Evolution of structure functions at NLO without PDFs

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National Centre for Nuclear Research

Theory Division seminar @ IFJ PAN 22.1.2026

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

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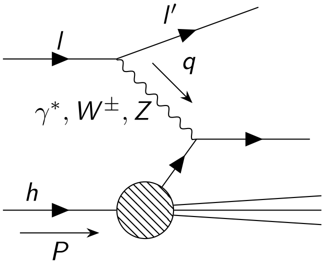
Deep Inelastic Scattering (DIS)

Cleanest environment to probe parton structure in nuclei



Kinematic variables

- Photon virtuality:

  $Q^2 = -q^2$

- Bjorken x:

  $x = \frac{Q^2}{2P\cdot q}$

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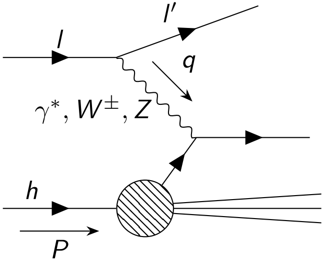

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$$x = \frac{Q^2}{2P\cdot q}$$

- Total cross sections expressed in terms of structure functions
- Collinear factorization:
  - Collinear partons carry fraction x of target's momentum (at infinite momentum frame)
  - Structure functions expressed in terms of parton distribution functions (PDFs)

$$F_i(x, Q^2) = \sum_j C_{ij}(Q^2, \mu^2) \otimes f_j(\mu^2)$$

$j = q, \bar{q}, g \qquad \mu =$ factorization scale

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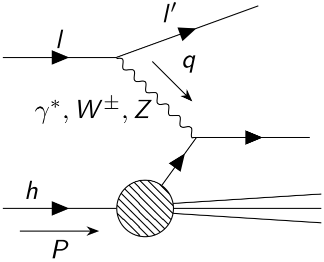

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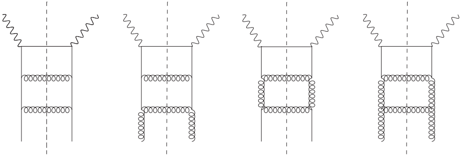
  - PDFs are fitted to DIS data (to structure functions)
  - Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution: PDFs to higher scales

DGLAP evolution

- Resums $\alpha_s \log\left(Q^2/\mu^2\right) \sim 1$ terms
- At LO: leading log (LL) level, only $\alpha_s^m \log^n(Q^2)$ with $n = m$ are considered, subleading terms with $n < m$ are neglected
  $\longrightarrow$ resumming ladder diagrams



$$\frac{\mathrm{d} f_i(\mu^2)}{\mathrm{d}\log(\mu^2)} = \sum_j \frac{\alpha_s}{2\pi} P_{ij} \otimes f_j(\mu^2) \quad i,j = q, \bar{q}, g$$

- Splitting functions P_{ij} perturbatively calculated $P_{ij}(z) = P_{ij}^{(0)}(z) + \frac{\alpha_s}{2\pi} P_{ij}^{(1)}(z) + \ldots$
- At LO scheme independent, at higher orders factorization scheme and scale dependent

DGLAP evolution

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$$i, j = q, \bar{q}, g$$

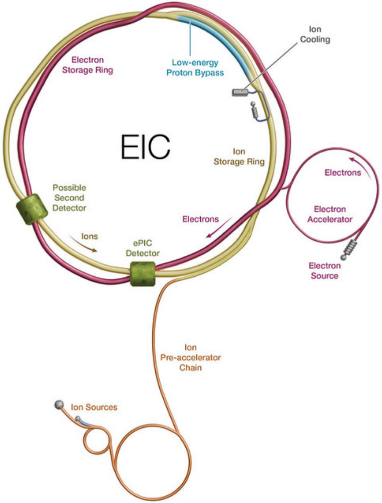
Usually one sets $\mu^2 = Q^2$

PDFs parametrizated at initial scale Q_0

Global analysis:

- Run DGLAP evolution
- Fit to data
- Tune parameters
- Repeat

DIS colliders



Only DIS collider experiment:

- HERA (Hadron-Elektron-Ringanlage) at DESY in Hamburg
- 1992-2007
- Lepton-proton collisions with center-of-mass energy 320 GeV

Near future:

- EIC (Electron-Ion Collider) at BNL in NY
- $\sim$ 2035
- Center-of-mass energy lower than at HERA (20 – 140 GeV)
- Advantages: nuclear targets, high luminosity, and polarized beams

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  - Avoiding the problems with PDFs
  - More straightforward to compare to experimental data

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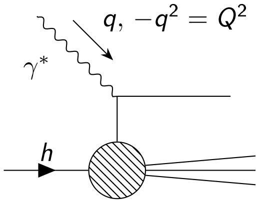
- The novelty of our work:
  - Momentum space
  - Full three-flavor basis at NLO
- NLO physical basis 2412.09589 continuation for LO work 2304.06998

Straightforward example with only two observables



$$F_i(x, Q^2) = \sum_j C_{F_i f_j}(Q^2, \mu^2) \otimes f_j(\mu^2),$$

where $F_i = F_2, F_{\mathrm{L}} / \frac{\alpha_{\mathrm{s}}}{2\pi}$, and $f_j = \Sigma, g$

Quark singlet:
$\Sigma(x, \mu^2) = \sum_q^{n_{\mathrm{f}}} \left[q(x, \mu^2) + \overline{q}(x, \mu^2) \right]$, $n_{\mathrm{f}} = 3$
Gluon PDF: $g(x, \mu^2)$

First step: invert the linear mapping (difficult because $f \otimes g = \int_x^1 \frac{\mathrm{d}z}{z} f(z) g\left(\frac{x}{z}\right)$)

$$f_j(\mu^2) = \sum_i C^{-1}_{F_i f_j}(Q^2, \mu^2) \otimes F_i(Q^2) + \mathcal{O}(\alpha_{\mathrm{s}}^2)$$

DGLAP evolution in physical basis

$$\frac{\mathrm{d} F_i(x, Q^2)}{\mathrm{d} \log(Q^2)} = \sum_j \frac{\mathrm{d} C_{F_i f_j}(Q^2, \mu^2)}{\mathrm{d} \log(Q^2)} \otimes f_j(\mu^2)$$

$$= \sum_j \frac{\mathrm{d} C_{F_i f_j}(Q^2, \mu^2)}{\mathrm{d} \log(Q^2)} \otimes \sum_k C^{-1}_{F_k f_j}(Q^2, \mu^2) \otimes F_k(Q^2) + \mathcal{O}(\alpha_{\mathrm{s}}^3)$$

Scheme and scale dependence at NLO

DGLAP evolution in physical basis:

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$$= \sum_k \mathcal{P}_{ik} \otimes F_k(Q^2) + \mathcal{O}(\alpha_\mathrm{s}^3)$$

Kernels $\mathcal{P}_{ik}$ are independent of the factorization scheme and scale

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$\mathcal{P}_{ij}$'s determined by:

- Splitting functions
- Coefficient functions
  $\longrightarrow$ The scheme and scale dependence exactly cancels between these two

Inverting the gluon PDF at NLO

Simple example without quarks

Invert $g(x)$ from $\widetilde{F}_{\mathrm{L}} = C^{(1)}_{F_{\mathrm{L}}g} \otimes g + \frac{\alpha_{\mathrm{s}}}{2\pi} C^{(2)}_{F_{\mathrm{L}}g} \otimes g$ where $\widetilde{F}_{\mathrm{L}}(x, Q^2) \equiv \frac{2\pi}{\alpha_{\mathrm{s}}} \frac{F_{\mathrm{L}}(x,Q^2)}{x}$

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Get $C^{(1)}_{F_{\mathrm{L}}g} \otimes g$ from $\widetilde{F}_{\mathrm{L}}$: $C^{(1)}_{F_{\mathrm{L}}g} \otimes g = \widetilde{F}_{\mathrm{L}} - \frac{\alpha_s}{2\pi} C^{(2)}_{F_{\mathrm{L}}g} \otimes g$

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$$g(x) = \hat{P}(x) \left[\widetilde{F}_{\mathrm{L}}(x) - \frac{\alpha_{\mathrm{s}}}{2\pi} C^{(2)}_{F_{\mathrm{L}}g} \otimes g \right]$$

