Spin polarization in a relativistic fluid
Dissipative contributions

[based on MB, JHEP 07 (2025), 255; ArXiv:2502.15520]

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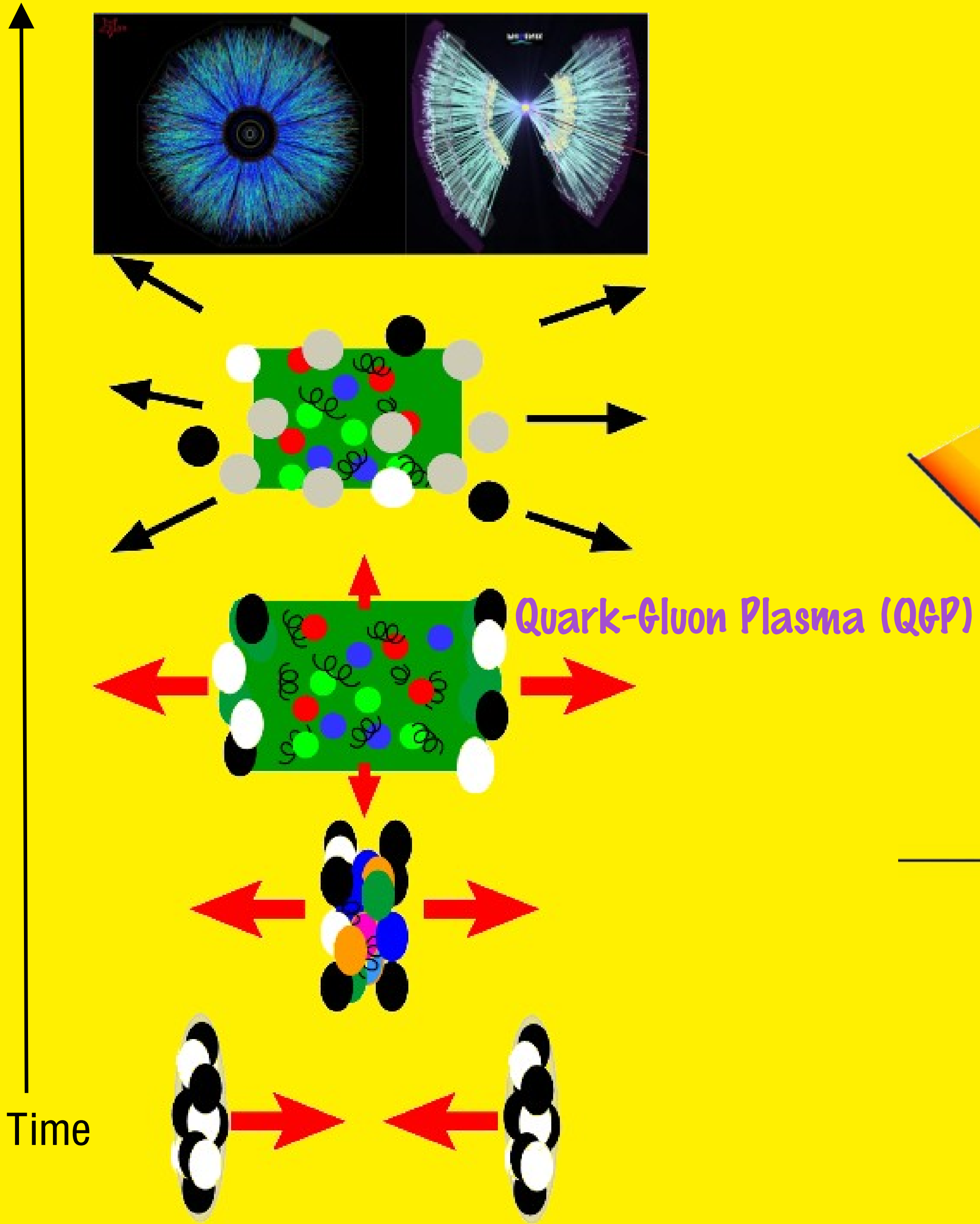
20/11/25

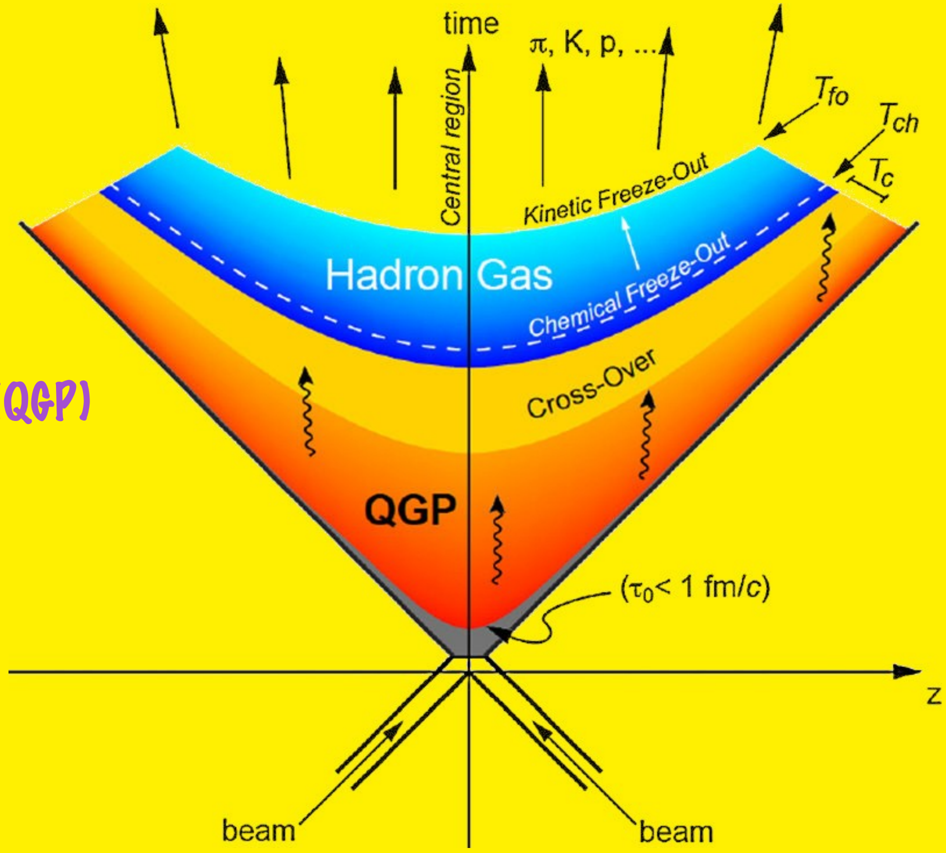
Outline

- Spin polarization and vorticity in the QGP and in heavy-ion collisions

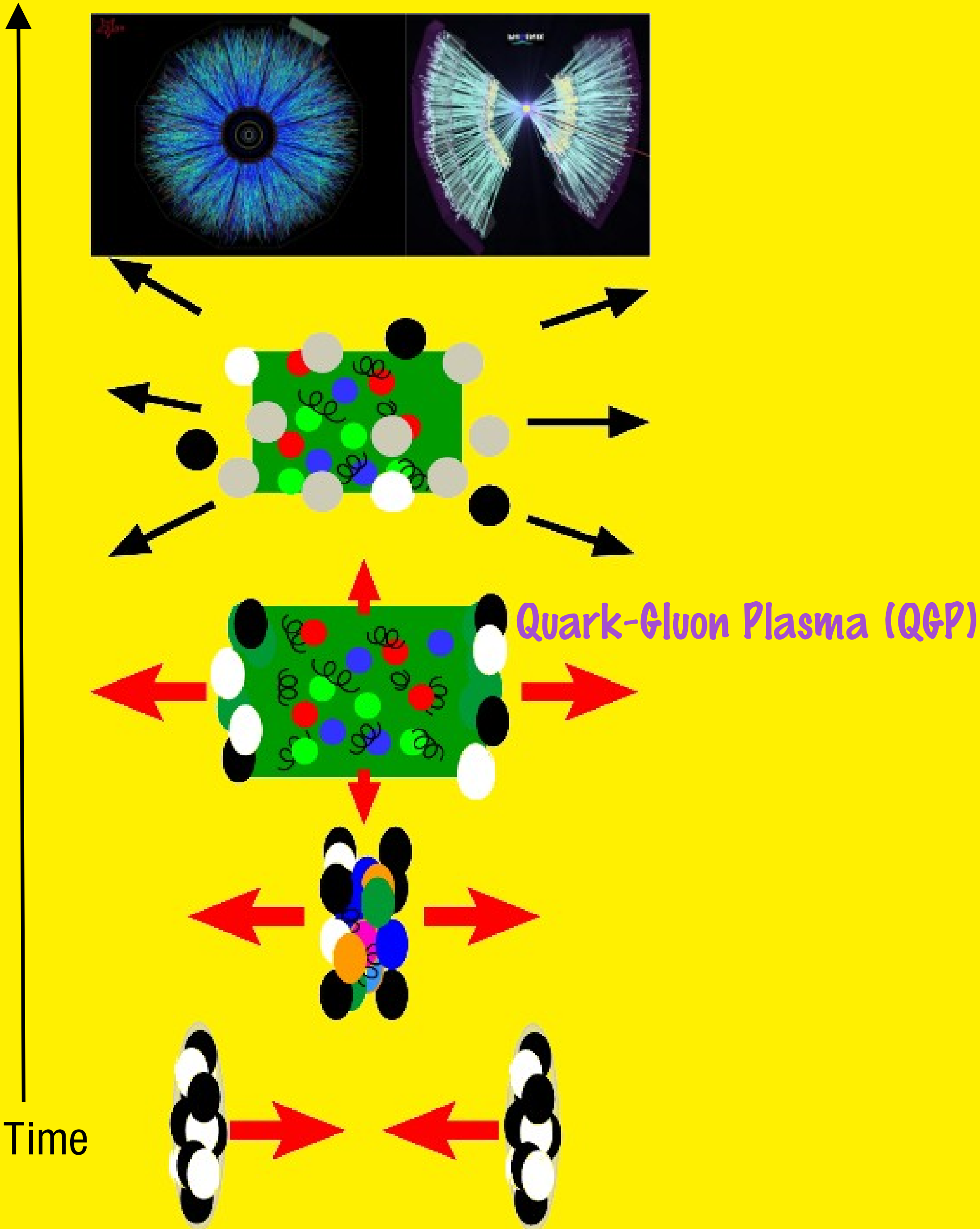
- Dissipative effects in spin polarization

Relativistic Heavy-ion Collisions (HIC)

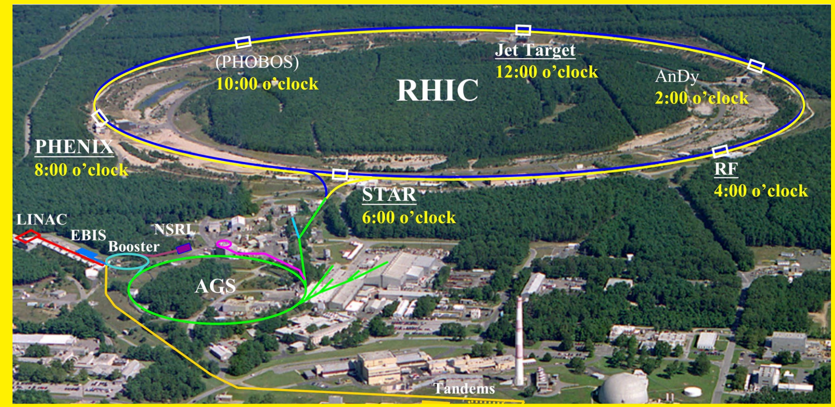




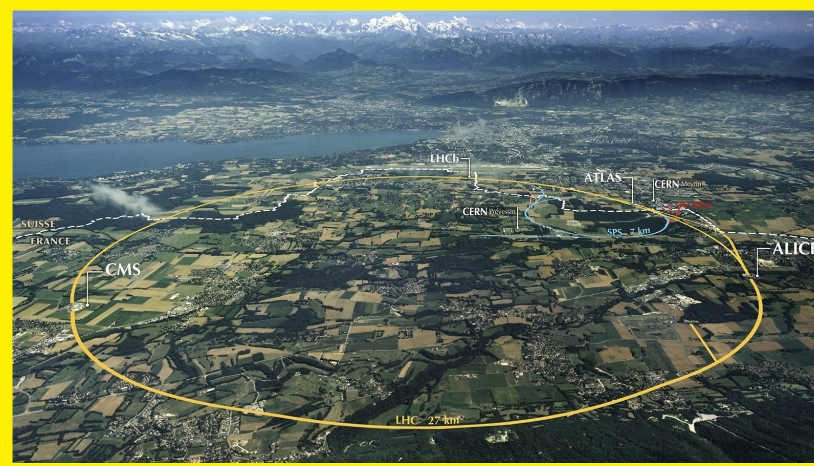
Relativistic Heavy-ion Collisions (HIC)



