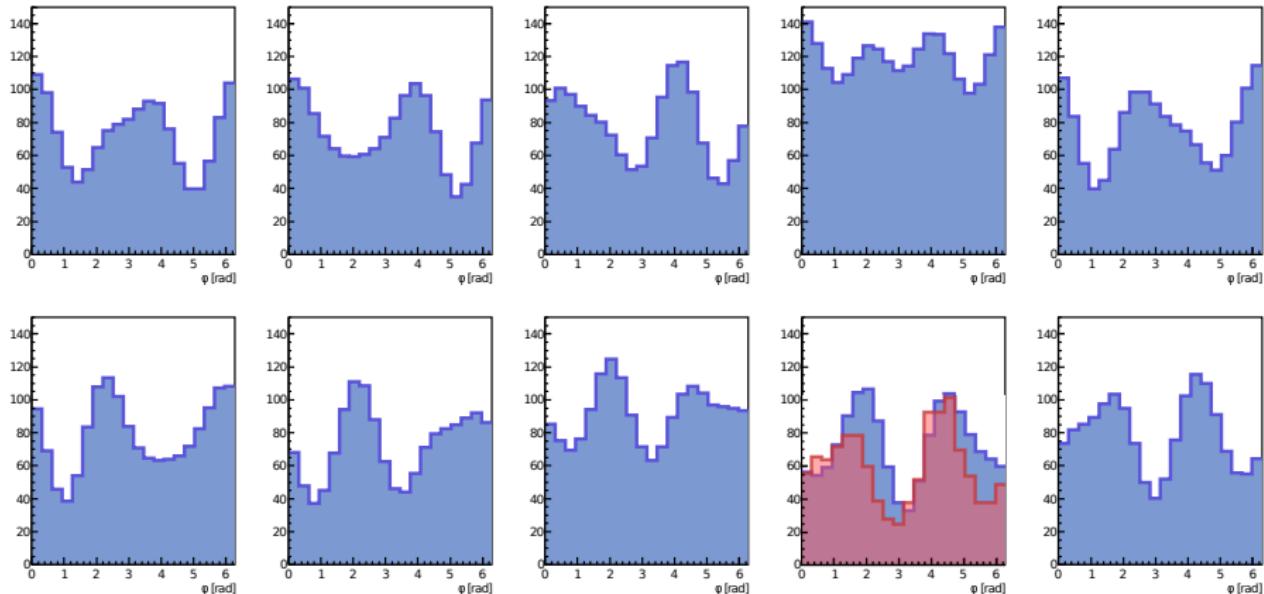
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 8



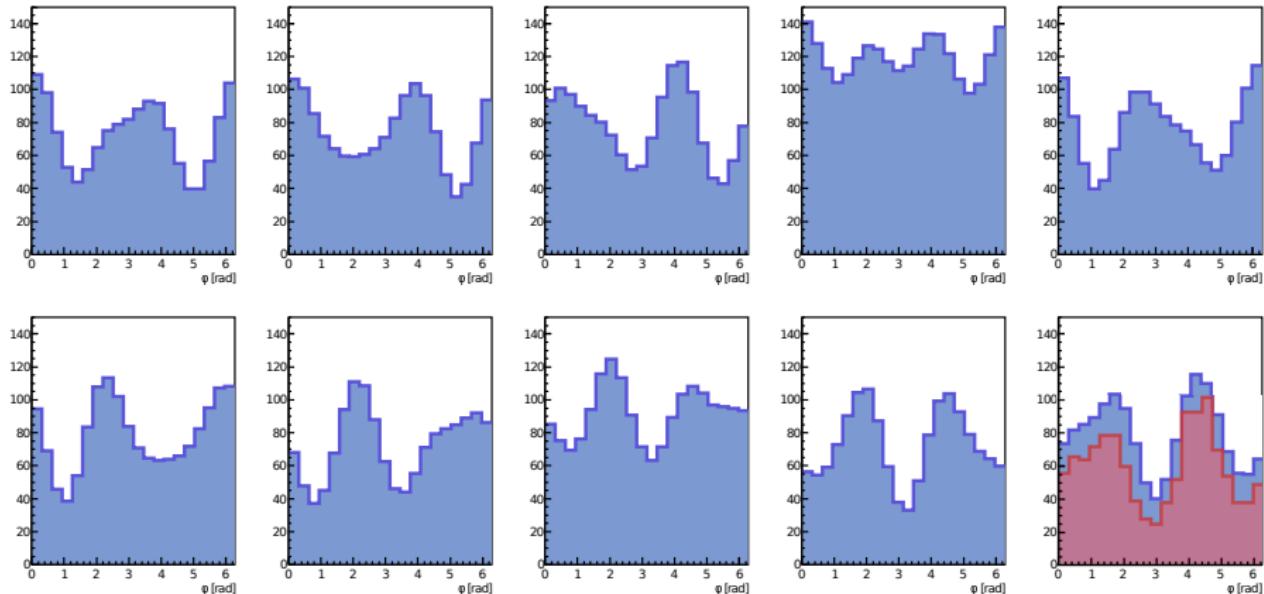
# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 9



