Initial energy density from new, exact solutions of relativistic hydrodynamics

T. Csörgő[1,2], G. Kasza[2]

[1] Wigner RCP, Budapest, Hungary
[2] EKU KRC, Gyöngyös, Hungary

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Introduction

- Initial energy density: Bjorken´s formula widely used
- Problems: phenomenological formula, lack of acceleration
- Finite size correction known for a special EoS (CNC)
- Was Bjorken´s formula correct?
- New and exact solution of relativistic hydro
- Evaluate initial energy density exactly
- Tested on non-relativistic, exact solutions, but ...
- ... Bjorken´s formula is not reproduced: why?

Perfect fluid hydrodynamics

- Energy momentum conservation:

$$\partial_\nu T^{\mu\nu} = 0$$

- Energy-momentum tensor (perfect fluid):

$$T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu - p g^{\mu\nu}$$

- Entropy conservation:

$$\partial_\mu \left(\sigma u^\mu\right) = 0$$

- Equation of State (EoS): closes the system of equations:

$$\varepsilon = \kappa p$$

$$\kappa = c_s^{-2}$$

- Here κ and the speed of sound (c_s) are constants (independent of T)

Some of the important, exact solutions

- Exact and analytic solutions are important: connect initial/final states
- Some of the important solutions for this presentation:
  - Landau-Khalatnikov solution (1+1 dim)

    L. D. Landau, Izv. Akad. Nauk Ser. Fiz.17, 51 (1953)
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  - CNC solution (1+d dim)

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  - CKCJ solution (1+1 dim)

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Bjorken-estimate

- Original idea: energy density measurable from dE/dy
- Famous result for initial ε :

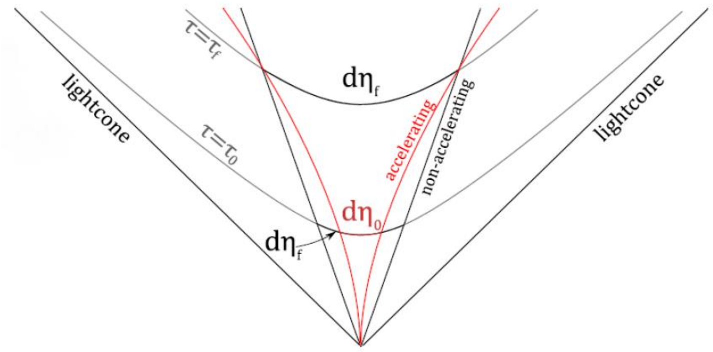
$$\varepsilon_0^{Bj} = \frac{dE}{dV_0} = \frac{1}{R^2\pi}\frac{dE}{dz_0} = \frac{\langle E \rangle}{R^2\pi\tau_0}\left.\frac{dN}{dy}\right|_{y=0} = \frac{\langle m_T \rangle}{R^2\pi\tau_0}\left.\frac{dN}{dy}\right|_{y=0}$$

J. D. Bjorken, Phys.Rev.D27 (1983)

- But: dN/dy is not flat!
- Bjorken solution lacks finiteness and acceleration!
- From an accelerating solution of relativistic hydrodynamics:

$$y \neq \eta_x$$

$$\eta_x^{init} \neq \eta_x^{fin}$$



Advanced (CNC) estimation

- Improved Bjorken formula:

$$\varepsilon_0 = \varepsilon_0^{Bj} \frac{dy}{d\eta_x^{fin}} \frac{d\eta_x^{fin}}{d\eta_x^{init}}$$

- From the 1+1d CNC solution:

$$\varepsilon_0^{CNC}(\lambda) = \varepsilon_0^{Bj} (2\lambda - 1) \left(\frac{\tau_f}{\tau_0}\right)^{\lambda - 1}$$

T. Csörgő, M. I. Nagy, M. Csanád, Phys. Rew. C77 024908 (2008)

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- Problem: valid for an unrealistic, superhard EoS ($c_s = 1$)
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Conjecture for $c_s < 1$ ($\kappa > 1$)

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CKCJ solution

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Conjecture

- Based on the 1+1d CNC result
- The conjecture was determined by four requirements:
  - It has to reproduce the Bjorken formula for $\lambda \to 1$
  - It has to reproduce the CNC formula for $\kappa \to 1$
  - It has to follow the known hydro behaviour, corresponding to exact solutions valid for any temperature independent EoS:

$$\varepsilon(\tau) \propto (\tau_0/\tau)^{\frac{1}{\kappa}}$$

  - It should approximately reproduce the results of numerical hydro calculations
- Occam´s razor:

$$\varepsilon_0^c(\kappa,\lambda) = \varepsilon_0^{Bj}\,(2\lambda-1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{(\lambda-1)\left(1-\frac{1}{\kappa}\right)}$$

