Perturbative solutions of relativistic hydrodynamics

arXiv:1711.05446

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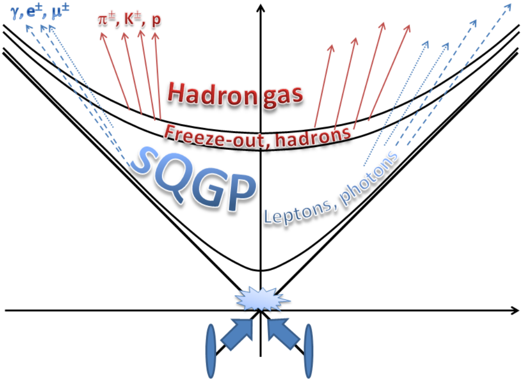
This was supposed to be Bálint's talk, but he is a 2nd year undergrad student, and unfortunately he has four exams this week...

Hydrodynamics in high energy physics

- Strongly interacting QGP created at RHIC & LHC
- A hot, expanding, strongly interacting, (nearly) perfect fluid
- Hadrons created at the "chemical" freeze-out
- Hadron distributions decouple at "kinetic" freeze-out
- Photons and leptons "shine through"



Equations of relativistic perfect hydrodynamics

Looking for u^μ ,p, ϵ fields

Assumptions:

- no viscosity
- no heat conduction
- local energy-momentum conservation

Energy-pressure connection: EoS

Locally conserved entropy density σ
$\rightarrow$ temperature: $T = (\epsilon + p)/\sigma$

Or locally conserved charge density n
$\rightarrow$ temperature: $T = p/n$

Energy, momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$

Equation of state (EoS)

$$\epsilon = \kappa p$$

Continuity equation

$\partial_\mu(\sigma u^\mu) = 0$ or $\partial_\mu(nu^\mu) = 0$

Known solutions for relativistic hydrodynamics

- Many frameworks for numerical calculations
- Exact, analytic solutions important: connect initial/final state
- Famous 1+1D solutions: Landau–Khalatnikov & Hwa–Bjorken

  L. D. Landau, Izv. Akad. Nauk Ser. Fiz. **17**, 51 (1953)
  I.M. Khalatnikov, Zhur. Eksp. Teor. Fiz. **27**, 529 (1954)
  R. C. Hwa, Phys. Rev. **D 10**, 2260 (1974)
  J. D. Bjorken, Phys. Rev. **D 27**, 140 (1983)
- Discovery of sQGP $\rightarrow$ Many new solutions

  a review: de Souza, Koide, Kodama, Prog. Part. Nucl. Phys. **86**, 35 (2016)

  See the WPCF2018 talks by T. Csörgő, G. Kasza, Z. Jiang
- First truly 3D relativistic solution: **Hubble-flow**

  Csörgő, Csernai, Hama, Kodama, Heavy Ion Phys. **A21**, 73, nucl-th/0306004
- Describes well experimental data

  Csanád, Vargyas, Eur. Phys. J. **A 44**, 473 (2010) nucl-th/09094842
- Lack of non-spherical 3D, accelerating solutions so far

How to describe *almost* known cases?

Perturbations on top of a known solution!

Perturbative handling of relativistic hydrodynamics

Perturbed fields:

- Start: a known solution
- Fields: u^μ, p, n
- $u^\mu \to u^\mu + \delta u^\mu$
- $p \to p + \delta p$
- $n \to n + \delta n$
- Similarly for $n \to \sigma$

Equations for perturbations:

- Substitute perturbations into hydro
- Substract 0^{th} order equations
- Neglect 2^{nd} or higher order perturbations
- Remainder: perturbed equation
- Solution yields perturbations
- Resulting fields: $\delta u^\mu, \delta n, \delta p$

Orthogonality criterion for the flow field

$$(u^\mu + \delta u^\mu)(u_\mu + \delta u_\mu) = 1 \qquad \Rightarrow \qquad u^\mu \delta u_\mu = 0$$

Perturbations on a standing fluid: waves

Known solution: Standing fluid

- $u^\mu = (1,0,0,0)$
- $p =$ const.
- $n =$ const.

