Revisiting Lithium-7 Constraints with a Light Decoupled Dark Sector

by

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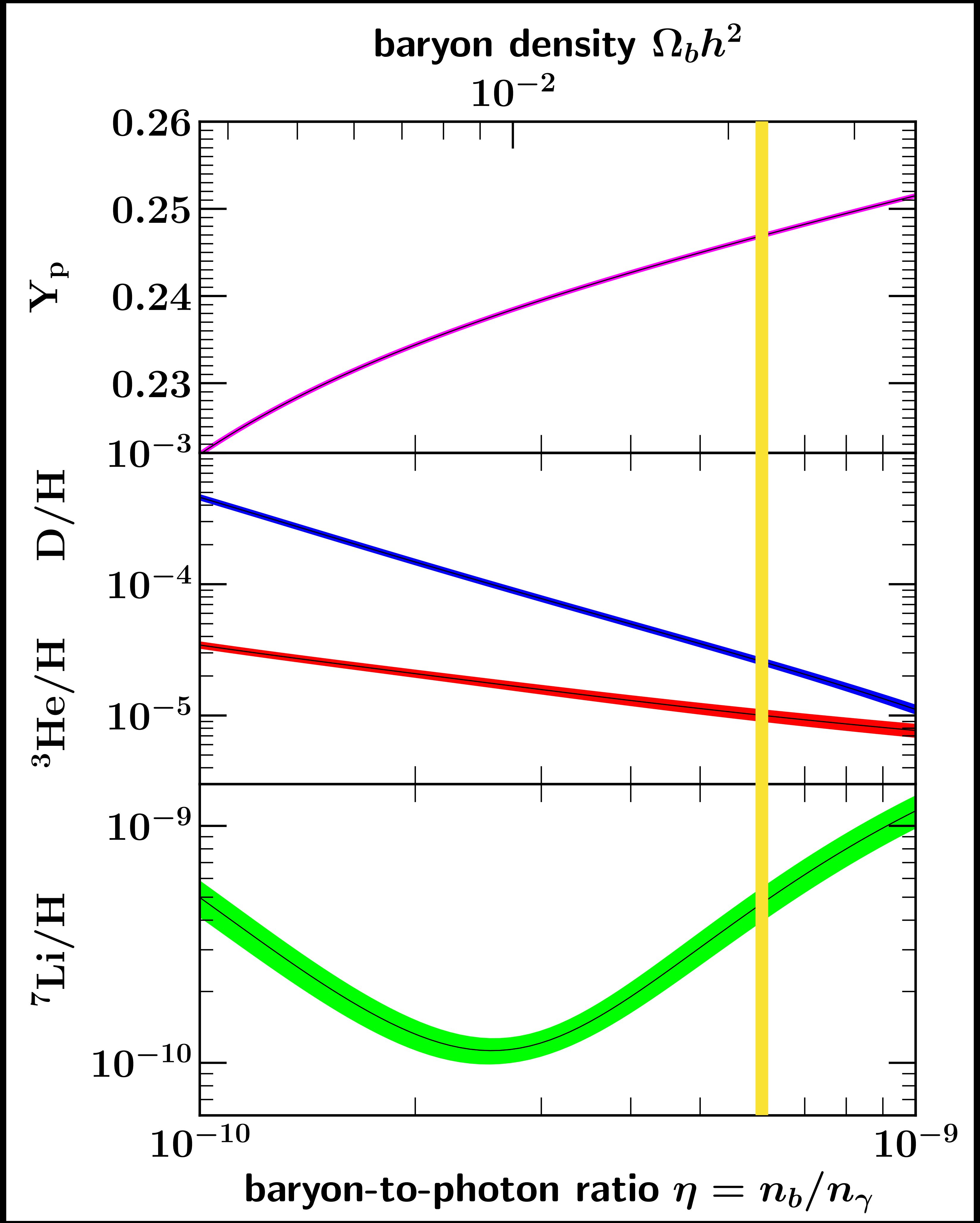
Standard Big-Bang Nucleosynthesis

- Controlled almost entirely by the baryon-to-photon ratio
- Particle content, nuclear reaction rates, weak interactions, and expansion rate are fixed
- Once η is known, BBN predicts all light-element abundances

$$\eta = \frac{n_b}{n_\gamma}$$

WMAP: $(6.19 \pm 0.14) \times 10^{-10}$ (2012)

PLANCK: $(6.104 \pm 0.058) \times 10^{-10}$ (2018)



[1] arXiv:1912.01132

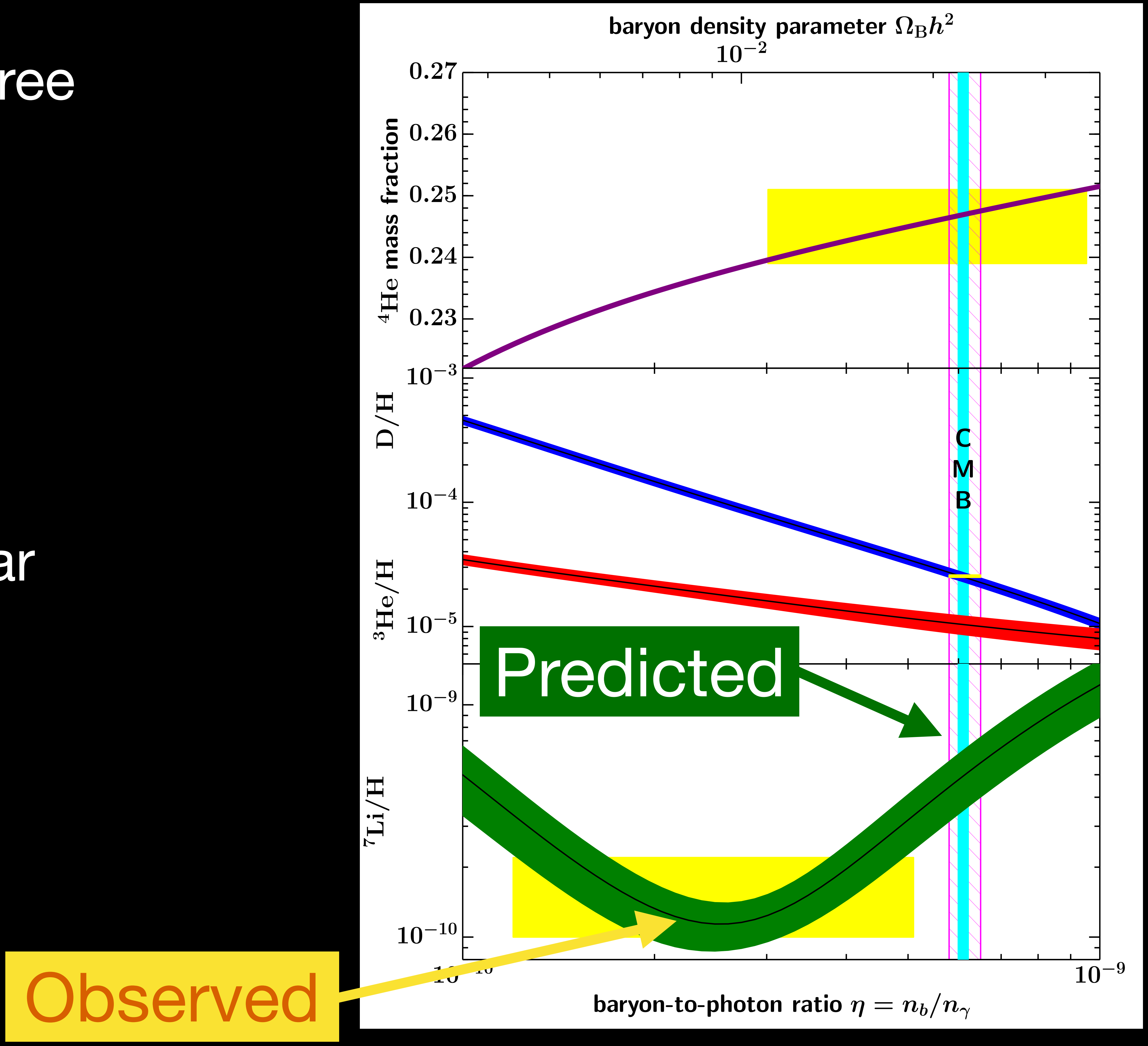
BBN Predictions vs Observations

- Using η from the CMB, BBN gives parameter-free abundance predictions
- $\mathrm{D/H}$ and $^4\mathrm{He}$ agree well with observations

$$\mathrm{D/H} = (2.547 \pm 0.025) \times 10^{-5}$$
$$\mathrm{Yp} = 0.245 \pm 0.003$$

- $^3\mathrm{He}$ is not a clean primordial observable (stellar processing)
- $^7\mathrm{Li}$ is overproduced by a factor of 3–4

Predicted $\mathrm{Li/H} \approx 5.3 \times 10^{-10}$
Observed $\mathrm{Li/H} = (1.6 \pm 0.3) \times 10^{-10}$



PDG 2024

Cosmological Lithium Problem

- Four possible solutions
  - Astrophysical effects: stellar mixing or lithium depletion
  - Nuclear physics: revise nuclear reactions
  - Observational systematics
  - Non-standard cosmology (our case)

We assume that this discrepancy points to some new physics which changes the theoretical predictions of the lithium abundance.

Framework

- Relevant reactions:

1. $^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu_e$ — This is how Li is produced post BBN

2. $^7\mathrm{Be} + \gamma \rightarrow {}^3\mathrm{He} + {}^4\mathrm{He}$ — We can make sure we have less ^7Be

   $E_\gamma^{\mathrm{thr}}(^7\mathrm{Be}) = 1.59\ \mathrm{MeV}$ **

3. $\mathrm{D} + \gamma \rightarrow n + p$ — Need to avoid spoiling D, ^4He and ^3He

   $E_\gamma^{\mathrm{thr}}(\mathrm{D}) = 2.22\ \mathrm{MeV}$ **

- Implication: Injected photons must satisfy $1.59 \lesssim E_\gamma \lesssim 2.22\ \mathrm{MeV}$

Goal: Reduce $^7\mathrm{Be}$ without disturbing D or $^4\mathrm{He}$.

