Fragmentation processes at hadron colliders at NNLO+NNLL

Based on work by

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IFJ PAN Division IV seminar
Kraków, Poland, 13 November 2025

UNIVERSITY OF CAMBRIDGE

An overview of some recent calculations of fragmentation-style processes performed at next-to-next-to-leading order in QCD

Part I:

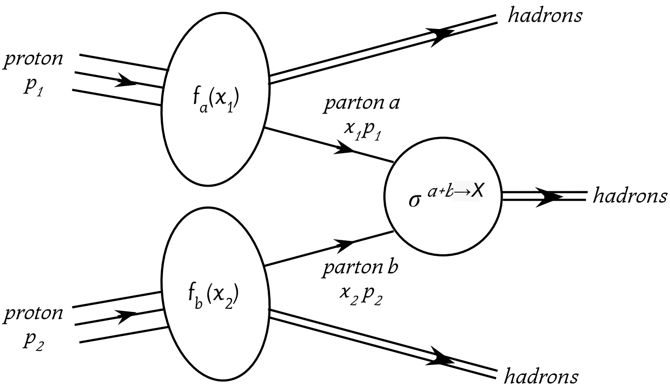
Light hadron fragmentation

arXiv:2503.11489

in collaboration with

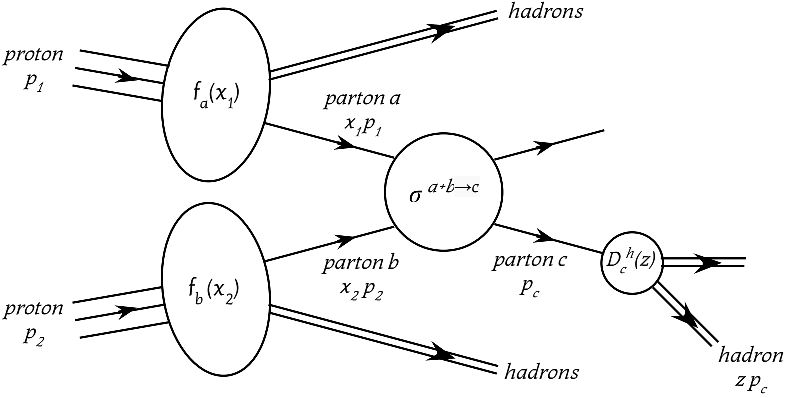
Michał Czakon, Alexander Mitov, René Poncelet

Standard QCD collinear factorisation



$$\sigma^{pp\to X}(p_1,p_2) = \sum_{a,b} \iint_0^1 f_a(x_1) f_b(x_2)$$
$$\sigma^{a+b\to X}(p_a = x_1 p_1, p_b = x_2 p_2)\, dx_1\, dx_2 + \mathcal{O}(m_p^n/Q^n)$$

Final-state factorisation



$$\sigma^{pp\to h+X}(p_1,p_2,p_h) = \sum_{a,b,c} \iiint_0^1 f_a(x_1) f_b(x_2) D^{\mathrm{NP}}_{c\to h}(z)\delta(p_h - zp_c)$$

$$\sigma^{a+b\to c}(p_a = x_1 p_1, p_b = x_2 p_2, p_c)\,dx_1\,dx_2\,dz + \mathcal{O}(m_p^n/Q^n, m_h^m/E_h^m)$$

The goal

- Describe $p\,p \to h + X$ at NNLO
- Until now: no NNLO predictions for the LHC available
- Final roadblock for global NNLO FF fits
- Previous fits either limited to NLO (poor theory precision) or to SIA data only (poor sensitivity to gluon FF)
- Since two years ago: can also include SIDIS data at NNLO
  Goyal, Moch, Pathak, Rana, Ravindran (2023, 2025); Bonino, Gehrmann, Stagnitto (2024); Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024, 2025); Goyal et al. (2024); Ahmed et al. (2024)
- Now: can include measurements from any collider!
- Note: only hadron colliders are sensitive to gluon FFs at LO