•GSI@Darmstadt: HADES/FAIR
Au-Au, Ag-Ag $\sqrt{S_{NN}} = 2, \ldots, 5$ GeV



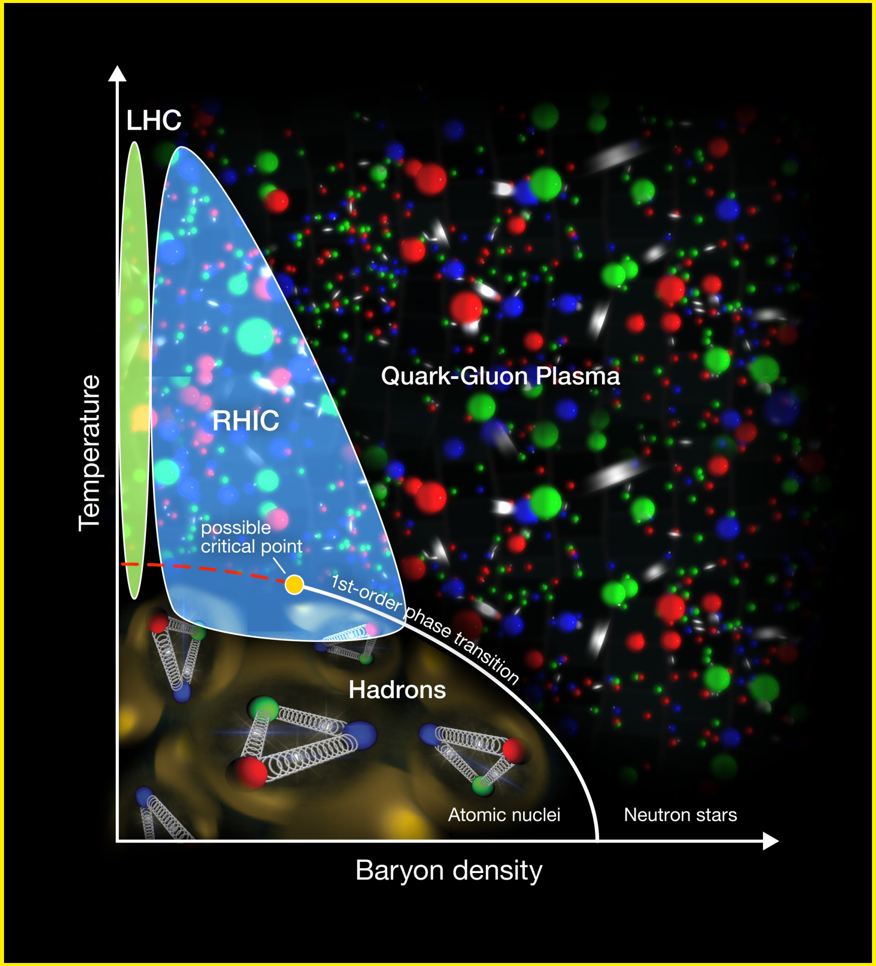
•RHIC: Relativistic Heavy Ion Collider
Au-Au $\sqrt{S_{NN}} = 7.7, \ldots, 200$ GeV



•LHC: Large Hadron Collider, ALICE
Pb-Pb $\sqrt{S_{NN}} = 2.76,\ 5.5$ TeV

Questions addressed by HIC

- Mapping the QCD phase diagram (Critical point?)
- Nature and properties of the quark-gluon plasma (QGP)/QCD
  - Strongly coupled fluid (viscosity, etc..)
  - Equation of State
  - Thermalization
- Small bang: Early universe
- Local parity violations in QCD?
  - Chiral Magnetic Effect



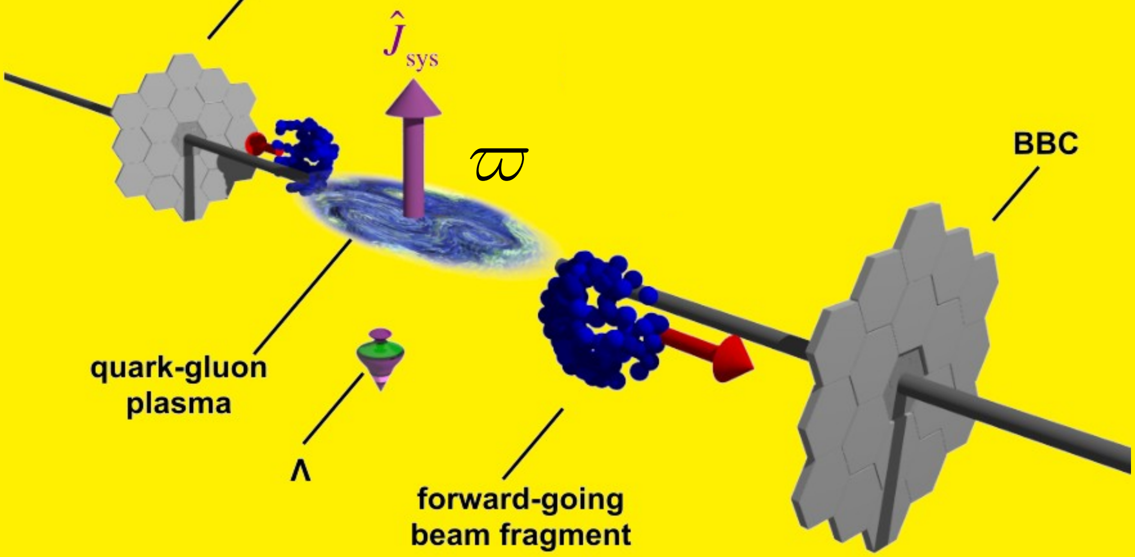
Rich phenomenology

Peripheral collisions → Angular momentum → Thermal Vorticity of the fluid: ϖ

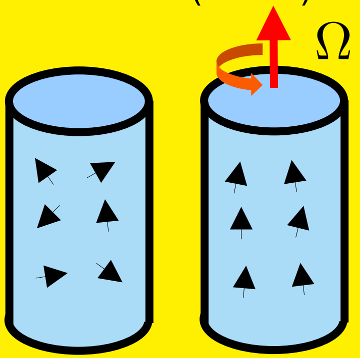
Include acceleration, rotation and gradients of temperature

$$\varpi^{\mu\nu} = -\frac{1}{2}\left(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu\right)$$

$$\beta^\mu = u^\mu / T$$



Global polarization w.r.t. reaction plane
Similar to Barnett effect (1915)



Ferromagnetic → Magnetization

- Polarization estimated at quark level by spin-orbit coupling

  Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301

- By local thermodynamic equilibrium of the spin degrees of freedom

  F. Becattini, F. Piccinini, Ann. Phys. 323 (2008) 2452;
  F. Becattini, F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906

Spin ∝ (thermal) vorticity

Peripheral collisions → Angular momentum → Thermal Vorticity of the fluid: ϖ

Include acceleration, rotation and gradients of temperature

$$\varpi^{\mu\nu} = -\frac{1}{2}\left(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu\right)$$

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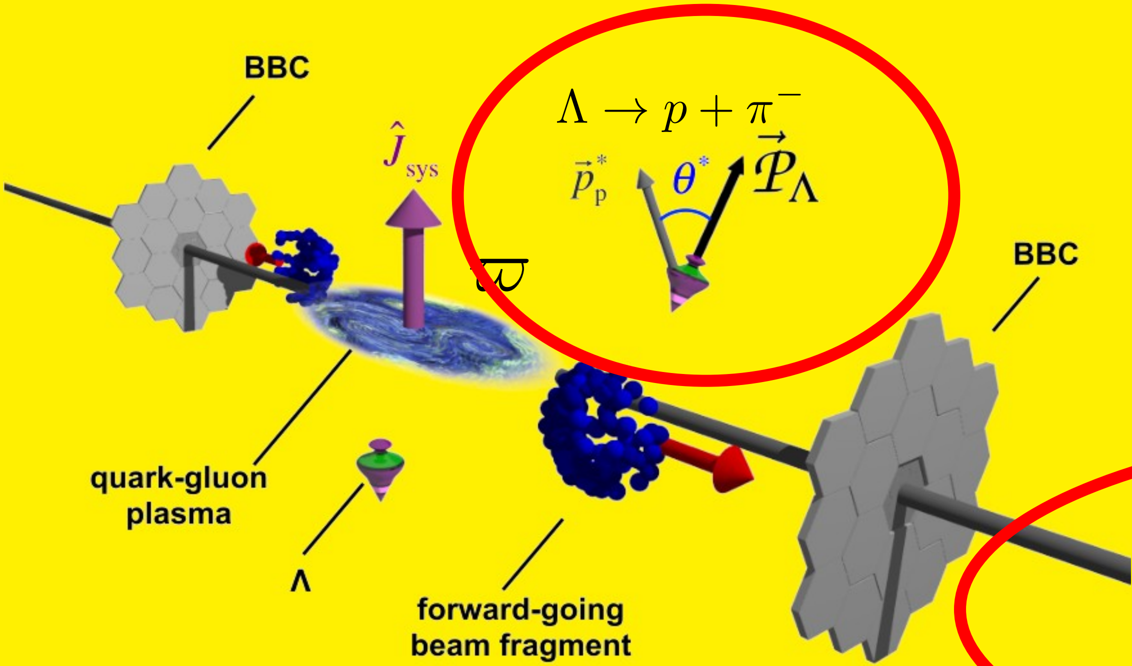
Global polarization w.r.t. reaction plane



Spin Polarization measurement

"self-analysing" Λ hadron by weak decay

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta^*} = \frac{1}{2}\left(1 + \alpha_\Lambda |\boldsymbol{\mathcal{P}}_\Lambda| \cos\theta^*\right)$$

- Polarization estimated at quark level by spin-orbit coupling

  Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301

- By local thermodynamic equilibrium of the spin degrees of freedom

  F. Becattini, F. Piccinini, Ann. Phys. 323 (2008) 2452;
  F. Becattini, F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906