These yield e.g.:

- $\partial_\mu u^\mu = 0$, $\partial_\mu p = 0$
- $u^\mu \partial_\mu = \partial_0$
- $Q^{\mu\nu} = (u^\mu u^\nu - g^{\mu\nu})$, $Q^{\mu\nu}\partial_\mu = (0, \nabla)$

Perturbed Energy equation

$$\kappa \partial_0 \delta p + (\kappa + 1) p \partial_\mu \delta u^\mu = 0$$

Perturbed Euler equation

$$(\kappa + 1) p \partial_0 \delta u^\nu - Q^{\mu\nu} \partial_\mu \delta p = 0$$

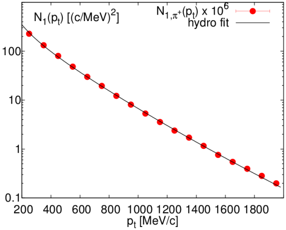
Wave equation for the pressure

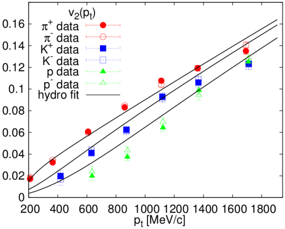
$$\partial_0^2 \delta p = \frac{1}{\kappa} \Delta \delta p$$

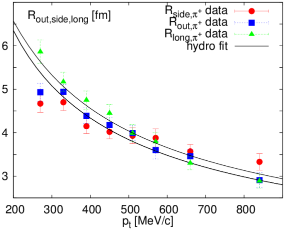
$\rightarrow$ sound waves!

A realistic solution: Hubble-flow

- First exact, analytic and truly 3D relativistic solution
  $$u^\mu = \frac{x^\mu}{\tau},\ n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{N}(S),\ p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}}$$
  Csörgő, Csernai, Hama, Kodama, Heavy Ion Phys. **A21**, 73 (2004), nucl-th/0306004
- Scaling variable: $u_\mu \partial^\mu S = 0$, multipole solutions also possible
  Csanád, Szabó, Phys. Rev. **C 90**, 054911 (2014), arXiv:1405.3877
- Describes data (spectra, v_n, HBT)
  Data: PHENIX, PRC**69**(2004), PRL**91**(2003), PRL**93**(2004)
  Calculation: Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842







Perturbed equations for the Hubble-flow solution

Equations for first order perturbations δu^μ, δp, δn:

Euler equation

$$\frac{\partial_\mu \delta p}{(\kappa+1)p}\left[g^{\mu\nu} - u^\mu u^\nu\right] = \frac{\kappa-3}{\tau\kappa}\delta u^\nu + u^\mu \partial_\mu \delta u^\nu \qquad (1)$$

Energy equation

$$\kappa u^\mu \partial_\mu \delta p + \frac{3(\kappa+1)}{\tau}\delta p = -(\kappa+1)p\partial_\mu \delta u^\mu \qquad (2)$$

Continuity equation

$$\delta u^\mu n \frac{\mathcal{N}'(S)}{\mathcal{N}(S)}\partial_\mu S + u^\mu \partial_\mu \delta n + \frac{3\delta n}{\tau} + n\partial_\mu \delta u^\mu = 0 \qquad (3)$$

Note: idea related to the work of Shi, Liao and Zhuang
Phys.Rev. C90 (2014) no.6, 064912 [arXiv:1405.4546]

A class of perturbative solutions on top of Hubble flow

A class of possible solutions of the perturbative equations

$$u^\mu = \frac{x^\mu}{\tau} \quad \rightarrow \quad \delta u^\mu = \delta \cdot F(\tau) g(x_\nu) \chi(S) \partial^\mu S \tag{4}$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}} \quad \rightarrow \quad \delta p = \delta \cdot p_0 \pi(S) \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}} \tag{5}$$

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^{3} \mathcal{N}(S) \quad \rightarrow \quad \delta n = \delta \cdot n_0 \left(\frac{\tau_0}{\tau}\right)^{3} h(x_\nu) \nu(S) \tag{6}$$

- Assume a given solution $(S, \mathcal{N}(S))$
- Choose perturbations $(\delta p, \delta u^\mu, \delta n)$ as above
- When are these perturbed fields also solutions?
- What are the restrictions for the F, g, h, χ, π, ν functions?

Which perturbations provide solutions?