Plug in $g(x) = \hat{P}(x) \widetilde{F}_{\mathrm{L}}(x) + \mathcal{O}(\alpha_{\mathrm{s}})$ to the right hand side

$$g(x) = \hat{P}(x) \widetilde{F}_{\mathrm{L}}(x) - \frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi} \hat{P}(x) \left[C^{(2)}_{F_{\mathrm{L}}g} \otimes \hat{P} \widetilde{F}_{\mathrm{L}} \right] + \mathcal{O}(\alpha_{\mathrm{s}}^2)$$

Six observable basis

- Full three-flavor basis: $u, \bar{u}, d, \bar{d}, s = \bar{s}$, and g
  $\longrightarrow$ Need six linearly independent DIS structure functions

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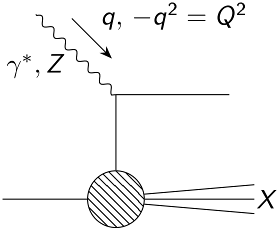
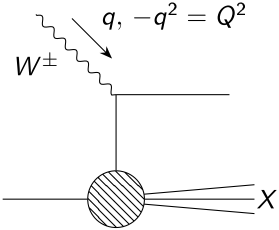
- We choose the NLO structure functions:





Neutral current γ^*, Z

- γ^* exhange $\rightarrow F_2$ and F_{L}
- Z boson exhange $\rightarrow F_3$

Charged current $W^\pm$

- W^- exhange $\rightarrow F_3^{\mathrm{W}^-}$ and $F_{2\mathrm{c}}^{\mathrm{W}^-}$
- $\Delta F_2^W = F_2^{\mathrm{W}^-} - F_2^{W^+}$

Comparison with conventional DGLAP evolution

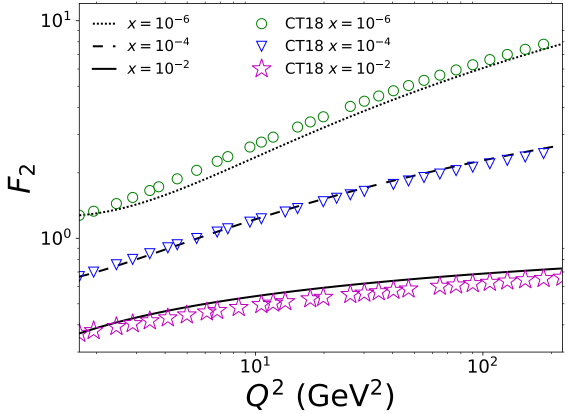
Physical basis evolution

- Renormalization scheme in $\alpha_s(\mu_r^2)$
- Perturbative truncation
  $\longrightarrow$ sum rule not exact
- Parametization of observable quantities

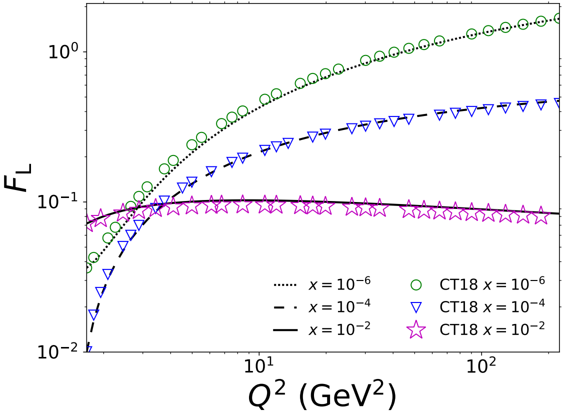
Evolution with PDFs

- Factorization scheme and scale
- Renormalization scheme in $\alpha_s(\mu_r^2)$
- Easy to enforce an exact sum rule
- Parametization of non-observable quantities