Spin $\propto$ (thermal) vorticity

Agreement between hydrodynamic **predictions** and the data

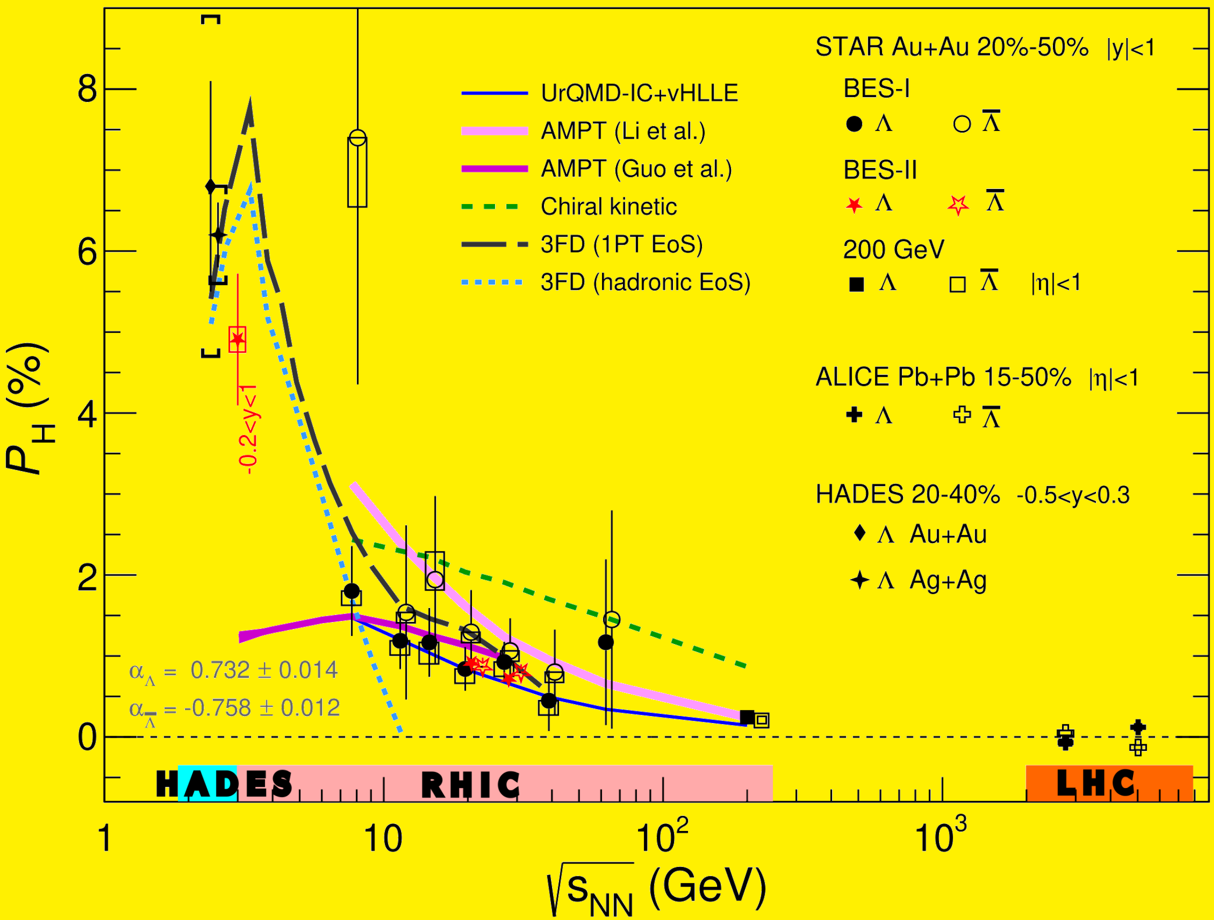
[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)]

Global Spin Polarization ◀ Integrate over momenta

$$P_H^\mu(p) = \frac{1}{4m_H}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

$$\varpi^{\mu\nu} = -\frac{1}{2}\left(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu\right)$$

$$n_F = \left(\mathrm{e}^{\beta\cdot p-\zeta}+1\right)^{-1}$$

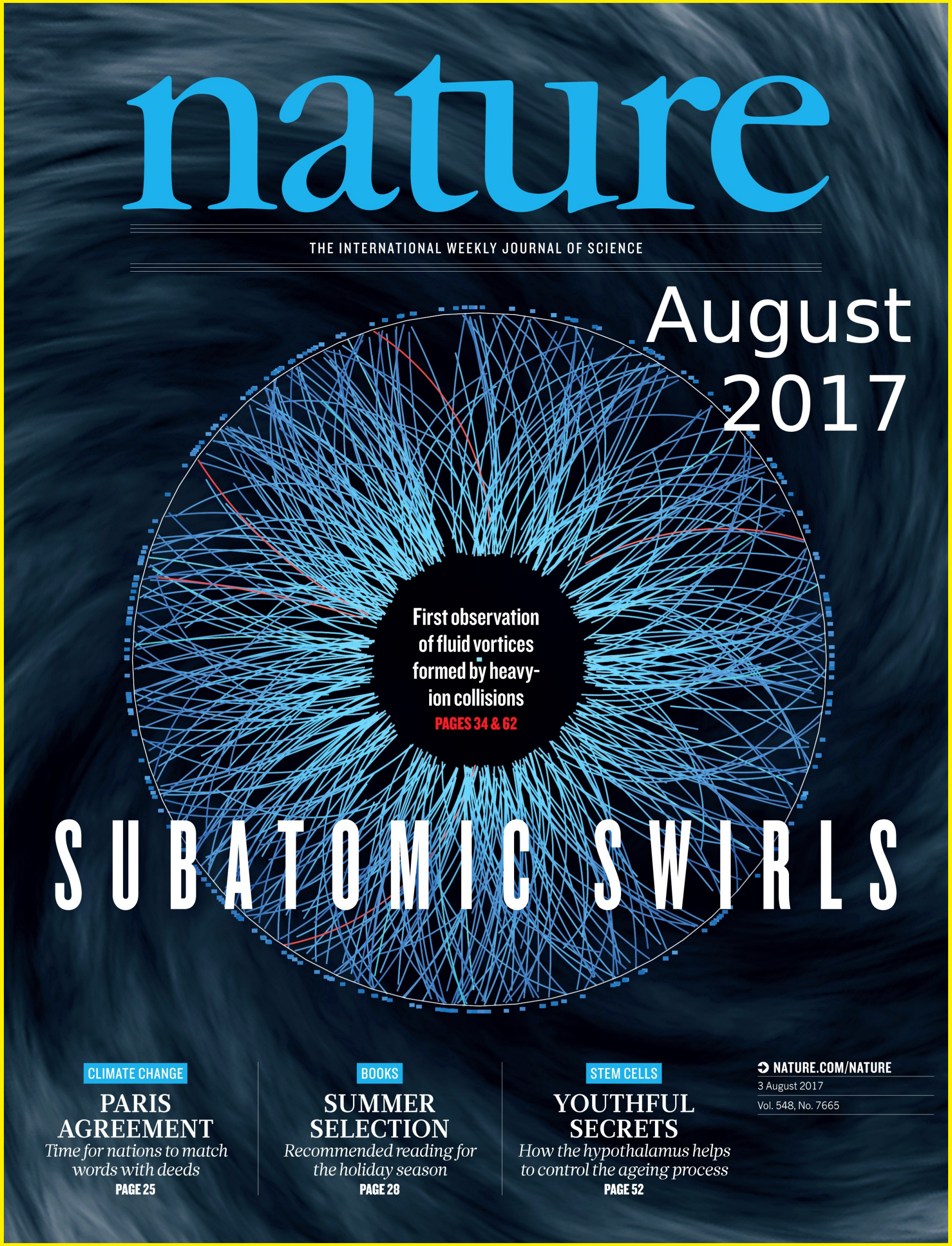


[F. Becattini, MB, T. Niida, S. Pu, A. H. Tang and Q. Wang, Int. J. Mod. Phys. E 33 (2024) no.06, 2430006]

Σ is a 3D hyper-surface where the hadron is formed.

Different models of the collision, same formula for polarization

Spin polarization: new chapter in heavy ion collisions



[STAR, Nature 548 (2017)]

- Quantum effect in hydrodynamics
  similar to superfluidity
- Relativistic spin, w/o magnetic field
  similar to spintronic
  [Takahashi et al, Nature Physics 12, 52–56 (2016)]
- Probe different properties of the QGP
  → **Most Vorticous fluid ever observed**

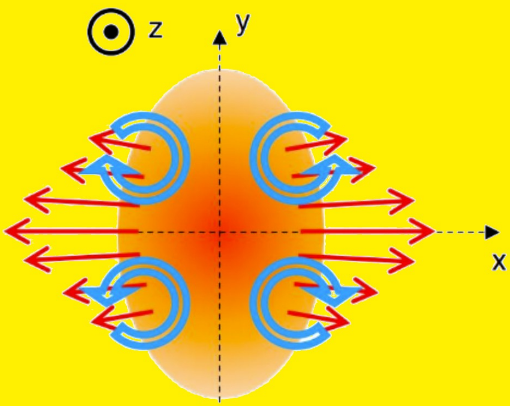
$$\omega \simeq 10^{22}\ \mathrm{Hz}$$

[STAR, Nature 548 (2017)]

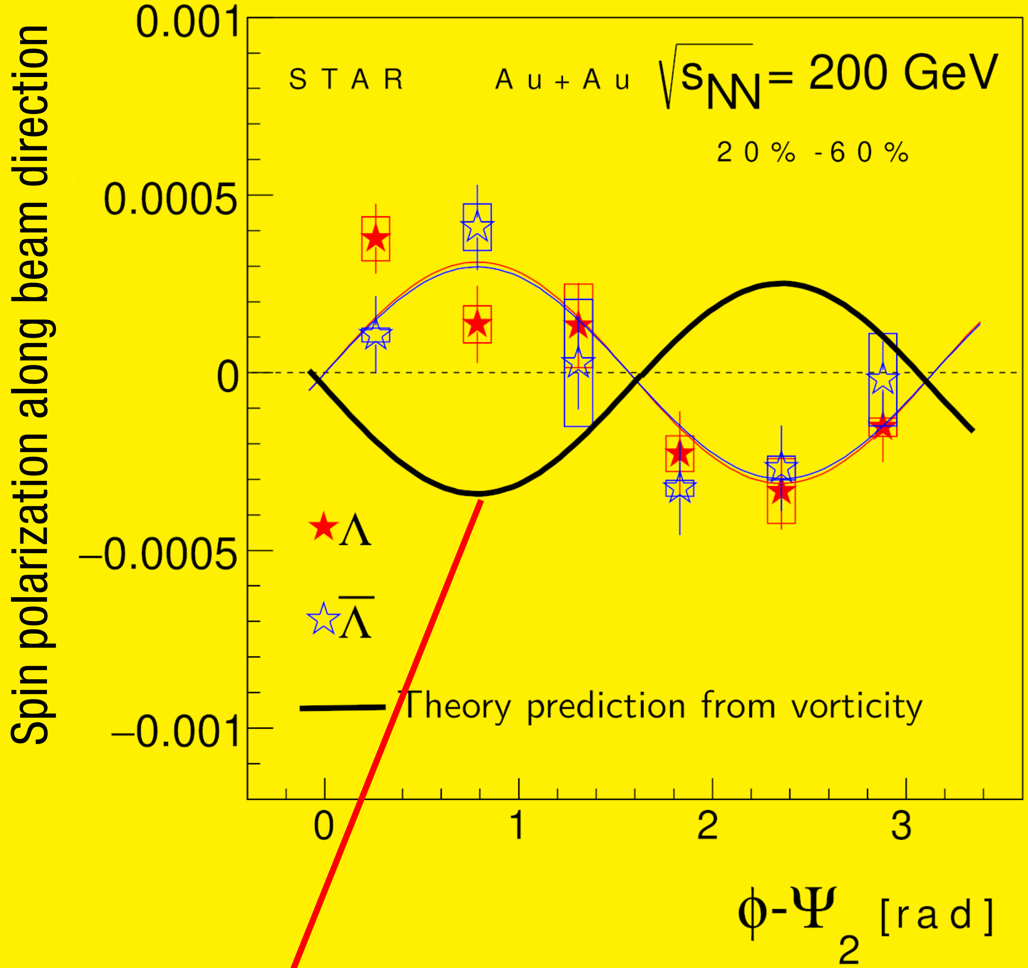
- Promising ground for new discoveries and new probes of the QGP

Local spin polarization

- "Local": Momentum dependent polarization (along beam direction)



STAR, PRL 123 no. 13 (2019)



Sign puzzle for spin polarization!

