The light scalar κ: its nature and its role at nonzero temperature

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Outline

From quarks and gluons to hadrons

A simple example: K*(892)

$K_0^*(1430)$ and the light k= $K_0^*(800)$ as a companion pole

(Same mechanism for other states)

The k at nonzero T in thermal models

Summary

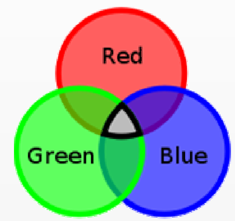
From QCD Lagrangian to baryons and mesons

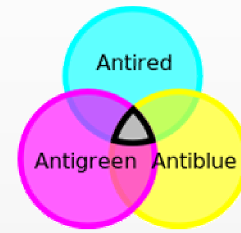
Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



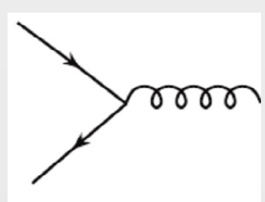


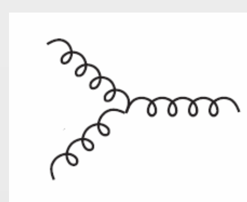
$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \; i = u,d,s \; ,...$$

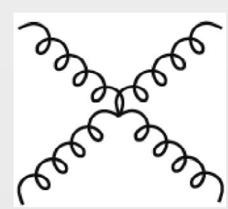
8 type of gluons (R$\overline{\text{G}}$,B$\overline{\text{G}}$,…)

$$\boxed{\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}}$$

$$A_\mu^a \; ; \; a = 1,...,8$$







Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are white and are called hadrons.

Hadrons can be:

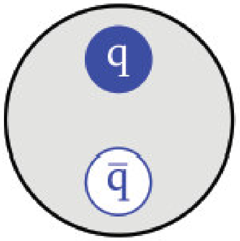
Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

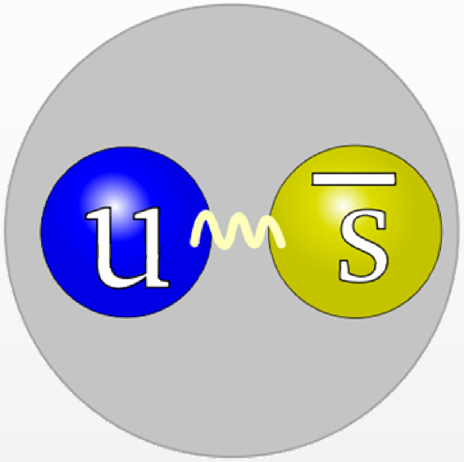
Conventional mesons

- Quark: u,d,s,... R,G,B
- Conventional mesons: quark − antiquark bound states.



- $|color>= \sqrt{1/3}(\bar{R}R + \bar{G}G + \bar{B}B)$
- With $q\bar{q}$ states we can understand a lot of QCD, but not everything.

Example of conventional quark-antiquark states: the K*(892) and the K mesons



In the vector channel: K*(892) (brother of the rho meson)

In the pseudoscalar channel: positively charged kaon.

In the scalar channel, the situation is more complicated:
scalar kaons K0*(1430) and K0*(800) (see later).

Classification of some conventional light mesons

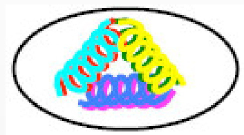
State	S	L	J	P	C	J^{PC}	Mesons	Name
$^1S_0$	0	0	0	-	+	0^{-+}	π η η' K	pseudoscalar
$^3S_1$	1	0	0	-	-	1^{--}	ρ ω ϕ K^*	vector
$^1P_1$	0	1	1	+	-	1^{+-}	b_1 h_1 h_1' K_1	pseudo-vector
$^3P_0$	1	1	0	+	+	0^{++}	a_0 f_0 f_0' K_0^*	scalar
$^3P_1$	1	1	1	+	+	1^{++}	a_1 f_1 f_1' K_1	axial vector
$^3P_2$	1	1	2	+	+	2^{++}	a_2 f_2 f_2' K_2^*	tensor

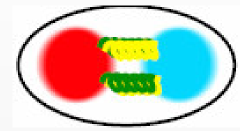
- Not all quantum numbers are permitted for a quark - antiquark states.

$$J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, \cdots$$

are exotic quantum numbers.

Non-conventional mesons: theoretical expectations

1) Glueballs 

2) Hybrids 

3) Four-quark states

Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

Companion poles (another type of dynamical generation)

Companion poles

- In the following papers the idea of companion poles was discussed.

N. A. Törnqvist, *Understanding the scalar meson $q\bar{q}$ nonet*, *Z. Phys. C* **68** (1995) 647 [arXiv:hep-ph/9504372]; N. A. Törnqvist and M. Roos, *Confirmation of the Sigma Meson*, *Phys. Rev. Lett.* **76** (1996) 1575 [arXiv:hep-ph/9511210].

M. Boglione and M. R. Pennington, *Dynamical generation of scalar mesons*, *Phys. Rev. D* **65** 114010 (2002) [arXiv:hep-ph/0203149].

- In particular, the states $K_0^*(892)$ and $a_0(980)$ represents a nice example (see later on).

Related ideas studied by E. Oset, J. Pelaez, G. Rupp, Van Beveren,...

Loops in a simple ‘boring’ example: K*(892)

based on M. Soltysiak, T. Wolkanowski and F. G.,
Large-Nc pole trajectories of the vector kaon K*(892)
and of the scalar kaons K0*(800) and K0*(1430),"
Acta Phys. Polon. Supp.9 (2016) 321
[arXiv:1604.01636 [hep-ph]].

K*(892) from PDG

$K^*(892)$ $\qquad I(J^P) = \frac{1}{2}(1^-)$

$K^*(892)^\pm$ hadroproduced mass $m = 891.66 \pm 0.26$ MeV
$K^*(892)^\pm$ in τ decays mass $m = 895.5 \pm 0.8$ MeV
$K^*(892)^0$ mass $m = 895.81 \pm 0.19$ MeV $\quad (S = 1.4)$
$K^*(892)^\pm$ hadroproduced full width $\Gamma = 50.8 \pm 0.9$ MeV
$K^*(892)^\pm$ in τ decays full width $\Gamma = 46.2 \pm 1.3$ MeV
$K^*(892)^0$ full width $\Gamma = 47.4 \pm 0.6$ MeV $\quad (S = 2.2)$

$K^*(892)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$K\pi$	~ 100 %		289
$K^0\gamma$	$(2.46 \pm 0.21) \times 10^{-3}$		307
$K^\pm\gamma$	$(9.9 \pm 0.9) \times 10^{-4}$		309
$K\pi\pi$	$< 7 \times 10^{-4}$	95%	223

A simple model for K*(892)

- Lagrangian:

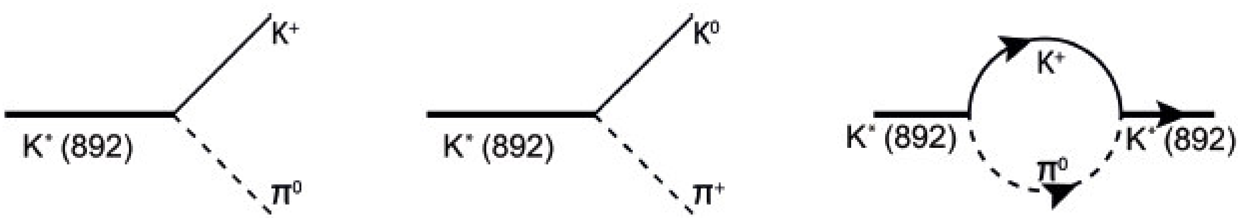
$$\boxed{\mathcal{L}_v = cK^*(892)^+_\mu \partial^\mu K^- \pi^0 + \dots}$$

- Decay width:

$$\Gamma_{K^*}(m) = 3\frac{|\vec{k}_1|}{8\pi m^2}\frac{c^2}{3}\left[-M_\pi^2 + \frac{(m^2+M_\pi^2-M_K^2)^2}{4m^2}\right]F_\Lambda(m)$$

where:

$$F_\Lambda(m) = e^{-2|\vec{k}_1|^2/\Lambda^2}$$

- 

$$\mathrm{Re}\,\Pi(s) = \frac{1}{\pi}\int \mathrm{d}s' \,\frac{-\mathrm{Im}\,\Pi(s')}{s-s'}$$

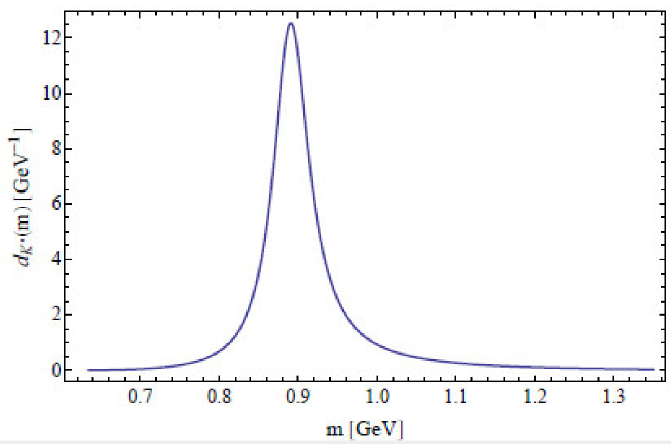
Form factor: it can be included in the Lagrangian by making it nonlocal.
Even if it cuts the three-momentum, a covariant generalization is possible.

M. Soltysiak and F. Giacosa, ``A covariant nonlocal Lagrangian for the description of the scalar kaonic sector,''
Acta Phys.\ Polon.\ Supp.\ {\bf 9} (2016) 467 [arXiv:1607.01593 [hep-ph]].