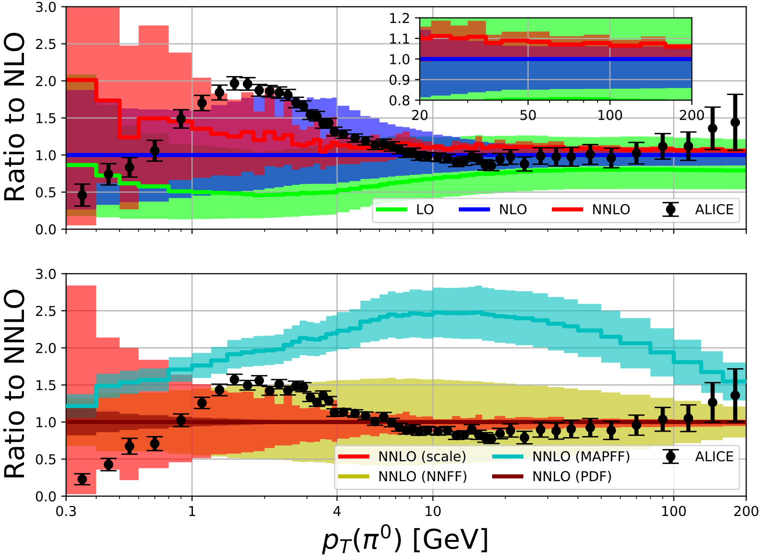
Implementation

- STRIPPER framework: designed to handle the real phase space of any NNLO cross section (without fragmentation)
- Underlying technology: sector-improved residue subtraction scheme

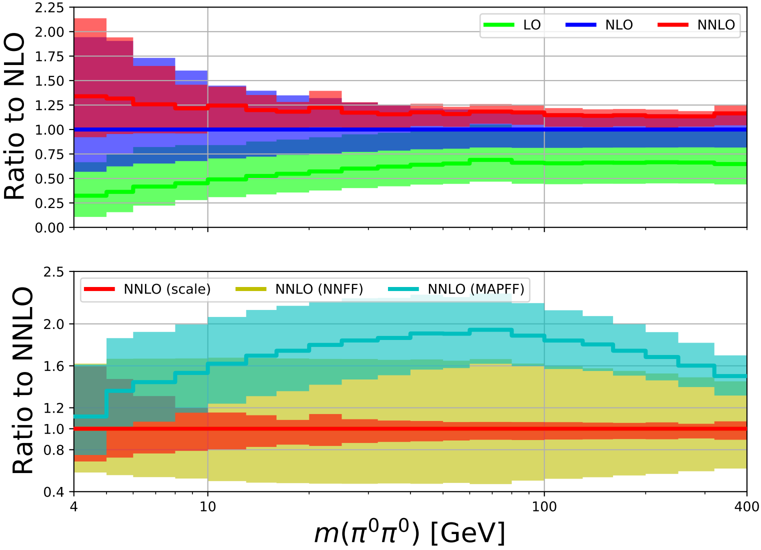
  Czakon (2010, 2011); Czakon, Heymes (2014); Czakon, van Hameren, Mitov, Poncelet (2019)
- Extended to support fragmentation a few years ago

  Czakon, TG, Mitov, Poncelet (2021)
- This involved:
  - Staying differential in longitudinal momentum fraction ('distinguishable' collinear partons)
  - Being able to convolve with arbitrary FFs
  - Adding collinear renormalisation counterterms (as for PDFs)
- As the base code: fully general implementation
- I.e.: any process with any number of identified hadrons supported!
- Here: light hadron fragmentation!

