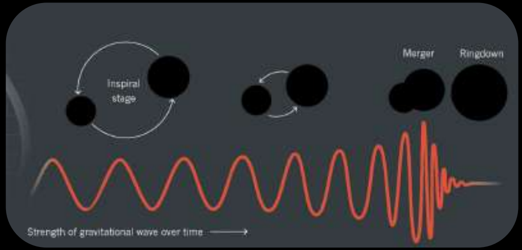
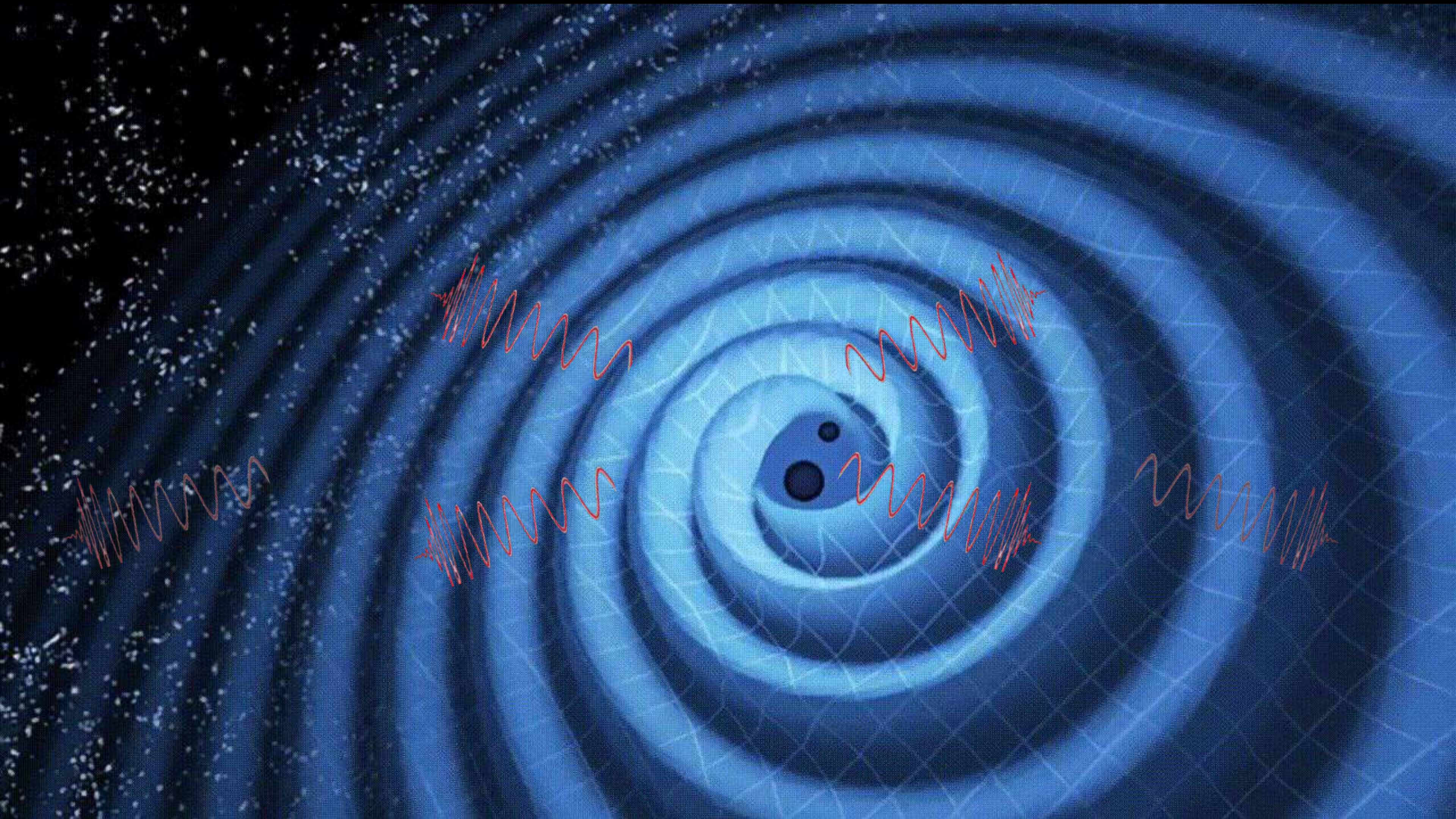
Gravitational Waves: Principles, Detection, and Recent Discoveries

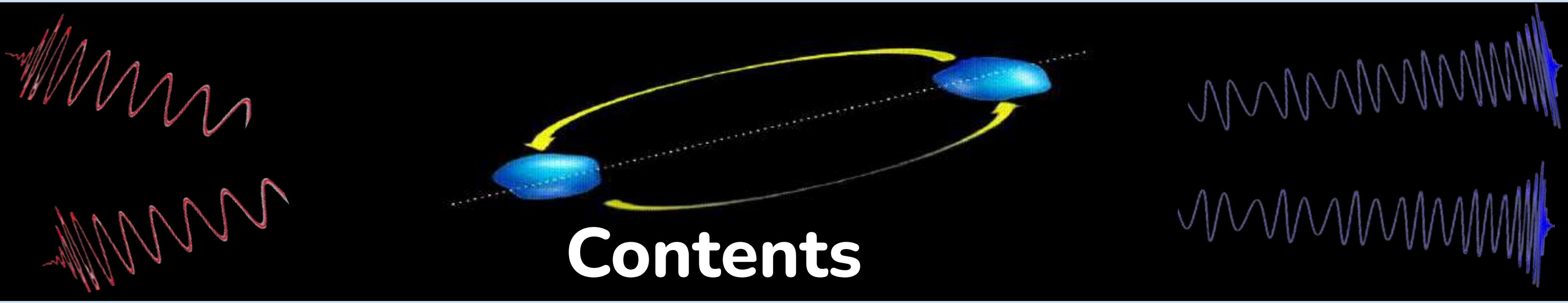
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



$$h = A e^{i\Psi}$$

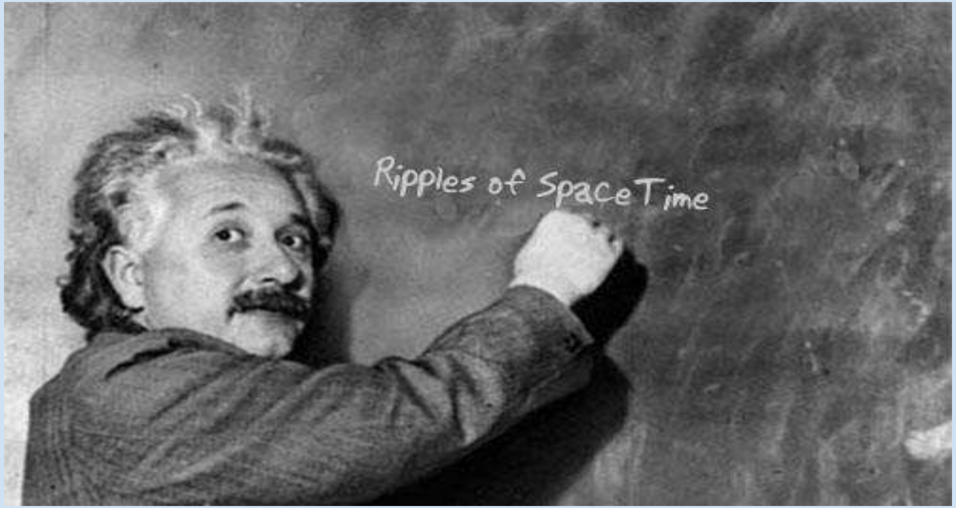
Syed Naqvi, Post-Doc, NZ-15
Einstein Telescope member (David E. Alvarez-Castillo group)



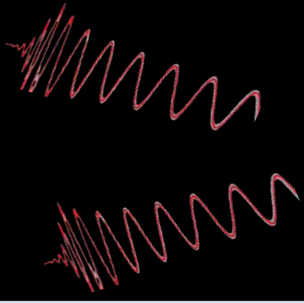


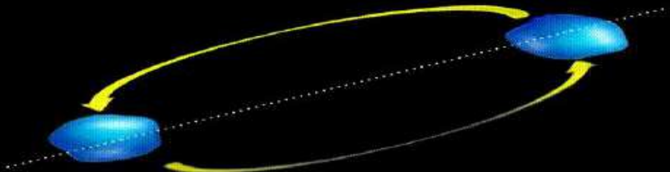
Contents:

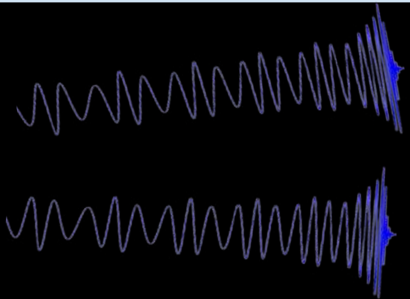
- **Einstein’s journey to the famous quadrupole formula**



$$h \sim \ddot{Q}$$



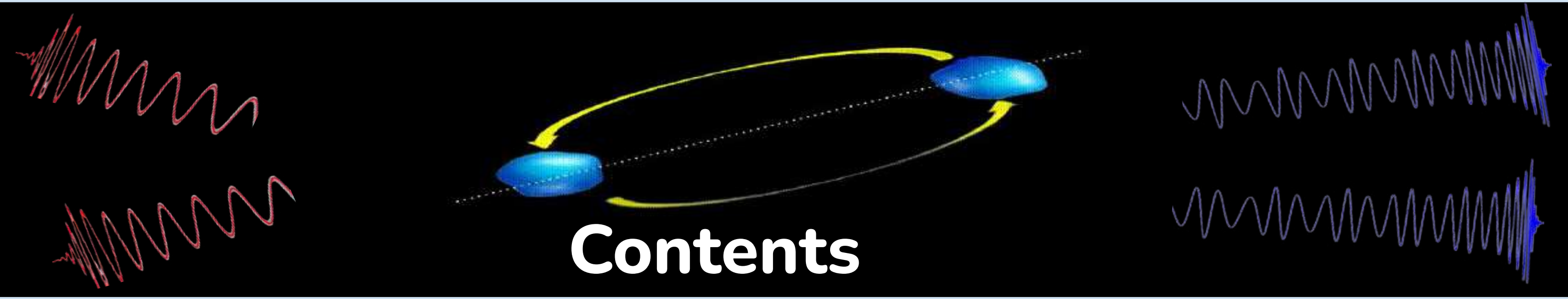




Contents
:

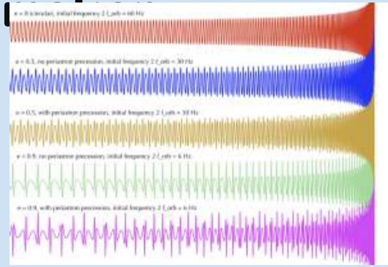
- **Einstein’s journey to the famous quadrupole formula**

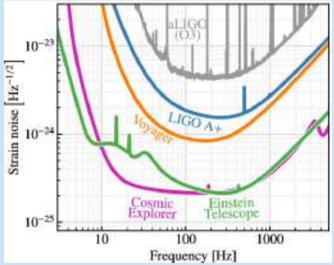
- **Weber bar’s to current Interferometric detectors**

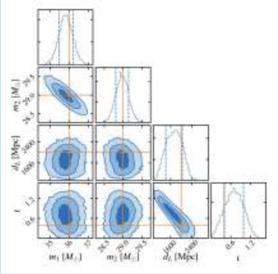


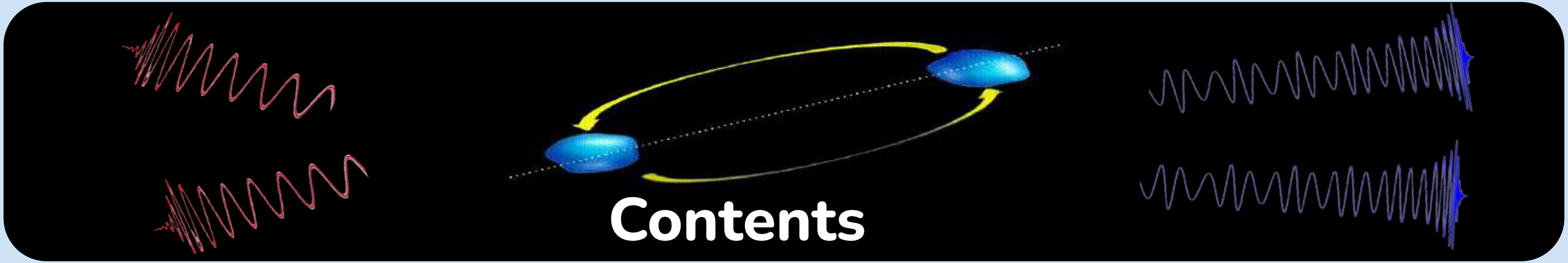
Contents
:

- **Einstein's journey to the famous quadrupole formula**
- **Weber bar's to current Interferometric detectors**
- **Gravitational Waves: Template banks + Parameter Estimation**







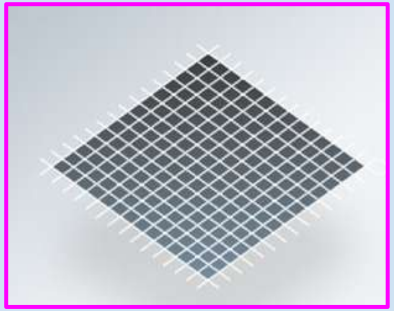


Contents :

- **Einstein’s journey to the famous quadrupole formula**
- **Weber bar’s to current Interferometric detectors**
- **Gravitational Waves: Template banks + Parameter Estimation**
- **Recent results : O3 i O4 Catalogue**

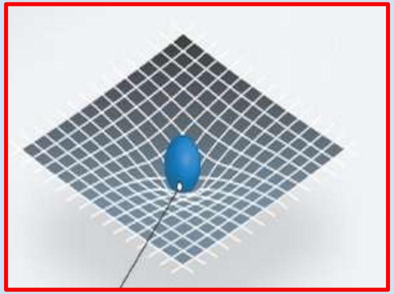
From Special Relativity to General Relativity

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$



Space + Time
=>
Spacetime

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$



Gravity=>
Bending of
spacetime

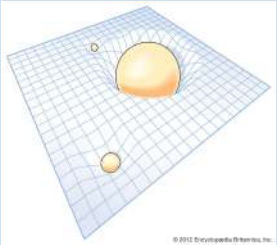
Einstein Field Equations : Meme Version

- Non-linear nature, many types of spacetimes(compact objects, universe expanding etc . . .)