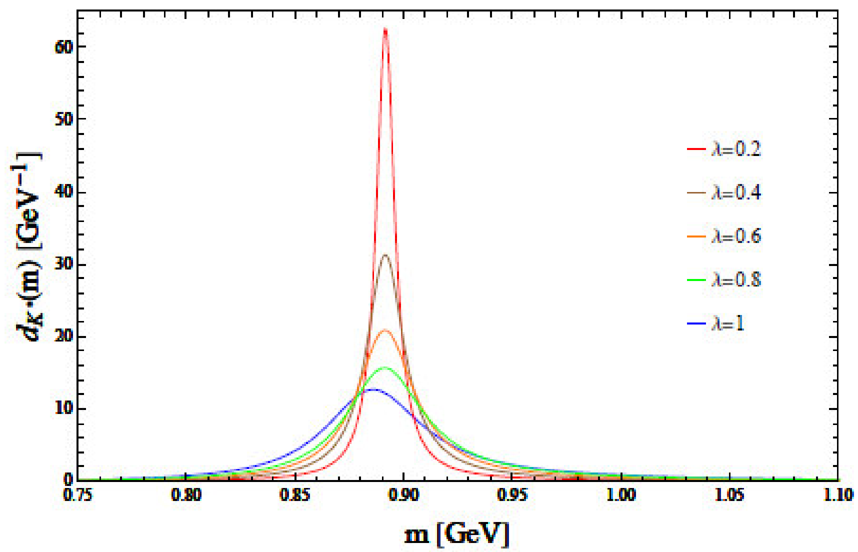
Spectral function

Spectral function $d_{K^*}(m)dm$ determines the probability that $K^*(892)$ has a mass beetwen m and $m + dm$.

- Specral function:
  $d_{K^*}(m) = \frac{2m}{\pi} |\operatorname{Im} \Delta_{K^*}(p^2 = m^2)|$
- normalization condition:
  $\int_0^\infty d_{K^*}(m)\mathrm{dm} = 1.$



Large-Nc study of K*(892)

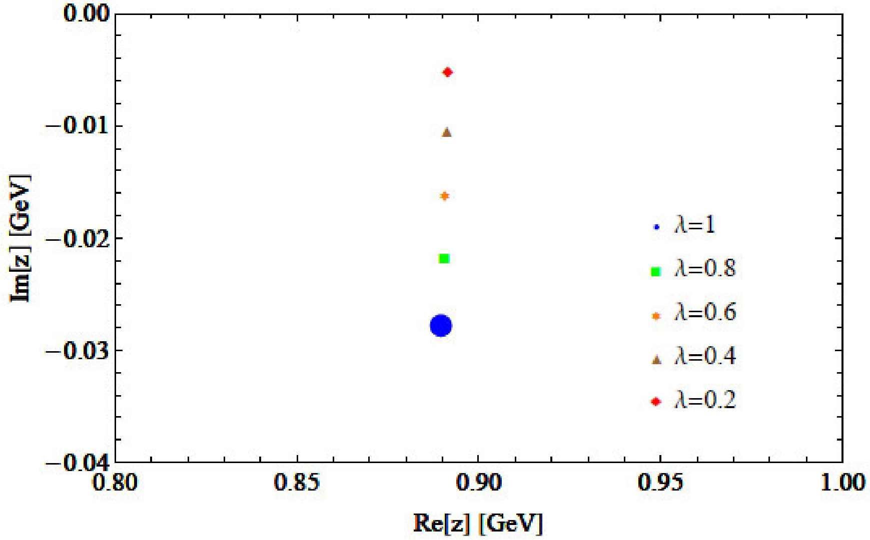


1604.01636

$c \to \sqrt{\lambda}c, \quad \lambda \equiv \frac{3}{N_c} \quad N_c$ is the number of colors

For large-N_c the spectral function tends to a Dirac$-\delta$, as expected.

Pole position of K*(892)



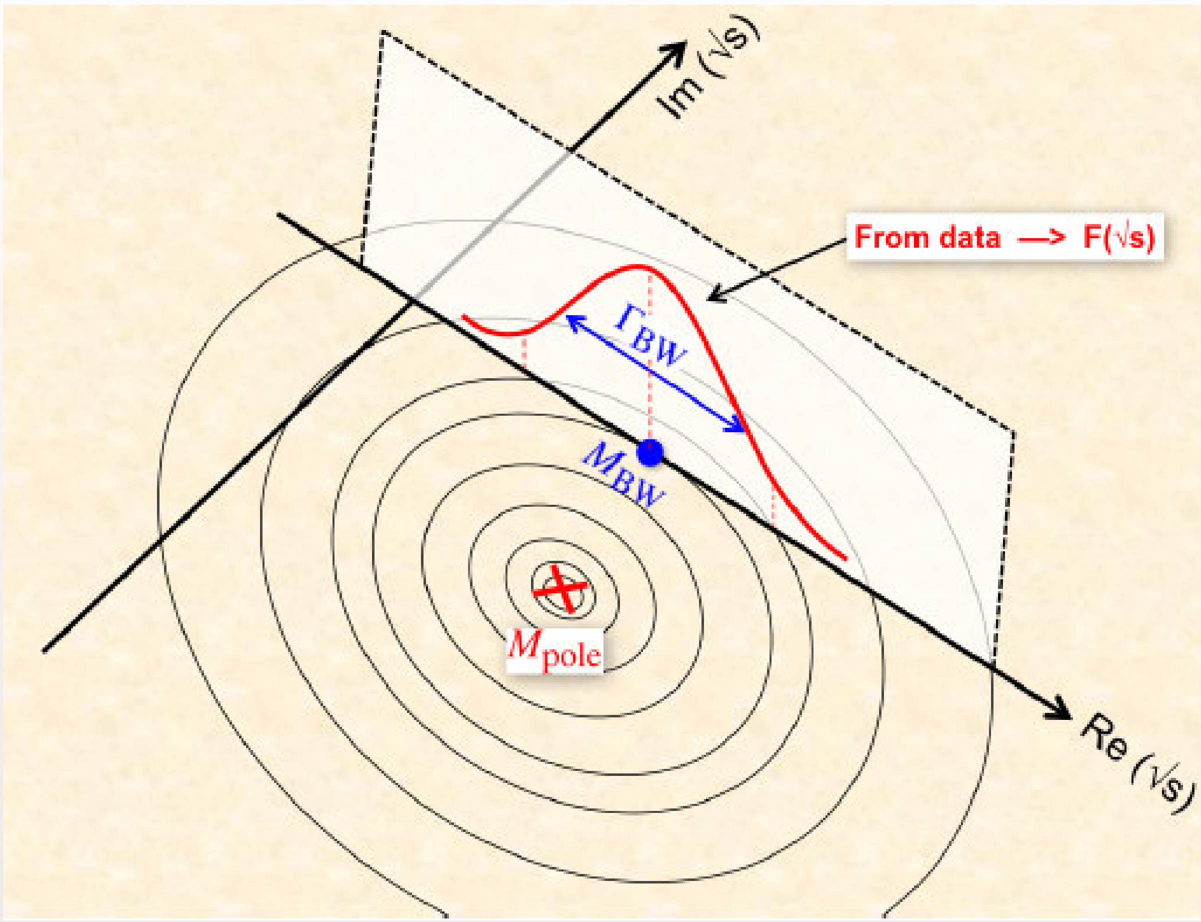
$K^*(892): 0.89 - 0.028i\ (GeV)$

For large N_c the pole tends to the real axis.

1604.01636

- It behaves like a Breit-Wigner resonance.
- one peak – one single pole.
- Large$-N_c$ in agreement with $q\bar{q}$.

Narrow state, nice corresponce



K0*(800) as a companion pole of K0*(1430)

based on M. Soltysiak, T. Wolkanowski and F. G.,
$K_0^*(800)$ as a companion pole of $K_0^*(1430)$,'
Nucl. Phys. B 909 (2016) 418
[arXiv:1512.01071 [hep-ph]].

$K_0^*(1430)$ and $K_0^*(800)$ from PDG

$K_0^*(1430)$ [nn]

$I(J^P) = \frac{1}{2}(0^+)$

Mass $m = 1425 \pm 50$ MeV
Full width $\Gamma = 270 \pm 80$ MeV

$K_0^*(1430)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$K\pi$	(93 ± 10) %	619
$K\eta$	$(8.6^{+2.7}_{-3.4})$ %	486

$K_0^*(800)$ or κ

$I(J^P) = \frac{1}{2}(0^+)$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$K_0^*(800)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
682 ±29 OUR AVERAGE		Error includes scale factor of 2.4. See the ideogram below.		

