Standard Model

Lecture 5
Electroweak interactions

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Lecture 5

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Structure of Standard Model

Structure of Standard Model

Weak force - chiral gauge theory $\Rightarrow$ breaks parity

$\rightarrow$ gauge bosons interact differently with left- and right-handed fermions

- Standard Model built on gauge group:

$$G = U(1) \times SU(2) \times SU(3)$$

$\rightarrow$ *U(1) = $U(1)_Y$* - **hypercharge** *(not electromagnetism)*, charges denoted as $Y = 2(Q - I_3)$

(Gell-Mann-Nishijima)

$\rightarrow$ hypercharges are fractional

$\Rightarrow$ reproduce convention for electric charges

Fermions of single generation

	$U(1)$	$SU(2)$	$SU(3)$
Q_L	$\frac{1}{6}$	2	3
L_L	$-\frac{1}{2}$	2	1
u_R	$\frac{2}{3}$	1	3
d_R	$-\frac{1}{3}$	1	3
e_R	-1	1	1

Structure of Standard Model

- Each field transforms in fundamental representations of *SU(2)* or *SU(3)* *(denoted: **2** & **3**)* or in singlet representation *(denoted: **1**)*

  → **1** for non-Abelian group ⇒ fields neutral under the force

  → 1 for *U(1)* ⇒ field experiences the force *(0 - fields neutral under U(1))*

  → $(\mathbf{R_2}, \mathbf{R_3})_Y$ ⇒ $\mathbf{R_2}$, $\mathbf{R_3}$ - representations of *SU(2)* & *SU(3)*, *Y* - hypercharge

- Weyl fermions - left-handed *(L)* or right-handed *(R)* *(opposite for conjugate fermion)*

- Quarks - fermions transforming in **3** of *SU(3)*

  → true for right-handed quarks (u_R, d_R)

  → single left-handed quark Q_L transforms in **2** of *SU(2)*

  Only left-handed fermions transform in **2** of *SU(2)*

- *SU(2)* gauge structure of left-handed fermions:

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \text{and} \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

  → for Q_L: *SU(2)* doublet as left-handed *u*-quark & left-handed *d*-quark *(left-handed quark)*

  → For L_L: *SU(2)* doublet as left-handed neutrino & left-handed electron *(left-handed lepton)*

  Higgs field spontaneously breaks *SU(2)* gauge symmetry

Structure of Standard Model

- Include gauge degrees of freedom

→ 15 fermions *(Q_L: 2 × 3 = 6 ⇒ total: 6+2+3+3+1=15)*

Right-handed neutrino:

	$U(1)$	$SU(2)$	$SU(3)$
ν_R	0	1	1

→ if right-handed neutrino exists (?)

⇒ not interact with any force → hard to detect (**sterile neutrino**)

→ interaction with particles only through Higgs field

⇒ neutrinos get masses

Higgs boson *H*:

	$U(1)$	$SU(2)$	$SU(3)$
H	$\frac{1}{2}$	**2**	**1**

→ spin 0 particle

→ discovered in 2012 by CMS & ATLAS

Anomaly cancellation

Standard Model - chiral gauge theory

→ check consistency

→ only charged fermions contribute to anomalies

- *SU(3)*^{3} anomaly ⇒ cancellation condition: $\sum_{\text{left-handed}} A(R) = \sum_{\text{right-handed}} A(R)$

→ fermions: singlets with $A(\mathbf{1}) = 0$ or fundamental representation with $A(\mathbf{3}) = 1$

→ 2 right-handed u_R, d_R + 1 left-handed quark Q_L

⇒ computing anomaly: sum over *SU(2)* gauge index

$$\sum A(R) = 2 \qquad \text{for left-handed and right-handed quarks}$$

- No perturbative *SU(2)*^{3} anomaly
- Witten anomaly ⇒ cancellation condition: even number of *SU(2)* doublets

→ 3 in Q_L *(sum over SU(3) indices)* & 1 doublet in L_L

- *U(1)*^{3} anomaly ⇒ cancellation condition: $\sum_{\text{left-handed}} Y^3 = \sum_{\text{right-handed}} Y^3$

$$\sum_{\text{left-handed}} Y^3 = 6 \times \left(\frac{1}{6}\right)^3 + 2 \times \left(-\frac{1}{2}\right)^3 = -\frac{2}{9} \qquad \sum_{\text{right-handed}} Y^3 = 3 \times \left(\frac{2}{3}\right)^3 + 3 \times \left(-\frac{1}{3}\right)^3 + (-1)^3 = -\frac{2}{9}$$

multiply by dimension of representation of non-Abelian factors

Anomaly cancellation

- Mixed anomaly $SU(2)^2 \times U(1)$

  → cancellation condition: $\sum_{\text{left-handed}} Y = \sum_{\text{right-handed}} Y$ sum only over fermions in **2** of *SU(2)*

  → satisfied by: $SU(2)^2 \times U(1): \quad 3 \times \left(\frac{1}{6}\right) + \left(-\frac{1}{2}\right) = 0$

- Mixed anomaly $SU(3)^2 \times U(1)$

  → cancellation condition: $\sum_{\text{left-handed}} Y = \sum_{\text{right-handed}} Y$ sum over quarks in **3** of *SU(3)*

