The light scalar $\kappa$: its nature and its role at nonzero temperature

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in collaboration with
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Wojciech Broniowski (UJK +IFJ), Viktor Begun (ex UJK, now WUT)

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Outline

From quarks and gluons to hadrons
A simple example: \( K^*(892) \)
\( K_0^*(1430) \) and the light \( k = K_0^*(800) \) as a companion pole
(Same mechanism for other states)
The \( k \) at nonzero \( T \) in thermal models
Summary
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From QCD Lagrangian to baryons and mesons

Born  Giuseppe Lodovico Lagrangia
       25 January 1736
       Turin

Died  10 April 1813 (aged 77)
       Paris

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The QCD Lagrangian

Quark: $u,d,s$ and $c,b,t$ \( R,G,B \)

$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix}; \quad i = u,d,s,...$$

8 type of gluons \((R\overline{G}, B\overline{G},...\))

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i)q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$

$$A_\mu^a; \quad a = 1,..., 8$$

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Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‘colored’ state has been seen.

Confinement: physical states are white and are called hadrons.

Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is not necessarily a quark-antiquark state.
A quark-antiquark state is a conventional meson.

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Conventional mesons

- Quark: u, d, s, ... R, G, B

- $|\text{color}| = \sqrt{1/3(\bar{R}R + \bar{G}G + \bar{B}B)}$
- With $q\bar{q}$ states we can understand a lot of QCD, but not everything.
Example of conventional quark-antiquark states: the $K^*(892)$ and the $K$ mesons

In the vector channel: $K^*(892)$ (brother of the rho meson)

In the pseudoscalar channel: positively charged kaon.

In the scalar channel, the situation is more complicated: scalar kaons $K^0*(1430)$ and $K^0*(800)$ (see later).
Classification of some conventional light mesons

<table>
<thead>
<tr>
<th>State</th>
<th>S</th>
<th>L</th>
<th>J</th>
<th>P</th>
<th>C</th>
<th>J^{PC}</th>
<th>Mesons</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1S_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>0^{-+}</td>
<td>$\pi,\eta,\eta'$</td>
<td>K</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>1^{--}</td>
<td>$\rho,\omega,\phi$</td>
<td>$K^*$</td>
</tr>
<tr>
<td>$^1P_1$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>+</td>
<td>-</td>
<td>1^{-+}</td>
<td>$b_1,h_1,h_1'$</td>
<td>$K_1$</td>
</tr>
<tr>
<td>$^3P_0$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0^{++}</td>
<td>$a_0,f_0,f_0'$</td>
<td>$K_0^*$</td>
</tr>
<tr>
<td>$^3P_1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>+</td>
<td>+</td>
<td>1^{++}</td>
<td>$a_1,f_1,f_1'$</td>
<td>$K_1$</td>
</tr>
<tr>
<td>$^3P_2$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>+</td>
<td>+</td>
<td>2^{++}</td>
<td>$a_2,f_2,f_2'$</td>
<td>$K_2^*$</td>
</tr>
</tbody>
</table>

- Not all quantum numbers are permitted for a quark - antiquark states.

\[ J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, \ldots \]

are exotic quantum numbers.
Non-conventional mesons: theoretical expectations

1) Glueballs

Compact diquark-antidiquark states

2) Hybrids

3) Four-quark states

Molecular states (a type of dynamical generation)

Companion poles (another type of dynamical generation)
Companion poles

• In the following papers the idea of companion poles was discussed.


• In particular, the states $K_0^*(892)$ and $a_0(980)$ represents a nice example (see later on).