T. Csörgő, M. Csanád, M.I. Nagy: J.Phys.G35:104128 (2008)

M. Csanád, T. Csörgő, Z. Jiang, C.B. Jang Universe 3, no. 1, 9 (2017)

CKCJ (new) solution

- Generalization of the 1+1d CNC result for $c_s < 1$ ($\kappa > 1$)

$$
\begin{aligned}
\eta_x(H) &= \Omega(H) - H \\
\Omega(H) &= \frac{\lambda}{\sqrt{\lambda-1}\sqrt{\kappa-\lambda}} \arctan\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}} \tanh(H)\right) \\
\sigma(\tau, H) &= \sigma_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda} \mathcal{V}_\sigma(s) \left[1 + \frac{\kappa-1}{\lambda-1} \sinh^2(H)\right]^{-\frac{\lambda}{2}} \\
T(\tau, H) &= T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{\kappa}} \mathcal{T}(s) \left[1 + \frac{\kappa-1}{\lambda-1} \sinh^2(H)\right]^{-\frac{\lambda}{2\kappa}} \\
\mathcal{T}(s) &= \frac{1}{\mathcal{V}_\sigma(s)} \\
s(\tau, H) &= \left(\frac{\tau_0}{\tau}\right)^{\lambda-1} \sinh(H) \left[1 + \frac{\kappa-1}{\lambda-1} \sinh^2(H)\right]^{-\lambda/2}
\end{aligned}
$$

- Accelerating solution (if $\lambda > 1$) with realistic equation of state
- Limited to a finite rapidity range (see T. Csörgő´s talk)

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Exact result for the initial energy density

- CKCJ solution: generalizes 1+1d CNC solution for $c_s < 1$
- Instead of a conjecture → an exact result
- Expectation:
  - Bjorken´s formula in the $\lambda \to 1$ limit
  - CNC´s formula in the $\kappa \to 1$ limit

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- Question #1: Can these expectations be satisfied with an exact calculation?

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- Question #1: Can these expectations be satisfied with an exact calculation?
- Question #2: If cannot, what does it mean for the Bjorken estimate?

Exact result for the initial energy density

- CKCJ solution is valid in the midrapidity region
- Energy content of a coordinate-rapidity slice:

$$\Delta p^0 = R^2\pi \int_{-\eta_x}^{\eta_x} d\eta_x' \frac{d\Sigma_\nu}{d\eta_x'} T^{0\nu}(x)$$

- Particle content of a coordinate-rapidity slice:

$$\Delta N = R^2\pi \int_{-\eta_x}^{\eta_x} d\eta_x' \frac{d\Sigma_\nu}{d\eta_x'} n(x) u^\nu$$

- The volume element of the freeze-out hypersurface (at midrapidity):

$$\frac{d\Sigma_\nu}{d\eta_x} \approx \tau_f \left(1, 0, 0, \lambda\eta_x\right)$$

Exact result for the initial energy density

- The coordinate-rapidity densities are:

$$\frac{dE}{d\eta_x} = \lim_{\Delta\eta_x \to 0} \frac{\Delta E}{\Delta\eta_x} = R^2 \pi \tau_f \varepsilon_f$$

$$\frac{dN}{d\eta_x} = \lim_{\Delta\eta_x \to 0} \frac{\Delta N}{\Delta\eta_x} = R^2 \pi \tau_f n_f$$

- Average transverse mass is:

$$\langle m_T \rangle = \frac{\left(\frac{dE}{d\eta_x}\right)}{\left(\frac{dN}{d\eta_x}\right)} = \frac{dE}{dN} = \frac{\varepsilon_f}{n_f}$$

Exact result for the initial energy density

- The rapidity distribution is:

$$\left.\frac{dN}{dy}\right|_{y=0} = \frac{dN}{d\eta_x}\frac{d\eta_x}{dy} = \frac{R^2\pi\tau_f}{2\lambda-1}n_f$$

Jacobian is given by saddle-point calculation

- Initial energy density from the CKCJ solution:

$$\varepsilon_0 = \varepsilon_f\left(\frac{\tau_f}{\tau_0}\right)^{\lambda\left(1+\frac{1}{\kappa}\right)} = \langle m_T\rangle n_f\left(\frac{\tau_f}{\tau_0}\right)^{\lambda\left(1+\frac{1}{\kappa}\right)}$$

- Using the rapidity density:

$$\varepsilon_0 = (2\lambda-1)\frac{\langle m_T\rangle}{R^2\pi\tau_0}\left.\frac{dN}{dy}\right|_{y=0}\left(\frac{\tau_f}{\tau_0}\right)^{\lambda\left(1+\frac{1}{\kappa}\right)-1}$$