  → satisfied by: $SU(3)^2 \times U(1): \quad 2 \times \left(\frac{1}{6}\right) = \frac{2}{3} - \frac{1}{3}$

- Couple to gravity

  → cancellation condition: $\sum_{\text{left-handed}} Y = \sum_{\text{right-handed}} Y$ sum over all fermions

$$\sum_{\text{left-handed}} Y = 6 \times \frac{1}{6} + 2 \times \left(-\frac{1}{2}\right) = 0 \qquad \sum_{\text{right-handed}} Y = 3 \times \frac{2}{3} + 3 \times \left(-\frac{1}{3}\right) - 1 = 0$$

Unification in Standard Model

→ quarks, electron & neutrino unified ⇒ need for mathematical consistency

Yukawa interactions

- Standard Model - chiral gauge theory $\Rightarrow$ no gauge invariant mass terms for fermions

  $\rightarrow$ left- and right-handed fermions not transform same way under gauge symmetry

  $\rightarrow$ all SM fermions naturally massless **(?)**

  All fermions interact with Higgs boson

  $\rightarrow$ quantum numbers $(\mathbf{2}, \mathbf{1})_{1/2}$ under gauge group

  $\Rightarrow$ couplings restricted to fermions *(couple to all fermions through Yukawa couplings)*

- Quarks

  $\rightarrow$ fermion bilinears *(Lorentz scalars & singlets)* under *SU(3)*: $\bar{Q}_L + u_R \,/\, d_R$

  $\rightarrow$ $\bar{Q}_L u_R$: $(\bar{\mathbf{2}}, \mathbf{1})_{+1/2}$ $\quad$ $\bar{Q}_L d_R$: $(\bar{\mathbf{2}}, \mathbf{1})_{-1/2}$

  $\Rightarrow$ Yukawa term: contract with H or $H^\dagger$

SU(2) representations

$\rightarrow$ *SU(2)* vectors x^a and z^a, a = 1, 2, transforming in **2** of *SU(2)* $\Rightarrow$ 2 ways to form singlets

mesons: $x^\dagger z = \bar{x}_a z^a$ $\qquad$ baryons: $xz = \epsilon_{ab} x^a z^b$

$\rightarrow$ representation **2** pseudoreal: x^a in **2** $\rightarrow$ can form $\epsilon_{ab} x^b$ in $\bar{\mathbf{2}}$

Yukawa interactions

- $\bar{Q}_L$ naturally in $\bar{\mathbf{2}}$ $\Rightarrow$ contract with H sitting in **2**

  $\rightarrow$ need ϵ if contract with $H^\dagger$

$$\tilde{H}^a = \epsilon^{ab} H_b^\dagger$$

- Gauge invariant Yukawa couplings with quarks:

$$\mathcal{L}_{\text{Yuk}} = -y^d\, \bar{Q}_L H d_R - y^u\, \bar{Q}_L \tilde{H} u_R + \text{h.c.}$$

y^d and y^u are Yukawa coupling constants

  $\rightarrow$ both terms neutral under $U(1)_Y$

- Yukawa couplings with leptons

  $\rightarrow$ $\bar{L}_L e_R$: $(\bar{\mathbf{2}}, \mathbf{1})_{-1/2}$ *(if right-handed neutrino exists)* $\bar{L}_L \nu_R$: $(\bar{\mathbf{2}}, \mathbf{1})_{+1/2}$

  Gauge invariant Yukawa interactions with Higgs:

$$\mathcal{L}_{\text{Yuk}} = -y^e\, \bar{L}_L H e_R - y^\nu\, \bar{L}_L \tilde{H} \nu_R + \text{h.c.}$$

y^e and y^ν are Yukawa coupling constants

  $\rightarrow$ neutrino Yukawa term ($H^\dagger$) $\Rightarrow$ *SU(2)* gauge indices contracted with ϵ_{ab}

- Majorana mass term *(for right-handed neutrino)*: $\mathcal{L}_{\text{Maj}} = M\, \nu_R \nu_R + \text{h.c.}$

Three generations

3 generations of fermions

Add flavour index *i* = 1, 2, 3 to each fermion

- Quarks *(up & down, strange & charm, bottom & top):*

$$d_R^i = \{ d_R \, , \; s_R \, , \; b_R \} \quad : \quad (1,3)_{-1/3}$$
$$u_R^i = \{ u_R \, , \; c_R \, , \; t_R \} \quad : \quad (1,3)_{2/3}$$

→ *SU(2)* doublets: $Q_L^i = \left\{ \begin{pmatrix} u_L \\ d_L \end{pmatrix} , \begin{pmatrix} c_L \\ s_L \end{pmatrix} , \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right\} \quad : \quad (2,3)_{1/6}$

- Leptons *(electron, muon, tau):* $e_R^i = \{ e_R \, , \; \mu_R \, , \; \tau_R \} \quad : \quad (1,1)_{-1}$

→ *SU(2)* doublets: $L_L^i = \left\{ \begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix} , \begin{pmatrix} \nu_L^\mu \\ \mu_L \end{pmatrix} , \begin{pmatrix} \nu_L^\tau \\ \tau_L \end{pmatrix} \right\} \quad : \quad (2,1)_{-1/2}$