Results: π^0 p_T spectrum at 8 TeV LHC (ALICE)

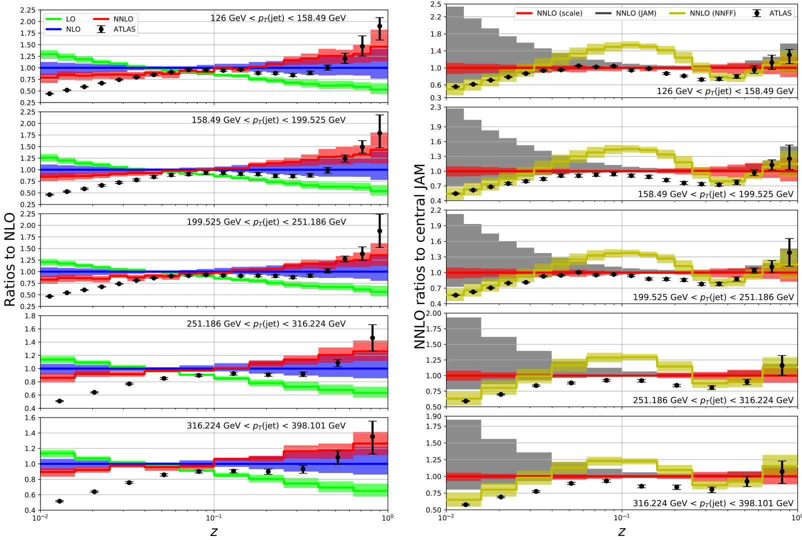


Results: $\pi^0\pi^0$ invariant mass spectrum at 13 TeV LHC



Results: charged hadrons at 5.02 TeV LHC (ATLAS)

Jets with $R = 0.4$ anti-k_T, $z = p_T(h)\cos(\Delta R)/p_T(\text{jet})$



Part II:

$p\, p \longrightarrow W^{\pm} D^{\mp}$ at NNLO

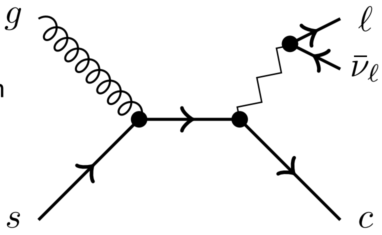
arXiv:2510.24525

in collaboration with

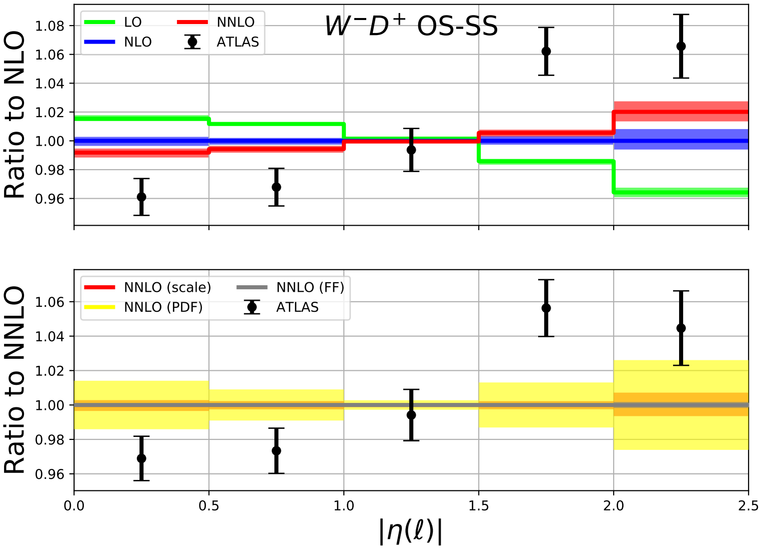
René Poncelet and Miha Muškinja

The goal

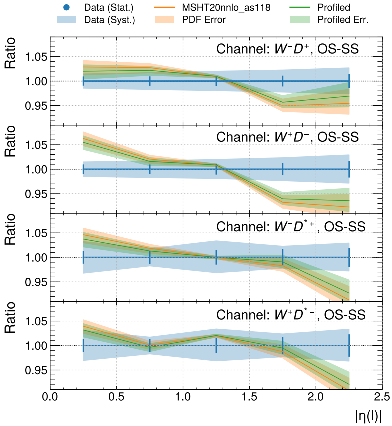
- Describe $p\,p \to W^{\pm} D^{\mp}$ at NNLO
- Interesting for strange-quark PDF fits
- Experimentally and theoretically cleaner than $W + c$-jet
- ATLAS measurement for both W^{+} and W^{-} (arXiv:2302.00336)
- $\Rightarrow$ Can constrain $s - \bar{s}$ asymmetry!
- Study normalised lepton rapidity spectrum
- Very sensitive to strange PDFs,
  other theory errors drop out