Related ideas studied by E. Oset, J. Pelaez, G. Rupp, Van Beveren,...
Loops in a simple ‘boring’ example: $K^*(892)$

based on M. Soltysiak, T. Wolkanowski and F. G.,
Large-Nc pole trajectories of the vector kaon $K^*(892)$
and of the scalar kaons $K^0(800)$ and $K^0(1430)$,
[arXiv:1604.01636 [hep-ph]].
**K*(892) from PDG**

*J^P = \frac{1}{2}(1^-)*

- **K*(892)^±** hadroproduced mass $m = 891.66 \pm 0.26$ MeV
- **K*(892)^±** in $\tau$ decays mass $m = 895.5 \pm 0.8$ MeV
- **K*(892)^0** mass $m = 895.81 \pm 0.19$ MeV $(S = 1.4)$
- **K*(892)^±** hadroproduced full width $\Gamma = 50.8 \pm 0.9$ MeV
- **K*(892)^±** in $\tau$ decays full width $\Gamma = 46.2 \pm 1.3$ MeV
- **K*(892)^0** full width $\Gamma = 47.4 \pm 0.6$ MeV $(S = 2.2)$

### K*(892) Decay Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\Gamma_i/\Gamma$</th>
<th>Confidence Level</th>
<th>$p$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^\pi$</td>
<td>$\sim 100$</td>
<td>%</td>
<td>289</td>
</tr>
<tr>
<td>$K^0\gamma$</td>
<td>$(2.46 \pm 0.21) \times 10^{-3}$</td>
<td></td>
<td>307</td>
</tr>
<tr>
<td>$K^\pm\gamma$</td>
<td>$(9.9 \pm 0.9) \times 10^{-4}$</td>
<td></td>
<td>309</td>
</tr>
<tr>
<td>$K^\pi\pi$</td>
<td>$&lt; 7 \times 10^{-4}$</td>
<td>95%</td>
<td>223</td>
</tr>
</tbody>
</table>
A simple model for $K^*(892)$

- Lagrangian:

$$\mathcal{L}_v = cK^*(892)_{\mu} \partial^\mu K^- \pi^0 + \ldots$$

- Decay width:

$$\Gamma_{K^*}(m) = 3 \frac{|\vec{k}_1|}{8\pi m^2} \frac{e^2}{3} \left[ -M_{\pi}^2 + \frac{(m^2 + M_{\pi}^2 - M_{K^*}^2)^2}{4m^2} \right] F_{\Lambda}(m)$$

where:

$$F_{\Lambda}(m) = e^{-2|\vec{k}_1|^2/\Lambda^2}$$

Form factor: it can be included in the Lagrangian by making it nonlocal. Even if it cuts the three-momentum, a covariant generalization is possible.


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Spectral function $d_{K^*}(m)dm$ determines the probability that $K^*(892)$ has a mass between $m$ and $m + dm$.

- Spectral function:
  $$d_{K^*}(m) = \frac{2m}{\pi} |\text{Im} \Delta_{K^*}(p^2 = m^2)|$$

- Normalization condition:
  $$\int_0^\infty d_{K^*}(m)dm = 1.$$
Large-Nc study of $K^*(892)$

$c \rightarrow \sqrt{\lambda}c, \quad \lambda \equiv \frac{3}{N_c} \quad N_c$ is the number of colors

For large-$N_c$ the spectral function tends to a Dirac-$\delta$, as expected.
Pole position of $K^*(892)$

$K^*(892) : 0.89 - 0.028i \ (GeV)$

For large $N_c$ the pole tends to the real axis.

- It behaves like a Breit-Wigner resonance.
- one peak — one single pole.
- Large $N_c$ in agreement with $q\bar{q}$. 
Narrow state, nice corresponce
K0*(800) as a companion pole of K0*(1430)

based on M. Soltysiak, T. Wolskanowski and F. G.,
K0*(800) as a companion pole of K0*(1430),'
$K_0^*(1430)$ and $K_0^*(800)$ from PDG

$K_0^*(1430)$

$\quad I(J^P) = \frac{1}{2}(0^+)$

Mass $m = 1425 \pm 50$ MeV
Full width $\Gamma = 270 \pm 80$ MeV

$K_0^*(1430)$ DECAY MODES

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
<th>$p$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K\pi$</td>
<td>$(93 \pm 10)$%</td>
<td>619</td>
</tr>
<tr>
<td>$K\eta$</td>
<td>$(8.6 \pm 2.7)$%</td>
<td>486</td>
</tr>
</tbody>
</table>