Exact result for the initial energy density

- The exact result from the CKCJ solution is:

$$\varepsilon_0(\kappa,\lambda) = \varepsilon_0^{Bj}\,(2\lambda-1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda\left(1+\frac{1}{\kappa}\right)-1}$$

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- But: in the $\kappa \rightarrow 1$ limit, CNC´s formula is not reproduced

Exact result for the initial energy density

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$$\varepsilon_0(1,\lambda) = \varepsilon_0^{Bj}\,(2\lambda-1)\left(\frac{\tau_f}{\tau_0}\right)^{2\lambda-1}$$

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- Problem with the Bjorken-estimation:

The work done by the pressure was not evaluated explicitely!

Exact result for the initial energy density

Once thermal equilibrium is established and hydrodynamic expansion of the fluid commences, the t^{-1} time dependence of the energy density ϵ will not be valid (although we shall calculate a similar behavior $\epsilon \sim t^{-n}$ with $1 \leq n \leq \frac{4}{3}$). Thus the time t appearing in the expression (8) for energy density should be interpreted as an *initial* time for imposition of boundary conditions for hydrodynamic flow.

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$$E = N \frac{d\langle E\rangle}{dy} \Delta y = N \frac{d\langle E\rangle}{dy} \frac{1}{2} \left[\frac{2d}{t} \right] \qquad (3)$$

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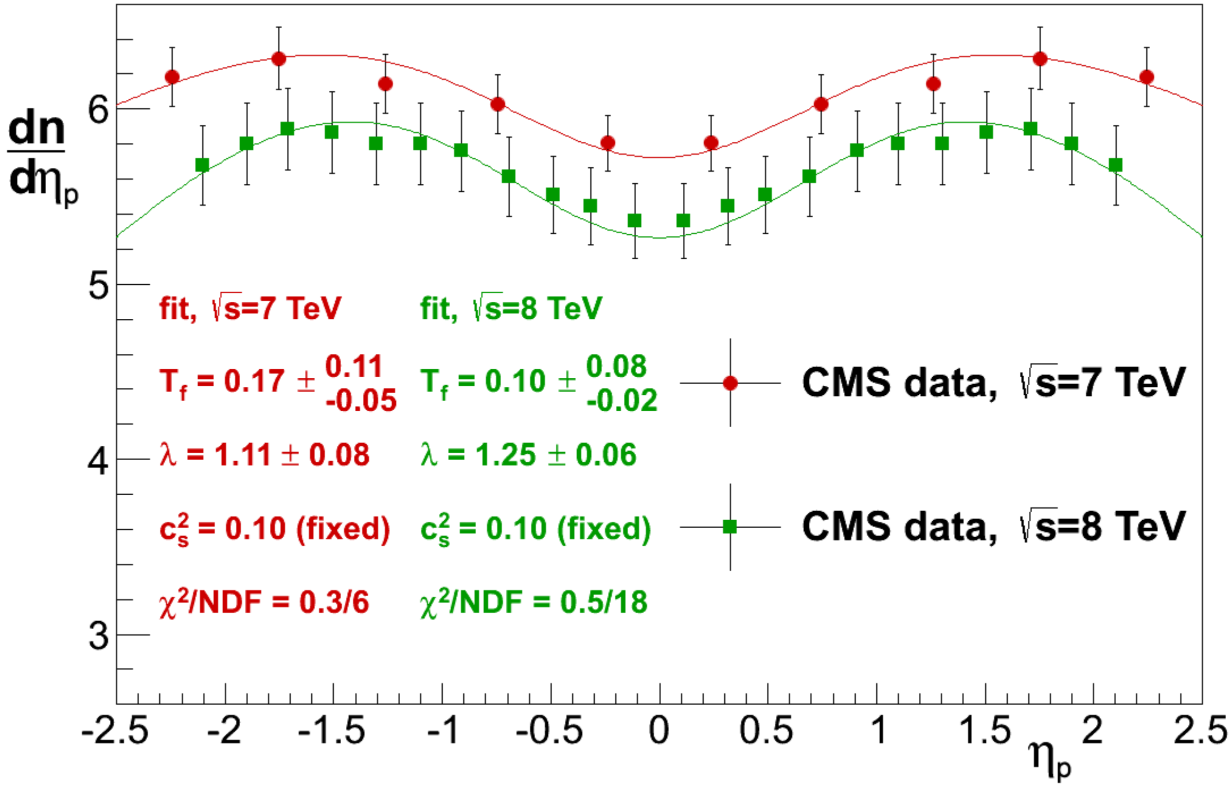
If p=0, the correction of the advanced estimation vanishes, the Bjorken formula and its $\lambda > 1$ correction is reproduced.