** Keep note of the energy thresholds

A Simple Example

- Assume a particle ϕ (decaying particle in DS) that decays into:

$$\phi \to \gamma\gamma \quad \text{or} \quad \cancel{\phi \to e^+e^-}$$

- For $\phi \to \gamma\gamma$:

$$E_\gamma = \frac{m_\phi}{2}, \quad 1.59 \lesssim E_\gamma \lesssim 2.22 \ \mathrm{MeV} \quad \Rightarrow 3.17 \lesssim m_\phi \lesssim 4.44 \ \mathrm{MeV}$$

$$\text{or} \quad m_\phi \simeq 4.4 \ \mathrm{MeV} \quad \Rightarrow 6 \times 10^3 \lesssim \tau_\phi \lesssim 7 \times 10^{10} \ \mathrm{s}$$

- For $\phi \to e^+e^-$, photodisintegration is far less efficient and in this case will not work!

Framework

- Dark sector is assumed to be decoupled from SM thermal bath.
- Fermionic DM freezes out in the hidden bath into a lighter scalar particle before Big Bang Nucleosynthesis (BBN).
- The scalar eventually decays electromagnetically:

$$\phi \to \gamma\gamma \quad \text{(or subdominantly } \phi \to e^+e^-)$$

- Reduce $^7\mathrm{Be}$ (and therefore $^7\mathrm{Li}$) abundance without upsetting $\mathrm{D/H}$, Y_p, or $^3\mathrm{He/D}$.
- We explore both DM and SM sectors, adjusting lifetimes and abundances to identify regions that can resolve the lithium problem.

Based on [2] arXiv:2011.06519

Methodology

Populating the hidden sector
$h \to \phi\phi$

Equilibrium within the hidden sector
$\phi\phi \leftrightarrow \chi\chi$

We track the evolution of ϕ

Other quantities

Around $T \sim \frac{m_h}{3}$

Dark Matter Freezes out (T_{cd})

How? Iterative method!