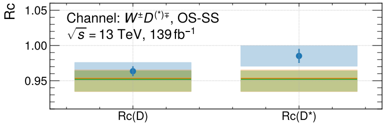


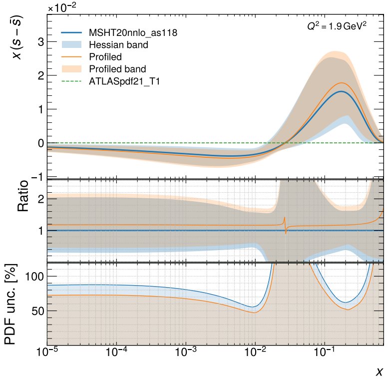
Results: normalised lepton rapidity spectrum



Results: PDF profiling







- Slightly increased $s - \bar{s}$ asymmetry after inclusion of $W^{\pm} D^{\mp}$ data

Part III:

Open bottom at NNLO+NNLL

arXiv:2411.09684

in collaboration with

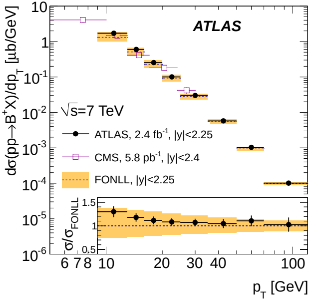
Michał Czakon, Alexander Mitov, René Poncelet

The goal

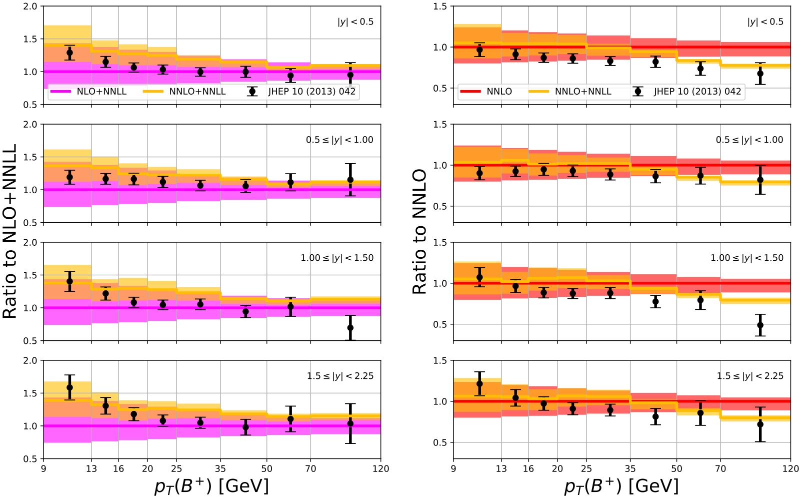
- Want to describe the process $pp \to b/\bar{b} + X$ with high precision
- At high energies, the b-quark is effectively almost massless
- $\Rightarrow$ Quasi-collinear singularities ($b \to b\, g$, $g \to b\, \bar{b}$, etc.)
- These introduce large logarithms $\sim \ln p_{T,b}/m_b$
- So: want to perform resummation (using DGLAP)
- We essentially extend FONLL to NNLO+NNLL
- Skip details here $\Rightarrow$ see backup slides