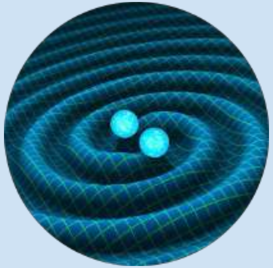
Different spacetimes represent different settings...

$$\boxed{ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu}$$

Pierogi Ruskie: Spacetimes of heavy objects Black Holes, NS etc....

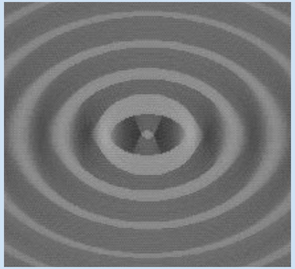


Pierogi ze szpinakiem i serem feta: Spacetimes having radiation/Grav Waves from astrophysical sources

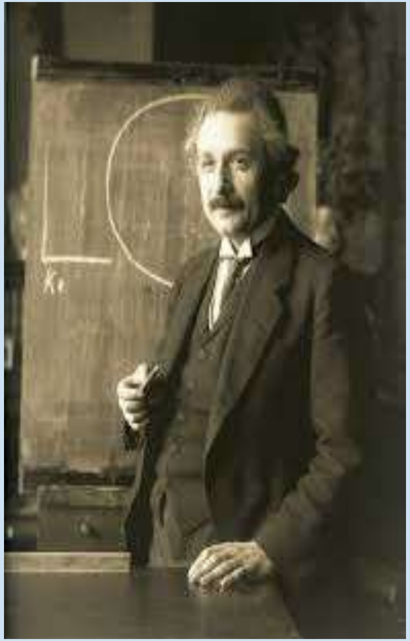


⋮

Indyjskie pierogi: Spacetimes having Standing Grav. Waves



From doubt...



Einstein

Rosen

- ❖ 1905 : Henri Poincare
- ❖ 1915-16: Einstein linearized gravity
- ❖ Issues :
  > Plane GWs in full theory?

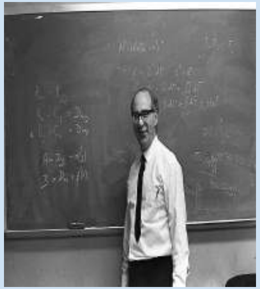
  > Do full Einstein Equation have solution which can be interpreted a GW?

  > Do GWs carry energy?
  >......
- ❖ 1937 : Einstein- Rosen metric, exact solution but with singularities

To belief...

Bondi

Pirani



Robinson

Trautmann

❖ **1959, Bondi-Pirani-Robinson :**

> the plane wave in the full theory is defined ✓

> found as a class of solutions of Einstein fields equations ✓

> they carry energy in a form of a sandwich wave which affects test particles ✓

❖ **1958, Andrzej Trautman:**

> Radiation is nonlocal

> defining GW in full Einstein theory = boundary conditions at infinity

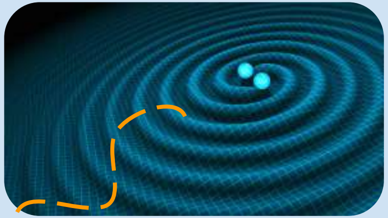
Credit: Green light for GW search, Hill & Nurowksi

GWs = Ripples of spacetime = Perturbations

Einstein Field Equations in LINEARIZED form : *weak field*

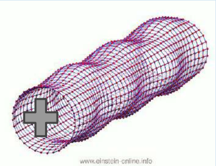
$$||h_{ab}|| << 1$$

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu} = \cancel{\frac{8\pi G}{c^4}}\, T_{\mu\nu}$$



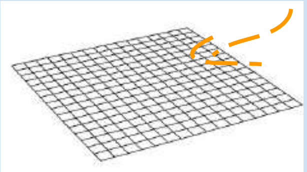
Metric Perturbations

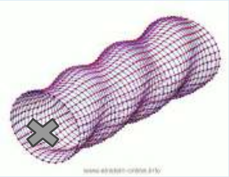
$$g_{ab} = \eta_{ab} + h_{ab}$$



Wave Equation
(TT gauge)

$$\Box \bar{h}_{ab} = 0$$





Hartle Book: Rough GW and EM analogy

	Linearized Gravitation	Electromagnetism
Basic "potentials"	Linearized metric perturbation $h_{\alpha\beta}(x)$	Vector and scalar potentials $(\Phi(t,\vec{x}),\ \vec{A}(t,\vec{x}))$
Field quantities	Linearized Riemann curvature $\delta R_{\alpha\beta\gamma\delta}(x)$	Electric and magnetic fields $\vec{E}(t,\vec{x}),\ \vec{B}(t,\vec{x})$
Gauge transformation leading to new potentials but the same fields	$h_{\alpha\beta} \to h_{\alpha\beta} - \partial_\alpha \xi_\beta - \partial_\beta \xi_\alpha$	$\vec{A} \to \vec{A} + \nabla\Lambda$ $\Phi \to \Phi - \partial\Lambda/\partial t$
Example of a gauge condition	Lorentz gauge $\partial_\beta h_\alpha^\beta - \frac{1}{2}\partial_\alpha h_\beta^\beta = 0$	Lorentz condition $\vec{\nabla}\cdot\vec{A} + \partial\Phi/\partial t = 0$
Field equations simplified by the gauge condition	$\Box h_{\alpha\beta} = 0$	Maxwell's equations $\Box\vec{A} = 0$ $\Box\Phi = 0$

Generation of G.Ws = Quadrupole radiation

Including matter source term

$$\Box \bar{h}_{ab} = -16\pi T_{ab} \quad \Longrightarrow \quad \bar{h}_{ab}(t, \mathbf{x}) = 4 \int d^3x' \frac{T_{ab}\left(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}'\right)}{|\mathbf{x} - \mathbf{x}'|}$$

Quadrupole Formula
(in units c,G)

$$\bar{h}^{ij}(t, r) = \frac{2}{r}\frac{G}{c^4}\frac{d^2}{dt^2} Q^{ij}\left(t - \frac{r}{c}\right)$$

Reduced Quadrupole Moment

$$Q^{jk} = I^{jk} - \frac{1}{3}\delta^{jk}\delta_{lm}I^{lm}$$

$$I^{jk} = \int d^3x\, \rho(t, \vec{x}) x^j x^k .$$

Moment of Inertia tensor

$$I_p = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & I_{-yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

Generation of G.Ws = Quadrupole radiation

Including matter source term

$$\Box \bar{h}_{ab} = -16\pi T_{ab} \Longrightarrow \bar{h}_{ab}(t, \mathbf{x}) = 4 \int d^3x' \frac{T_{ab}\left(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}'\right)}{|\mathbf{x} - \mathbf{x}'|}$$

Quadrupole Formula
(in units c,G)

$$\bar{h}^{ij}(t, r) = \frac{2}{r}\frac{G}{c^4}\frac{d^2}{dt^2}Q^{ij}\left(t - \frac{r}{c}\right)$$

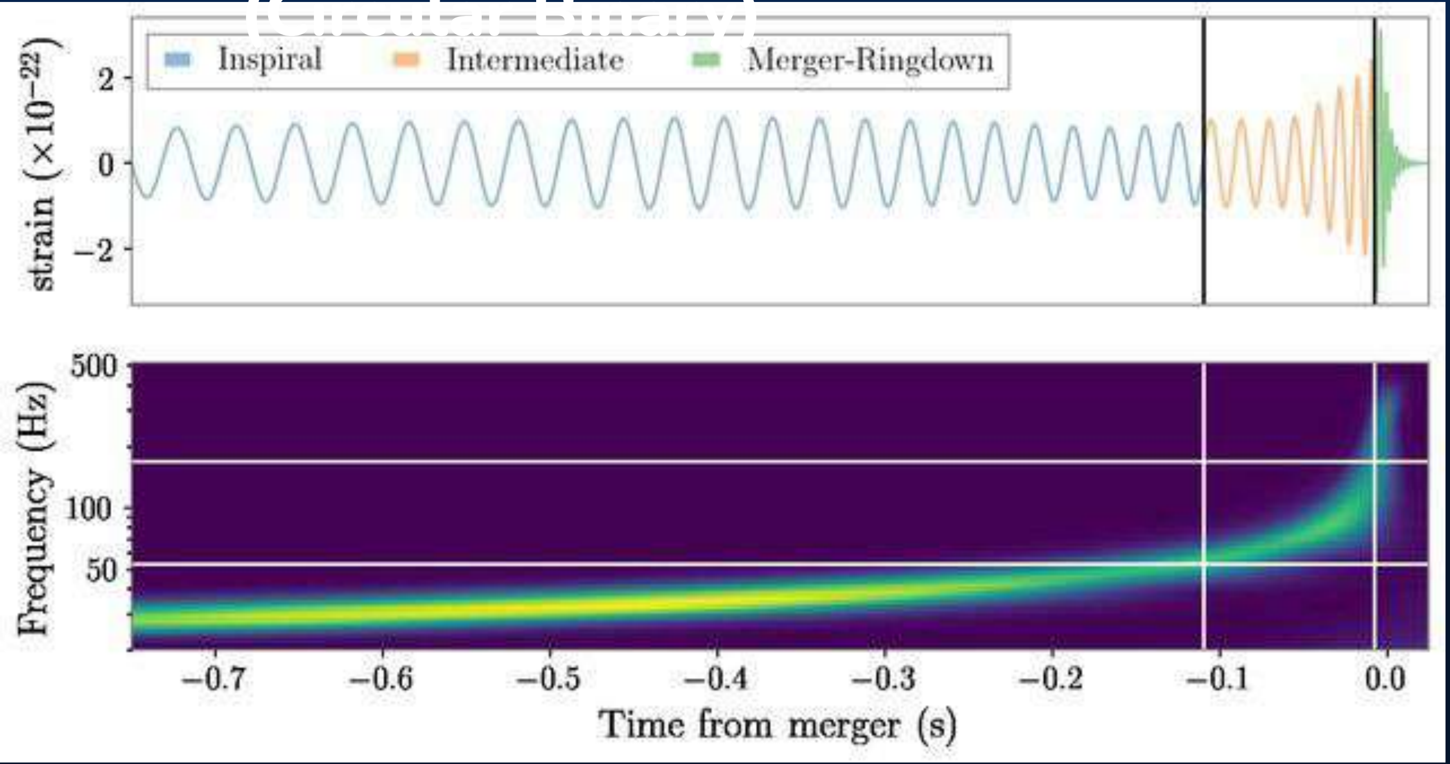
GW Luminosity

$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5}\boxed{\frac{G}{c^5}}\langle \frac{\partial^3 Q_{ij}}{\partial t^3}\frac{\partial^3 Q_{ij}}{\partial t^3}\rangle$$

Energy carried away by GWs per second ~ Peak luminosity @ BH merger : **10^{50} watts!!!**