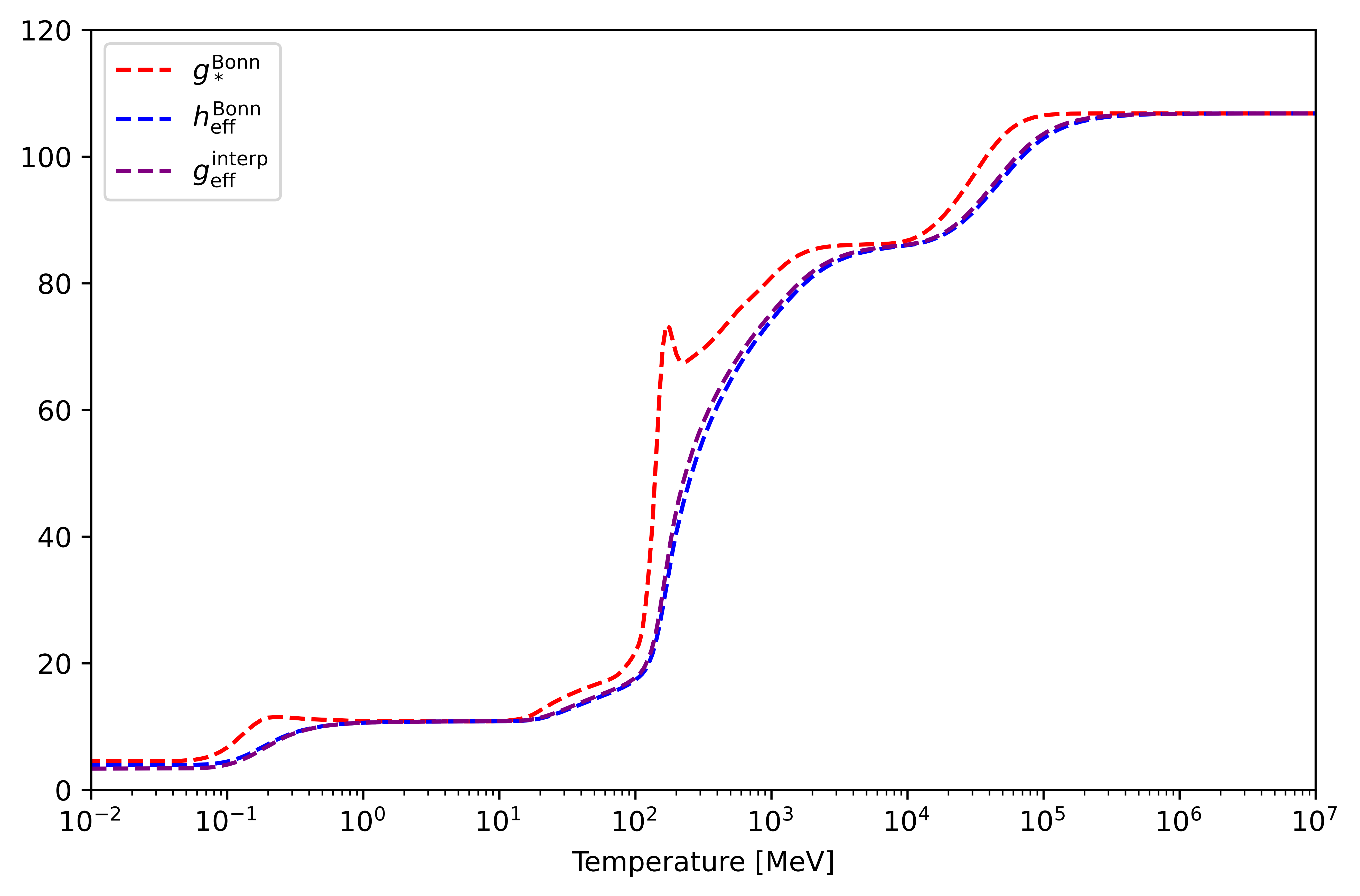
Decreasing T or increasing time

Based on [2] arXiv:2011.06519

Methodology

Standard cosmology

$h_{eff}(T),\ g_{eff}(T)$



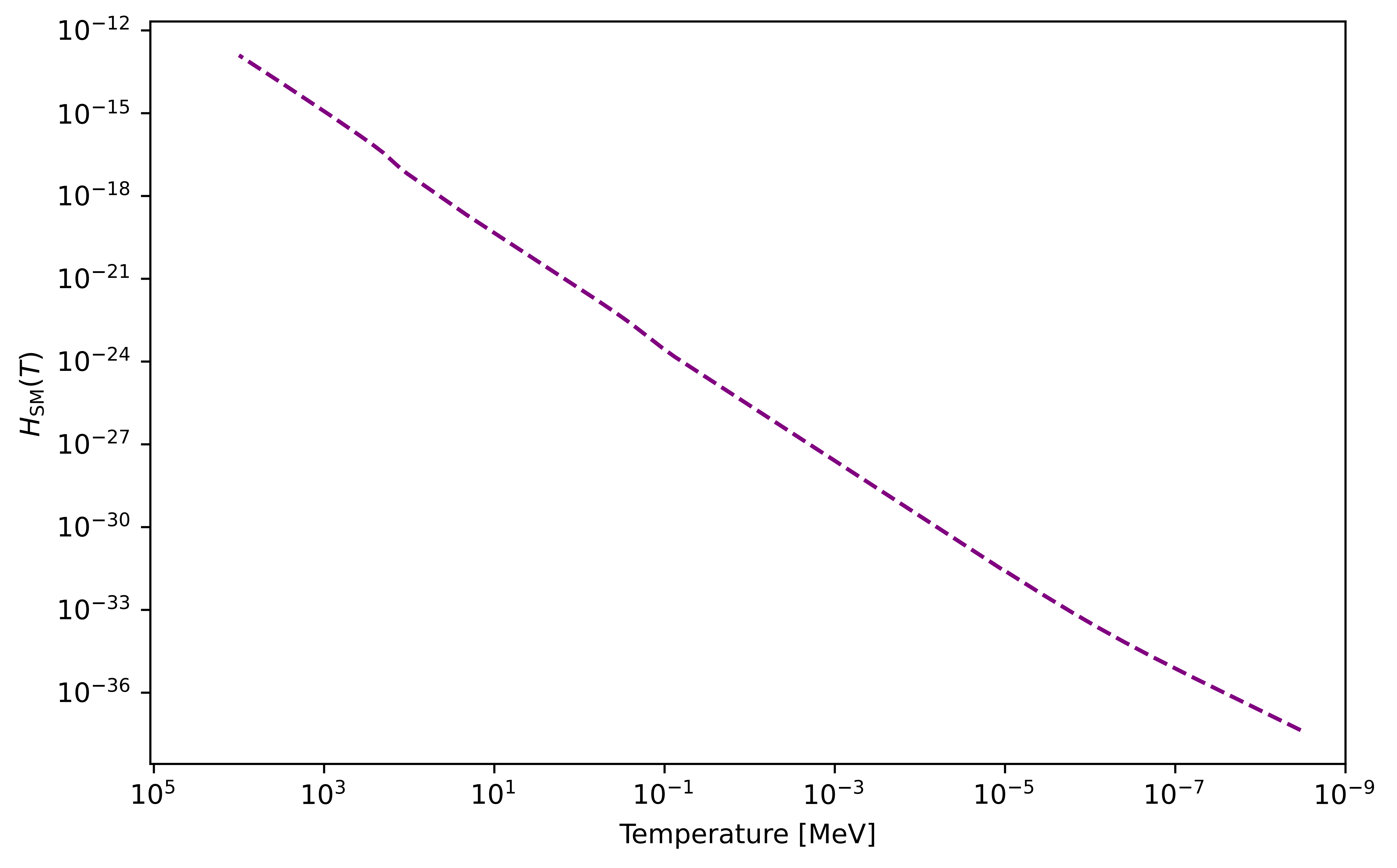
arXiv:1503.03513

Methodology

Standard cosmology

$h_{eff}(T),\ g_{eff}(T)$

$H^{(0)}(T),\ \rho_{SM}^{(0)}(T),\ P_{SM}^{(0)}(T)$



Methodology

Standard cosmology

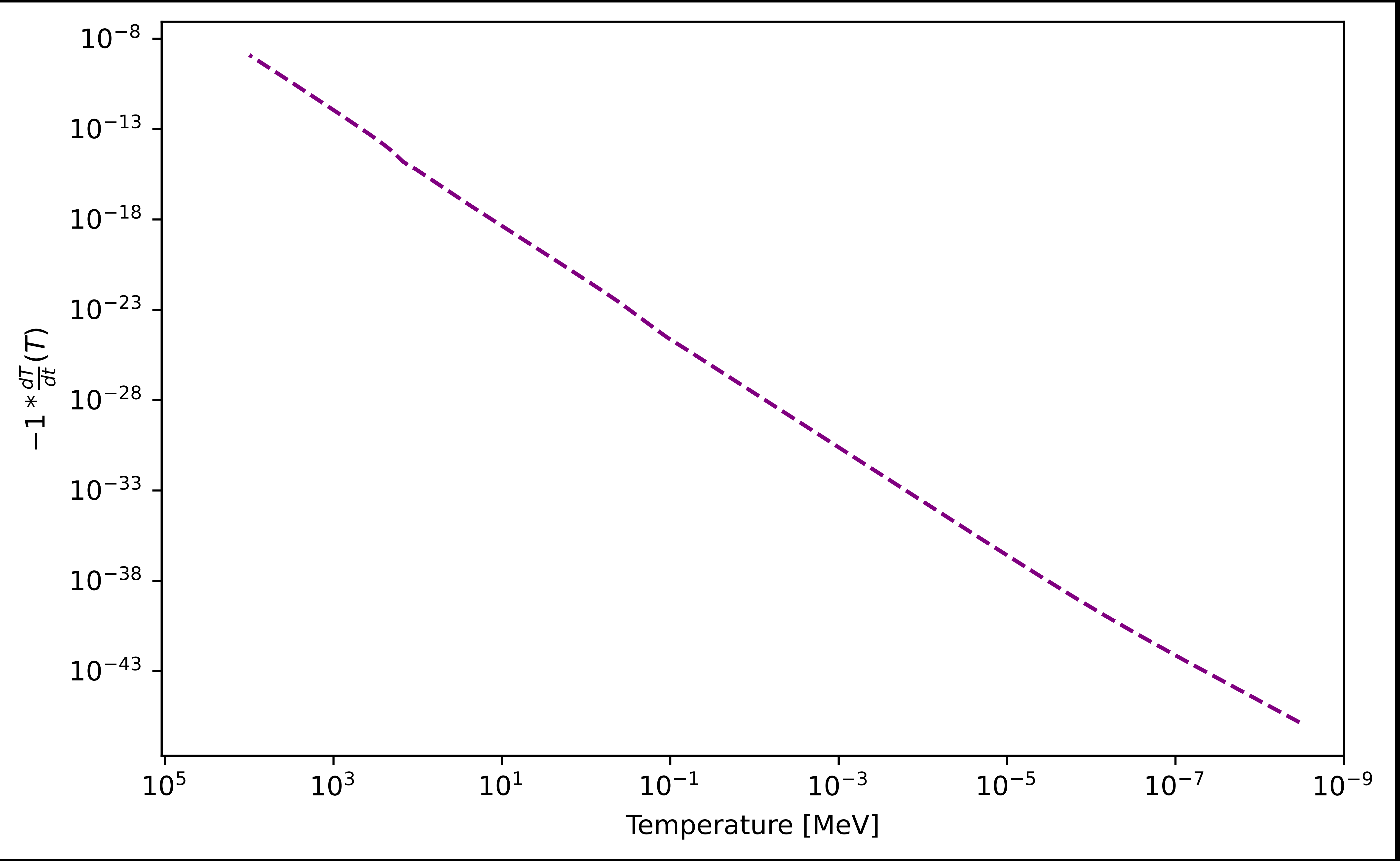
$h_{eff}(T),\ g_{eff}(T)$

$\downarrow$

$H^{(0)}(T),\ \rho^{(0)}_{SM}(T),\ P^{(0)}_{SM}(T)$

$\downarrow$

$$\frac{\mathrm{d}T^{(0)}}{\mathrm{d}t}(T)$$



Adding Dark Sector

Standard cosmology

$$h_{eff}(T),\ g_{eff}(T)$$

$$\downarrow$$

$$H^{(0)}(T),\ \rho_{SM}^{(0)}(T),\ P_{SM}^{(0)}(T)$$

$$\downarrow$$

$$\frac{\mathrm{d}T^{(0)}}{\mathrm{d}t}(T)$$

$$+$$

Dark sector

Based on [2] arXiv:2011.06519

Dark Sector

- We consider bosons dark-sector particle ϕ with $\{m_\phi, \tau_\phi, g_\phi, \zeta_{\rm cd}\}$
- Chemically decouples at $\{t_{\rm cd}\ T_{\rm D,cd}\}$ where $T_{\rm D,cd} = \zeta_{\rm cd} T_{\rm cd}$
- Phase-space distribution function:

$$f_\phi(T_{cd}, p) = \left(\exp\left[\frac{\sqrt{p_\phi^2 + m_\phi^2}}{\zeta_{cd} T_{cd}} \right] - 1 \right)^{-1}$$

- After decoupling:
  - Decays removes ϕ
  - Inverse decays produces ϕ
- We track the evolution by writing:

$$\frac{\partial f_\phi(T,p)}{\partial t} - H(t)\, p \frac{\partial f_\phi(T,p)}{\partial p} = \frac{1}{2E_\phi} \int \frac{d^3 p_z}{(2\pi)^3 2E_z} \frac{d^3 p_{\bar z}}{(2\pi)^3 2E_{\bar z}} \,|\mathscr{M}_{\phi\to z\bar z}|^2 (2\pi)^4 \delta^{(4)}(p - p_z - p_{\bar z}) \Big[-f_\phi (1 \pm f_z)(1 \pm f_{\bar z}) + f_z f_{\bar z}(1 + f_\phi) \Big]$$

Based on [2] arXiv:2011.06519

Dark Sector

- Can be re-written as:

$$\frac{\partial f_\phi(T,p)}{\partial t} - H(T)\,p\,\frac{\partial f_\phi(T,p)}{\partial p} = -D_z^{\pm}(T,p)\left[f_\phi(T,p) - \bar{f}_\phi(T,p)\right]$$

Decay term

Production term

where:

$$D_z^{\pm}(t,p) = \frac{m_\phi}{E_\phi \tau_\phi}\left[1 + \frac{1}{\beta_z p}\ln\left(\frac{1 \mp \exp[-(E_\phi + \beta_z p)/(2T)]}{1 \mp \exp[-(E_\phi - \beta_z p)/(2T)]}\right)\right]_{T=T(t)} \qquad \beta_z = \sqrt{1 - 4m_z^2/m_\phi^2} \qquad \frac{1}{\tau_\phi} = \frac{\beta_z}{16\pi m_\phi}\left|\mathcal{M}_{\phi\to z\bar{z}}\right|^2$$

- The formal solution will be:

$$f_\phi(t,p) = f_\phi\left(t_{\rm cd}, \frac{pR(t)}{R(t_{\rm cd})}\right)\exp\left(-\int_{t_{\rm cd}}^{t} dt'\, D_z^{\pm}\left(t', \frac{pR(t)}{R(t')}\right)\right) + \int_{t_{\rm cd}}^{t} dt'\,\bar{f}_\phi\left(t', \frac{pR(t)}{R(t')}\right)\exp\left(-\int_{t'}^{t} dt''\, D_z^{\pm}\left(t'', \frac{pR(t)}{R(t'')}\right)\right)$$

$$\frac{\partial f_\phi(T,p)}{\partial t} - H(t)\,p\frac{\partial f_\phi(T,p)}{\partial p} = \frac{1}{2E_\phi}\int \frac{d^3p_z}{(2\pi)^3 2E_z}\frac{d^3p_{\bar{z}}}{(2\pi)^3 2E_{\bar{z}}}\,|\mathcal{M}_{\phi\to z\bar{z}}|^2 (2\pi)^4\delta^{(4)}(p - p_z - p_{\bar{z}})\left[-f_\phi(1 \pm f_z)(1 \pm f_{\bar{z}}) + f_z f_{\bar{z}}(1 + f_\phi)\right]$$

Based on [2] arXiv:2011.06519

Adding Dark Sector

Standard cosmology

$h_{eff}(T),\ g_{eff}(T)$

$\downarrow$

$H^{(0)}(T),\ \rho_{SM}^{(0)}(T),\ P_{SM}^{(0)}(T)$

$\downarrow$

$$\frac{\mathrm{d}T^{(0)}}{\mathrm{d}t}(T)$$

+

Dark sector

$m_\phi \quad T_{cd}$

$\tau_\phi \quad \zeta_{cd}$

$\longrightarrow$ Boltzmann equation $f_\phi(T,p)$

$$n_\phi(t) = g_\phi \int \frac{d^3p}{(2\pi)^3} f_\phi(t,p)$$

$$\rho_\phi(t) = g_\phi \int \frac{d^3p}{(2\pi)^3} E_\phi f_\phi(t,p)$$

$$\dot{q}_\phi(t) = \dot{\rho}_\phi(t) + 3H(t)\left[\rho_\phi(t) + P_\phi(t)\right]$$