Perturbations:

$$\delta u^\mu = \delta \cdot F(\tau) g(x_\nu) \chi(S) \partial^\mu S$$

$$\delta p = \delta \cdot p_0 \pi(S) \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}}$$

$$\delta n = \delta \cdot n_0 \left(\frac{\tau_0}{\tau}\right)^{3} h(x_\nu) \nu(S)$$

Restrictions for $\chi(S), \nu(S), \pi(S), h(x_\nu), g(x_\nu)$

$$\frac{\chi'(S)}{\chi(S)} = -\frac{\partial_\mu \partial^\mu S}{\partial_\mu S \partial^\mu S} - \frac{\partial_\mu S \partial^\mu \ln g(x_\nu)}{\partial_\mu S \partial^\mu S} \tag{7}$$

$$\frac{\pi'(S)}{\chi(S)} = (\kappa+1)\left[F(\tau)\left(u^\mu \partial_\mu g(x_\nu) - \frac{3g(x_\nu)}{\kappa\tau}\right) + F'(\tau) g(x_\nu)\right] \tag{8}$$

$$\frac{\nu(S)}{\chi(S)\mathcal{N}'(S)} = -\frac{F(\tau) g(x_\nu) \partial_\mu S \partial^\mu S}{u^\mu \partial_\mu h(x_\nu)} \tag{9}$$

How to find a concrete solution?

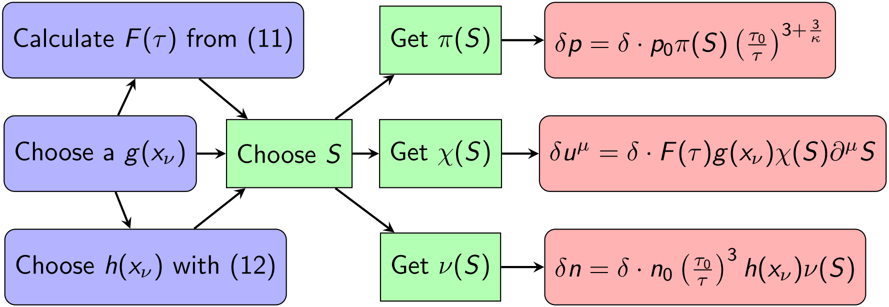
The way of finding a concrete solution

Restriction equations

$$\frac{\chi'(S)}{\chi(S)} = -\frac{\partial_\mu \partial^\mu S}{\partial_\mu S \partial^\mu S} - \frac{\partial_\mu S \partial^\mu \ln g(x_\nu)}{\partial_\mu S \partial^\mu S} \tag{10}$$

$$\frac{\pi'(S)}{\chi(S)} = (\kappa+1)\left[F(\tau)\left(u^\mu \partial_\mu g(x_\nu) - \frac{3g(x_\nu)}{\kappa\tau}\right) + F'(\tau) g(x_\nu)\right] \tag{11}$$

$$\frac{\nu(S)}{\chi(S)\mathcal{N}'(S)} = -\frac{F(\tau) g(x_\nu) \partial_\mu S \partial^\mu S}{u^\mu \partial_\mu h(x_\nu)} \tag{12}$$



A specific subclass of solutions

To obtains a concrete solution → fix the $g(x_\nu)$, $F(\tau)$, $h(x_\nu)$ functions:

$$g(x_\nu) = 1,$$

$$F(\tau) = \tau + c\tau_0 \left(\frac{\tau}{\tau_0}\right)^{\frac{3}{\kappa}}$$

$$h(x_\nu) = \begin{cases} \ln\left(\frac{\tau}{\tau_0}\right) + c\frac{\kappa}{3-\kappa}\left(\frac{\tau}{\tau_0}\right)^{\frac{3}{\kappa}-1} & \text{if } \kappa \neq 3 \\ (1+c)\ln\left(\frac{\tau}{\tau_0}\right) & \text{if } \kappa = 3 \end{cases}$$

Remainging restrictions:

- $u_\mu \partial^\mu S = 0$
- $\frac{\partial_\mu \partial^\mu S}{\partial_\mu S \partial^\mu S}$ may depend only on S
- $\tau^2 \partial_\mu S \partial^\mu S$ may also depend only on S

Scaling variables found so far: $S = \frac{r^m}{t^m}$, $S = \frac{r^m}{\tau^m}$, $S = \frac{\tau^m}{t^m}$

An example case

Scaling variable

$$S = \frac{r^m}{t^m} \qquad (13)$$

Perturbations

$$\delta p = \delta \cdot p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}} \pi(S)$$

$$\delta u^\mu = \delta \cdot \left(\tau + c\tau_0 \left(\frac{\tau}{\tau_0}\right)^{\frac{3}{\kappa}}\right) \partial^\mu S \chi(S)$$

$$\delta n = \delta \cdot n_0 \left(\frac{\tau_0}{\tau}\right)^3 \left(\ln\left(\frac{\tau}{\tau_0}\right) + c\frac{\kappa}{3-\kappa}\left(\frac{\tau}{\tau_0}\right)^{\frac{3}{\kappa}-1}\right) \nu(S)$$