$K_0^*(800)$

$\quad I(J^P) = \frac{1}{2}(0^+)$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$K_0^*(800)$ MASS

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$682 \pm 29$</td>
<td>OUR AVERAGE</td>
<td></td>
<td></td>
<td>Error includes scale factor of 2.4. See the ideogram below.</td>
</tr>
</tbody>
</table>

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Simple Lagrangian for $K^0\ast(1430)$

- Lagrangian:
  \[ \mathcal{L}_{int} = aK_0^* K^- \pi^0 + bK_0^* \partial_\mu K^- \partial^\mu \pi^0 + \ldots \]

- Decay width:
  \[ \Gamma_{K_0^*}(m) = 3 \frac{|k_1|}{8\pi m^2} \left[ a - \frac{b}{2} \frac{m^2 - M_{K}^2 - M_{\pi}^2}{2} \right]^2 F_\Lambda(m) \]

where:

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Propagator of $K_0^*(1430)$

- Propagator of the scalar kaonic field:
  \[
  \Delta_{K_0^*}(p^2 = m^2) = \frac{1}{m^2 - M_0^2 + \Pi(m^2) + i\varepsilon}
  \]
  where $M_0$ is the bare mass of the scalar.

- Spectral function:
  \[
  d_{K_0^*}(m) = \frac{2m}{\pi} |\text{Im} \Delta_{K_0^*}(p^2 = m^2)|
  \]
  \[
  \int_0^\infty d_{K_0^*}(m)dm = 1.
  \]

According to the optical theorem, $\text{Im} \Pi(m) = m\Gamma_{K_0^*}(m)$.
$$\delta(m) = \frac{1}{2} \arccos \left[ 1 - \pi \Gamma_{K^*}(m) d_{K^*}(m) \right] .$$

**Fit**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$1.60 \pm 0.22 \text{ GeV}$</td>
</tr>
<tr>
<td>$b$</td>
<td>$-11.16 \pm 0.82 \text{ GeV}^{-1}$</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$0.496 \pm 0.008 \text{ GeV}$</td>
</tr>
<tr>
<td>$M_0$</td>
<td>$1.204 \pm 0.008 \text{ GeV}$</td>
</tr>
</tbody>
</table>

$$\chi^2 / \text{d.o.f.} = 1.25$$
Phase shifts/2

only nonderivative

\[ \mathcal{L}_{int} = a K_0^{*+} K^- \pi^0 + \ldots \]

only derivative

\[ \mathcal{L}_{int} = b K_0^{*+} \partial_\mu K^- \partial^\mu \pi^0 + \ldots \]

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Spectral function of $K_0^*(1430)$

Is there a $K_0^*(800)$ or not?

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Comparison of the spectral functions of $K^*(892)$ and $K_0^*(1430)$.
Large-$N_c$
Poles

\[ K_0^*(1430) : (1.413 \pm 0.057) - (0.127 \pm 0.011)i \text{ (GeV)} \]
\[ K_0^*(800) : (0.745 \pm 0.029) - (0.263 \pm 0.027)i \text{ (GeV)} \]
Pole trajectories in the light scalar kaonic system

The additional companion pole on the left, corresponding to the light $k$, disappears for $N_c$ larger than 12.4.

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Pole trajectories/2

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Considerations on the scalar kaonic system

- Scalar kaon: out of one "seed" state → 2 poles appear
  - $K_0^*(1430)$ corresponds to a peak
  - $K_0^*(800)$ "no peak" but there is a pole.
- We determined the position of the poles
  - for vector kaon ( 0.89 - 0.028i (GeV))
  - for scalar kaons
    - $K_0^*(1430) : (1.413 \pm 0.057) - (0.127 \pm 0.011)i$ (GeV)
    - $K_0^*(800) : (0.745 \pm 0.029) - (0.263 \pm 0.027)i$ (GeV)
- $K^*(892)$ is a quark-antiquark state.
- $K_0^*(1430)$ is predominantly a quark-antiquark state.
- $K_0^*(800)$ is a dynamically generated state.
Same phenomenon for other states: past and ongoing works

- $a_0(980)$ as a companion pole of $a_0(1450)$
  T. Wolskanowski, F.G. and D. H. Rischke, $a_0(980)$ revisited,

- Non-Breit-Wigner lineshape of the resonance $\Psi(3770)$. Interestingly, two poles are connected to it.