Simple Lagrangian for K0*(1430)

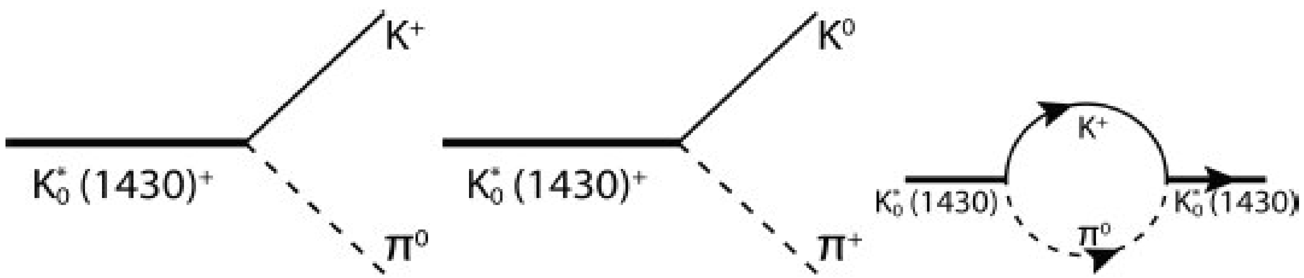
- Lagrangian:

$$\mathcal{L}_{int} = aK_0^{*+}K^-\pi^0 + bK_0^{*+}\partial_\mu K^-\partial^\mu\pi^0 + \ldots$$

- Decay width:

$$\Gamma_{K_0^*}(m) = 3\frac{|\vec{k}_1|}{8\pi m^2}\left[a - b\frac{m^2 - M_K^2 - M_\pi^2}{2}\right]^2 F_\Lambda(m)$$

where:



Propagator of K0*(1430)

- Propagator of the scalar kaonic field:

  $\Delta_{K_0^*}(p^2=m^2)=\frac{1}{m^2-M_0^2+\Pi(m^2)+i\varepsilon}$ where M_0 is the bare mass of the scalar

- Specral function:

$$d_{K_0^*}(m)=\frac{2m}{\pi}|\,\mathrm{Im}\,\Delta_{K_0^*}(p^2=m^2)|$$

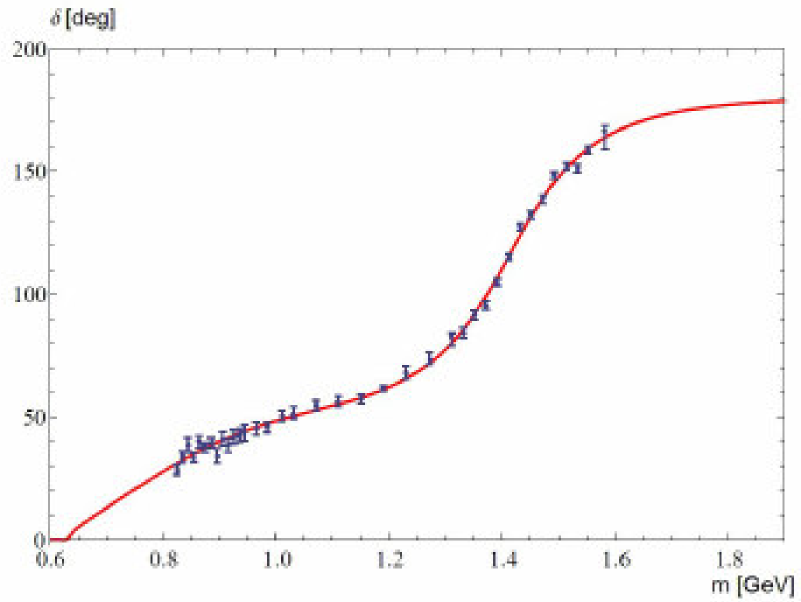
normalization condition:

$$\int_0^\infty d_{K_0^*}(m)\mathrm{dm}=1.$$

According to the optical theorem, $\mathrm{Im}\,\Pi(m)=m\Gamma_{K_0^*}(m)$.

Phase shift

$$\delta(m) = \frac{1}{2}\arccos\left[1 - \pi\Gamma_{K_0^*}(m) d_{K_0^*}(m)\right] \ .$$