The functions of the scaling variable:

$$\chi(S) = \left(\frac{r}{t}\right)^{-m-1} \qquad (14)$$

$$\pi(S) = -\frac{(\kappa+1)(\kappa-3)}{\kappa} m \left(\frac{r}{t}\right)^{-1} \qquad (15)$$

$$\nu(S) = m^2 \left(\frac{r}{t}\right)^{m-1} \left(\left(\frac{r}{t}\right)^2 - 1\right)\left(1 - \left(\frac{r}{t}\right)^{-2}\right) \mathcal{N}'\left(\frac{r^m}{t^m}\right) \qquad (16)$$

A concrete solution with $S = t/r$, $\mathcal{N}(\mathcal{S}) = exp(-S^{-2})$

Choice of scaling variable

$$S = \frac{t}{r} \tag{17}$$

Gaussian Hubble-flow density profile: $\mathcal{N}(S) = e^{-\frac{r^2}{t^2}} = e^{-S^{-2}}$

The functions appearing in the perturbations:

$$\chi(S) = 1 \tag{18}$$

$$\pi(S) = \frac{(\kappa+1)(\kappa-3)}{\kappa}\left(\frac{t}{r}\right) \tag{19}$$

$$\nu(S) = 2\left(\frac{t}{r}\right)^{-3}\left(1-\left(\frac{t}{r}\right)^2\right)^2 \mathcal{N}\left(\frac{t}{r}\right) \tag{20}$$

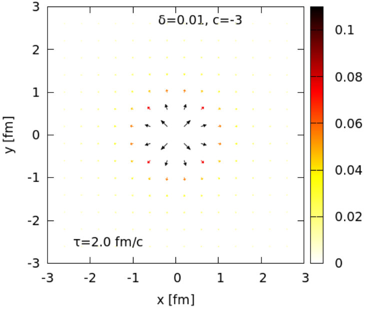
Let us chose parameters which describe v_2, $N(p_T)$, R_{HBT}

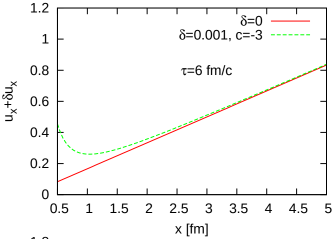
Csanád, Vargyas, Eur. Phys. J. **A 44**, 473

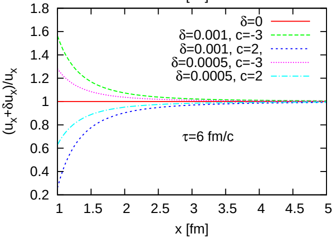
Four-velocity perturbation

$$\delta u^\mu = \delta \cdot \left(\tau + c\tau_0 \left(\frac{\tau}{\tau_0}\right)^{\frac{3}{\kappa}}\right) \partial^\mu S$$

$$u^\mu = \frac{x^\mu}{\tau}$$



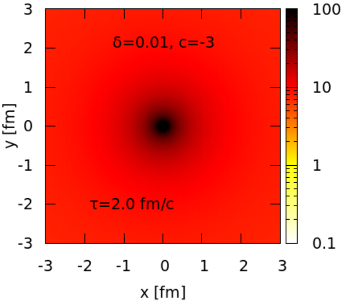


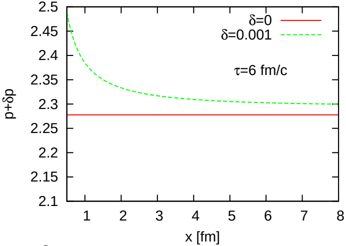


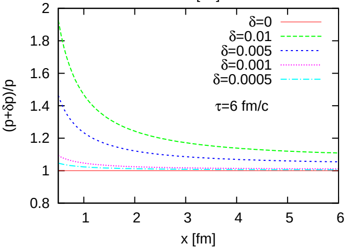
Pressure perturbation

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}}$$

$$\delta p = \delta \cdot p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}} \frac{(\kappa+1)(\kappa-3)}{\kappa} S$$