- Ongoing and future works:
  $\Psi(4040)$ as seed state, $Y(4008)$ as its dynamically generated companion poles.
  $\Psi(4160)$ line shape and its companion poles (eventually also above its mass)
  $X(3872)$ as a companion pole of a seed charm-anticharm state.
  $Ds(2317)$ as a companion pole of a seed charm-starnge state.
The light k in heavy ion collisions

based on W. Broniowski, F.G., V. Begun,
Cancellation of the sigma meson in thermal models
[arXiv:1506.01260 [nucl-th]].
At the freeze-out, the emission of hadrons is well described by thermal models. Question: does the light k and its brother, f₀(500), play a role? Light states, at first yes, but the answer is unexpected.
Theoretical description of a thermal gas

\[ \ln Z = \sum_k \ln Z^\text{stable}_k + \sum_k \ln Z^\text{res}_k \]

\[ \ln Z^\text{stable},_k = f_k V \int \frac{d^3p}{(2\pi)^3} \ln \left[ 1 \pm e^{-E_p/T} \right]^{\pm 1} \]

\[ E_p = \sqrt{p^2 + M_k^2} \]
Theoretical description of a thermal gas: unstable particles

\[ \ln Z_k^{\text{res}} = f_k V \int_0^\infty d_k(M) \, dM \int \frac{d^3p}{(2\pi)^3} \ln \left[ 1 - e^{-E_p/T} \right]^{-1} \]

At first, the function \(d_k(m)\) can be interpreted as a mass probability density. Namely, a resonance does not have a definite mass but a mass distribution. If not too broad, \(d_k(m)\) well described by a Breit-Wigner function. (This is not the case for the kaonic system and also not for \(f_0(500)\).)
The density function can be directly extracted from two-body scattering data (phase shifts).

\[ d_k(M) = \frac{d\delta_k(M)}{\pi dM} \]

Recall from scattering theory:

\[ \frac{e^{2i\delta_k} - 1}{2i} = a_k = \frac{-\sqrt{s}\Gamma(\sqrt{s})}{s - m^2 + i\sqrt{s}\Gamma(\sqrt{s})} \]

This is a model-independent way of taking the resonances into account.

Indeed, it is a justification of the validity of thermal gas models.

But it is even more, since it allows also to include repulsions in some channels.
Theoretical description of a thermal gas: QCD

\[
\ln Z = \ln Z_\pi + \ln Z_K + \ln Z_{(1/2,0++)} + \ln Z_{(3/2,0++)} + \ln Z_{(0,0++)} + \ln Z_{(2,0++)} + \ln Z_{(1,1--)} + \ldots
\]

\[
\ln Z_{(I,J)} = (2I + 1)(2J + 1) \int_0^{A_0} \frac{d\delta_{(I,J)}}{\pi dm} \int_p \ln \left[ 1 - e^{-\frac{\sqrt{p^2 + m^2}}{T}} \right]^{-1}
\]

For many resonances the Breit-Wigner approximation is valid

\[
\frac{d\delta_{I,J}}{\pi dM} \sim \sum_k \frac{\Gamma_{IJ,k}}{2\pi} \left[ (M - M_{IJ,k})^2 + \frac{\Gamma_{IJ,k}^2}{4} \right]^{-1}
\]

However, this approximation does not hold for the light \( k \) (and also not for \( f_0(500) \)).