Based on [2] arXiv:2011.06519

Adding Dark Sector

Standard cosmology

$h_{eff}(T),\ g_{eff}(T)$

$H^{(0)}(T),\ \rho_{SM}^{(0)}(T),\ P_{SM}^{(0)}(T)$

$\frac{\mathrm{d}T^{(0)}}{\mathrm{d}t}(T)$

Dark sector

$m_\phi \quad T_{cd}$

$\tau_\phi \quad \zeta_{cd}$

Boltzmann equation $f_\phi(T,p)$

$$\frac{dT}{dt} = -\frac{\dot{q}_\phi(t) + 3H(T)\left[\rho_{\rm SM}(T) + P_{\rm SM}(T)\right]}{d\rho_{\rm SM}(T)/dT}, (T \gtrsim T_{\nu d})$$

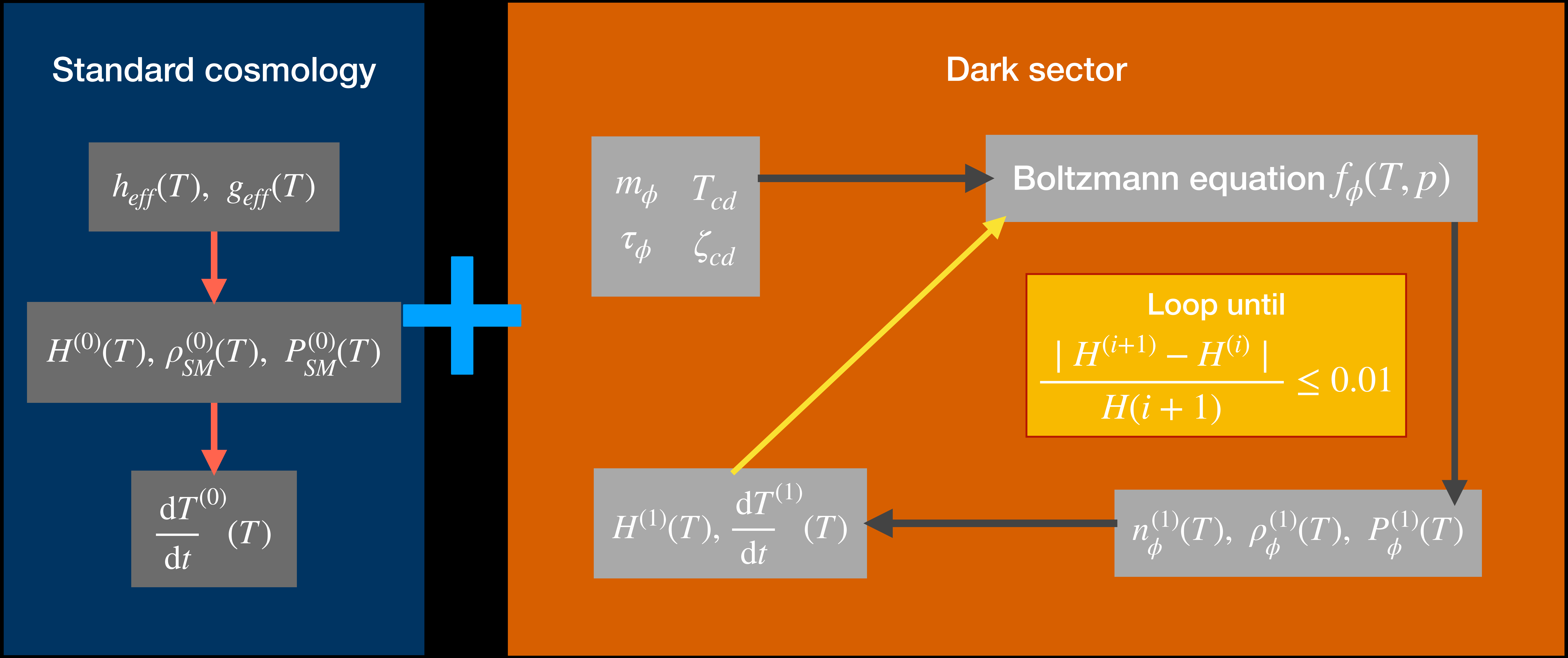
$$\frac{dT}{dt} = -\frac{\dot{q}_\phi(t) + 4H(T)\rho_\gamma(T) + 3H(T)\left[\rho_{e^\pm}(T) + P_{e^\pm}(T)\right]}{d\rho_\gamma(T)/dT + d\rho_{e^\pm}(T)/dT}, (T \lesssim T_{\nu d})$$

$$H(t) = \sqrt{\frac{8\pi G}{3}\left[\rho_{\rm SM}(t) + \rho_\phi(t)\right]}$$

$n_\phi^{(1)}(T),\ \rho_\phi^{(1)}(T),\ P_\phi^{(1)}(T)$

Based on [2] arXiv:2011.06519

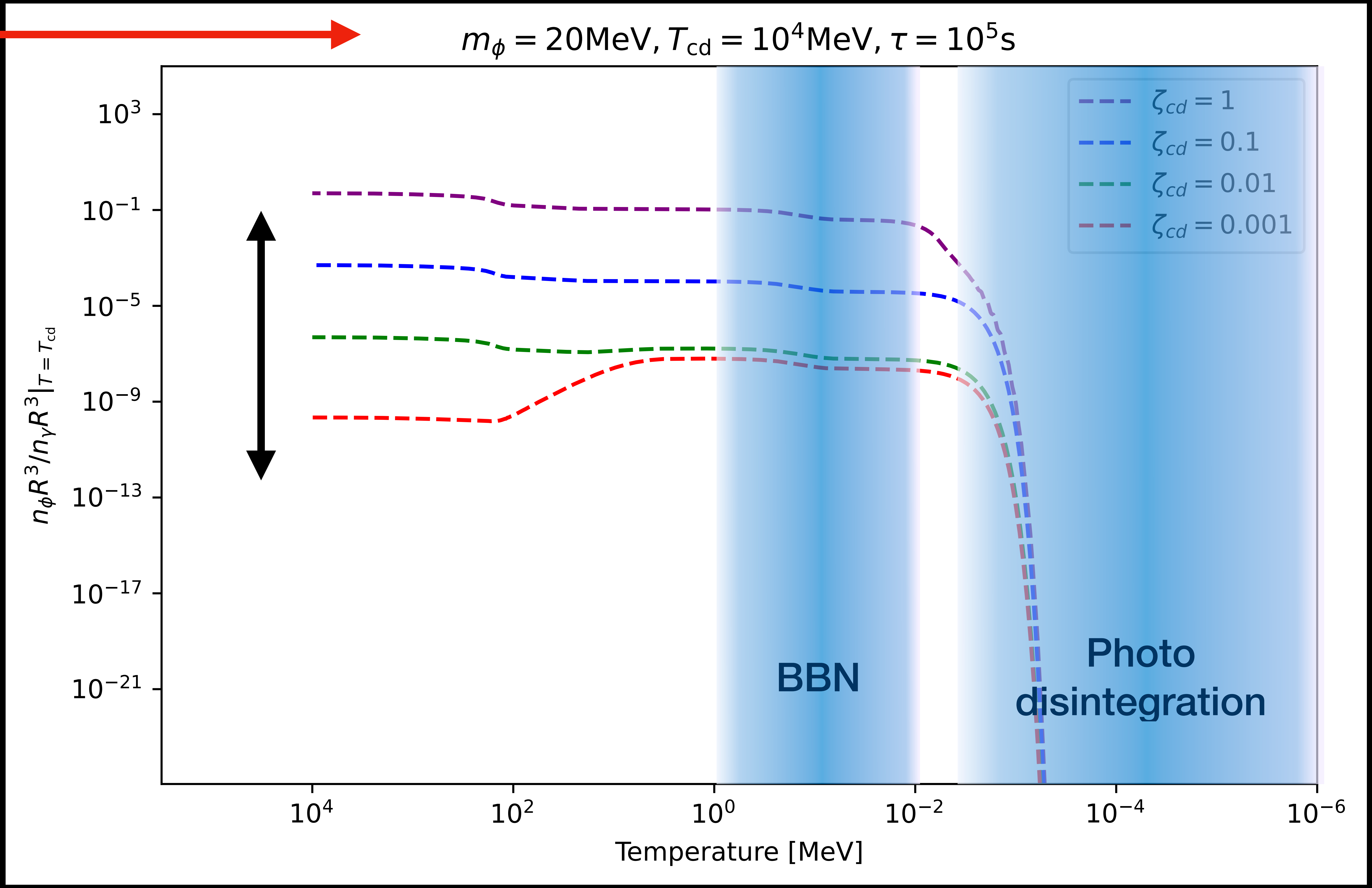
Adding Dark Sector



Based on [2] arXiv:2011.06519

Adding Dark Sector

Initial conditions

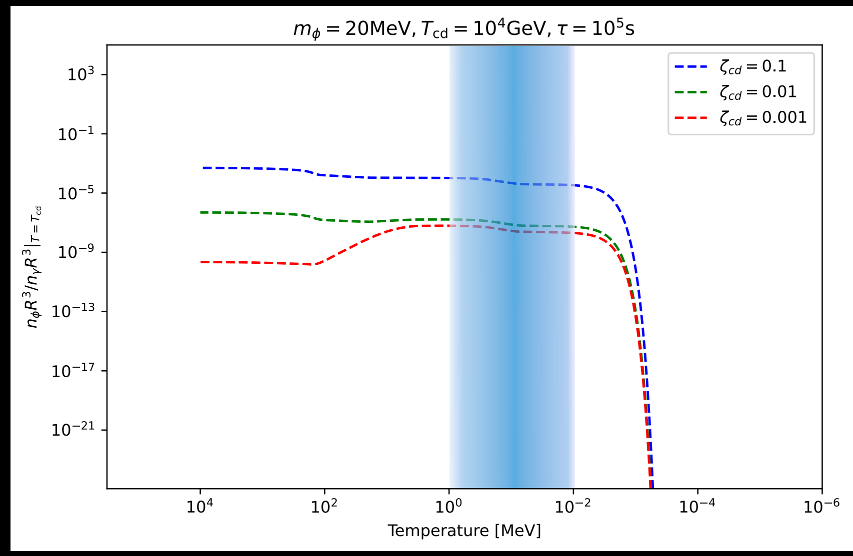


$$f_\phi(T_{\rm cd}, p) = \left[\exp\left(\frac{\sqrt{p^2 + m_\phi^2}}{\zeta_{\rm cd}\, T_{\rm cd}}\right) - 1\right]^{-1}$$