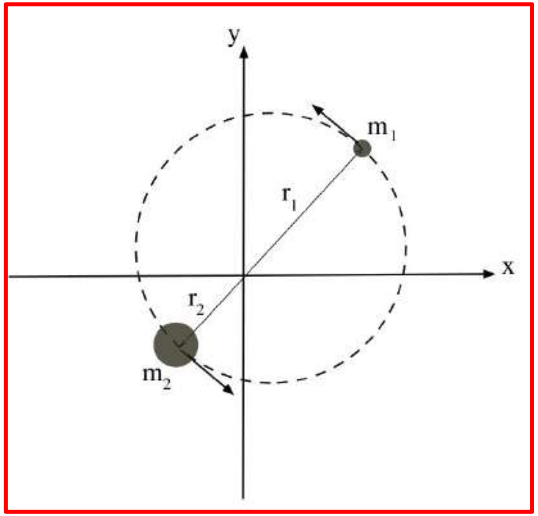
Gravitational Waveform: Time Domain vs Freq Domain

(Circular Binary)



“Chirp/ćwierkanie” Signal

GWs from binary system in Circular orbit

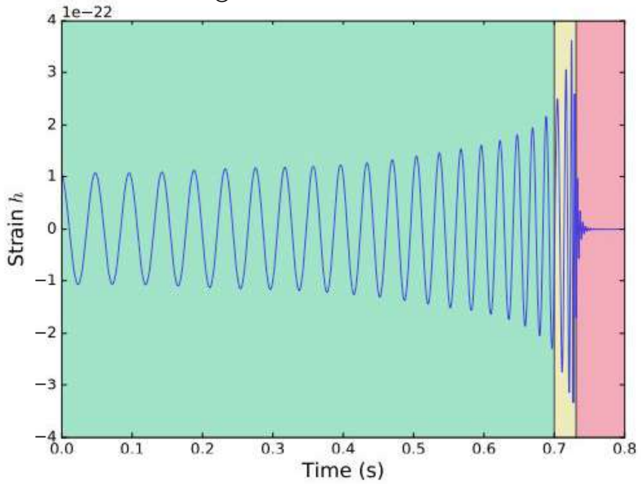


Radiated power:
Evaluated from Reduced Quadrupole Moment

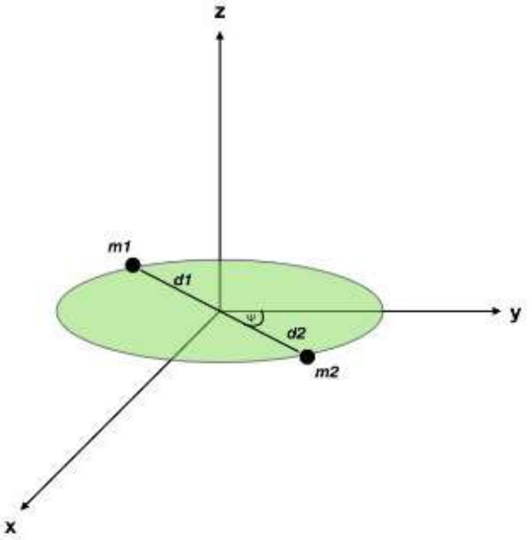
$$< E_{GW} > \approx < \dot{h}_{+}^{2} + \dot{h}_{\times}^{2} >$$

$$< E_{GW} > = \frac{32}{5}\frac{G^4}{c^5}\frac{m_1^2 m_2^2 (m_1 + m_2)}{a}$$

35-30 $M_\odot$: quasi circular case



GWs from binary system in Eccentric orbit

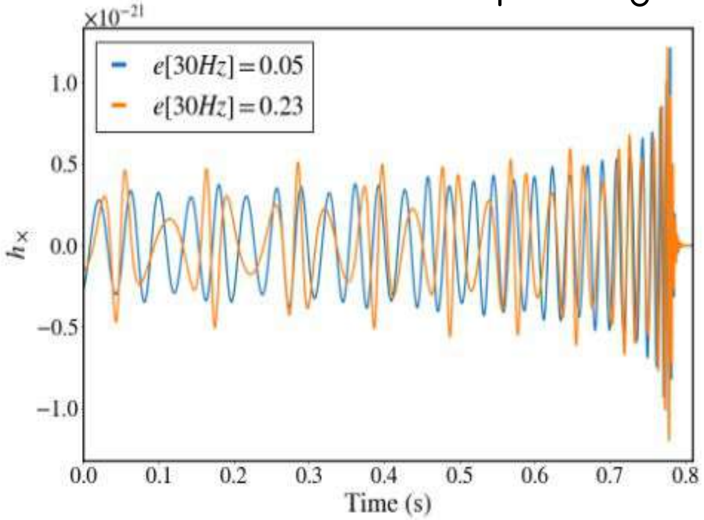


Radiated power:
Evaluated from Reduced Quadrupole Moment

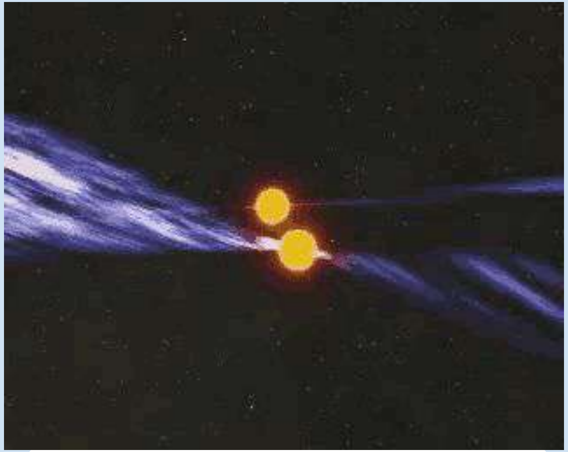
$$< E_{GW} > \approx < \dot{h}_+^2 + \dot{h}_\times^2 >$$

$$< E_{GW} > = \frac{32}{5} \frac{G^4}{c^5} \frac{m_1^2 m_2^2 (m_1 + m_2)}{a} f(e)$$

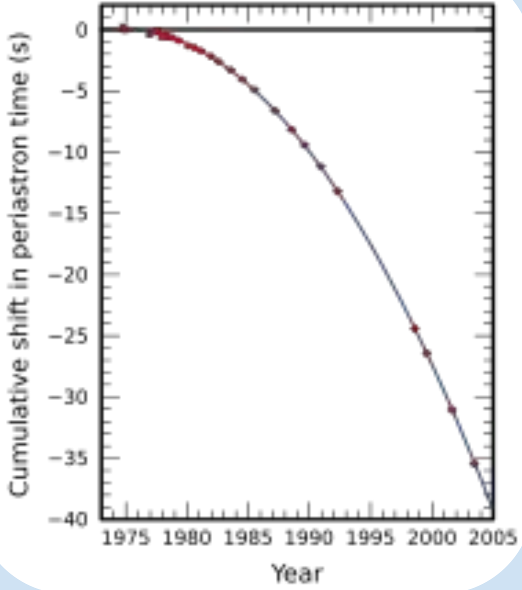
Non-spinning, eccentric, M_T = 50$M_\odot$



First Indirect GW detection:
Hulse–Taylor binary pulsar (PSR 1913+16)



- Discovered in 1974; binary neutron-star system: one star is a millisecond pulsar
- Orbital period: ~7.75 hours
- Eccentric orbit → strong relativistic effects
- Provided the first evidence that gravitational waves carry energy



PSR 1913+16

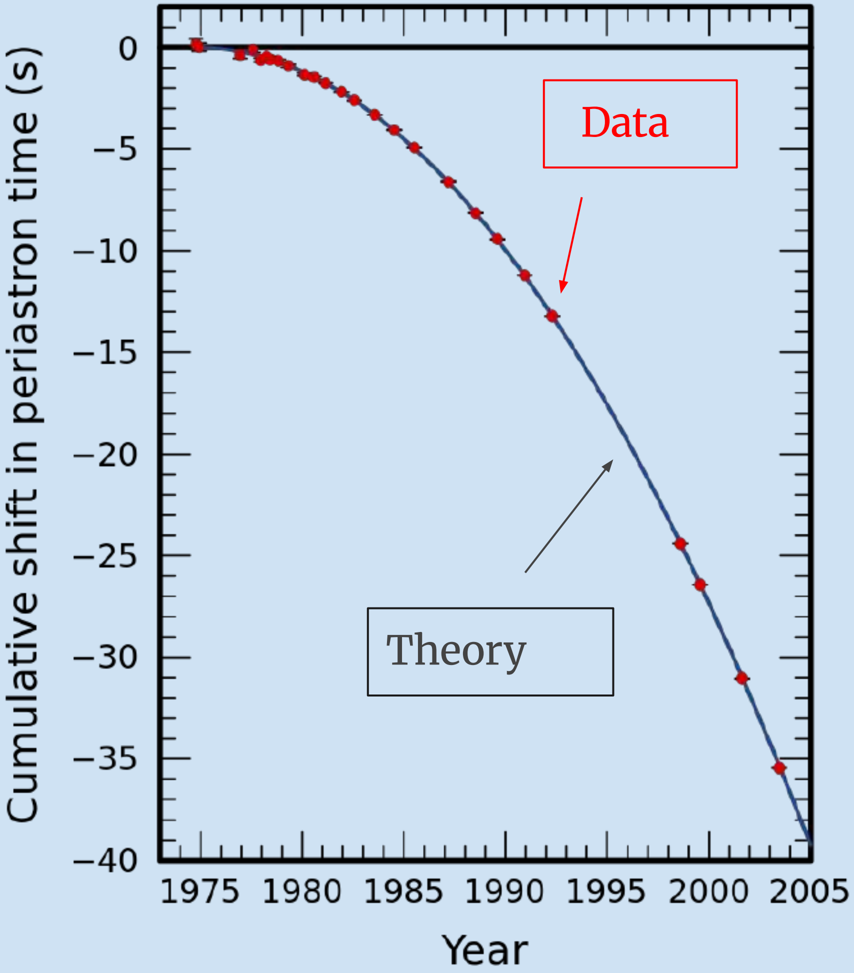
$$l_0 \sim 10^{11} cm$$

$$\nu_{GW} \sim 10^5 Hz$$

$$\lambda_{GW} = \frac{c}{\nu_{GM}} \sim 10^{14} cm$$

$$(\lambda_{GW} >> l_0)$$

(slow-motion approximation,
on which the quadrupole formalism is based)



Hulse-Taylor binary pulsar
e_o ~ 0.617 and $f(e_o)$ ~ 0.184,

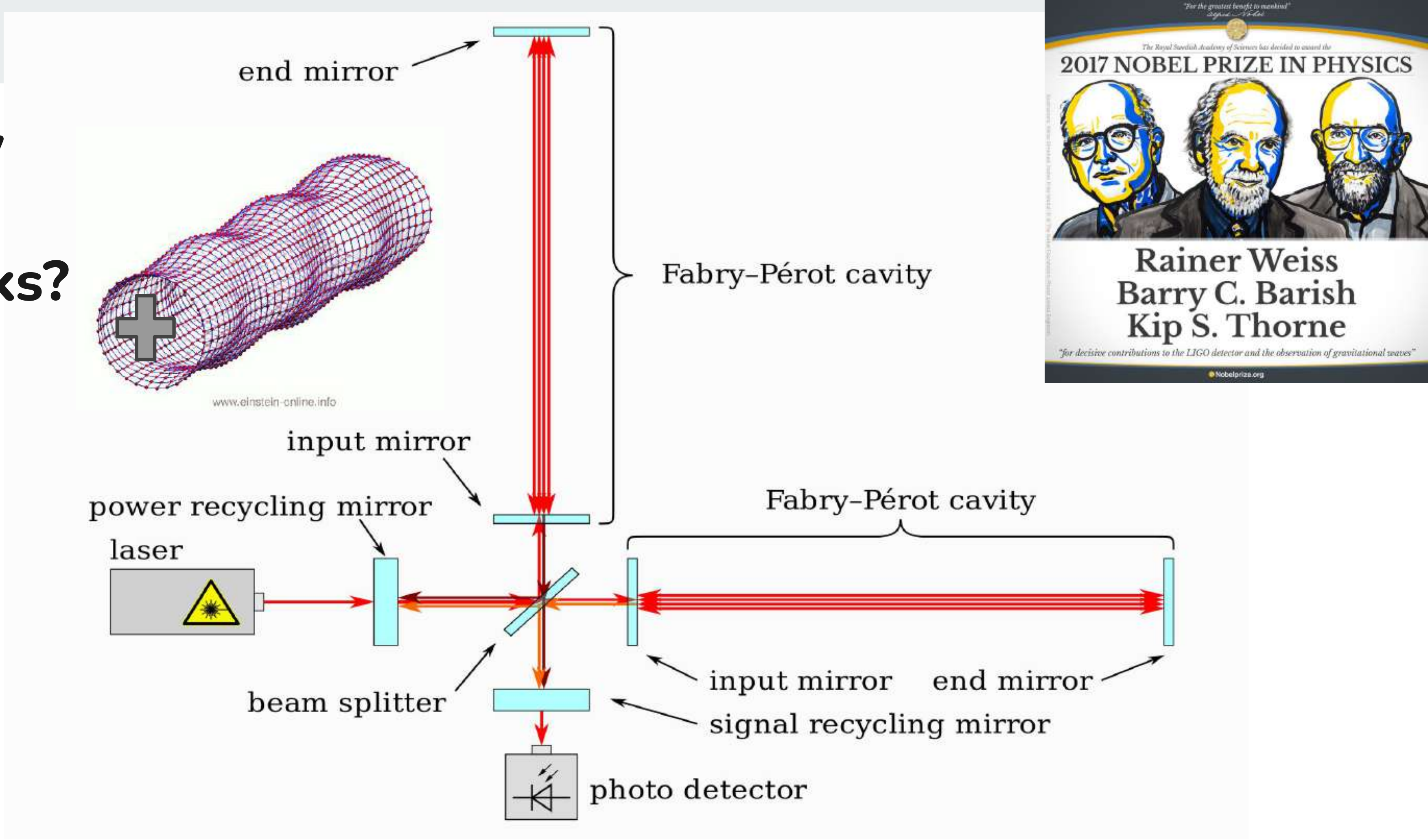
Time to coalescence
shorter by a factor $l/f(e_o)$ ~ 5.4, compared
to a binary on a circular
orbit with the same period.