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Theoretical description of a thermal gas: QCD

\[ \ln Z = \ln Z_\pi + \ln Z_K + \ln Z_{(1/2,0++)} + \ln Z_{(3/2,0++)} + \ln Z_{(0,0++)} + \ln Z_{(2,0++)} + \ln Z_{(1,1--)} + \ldots \]

\[ \ln Z_{(1/2,0++)} + \ln Z_{(3/2,0++)} = \int_0^{\Lambda_0} dm \left[ 2 \frac{d\delta_{(1/2,0)}}{\pi dm} + 4 \frac{d\delta_{(3/2,0)}}{\pi dm} \right] \int_p \ln \left[ 1 - e^{-\frac{\sqrt{p^2 + m^2}}{T}} \right]^{-1} \]

Simple and model-independent procedure: **just use scattering data!**
The scalar kaonic resonace $K_0^*(800)$: (partial) cancellation in thermal models

The total contribution from is the red curve: $\ln Z_{(1/2,0)} + \ln Z_{(3/2,0)}$ cancellation is evident: easiest thing to do is to forget about the k. (Eventually, visible in correlations).

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The f$_0$(500) spectral function and the isotensor repulsion/1

The total contribution from J=0 is the red curve: $\ln Z(0,0) + \ln Z(2,0)$

$\ln Z(0,0)$ is the contribution of f$_0$(500). It is indeed nonzero and even non-negligible, but it is almost exactly cancelled by the isotensor repulsion. Thermal models however usually neglect repulsions.

Either you take into account both l=0 and l =2, or –simply- you neglect both of them.
Conclusions

The light $k$ exists
(as a companion dynamically generated pole)
but
can be neglected in thermal models at nonzero $T$
Thank You
Cutoff function/1

- the cutoff parameter $\Lambda$ does not exist at the Lagrangian level
- it can be implemented by using a non-local interaction term (if $f_\Lambda(q) = f_\Lambda(|q|)$), e.g.

$$\mathcal{L}_{\text{int}} = gS(x)\phi^2(x) \rightarrow \mathcal{L}_{\text{int}} = gS(x) \int d^4y \phi(x+y/2)\phi(x-y/2)\Phi(y)$$

- changes also the tree-level result for the decay width:

$$\Gamma_{\text{tree}}(s) \rightarrow \Gamma_{\text{tree}}(s) \cdot f_\Lambda^2(\rho S_{\phi\phi})$$

- our choice:

**Regularization function in our case**

$$f_\Lambda(q) = \exp\left(-|q|^2/\Lambda^2\right)$$
The contribution of the loop $\Pi(m^2)$ in which the particles $\varphi_1$ and $\varphi_2$ circulate as calculated from the original local Lagrangian (1) reads

$$\Pi(m^2) = \int \frac{d^4k_1}{(2\pi)^4} \frac{[a-b(k_1 \cdot k_2)]^2}{[k_1^2 - m_1^2 + i\varepsilon][k_2^2 - m_2^2 + i\varepsilon]} ,$$

(6)

where the constraint $k_2 = p - k_1$ is understood and $p$ is the momentum of the unstable particle $\Sigma$. In its reference frame $p = (m, 0)$. As mentioned above, this loop contribution is divergent (with $\Lambda^4$). The substitution (4) makes it convergent thanks to the form-factor:

$$\Pi(m^2) \rightarrow \Pi_\Lambda(m^2) = \int \frac{d^4k_1}{(2\pi)^4} \frac{[a-b(k_1 \cdot k_2)]^2 f_\Lambda^2(\vec{k}_1^2)}{[k_1^2 - m_1^2 + i\varepsilon][k_2^2 - m_2^2 + i\varepsilon]} .$$

(7)

At this point, one may object that the form factor breaks covariance, since it depends on the three-momentum only. We will show in the next section that this is not necessarily the case. Once the form factor is in-
Quark model(s) and their QFT extensions

Mesons in a Relativized Quark Model with Chromodynamics
S. Godfrey, Nathan Isgur
Published in Phys.Rev. D32 (1985) 189-231

Mesonic loops e.g. included into
A Low Lying Scalar Meson Nonet in a Unitarized Meson Model
E. van Beveren, T. A. Rijken, K. Metzger, C.~Dullemond, G.~Rupp and J. E.~Ribeiro,
Z. Phys. C 30 (1986) 615
Meson spectroscopy: too much excitement and too few excitations
G. Rupp, S. Coito and E. van Beveren,

NJL: quark-based model with
chiral symmetry and SSB
chiral condensate
Effective quark mass
Mesons as quarkonia (pion: ok)