where,

$$\zeta_{cd} = \frac{T_{{\rm DS},\,cd}}{T_{{\rm SM},\,cd}}$$

Based on [2] arXiv:2011.06519



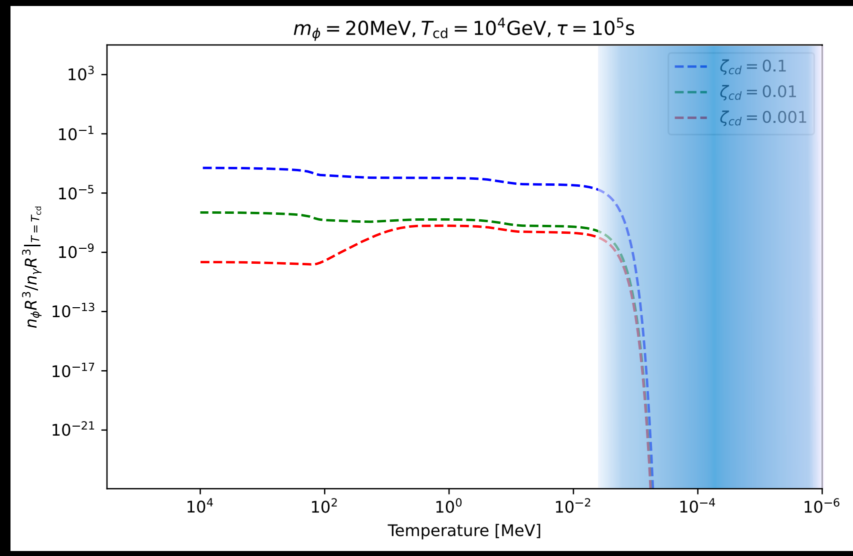
Packages

- AlterBBN [3] is a C program that is used for the calculation of abundance of the elements in from BBN.
- It tracks about 202 nuclear reactions.
- It approximately starts at 2.3 MeV for safety, nucleosynthesis really kicks in at 0.1 MeV and the code runs down until the network has frozen, typically near 0.01 MeV, as enforced by its built-in convergence tests.

```
./alter_standmod.x 0.1 3 0.1 4 0 0.  0.
returns:
               Yp        H2/H       He3/H      Li7/H      Li6/H      Be7/H
 value:    2.652e-01  4.249e-05  1.197e-05  3.244e-10  2.007e-14  2.754e-10
  +/- :    3.243e-04  1.343e-06  1.978e-07  2.380e-11  1.996e-14  2.305e-11

Excluded by BBN constraints (conservative limits)
Excluded by BBN constraints (chi2 including correlations)
```

arXiv:1806.11095



Packages

- ACROPOLIS [4] is a Python code that calculates how late-time energy injections alter light element abundances after BBN.
- It takes BBN outputs (e.g. from AlterBBN) as input and applies photodisintegration from decays or annihilations.
- Using 17 nuclear reactions, it predicts the final abundances of 9 key nuclei.

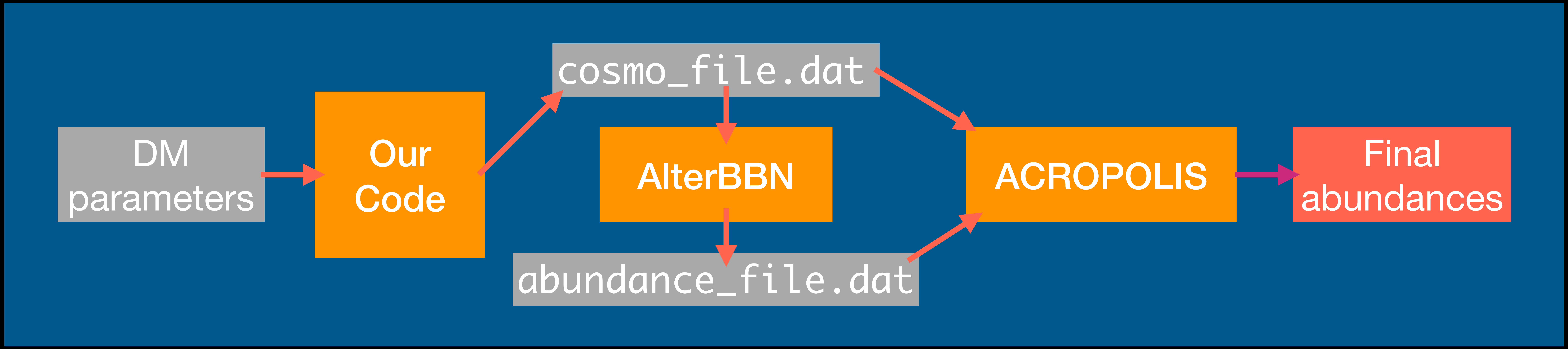
```
./decay 10 1e5 10 1e-10 0 1
ACROPOLIS v1.3.1 (https://acropolis.hepforge.org)

INFO   : Extracting and reading database files.
INFO   : Finished after 41.7ms.
INFO   : Calculating non-thermal spectra and reaction rates.
INFO   : Finished after 10.2s.
INFO   : Calculating final transfer matrix.
INFO   : Finished after 279.0ms.

     |     mean     |     high     |     low
-------------------------------------------------
   n | 0.000000e+00 | 0.000000e+00 | 0.000000e+00  [decayed]
   p | 7.525079e-01 | 7.525734e-01 | 7.524425e-01
  H2 | 1.837466e-05 | 1.791801e-05 | 1.883512e-05
  H3 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00  [decayed]
 He3 | 7.755380e-06 | 7.835375e-06 | 7.686027e-06
 He4 | 6.185801e-02 | 6.184180e-02 | 6.187418e-02
 Li6 | 8.104705e-15 | 2.626981e-14 | 1.260031e-15
 Li7 | 4.086930e-10 | 4.375372e-10 | 3.825925e-10
 Be7 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00  [decayed]
```

arXiv:[2011.06518]

Workflow



cosmo_file.dat

1. Time t [seconds (s)]
2. Photon temperature T_γ [MeV]
3. Time-temperature relation $[\text{MeV}]^2$ ← $\left(\frac{dT}{dt}\right)$
4. Neutrino temperature T_ν [MeV]
5. Hubble rate H [MeV]

BBN Abundance Criteria

- Abundance ratios: $Y_p = 4 \times \frac{N_{\mathrm{He}^4}}{N_H}, \quad Y_{\mathrm{D/H}} = \frac{N_D}{N_H}, \quad Y_{^3\mathrm{He/D}} = \frac{N_{^3\mathrm{He}} + N_T}{N_D}, \quad Y_{^7\mathrm{Li/H}} = \frac{N_{^7\mathrm{Li}} + N_{^7\mathrm{Be}}}{N_H}$
- Uncertainty for a ratio, $\mathrm{R} = \frac{\mathrm{A}}{\mathrm{B}}$ $\longrightarrow$ $\left(\frac{\sigma_R}{R}\right)^2 = \left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2$
    - Example: deuterium-to-hydrogen ratio $\sigma_{\mathrm{D/H}} = Y_{\mathrm{D/H}}\sqrt{\frac{\sigma_D^2}{N_D^2} + \frac{\sigma_H^2}{N_H^2}}$
- Comparison with observations: $\sigma^2_{\mathrm{tot}} = \sigma^2_{\mathrm{obs}} + \sigma^2_{\mathrm{model}},$

$$Z = \frac{\mathrm{R}_{\mathrm{model}} - \mathrm{R}_{\mathrm{obs}}}{\sigma_{\mathrm{tot}}}$$

z-score

$$Z_{D/H} = \frac{\frac{N_D}{N_H} - \left(Y_{D/H}\right)_{\mathrm{obs}}}{\sqrt{\left(\frac{N_D}{N_H}\right)^2 \left(\frac{\sigma_D^2}{N_D^2} + \frac{\sigma_H^2}{N_H^2}\right) + \sigma^2_{\mathrm{obs}}}}.$$

$N_i \equiv$ number density of nucleus i,
$\sigma_i \equiv$ statistical uncertainty.

BBN Abundance Criteria

- Allowed vs excluded regions (95% CL)
  - One-sided criterion: $z < 1.95996 \qquad (95\,\%$ CL)
  - Two-sided criterion $|z| < 1.95996$
- Choice of sidedness:

$Z_{Y_p}, Z_{D/H}, Z_{^7Li/H}$ are evaluated with a two-sided test,

$Z_{^3He/D}$ is evaluated with a one-sided test.

```
Running whole pipeline with these parameters: Namespace
(mass=20.0, zeta=0.01, g_phi=1, tau=100000.0, T_cd=1000
0.0, T_min=1e-05, particle='gamma', temp_steps=2001, mo
m_steps=101, n_cpus=100, sm_plots=1, mom_plots=1, debug
=False, console=True)
=====> Running Alterbbn!
AlterBBN v2.0
===> AlterBBN ran successfully!
=====> Running acropolis-1.3.1!
0.0 0.7556668154948342 1.8305986536453805e-05 0.0 7.773
353192216743e-06 0.06106831243605245 2.4755940340475237
e-14 3.9083225001649117e-10 0.0 0.0 0.7557355077515925
1.7915603764658448e-05 0.0 7.832016116768279e-06 0.0610
5129052172306 4.414587702471726e-14 4.1615613237048634e
-10 0.0 0.0 0.7555984480098051 1.8706118728889256e-05 0
.0 7.720971943381608e-06 0.06108524356784159 1.68937683
41604907e-14 3.676000998433343e-10 0.0
===> acropolis-1.3.1 ran successfully!
=====> Run complete
+-------------+------------+-------------+
| Observable  |  z-score   |  Decision   |
+-------------+------------+-------------+
|     Yp      |  -0.24219  |   ALLOWED   |
|    D / H    |  -2.10157  | NOT ALLOWED |
|   He3 / D   |  -2.69704  |   ALLOWED   |
|    Li / H   |   8.31559  | NOT ALLOWED |
+-------------+------------+-------------+
x=2.00000e+01, y=1.00000e+05 -> NOT ALLOWED
```

Pipeline is ready and we are in testing phase!