Exact result for the initial energy density

- Parameters: fits to CMS p+p data at 7 TeV



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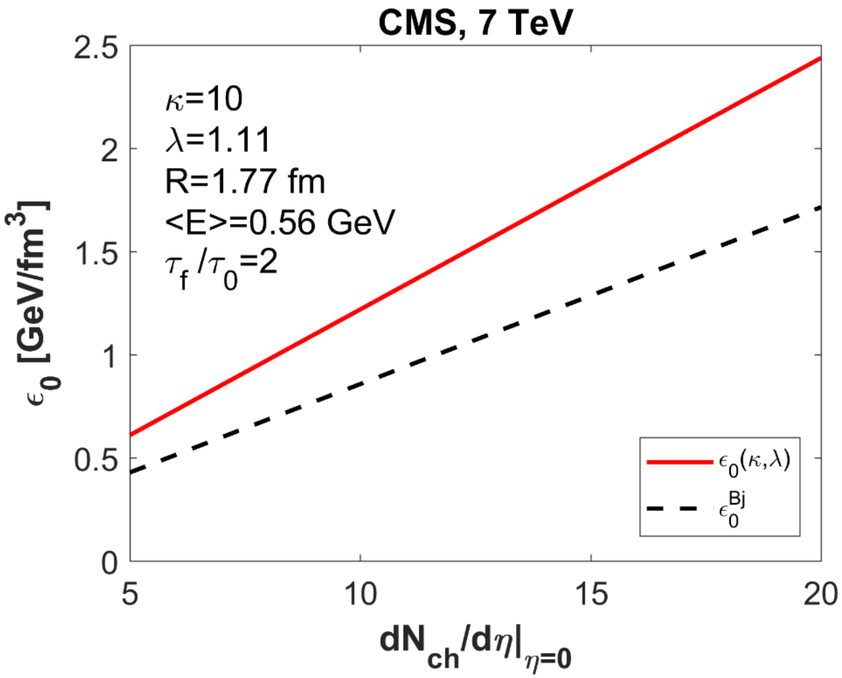