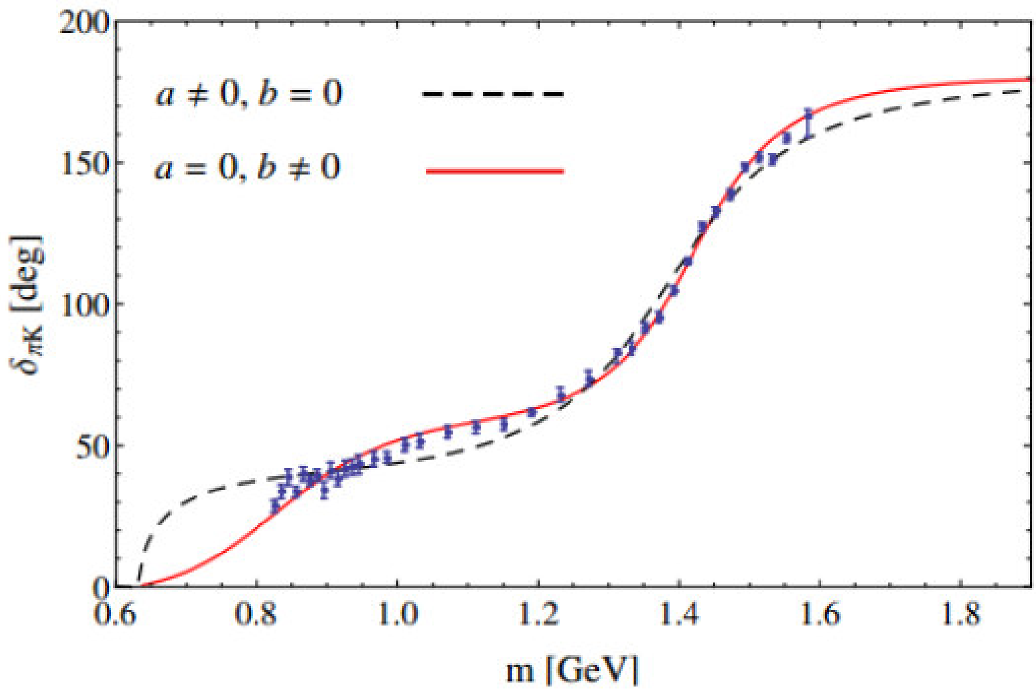


Fit

a	1.60± 0.22 GeV
b	-11.16±0.82 GeV^{-1}
Λ	0.496±0.008 GeV
M_0	1.204±0.008 GeV

$\chi^2/d.o.f. = 1.25$

Phase shifts/2

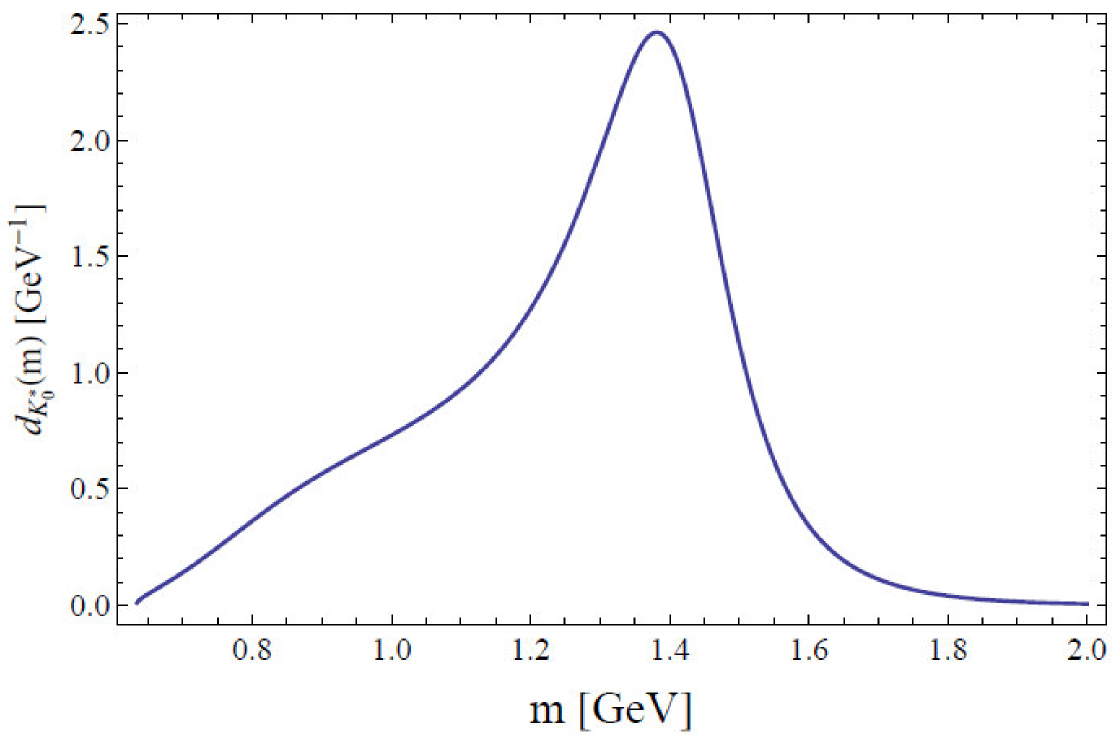


only nonderivative
$\mathcal{L}_{int} = aK_0^{*+}K^-\pi^0 + \ldots$

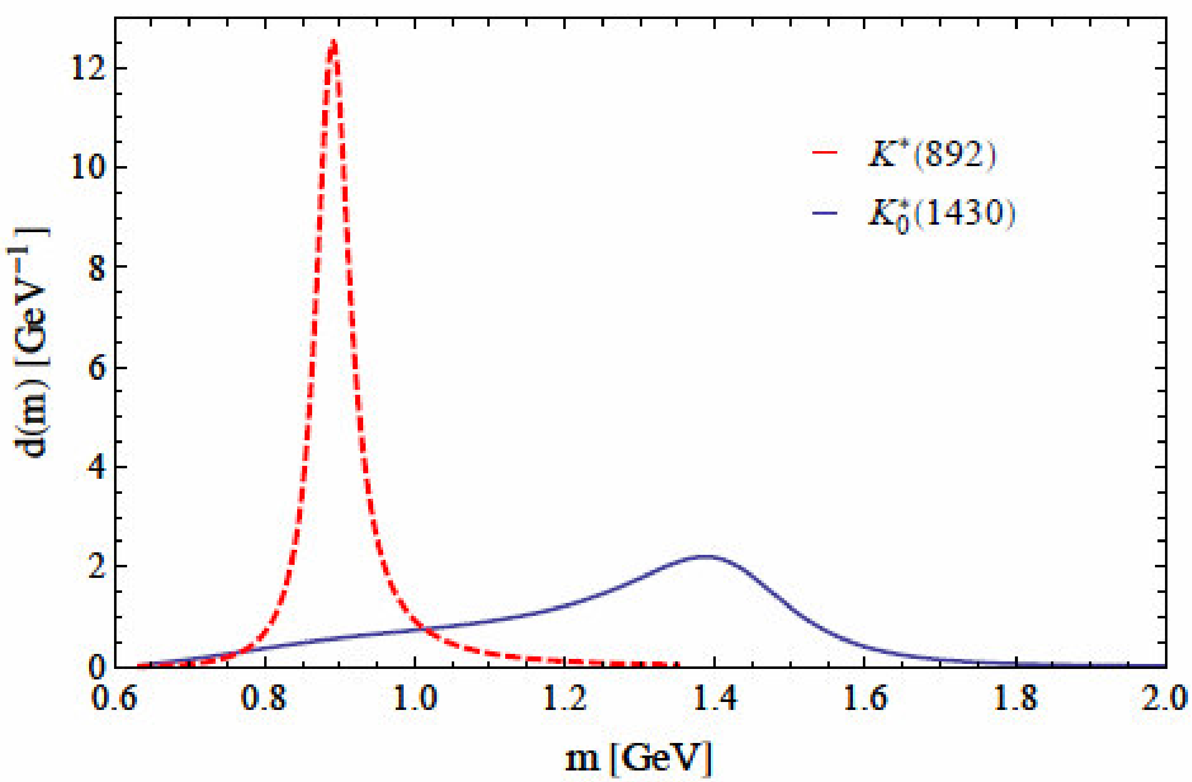
only derivative
$\mathcal{L}_{int} = bK_0^{*+}\partial_\mu K^-\partial^\mu\pi^0 + \ldots$

Spectral function of K0*(1430)

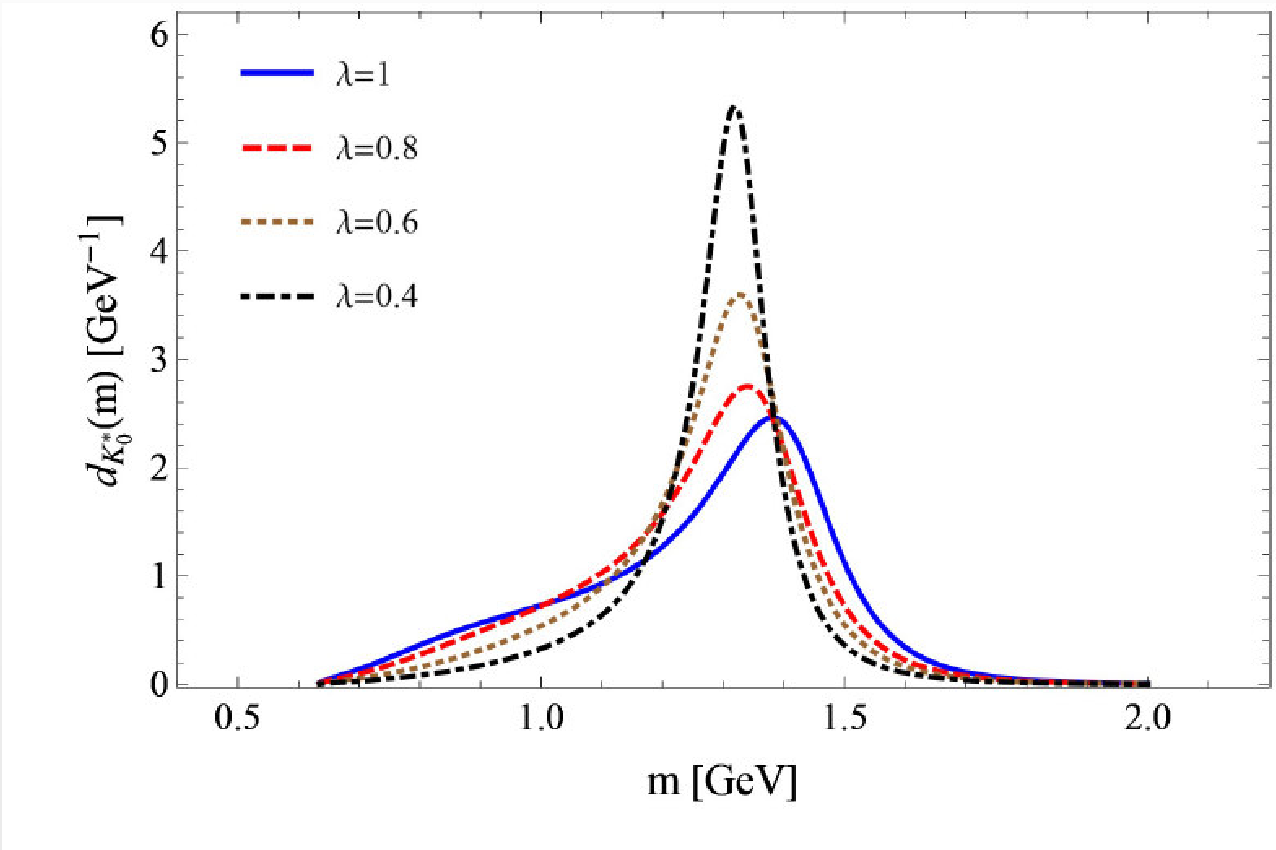
Is there a $K_0^*(800)$ or not?



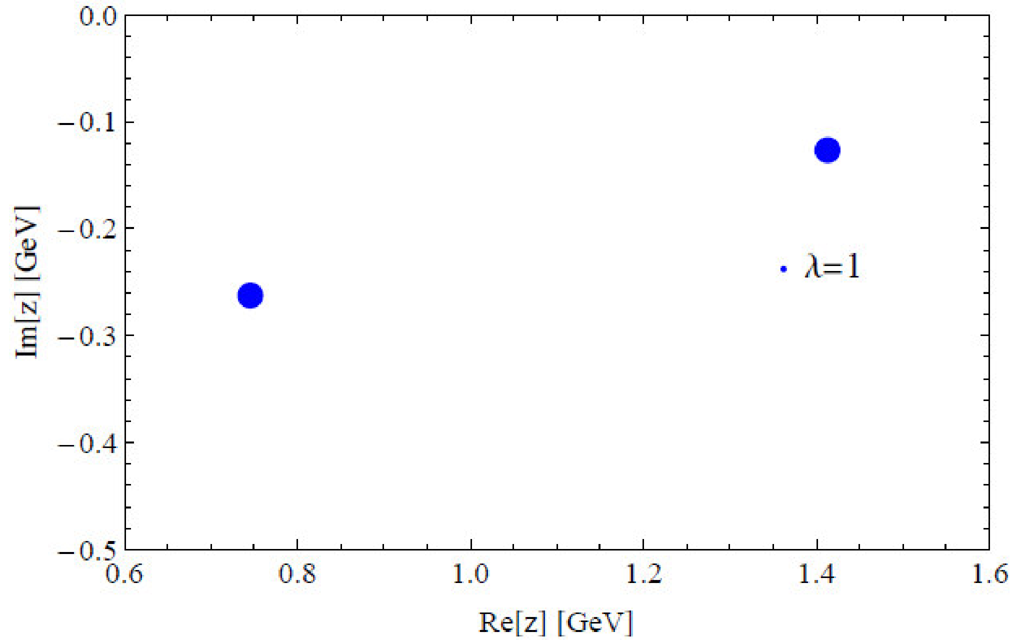
Comparison of the spectral functions of K*(892) and K0*(1430)