Comparison with conventional DGLAP evolution



F_2



F_{L}

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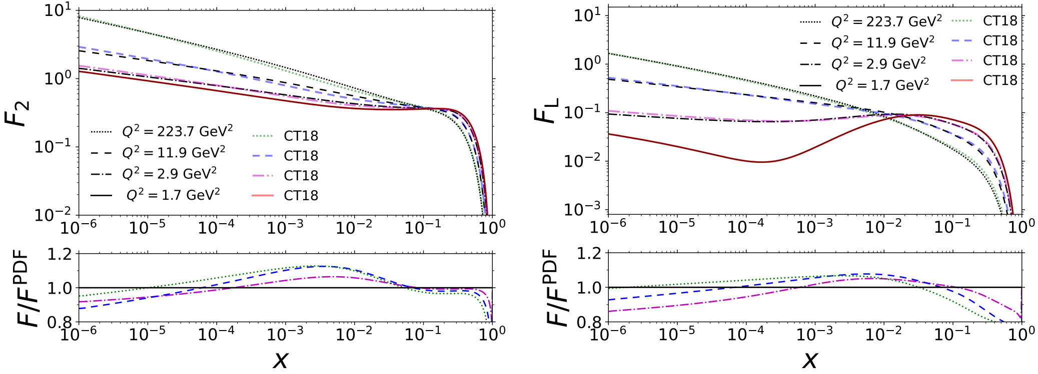
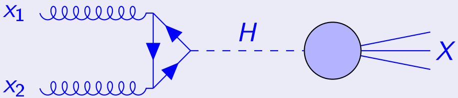


F_2/F_2^{CT18} $F_{\mathrm{L}}/F_{\mathrm{L}}^{\mathrm{CT18}}$

- Similar Q^2 evolution
- Differences in values from:
  - uncertainty in PDFs from scheme and scale (error band not shown)
  - perturbative truncation

Cross sections in terms of physical basis

Example of Higgs production by gluon fusion



$$\sigma(p+p \longrightarrow \mathrm{H}+X) = \int \mathrm{d}x_1 \mathrm{d}x_2 g(x_1,\mu) g(x_2,\mu) \hat{\sigma}_{gg\to H+X}(x_1, x_2, \frac{m_H^2}{\mu^2}),$$

where m_H is the Higgs mass, $g(x_1,\mu)$ and $g(x_2,\mu)$ are the gluon PDFs

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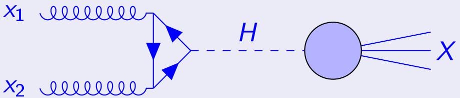

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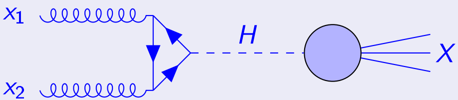

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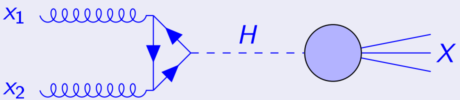

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Harland-Lang and Thorne 1811.08434:

explicit μ dependence vanishes and terms $\log\left(Q^2/m_H^2\right)$ are left behind

$\longrightarrow$ no need to choose relation between μ and Q or m_H

Cross sections in pysical basis

Cancellation of factorization scheme and scale requires separating between the terms corresponding to different perturbative orders

$$\sigma = \sum_{ij} f_i \otimes \sigma_{ij} \otimes f_j$$
$$= \sum_{ij} \Big\{ f_i^{(0)} \otimes \sigma_{ij}^{(0)} \otimes f_j^{(0)}$$
$$+ \alpha_s \left[f_i^{(0)} \otimes \sigma_{ij}^{(1)} \otimes f_j^{(0)} + f_i^{(1)} \otimes \sigma_{ij}^{(0)} \otimes f_j^{(0)} + f_i^{(0)} \otimes \sigma_{ij}^{(0)} \otimes f_j^{(1)} \right] \Big\} + \mathcal{O}(\alpha_s^2)\,,$$

where $f_i^{(0)}$ and $f_i^{(1)}$ denote the corresponding LO and NLO physical-basis counterparts for the PDFs

Establish "PDF sets" of physical-basis counterparts for PDFs at LO and NLO

$\longrightarrow$ use existing codes to calculate LHC cross sections in physical basis

Including final-state heavy flavours

$$F_i(x, Q^2) = \sum_j \left[C_{ij}\left(\frac{\mu^2}{Q^2}\right) + C_{ij}\left(\frac{m_q^2}{Q^2}, \frac{\mu^2}{Q^2}\right)\right] \otimes f_j(\mu^2)$$

- final-state particles on mass shell $\rightarrow$ need to change the momentum fraction

  Rescaling variable: $\chi = x\left(1 + \frac{m_q^2}{Q^2}\right)$

Treatment of this scaling depends on the chosen heavy-flavour scheme. E.g. in S-ACOT scheme