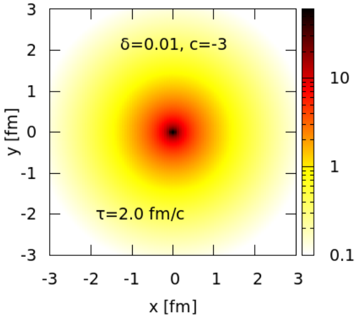


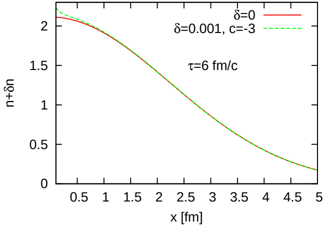


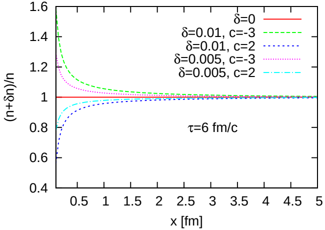
Density perturbation

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{N}(S)$$

$$\delta n = \delta \cdot 2bn_0 \left(\frac{\tau_0}{\tau}\right)^3 \left(\ln\left(\frac{\tau}{\tau_0}\right) + c\frac{\kappa}{3-\kappa}\left(\frac{\tau}{\tau_0}\right)^{\frac{3}{\kappa}-1}\right) S^{-3}\left(1-S^2\right)^2 \mathcal{N}(S)$$







Calculation of observables

- Source function → Jüttner-distribution:

$$S(x,p)d^4x = Nn(x)\exp\left(-\frac{p_\mu u^\mu}{T}\right)H(\tau)p_\mu d^3\Sigma^\mu(x^\mu)d\tau \tag{21}$$

- The Cooper–Frye factor: $p_\mu d^3\Sigma^\mu(x^\mu) = \frac{p_\mu u^\mu}{u^0}d^3x$
- Freeze out at constant proper time → $H(\tau) = \delta(\tau - \tau_0)$
- The perturbed source function:

$$S(x,p) = Nn(x)\exp\left(-\frac{p_\mu u^\mu}{T}\right)\delta(\tau-\tau_0)\frac{p_\mu u^\mu}{u^0}\cdot(1+\Delta)d\tau dx^3$$

$$\Delta = \left[\frac{\delta u^0}{u^0} + \frac{p_\mu \delta u^\mu}{p_\nu u^\nu} - \frac{p_\mu \delta u^\mu}{T} + \frac{p_\mu u^\mu \delta T}{T^2} + \frac{\delta n}{n}\right]$$

- Single-particle distribution:

$$N_1(p) = \int S(x,p)d^4x \tag{22}$$

Single particle transverse momentum distribution

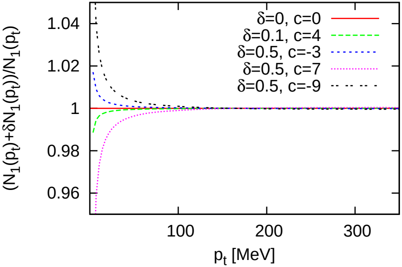
Two component Gaussian:

$$N(p) = Nn_0\mathcal{E}_1\mathcal{V}_1(1+\mathcal{P}_1+\mathcal{P}_2+\mathcal{P}_3) + Nn_0\mathcal{E}_2\mathcal{V}_2(\mathcal{P}_4+\mathcal{P}_5)$$

$$\mathcal{E}_1 = \exp\left[-\frac{E^2+m^2}{2ET_0} - \frac{p^2}{2ET_{\text{eff}}}\right] \qquad \mathcal{E}_2 = \exp\left[-\frac{E^2+m^2}{2ET_0} - \frac{p^2}{2ET_{\text{eff},\delta}}\right]$$

Used parameters: describes hadronic & photonic data $(v_2, R_{HBT}, N(p_T))$

M. Csanád, M. Vargyas, Eur. Phys. J. **A 44**, 473 (2010)



Effective temperatures:

$$T_{\text{eff}} = T_0 + \frac{T_0 E \dot{R}_0^{\,2}}{2b(T_0 - E)}$$

$$T_{\text{eff},\delta} = T_0 + \frac{T_0 E \dot{R}_0^{\,2}}{2b(2T_0 - E)}$$

Hubble-flow results very stable against perturbations!