→ right-handed neutrino **(?)***:* $\nu_R^i = \{ \nu_R^e \, , \; \nu_R^\mu \, , \; \nu_R^\tau \} \quad : \quad (1,1)_0$

Gauge interactions ⇒ all generations experience same forces

- Yukawa couplings

→ inter-generation mixing

$$\mathcal{L}_{\text{Yuk}} = -y_{ij}^d \, \bar{Q}_L^i H d_R^j - y_{ij}^u \, \bar{Q}_L^i \tilde{H} u_R^j - y_{ij}^e \, \bar{L}_L^i H e_R^j - y_{ij}^\nu \, \bar{L}_L^i \tilde{H} \nu_R^j + \text{h.c.}$$

SM Lagrangian

Standard Model Lagrangian

→ terms must be Lorentz invariant & gauge invariant

full Lagrangian: $\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm fermi} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yuk}$

- Gauge field kinetic term:

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \mathrm{Tr}\, W_{\mu\nu} W^{\mu\nu} - \frac{1}{2} \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu}$$

$SU(3)$ gluon field strength $G_{\mu\nu}$

$SU(2)$ gauge field strength $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu]$

$U(1)$ hypercharge field strength $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

- Kinetic terms for fermions:

$$\mathcal{L}_{\rm fermi} = -i \sum_{i=1}^{3} \left(\bar{Q}_L^i \bar{\sigma}^\mu \mathcal{D}_\mu Q_L^i + \bar{L}_L^i \bar{\sigma}^\mu \mathcal{D}_\mu L_L^i + \bar{u}_R^i \sigma^\mu \mathcal{D}_\mu u_R^i + \bar{d}_R^i \sigma^\mu \mathcal{D}_\mu d_R^i + \bar{e}_R^i \sigma^\mu \mathcal{D}_\mu e_R^i + \bar{\nu}_R^i \sigma^\mu \partial_\mu \nu_R^i \right)$$

→ kinetic term for Q_L charged under each of 3 gauge fields:

$$\mathcal{D}_\mu Q_L = \partial_\mu Q_L - i g_s G_\mu Q_L - i g W_\mu Q_L - \frac{i}{6} g' B_\mu Q_L$$

→ coupling constants: g_s - *SU(3)* strong force

g - *SU(2)* weak force

g' - *U(1)* hypercharge

SM Lagrangian

- Higgs term *(kinetic term & potential)*

$$\mathcal{L}_{\text{Higgs}} = \mathcal{D}_\mu H^\dagger \mathcal{D}^\mu H - \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2$$

→ Higgs kinetic term from gauge quantum numbers

$$\mathcal{D}_\mu H = \partial_\mu H - igW_\mu H - \frac{i}{2} g' B_\mu H$$

- Yukawa terms

$$\mathcal{L}_{\text{Yuk}} = -y^d_{ij}\, \bar{Q}^i_L H d^j_R - y^u_{ij}\, \bar{Q}^i_L \tilde{H} u^j_R - y^e_{ij}\, \bar{L}^i_L H e^j_R - y^\nu_{ij}\, \bar{L}^i_L \tilde{H} \nu^j_R + \text{h.c.}$$

Parameters of SM

→ 3 gauge couplings: g_s, g, g'

→ λ *(dimensionless Higgs coupling)* and v^2 *(vacuum expectation value)* in Higgs potential

→ Yukawa couplings *(see following lectures)*

Global symmetries

SM gauge group: $G = U(1) \times SU(2) \times SU(3)$

What are global symmetries of SM?

- **Assume:** Classical theory *(absence of Yukawa terms)*

  → $U(3)^5$ global symmetry *(3 - 3 generations, 5 - Q_L, L_L, u_R, d_R, e_R)*

  → $U(3)^6$ global symmetry *(if right-handed neutrinos)*

  Yukawa terms break symmetry

  → remaining approximate symmetries

  ⇒ isospin or eightfold way

- $U(1)_B \times U(1)_L$ - **baryon number & lepton number**

	Q_L	L_L	u_R	d_R	e_R	ν_R
$U(1)_B$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	0	0
$U(1)_L$	0	1	0	0	1	1

→ $U(1)_B$ only on quarks & $U(1)_L$ only on leptons

→ $U(1)_B$ & $U(1)_L$ act in same way on each generation

Chiral (ABJ) anomalies

U(1) - good symmetry of classical theory

- Quantum theory

  → chiral anomalies with $U(1)_B$ & $U(1)_L$

  → chiral anomaly with *SU(2)* gauge group *(only left-handed fermions carry SU(2) charge)*

  → chiral anomaly with $U(1)_Y$:

$$\sum_{\text{left}} BY^2 - \sum_{\text{right}} BY^2 = \frac{1}{3}\left(6 \times \left(\frac{1}{6}\right)^2 - 3 \times \left(\frac{2}{3}\right)^2 - 3 \times \left(-\frac{1}{3}\right)^2\right) = -\frac{1}{2} \qquad \sum_{\text{left}} LY^2 - \sum_{\text{right}} LY^2 = \left(2 \times \left(-\frac{1}{2}\right)^2 - (-1)^2\right) = -\frac{1}{2}$$