# Assigning event to event bin

Calculate Bayesian probability that the event belong to event bin 10



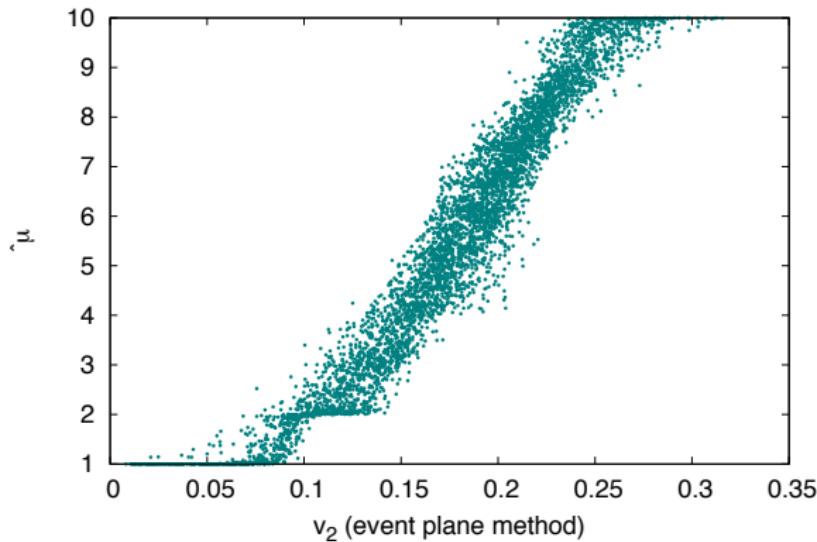
# Iterations of event bins

## Sorted events: Gradual change of event shape

- 2000 events, AMPT centrality 0–20%,  $\sqrt{s_{NN}} = 2.76$  TeV
- each frame averaged over 50 events and shifted by 10 events wrt previous frame
- change of colour = change of event bin

## Toy model: only elliptic flow

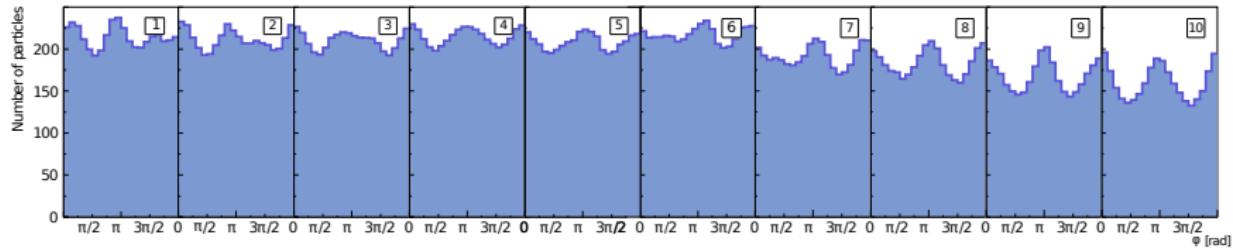
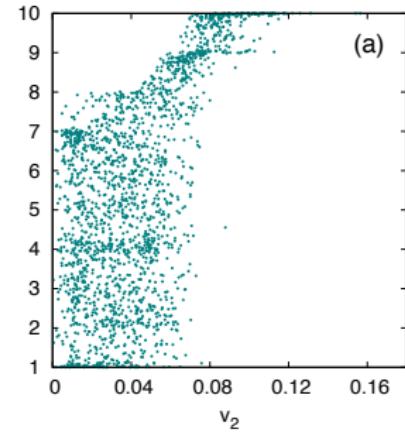
- in the toy model, azimuthal distribution of pions with only elliptic flow is generated
- correlation between  $v_2$  and  $\mu$ : 0.959
- $v_2$  is good sorting variable



# Sorted AMPT central events (LHC energy)

Event shape sorting goes beyond characterisation of events according to single variable (e.g.  $v_2$  or  $q_2$ )