Other sources for spin polarization

Spin polarization four-vector in a fluid:

$$S_\mu(p) = \frac{1}{8m\int \mathrm{d}\Sigma\cdot p\, n_F}\int \mathrm{d}\Sigma\cdot p\,\epsilon_{\mu\nu\rho\sigma}\left\{p^\nu\varpi^{\rho\sigma} + 2\frac{p^\rho}{p\cdot n}n^\sigma\left[p_\lambda\xi^{\nu\lambda} + \frac{1}{4}\partial^\nu\frac{\mu}{T} + p_\lambda\Theta^{\nu\lambda}\right] - 2p^\nu\mu_B F^{\rho\sigma}\right\}n_F(1-n_F)$$

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right) \qquad \Theta_{\mu\nu} = \mathfrak{S}_{\mu\nu} - \varpi_{\mu\nu}$$

- Thermal shear: **New effect in hydrodynamics!**
  essential for local spin polarization and to solve the sign puzzle

  [Becattini, MB, Palermo, Phys. Lett. B 820, 136519 (2021); Liu, Yin, JHEP 07 (2021); Becattini, MB, Inghirami, Karpenko, Palermo, PRL 127, 272302 (2021); Fu, Liu, Pang, Song, Yin, PRL 127 (2021) 14, 142301]

- Spin Hall Effect: Important at low energies

  [Liu, Yin, PRD 104, 054043 (2021)]

- Spin potential $\mathfrak{S}$: When spin is not yet equilibrated-> **Spin hydrodynamics!**

  [MB, PRC 105 (2022)]

- Electromagnetic field: small contribution

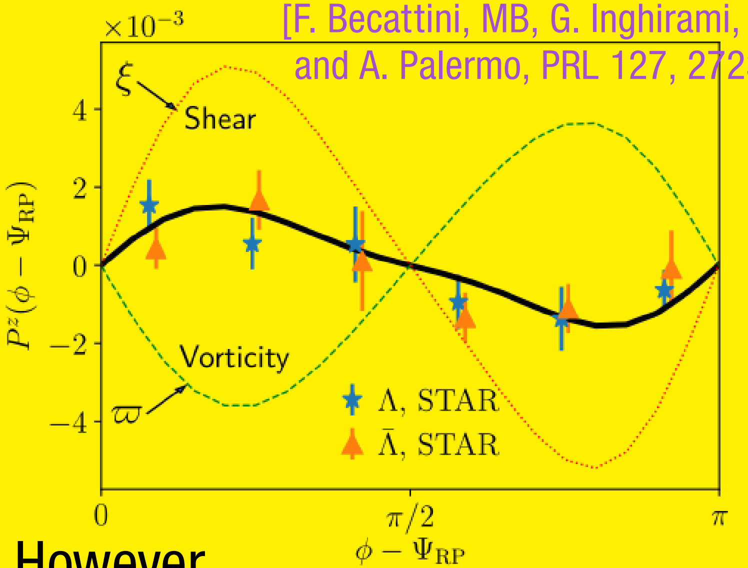
  [J.-H. Gao, Z.-T. Liang, S. Pu, Q. Wang, X.-N. Wang, PRL 109 (2012); MB, Nucl. Phys. A (2023)]

These are equilibrium and out-of-equilibrium non-dissipative effects.

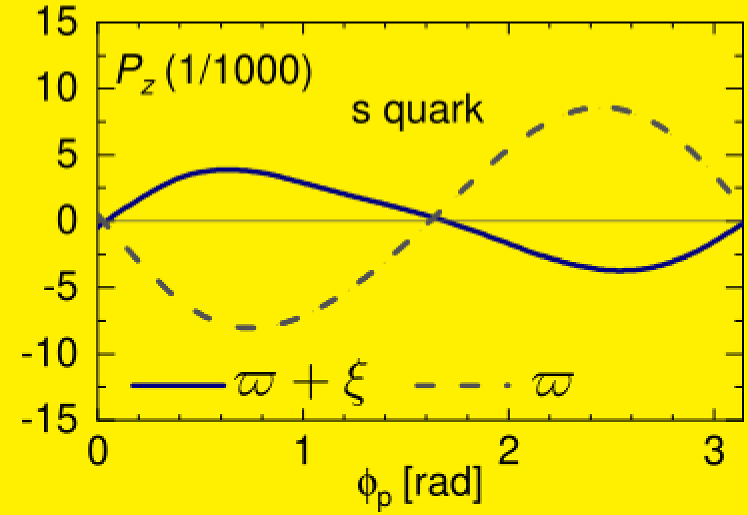
Solution to the sign puzzle?

Thermal shear-induced polarization can explain the local spin polarization data

[F. Becattini, MB, G. Inghirami, I. Karpenko, and A. Palermo, PRL 127, 272302 (2021)]



[Fu, Liu, Pang, Song, Yin, PRL 127 (2021)]



However,

- Isothermal local equilibrium vs strange quark scenario
- Different formulas for shear-induced polarization (kinetic vs statistical)
- Sensitivity of shear flow to properties of plasma (EoS and bulk viscosity)

[Jiang, Wu, Cao, Zhang, PRC 108, 064904 (2023); Palermo, Grossi, Karpenko and Becattini, EPJC 84 (2024)]

- Does spin reach equilibrium fast enough?

[M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov and H.-U. Yee, JHEP 08 (2022); D. Wagner, M. Shokri and D.H. Rischke, Phys. Rev. Res. 6 (2024)]

- Other effects and dissipative effects?

[N. Weickgenannt, D. Wagner, E. Speranza and D. H. Rischke, PRD 106 (2022); MB, JHEP 07 (2025)]

- Pseudo-gauge dependence?

[Becattini, Florkowski, Speranza, PLB 789 (2019); Speranza, Weickgenannt, EPJA 57 (2021)]

Spin hydrodynamics

Include spin in an hydrodynamic picture.

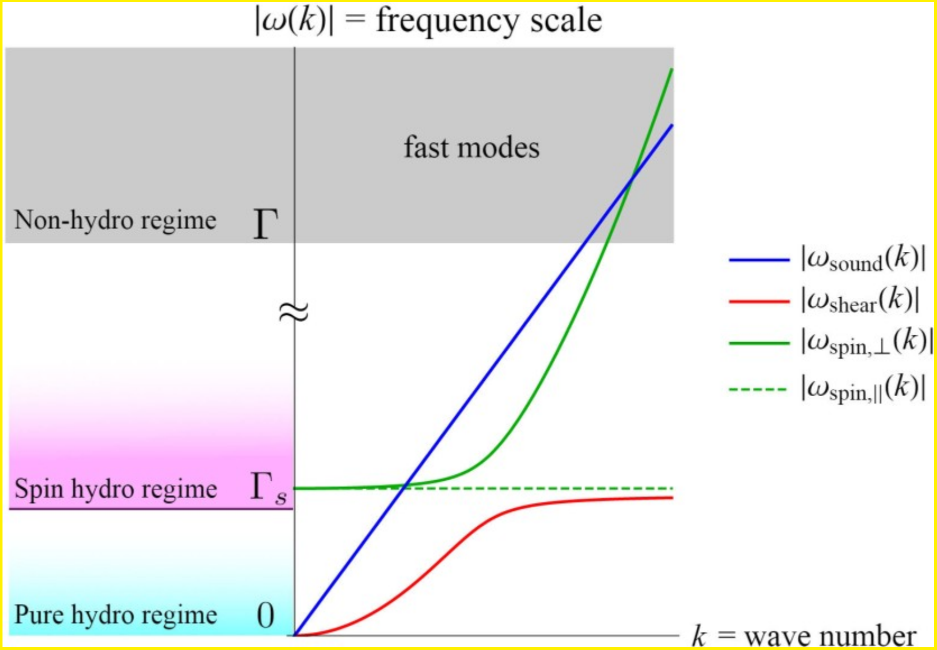
[W. Florkowski, B. Friman, A. Jaiswal and E. Speranza, Phys. Rev. C 97 (2018) no.4]

Spin hydrodynamics is necessary when the spin relaxation time scale is much longer than the time scale of e.g. kinetic equilibration.

[Becattini, Florkowski, Speranza, PLB 789 (2019); M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, JHEP 11 (2021)]

Reviews:

[X. G. Huang, Nucl. Sci. Tech. 36 (2025) no.11, 208;
W. Florkowski, A. Kumar and R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019)
W. Florkowski, B. Friman, A. Jaiswal and E. Speranza, Phys. Rev. C 97 (2018) no.4]



Include the spin tensor $S^{\lambda,\mu\nu}$ and the spin potential $\mathfrak{S}$ in the hydro equations:

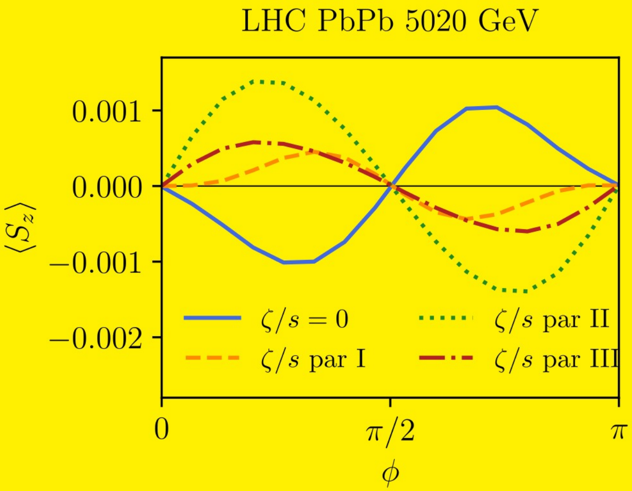
$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j^\mu = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

$$T^{\mu\nu} = T^{\mu\nu}(\beta, \zeta, \mathfrak{S}), \quad j^\mu = j^\mu(\beta, \zeta, \mathfrak{S}), \quad S^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}(\beta, \zeta, \mathfrak{S})$$

What is the spin polarization good for?

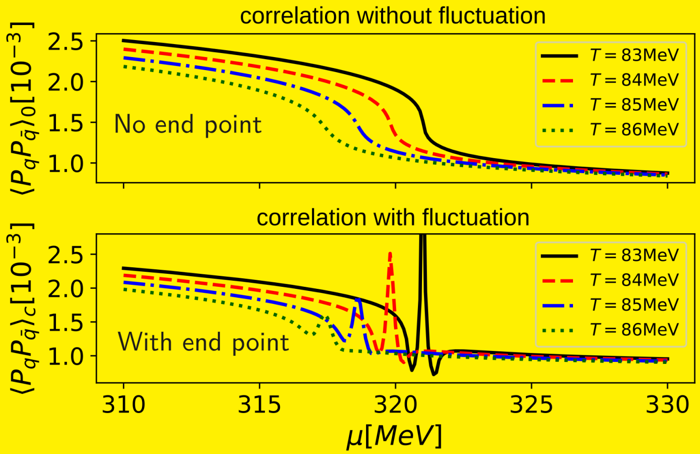
Spin polarization is the only observable that depends on the gradients of the hydrodynamic fields at the time of hadronization!

- Use of Λ polarization and spin-spin correlation to study vorticity structure and properties of the QGP



[L. Pang, H. Petersen, Q. Wang, and X.-N. Wang, PRL 117 (2016);
X. L. Xia, H. Li, Z. B. Tang and Q. Wang, PRC 98 (2018);
S. Ryu, V. Jupic, C. Shen, PRC 104 (2021);
Palermo, Grossi, Karpenko and Becattini, EPJC 84 (2024)]

- Probe for a signature of the QCD critical point

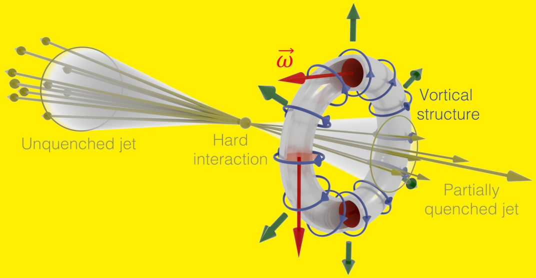


[S. K. Singh and J. e. Alam, EPJC {83 (2023)
H. L. Chen, W. j. Fu, X. G. Huang and G. L. Ma, PRL 135 (2025)]

What is the spin polarization good for?