  $U(1)_B$, $U(1)_L$ - not good symmetries of quantum theory

  → anomalies associated to gauge fields of weak force

  ⇒ good approximate symmetries *(not observed to be violated)*

- Baryon number violation

  → e.g. electroweak instanton - cannot allow proton to decay into positron

  → e.g. ^{3}He nucleus decay - 10^{173} years

  In practice - baryon number & lepton number are good symmetries

  ***B − L* is non-anomalous** *(same anomalies for $U(1)_B$ & $U(1)_L$)* ⇒ **global symmetry of SM**

Weak theta term

$\theta \approx 0$ for strong force (*SU(3)*)

- θ term for *U(1)* gauge group

  → Abelian gauge theories - θ term not affect local dynamics *(masses, x-sections etc.)*

  ⇒ no *U(1)* instantons

  → endow magnetic monopoles with electric charge through Witten effect **(?)**

  → no experimental insight

- θ term for *SU(2)* gauge group

$$S_\theta = \frac{g^2\theta_W}{16\pi^2}\int d^4x \ \mathrm{Tr}\, W_{\mu\nu}{}^\star W^{\mu\nu}$$

  → global $U(1)_B$ *(or* $U(1)_L$*)* ABJ anomaly

  ⇒ anomaly re-interpreted as shifting θ term

$$U(1)_B : \theta_W \to \theta_W + 3\alpha$$

  → θ_W unphysical *(not affect physics)*

  ⇒ use anomalous $U(1)_L$ symmetry to set $\theta_W = 0$ ⇒ **no weak CP problem**

Electroweak symmetry breaking

Electroweak symmetry breaking

- Higgs boson dynamics:

$$\mathcal{L}_{\text{Higgs}} = \mathcal{D}_\mu H^\dagger \mathcal{D}^\mu H - \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2$$

→ Higgs potential v^2 - breaks $U(1) \times SU(2)$ gauge symmetry

$\Rightarrow$ give masses to gauge bosons

→ Yukawa interactions with Higgs

$\Rightarrow$ give masses to fermions

- Maxwell & Yang-Mills terms for $U(1) \times SU(2)$ gauge fields:

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr}\, W_{\mu\nu} W^{\mu\nu} + \mathcal{L}_{\text{Higgs}}$$

→ Higgs covariant derivative:

$$\mathcal{D}_\mu H = \partial_\mu H - i g W_\mu H - \frac{i}{2} g' B_\mu H$$

→ in ground state of potential $\Rightarrow \quad H^\dagger H = v^2/2$

→ Higgs vacuum expectation value v (***vev***): $\quad \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

Electroweak symmetry breaking

- Parametrise fluctuations of Higgs

$$H = e^{i\xi^A(x)T^A} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

h(x) - real scalar
$T^A = ½\ \sigma^A$ - generators of *SU(2)*
$\xi^A(x)$ - would-be Goldstone bosons

- $\Omega(x) = e^{i\xi^A(x)T^A}$

→ covariant derivative:

$$\mathcal{D}_\mu H = \frac{1}{\sqrt{2}} \Omega \left(\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - i \left[g \left(\Omega^{-1} W_\mu \Omega + \frac{i}{g} \Omega^{-1} \partial_\mu \Omega \right) + \frac{g'}{2} B_\mu \right] \begin{pmatrix} 0 \\ v + h \end{pmatrix} \right)$$

⇒ field Ω can be eliminated by gauge transformation

- Unitary gauge *(SU(2) rotation):* $\xi^A(x) = 0$ or $\Omega = \mathbb{1}$

→ Lagrangian:

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \mathrm{Tr}\, W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} \partial_\mu h \partial^\mu h - \lambda h^2 \left(v + \frac{h}{2} \right)^2 + \boxed{\frac{1}{8} (v + h)^2 \left(g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (g W_\mu^3 - g' B_\mu)^2 \right)}$$

expand *SU(2)* gauge boson fields W_μ in terms of generators T^A + contract with Higgs field

→ masses from quadratic term

→ $\lambda\, v^2\, h^2$ - mass term for ***h*** ⇒ **Higgs boson**

$$M_h = \sqrt{2\lambda} v \approx 125\ \mathrm{GeV}$$

combination of vev *v* and Higgs coupling λ

Electroweak symmetry breaking

- Masses of gauge bosons

  → same mass for W^1_μ & W^2_μ: $M_W = vg/2$

  → complex combination:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$$

  ⇒ ensures $W^\pm$ electric charge ±1

  → experimentally measured mass: $M_W = \frac{gv}{2} \approx 80\ \mathrm{GeV}$

  combination of vev and *SU(2)* gauge coupling *g*

Weinberg angle *(weak mixing angle)*

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \quad \Longleftrightarrow \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

→ 2 linear combinations of gauge fields:

$$Z_\mu = \cos\theta_W\, W^3_\mu - \sin\theta_W\, B_\mu$$
$$A_\mu = \sin\theta_W\, W^3_\mu + \cos\theta_W\, B_\mu$$

→ experimentally measured *Z* boson mass:

$$M_Z = \frac{v}{2}\sqrt{g^2 + g'^2} \approx 91\ \mathrm{GeV}$$

Electroweak symmetry breaking

Masses of gauge bosons

→ combinations of Higgs vev v, Higgs coupling λ and gauge couplings g and g'