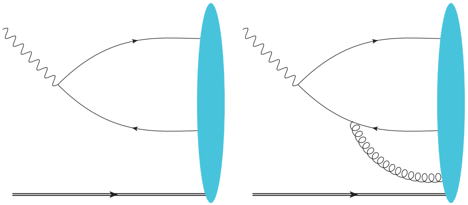
$$C \otimes f: \quad \int_x^1 \frac{\mathrm{d}z}{z} C(z) f\left(\frac{x}{z}\right) \longrightarrow \int_\chi^1 \frac{\mathrm{d}z}{z} C(z) f\left(\frac{\chi}{z}\right)$$

- $C^{(1)}_{F_{\mathrm{L}}g}(m_q^2/Q^2)$ is more complicated than in the massless case
  $\longrightarrow$ Can't invert gluon PDF the same way

DIS in gluon saturation regime

- Collinear framework does not include mechanism for taming down the gluon PDF growth at small x
- Gluon saturation: At small x gluon recombination becomes substantial
  $\longrightarrow$ gluons saturate
- Colour Glass Condensate (CGC):
  - classical colour field
  - non-linear evolution

DIS in gluon saturation regime



LO dipole picture   NLO correction

DIS in dipole picture:

- classical target seen as a "shock wave" at rest
- large photon plus light-cone momentum

$$\sigma_{T,L}^{\gamma^* A} = 2 \sum_f \int \mathrm{d}^2\mathrm{b}\, \mathrm{d}^2\mathrm{r}\, \mathrm{d}z \left|\Psi^{\gamma^* \to q\bar{q}}(\mathrm{r}, z, Q^2)\right|^2 N(\mathrm{b}, \mathrm{r}, x).$$

N = dipole amplitude
$\Psi^{\gamma^* \to q\bar{q}}$ =photon light front wave function
r = dipole transverse size, z = fraction of the photon plus momentum the quark carries

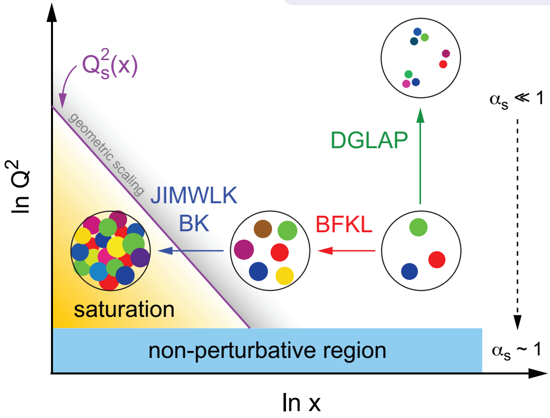
BK vs DGLAP evolution

DGLAP

- large Q^2, moderate x
- $\alpha_s \log\left(Q^2/\mu^2\right) \sim 1$
- linear evolution

BK

- moderate Q^2, small x
- $\alpha_s \log(1/x) \sim 1$
- non-linear evolution from gluon recombination



BK vs DGLAP evolution

- One of goals in EIC is to search for gluon saturation
  - saturation scale: $Q_s^2 \sim A^{1/3} x^{-\lambda} \rightarrow$ saturation effects stronger in nuclei

- To see saturation effects on experimental data we have to distinguish the genuine difference between DGLAP and BK dynamics

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- To see saturation effects on experimental data we have to distinguish the genuine difference between DGLAP and BK dynamics
- Both frameworks require input which are fitted to the same experimental data
  $\longrightarrow$ The results do not deviate dramatically and distinguishing DGLAP/BK dynamics is difficult

Comparison to BK-evolved $F_{2,\mathrm{L}}$ (work in preparation)

- Need to be as independent as possible from the initial condition parametrization

- "Force" collinear factorization and CGC $F_{2,\mathrm{L}}$ to agree in a line in (x, Q^2) plane

- Differences between the two frameworks outside the chosen line quantify signatures of gluon saturation

- **With differences we can quantify the precision needed at EIC and LHeC/FCC-he to distinguish saturation effects**

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- Matching in a common region of validity for both frameworks:
  - In a region $Q^2 \gg Q_s^2$ where saturation effects are moderate
  - With small enough $\alpha_s \log(Q^2)$ so that DGLAP evolution dynamics is reliable
  - Also, $\alpha_s \log(Q^2)$ can not be so large that DGLAP evolution would dominate

Comparison to BK-evolved $F_{2,\mathrm{L}}$ (work in preparation)