QCD phenomenology based on a chiral effective Lagrangian
Tetsuo Hatsuda, Teiji Kunihiro
Phys.Rept. 247 (1994) 221-367

The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking,
and hadrons as relativistic bound states
Reinhard Alkofer, Lorenz von Smekal

Baryons as relativistic three-quark bound states
G. Eichmann, H.~ Sanchis-Alepuz, R. Williams, R. Alkofer and C. S. Fischer,
Progr. Part. Nucl. Phys. 91 (2016) 1

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Example of conventional quark-antiquark states: the $\rho$ and the $\pi$ mesons

$\mathbf{u} \bar{\mathbf{d}}$

Vector channel: Rho-meson

$m_{\rho^+} = 775$ MeV

Pseudoscalar channel: Pion

$m_{\pi^+} = 139$ MeV

$M_u + M_d \approx 7$ MeV

Mass generation in QCD is a nonpert. phenomenon based on SSB

In the scalar channel, the situation is more complicated: a0(1450) and a0(980), see later on

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a_{0}(1450) and a_{0}(1470) from PDG

\[ I^{G}(J^{PC}) = 1^{-}(0^{++}) \]

Decays into \( \eta \pi, \eta' \pi, KK \)

\[ a_{0}(1450) \text{ MASS} \]

\begin{tabular}{lrrr}
\hline
VALUE (MeV) & EVTS & DOCUMENT ID & TECN \\
1474 & \pm 19 & OUR AVERAGE & \\
\hline
\end{tabular}

Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

\[ a_{0}(980) \text{ MASS} \]

\[ I^{G}(J^{PC}) = 1^{-}(0^{++}) \]

Decays into \( \eta \pi, KK \)

\[ a_{0}(980) \text{ MASS} \]

\begin{tabular}{lrrr}
\hline
VALUE (MeV) & DOCUMENT ID & TECN \\
980 & \pm 20 \text{ OUR ESTIMATE} & Mass determination very model dependent & \\
\hline
\end{tabular}

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Lagrangian for a0 system

\[ \mathcal{L}_{a_0 \eta \pi} = A_1 a_0^0 \eta \pi^0 + B_1 a_0^0 \partial_\mu \eta \partial^\mu \pi^0 \]
\[ \mathcal{L}_{a_0 \eta' \pi} = A_2 a_0^0 \eta' \pi^0 + B_2 a_0^0 \partial_\mu \eta' \partial^\mu \pi^0 \]
\[ \mathcal{L}_{a_0 K\bar{K}} = A_3 a_0^0 (K^0 \bar{K}^0 - K^- K^+) + B_3 a_0^0 (\partial_\mu K^0 \partial^\mu \bar{K}^0 - \partial_\mu K^- \partial^\mu K^+) \]

The field a0 corresponds (roughly) to a0(1450)
Spectral function, poles, and large-Nc

The additional companion pole on the left, corresponding to the $a_0(980)$, disappears for $N_c$ larger than 4.9.

Branching ratios for $a_0(1450)$ and $a_0(980)$ in agreement with PDG

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General properties

- $\Psi(3770)$
- D-wave state, first charmonium above DD threshold.

$\Psi(3770)$

$$I^G(J^{PC}) = 0^-(1^{--})$$

$\Psi(3770)$ MASS (MeV)

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3773.13 ± 0.35</td>
<td>OUR FIT</td>
<td>Error includes scale factor of 1.1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3778.1 ± 1.2</td>
<td>OUR AVERAGE</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\Psi(3770)$ WIDTH

<table>
<thead>
<tr>
<th>VALUE (MeV)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.2 ± 1.0</td>
<td>OUR FIT</td>
<td></td>
</tr>
<tr>
<td>27.5 ± 0.9</td>
<td>OUR AVERAGE</td>
<td></td>
</tr>
</tbody>
</table>