- simulated 2000 central 0–20% events from AMPT for  $\sqrt{s_{NN}} = 2.76$  TeV
- correlation between sorting variable  $\mu$  and elliptic flow  $v_2$

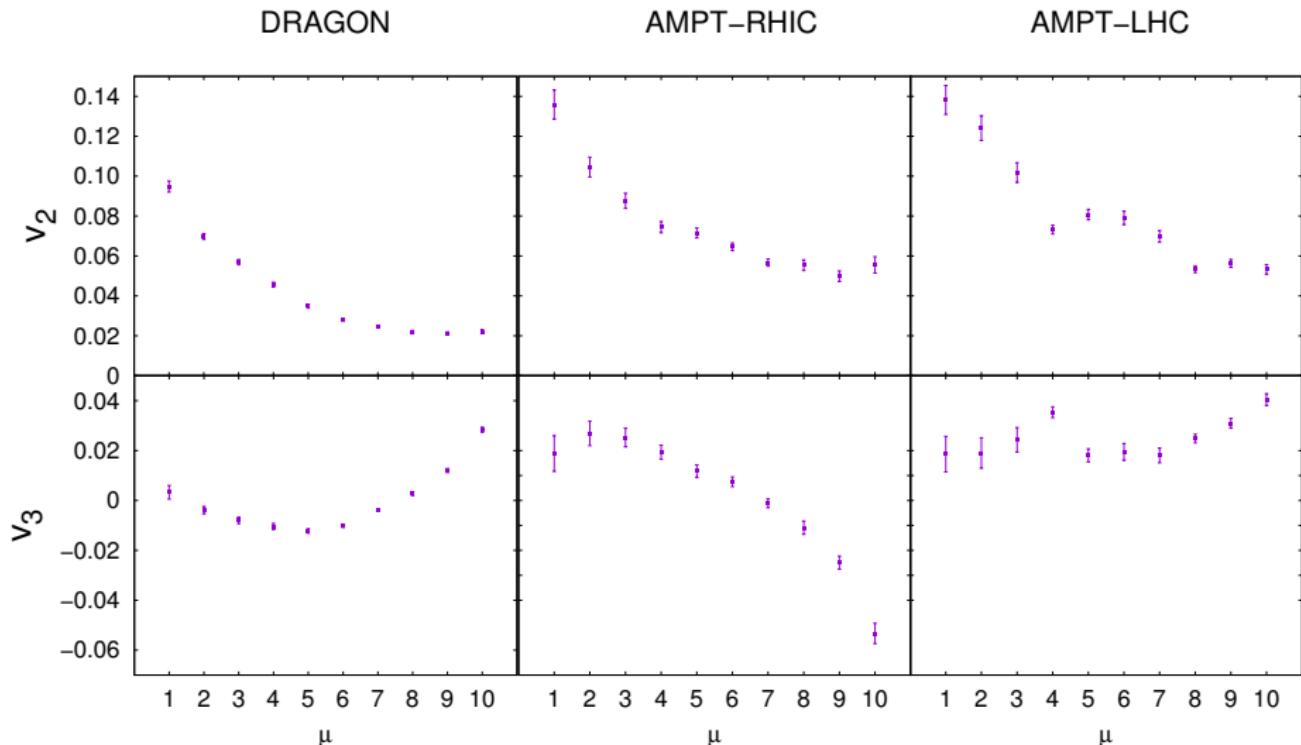


R. Kopečná, B. Tomášik: Eur. Phys. J. A 52 (2016) 115.

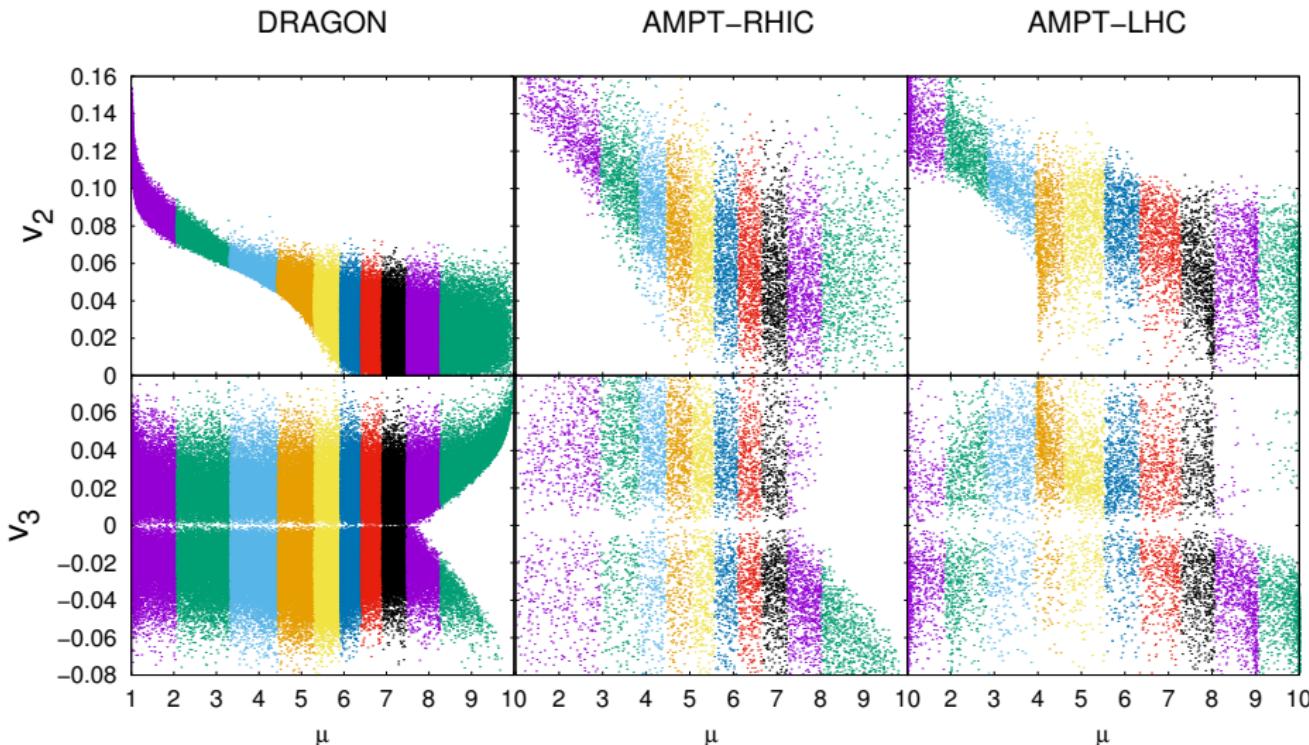
## Models which will be compared

- DRAGON** 150 000 events by DRAGON with anisotropies  
 $a_2, \rho_2 \in (-0.1; 0.1)$ ,  $a_3, \rho_3 \in (-0.03; 0.03)$   
(DRAGON is MC final state generator based on blast-wave model with included resonances)
- AMPT-RHIC** 10 000 events by AMPT in AuAu collisions with energy  