How to detect them? Past

-Joseph Weber and his resonant bars in 1960s

-By the early 2000s, a set of initial detectors was completed, including TAMA 300 in Japan, GEO 600 in Germany, the Laser Interferometer Gravitational-Wave Observatory (LIGO) in the United States, and Virgo in Italy.

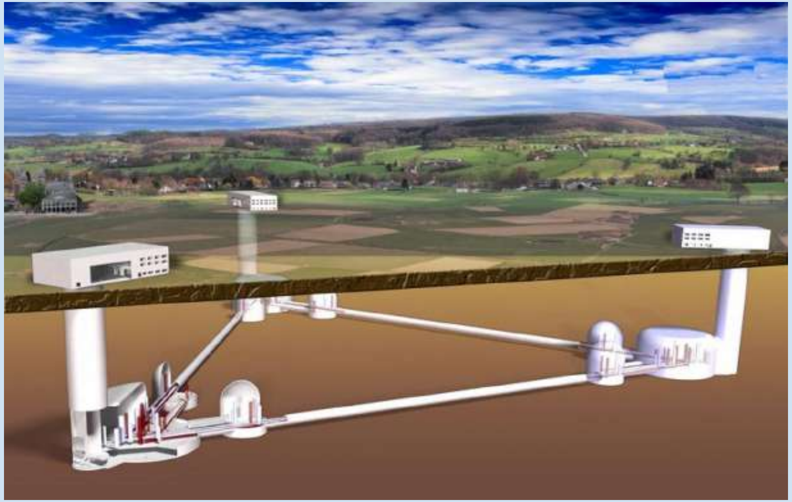
How It works?

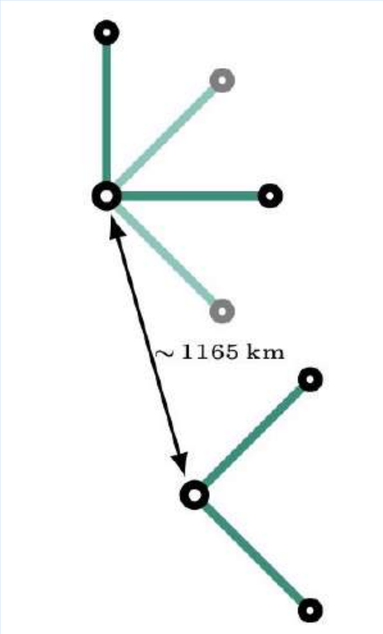


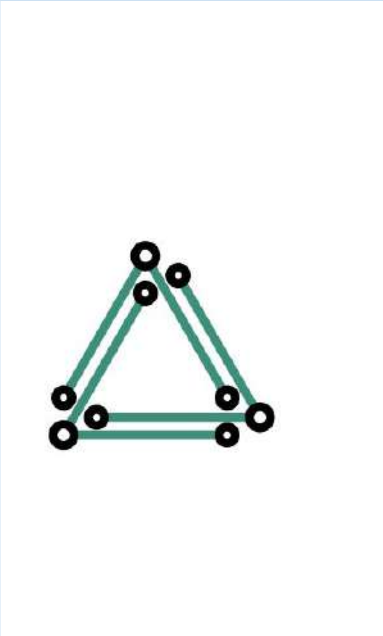
end mirror
Fabry–Pérot cavity
www.einstein-online.info
input mirror
power recycling mirror
laser
Fabry–Pérot cavity
beam splitter
input mirror
end mirror
signal recycling mirror
photo detector
"For the greatest benefit to mankind"
The Royal Swedish Academy of Sciences has decided to award the
2017 NOBEL PRIZE IN PHYSICS
Rainer Weiss
Barry C. Barish
Kip S. Thorne
"for decisive contributions to the LIGO detector and the observation of gravitational waves"
Nobelprize.org

How to detect them? Future   Ground Based

Einstein Telescope (~2035)   2L Config.   △ Config.





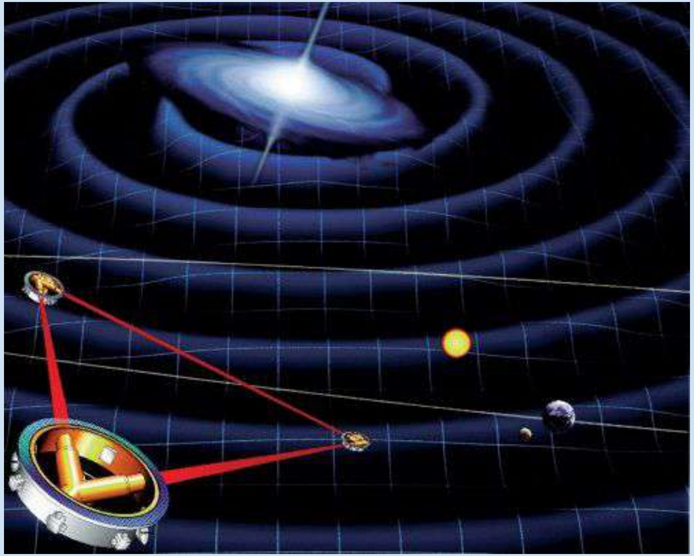


2 proposed config: L-shape(15km) vs Triangle(10km)

Lower frequencies (~1 Hz) → much earlier inspiral information

Three candidate sites: i) Italy (Sardina) ii) Meuse-Rhine Euroregion(Maastricht, Liège and Aachen)
iii) Germany (Saxony)

How to detect them? Future Space based



USA: LISA ;

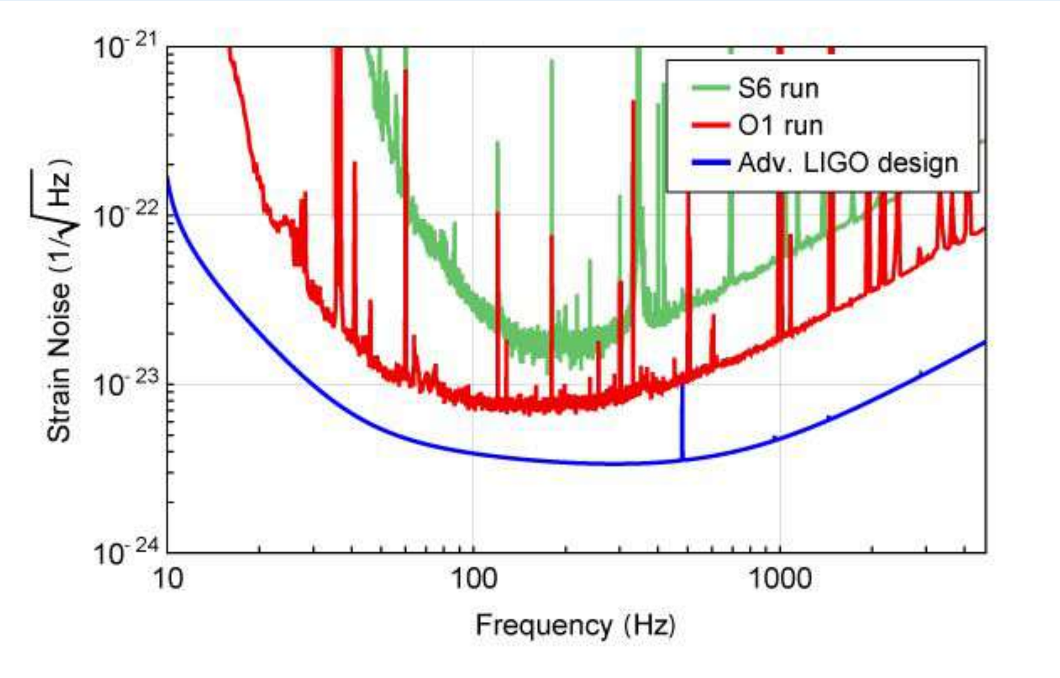
China: Tian-Qin

Operate in the milli-Hz to 1 Hz band

The Detector Noise : PSD $S_n(f)$

-2015 Advanced LIGO: the multi-stage optic suspensions and their attached internal seismic isolation platforms .

These subsystems produced a significant improvement on the low-frequency side of LIGO's noise spectrum

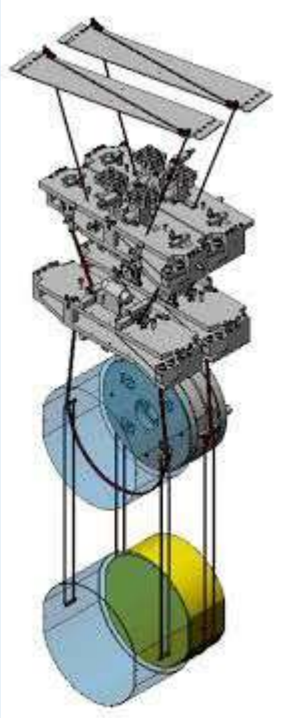


Noise Sources:

seismic & Newtonian noise at **low frequencies**,

suspension and thermal noise at **mid frequencies,**

Quantum (shot + radiation-pressure) noise at **high frequencies** — all combining to shape the detector's sensitivity curve.



The Detector Noise : PSD $S_n(f)$

$$S_n(f) = \frac{1}{L^2}\left[S_{x,\mathrm{seismic}}(f) + S_{x,\mathrm{suspension}}(f) + S_{x,\mathrm{thermal}}(f) + S_{x,\mathrm{quantum}}(f) + S_{x,\mathrm{coating}}(f) + \dots\right]$$