Mode | Fraction ($\Gamma_i/\Gamma$) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_1$</td>
<td>$D\bar{D}$</td>
</tr>
<tr>
<td>$\Gamma_2$</td>
<td>$D^0\bar{D}^0$</td>
</tr>
<tr>
<td>$\Gamma_3$</td>
<td>$D^+D^-$</td>
</tr>
</tbody>
</table>
Lagrangian and loops

\[ \mathcal{L}_{\psi D \bar{D}} = ig_{\psi D^0 \bar{D}^0} \psi_{\mu} \left( \partial^\mu D^0 \bar{D}^0 - \partial^\mu \bar{D}^0 D^0 \right) + ig_{\psi D^+ D^-} \psi_{\mu} \left( \partial^\mu D^+ D^- - \partial^\mu D^- D^+ \right) \]

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Fit to data

\[ \sigma_{e^+e^- \rightarrow DD} = \frac{\pi}{2E} g_{\psi e^+e^-}^2 \rho_{\psi}(E) \]

Fig. 5 Data: • BES [7], * BES [5], * BaBar [6] (the latter not used in the fit). Solid line: our fit (cf. Table 1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_\psi) (MeV)</td>
<td>3773.05 ± 0.95</td>
</tr>
<tr>
<td>(\Lambda) (MeV)</td>
<td>272.55 ± 1.17</td>
</tr>
<tr>
<td>(g_{\psi DD})</td>
<td>30.7 ± 4.8</td>
</tr>
<tr>
<td>(g_{\psi e^+e^-})</td>
<td>(1.062 ± 0.032) × 10^{-3}</td>
</tr>
<tr>
<td>(\chi^2)</td>
<td>20.52</td>
</tr>
<tr>
<td>(\chi^2/d.o.f)</td>
<td>0.86</td>
</tr>
</tbody>
</table>

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Position(s) of the poles of $\Psi(3770)$

Two poles are present in the complex plane

First pole: $E = 3776.8 - i12.3$ MeV,

hence

$m^\text{pole}_\psi \simeq 3776.8 \pm 1.0$ MeV and
$\Gamma^\text{pole}_\psi \simeq 24.6 \pm 2.0$ MeV,

Second pole: $E = 3741.2 - i18.5$ MeV

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The additional companion pole disappears for $N_c$ larger than 3.9.
Phase-shift formula

The radial wave function with angular momentum $l$ of a particle scattered by central potential $U(r)$ is

$$
\psi_l(r) \propto \sin[kr - l\pi/2 + \delta_l],
$$

where $k = |k|$ is the length of the three-momentum, and $\delta_l$ is the phase shift due the interaction with the potential. If we confine our system into a sphere of radius $R$, the condition $kR - l\pi/2 + \delta_l = n\pi$ with $n = 0, 1, 2, \ldots$ must be met, since $\psi_l(r)$ has to vanish at the boundary. Conversely, the number of states $n_0$ that one can has by limiting $k$ in the range $(0, k_0)$ is given by $n_0 = (k_0R - l\pi/2 + \delta_l)/\pi$. Then, the density of states that one can place between $k$ and $k + dk$ is given by

$$
\frac{d\eta_l}{dk} = \frac{R}{\pi} + \frac{1}{\pi} \frac{d\delta_l}{dk},
$$

where the first term describes the density of states $\frac{d\eta_{l,\text{free}}}{dk}$ in absence of interactions, while the second term $\frac{1}{\pi} \frac{d\delta_l}{dk}$ describes the effect of the interacting potential.

When translating the discussion from Quantum Mechanics to Quantum Field Theory, we replace the momentum $k$ with the invariant mass $m$, and the angular momentum $l$ with the pair $(I, J)$. Upon summing over the latter, one obtains the full density of states of an interacting pion gas as

$$
\frac{d\eta}{dm} = \delta(m - m_\pi) + \sum_{I, J} \frac{1}{\pi} \frac{d\delta_{I, J}(m)}{dm}.
$$

When plugging the previous equation into the general expression

$$
\ln Z_{I, J} = J_{(I, J)} \int_0^\infty \frac{d\eta_{I, J}}{dm} \int d^3p \frac{1}{(2\pi)^3} \ln \left[1 - e^{-\sqrt{p^2 + m^2}/T}\right]
$$

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