Large-Nc

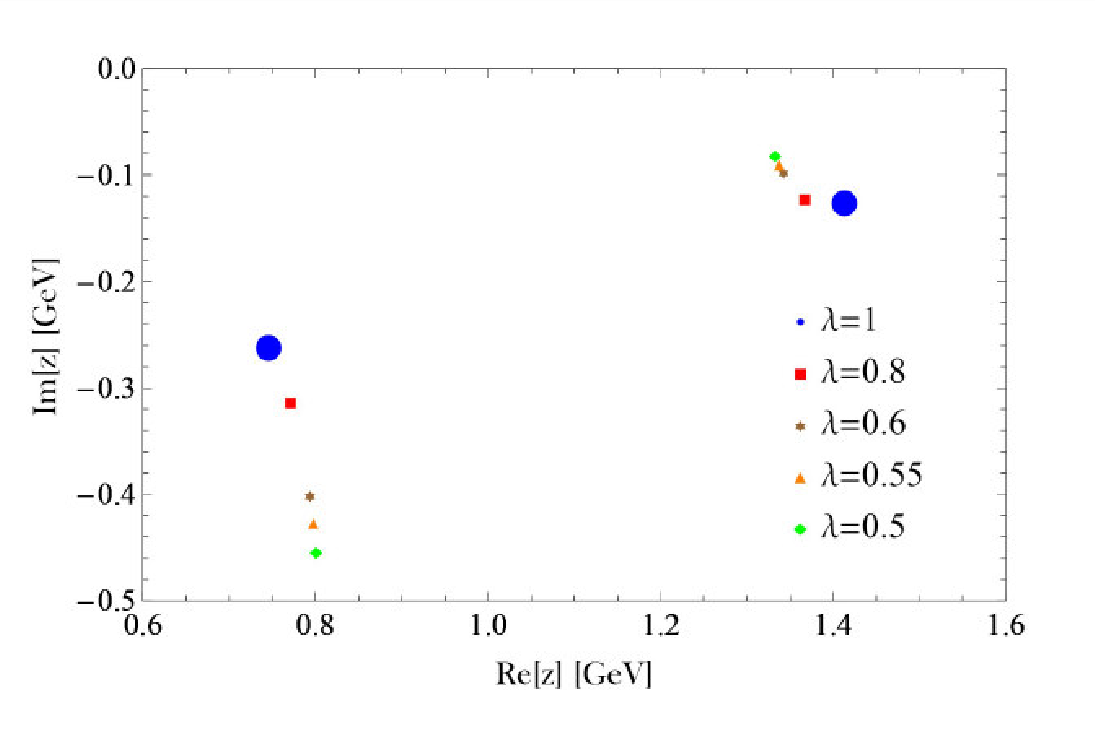


Poles



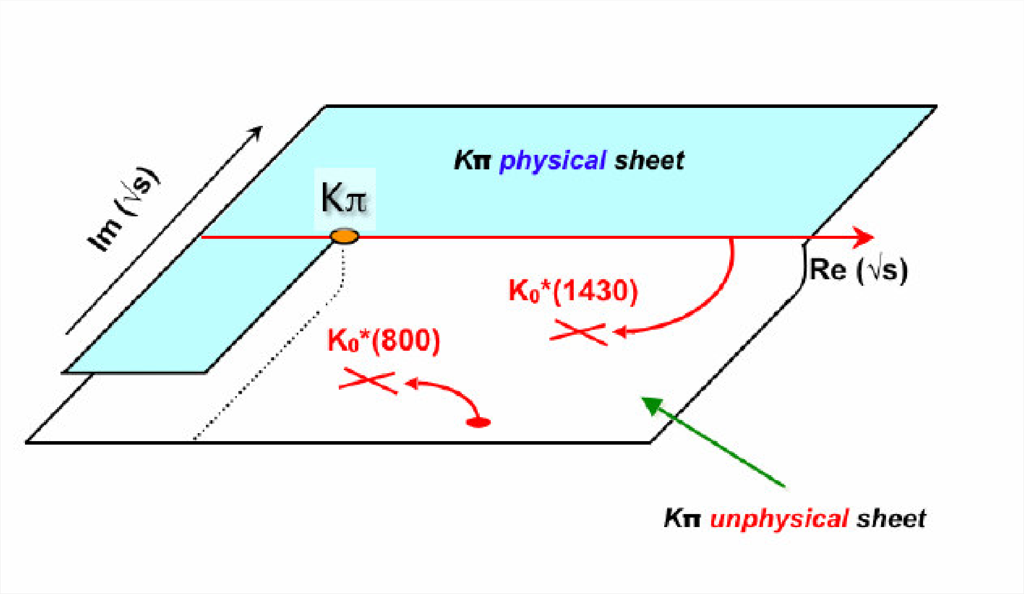
$$K_0^*(1430) : (1.413 \pm 0.057) - (0.127 \pm 0.011)\mathrm{i}\ (GeV)$$
$$K_0^*(800) : (0.745 \pm 0.029) - (0.263 \pm 0.027)\mathrm{i}\ (GeV)$$

Pole trajectories in the light scalar kaonic system



The additional companion pole on the left, corresponding to the light k, disappears for Nc larger then 12.4.

Pole trajectories/2



Considerations on the scalar kaonic system

- Scalar kaon: out of one "seed" state $\rightarrow$ 2 poles appear
  - $K_0^*(1430)$ corresponds to a peak
  - $K_0^*(800)$ "no peak" but there is a pole.
- We determined the position of the poles
  - for vector kaon ($0.89 - 0.028\text{i}$ (GeV))
  - for scalar kaons
    $K_0^*(1430) : (1.413 \pm 0.057) - (0.127 \pm 0.011)\text{i}$ (GeV)
    $K_0^*(800) : (0.745 \pm 0.029) - (0.263 \pm 0.027)\text{i}$ (GeV)
- $K^*(892)$ is a quark-antiquark state.
- $K_0^*(1430)$ is predominantly a quark-antiquark state.
- $K_0^*(800)$ is a dynamically generated state.

Same phenomenon for other states: past and ongoing works

- $a_0(980)$ as a companion pole of $a_0(1450)$
  T. Wolkanowski, F.G. and D. H. Rischke, $a_0(980)$ revisited,
  Phys. Rev. D 93 (2016) no.1, 014002 [arXiv:1508.00372 [hep-ph]].

- Non-Breit-Wigner lineshape of the resonance Ψ(3770). Interestingly, two poles are connected to it.

  S. Coito and F.G., Line-shape and poles of the Ψ(3770), arXiv:1712.00969 [hep-ph].

- Ongoing and future works:

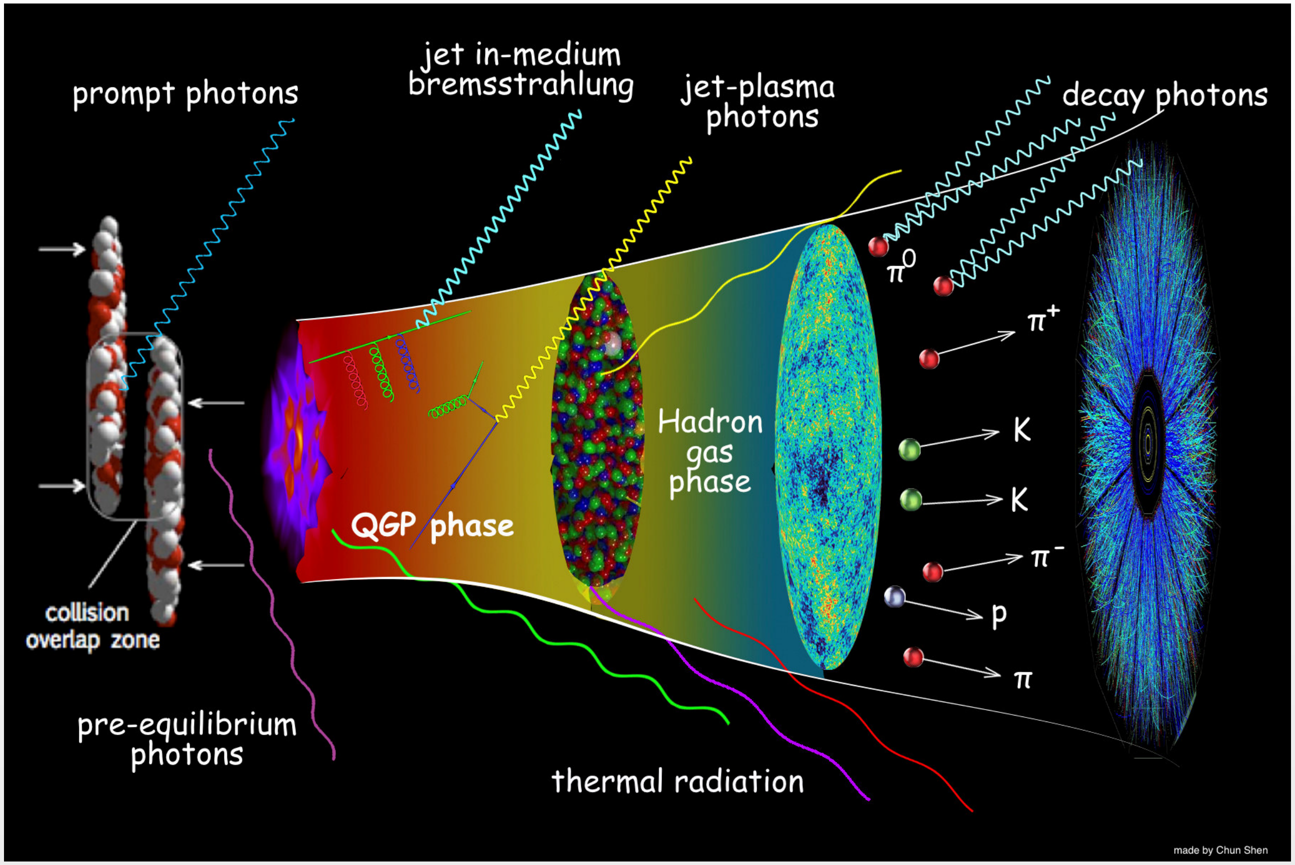
  Ψ(4040) as seed state, Y(4008) as its dynamically generated companion poles.
  Ψ(4160) line shape and its companion poles (eventually also above its mass)
  X(3872) as a companion pole of a seed charm-anticharm state
  Ds(2317) as a companion pole of a seed charm-starnge state.

The light k in heavy ion collisions

based on W. Broniowski, F.G., V. Begun,

Cancellation of the sigma meson in thermal models
Phys. Rev. C 92 (2015) no.3, 03490
[arXiv:1506.01260 [nucl-th]].

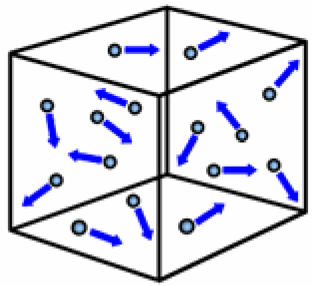
Heavy-ion collisions



At the freeze-out, the emission of hadrons is well described by thermal models.
Question: does the light k and its brother, $f_0(500)$, play a role?
Light states, at first yes,, but the answer is unexpected.

Theoretical description of a thermal gas

$$\ln Z = \sum_k \ln Z_k^{\text{stable}} + \sum_k \ln Z_k^{\text{res}}$$



$$\ln Z_k^{\text{stable,}} \quad = f_k V \int \frac{d^3p}{(2\pi)^3} \ln \left[1 \pm e^{-E_p/T}\right]^{\pm 1}$$

$$E_p = \sqrt{\vec{p}^2 + M_k^2}$$

Theoretical description of a thermal gas: unstable particles

$$\ln Z_k^{\mathrm{res}} = f_k V \int_0^\infty d_k(M)\, dM \int \frac{d^3p}{(2\pi)^3} \ln\left[1 - e^{-E_p/T}\right]^{-1}$$

At first, the function $d_k(m)$ can be interpreted as a mass probability density. Namely, a resonance does not have a definite mass but a mass distribution. If not too broad, $d_k(m)$ well described by a Breit-Wigner function. (This is not the case for the kaonic system and also not for $f_0(500)$.)