Mirror Thermal Noise

Temperature

Boltzmann constant

Geometrical coating thickness

Loss angle of coating

PSD of displacement

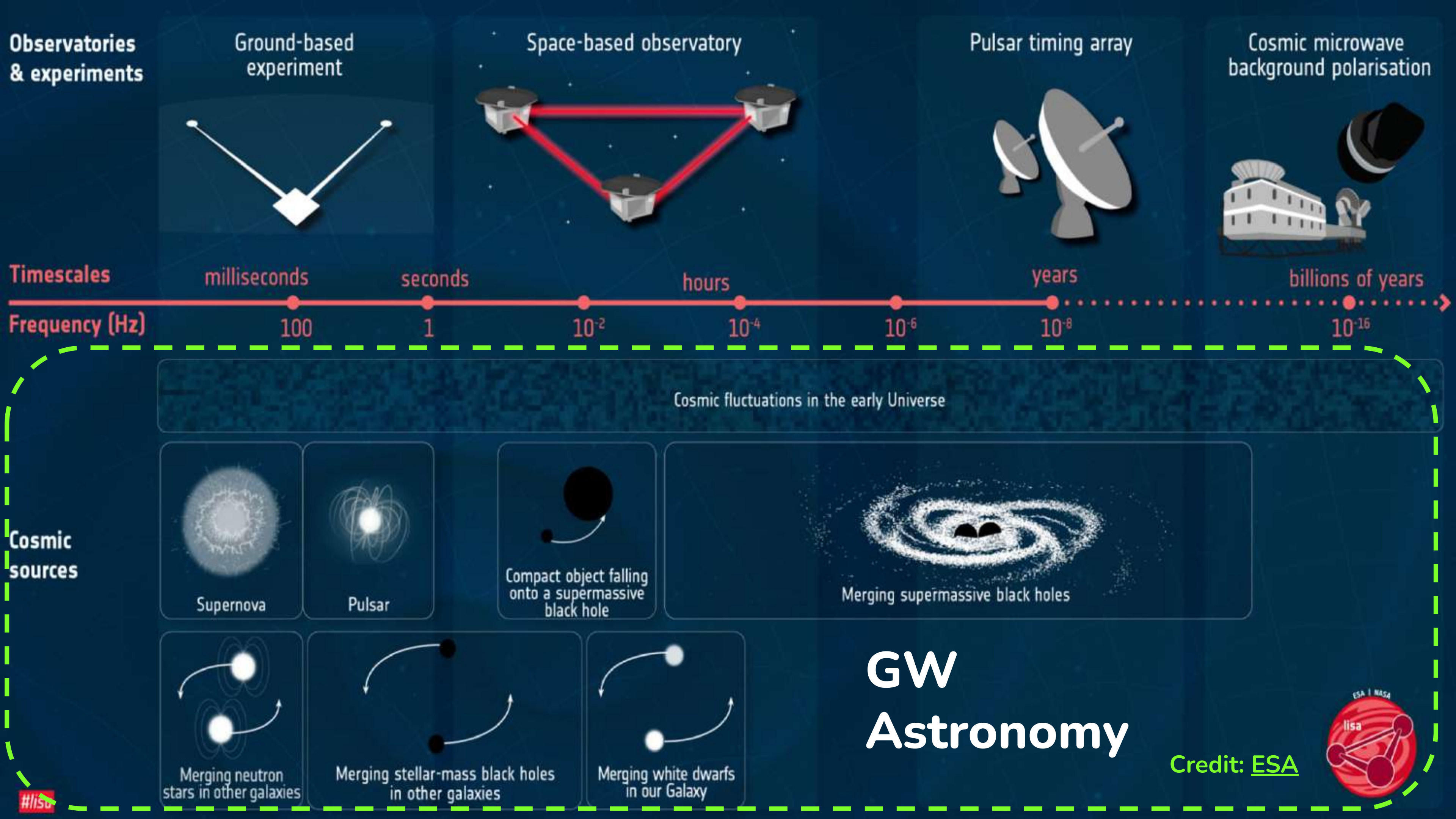
$$\boxed{S_x(f) = \frac{4k_\mathrm{B}T}{\pi^2 fY}\frac{d}{r_0^2}\left(\frac{Y'}{Y}\phi_{\parallel} + \frac{Y}{Y'}\phi_{\perp}\right)}$$

Harry et al, CQG 19, 897–917, 2002

Young's modulus of mirror substrate

laser beam radius

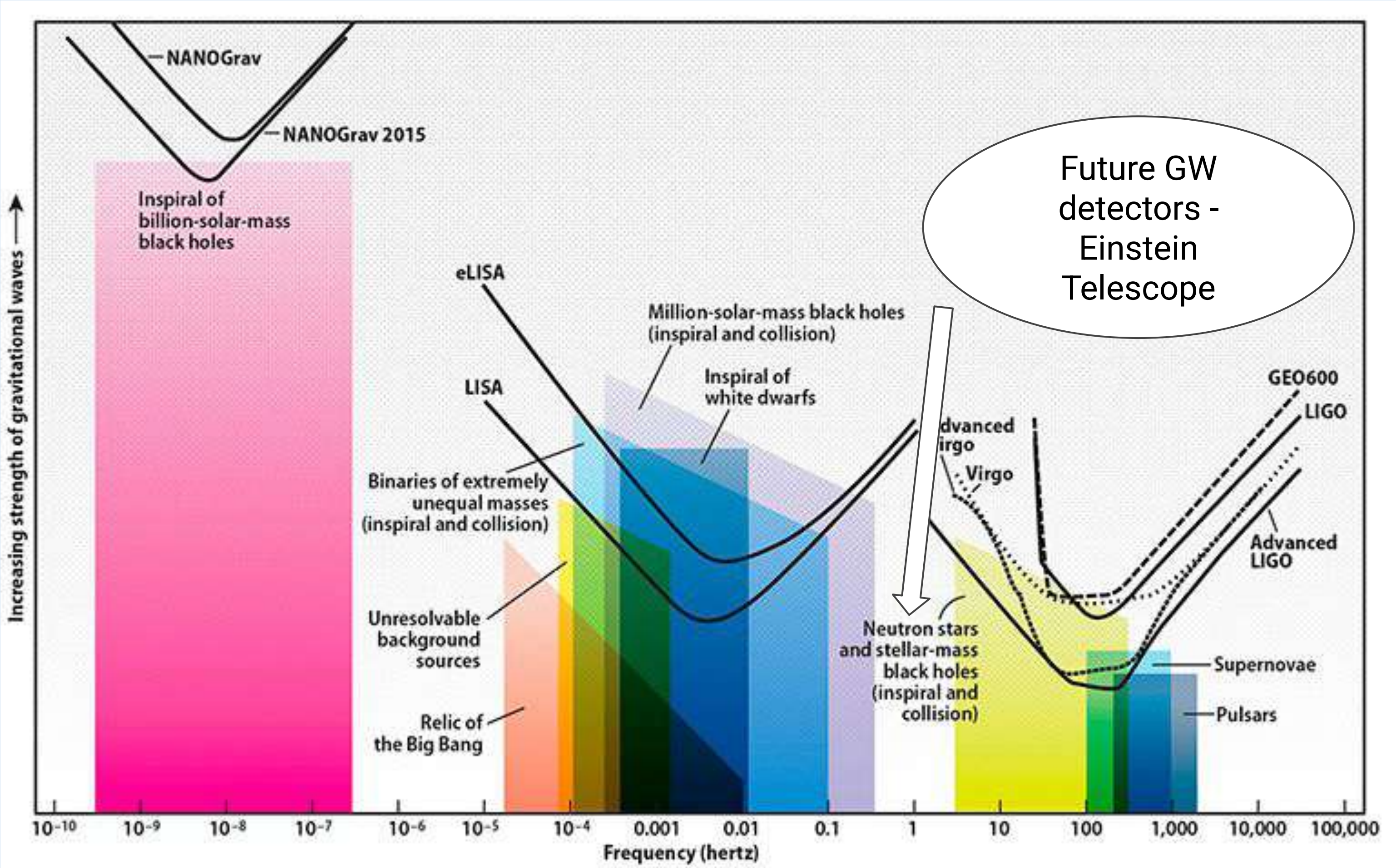
Young's modulus of coating



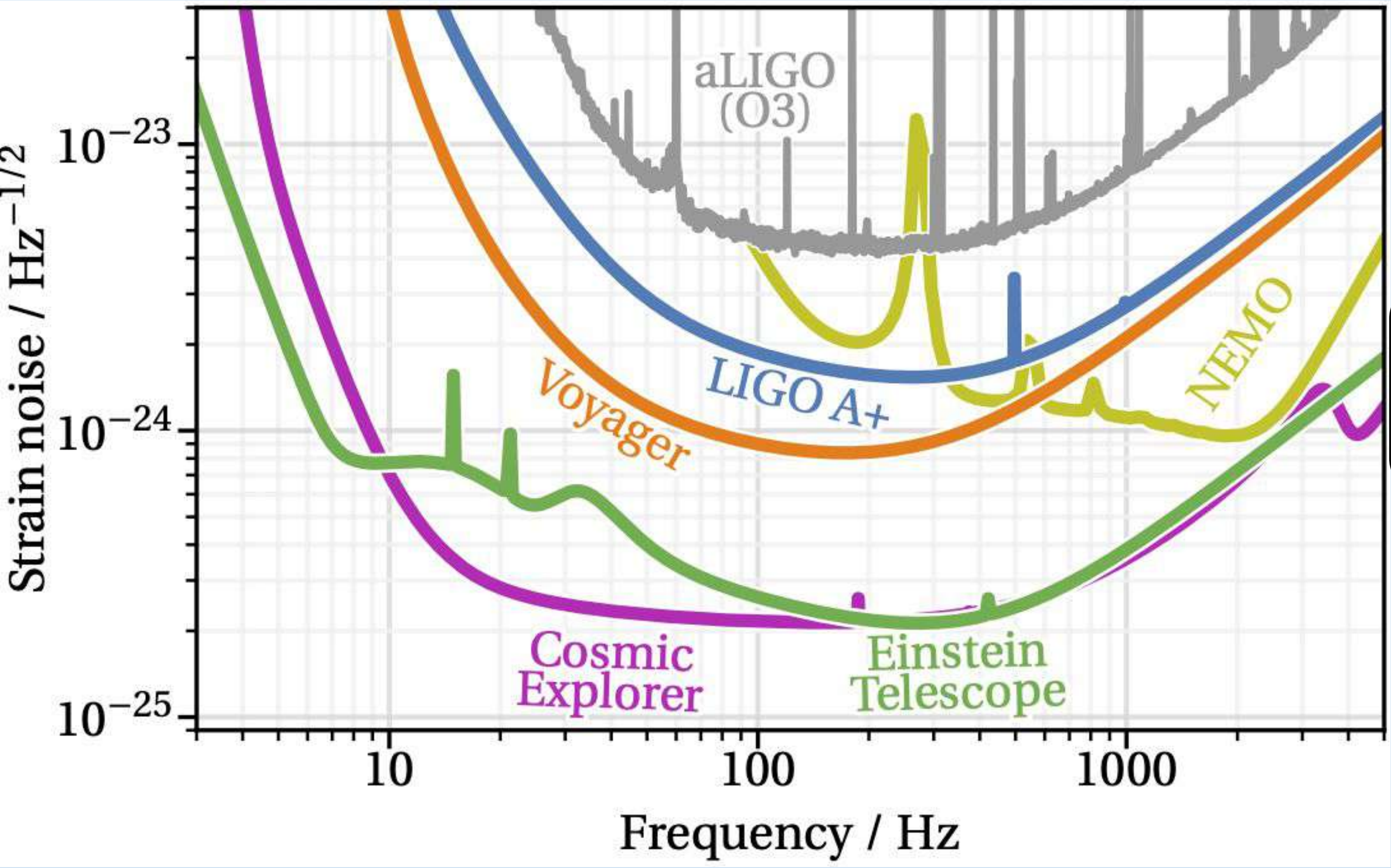
GW Astronomy

Credit: ESA

Future of GWs : A new window to the universe



Future of GWs : A new window to the universe



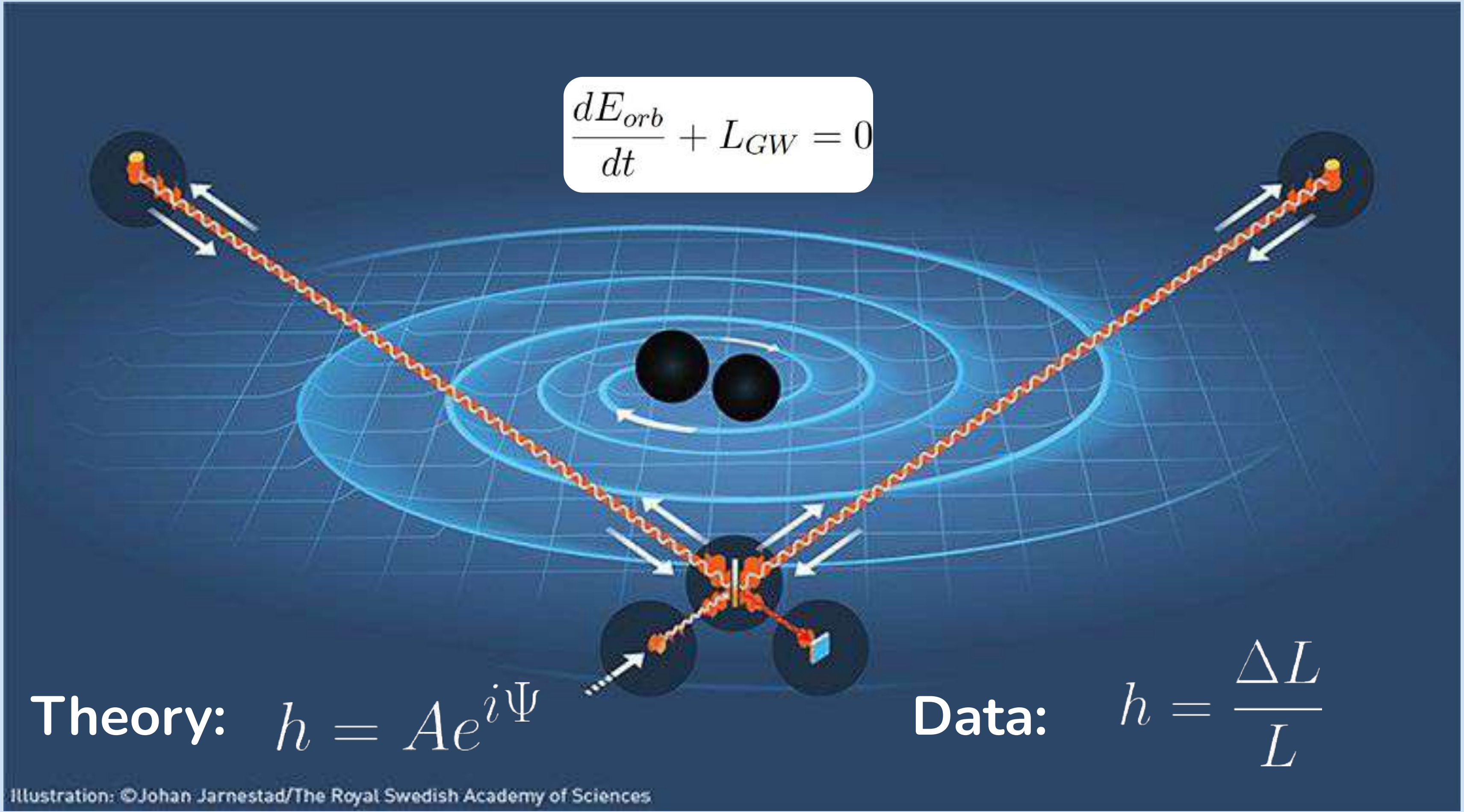
Lower freq sensitivity (1Hz)=> O(10^5-10^6/yr) + longer-duration signals