→ experimentally measured

$$M_W = M_Z \cos\theta_W < M_Z$$

Higgs boson *H* → doublet of *SU(2)*

- Higgs *H*: 2 complex *dof* or 4 real *dof*

  → scalar *h (Higgs boson)* + 3 gauge bosons W^1_μ , W^2_μ , Z_μ

  → Higgs discovered in 2012 *(LHC experiments)*

  → massive *W* and *Z* bosons discovered in 1983 *(LEP experiments)*

- Scales of masses

  → set by Higgs expectation value + dimensionless coupling constants

$$v \approx 250\ \text{GeV} \quad \text{and} \quad \lambda \approx 0.35 \quad \text{and} \quad g \approx 0.64 \quad \text{and} \quad g' \approx 0.34$$

→ Weinberg angle: $\cos\theta_W \approx 0.88 \implies \theta_W \approx 29^\circ \qquad \sin^2\theta_W \approx 0.223$

Electroweak theory

Massless gauge boson of *U(1)* × *SU(2)*

field A_μ → **photon**

- Photon massless ⇒ Higgs induces symmetry breaking

$$U(1)_Y \times SU(2) \to U(1)_{\mathrm{EM}}$$

Electroweak theory (EW) - *U(1)* × *SU(2)* sector of Standard Model

GSW - Glashow-Salam-Weinberg, 1960s

- Higgs *vev* transformation under general *U(1)* × *SU(2):*

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \longrightarrow e^{ig\alpha^A T^A} e^{ig'\beta Y} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

→ Higgs hypercharge *Y* = ½: $g\alpha^A T^A + g'\beta Y = \frac{g}{2} \begin{pmatrix} \alpha^3 + g'\beta/g & \alpha^1 - i\alpha^2 \\ \alpha^1 + i\alpha^2 & -\alpha^3 + g'\beta/g \end{pmatrix}$ $\qquad T^A = \frac{1}{2}\sigma^A$

- ⟨*H*⟩ invariant ⇒ choice of parameters: $\alpha^1 = \alpha^2 = 0$ and $g\alpha^3 = g'\beta$

→ unbroken generator $U(1)_{EM}$: $Q = T^3 + Y$ *(weak isospin + weak hypercharge)*

⇒ *Q* determines electric charge of the fields

Electroweak theory

- Electromagnetic coupling constant *e*

  → **look at** general covariant derivative for field transforming in fundamental of *SU(2)* with *Y*

$$\mathcal{D}_\mu = \partial_\mu - igW_\mu^A T^A - ig'YB_\mu \qquad T^\pm = (T^1 \pm iT^2)/\sqrt{2}$$

  → for fields Z_μ and A_μ : $\mathcal{D}_\mu = \partial_\mu - ig(W_\mu^+ T^+ + W_\mu^- T^-) - i(g\cos\theta_W T^3 - g'\sin\theta_W Y)Z_\mu - \boxed{ieQA_\mu}$

  EM coupling: $e = g\sin\theta_W = g'\cos\theta_W \qquad e \approx 0.30$

- **Fine structure constant**: $\alpha = e^2/4\pi \qquad \alpha \approx 1/137$

Electroweak sector bosons: Higgs, *W*, *Z*

- Higgs *h*: electrically neutral ⇒ *SU(2)* generator $T^3 = -1/2$ & $Y = +1/2$ ⇒ $Q = T^3 + Y = 0$
- *Z* boson: electrically neutral ⇒ no hypercharge & commutes with T^3
- W bosons → under *SU(2)* & $\alpha^1 = \alpha^2 = 0$ & $\alpha^3 = const$

$$\delta W_\mu = -ig[W_\mu, \alpha^3 T^3] = g\alpha^3(-W_\mu^1 T^2 + W_\mu^2 T^1) \qquad \delta W_\mu^1 = g\alpha^3 W_\mu^2 \text{ and } \delta W_\mu^2 = -g\alpha^3 W_\mu^1$$

  → *SU(2)*: part of $U(1)_{EM}$ transformation with $g\alpha^3 = e\alpha$

$$\delta W_\mu^\pm = \pm ie\alpha W_\mu^\pm$$

Boson $W^\pm_\mu$ has electric charge $Q = \pm 1$

Running of weak coupling

Gauge couplings of EW sector run with energy scale

- Hypercharge - *U(1)* gauge theory $\Rightarrow$ *g'* smaller as going to IR
- Non-Abelian *SU(2)* gauge symmetry

$\rightarrow$ include N_s scalars in fundamental representation

$\Rightarrow$ **Assume:** *SU(N_c)* gauge theory coupled to N_f massless Dirac fermions

$$\frac{1}{g^2(\mu)} = \frac{1}{g_0^2} - \frac{b_0}{(4\pi)^2}\log\frac{\Lambda_{UV}^2}{\mu^2} \qquad b_0 = \frac{11}{3}N_c - \frac{2}{3}N_f - \frac{1}{6}N_s$$

$\rightarrow$ applying to EW: $N_c = 2$ and $N_s = 1$ *(Higgs doublet)*

$N_f = 2$ Dirac fermions for each generation

Coefficient of one-loop β-function for weak force

$$b_0 = b_{\text{weak}} \qquad b_{\text{weak}} = \frac{11}{3}\times 2 - \frac{2}{3}\times 6 - \frac{1}{3} = 3$$