Thermal gas: connection to scattering data

R. Dashen, S.-K. Ma, and H. J. Bernstein, Phys.Rev. **187**, 345 (1969).
R. Dashen and R. Rajaraman, Phys.Rev. **D10**, 694 (1974).

W. Weinhold, B. Friman, and W. Noerenberg, Acta Phys.Polon. **B27**, 3249 (1996).
W. Weinhold, B. Friman, and W. Norenberg, Phys.Lett. **B433**, 236 (1998), arXiv:nucl-th/9710014 [nucl-th].

The density function can be directly extracted from two-body scattering data (phase shifts).

$$\boxed{d_k(M) = \frac{d\delta_k(M)}{\pi dM}}$$

Recall from scattering theory:

$$\frac{e^{2i\delta_k}-1}{2i} = a_k = \frac{-\sqrt{s}\Gamma(\sqrt{s})}{s-m^2+i\sqrt{s}\Gamma(\sqrt{s})}$$

This is a model-independent way of taking the resonances into account.

Indeed, it is a justification of the validity of thermal gas models.

But it is even more, since it allows also to include repulsions in some channels.

Theoretical description of a thermal gas: QCD

$$\ln Z = \ln Z_{\pi} + \ln Z_{K} + \ln Z_{(1/2,0++)} + \ln Z_{(3/2,0++)} + \ln Z_{(0,0++)} + \ln Z_{(2,0++)} + \ln Z_{(1,1--)} + \ldots$$

$$\ln Z_{(I,J)} = (2I+1)(2J+1)\int_0^{\Lambda_0} dm \frac{d\delta_{(I,J)}}{\pi dm}\int_p \ln\left[1 - e^{-\frac{\sqrt{p^2+m^2}}{T}}\right]^{-1}$$

For many resonances the Breit-Wigner approximation is valid

$$\frac{d\delta_{IJ}}{\pi dM} \simeq \sum_k \frac{\Gamma_{IJ,k}}{2\pi}\left[(M - M_{IJ,k})^2 + \frac{\Gamma_{IJ,k}^2}{4}\right]^{-1}$$

However, this approximation does not hold for the light k (and also not for $f_0(500)$).

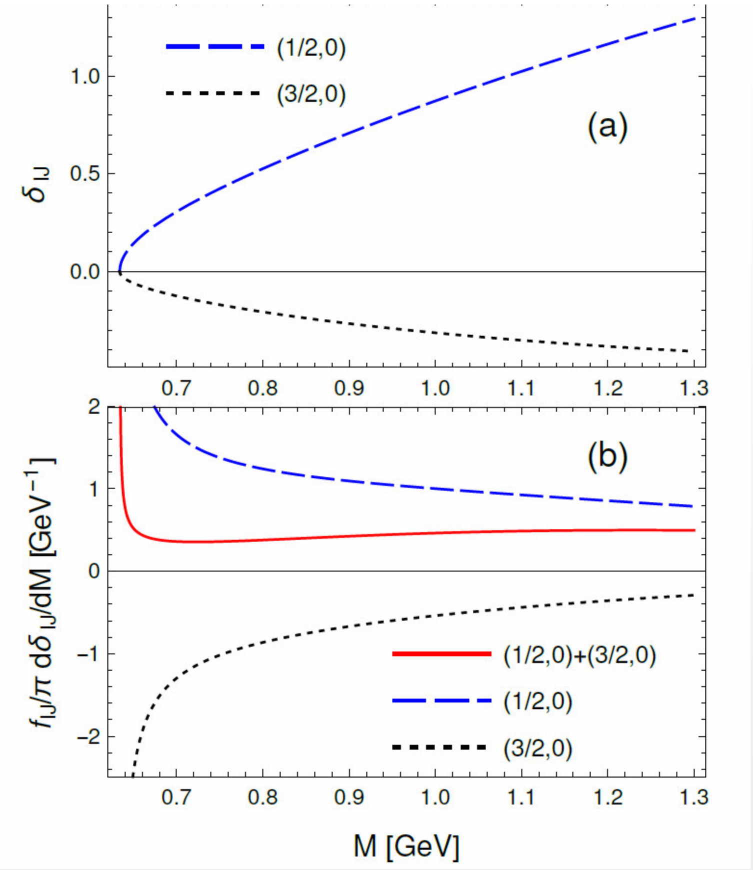
Theoretical description of a thermal gas: QCD

$$\ln Z = \ln Z_{\pi} + \ln Z_{K} + \boxed{\ln Z_{(1/2,0++)} + \ln Z_{(3/2,0++)}} + \ln Z_{(0,0++)} + \ln Z_{(2,0++)} + \ln Z_{(1,1--)} + \ldots$$

$$\ln Z_{(1/2,0++)} + \ln Z_{(3/2,0++)} = \int_0^{\Lambda_0} dm \left[2\frac{d\delta_{(1/2,0)}}{\pi dm} + 4\frac{d\delta_{(3/2,0)}}{\pi dm} \right] \int_p \ln \left[1 - e^{-\frac{\sqrt{p^2+m^2}}{T}} \right]^{-1}$$

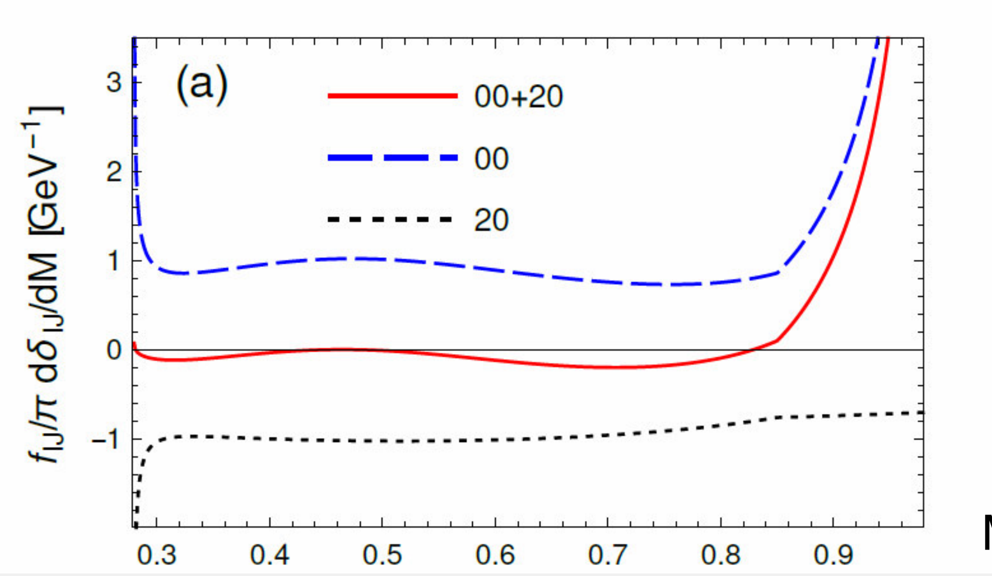
Simple and model-independent procedure: **just use scattering data!**

The scalar kaonic resonace $K_0^*(800)$: (partial) cancellation in thermal models



The total contribution from is the red curve: $\ln Z_{(1/2,0)} + \ln Z_{(3/2,0)}$
cancellation is evident: easiest thing to do is to forget about the k.
(Eventually, visible in correlations).

The f_0(500) spectral function **and** the isotensor repulsion/1



M [GeV]

arxiv: 1506.01260

The total contribution from J=0 is the red curve: $\ln Z_{(0,0)} + \ln Z_{(2,0)}$

$\ln Z_{(0,0)}$ is the contribution of f_0(500). It is indeed nonzero and even non-negligible, but it is almost exactly cancelled by the isotensor repulsion. Thermal models however usually neglect repulsions.

Either you take into account both I=0 and I =2, or –simply- you neglect both of them

Conclusions

The light k exists

(as a companion dynamically generated pole)

but

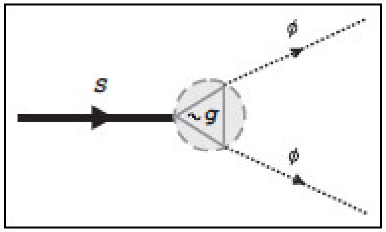
can be neglected in thermal models at nonzero T

Uniwersytet
Jana Kochanowskiego w Kielcach

Thank You

Cutoff function/1

- the cutoff parameter Λ does **not** exist at the Lagrangian level
- it can be implemented by using a non-local interaction term (if $f_\Lambda(q) = f_\Lambda(|\mathbf{q}|)$), e.g.



$$\mathcal{L}_{\text{int}} = gS(x)\phi^2(x) \;\rightarrow\; \mathcal{L}_{\text{int}} = gS(x)\int \mathrm{d}^4y\; \phi(x+y/2)\phi(x-y/2)\Phi(y)$$

- changes also the tree-level result for the decay width:

$$\Gamma^{\text{tree}}(s) \;\rightarrow\; \Gamma^{\text{tree}}(s)\cdot f_\Lambda^2(p_{S\phi\phi})$$

- our choice:

Regularization function in our case

$$f_\Lambda(q) = \exp\left(-|\mathbf{q}|^2/\Lambda^2\right)$$

Cutoff function/2

The contribution of the loop $\Pi(m^2)$ in which the particles φ_1 and φ_2 circulate as calculated from the original local Lagrangian (1) reads

$$\Pi(m^2) = \int \frac{d^4k_1}{(2\pi)^4} \frac{[a - b\,(k_1 \cdot k_2)]^2}{[k_1^2 - m_1^2 + i\varepsilon]\,[k_2^2 - m_2^2 + i\varepsilon]}\ , \tag{6}$$