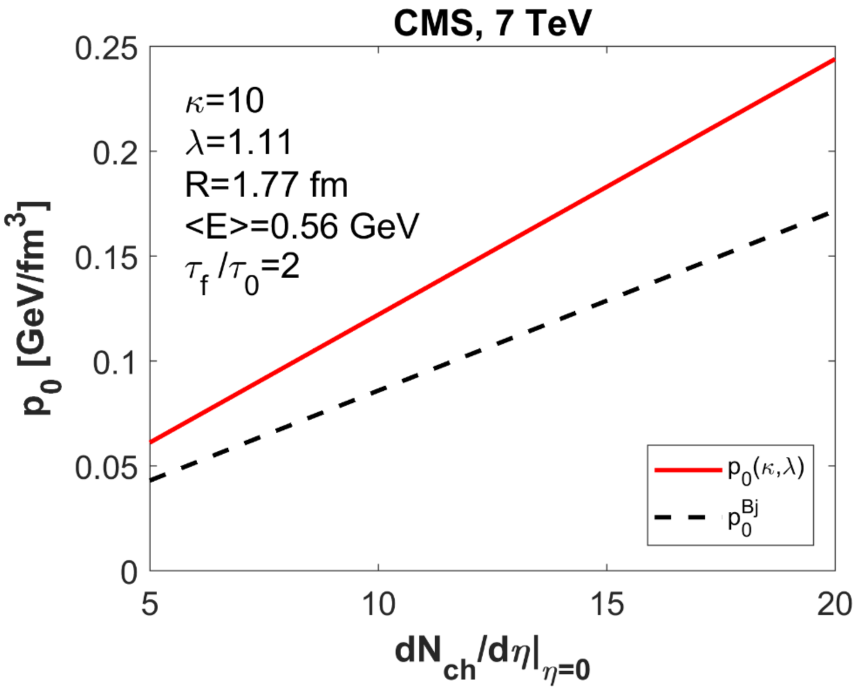

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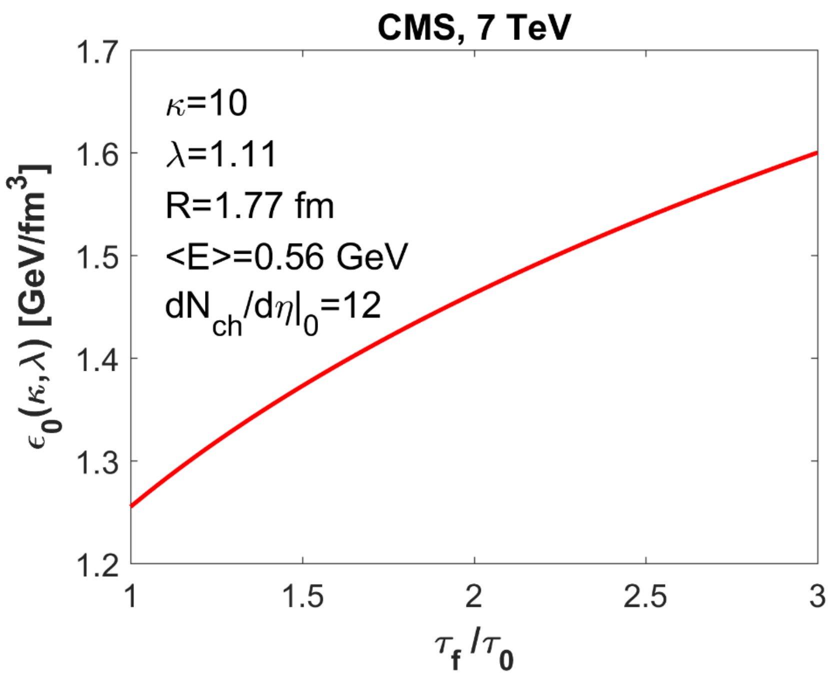
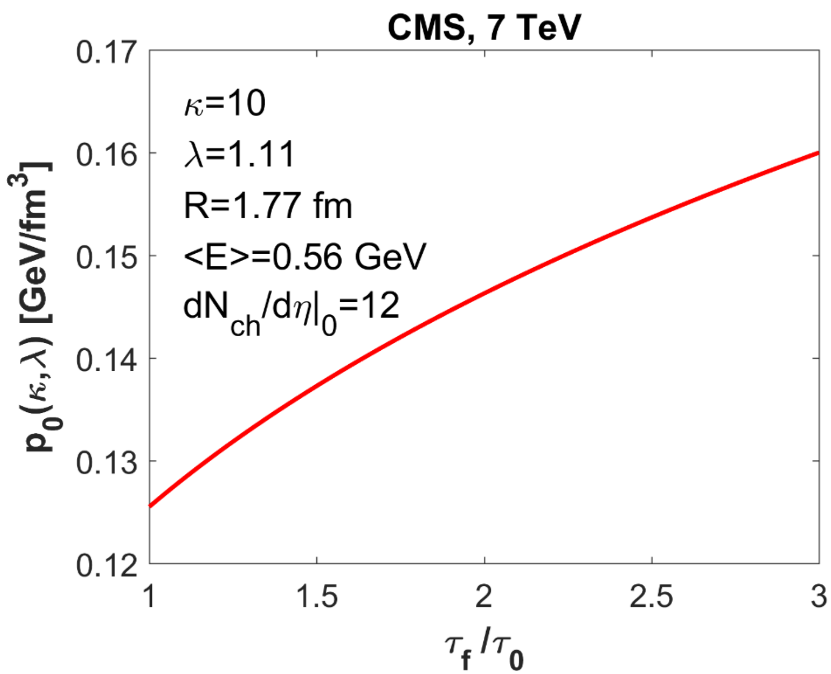
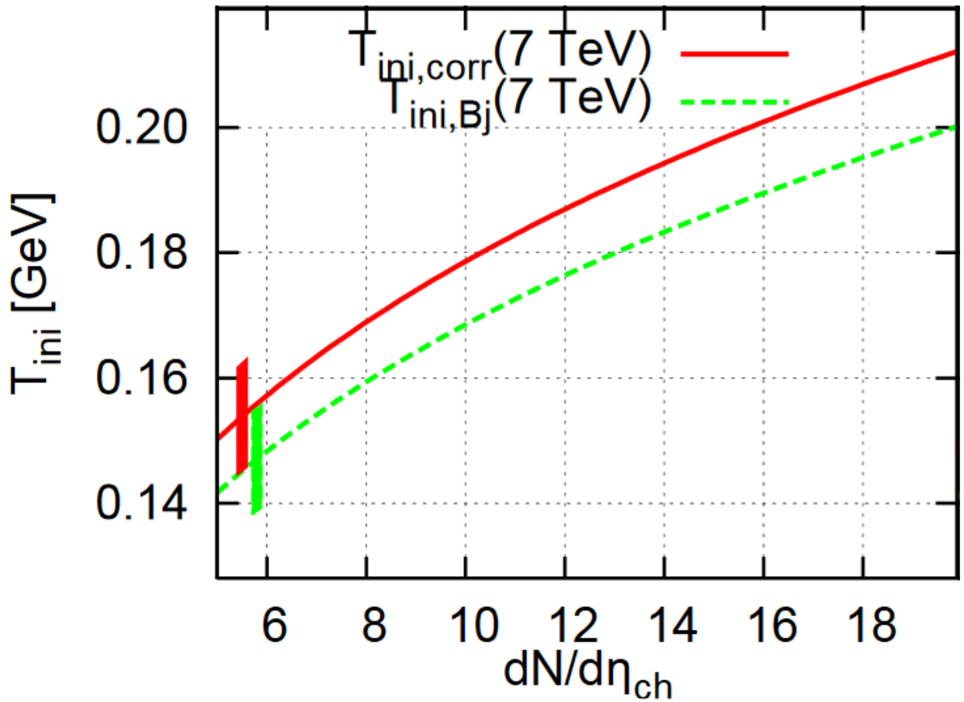