 $\sqrt{s_{NN}} = 200$  GeV, impact parameter 7 – 10 fm
- AMPT-LHC** 10 000 events by AMPT in PbPb collisions with energy  
 $\sqrt{s_{NN}} = 2760$  GeV, impact parameter 7 – 10 fm

# Anisotropic flow in similar events



# Anisotropic flow in similar events



## Femtoscopy: trivia

Correlation radii are parameters of the measured correlation function:

$$C(q, K) - 1 = \exp(-R_o^2(K)q_o^2 - R_s^2(K)q_s^2 - R_l^2(K)q_l^2)$$

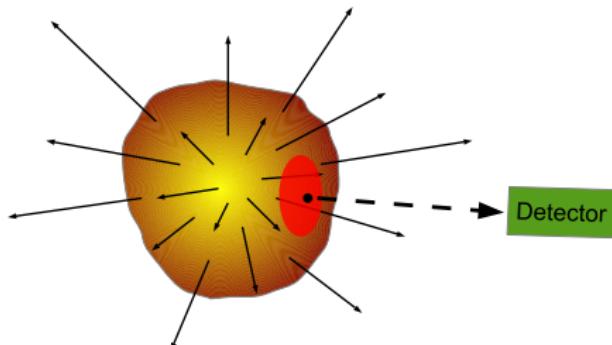
- $K$ : average pair momentum,  
 $q$ : relative pair momentum
- no cross terms at  
midrapidity at high energies
- out-side-long coordinate  
frame