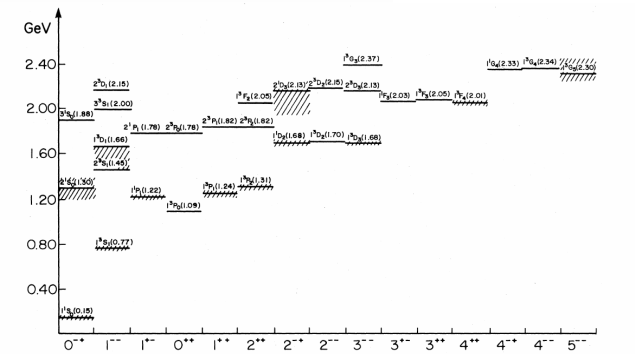
where the constraint $k_2 = p - k_1$ is understood and p is the momentum of the unstable particle S. In its reference frame $p = (m, \vec{0})$. As mentioned above, this loop contribution is divergent (with Λ^4). The substitution (4) makes it convergent thanks to the form-factor:

$$\Pi(m^2) \to \Pi_\Lambda(m^2) = \int \frac{d^4k_1}{(2\pi)^4} \frac{[a - b\,(k_1 \cdot k_2)]^2\, f_\Lambda^2(\vec{k}_1^2)}{[k_1^2 - m_1^2 + i\varepsilon]\,[k_2^2 - m_2^2 + i\varepsilon]}\ . \tag{7}$$

At this point, one may object that the form factor breaks covariance, since it depends on the three-momentum only. We will show in the next section that this is not necessarily the case. Once the form factor is in-

Quark model(s) and their QFT extensions

Mesons in a Relativized Quark Model with Chromodynamics
S. Godfrey, Nathan Isgur
Published in Phys.Rev. D32 (1985) **189-231**



Mesonic loops e.g. included into
A Low Lying Scalar Meson Nonet in a Unitarized Meson Model
E. van Beveren, T. A. Rijken, K. Metzger, C.~Dullemond, G.~Rupp and J. E.~Ribeiro,
Z. Phys. C **30** (1986) 615
Meson spectroscopy: too much excitement and too few excitations
G. Rupp, S. Coito and E. van Beveren,
Acta Phys. Polon. Supp. 5 (2012) 1007

QCD phenomenology based on a chiral effective Lagrangian
Tetsuo Hatsuda, Teiji Kunihiro
Phys.Rept. **247** (1994) 221-367

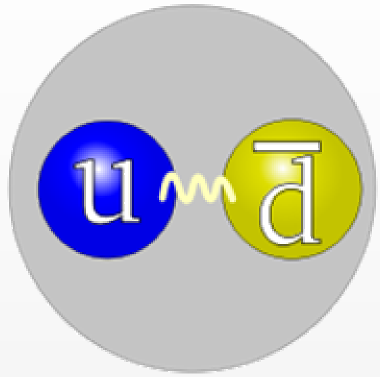
NJL: quark-based model with
chiral symmetry and SSB
chiral condensate
Effective quark mass
Mesons as quarkonia (pion: ok)

The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states
Reinhard Alkofer, Lorenz von Smekal
Phys.Rept. **353** (2001) 281

Baryons as relativistic three-quark bound states
G. Eichmann, H.~ Sanchis-Alepuz, R. Williams, R. Alkofer and C. S. Fischer,
Progr. Part. Nucl. Phys. **91** (2016) 1

DS:
quarks and gluons propagators from QCD
Condensates
Effective quark and gluon masses
Spectra of mesons as quarkonia (pion: ok) and baryons as qqq states

Example of conventional quark-antiquark states: the ρ and the π mesons



$$m_u + m_d \approx 7 \text{ MeV}$$

Vector channel: Rho-meson $m_{\rho^+} = 775 \text{ MeV}$

Mass generation in QCD is a nonpert. penomenon based on SSB

Pseudoscalar channel: Pion $m_{\pi^+} = 139 \text{ MeV}$

In the scalar channel, the situation is more complicated: a0(1450) and a0(980), see later on

a0(1450) and a0(1470) from PDG

$a_0(1450)$

$I^G(J^{PC}) = 1^-(0^{++})$

See minireview on scalar mesons under $f_0(500)$.

$a_0(1450)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1474 ±19 OUR AVERAGE				

Decays into ηπ, η'π, KK

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

$a_0(980)$

$I^G(J^{PC}) = 1^-(0^{++})$

See our minireview on scalar mesons under $f_0(500)$. (See the index for the page number.)

$a_0(980)$ MASS

VALUE (MeV)	*DOCUMENT ID*
980±20 OUR ESTIMATE	Mass determination very model dependent

Decays into ηπ, KK

Lagrangian for a0 system

$$\mathcal{L}_{a_0\eta\pi} = A_1 a_0^0 \eta \pi^0 + B_1 a_0^0 \partial_\mu \eta \partial^\mu \pi^0$$

$$\mathcal{L}_{a_0\eta'\pi} = A_2 a_0^0 \eta' \pi^0 + B_2 a_0^0 \partial_\mu \eta' \partial^\mu \pi^0$$

$$\mathcal{L}_{a_0 K\bar{K}} = A_3 a_0^0 (K^0 \bar{K}^0 - K^- K^+) + B_3 a_0^0 (\partial_\mu K^0 \partial^\mu \bar{K}^0 - \partial_\mu K^- \partial^\mu K^+)$$

The field a0 corresponds (roughly) to a0(1450)

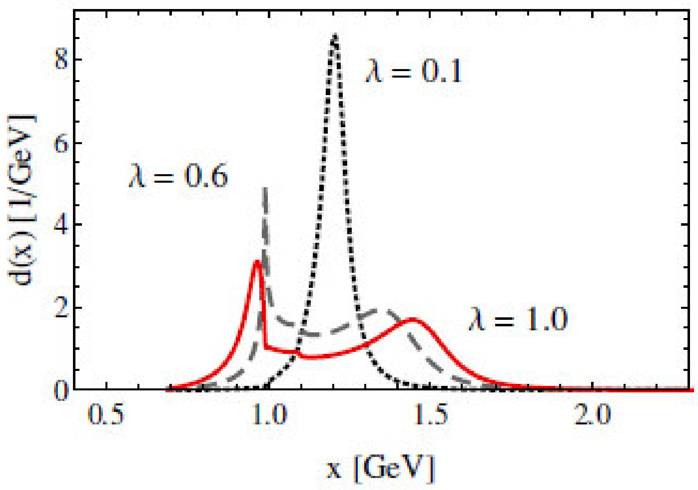
$a_0(1450)$ this leads to

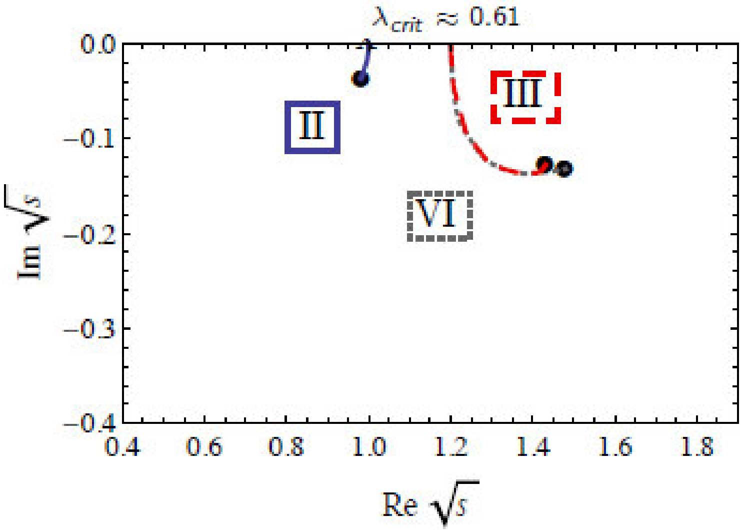
$$\frac{\Gamma^{\text{tree}}_{a_0\to\eta'\pi}}{\Gamma^{\text{tree}}_{a_0\to\eta\pi}} \simeq 0.44 \ , \quad \frac{\Gamma^{\text{tree}}_{a_0\to K\bar{K}}}{\Gamma^{\text{tree}}_{a_0\to\eta\pi}} \simeq 0.96 \ ,$$

which can be compared to the experimental values

$$\frac{\Gamma_{a_0\to\eta'\pi}}{\Gamma_{a_0\to\eta\pi}} = 0.35 \pm 0.16 \ , \quad \frac{\Gamma_{a_0\to K\bar{K}}}{\Gamma_{a_0\to\eta\pi}} = 0.88 \pm 0.23$$

Spectral function, poles, and large-Nc





Large-N_c behavior: $A_i^2 \to \lambda A_i^2$, $B_i^2 \to \lambda B_i^2$, $\lambda = \frac{3}{N_c}$

$$a_0(1450) : \; 1.480 - i0.132 \text{ GeV} ,$$
$$a_0(980) : \; 0.981 - i0.037 \text{ GeV} ,$$

1508.00372

The additional companion pole on the left, corresponding to the a0(980), disappears for Nc larger then 4.9.

Branching ratios for a0(1450) and a0(980) in agreement with PDG

General properties

- Ψ(3770)
- D-wave state, first charmonium above DD threshold.

$\psi(3770)$ $\qquad I^G(J^{PC}) = 0^-(1^{--})$

ψ(3770) MASS (MeV)

OUR FIT includes measurements of $m_{\psi(2S)}$, $m_{\psi(3770)}$, and $m_{\psi(3770)} - m_{\psi(2S)}$.