Comparison with an earlier estimate

- Parameters: fits to CMS p+p data at 7 TeV
- Conjectured $dN_{ch}/d\eta$ dependence of T_{ini}: $\varepsilon_{ini} \sim T_{ini}^4$
- Result: T_{ini} depends on $dN_{ch}/d\eta$



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- Temperature is independent of the pseudorapidity density!

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$dN_{ch}/d\eta$ dependence eliminated!

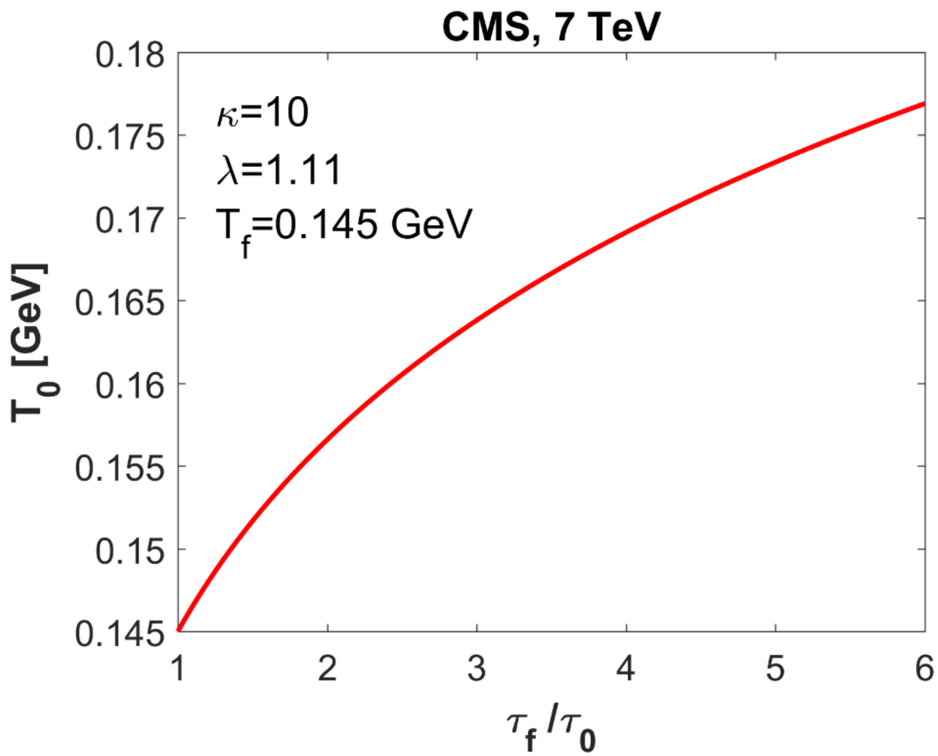
Exact result for the initial temperature

- Parameters: fits to CMS p+p data at 7 TeV
- Temperature is independent of the pseudorapidity density!
- Measuring T_0 (with direct photon spectra)

→ τ_f / τ_0 can be determined!

$$\frac{T_0}{T_f} = \left(\frac{\varepsilon_0}{\varepsilon_f}\right)^{\frac{1}{1+\kappa}}$$

$dN_{ch}/d\eta$ dependence eliminated!



Fundamental Thermodynamic Relation (λ=1)

- Fundamental equation:

$$dE = TdS - pdV$$

- Bjorken´s formula compared to the new formula (with λ=1):

$$W_{exp} = \varepsilon_0(\kappa,1) - \varepsilon_0^{Bj} = \varepsilon_0(\kappa,1)\left[1 - \left(\frac{\tau_f}{\tau_0}\right)^{-\frac{1}{\kappa}}\right]$$

- Estimation for Au+Au at RHIC (κ=10, τ_f/τ_i=10):

$$W_{exp} = \varepsilon_0(\kappa,1) - \varepsilon_0^{Bj} = 0.21 \cdot \varepsilon_0(\kappa,1)$$

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Significant correction!