Improved reach vs. 2G:
- SNR → 10×
- Redshift horizon z → 20×
- Detection rate → 100×

Our Group Goal

- **Neutron Stars**
- **oscillations of compact stars**
- **stochastic GWs**

Detecting Gravitational



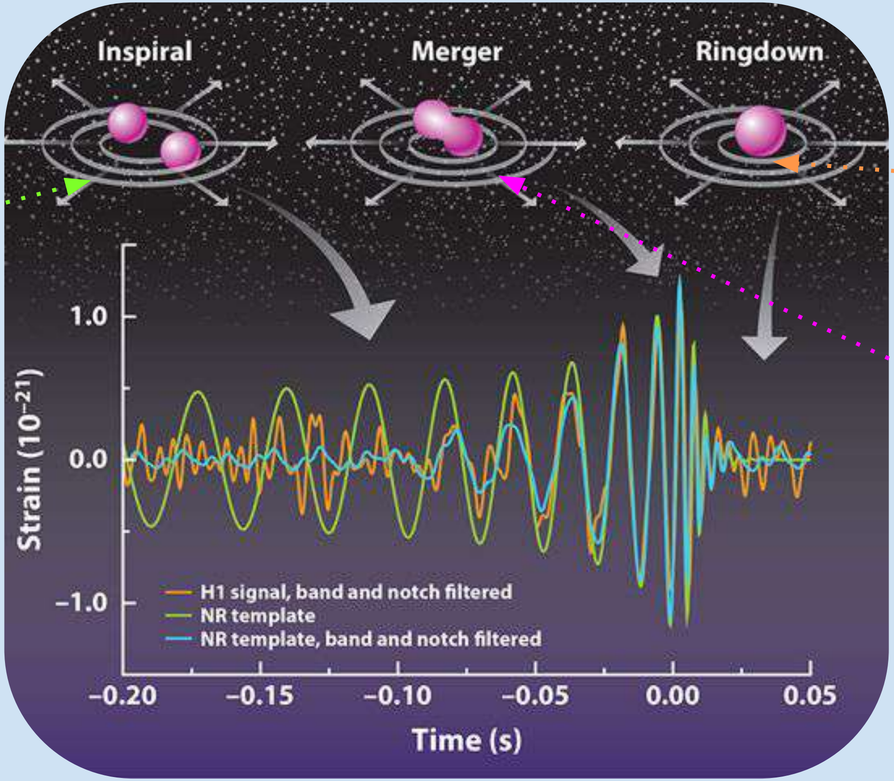
$$\frac{dE_{orb}}{dt} + L_{GW} = 0$$

Theory: $h = Ae^{i\Psi}$

Data: $h = \frac{\Delta L}{L}$

GWs : Methods to compute templates

Post-Newtonian Theory



BH Perturbation Theory

Numerical Rel.

$$h = Ae^{i\Psi}$$

the correct template/needle!

Gravitational Waves and the Effort to Detect them By Peter Shawhan

GWs : Data-Analysis

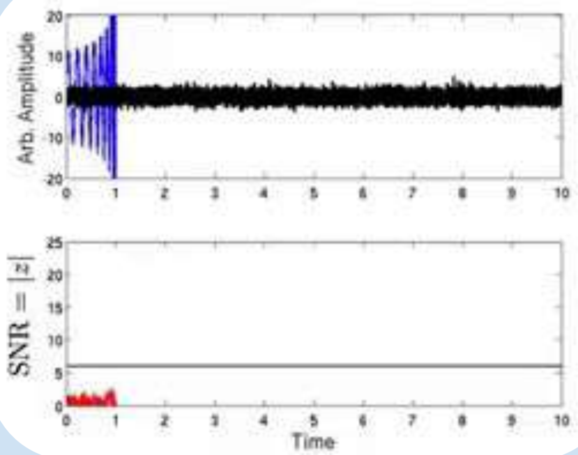
Parameter Estimation

$$h = Ae^{i\Psi}$$

GW Data-analysis => Matched Filtering

$$d(t) = h(t) + n(t)$$

$$z = \langle d|\hat{h}\rangle = 4\int_{\mathrm{f_{low}}}^{\mathrm{f_{high}}} \frac{\tilde{d}(f)\hat{h}^*(f)}{S_n(f)}df$$



- Cross–correlate with detector data: d(t) using the noise-weighted inner product → Signal-to-Noise Ratio (SNR)
- Template bank: Covers a grid of masses, spins, orientations Search = "scan over templates" to find the best match
- Noise is non-Gaussian and non-stationary → **Need ranking statistics & likelihood**

Aaron Zimmerman talk: Link
Designing a template bank to observe compact binary coalescences, Canton and Harry (arxiv:1705.01845)

GW
Parameter Estimation

Bayesian Analysis:
Comprehensive, reliable for realistic data, especially when the likelihood is non-Gaussian.

Given the detector data,:
What is the probability that the observed strain was produced by an astrophysical signal rather than by noise?

Fisher Analysis:
Quick, approximate, best for high SNR and gaussian noise

*References:
- *Post-Newtonian theory for gravitational waves, Living Reviews in Relativity, Luc Blanchet 2024*
- *Sources of Gravitational Waves: Theory and Observations, Alessandra Buonanno and B.S. Sathyaprakash*

Bayesian Analysis: Given the data, inferring parameters

Posterior: prob. distn. of param (θ)

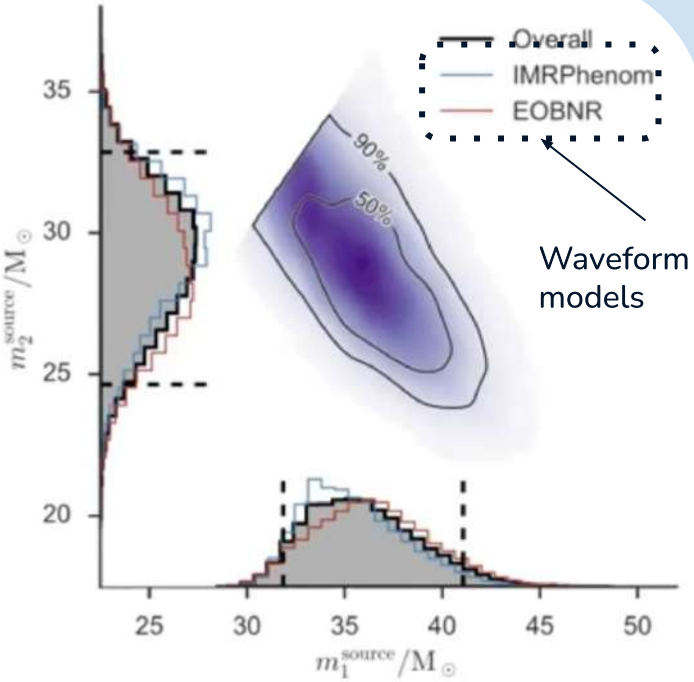
$$p(\theta|\vec{d}, M) = \frac{p(\vec{d}|\theta, M)p(\vec{\theta}, M)}{p(d|M)}$$

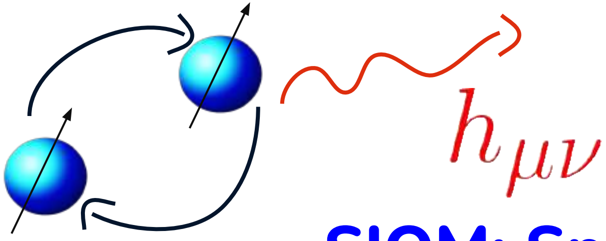
Likelihood: measures how well the data fits param

$$\Lambda(\mathcal{H}_1|s) = \frac{p(s|\mathcal{H}_1)}{p(s|\mathcal{H}_0)} = \boxed{e^{(s,h)}} e^{-(h,h)/2}$$

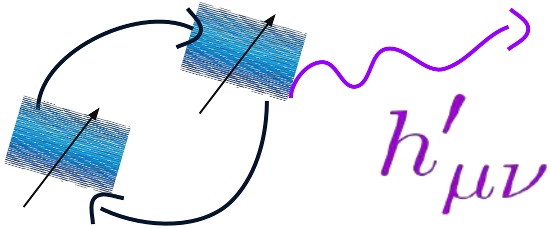
Prior: encodes our knowledge about param

Evidence: normalization



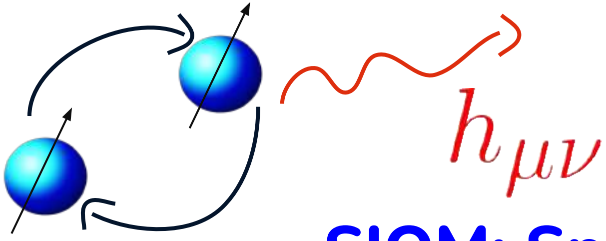


Testing the Compact binary nature

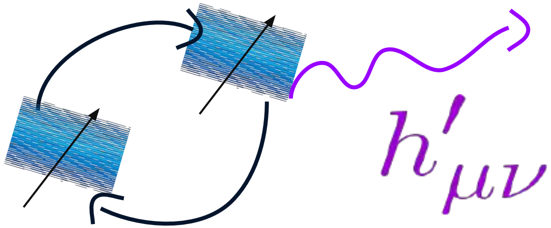


SIQM: Spin-Induced Quadrupole Moment

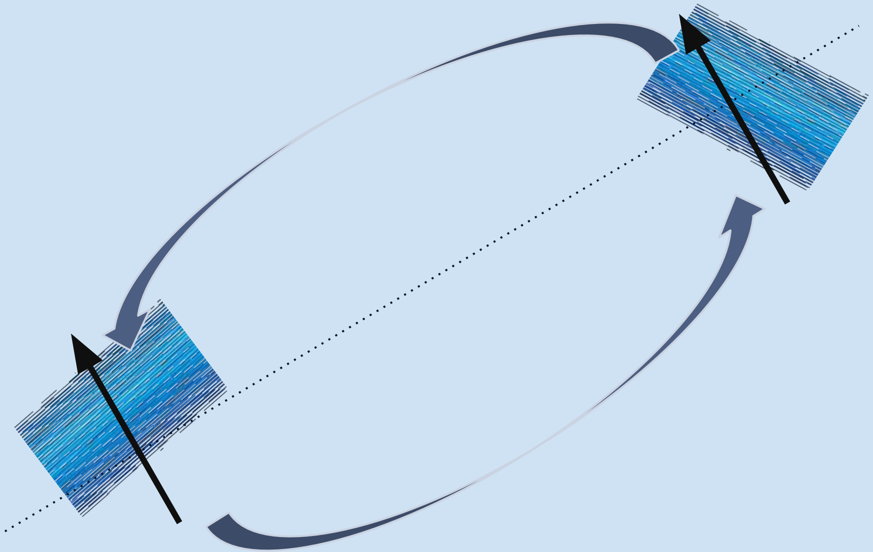
$$\left\{ \begin{array}{l} \kappa = 1, \text{ BH} \\ \kappa \neq 1, \text{ N.S, Exotic Compact Objects} \end{array} \right\}$$