Final step will be to start from the hidden sector Lagrangian itself to obtain DM parameters!

$\boxed{\text{Thank you!}}$

References

[1] B. D. Fields, K. A. Olive, T. Yeh, and C. Young, Big-Bang Nucleosynthesis after Planck, *JCAP* 03 (2020) 010 [arXiv:1912.01132]

[2] P. F. Depta, M. Hufnagel, and K. Schmidt-Hoberg, “Updated BBN constraints on electromagnetic decays of MeV-scale particles,” JCAP 04 (2021) 011 [arXiv:2011.06519]

[3] A. Arbey, J. Auffinger, K. P. Hickerson, and E. S. Jenssen, AlterBBN v2: A public code for calculating Big-Bang nucleosynthesis constraints in alternative cosmologies, Comput. Phys. Commun. 248 (2020) 106982, [1806.11095]

[4] P. F. Depta, M. Hufnagel, and K. Schmidt-Hoberg, ACROPOLIS: A generiC fRamework fOr Photodisintegration Of LIght elementS, JCAP 03 (2021) 061, [2011.06518].

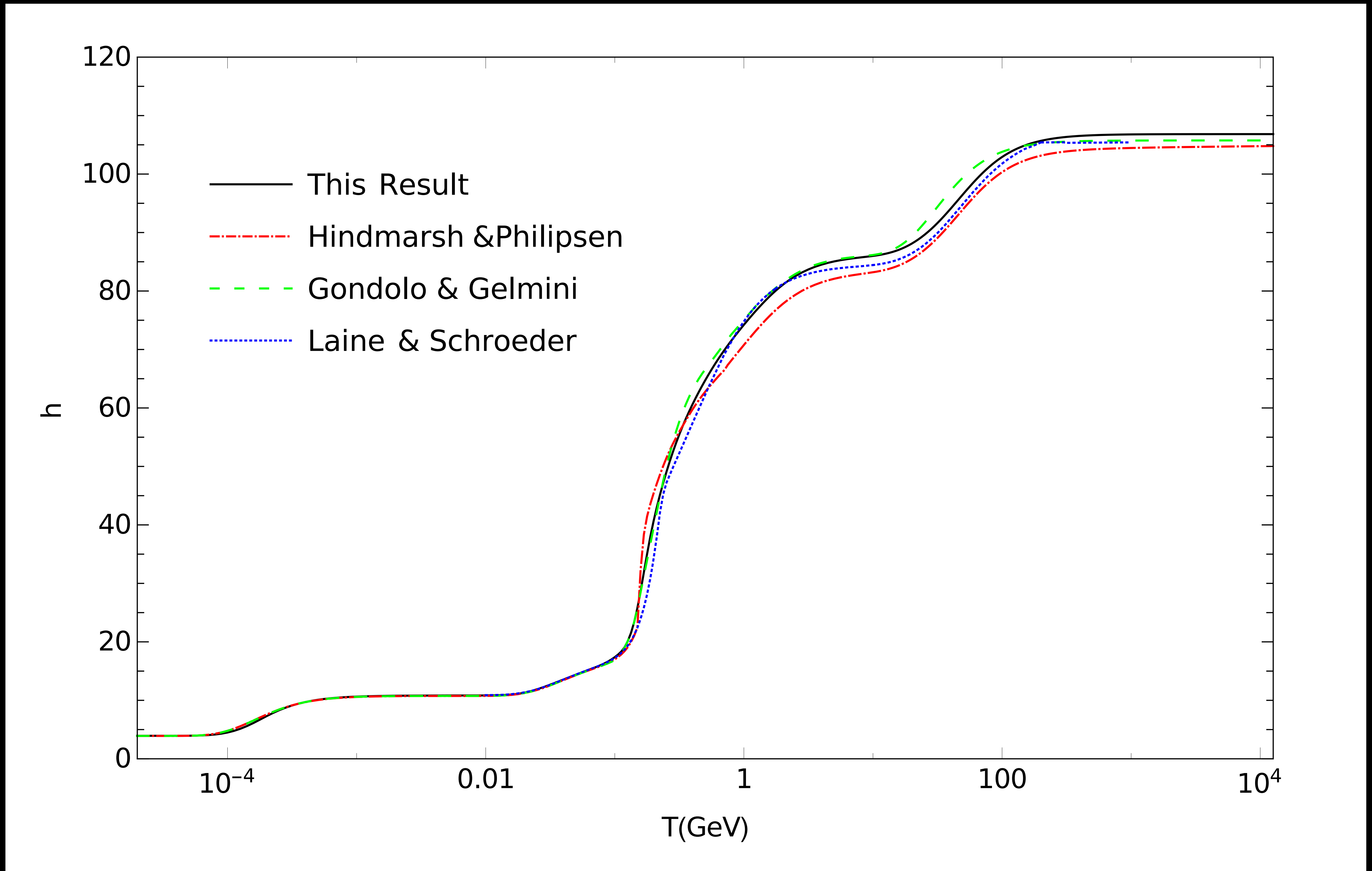
Backup

Lithium abundances

- Metal-poor halo stars: $(Li/H)_{\text{p,field}} = 1.23^{+0.68}_{-0.32} \times 10^{-10}$
- Globular clusters: $(Li/H)_{\text{p,GC}} = (2.19 \pm 0.28) \times 10^{-10}$
- WMAP: $^7Li/H = (5.24^{+0.71}_{-0.62}) \times 10^{-10}$

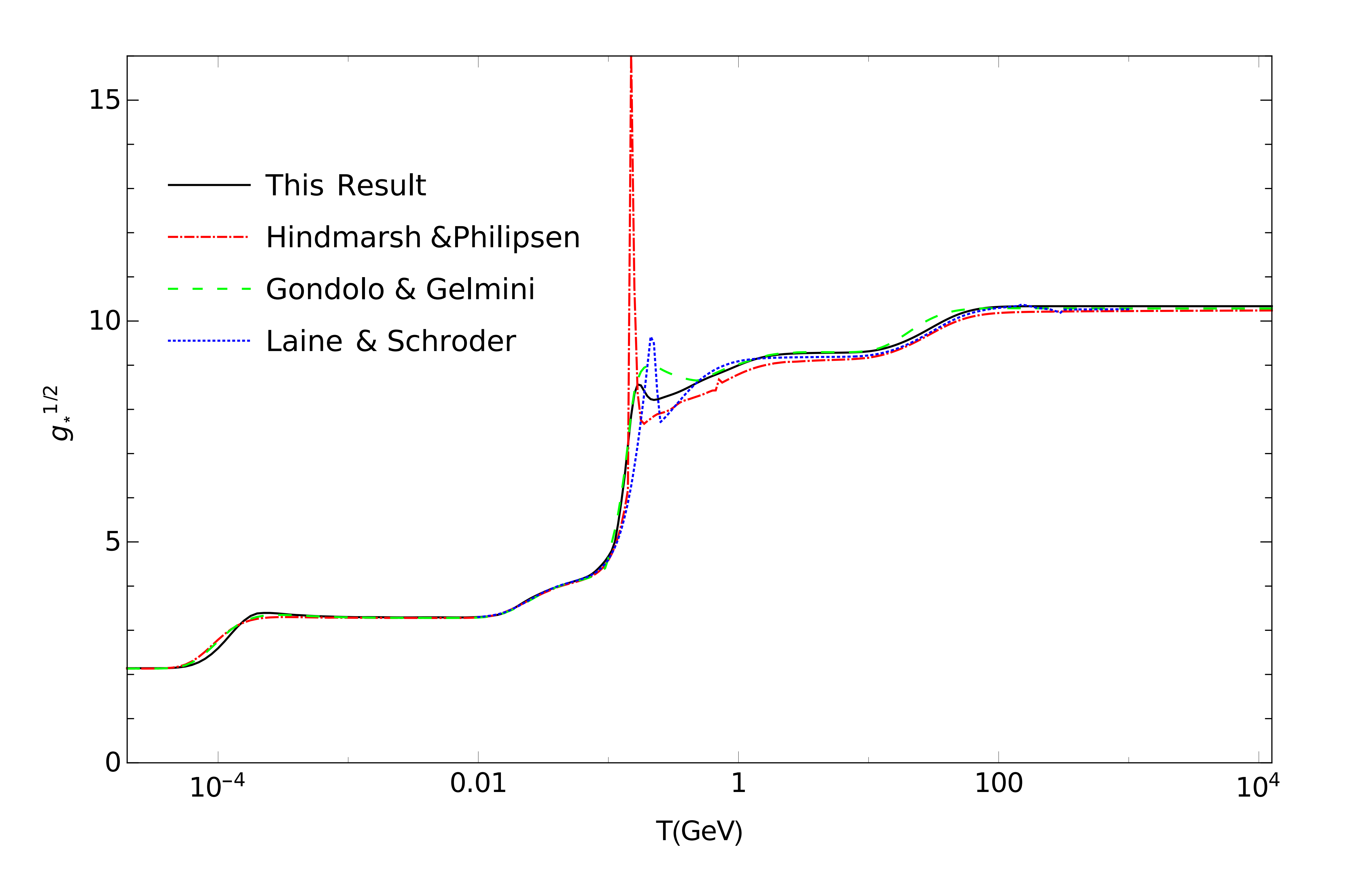
$$
\begin{aligned}
{}^7\mathrm{Li} + \gamma &\rightarrow {}^3\mathrm{H} + {}^4\mathrm{He}, \quad E_\gamma^{\mathrm{thr}} = 2.47\ \mathrm{MeV} \\
{}^3\mathrm{He} + \gamma &\rightarrow p + d, \quad E_\gamma^{\mathrm{thr}} = 5.49\ \mathrm{MeV} \\
{}^4\mathrm{He} + \gamma &\rightarrow {}^3\mathrm{H} + p, \quad E_\gamma^{\mathrm{thr}} = 19.8\ \mathrm{MeV} \\
{}^4\mathrm{He} + \gamma &\rightarrow {}^3\mathrm{He} + n, \quad E_\gamma^{\mathrm{thr}} = 20.6\ \mathrm{MeV}
\end{aligned}
$$