out perpendicular to  
beam, along  $K_t$

long beam direction

side perpendicular to  
out and long

The correlation radii measure the sizes of the homogeneity regions



## Azimuthal dependence of correlation radii

Correlation radii are customarily decomposed into Fourier series

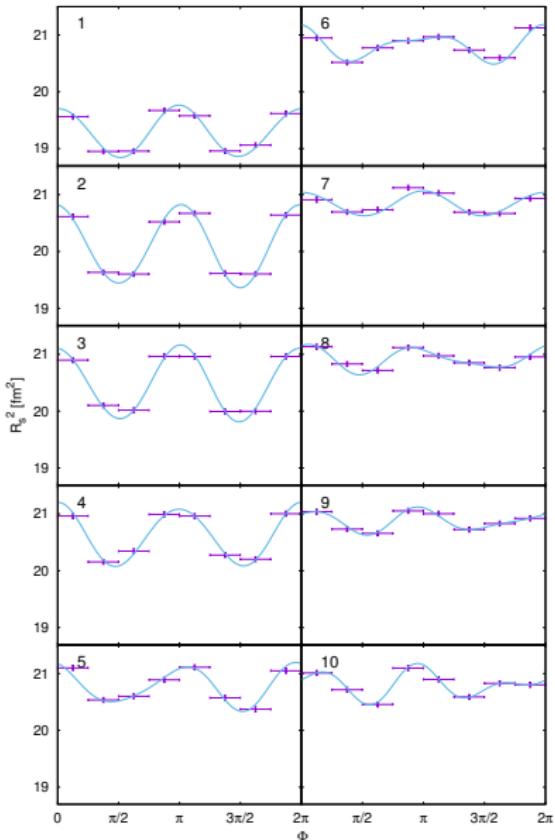
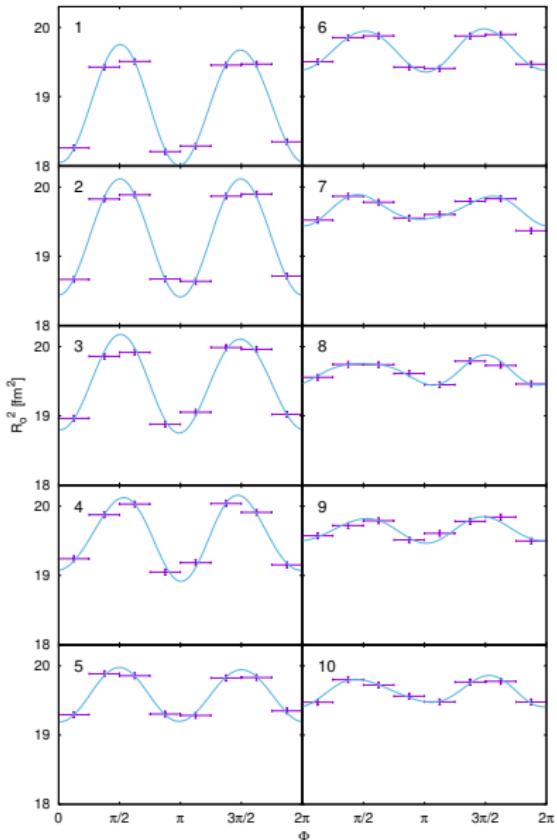
$$R_i^2(\phi) = R_{i,0}^2 + \sum_{n=1}^{\infty} R_{i,n}^2 \cos(n(\phi - \phi_n))$$

where  $i = o, s, l$ .