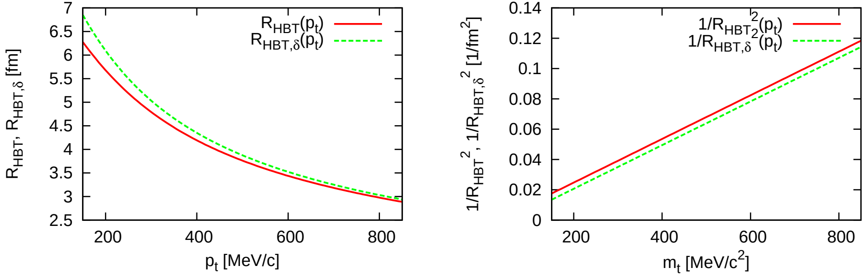
HBT-radii for $S = t/r$

Size of the source $\rightarrow$ HBT-radii

- $R^{-2} \propto m_t$ scaling

$$R^2_{\mathrm{HBT}} = \frac{T_0 \tau_0^2 (T_{\mathrm{eff}} - T_0)}{m_T T_{\mathrm{eff}}}$$

$$R^2_{\mathrm{HBT},\delta} = \frac{T_0 \tau_0^2 (T_{\mathrm{eff},\delta} - T_0)}{m_T T_{\mathrm{eff},\delta}}$$



Parameters: $\delta = 0.5$, $c = -3$

Summary

Hubble-flow

$$u^\mu = \frac{x^\mu}{\tau}$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}}$$

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^{3} \mathcal{N}(S)$$

Perturbations

$$\delta u^\mu = \delta \cdot F(\tau) g(x_\nu) \chi(S) \partial^\mu S$$

$$\delta p = \delta \cdot p_0 \pi(S) \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}}$$

$$\delta n = \delta \cdot n_0 \left(\frac{\tau_0}{\tau}\right)^{3} h(x_\nu) \nu(S)$$

Conclusions:

- A method of finding perturbations
- Analytic understanding of "ripples" possible
- Observables very stable in Hubble-flow case

Outlook:

- Hubble-flow with non-spherical symmetry?
- Other auxiliary functions, waves?
- Perturbations on top of other solutions?

Thank you for your attention!

Equations of non-relativistic hydrodynamics

Looking for (u, p, ρ) fields

Assumptions:

- zero viscosity
- zero heat conductivity

Euler-equation

$$\frac{\partial u}{\partial t} + (u\nabla)u = -\frac{1}{\rho}\nabla p$$

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla(\rho u) = 0$$

Equation of state

$p - \rho$ relation

Perturbations of nonrel standing fluid: waves

Known solution: Standing fluid

- $u = 0$
- $p =$ const.
- $\rho =$ const.

Sound speed from equation of state:

$$\frac{\delta p}{\delta \rho} = c^2$$

Perturbed Euler-equation

$$\frac{\partial \delta u}{\partial t} = -\frac{1}{\rho} \nabla \delta p$$

Perturbed continuity equation

$$\frac{\partial \delta \rho}{\partial t} + \rho \nabla \delta u = 0$$

Wave solution for pressure

$$\frac{\partial^2 p}{\partial t^2} = c^2 \Delta p$$

Decomposition of energy-momentum tensor

Two equations:

- Lorentz-orthogonal to u^μ
- Lorentz-perpendicular to u^μ

Euler equation

$$(\kappa + 1) p u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu) \partial_\nu p$$

Energy equation

$$\kappa u^\mu \partial_\mu p + (\kappa + 1) p \partial_\mu u^\mu = 0$$

Perturbative equations in general

Euler equation

$$(\kappa+1)\delta p u^\mu \partial_\mu u^\nu + (\kappa+1) p \delta u^\mu \partial_\mu u^\nu + (\kappa+1) p u^\mu \partial_\mu \delta u^\nu = (g^{\mu\nu} - u^\mu u^\nu)\partial_\mu \delta p - \delta u^\mu u^\nu \partial_\mu p - u^\mu \delta u^\nu \partial_\mu p \quad (23)$$

Energy equation

$$\kappa \delta u^\mu \partial_\mu p + \kappa u^\mu \partial_\mu \delta p + (\kappa+1)\delta p \partial_\mu u^\mu + (\kappa+1) p \partial_\mu \delta u^\mu = 0 \quad (24)$$

Continuity equation

$$u^\mu \partial_\mu \delta n + \delta n \partial_\mu u^\mu + \delta u^\mu \partial_\mu n + n \partial_\mu \delta u^\mu = 0 \quad (25)$$

Solving the energy equation

Pressure perturbation

$$\delta p = \delta \cdot p_0 \left(\frac{\tau_0}{\tau}\right)^{3+\frac{3}{\kappa}} \pi(S). \quad (26)$$