$\rightarrow$ $b_{\text{weak}} > 0$ $\Rightarrow$ *SU(2)* sector of SM asymptotically free *(flows to strong coupling in IR)*

- Strong coupling effects in weak sector

$\rightarrow$ Higgs mechanism gives masses to gauge bosons

$\Rightarrow$ freeze running of *g* at scale: $\mu \sim M_W$

Running of weak coupling

- Weak coupling: $\alpha_W = g^2/4\pi \approx 1/30$

  → particles decaying through weak force ⇒ emission of intermediate *W* or *Z* boson

  → large mass of *W*/*Z* bosons ⇒ small decay rate

- Weak and strong couplings at M_Z:

$$\alpha_s(M_Z) \approx 0.12 \quad \text{and} \quad \alpha_w(M_Z) \approx 0.034$$

  → asymptotic freedom ⇒ g_s and g_w smaller at higher energies

  → running of strong coupling: $b_{\text{strong}} = \frac{11}{3} \times 3 - \frac{2}{3} \times 6 = 7$

  $b_{\text{strong}} > b_{\text{weak}}$ - couplings converge at higher energies

- Convergence of weak and strong couplings

  → meet at scale μ: $\frac{1}{g_s^2} - \frac{b_{\text{strong}}}{(4\pi)^2} \log \frac{M_W^2}{\mu^2} = \frac{1}{g_w^2} - \frac{b_{\text{weak}}}{(4\pi)^2} \log \frac{M_W^2}{\mu^2}$

$$\mu = M_W \exp\left(\frac{2\pi}{b_{\text{strong}} - b_{\text{weak}}}\left(\frac{1}{\alpha_w} - \frac{1}{\alpha_s}\right)\right) \approx 2 \times 10^{16}\,\text{GeV}$$

Running of weak coupling

- Running is logarithmic - long time to converge

  → Planck scale: $M_{\rm pl} \sim 10^{19}\ {\rm GeV}$

- Unification

  → coupling constants converge ⇒ *SU(2)* and *SU(3)* forces within larger gauge group (?)

  ⇒ proposals: e.g. *SU(5)* and *SO(10)*

- Hypercharge coupling *g'* *(running)*

$$\alpha_Y = g'^2/4\pi$$

→ convergence:

$$\alpha_Y = \alpha_s \quad \text{at} \quad \mu \approx 5 \times 10^{19}\ {\rm GeV}$$
$$\alpha_Y = \alpha_w \quad \text{at} \quad \mu \approx 10^{21}\ {\rm GeV}\ .$$

3 convergence points do not meet!

→ unification of hypercharge coupling - regime of quantum gravity (?)

Fermion masses

Masses of fundamental fermions

→ Higgs gives mass to *W* and *Z* bosons

→ Higgs gives masses to fundamental fermions in SM - Yukawa interactions

- SM - chiral theory

→ left- and right-handed fermions transform differently under gauge group

→ no straightforward mass terms for fermions in SM

Any mass term violates gauge symmetry

→ **Yukawa terms** - gauge invariant interaction terms giving mass if Higgs field gets expectation value

- 1st generation

→ Yukawa couplings: $\mathcal{L}_{\text{Yuk}} = -y^d \bar{Q}_L H d_R - y^u \bar{Q}_L \tilde{H} u_R - y^e \bar{L}_L H e_R - y^\nu \bar{L}_L \tilde{H} \nu_R + \text{h.c.}$

H - Higgs doublet transforming in **2** of *SU(2)* gauge group
$\tilde{H}$ - conjugated Higgs doublet contracted with ϵ, transforming in **2**
y^d, y^u, y^e, y^ν - dimensionless Yukawa couplings

$$\tilde{H}^a = \epsilon^{ab} H_b^\dagger \quad \text{with} \quad a, b = 1, 2$$

Fermion masses

- Higgs vev:

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \Longrightarrow \quad \langle \tilde{H} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$$

→ distinguish 2 components of *SU(2)* doublets Q_L and L_L

⇒ different masses & charges under unbroken EM symmetry

→ diferent names for 2 components of doublets:

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \text{and} \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

- **Assume:** Yukawa couplings with only vev *v* *(ignoring interactions with Higgs fluctuations)*

$$\mathcal{L}_{\text{Yuk}} = -\frac{v}{\sqrt{2}} \left(y^d \, \bar{d}_L d_R + y^u \, \bar{u}_L u_R + y^e \, \bar{e}_L e_R + y^\nu \, \bar{\nu}_L \nu_R \right)$$

→ fermion mass: $m^X = \frac{1}{\sqrt{2}} y^X v \qquad X = d, u, e, \nu$

Scale of fundamental fermion masses set by Higgs vev

Fermion electric charges

- Electric charges of fermions

$$Q = T^3 + Y$$

→ hypercharges:

	Q_L	L_L	u_R	d_R	e_R
Y	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	-1

→ right-handed fermions uncharged under the *SU(2)* ⇒ *Q = Y*

- *SU(2)* doublets Q_L and L_L

→ T^3 eigenvalues: ±1/2 *(+ for upper component, − for lower component)*

$$u_L: \quad Q = \frac{1}{2} + \frac{1}{6} = \frac{2}{3} \quad \text{and} \quad d_L: \quad Q = -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3}$$
$$\nu_L: \quad Q = \frac{1}{2} - \frac{1}{2} = 0 \quad \text{and} \quad e_L: \quad Q = -\frac{1}{2} - \frac{1}{2} = -1$$

→ electric charges for left-handed fermions same as for right-handed

⇒ mass terms invariant under $U(1)_{EM} \subset SU(2) \times U(1)_Y$

Symmetry breaking → 4 Dirac fermions: *u, d, e, ν*

Weak decays

Electroweak currents

Gauge bosons coupling to fermions

→ fermion kinetic terms for single generation:

$$\mathcal{L}_{\text{fermi}} = -i\Big(\bar{Q}_L\bar{\sigma}^\mu\mathcal{D}_\mu Q_L + \bar{L}_L\bar{\sigma}^\mu\mathcal{D}_\mu L_L + \bar{u}_R\sigma^\mu\mathcal{D}_\mu u_R + \bar{d}_R\sigma^\mu\mathcal{D}_\mu d_R + \bar{e}_R\sigma^\mu\mathcal{D}_\mu e_R\Big)$$

no right-handed neutrino - neutral under all gauge symmetries

→ terms of interactions with EW gauge bosons *(ignore gluon fields)*

$$\begin{aligned}\mathcal{L}_{\text{kin}}\Big|_{\text{weak}} = &-\frac{g}{2}W^3_\mu\left(\bar{u}_L\bar{\sigma}^\mu u_L - \bar{d}_L\bar{\sigma}^\mu d_L + \bar{\nu}_L\bar{\sigma}^\mu\nu_L - \bar{e}_L\bar{\sigma}^\mu e_L\right)\\ &- \frac{g}{\sqrt{2}}W^+_\mu(\bar{u}_L\bar{\sigma}^\mu d_L + \bar{\nu}_L\bar{\sigma}^\mu e_L) - \frac{g}{\sqrt{2}}W^-_\mu(\bar{d}_L\bar{\sigma}^\mu u_L + \bar{e}_L\bar{\sigma}^\mu\nu_L)\\ &- g'B_\mu\Big(\frac{1}{6}\bar{u}_L\bar{\sigma}^\mu u_L + \frac{1}{6}\bar{d}_L\bar{\sigma}^\mu d_L - \frac{1}{2}\bar{\nu}_L\bar{\sigma}^\mu\nu_L - \frac{1}{2}\bar{e}_L\bar{\sigma}^\mu e_L\\ &\qquad + \frac{2}{3}\bar{u}_R\sigma^\mu u_R - \frac{1}{3}\bar{d}_R\sigma^\mu d_R - \bar{e}_R\sigma^\mu e_R\Big)\end{aligned}$$

→ if replace W^3_μ and B_μ with Z boson & photon fields:

$$\mathcal{L}_{\text{kin}}\Big|_{\text{weak}} = -\frac{e}{\sqrt{2}\sin\theta_W}(W^+_\mu J^\mu_+ + W^-_\mu J^\mu_-) - \frac{e}{\sin\theta_W\,\cos\theta_W}Z^\mu J^Z_\mu - eA^\mu J^{\text{EM}}_\mu$$

g & g' replaced with Weinberg angle $\tan\theta_W = g'/g$ and EM coupling $e = g\sin\theta_W = g'\cos\theta_W$

J^{EM} - currents interacting with gauge fields

Electroweak currents

- EM current coupling to photon

$$J_\mu^{\rm EM} = \frac{2}{3}(\bar{u}_L\bar{\sigma}_\mu u_L + \bar{u}_R\sigma^\mu u_R) - \frac{1}{3}(\bar{d}_L\bar{\sigma}_\mu d_L + \bar{d}_R\sigma^\mu d_R) - (\bar{e}_L\bar{\sigma}_\mu e_L + \bar{e}_R\sigma^\mu e_R) = \left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \bar{e}\gamma_\mu e\right)$$

→ each fermion multiplied by its electric charge

→ in terms of Dirac spinors *u*, *d*, *e* & γ^μ matrices

⇒ vector-like current

- EM current coupling to *Z* boson

$$J_\mu^Z = \frac{1}{2}(\bar{u}_L\bar{\sigma}_\mu u_L - \bar{d}_L\bar{\sigma}_\mu d_L + \bar{\nu}_L\bar{\sigma}_\mu \nu_L - \bar{e}_L\bar{\sigma}_\mu e_L) - \sin^2\theta_W\, J_\mu^{\rm EM}$$

- EM current coupling to *W* bosons

$$J_\mu^+ = \bar{u}_L\bar{\sigma}_\mu d_L + \bar{\nu}_L\bar{\sigma}_\mu e_L \quad \text{and} \quad J_\mu^- = \bar{d}_L\bar{\sigma}_\mu u_L + \bar{e}_L\bar{\sigma}_\mu \nu_L$$

Currents for *W* and *Z* bosons are chiral ⇒ different for left- and right-handed fermions