Fundamental Thermodynamic Relation (κ=1)

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- CNC´s formula compared to the new formula (with $\kappa = 1$):

$$W_{exp} = \varepsilon_0(1,\lambda) - \varepsilon_0^{CNC}(\lambda) = \varepsilon_0(1,\lambda)\left[1 - \left(\frac{\tau_f}{\tau_0}\right)^{-\lambda}\right]$$

- Estimation for Au+Au at RHIC ($\lambda = 1.1$, $\tau_f / \tau_i = 10$):

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Order of magnitude correction!

How good was the CNC conjecture?

- CNC´s conjectured estimation:

$$\varepsilon_0^c(\kappa,\lambda) = \varepsilon_0^{Bj}\,(2\lambda-1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{(\lambda-1)\left(1-\frac{1}{\kappa}\right)}$$

- Advanced, exact formula:

$$\varepsilon_0(\kappa,\lambda) = \varepsilon_0^{Bj}\,(2\lambda-1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{\frac{\lambda}{\kappa}}$$

- Only 5% correction with $\tau_f/\tau_i=10$, $\lambda=1.1$ and $\kappa=10$:

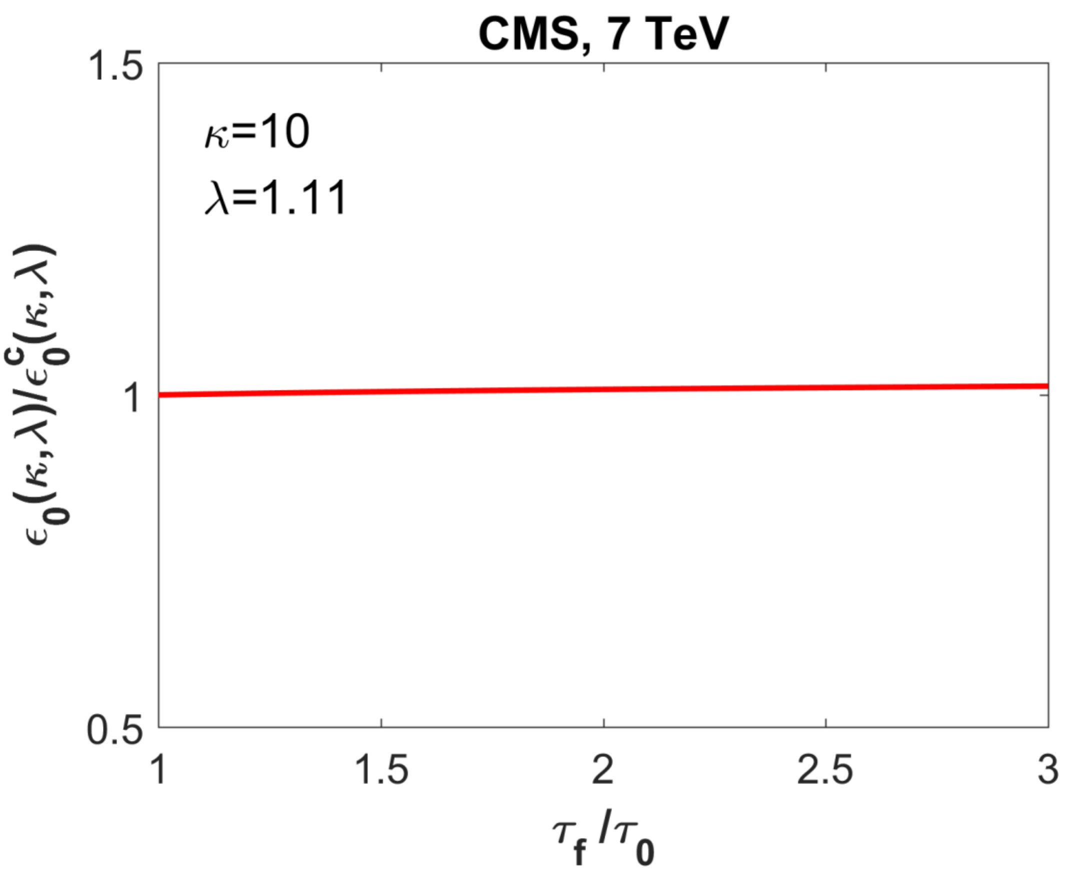
$$\frac{\varepsilon_0(\kappa,\lambda)}{\varepsilon_0^c(\kappa,\lambda)} = \left(\frac{\tau_f}{\tau_0}\right)^{\frac{2\lambda-1}{\kappa}-\lambda+1} \approx 10^{0.02} = 1.05$$

- Reason: neither Bjorken´s nor CNC´s formula was correct
- Taking into account W_{exp} → Occam´s razor still works!