- Investigate the energy loss of highly energetic partons in the QGP

[W. Serenone et al, PLB 820 (2021); V. H. Ribeiro, et al. PRC 109 (2024)]



- Probe local parity violation ζ_A (complementary to the Chiral Magnetic Effect);

$$S^\mu(p) = S^\mu_{\text{hydro}}(p) + \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \zeta_A n_F\,(1-n_F)}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F} \frac{\varepsilon p^\mu - m^2 \hat{t}^\mu}{m\varepsilon}$$

[F. Du, L. E. Finch and J. Sandweiss, PRC 78 (2008) F. Becattini, MB, A. Palermo, G. Prokhorov, PLB 822 (2021)]

- Quantum thermal effects in gravity and breaking of the Einstein equivalence principle at finite temperature

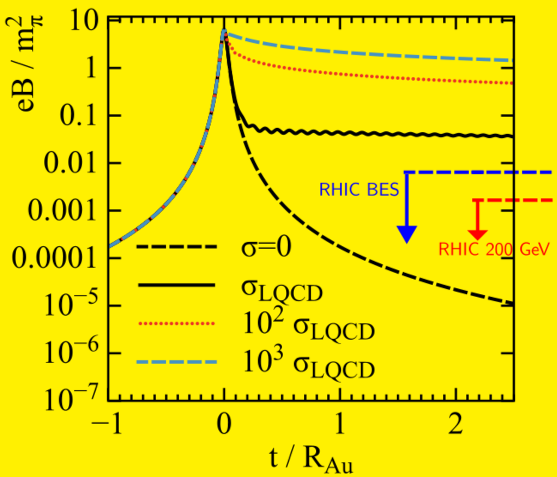
[MB, D. Kharzeev, Phys. Rev. D 103 (2021) 11, 116005]

$$\mathbf{S} \sim g_\Omega\, \boldsymbol{\omega} \quad g_\Omega = 1 - \frac{N_c^2-1}{2}\frac{1}{6}\frac{g^2T^2}{m^2},\, T \ll m$$

- Final time magnetic field intensity and conductivity of QGP

$$P_{\bar{\Lambda}} - P_\Lambda = \frac{2|\mu_\Lambda|B}{T}$$

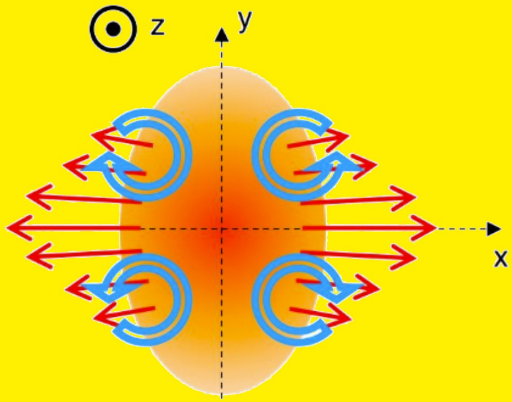
[L. McLerran and V. Skokov, Nucl. Phys. A 929 (2014); MB, Nucl. Phys. A (2023)]

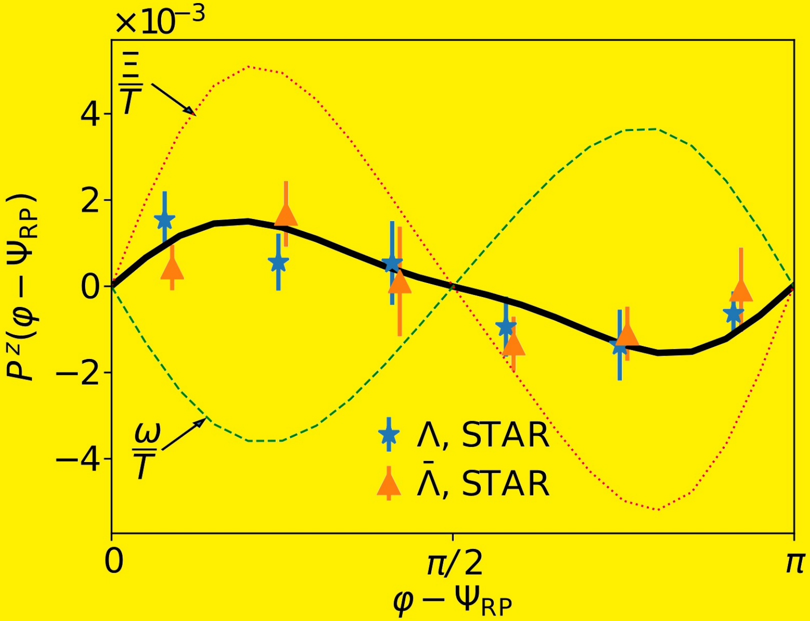


Dissipative effects in spin polarization

[based on MB, JHEP 07 (2025), 255; ArXiv:2502.15520]

Local spin polarization





F. Becattini, MB, G. Inghirami, I. Karpenko, and A. Palermo, PRL 127, 272302 (2021)

"Local": Momentum dependent polarization (along beam direction)

- Explained by incorporating shear effects: **are these dissipative**?
- However, the picture of equilibrated spins might not be complete

J.I. Kapusta, E. Rrapaj and S. Rudaz, PRCC 101 (2020)
S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar and R. Ryblewski, PLB 814 (2021)
M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov and H.-U. Yee, JHEP 08 (2022) 263
D. Wagner, M. Shokri and D.H. Rischke, Phys. Rev. Res. 6 (2024)

→ Develop Spin hydrodynamic and include a Spin potential

DISSIPATIVE EFFECTS?

Dissipative contributions

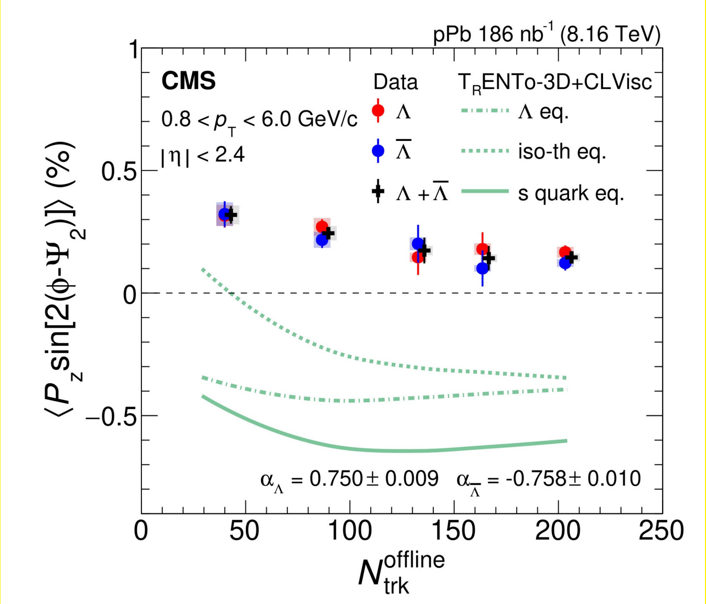
This talk goal: extend the spin polarization formula to dissipative effects

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\partial_\rho\beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F} + \text{Local (out-of-equilibrium) effects}$$

+ DISSIPATIVE EFFECTS

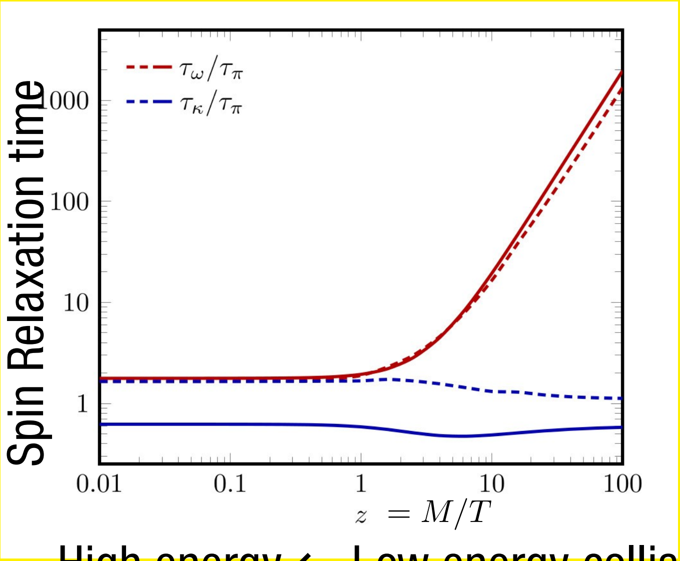
Where dissipative effects might be relevant?

Proton-Lead



Data: CMS, 2502.07898
Prediction: Yi, Wu, Zhu, Pu, Qin, PRC (2025)
(No spin hydro and no dissipative contributions)

Low energy



High energy ← Low energy collision

D. Wagner, M. Shokri and D.H. Rischke, Phys. Rev. Res. 6 (2024)

Early dynamics

S. K. Singh, R. Ryblewski and W. Florkowski, PRC 111 (2025)

Polarization from Wigner function

F. Becattini, MB, T. Niida, S. Pu, A. H. Tang and Q. Wang,
Int. J. Mod. Phys. E 33 (2024) no.06, 2430006

The definition of spin vector is based on the Pauli-Lubanski vector:

$$\widehat{S}^{\mu} = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\widehat{J}_{\nu\rho}\widehat{P}_{\sigma}$$

The covariant Wigner function:

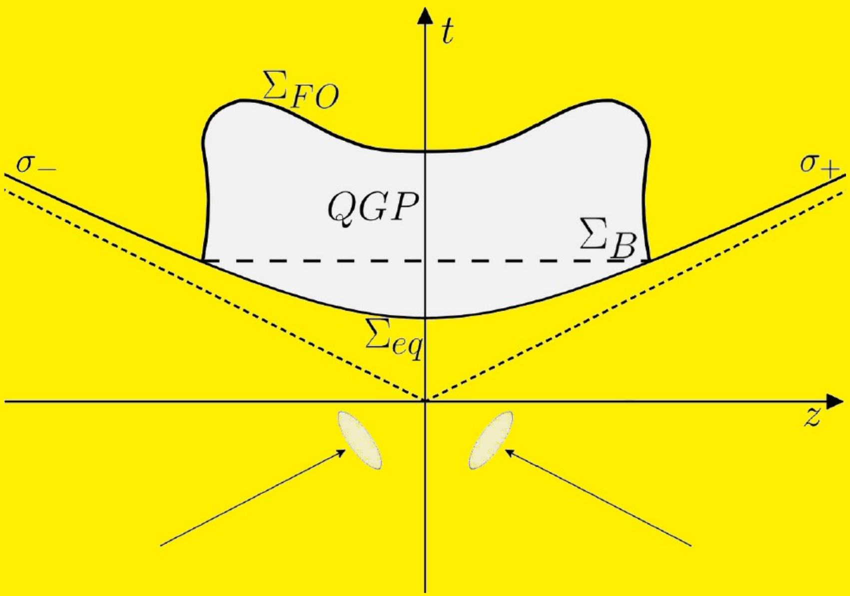
$$W(x,k)_{AB} = \frac{1}{(2\pi)^4}\int \mathrm{d}^4 y\, \mathrm{e}^{-\mathrm{i}k\cdot y}\langle :\bar{\Psi}_B(x+y/2)\Psi_A(x-y/2):\rangle$$

where: $\langle \widehat{X} \rangle = \mathrm{tr}\left(\widehat{\rho}\,\widehat{X}\right)$

It allows to calculate the mean spin vector:

$$S^{\mu}(k) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot k\ \mathrm{tr}_4\left(\gamma^{\mu}\gamma^{5}W_{+}(x,k)\right)}{\int \mathrm{d}\Sigma\cdot k\ \mathrm{tr}_4 W_{+}(x,k)} = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot k\ \mathcal{A}^{\mu}_{+}(x,k)}{\int \mathrm{d}\Sigma\cdot k\ \mathcal{F}_{+}(x,k)}$$