Four-velocity perturbation

$$\delta u^\mu = \delta \cdot F(\tau) g(x_\mu) \partial^\mu S \cdot \chi(S) \quad (27)$$

- Orthogonality satisfied ($\delta u_\mu u^\mu = 0$)

Energy equation

$$\frac{\chi'(S)}{\chi(S)} = -\frac{\partial_\mu \partial^\mu S}{\partial_\mu S \partial^\mu S} - \frac{\partial_\mu S \partial^\mu \ln g(x_\mu)}{\partial_\mu S \partial^\mu S} \quad (28)$$

Right side is a function of S!

Solution of the Euler equation

Using (26) and (27) perturbations:

Euler equation:

$$\frac{\pi'(S)}{\chi(S)} = (\kappa + 1)\left[F(\tau)\left(u^\mu \partial_\mu g(x_\mu) - \frac{3g(x_\mu)}{\kappa\tau}\right) + F'(\tau)g(x_\mu)\right] \tag{29}$$

- Right side is a function of S
- Restriction for S, $g(x_\mu)$, $F(\tau)$

Solving the continuity equation

Using (27) form of perturbation

The particle density perturbation

$$\delta n = \delta \cdot n_0 \left(\frac{\tau_0}{\tau}\right)^3 h(x_\mu)\nu(S) \qquad (30)$$

Continuity equation

$$\frac{\nu(S)}{\chi(S)\mathcal{N}'(S)} = -\frac{F(\tau)g(x_\mu)\partial_\mu S \partial^\mu S}{u^\mu \partial_\mu h(x_\mu)} \qquad (31)$$

Right side is a funciton of S

- Restriction for S, $h(x_\mu)$, $F(\tau)$

Scaling variable $S = r^m/\tau^m$

Scaling variable $S = r^m/\tau^m$

$$\chi(S) = \frac{S^{-\frac{m+1}{m}}}{\sqrt{S^{\frac{2}{m}}+1}} \tag{32}$$

$$\pi(S) = \frac{(\kappa+1)(\kappa-3)}{\kappa}\left[\pi_0 - m\sqrt{1+S^{-\frac{2}{m}}}\right], \tag{33}$$

$$\nu(S) = m^2 S^2 \left[S^{-\frac{2}{m}}+1\right]\frac{S^{-\frac{m+1}{m}}}{\sqrt{S^{\frac{2}{m}}+1}}\mathcal{N}'(S) \tag{34}$$

Scaling variable $S = \tau^m / t^m$

Scaling variable $S = \tau^m / t^m$

$$\chi(S) = \frac{S^{\frac{2}{m}-1}}{\left(1 - S^{\frac{2}{m}}\right)^{\frac{3}{2}}} \tag{35}$$

$$\pi(S) = \frac{(\kappa+1)(\kappa-3)}{\kappa}\left(\pi_0 + \frac{m}{\sqrt{1 - S^{\frac{2}{m}}}}\right) \tag{36}$$

$$\nu(S) = m^2 S^2 \frac{S^{\frac{2}{m}-1}}{1 - S^{\frac{2}{m}}} \mathcal{N}'(S) \tag{37}$$

Perturbative equations

Perturbed fields

- $u \rightarrow u + \delta u$
- $p \rightarrow p + \delta p$
- $\rho \rightarrow \rho + \delta \rho$

Perturbed equations

- first order perturbation
- using another solution

Source function

$$S(x,p) = N\delta(\tau-\tau_0)d\tau d^3 x n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{N}(S)$$

$$\exp\left[-\frac{Et-xp_x-yp_y-zp_z}{\tau T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{3}{\kappa}}}\mathcal{N}(S)\right]\left(E-\frac{xp_x+yp_y+zp_z}{t}\right)\cdot$$

$$\cdot\left[1+\delta\left(-\frac{(\tau+c\tau_0^{\frac{\kappa-3}{\kappa}}\tau^{\frac{3}{\kappa}})\partial^0 S\chi(S)\tau}{t}+\right.\right.$$

$$+\frac{(\tau+c\tau_0^{\frac{\kappa-3}{\kappa}}\tau^{\frac{3}{\kappa}})\chi(S)t}{Et-xp_x-yp_y-zp_z}p_\mu\partial^\mu S+$$

$$+\frac{(Et-xp_x-yp_y-zp_z)(\mathcal{N}(S)\pi(S)-h(x,y,z,t)\nu(S))}{\tau T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{3}{\kappa}}}+$$