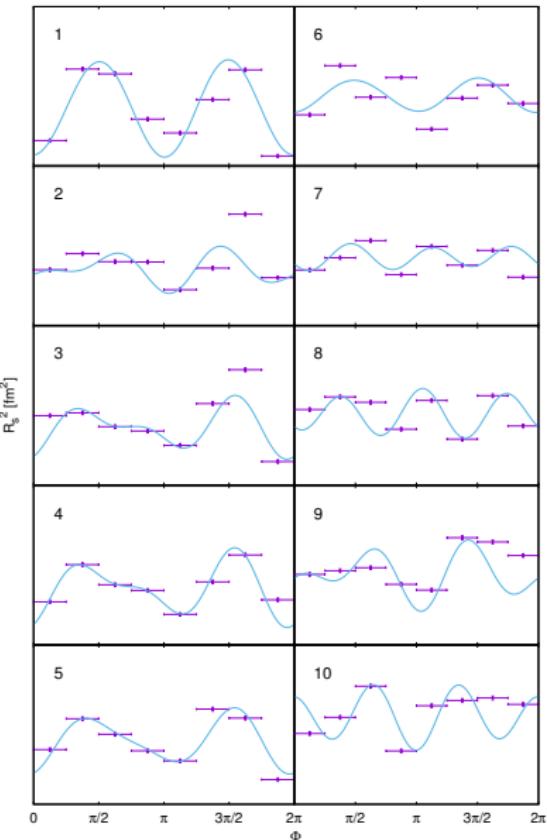
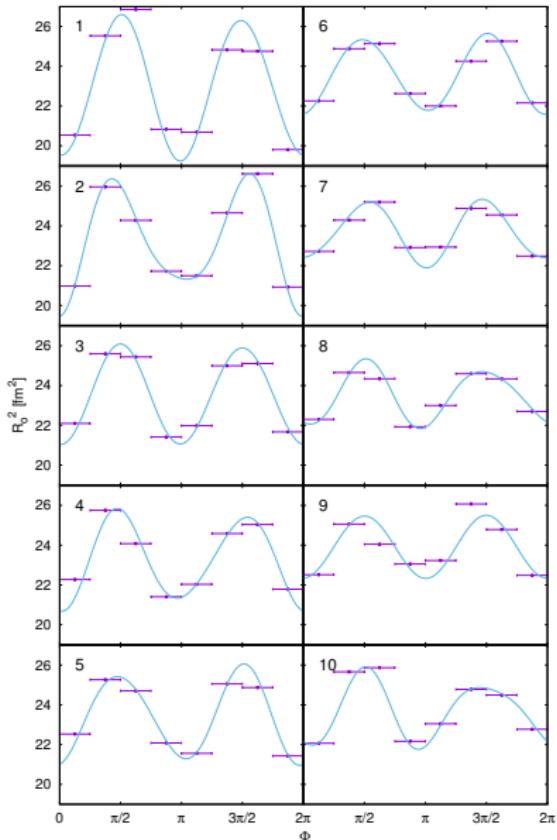
Real effect of oscillations: amplitudes divided by 0th term allow to scale out the absolute sizes

$$R_i^2(\phi) = R_{i,0}^2 \left( 1 + \sum_{n=1}^{\infty} \frac{R_{i,n}^2}{R_{i,0}^2} \cos(n(\phi - \phi_n)) \right)$$

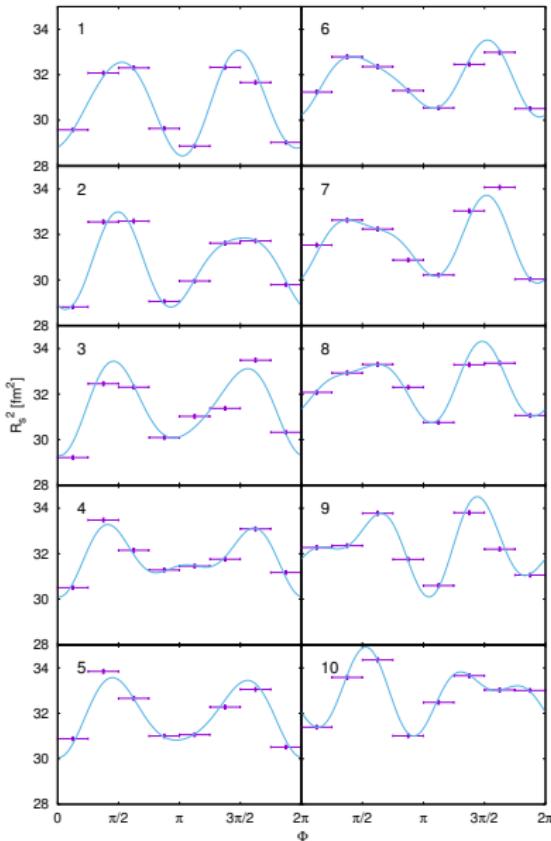
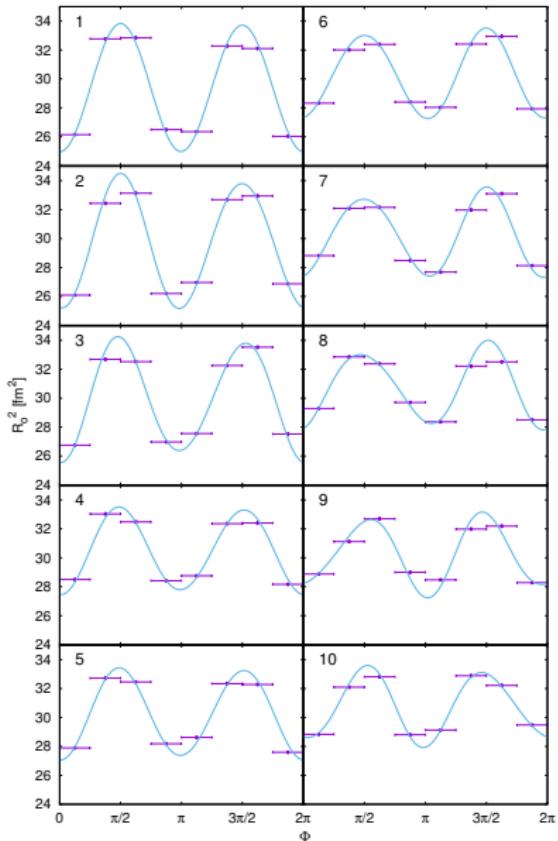
# Correlation radii from DRAGON in different event bins