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
3773.13±0.35 OUR FIT Error includes scale factor of 1.1.				
3778.1 ±1.2 OUR AVERAGE				

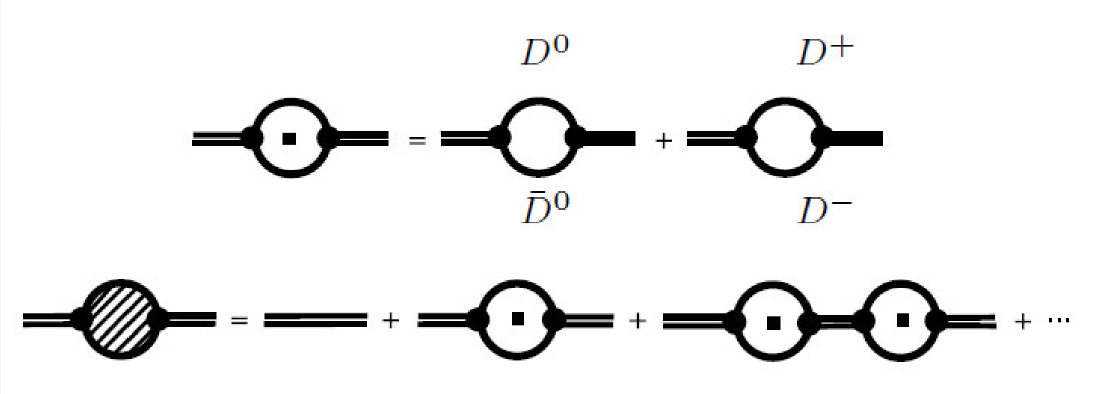
ψ(3770) WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*
27.2± 1.0 OUR FIT		
27.5± 0.9 OUR AVERAGE		

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D\overline{D}$	$(93 {}^{+8}_{-9})$ %
Γ_2	$D^0\overline{D}^0$	$(52 {}^{+4}_{-5})$ %
Γ_3	D^+D^-	(41 ± 4) %

Lagrangian and loops

$$\mathcal{L}_{\psi D\bar{D}} = ig_{\psi D^0 \bar{D}^0}\psi_\mu\left(\partial^\mu D^0 \bar{D}^0 - \partial^\mu \bar{D}^0 D^0\right) + ig_{\psi D^+ D^-}\psi_\mu\left(\partial^\mu D^+ D^- - \partial^\mu D^- D^+\right)$$



Fit to data

$$\sigma_{e^+e^-\to D\bar{D}} = \frac{\pi}{2E} g^2_{\psi e^+e^-} d_\psi(E)$$

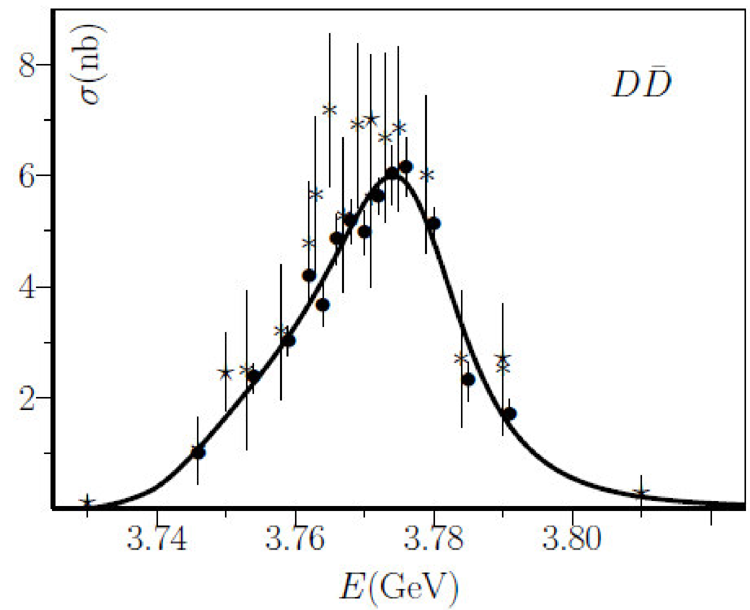


Fig. 5 Data: • BES [7], ∗ BES [5], ⋆ BaBar [6] (the latter not used in the fit). Solid line: our fit (cf. Table 1).

m_ψ (MeV)	3773.05 ± 0.95
Λ (MeV)	272.55 ± 1.17
$g_{\psi D\bar{D}}$	30.7 ± 4.8
$g_{\psi e^+e^-}$	$(1.062 \pm 0.032) \times 10^{-3}$
χ^2	20.52
$\chi^2/d.o.f$	0.86

Position(s) of the poles of Ψ(3770)

Two poles are present in the complex plane

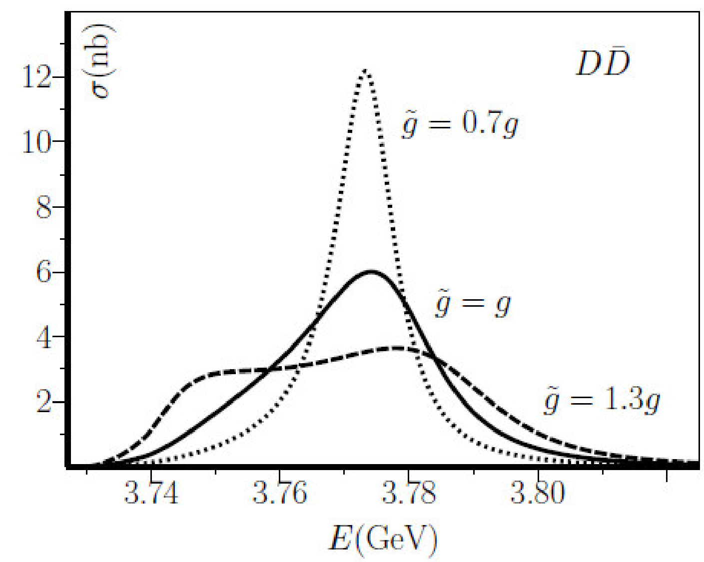
First pole: $E = 3776.8 - i12.3$ MeV,

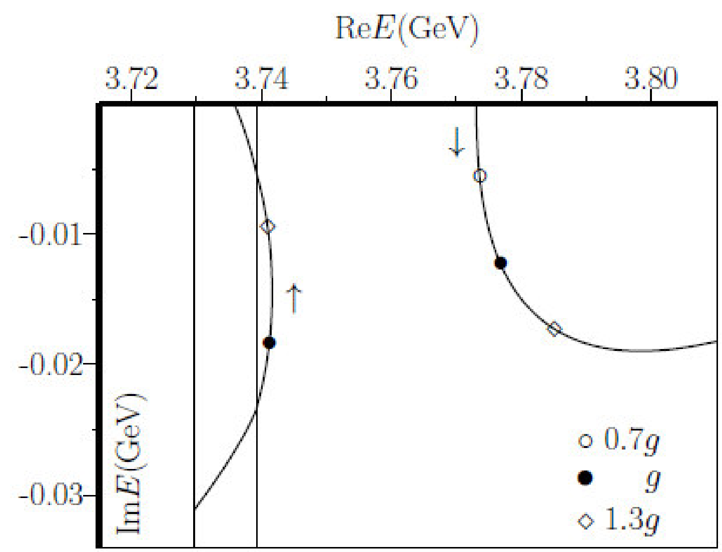
hence

$m_\psi^{\text{pole}} \simeq 3776.8 \pm 1.0$ MeV and
$\Gamma_\psi^{\text{pole}} \simeq 24.6 \pm 2.0$ MeV,

Second pole: $E = 3741.2 - i18.5$ MeV

Pole trajectories of Ψ(3770)





Second pole: $E = 3741.2 - i18.5$ MeV

The additional companion pole disappears for Nc larger then 3.9.

Phase-shift formula

The radial wave function with angular momentum l of a particle scattered by central potential $U(r)$ is

$$\psi_l(r) \propto \sin[kr - l\pi/2 + \delta_l] \ , \tag{6}$$

where $k = \left|\vec{k}\right|$ is the length of the three-momentum, and δ_l is the phase shift due the interaction with the potential. If we confine our system into a sphere of radius R, the condition $kR - l\pi/2 + \delta_l = n\pi$ with $n = 0, 1, 2, ...$must be met, since $\psi_l(r)$ has to vanish at the boundary. Conversely, the number of states n_0 that one can has by limiting k in the range $(0, k_0)$ is given by $n_0 = (k_0 R - l\pi/2 + \delta_l)/\pi$. Then, the density of states that one can place between k and $k + dk$ is given by

$$\frac{dn_l}{dk} = \frac{R}{\pi} + \frac{1}{\pi}\frac{d\delta_l}{dk} \ , \tag{7}$$

where the first term describes the density of states $\frac{dn_l^{free}}{dk}$ in absence of interactions, while the second term $\frac{1}{\pi}\frac{d\delta_l}{dk}$ describes the effect of the interacting potential. When translating the discussion fro Quantum Mechanics to Quantum Field Theory, we replace the momentum k with the invariant mass m, and the angular momentum l with the pair (I, J). Upon summing over the latter, one obtains the full density of states of an interacting pion gas as

$$\frac{dn}{dm} = \delta(m - m_\pi) + \sum_{I,J} \frac{1}{\pi} \frac{d\delta_{I,J}(m)}{dm} \ . \tag{8}$$

When plugging the previous equation into the general expression

$$\ln Z_{(I,J)} = f_{(I,J)}^{\deg} \int_0^\infty \frac{dn_{(I,J)}}{dm} dm \int \frac{d^3p}{(2\pi)^3} \ln\left[1 - e^{-\sqrt{\vec{p}^2 + m^2}/T}\right] \tag{9}$$