Testing the Compact binary nature



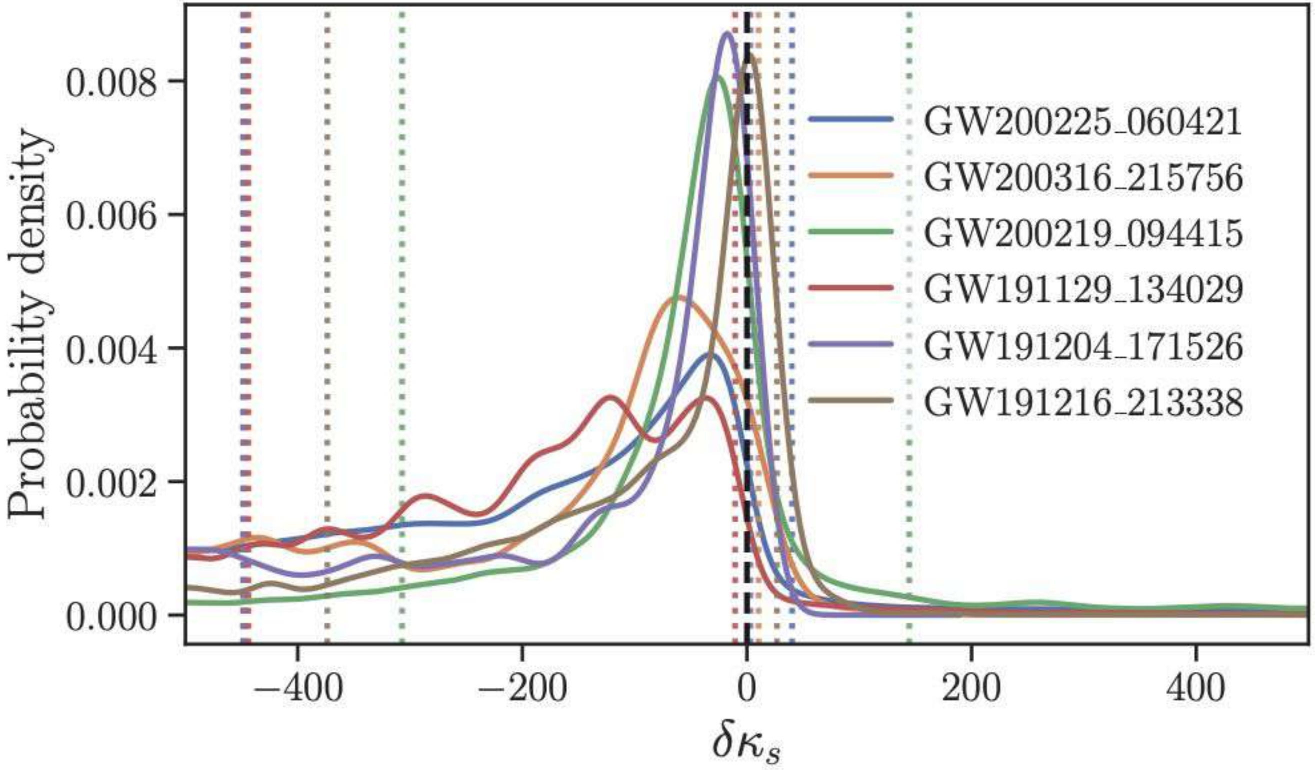
SIQM: Spin-Induced Quadrupole Moment



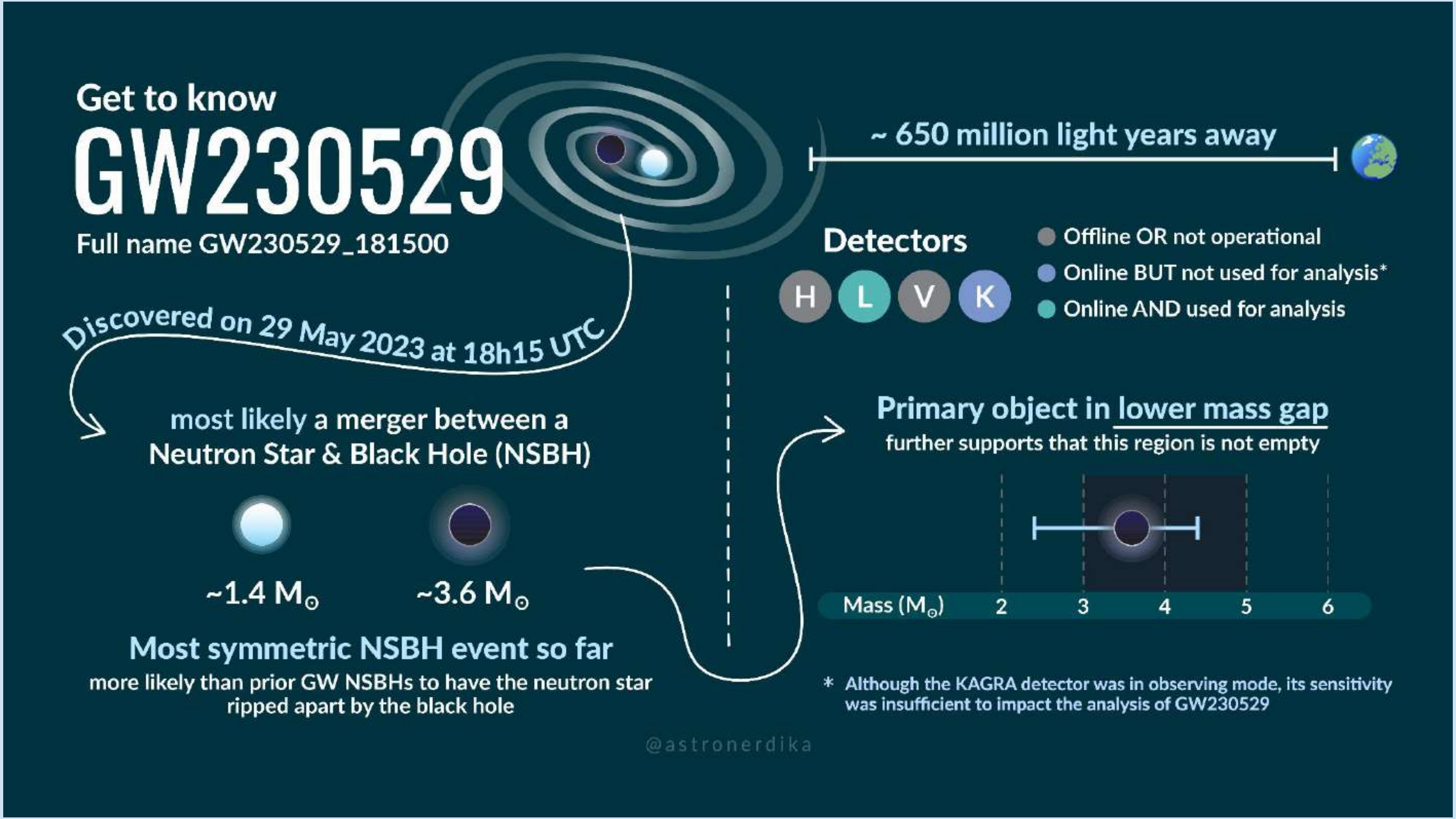
No-hair theorem test	Is the object truly a Kerr BH?
Astrophysical identity test	Neutron star vs boson star vs BH
Modified gravity/GR test	Deviations from GR predictions

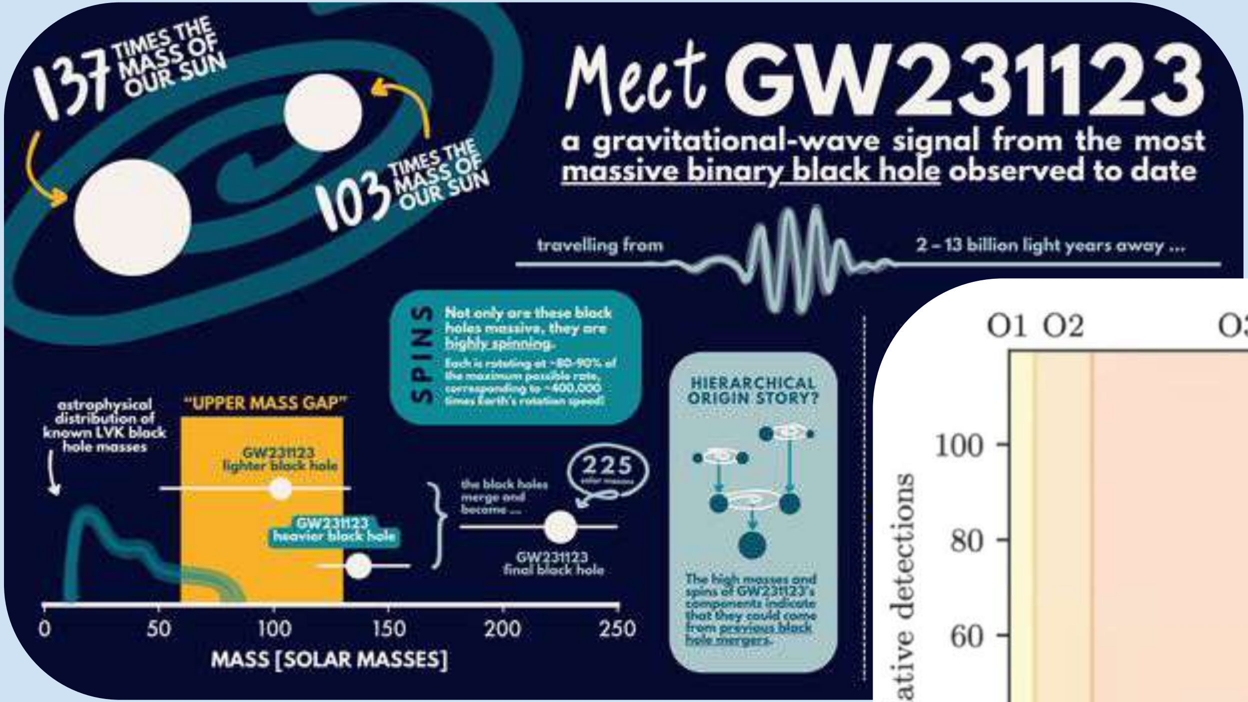
Constrain on kappa GWTC 3 : Waveform IMRPhenomPv2

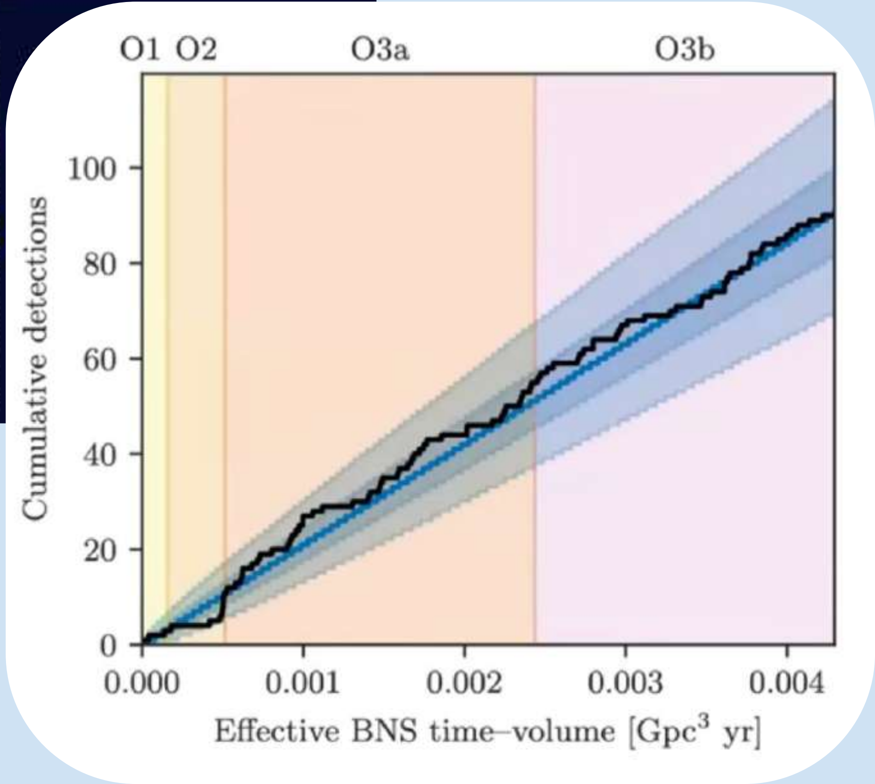
Parameterised deviations $\kappa s = 1 + \delta\kappa s$ assuming $\kappa a = 0$.