Backup

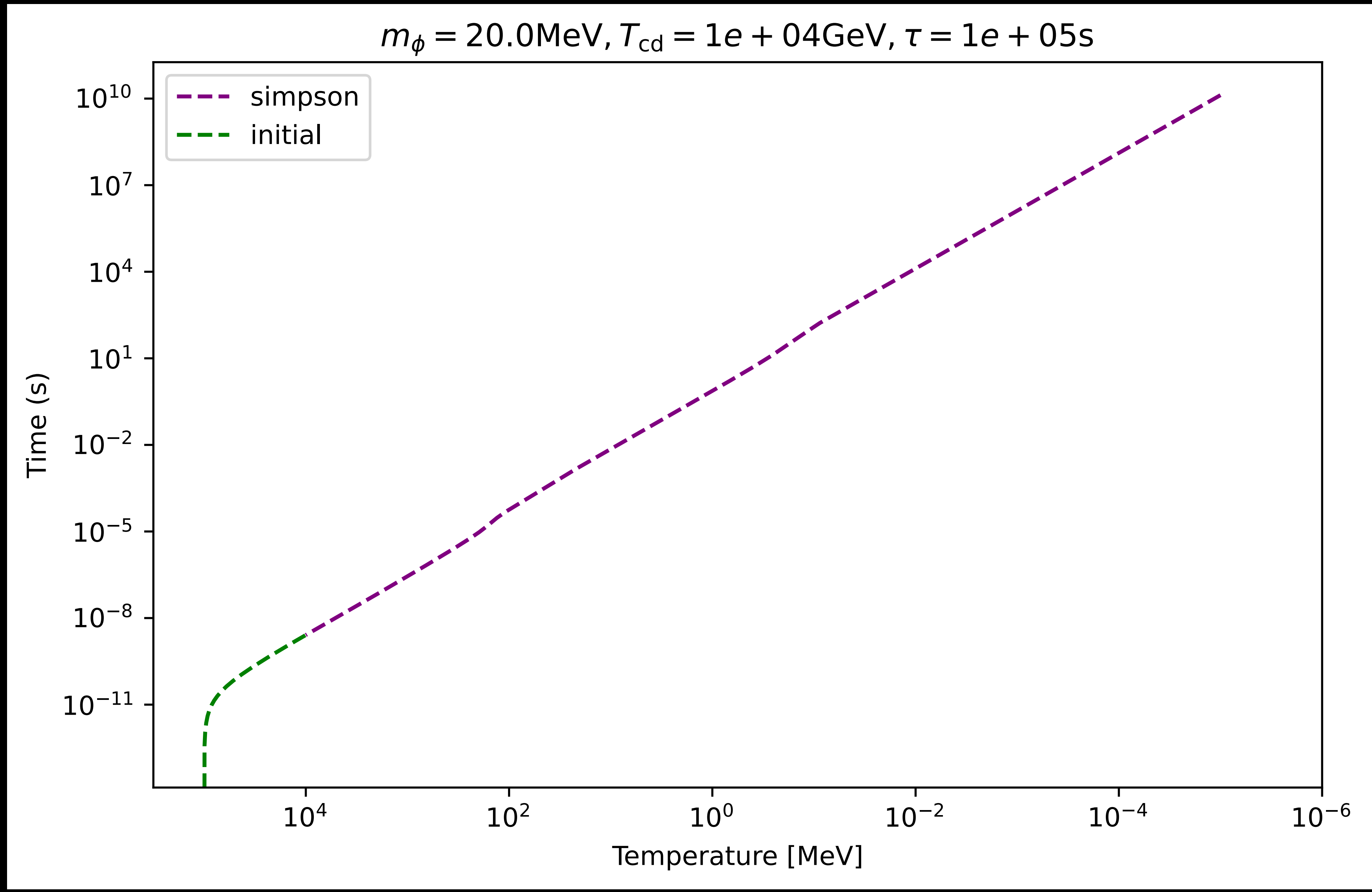


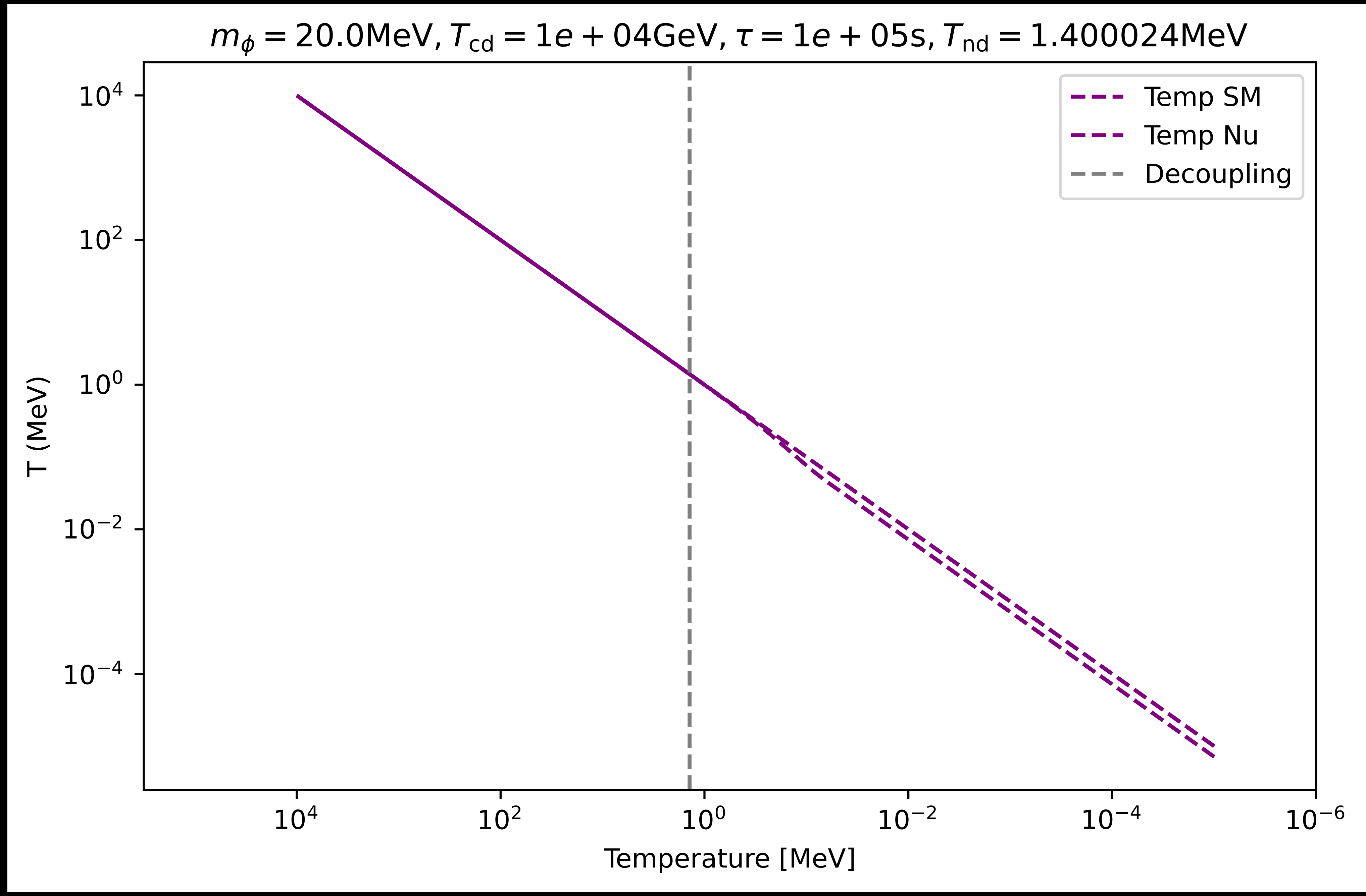
arXiv:1503.03513

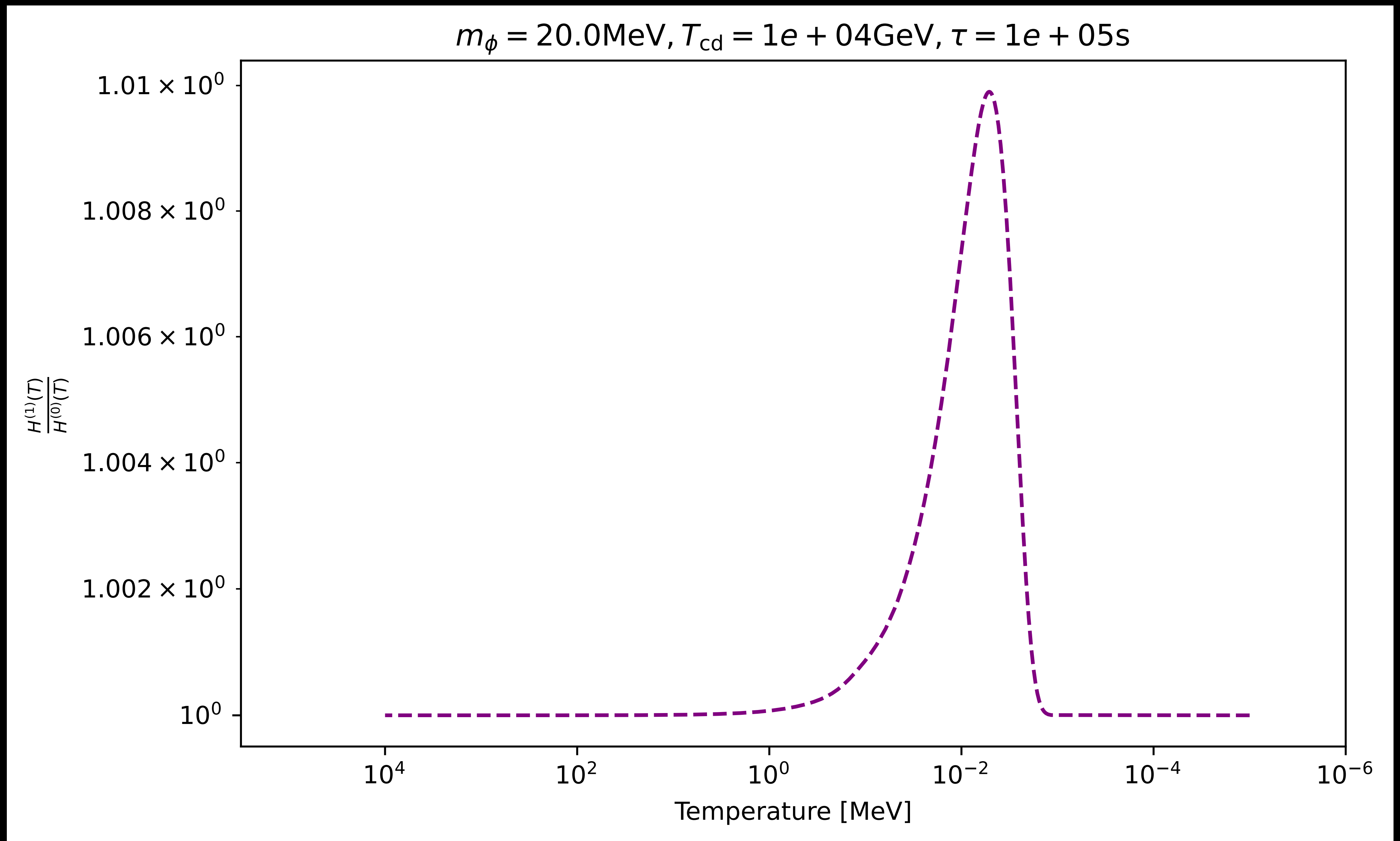
arXiv:1503.03513



Backup


$m_\phi = 20.0\text{MeV}, T_{\text{cd}} = 1e+04\text{GeV}, \tau = 1e+05\text{s}$
simpson
initial
Time (s)
Temperature [MeV]


$m_\phi = 20.0\text{MeV}, T_{\text{cd}} = 1e+04\text{GeV}, \tau = 1e+05\text{s}, T_{\text{nd}} = 1.400024\text{MeV}$
Temp SM
Temp Nu
Decoupling
T (MeV)
Temperature [MeV]


$m_\phi = 20.0\text{MeV}, T_{\text{cd}} = 1e+04\text{GeV}, \tau = 1e+05\text{s}$
$\frac{H^{(1)}(T)}{H^{(0)}(T)}$
Temperature [MeV]

Backup

Equation dump for radiation domination universe!

$$g_{\text{eff}}(T) = \left[\frac{h_{\text{eff}}(T)}{\sqrt{g_\star(T)}}, \left(1 + \frac{T}{3}, \frac{h'_{\text{eff}}(T)}{h_{\text{eff}}(T)}\right)\right]^2$$

$$H(T) = \sqrt{\frac{4\pi^3}{45}} \frac{h_{\text{eff}}(T)\,T^2}{\sqrt{g_*(T)}\,M_{\text{Pl}}} \left[1 + \frac{T}{3}\frac{h'_{\text{eff}}(T)}{h_{\text{eff}}(T)}\right]$$

$$\rho_{\text{SM}}(T) = \frac{\pi^2}{30}\, g_{\text{eff}}(T)\, T^4$$

$$p_{\text{SM}}(T) = \frac{1}{3}\,\rho_{\text{SM}}(T)$$

$$\frac{d\rho_{\text{SM}}}{dT} = \frac{\pi^2}{30}\left[g'_{\text{eff}}(T)\,T + 4\,g_{\text{eff}}(T)\right]T^3$$

$$\frac{dT}{dt} = \begin{cases} -3\,H(T)\,\dfrac{\rho_{\text{SM}}(T)+p_{\text{SM}}(T)}{\dfrac{d\rho_{\text{SM}}}{dT}} & T \geq T_{\nu,\text{dec}}, \\[2ex] -3\,H(T)\,\dfrac{\rho_{\text{EM}}(T)+p_{\text{EM}}(T)}{\dfrac{d\rho_{\text{EM}}}{dT}} & T < T_{\nu,\text{dec}}\,. \end{cases}$$

$$n_\gamma(T) = \frac{2\,\zeta(3)\,T^3}{\pi^2}$$

$$u_{\min} = \frac{m_e}{T}, \quad I_+(T) = \frac{8\cdot 30}{2\cdot 7\,\pi^4}\int_{u_{\min}}^{\infty} \frac{u^2\sqrt{u^2-u_{\min}^2}}{e^{u+\zeta_e(T)}+1}\,du, \quad J_+(T) = \frac{8\cdot 30}{2\cdot 7\,\pi^4}\int_{u_{\min}}^{\infty}\frac{(u^2-u_{\min}^2)^{3/2}}{e^{u+\zeta_e(T)}+1}\,du$$

$$\delta n = 10^{-10}\,n_\gamma(T), \quad \mu_e(T) = T\,\sinh^{-1}\left[\frac{\delta n}{2\,g_e}\left(\frac{2\pi}{m_e\,T}\right)^{3/2} e^{m_e/T}\right], \quad \zeta_e(T) = \frac{\mu_e(T)}{T}$$

$$\rho_{\text{EM}}(T) = \frac{\pi^2}{30}\left[2 + 4\cdot\frac{7}{8}\,I_+(T)\right]T^4$$