Goal

Set BK-evolved F_2 and F_L as initial condition for (2-observable) physical basis DGLAP evolution
$\longrightarrow$ compare BK vs. DGLAP dynamics

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    $P_{qq} \otimes F_2 = \int_x^1 \frac{\mathrm{d}z}{z} P_{qq}(z) F_2\left(\frac{x}{z}\right)$
    $\longrightarrow$ need $F_{2,\mathrm{L}}$ initial values up to $x = 1$
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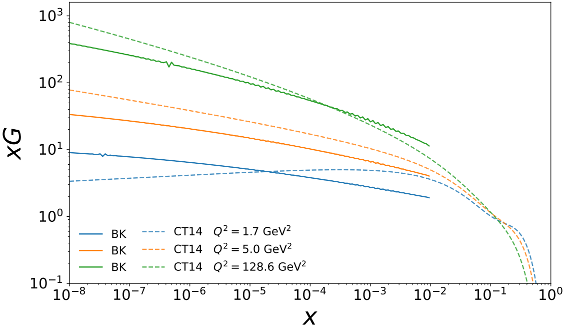
BK-improved initial condition:

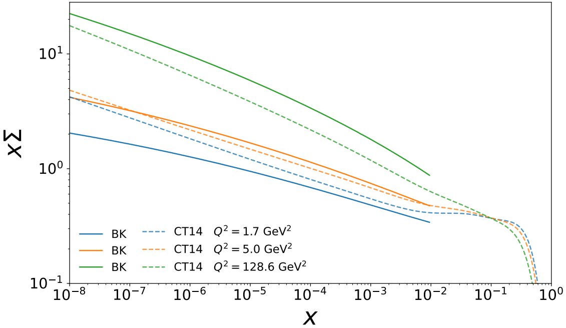
- Initial values for $F_{2,\mathrm{L}}$:
  - at $x \leq 10^{-2}$ from BK/dipole picture
  - at $x > 10^{-2}$ from DGLAP/collinear factorization
- Match $F_{2,\mathrm{L}}$ at the threshold

PDFs from BK-evolved structure functions

Now we have analytical form to calculate gluon PDF and quark singlet from F_2 and F_L in dipole picture



LO gluon



LO quark singlet

- Weaker x-evolution with BK-evolved $F_{2,L}$
- Bigger difference in gluon than in quark singlet

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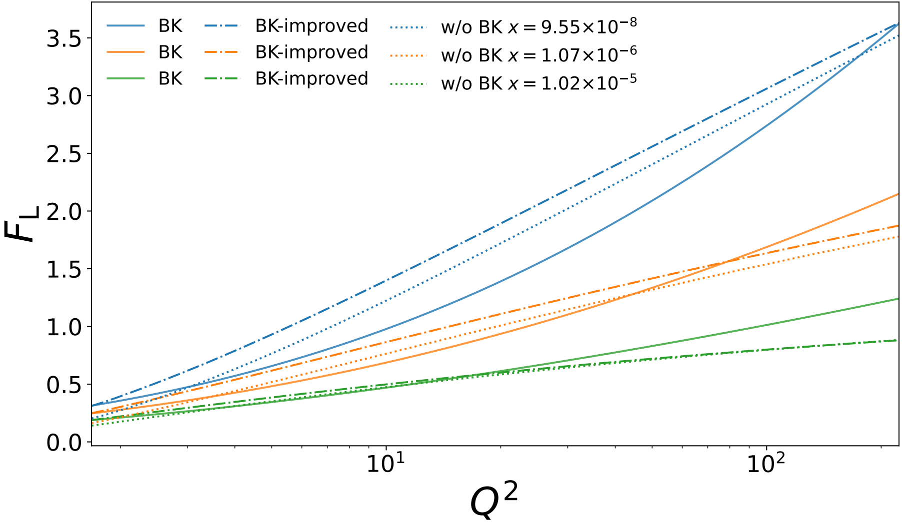
  $\longrightarrow$ Scheme and scale dependence avoided in the physical basis
- What next:
  - BK vs. DGLAP comparison
  - Express LHC cross sections, e.g. Drell-Yan, in physical basis
  - Include heavy quarks

Backup: Comparison to BK-evolved F_L (work in preparation)



Backup: Comparison to BK-evolved F_2 (work in preparation)