Open bottom production at the LHC

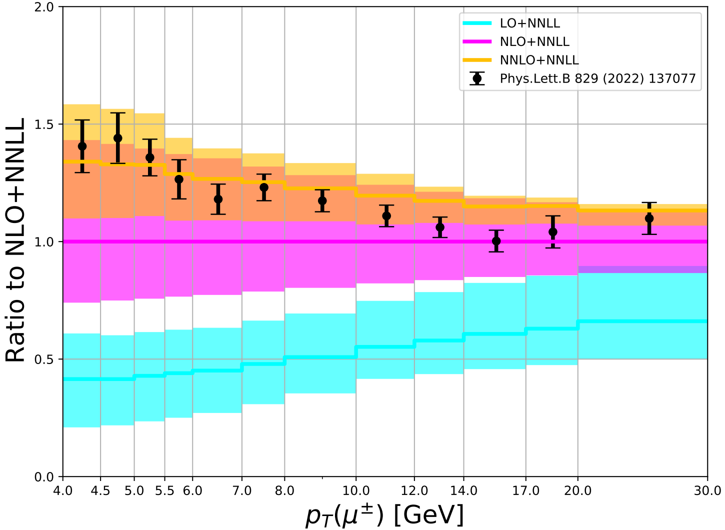
- Very nice data available from (all four) LHC experiments
- Not just *B*-hadron-level data, but also many decay modes
- General picture at NLO+NLL (FONLL):
    - Large theory uncertainty at low p_T
    - Shape difference (but within uncertainty)
    - More precise at high p_T
    - Experiment generally much more precise
- High time the predictions were updated!



B^+ spectrum at 7 TeV LHC (ATLAS)



$\mu^{\pm}$ from B-hadrons at 5.02 TeV (ATLAS)



Part IV:

Small-radius jets at NNLO+NNLL

arXiv:2503.21866

in collaboration with

Kyle Lee, Ian Moult, René Poncelet, Xiaoyuan Zhang

Goal and background

- High-precision predictions for small-radius jet production
- E.g. small R for anti-k_T jets
- As $R \to 0$, cross section develops large logs $\ln^m R$
- Factorisation very similar to fragmentation: Lee, Moult, Zhang (2024)

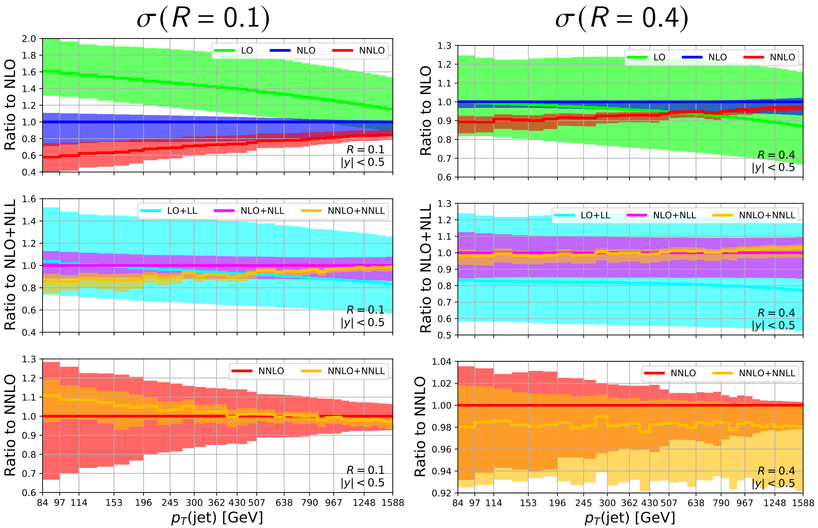
$$\frac{d\sigma_{\text{jet}}}{dp_T}(p_T, R) = \sum_i \int_0^1 \frac{dz}{z} \frac{d\sigma_i}{dp_T}(p_T/z, \mu_J) J_i \left(z, \ln \frac{p_T R}{z\, \mu_J}\right) + \mathcal{O}(R^2 \ln^m R)$$

- J_i is the 'FF' for producing a jet with radius R from parton i
- $\Rightarrow$ Swap out e.g. pion FF for 'jet FF' to get jet p_T spectra
- We used this to resum $\ln^m R$ to NNLL and match to NNLO