Non equilibrium statistical operator (Zubarev theory)



D.N. Zubarev, et al, Theor. Math. Phys. 1979, 40, 821
C.G. van-Weert, Ann. Phys. 1982, 140, 133
F. Becattini, MB, E. Grossi, Particles 2 (2019) 2, 197-207;
MB, Lect. Notes Phys. 987 (2021) 53-93.

$$\beta^\nu = \frac{u^\nu}{T}$$

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{eq}} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\,\widehat{j}^\mu\right)\right]$$

With the Gauss's theorem:

$$\widehat{\rho} = \frac{1}{Z}\exp\left[\underbrace{-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\,\widehat{j}^\mu\right)}_{\text{Local thermal equilibrium}} + \underbrace{\int_{\Omega} \mathrm{d}\Omega \left(\widehat{T}^{\mu\nu}\nabla_\mu\beta_\nu - \widehat{j}^\mu\nabla_\mu\zeta\right)}_{\text{Dissipative}}\right]$$

Entropy and dissipative part

D.N. Zubarev, A.V. Prozorkevich, S.A. Smolyanskii, Theor. Math. Phys. 1979, 40, 821
C.G. van-Weert, Ann. Phys. 1982, 140, 133
F. Becattini, M. B., E. Grossi, Particles 2 (2019) 2, 197-207; 1902.01089

$$\widehat{\rho}_{\rm LE}(\tau) = \frac{1}{Z}\exp\left[-\int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\,\widehat{j}^\mu\right)\right]$$

Entropy: $S(\tau) = -{\rm tr}\left(\widehat{\rho}_{\rm LE}(\tau)\log\widehat{\rho}_{\rm LE}(\tau)\right) = \int_{\Sigma(\tau)} {\rm d}\Sigma_\mu s^\mu$

$$= \log Z_{\rm LE} + \int_{\Sigma(\tau)} {\rm d}\Sigma_\mu \left(\langle\widehat{T}^{\mu\nu}\rangle_{\rm LE}\beta_\nu - \langle\widehat{j}^\mu\rangle_{\rm LE}\zeta\right)$$

$$\nabla\cdot s = \left(T^{\mu\nu} - \langle\widehat{T}^{\mu\nu}\rangle_{\rm LE}\right)\nabla_\mu\beta_\nu - \left(j^\mu - \langle\widehat{j}^\mu\rangle_{\rm LE}\right)\nabla_\mu\zeta$$

Definition: the part of a thermal average is **dissipative** if it is coming from the dissipative part of the statistical operator.

Only non-reversible effects (that are T-odd) are found to be dissipative, e.g. shear viscosity.

Caveat: A different definition is used in Quantum kinetic theory.

Hydrodynamic Limit

$$W(x,k) = \mathrm{tr}\left(\widehat{\rho}\,\widehat{W}(x,k)\right)$$

Expand the β, ζ and all the hydrodynamic fields from the point x where the Wigner operator is to be evaluated. For instance:

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda \beta_\nu(x)(y-x)^\lambda + \cdots$$

This gives at leading order

$$\int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}(y)\beta_\nu = \beta_\nu(x)\int_\Sigma \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu}(y) = \beta_\nu(x)\widehat{P}^\nu$$

And the local thermal equilibrium (LTE) part is approximated as

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \cdots\right]$$

$$\widehat{J}^{\mu\nu}_x = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}^{\lambda\mu}(y)\right]$$

$$\widehat{Q}^{\mu\nu}_x = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}^{\lambda\mu}(y)\right]$$

Linear response theory

In general, we obtain $\widehat{\rho} \simeq \frac{1}{Z} \exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q} + \widehat{B}_{\mathcal{U}} + \widehat{C}_{\mathcal{U}} + \cdots\right]$

$$\widehat{B}_{\mathcal{U}} = b_{\mathcal{U}}\, \mathcal{U}_{(\alpha)}(x)\, \widehat{\mathcal{B}}^{(\alpha)}_{\mathcal{U}} \qquad \widehat{C}_{\mathcal{U}} = c_{\mathcal{U}} \int_\Omega \mathrm{d}\Omega\, \mathcal{U}_{(\alpha)}(x_2)\, \widehat{\mathcal{C}}^{(\alpha)}_{\mathcal{U}}(x_2)$$

Where $\mathcal{U}_{(\alpha)}(x)$ is a generic hydrodynamic fields.

Using linear response theory, thermal averages reduce to the equilibrium ones:

$$\left\langle \widehat{O} \right\rangle_{\beta(x)} = \mathrm{tr}\left[\widehat{\rho}_{Eq}\, \widehat{O}\right], \quad \widehat{\rho}_{Eq} = \frac{1}{Z_{Eq}} \exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q}\right]$$

Using linear response theory:

$$W(x,k) = \langle \widehat{W}(x,k)\rangle_{\beta(x)} + \Delta_{\mathcal{U},\mathrm{LTE}} W(x,k) + \Delta_{\mathcal{U},\mathrm{D}} W(x,k) + \cdots$$

where:

$$\Delta_{\mathcal{U},\,\mathrm{D}} W(x,k) = \mathcal{U}_{(\alpha)}(x)\, c_{\mathcal{U}} \left(\widehat{W},\, \widehat{\mathcal{C}}^{(\alpha)}_{\mathcal{U}}\right)_{\mathrm{D}} \qquad \Delta_{\mathcal{U},\,\mathrm{LTE}} W(x) = \mathcal{U}_{(\alpha)}(x)\, b_{\mathcal{U}} \left(\widehat{W},\, \widehat{\mathcal{B}}^{(\alpha)}_{\mathcal{U}}\right)_{\mathrm{LTE}}$$

$$\left(\widehat{X},\, \widehat{Y}\right)_{\mathrm{D}} = \frac{\mathrm{i}}{|\beta(x)|} \int_{-\infty}^{t} \mathrm{d}^4 x_2 \int_{-\infty}^{t_2} \mathrm{d}s \left\langle \left[\widehat{X}(x),\, \widehat{Y}(s,\, \mathbf{x}_2)\right]\right\rangle_{\beta(x)}$$

$$\left(\widehat{X},\, \widehat{Y}\right)_{\mathrm{LTE}} = \int_0^{|\beta|} \frac{\mathrm{d}\tau}{|\beta(x)|} \langle \widehat{Y}_{[\tau/|\beta|]} \widehat{X}(x)\rangle_{\beta(x),\,\mathrm{c}}$$

$$\widehat{Y}_{[\tau/|\beta|]} = \mathrm{e}^{\frac{\tau}{|\beta|}\left(\beta(x)\cdot\widehat{P} - \zeta(x)\widehat{Q}\right)}\, \widehat{Y}\, \mathrm{e}^{-\frac{\tau}{|\beta|}\left(\beta(x)\cdot\widehat{P} - \zeta(x)\widehat{Q}\right)}$$

Spin hydrodynamics

We consider a general case without specifying the underlying QFT

[MB,JHEP 07 (2025), 255]

$$\widehat{\rho} = \frac{1}{Z} \exp\left\{ -\int_\Sigma \mathrm{d}\Sigma_\mu(y) \left(\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y) - \zeta_A(y)\widehat{j}^\mu_A(y) - \frac{1}{2}\mathfrak{S}_{\lambda\nu}(y)\widehat{S}^{\mu\lambda\nu}(y) \right)\right.$$

$$\left. + \int_\Omega \mathrm{d}\Omega \left[\widehat{T}^{\mu\nu}_S \xi_{\mu\nu} + \widehat{T}^{\mu\nu}_A \left(\mathfrak{S}_{\mu\nu} - \varpi_{\mu\nu}\right) - \widehat{j}^\mu \nabla_\mu \zeta - \nabla_\mu \left(\zeta_A \widehat{j}^\mu_A\right) - \frac{1}{2}\widehat{S}^{\mu\lambda\nu}\nabla_\mu \mathfrak{S}_{\lambda\nu} \right] \right\},$$

$$T^{\mu\nu}_S = \frac{1}{2}\left(T^{\mu\nu} + T^{\nu\mu}\right), \quad T^{\mu\nu}_A = \frac{1}{2}\left(T^{\mu\nu} - T^{\nu\mu}\right)$$

Where:

Thermal vorticity $\varpi_{\rho\sigma}(x) = -\frac{1}{2}\left[\partial_\rho \beta_\sigma(x) - \partial_\sigma \beta_\rho(x)\right]$

Thermal shear $\xi_{\rho\sigma}(x) = \frac{1}{2}\left[\partial_\rho \beta_\sigma(x) + \partial_\sigma \beta_\rho(x)\right]$

Spin potential $\mathfrak{S}_{\mu\nu}$ Axial chemical potential $\zeta_A = \mu_A/T$

- Hydrodynamic limit: expand the hydrodynamic fields
- The equilibrium has a residual SO(3) symmetry
- → Decompose the hydrodynamic fields into irreducible components
- Write down all the possible first order contribution to the Wigner function

$$\beta \sim u \sim \zeta \sim \mathcal{O}(\partial^0), \quad \zeta_A \sim \mathfrak{S} \ll \zeta, \quad \varpi \sim \xi \sim \partial\zeta \sim \partial\zeta_A \sim \partial\mathfrak{S} \sim \mathcal{O}(\partial^1)$$

- Use linear response theory to obtain the first order LTE and dissipative correction to Wigner function
- Match the linear response with the first order expression

→ Obtain the thermal and transport coefficients as Kubo formulas

Decomposition of the hydro fields

Gradients of chemical potential

$$\partial^\rho \zeta = u^\rho D\zeta + r^\rho, \quad r^\rho = \partial^{\langle\rho\rangle}\zeta$$

Gradients of axial chemical potential

$$\partial^\rho \zeta_A = u^\rho D\zeta_A + r_A^\rho, \quad r_A^\rho = \partial^{\langle\rho\rangle}\zeta_A$$

Thermal vorticity

Acceleration Rotation

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) = \alpha_\mu u_\nu - \alpha_\nu u_\mu + \epsilon_{\mu\nu\rho\sigma} w^\rho u^\sigma$$

Thermal shear

Shear tensor

$$\xi_{\rho\sigma} = u_\rho u_\sigma D\beta + \frac{\Delta_{\rho\sigma}}{3}\beta\theta + \frac{1}{2}\left(u_\rho \Delta^\tau_\sigma + u_\sigma \Delta^\tau_\rho\right)\left(\beta D u_\tau + \partial_\tau \beta\right) + \Delta^{\lambda\tau}_{\rho\sigma}\beta\sigma_{\lambda\tau}$$

Spin potential

$$\mathfrak{S}_{\mu\nu} = \mathfrak{a}_\mu u_\nu - \mathfrak{a}_\nu u_\mu + \epsilon_{\mu\nu\rho\sigma}\mathfrak{w}^\rho u^\sigma$$

Gradients of spin potential

$$\begin{aligned}\partial^\lambda \mathfrak{S}^{\mu\nu} =& u^\lambda\left(f^\mu u^\nu - f^\nu u^\mu\right) + \epsilon^{\lambda\mu\nu\rho}\Upsilon_\rho + \left(\Delta^{\lambda\mu}u^\nu - \Delta^{\lambda\nu}u^\mu\right) I \\ &+ \left(\epsilon^{\lambda\mu\alpha\beta}u^\nu - \epsilon^{\lambda\nu\alpha\beta}u^\mu\right) u_\alpha (I_\beta - \Upsilon_\beta) + \left(I_S^{\lambda\mu}u^\nu - I_S^{\lambda\nu}u^\mu\right) \\ &+ \varphi\,\epsilon^{\lambda\mu\nu\rho}u_\rho + \Phi^{\lambda,\mu\nu}_{S12} + \Phi^{\lambda,\mu\nu}_{S13}\end{aligned}$$