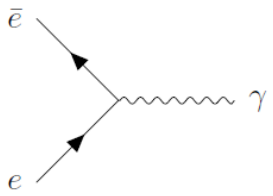
Feynman diagrams

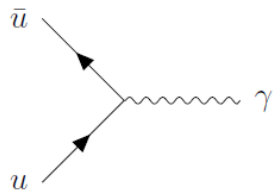
Feynman rules for EW sector

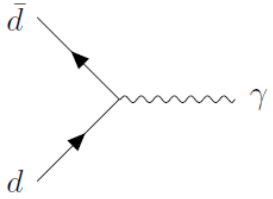
- γ coupling to u, d, e with coupling constant: $e\,q$ (q - charge)

→ Feynman diagrams:

ν neutral - no coupling

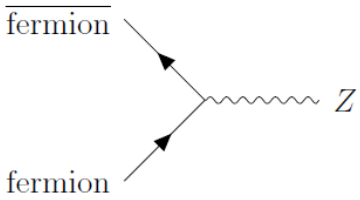






- *Z* boson coupling *(including neutrino)*

→ Feynman diagrams:

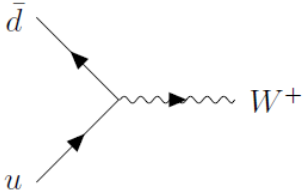


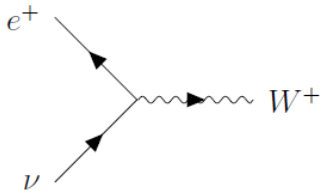
overall factor: $e/\sin\theta_W\ \cos\theta_W$

\+ different coefficients for fermion species

- *W* boson relates 2 different fermions

→ Feynman diagrams:



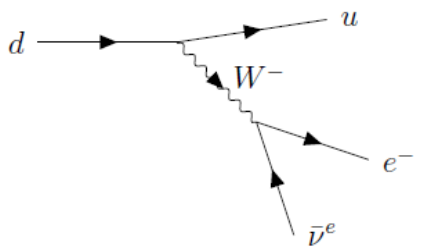


- 2 fermions with electric charges differ by ±1
- arrow on gauge boson propagator - complex spin 1 field
- arrow going other way - anti-particle W^-

Weak processes

- Weak force first observed in β-decay

$$n \rightarrow p + e^- + \bar{\nu}^e$$

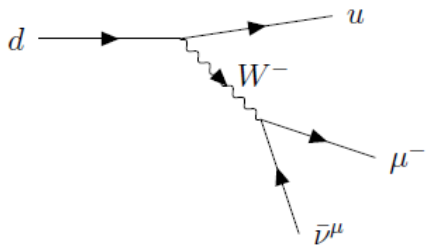


→ neutron *(udd, lifetime = 10 min)* to proton *(uud)*:

→ $m_d > m_u + m_e + m_{\nu e}$

⇒ *d*-quarks decay into *u*-quarks

- Charged pion $\pi^{\pm}$



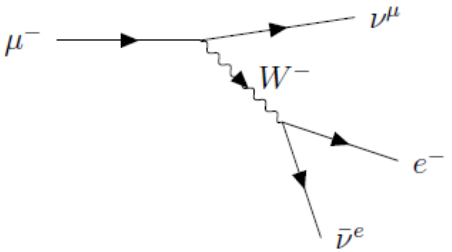
→ 99.99% - through weak force to muon and anti-neutrino

→ outgoing *u*-quark combines with anti-*u*-quark in pion

⇒ rapidly decay into photons

→ $\pi^{\pm}$ lifetime ~ 10^{-8} s

Resulting muons



→ through weak force to electrons and neutrinos

→ muon lifetime: 2×10^{-6} s

4-Fermi theory

- Weak force mediated by *W* & *Z*

  → if $E \ll M_W,\ M_Z$ - possible to ignore gauge bosons

- Ignore *W* and *Z* bosons

  → linear & quadratic terms in *W* bosons *(ignore h, but keep vev)*

$$\mathcal{L}_{\text{weak}} = -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) + \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} - \frac{g}{\sqrt{2}}(W^+_\mu J^\mu_+ + W^-_\mu J^\mu_-)$$

  → low energy ⇒ neglect kinetic terms for *W* bosons

$$\mathcal{L}_{\text{weak}} \approx \frac{g^2 v^2}{4}\left(W^+_\mu - \frac{2\sqrt{2}}{g v^2} J_{-\mu}\right)\left(W^{-\mu} - \frac{2\sqrt{2}}{g v^2} J^\mu_+\right) - \frac{2}{v^2} J_{+\mu} J^\mu_-$$

  → path integral over *W* bosons - first term = 0 ⇒ only current-current interaction

  4-fermion interaction: $\mathcal{L}_{\text{weak}} = -\frac{4 G_F}{\sqrt{2}} J_{+\mu} J^\mu_-$ $\qquad G_F = \frac{1}{\sqrt{2} v^2} \approx 1.16 \times 10^{-5}\ \text{GeV}^{-2}$

  G_F - Fermi coupling ⇒ direct measurement of Higgs vev

  → **$[G_F] = -2$** ⇒ 4-Fermi term irrelevant in renormalisation group sense

  → 4-fermion interactions - Fermi in 1933

  QFT: 4-fermion couplings allow one type of field to transmute into the others