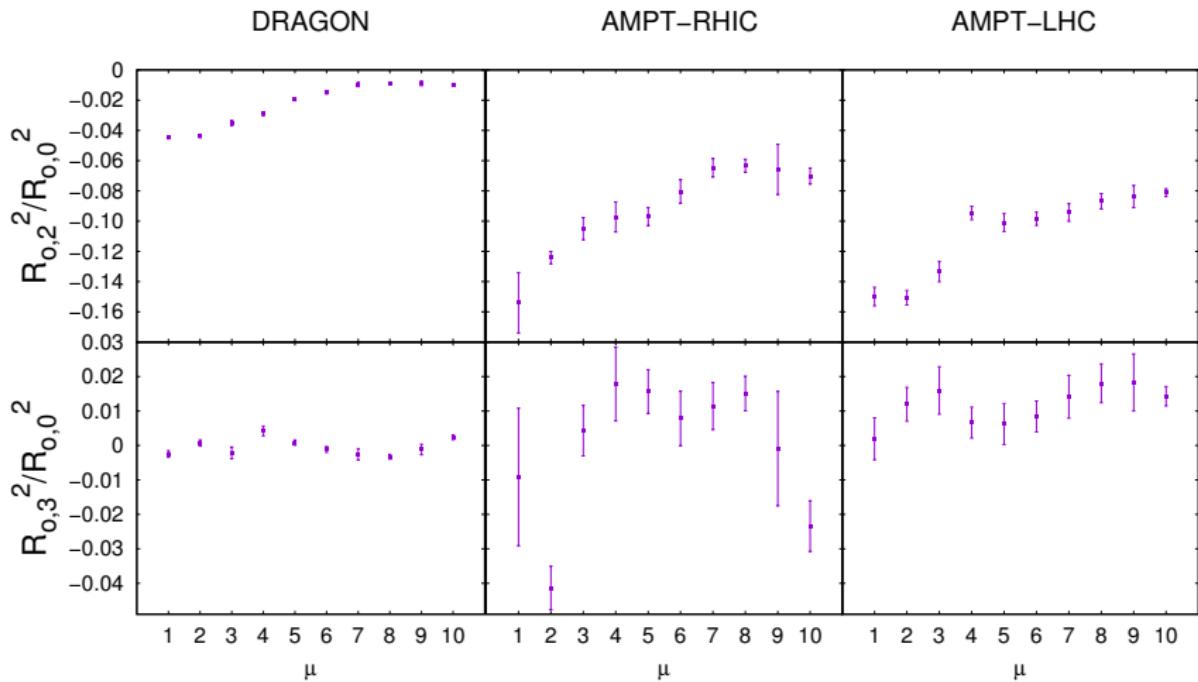
# Correlation radii from AMPT-RHIC in different event bins