Notation:

$$\Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu, \quad V^{\langle\rho\rangle} = V^\rho_\perp = \Delta^\rho_\lambda V^\lambda, \quad D = u^\mu\partial_\mu, \quad \theta = \partial_\mu u^\mu,$$

$$\Delta_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\Delta_{\mu\rho}\Delta_{\nu\sigma} + \Delta_{\mu\sigma}\Delta_{\nu\rho}\right) - \frac{1}{3}\Delta_{\mu\nu}\Delta_{\rho\sigma} \qquad Q^{\mu\nu} = \frac{\Delta^{\mu\nu}}{3} - \frac{k^\mu_\perp k^\nu_\perp}{k^2_\perp}$$

Axial Wigner function

[MB, JHEP 07 (2025), 255]

The non-dissipative part:

Chiral imbalance

Spin Hall Effect

Thermal vorticity

Spin potential

Acceleration and grad. of temperature

Shear tensor

$$\Delta_{\rm LTE}\mathcal{A}^\mu_+ = \left[a_{\zeta_A u}u^\mu + a_{\zeta_A k}\frac{k^\mu_\perp}{(k\cdot u)}\right]\zeta_A + a^c_{r\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\partial_{\langle\rho\rangle}\zeta$$
$$-a_{\varpi}\frac{2\widetilde{\varpi}^{\mu\nu}k_\nu}{(2\pi)^3} - (a_{wu}-a_{w\Delta})\frac{(k\cdot w)}{(k\cdot u)}u^\mu + (a_{\alpha\epsilon}-a_{w\Delta})\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\alpha_\rho + a_{wk}Q^{\mu\rho}w_\rho$$
$$-a_{\mathfrak{S}-\varpi}\frac{2(\widetilde{\mathfrak{S}}^{\mu\nu}-\widetilde{\varpi}^{\mu\nu})k_\nu}{(2\pi)^3} - (a_{\mathfrak{w}-wu}-a_{\mathfrak{w}-w\Delta})\frac{k^\rho_\perp u^\mu}{(k\cdot u)}(\mathfrak{w}-w)_\rho$$
$$+(a_{\mathfrak{a}-\alpha\epsilon}-a_{\mathfrak{w}-w\Delta})\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(\mathfrak{a}-\alpha)_\rho + a_{\mathfrak{w}-wk}Q^{\mu\rho}(\mathfrak{w}-w)_\rho$$
$$+a_{q\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(\beta D u_\rho + \partial_\rho\beta) + a_{\sigma\epsilon}\epsilon^{\mu\nu\alpha\rho}k^\sigma_\perp\frac{u_\nu k_\alpha}{(k\cdot u)}\beta\sigma_{\rho\sigma}$$

Thermal vorticity → main effect for global spin polarization
[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)]

Thermal shear → needed to explain local spin polarization
[F. Becattini, MB, A. Palermo, PLB 820 (2021)]
[S. Liu, Y. Yin, JHEP 07 (2021) 188]

Chiral imbalance → Can be used to probe topological charge in alternative to CME
[F. Becattini, M. Buzzegoli, A. Palermo and G. Prokhorov, PLB 822 (2021)]

Spin potential → Additional contribution
[MB, PRC 105 (2022)]

Remind that spin polarization is $S^\mu(k) = \frac{1}{8m}\frac{\int {\rm d}\Sigma\cdot k\;\mathcal{A}^\mu_+(x,k)}{\int {\rm d}\Sigma\cdot k\;n_f(\beta(x)\cdot k)}$

Axial Wigner function

[MB, JHEP 07 (2025), 255]

NEW IN THIS WORK The dissipative part:

$$\Delta_{\mathrm{D}}\mathcal{A}^\mu_+ = \left[\bar{a}_{D\zeta_A u}u^\mu + \bar{a}_{D\zeta_A k}\frac{k^\mu_\perp}{(k\cdot u)}\right] D\zeta_A + \left[\bar{a}_{r_A u}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar{a}_{r_A\Delta}\Delta^{\mu\rho} + \bar{a}_{r_A k}Q^{\mu\rho}\right]\partial_{\langle\rho\rangle}\zeta_A$$

Gradients of chiral imbalance

$$+\,\bar{a}_{f\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}f_\rho + \left[\bar{a}_{\Upsilon u}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar{a}_{\Upsilon\Delta}\Delta^{\mu\rho} + \bar{a}_{\Upsilon k}Q^{\mu\rho}\right]\Upsilon_\rho$$
$$+\left[\bar{a}_{I-\Upsilon u}\frac{k^\rho_\perp u^\mu}{(k\cdot u)} + \bar{a}_{I-\Upsilon\Delta}\Delta^{\mu\rho} + \bar{a}_{I-\Upsilon k}Q^{\mu\rho}\right](I_\rho - \Upsilon_\rho) + \left[\bar{a}_{\varphi u}u^\mu + \bar{a}_{\varphi k}\frac{k^\mu_\perp}{(k\cdot u)}\right]\varphi + \bar{a}_{I_S\epsilon}\,\epsilon^{\mu\nu\alpha\rho}k^\sigma_\perp\frac{u_\nu k^\perp_\alpha}{(k\cdot u)^2}I_{S\,\rho\sigma}$$
$$+\left[\bar{a}_{\mathrm{S}12\Delta\epsilon}\Delta^{\mu\tau}\epsilon^{\lambda\nu\rho\sigma}\frac{u_\lambda k^\perp_\nu}{(k\cdot u)} + \bar{a}_{\mathrm{S}12\epsilon}\epsilon^{\mu\nu\rho\sigma}\frac{k^\tau_\perp u_\nu}{(k\cdot u)}\right]\Phi^{S12}_{\tau,\rho\sigma} + \left[\bar{a}_{\mathrm{S}13\Delta\epsilon}\Delta^{\mu\tau}\epsilon^{\lambda\nu\rho\sigma}\frac{u_\lambda k^\perp_\nu}{(k\cdot u)} + \bar{a}_{\mathrm{S}13\epsilon}\epsilon^{\mu\nu\rho\sigma}\frac{k^\tau_\perp u_\nu}{(k\cdot u)}\right]\Phi^{S13}_{\tau,\rho\sigma}$$

Gradients of spin potential

Decomposition of the spin potential gradients

$$\partial^\lambda\mathfrak{S}^{\mu\nu} = u^\lambda\left(f^\mu u^\nu - f^\nu u^\mu\right) + \epsilon^{\lambda\mu\nu\rho}\Upsilon_\rho + \left(\Delta^{\lambda\mu}u^\nu - \Delta^{\lambda\nu}u^\mu\right)I + \left(I_S^{\lambda\mu}u^\nu - I_S^{\lambda\nu}u^\mu\right)$$
$$+\left(\epsilon^{\lambda\mu\alpha\beta}u^\nu - \epsilon^{\lambda\nu\alpha\beta}u^\mu\right)u_\alpha(I_\beta - \Upsilon_\beta) + \varphi\,\epsilon^{\lambda\mu\nu\rho}u_\rho + \Phi^{\lambda,\mu\nu}_{S12} + \Phi^{\lambda,\mu\nu}_{S13}$$

Remind that spin polarization is $S^\mu(k) = \dfrac{1}{8m}\dfrac{\int \mathrm{d}\Sigma\cdot k\;\mathcal{A}^\mu_+(x,k)}{\int \mathrm{d}\Sigma\cdot k\; n_f(\beta(x)\cdot k)}$

Axial Wigner function

$$\downarrow$$

(Local) Spin polarization

- All dissipative coefficients are odd under time-reversal
- No dissipative contribution from: shear tensor, rate of expansion or gradients of temperature because they break parity symmetry
- The only dissipative contributions at first order are given by the chiral imbalance and by gradients of the spin potential

- For interacting fields, there could be more contributions proportional to thermal vorticity at LTE!

$$\Delta_{\varpi,\mathrm{LTE}}\mathcal{A}^\mu_+ = -a_\varpi \frac{2\widetilde{\varpi}^{\mu\nu}k_\nu}{(2\pi)^3} \underbrace{-(a_{wu}-a_{w\Delta})\frac{(k\cdot w)}{(k\cdot u)}u^\mu + (a_{\alpha\epsilon}-a_{w\Delta})\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\alpha_\rho + a_{wk}Q^{\mu\rho}w_\rho}_{=0 \text{ for free Dirac field}}$$

$$a^{\mathrm{Free}}_\varpi = \delta(k^2-m^2)\theta(k\cdot u)n_F(\beta\cdot k)\left(1-n_F(\beta\cdot k)\right)$$

$\rightarrow$ Radiative corrections: [MB, D. Kharzeev, PRD 103 (2021)][S. Fang, S. Pu and D. L. Yang, 2503.13320]

The transport and thermal coefficients

Examples, LTE thermal-vorticity: $a_{\varpi} = -\frac{(2\pi)^3}{6(k\cdot u)}\left(\widehat{\mathcal{A}}^{\mu}_{+},\, \widehat{J}_{x\,\mu}\right)_{\rm LTE}$ $\quad \widehat{J}^{\rho}_{x} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\rho}u_{\alpha}\widehat{J}_{x\,\beta\gamma}$

LTE shear tensor: $a_{\sigma\epsilon} = -\frac{(k\cdot u)^2}{4(k_\perp^2)^2}\left(\widehat{\mathcal{A}}^{\mu}_{+},\, u^{\lambda}\epsilon_{\lambda\mu\tau\rho}k^{\perp}_{\sigma}k^{\tau}_{\perp}\widehat{\pi}^{\rho\sigma}_{\Xi}\right)_{\rm LTE}$

$$\widehat{\pi}^{\mu\nu}_{\Xi} = \Delta^{\mu\nu}_{\alpha\beta}\int_{\Sigma}{\rm d}\Sigma_{\lambda}(y)\left[(y-x)^{\alpha}\widehat{T}^{\lambda\beta}(y) + (y-x)^{\beta}\widehat{T}^{\lambda\alpha}(y)\right]$$

Dissipative spin potential: $\bar{a}_{\Upsilon u} = \frac{(k\cdot u)}{k_\perp^2}\left(u_{\mu}\widehat{\mathcal{A}}^{\mu},\, k^{\perp}_{\rho}\widehat{\mathcal{C}}^{\rho}_{\Upsilon}\right)_{\rm D}$

$$\widehat{\mathcal{C}}_{\partial\mathfrak{S}}(x_2) = \widehat{S}^{\tau,\rho\sigma}(x_2) - 2(x_2-x)^{\tau}\widehat{T}^{\rho\sigma}_{A}(x_2) \to \widehat{\mathcal{C}}^{\rho}_{\Upsilon}$$

Classification of coefficients

The coefficient have been classified according to their properties under discrete transformations: P parity conjugation, T time reversal and C charge conjugation

	$a_{\mathcal{U}},\, v_{\mathcal{U}}$	$a^c_{\mathcal{U}},\, v^c_{\mathcal{U}}$	$\bar{a}_{\mathcal{U}},\, \bar{v}_{\mathcal{U}}$	$\mathfrak{a}_{\mathcal{U}},\, \mathfrak{v}_{\mathcal{U}}$	$\bar{a}^c_{\mathcal{U}},\, \bar{v}^c_{\mathcal{U}}$	$\mathfrak{a}^c_{\mathcal{U}},\, \mathfrak{v}^c_{\mathcal{U}}$	$\bar{\mathfrak{a}}_{\mathcal{U}},\, \bar{\mathfrak{v}}_{\mathcal{U}}$	$\bar{\mathfrak{a}}^c_{\mathcal{U}},\, \bar{\mathfrak{v}}^c_{\mathcal{U}}$
P	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$-$
T	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$-$
C	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$-$

Example, a coefficient is chiral if its parity under charge conjugation is odd, i.e.

$$\widehat{\mathsf{P}}\,\widehat{O}\,\widehat{\mathsf{P}}^{-1} = \eta_O\widehat{O}, \quad \widehat{\mathsf{P}}\,\widehat{B}\,\widehat{\mathsf{P}}^{-1} = \eta_B\widehat{B}, \quad \mathfrak{a} = \langle\widehat{O}\,\widehat{B}\rangle_{\beta}\ \text{if}\ \eta_O\,\eta_B = -1$$

To have a non-vanishing chiral coefficient, a chiral imbalance or parity violating interactions are needed!