How good was the CNC conjecture?

- Parameters: fits to CMS p+p data at 7 TeV

T. Csörgő, G. Kasza, M. Csanád, Z. Jiang, arXiv:1805.01427 (2018)



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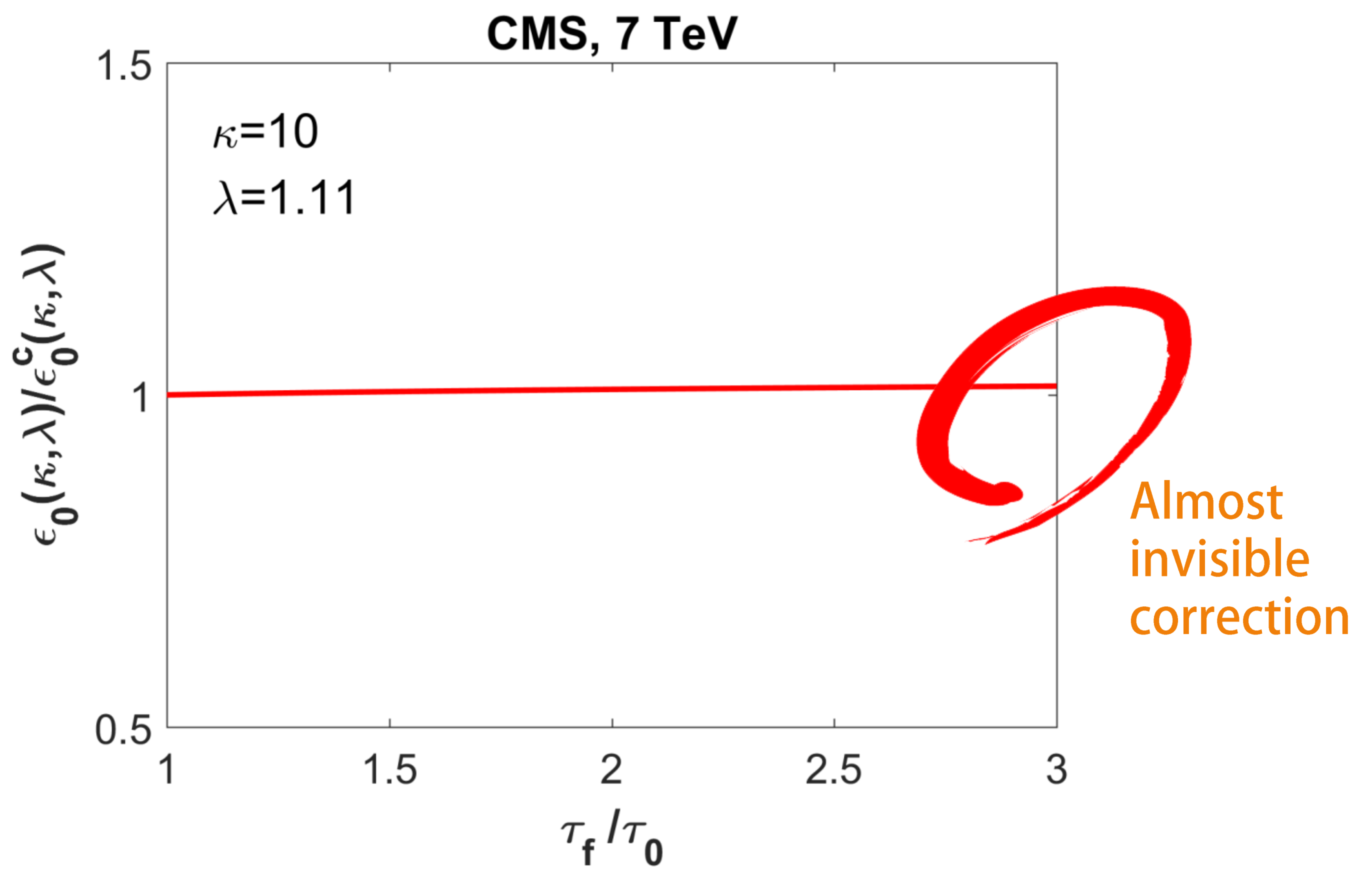

Summary & Outlook

- New, accelerating solution of relativistic hydro with realistic EoS
- Initial energy density recalculated exactly
- Bjorken´s estimate lacks not only the acceleration, but also the contribution of the work
- CNC´s advanced estimate also lacks the contribution of the work
- The CNC conjecture is numerically good, but analytically inaccurate
- Exact result for T_0 is independent of $dN_{ch}/d\eta$
  → measuring τ_f / τ_0 is possible
- Plans: further generalizations of the CKCJ solution