$$p_{\text{EM}}(T) = \frac{\pi^2}{90}\left[2 + 4\cdot\frac{7}{8}\,J_+(T)\right]T^4$$

$$\frac{d\rho_{\text{EM}}}{dT} = \frac{\pi^2}{30}T^3\cdot 4\left[\frac{7}{8}T\,\frac{dI_+}{dT} + 2 + 4\cdot\frac{7}{8}\,I_+(T)\right]$$

Backup

Equations dump for dark sector universe!

$$f_\phi(T_{cd}, p) = \left(\exp\frac{\sqrt{p_\phi^2 + m_\phi^2}}{\zeta_{cd} T_{cd}} - 1\right)^{-1} \qquad \beta_z = \sqrt{1 - 4m_z^2/m_\phi^2} \qquad \frac{1}{\tau_\phi} = \frac{\beta_z}{16\pi m_\phi}\left|\mathscr{M}_{\phi\to z\bar{z}}\right|^2$$

$$\frac{\partial f_\phi(T,p)}{\partial t} - H(t)\,p\frac{\partial f_\phi(T,p)}{\partial p} = \frac{1}{2E_\phi}\int\frac{d^3p_z}{(2\pi)^3 2E_z}\frac{d^3p_{\bar{z}}}{(2\pi)^3 2E_{\bar{z}}}\,|\mathscr{M}_{\phi\to z\bar{z}}|^2\,(2\pi)^4\delta^{(4)}(p - p_z - p_{\bar{z}})\Big[-f_\phi(1 \pm f_z)(1 \pm f_{\bar{z}}) + f_z f_{\bar{z}}(1 + f_\phi)\Big]$$

$$\frac{\partial f_\phi(T,p)}{\partial t} - H(T)\,p\,\frac{\partial f_\phi(T,p)}{\partial p} = -D_z^\pm(T,p)\left[f_\phi(T,p) - \bar{f}_\phi(T,p)\right] \qquad D_z^\pm(t,p) = \frac{m_\phi}{E_\phi\tau_\phi}\left[\frac{1}{\beta_z p}\ln\left(\frac{1 \mp \exp[-(E_\phi + \beta_z p)/(2T)]}{1 \mp \exp[-(E_\phi - \beta_z p)/(2T)]}\right)\right]_{T=T(t)}$$

$$f_\phi(t,p) = f_\phi\left(t_{\rm cd}, \frac{pR(t)}{R(t_{\rm cd})}\right)\exp\left(-\int_{t_{\rm cd}}^t dt'\,D_z^\pm\left(t', \frac{pR(t)}{R(t')}\right)\right) + \int_{t_{\rm cd}}^t dt'\,\bar{f}_\phi\left(t', \frac{pR(t)}{R(t')}\right)\exp\left(-\int_{t'}^t dt''\,D_z^\pm\left(t'', \frac{pR(t)}{R(t'')}\right)\right)$$

$$n_\phi(t) = g_\phi\int\frac{d^3p}{(2\pi)^3}\,f_\phi(t,p) \qquad H(t) = \sqrt{\frac{8\pi G}{3}\left[\rho_{\rm SM}(t) + \rho_\phi(t)\right]} \qquad \frac{dT}{dt} = -\frac{\dot{q}_\phi(t) + 3H(T)\big[\rho_{\rm SM}(T) + P_{\rm SM}(T)\big]}{d\rho_{\rm SM}(T)/dT}, (T \gtrsim T_{\nu d})$$

$$\rho_\phi(t) = g_\phi\int\frac{d^3p}{(2\pi)^3}\,E_\phi f_\phi(t,p) \qquad \dot{q}_\phi(t) = \dot{\rho}_\phi(t) + 3H(t)\big[\rho_\phi(t) + P_\phi(t)\big] \qquad \frac{dT}{dt} = -\frac{\dot{q}_\phi(t) + 4H(T)\rho_\gamma(T) + 3H(T)\big[\rho_{e^\pm}(T) + P_{e^\pm}(T)\big]}{d\rho_\gamma(T)/dT + d\rho_{e^\pm}(T)/dT}, (T \lesssim T_{\nu d})$$