Axial Wigner function with parity breaking

[MB, JHEP 07 (2025), 255]

NEW IN THIS WORK The chiral non-dissipative part:

$$\Delta_{\mathrm{LTE},\chi}\mathcal{A}^\mu_+ = \mathfrak{a}_{r_A\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\partial_{\langle\rho\rangle}\zeta_A$$

Chiral spin Hall effect

$$+\left[\mathfrak{a}_{fu}\frac{k^\rho_\perp u^\mu}{(k\cdot u)}+\mathfrak{a}_{f\Delta}\Delta^{\mu\rho}+\mathfrak{a}_{fk}Q^{\mu\rho}\right]f_\rho+\mathfrak{a}_{\Upsilon\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\Upsilon_\rho$$
$$+\left[\mathfrak{a}_{Iu}u^\mu+\mathfrak{a}_{Ik}\frac{k^\mu_\perp}{(k\cdot u)}\right]I+\mathfrak{a}_{I-\Upsilon\epsilon}\,\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(I_\rho-\Upsilon_\rho)+\left[\mathfrak{a}_{I_Su}\frac{k^\rho_\perp k^\sigma_\perp}{k^2_\perp}u^\mu+\mathfrak{a}_{I_S\Delta}\frac{\Delta^{\mu\rho}k^\sigma_\perp}{(k\cdot u)}+\mathfrak{a}_{I_Sk}\frac{Q^{\mu\rho}k^\sigma_\perp}{(k\cdot u)}\right]I_{S\,\rho\sigma}$$
$$+\left[\mathfrak{a}_{\mathrm{S12}\Delta}\Delta^{\tau\rho}\Delta^{\mu\sigma}+\mathfrak{a}_{\mathrm{S12k}}Q^{\tau\rho}\Delta^{\mu\sigma}\right]\Phi^{S12}_{\tau,\rho\sigma}+\left[\mathfrak{a}_{\mathrm{S13}\Delta}\Delta^{\tau\sigma}\Delta^{\mu\rho}+\mathfrak{a}_{\mathrm{S13k}}Q^{\tau\sigma}\Delta^{\mu\rho}\right]\Phi^{S13}_{\tau,\rho\sigma}.$$

Gradients of spin potential

NEW IN THIS WORK The chiral dissipative part:

$$\Delta_{\mathrm{D},\chi}\mathcal{A}^\mu_+ = \left[\bar{\mathfrak{a}}^c_{D\zeta u}u^\mu+\bar{\mathfrak{a}}^c_{D\zeta k}\frac{k^\mu_\perp}{(k\cdot u)}\right]D\zeta+\left[\bar{\mathfrak{a}}^c_{ru}\frac{k^\rho_\perp u^\mu}{(k\cdot u)}+\bar{\mathfrak{a}}^c_{r\Delta}\Delta^{\mu\rho}+\bar{\mathfrak{a}}^c_{rk}Q^{\mu\rho}\right]\partial_{\langle\rho\rangle}\zeta$$

Gradients of chemical potential

$$+\bar{\mathfrak{a}}_{\mathfrak{a}-\alpha\Delta}\Delta^{\mu\rho}(\mathfrak{a}-\alpha)_\rho-\bar{\mathfrak{a}}_{\mathfrak{a}-\alpha u}\frac{k^\rho_\perp u^\mu}{(k\cdot u)}(\mathfrak{a}-\alpha)_\rho+\bar{\mathfrak{a}}_{\mathfrak{w}-w\epsilon}\epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}(\mathfrak{w}-w)_\rho$$
$$+\bar{\mathfrak{a}}_{\mathfrak{a}-\alpha k}Q^{\mu\rho}(\mathfrak{a}-\alpha)_\rho$$

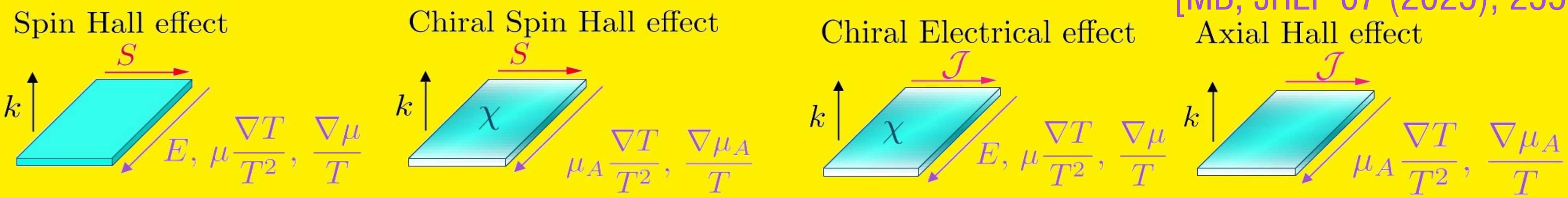
Spin potential

$$+\left[\bar{\mathfrak{a}}_{D\beta u}u^\mu+\bar{\mathfrak{a}}_{D\beta k}\frac{k^\mu_\perp}{(k\cdot u)}\right]D\beta+\left[\bar{\mathfrak{a}}_{\theta u}u^\mu+\bar{\mathfrak{a}}_{\theta k}\frac{k^\mu_\perp}{(k\cdot u)}\right]\beta\theta$$
$$+\left[\bar{\mathfrak{a}}_{qu}\frac{k^\rho_\perp u^\mu}{(k\cdot u)}+\bar{\mathfrak{a}}_{q\Delta}\Delta^{\mu\rho}+\bar{\mathfrak{a}}_{qk}Q^{\mu\rho}\right](\beta Du_\rho+\partial_\rho\beta)+\bar{\mathfrak{a}}_{\sigma u}(k\cdot u)\frac{k^\rho_\perp k^\sigma_\perp}{k^2_\perp}u^\mu\,\beta\sigma_{\rho\sigma}$$
$$+\bar{\mathfrak{a}}_{\sigma\Delta}\Delta^{\mu\rho}k^\sigma_\perp\beta\sigma_{\rho\sigma}+\bar{\mathfrak{a}}_{\sigma k}Q^{\mu\rho}k^\sigma_\perp\,\beta\sigma_{\rho\sigma}$$

Shear tensor

Chiral Spin Hall Effects

[MB, JHEP 07 (2025), 255]



Axial part of Wigner function:

SHE, Chiral SHE

$$\Delta_{\rm SHE}\mathcal{A}^\mu_+(x,k) = \epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\left[a^c_{r\epsilon}(k)\partial_\rho\zeta + \mathfrak{a}_{r_A\epsilon}(k)\partial_\rho\zeta_A\right]$$

Vector part of Wigner function:

Chiral electrical effect, Axial Hall effect

$$\Delta_{\rm SHE}\mathcal{V}^\mu_+(x,k) = \epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\left[\mathfrak{v}_{r\epsilon}(k)\partial_\rho\zeta + v^c_{r_A\epsilon}(k)\partial_\rho\zeta_A\right]$$

The currents are vanishing: $j^\mu_{\rm SHE} = \int {\rm d}^4k\,\Delta_{\rm SHE}\mathcal{V}^\mu(x,k) = 0, \quad j^\mu_{\rm A,SHE} = \int {\rm d}^4k\,\Delta_{\rm SHE}\mathcal{A}^\mu(x,k) = 0$

But the spin vector is

$$\mathbf{S}(k) = \mathbf{k}\times\left\langle\left\langle a^c_{r\epsilon}\left(\frac{\boldsymbol{\nabla}\mu}{T} + \mu\boldsymbol{\nabla}\frac{1}{T}\right)\right\rangle\right\rangle + \mathbf{k}\times\left\langle\left\langle \mathfrak{a}_{r_A\epsilon}\left(\frac{\boldsymbol{\nabla}\mu_A}{T} + \mu_A\boldsymbol{\nabla}\frac{1}{T}\right)\right\rangle\right\rangle$$

It can be used to probe anisotropies in the topological charge

Conclusions

- All possible first order dissipative effects on spin polarization have been classified
- Only the gradients of spin potential contribute without breaking the parity symmetry
- Outlook: estimate the phenomenological impact,

  for instance, compute the transport coefficients

- With interactions there could be additional contributions even at LTE
- Chiral Spin Hall Effect is a LTE effect contributing to local spin polarization

Thank you for the attention!

BACKUP SLIDES

Chiral Spin Hall Effect
free field

[MB, 2502.15520]

Axial part of Wigner function:

SHE, Chiral SHE

$$\Delta_{\rm SHE}\mathcal{A}^\mu_+(x,k) = \epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\left[a^c_{r\epsilon}(k)\partial_\rho\zeta + \mathfrak{a}_{r_A\epsilon}(k)\partial_\rho\zeta_A\right]$$

Vector part of Wigner function:

Chiral electrical effect, Axial Hall effect

$$\Delta_{\rm SHE}\mathcal{V}^\mu_+(x,k) = \epsilon^{\mu\nu\rho\sigma}\frac{k^\perp_\nu u_\sigma}{(k\cdot u)}\left[\mathfrak{v}_{r\epsilon}(k)\partial_\rho\zeta + v^c_{r_A\epsilon}(k)\partial_\rho\zeta_A\right]$$

$$a^c_{r\epsilon} = -\frac{\delta(k^2)\theta(k\cdot u)}{(2\pi)^3}\left[n_F^R(x,k)(1-n_F^R(x,k)) + n_F^L(x,k)(1-n_F^L(x,k))\right],$$

$$\mathfrak{a}_{r_A\epsilon} = -\frac{\delta(k^2)\theta(k\cdot u)}{(2\pi)^3}\left[n_F^R(x,k)(1-n_F^R(x,k)) - n_F^L(x,k)(1-n_F^L(x,k))\right],$$

$$\mathfrak{v}_{r\epsilon} = -\frac{\delta(k^2)\theta(k\cdot u)}{(2\pi)^3}\left[n_F^R(x,k)(1-n_F^R(x,k)) - n_F^L(x,k)(1-n_F^L(x,k))\right],$$

$$v^c_{r_A\epsilon} = -\frac{\delta(k^2)\theta(k\cdot u)}{(2\pi)^3}\left[n_F^R(x,k)(1-n_F^R(x,k)) + n_F^L(x,k)(1-n_F^L(x,k))\right],$$

$$n_F^\chi(x,k) = \frac{1}{\mathrm{e}^{\beta(x)\cdot k-\zeta(x)-\chi\zeta_A(x)}+1}, \quad \chi = \begin{cases} +1 & R \\ -1 & L\end{cases}$$

Kubo formulas in momentum space

$$\left(\widehat{X},\,\widehat{Y}\right)_{\rm D} = \frac{\rm i}{|\beta(x)|}\int_{-\infty}^{t} {\rm d}^4x_2 \int_{-\infty}^{t_2} {\rm d}s\,\left\langle\left[\widehat{X}(x),\,\widehat{Y}(s,\,x_2)\right]\right\rangle_{\beta(x)}$$

$$G^{R}_{\widehat{X}\widehat{Y}}(x-x_2) = -\,{\rm i}\theta(x-x_2)\left\langle\left[\widehat{X}(x),\,\widehat{Y}(x_2)\right]\right\rangle_{\beta(x)},$$

$$G^{R}_{\widehat{X}\widehat{Y}}(x) = \int \frac{{\rm d}^4p}{(2\pi)^3}{\rm e}^{-{\rm i}p\cdot x}G^{R}_{\widehat{X}\widehat{Y}}(p).$$

$$\left(\widehat{X},\,\widehat{Y}\right)_{\rm D} = -\frac{1}{|\beta(x)|}u^{\lambda}\lim_{p\cdot u\to 0}\lim_{p_\perp\to 0}\frac{\partial}{\partial p^{\lambda}}{\rm Im}\,G^{R}_{\widehat{X}\widehat{Y}}(p)$$