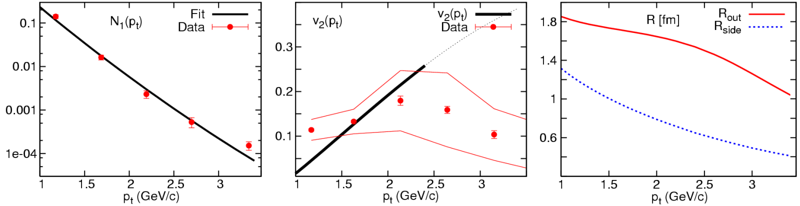
$$\left.\left.+\frac{h(x,y,z,t)\nu(S)}{\mathcal{N}(S)}\right)\right]$$

Photons from Hubble-flow solution

- Photons and leptons are created throughout the evolution
- *Their distribution reveals information about the EoS!*
- Compared to PHENIX data (spectra and flow) successfully
- Predicted photon HBT radii

  Csanád, Májer, Central Eur. J. Phys. **10** (2012), arXiv:1101.1279

  Data: PHENIX Collaboration, arXiv:0804.4168 and arxiv:1105.4126



- Average EoS: $c_s = 0.36 \pm 0.02_{stat} \pm 0.04_{syst}$ (i.e. $\kappa = 7.7$)
- Compatible with soft dilepton data as well

Single-particle distribution

$$N(p) = Nn_0\mathcal{E}_1\mathcal{V}_1(1+\mathcal{P}_1+\mathcal{P}_2+\mathcal{P}_3) + Nn_0\mathcal{E}_2\mathcal{V}_2(\mathcal{P}_4+\mathcal{P}_5) \tag{38}$$

The newly introduced functions:

$$\mathcal{E}_1 = \exp\left[-\frac{E^2+m^2}{2ET_0} - \frac{p^2}{2ET_{eff}}\right], \qquad \mathcal{V}_1 = \sqrt{\frac{2\pi T_0\tau_0^2}{E}\left(1-\frac{T_0}{T_{eff}}\right)}^3 \left(E - \frac{p^2}{E}\left(1-\frac{T_0}{T_{eff}}\right)\right), \tag{39}$$

$$\mathcal{E}_2 = \exp\left[-\frac{E^2+m^2}{2ET_0} - \frac{p^2}{2ET_{eff,\delta}}\right], \qquad \mathcal{V}_2 = \sqrt{\frac{2\pi T_0\tau_0^2}{E}\left(1-\frac{T_0}{T_{eff,\delta}}\right)}^3 \left(E - \frac{p^2}{E}\left(1-\frac{T_0}{T_{eff,\delta}}\right)\right). \tag{40}$$

The perturbative terms are:

$$\mathcal{P}_1 = -\frac{\delta(1+c)\tau_0^2}{r_1\sqrt{\tau_0^2+r_1^2}}, \qquad \mathcal{P}_2 = \frac{\delta(1+c)\tau_0}{E-\frac{p^2\rho_1^2}{\sqrt{\tau_0^2+r_1^2}}}\left(\frac{E}{r_1} - (p^2\rho_1^2)\frac{\sqrt{\tau_0^2+r_1^2}}{r_1^3}\right), \tag{41}$$

$$\mathcal{P}_3 = \frac{\delta 2bc\kappa}{(3-\kappa)R_0^2}\left(\frac{r_1}{\sqrt{\tau_0^2+r_1^2}}\right)^3\left(\frac{\tau_0}{r_1}\right)^4, \qquad \mathcal{P}_5 = -\frac{\delta(\tau_0+c\tau_0)}{T_0}\left(\frac{E}{r_2} - (p^2\rho_2^2)\frac{\sqrt{\tau_0^2+r_2^2}}{r_2^3}\right), \tag{42}$$

$$\mathcal{P}_4 = \frac{\delta 2bE\sqrt{\tau_0^2+r_2^2} - p^2\rho_2^2}{\dot{R}_0^{\,2}\tau_0 T_0}\left(\frac{(\kappa+1)(\kappa-3)}{\kappa}\frac{\tau_0^2+r_2^2}{r_2} - \frac{c\kappa}{3-\kappa}\tau_0\right)\left(\frac{r_2}{\sqrt{\tau_0^2+r_2^2}}\right)^3\left(\frac{\tau_0}{r_2}\right)^4. \tag{43}$$