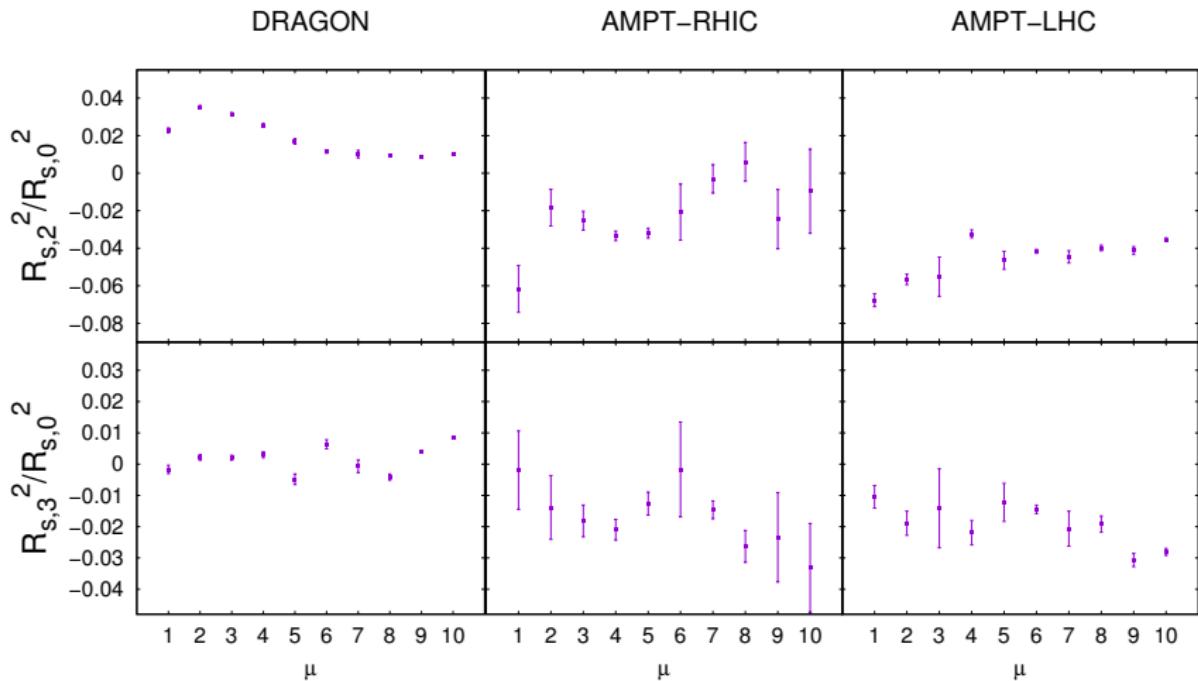
# Correlation radii from AMPT-LHC in different event bins



# Oscillation amplitudes for $R_o^2$



# Oscillation amplitudes for $R_s^2$



**INDIAN-SUMMER  
SCHOOL OF PHYSICS**  
Prague, Czech Republic

# Phenomenology of Hot and Dense Matter for Future Accelerators

September 3–7, 2018

Registration deadline:  
**June 1, 2018**

[iss2018.fjfi.cvut.cz](http://iss2018.fjfi.cvut.cz)

Paul Romatschke:  
**Hydrodynamic evolution, flow, bulk properties**

Nestor Armesto:  
**Hadron structure phenomenology**

Konrad Tywoniuk:  
**Jet suppression and energy loss**

Tuomas Lappi:  
**Initial stages of the collision, low-x physics**

Mateusz Płoskon:  
**Overview of experimental results, EIC perspective**

Chihiro Sasaki:  
**Theory of the deconfinement transition**

Selected students from COST member states will be given a grant to cover expenses financed by the THOR COST action CA15213. Attendance is limited to 45 participants.

**Venue:**

Faculty of Nuclear Sciences  
and Physical Engineering  
Czech Technical University in Prague  
Brehova 7, 11519 Prague 1

**Organizers**

Jan Cepila (chair), Boris Tomášik,  
Jesus Guillermo Contreras, Jaroslav Bielčík,  
Miroslav Myska, Jana Bielčíková,  
Jiří Mareš, Jiří Adam, Petr Bydžovský



# What is it good for?

- More selective comparison of data to theory.
- Better insight into the dynamics of evolution and freeze-out.
- Looking for and selecting rare events.
- Construction of mixed-events background for correlation functions.
- Better selection of the environment for other processes, e.g. jet quenching.
- ...

Published in

R. Kopečná, B. Tomášik: Eur. Phys. J. A **52** (2016) 115.  
J. Cimerman, B. Tomášik: in preparation.

# BACKUP SLIDES

# Event Shape Sorting: the algorithm

We will sort events according to their histograms in azimuthal angle.

- ① (Rotate the events appropriately)
- ② Sort your events as you wish
- ③ Divide sorted events into quantiles (we'll do deciles)
- ④ Determine average histograms in each quantiles
- ⑤ For each event  $i$  calculate Bayesian probability  $P(i|\mu)$  that it belongs to quantile  $\mu$
- ⑥ For each event calculate average  $\bar{\mu} = \sum_{\mu} \mu P(i|\mu)$
- ⑦ Sort events according to their values of  $\bar{\mu}$
- ⑧ If order of events changed, return to 3. Otherwise sorting converged.

S. Lehmann, A.D. Jackson, B. Lautrup, arXiv:physics/0512238

S. Lehmann, A. D. Jackson and B. E. Lautrup, *Scientometrics* **76** (2008) 369  
[physics/0701311 [physics.soc-ph]]