Tests of General Relativity with GWTC-3. LVK 2021


Get to know
GW230529
Full name GW230529_181500
Discovered on 29 May 2023 at 18h15 UTC
most likely a merger between a
Neutron Star & Black Hole (NSBH)
~1.4 $M_\odot$
~3.6 $M_\odot$
Most symmetric NSBH event so far
more likely than prior GW NSBHs to have the neutron star ripped apart by the black hole
~ 650 million light years away
Detectors
H L V K
Offline OR not operational
Online BUT not used for analysis*
Online AND used for analysis
Primary object in lower mass gap
further supports that this region is not empty
Mass ($M_\odot$) 2 3 4 5 6
* Although the KAGRA detector was in observing mode, its sensitivity was insufficient to impact the analysis of GW230529
@astronerdika



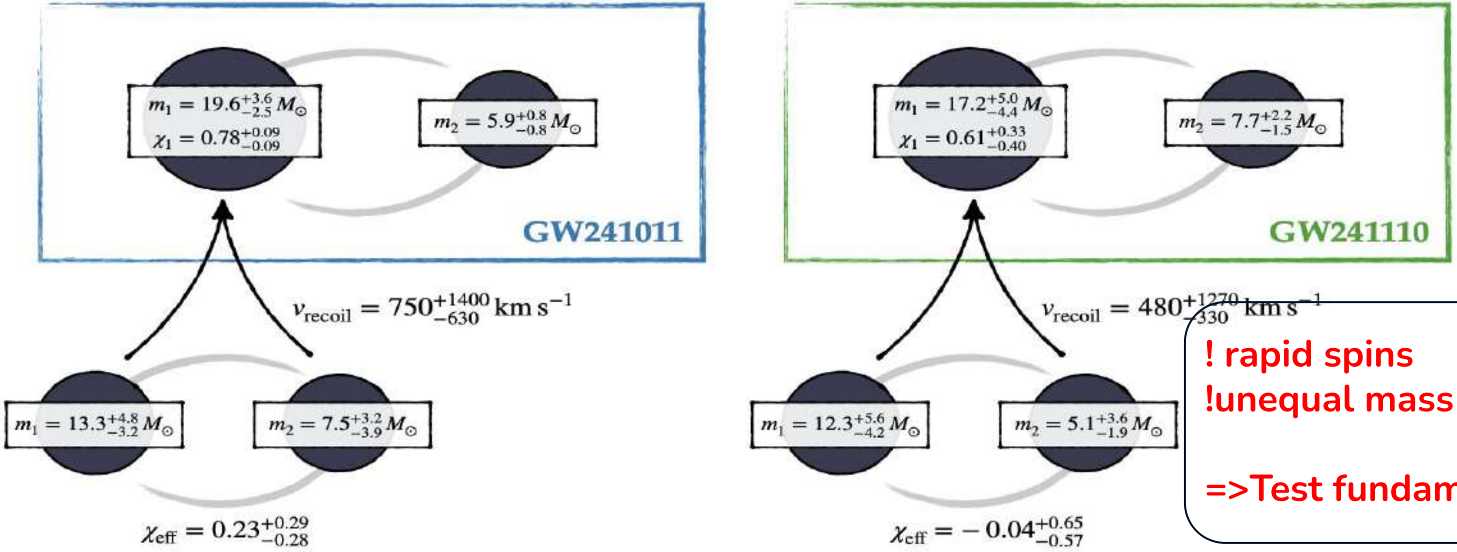


Now O4a: 128 new significant GW signal candidates!!!

The Astrophysical Journal Letters, 993:L21 (45pp), 2025 November 1
https://doi.org/10.3847/2041-8213/ae0d54

GW241011 and GW241110: Exploring Binary Formation and Fundamental Physics with Asymmetric, High-spin Black Hole Coalescences

Involve the mergers of second-generation black holes in dense stellar environments, such as globular, nuclear, and young massive star clusters

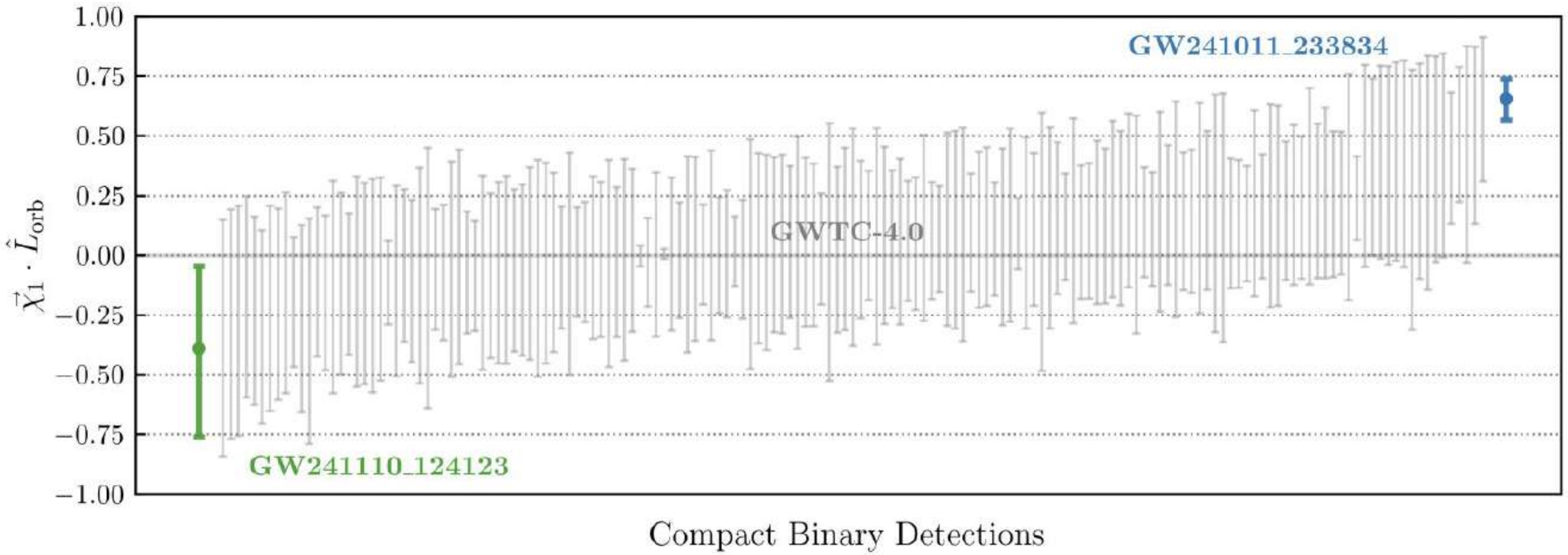


! rapid spins
!unequal mass ratio

=>Test fundamental physics

The Astrophysical Journal Letters, 993:L21 (45pp), 2025 November 1 https://doi.org/10.3847/2041-8213/ae0d54

GW241011 and GW241110: Exploring Binary Formation and Fundamental Physics with Asymmetric, High-spin Black Hole Coalescences



THE ASTROPHYSICAL JOURNAL LETTERS, 993:L21 (45pp), 2025 November 1 https://doi.org/10.3847/2041-8213/ae0d54

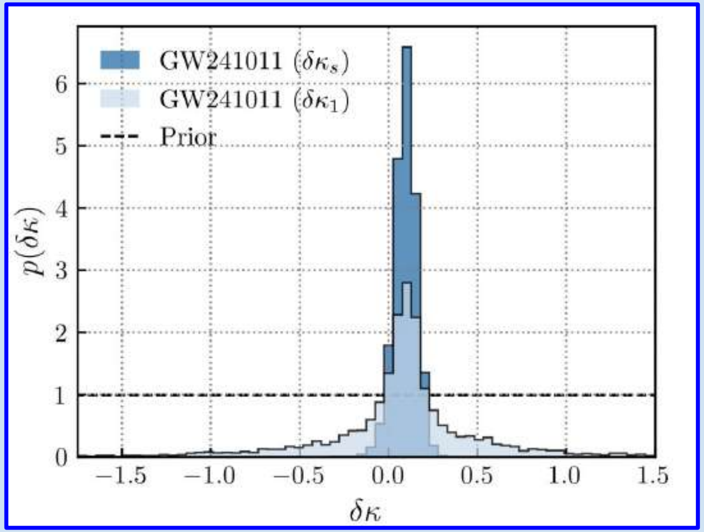
GW241011 and GW241110: Exploring Binary Formation and Fundamental Physics with Asymmetric, High-spin Black Hole Coalescences

Test fundamental physics

Define $\kappa_1 = 1 + \delta\kappa_1$ and $\kappa_2 = 1 + \delta\kappa_2$ as the spin-induced quadrupole coefficients of each compact object in GW241011's source binary.

Repeat parameter estimation with a modified IMRPHENOMXPHM waveform model allowing for nonzero δκ1 and δκ2

Black Hole Spin-induced Quadrupole Moment



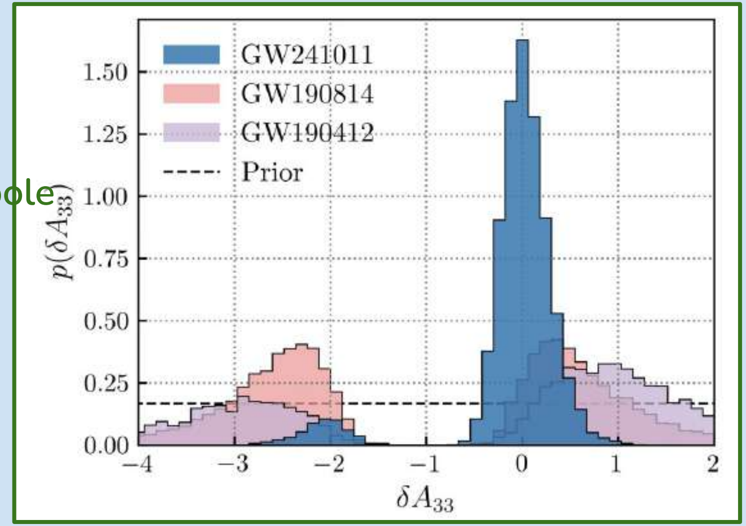
The Astrophysical Journal Letters, 993:L21 (45pp), 2025 November 1 https://doi.org/10.3847/2041-8213/ae0d54

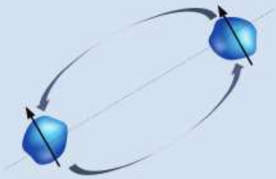
GW241011 and GW241110: Exploring Binary Formation and Fundamental Physics with Asymmetric, High-spin Black Hole Coalescences

Test fundamental physics

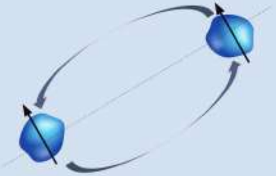
Radiation beyond the Quadrupole Approximation

- Typically, GW signals are dominated by the quadrupole $(\ell, m) = (2, \pm 2)$ multipole.
- Higher-order multipoles such as $(2, \pm 1)$ and $(3, \pm 3)$ become increasingly relevant for systems with unequal masses





Conclusion



Current Detections

➢ High-mass, asymmetric events enable higher-mode measurements

- GW241011 and GW241110 have mass ratios and orientations that make the (3,3) mode measurable, allowing precision tests of GR beyond the (2,2) mode.

➢ Isolated binary evolution struggles to explain the spins

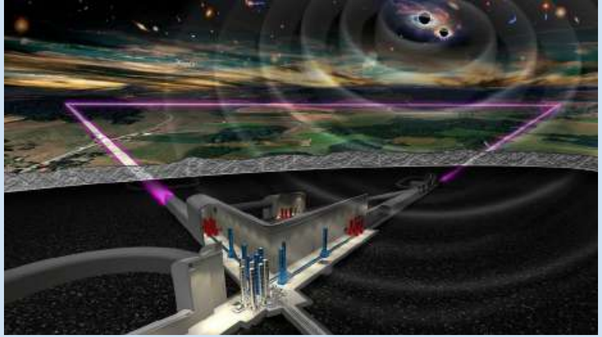
- Standard stellar evolution predicts low and aligned spins → tension with observed primary spins.

- Requires exotic channels.

Implications of GW241011 for rotating exotic compact objects **arXiv:2511.17341**

N. V. Krishnendu, Tamara Evstafyeva, Aditya Vijaykumar, William E. East, Rimo Das, Sayantani Datta, Nils Siemonsen, Nami Uchikata, Poulami Dutta Roy, Anuradha Gupta, Ish Gupta, Syed U. Naqvi, Manuel Piarulli, Muhammed Saleem, Elise M. Sänger, Pratyusava Baral, Sajad A. Bhat, Thomas A. Callister, Mattia Emma, Carl-Johan Haster

Conclusion



Future Detections : Einstein Telescope

➢ A Leap Beyond Current Detectors

- ET's underground, cryogenic, 10× improved sensitivity will reveal signals far below the LIGO–Virgo noise floor.

- Access to 1–10 Hz band opens an entirely new discovery space (IMBH inspirals, early inspiral phases, massive binaries).

➢ A Complete Census of Compact Objects

- Detects stellar-mass merger in the universe up to redshift 20+.

- Allows population studies with orders-of-magnitude larger statistics, enabling:
  - BH formation channels
  - Cosmic evolution of spins, masses, eccentricities

Conclusion

Thank You!

➤ Key Challenges

Ahead Data volumes ×100 larger → need new data analysis pipelines (fast inference, ML tools).

Theory and waveform modeling must improve to match ET’s precision.

Environmental noise control (seismic, Newtonian noise) is critical.