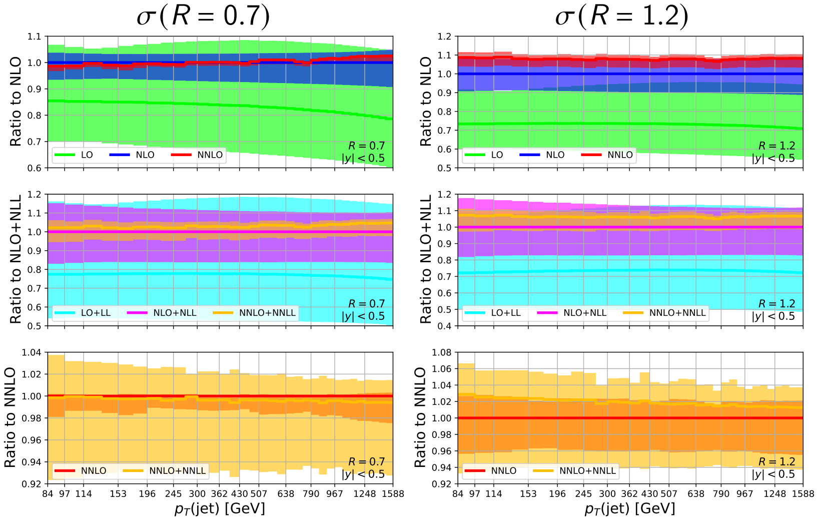
The measurement

- Ultimately want to compare to data
- arXiv:2005.05159: '3D' measurement of inclusive jets by CMS
- Double-differential in p_T and y for $R = 0.1, 0.2, ..., 1.2$
- Absolute spectra not provided; only ratio's w.r.t. $R = 0.4$
- Will use same binning and cuts to facilitate comparison
- Non-perturbative effects very large - must be included!
- NP corrections provided by CMS (obtained using parton showers)
- Important additional source of uncertainty

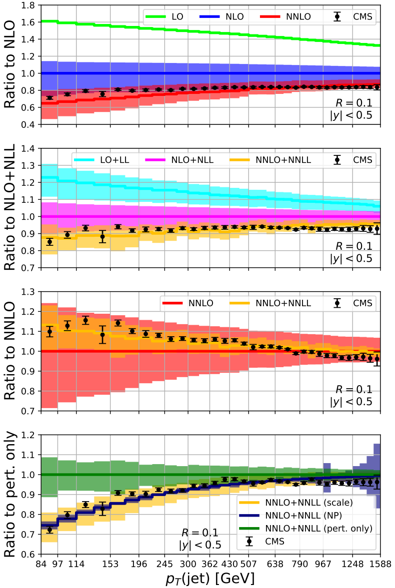
Results: cross sections at 13 TeV LHC

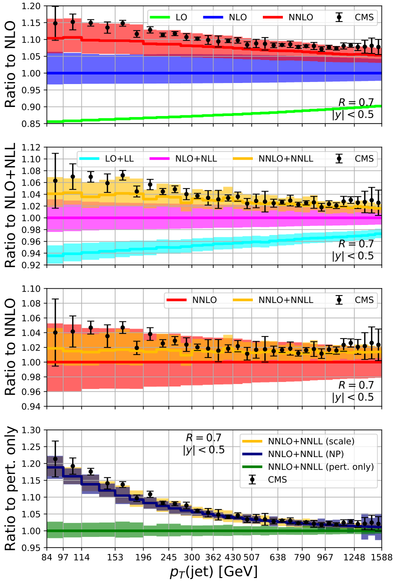


Results: cross sections at 13 TeV LHC



$\sigma(R=0.1)/\sigma(R=0.4)$ $\sigma(R=0.7)/\sigma(R=0.4)$





Conclusion & outlook

- First NNLO calculation of light hadron production at LHC
- Finally enables global FF fits at NNLO
- First NNLO calculation of $W^{\pm}D^{\mp}$ production at LHC
- Can be used to constrain strange-quark PDFs
- First NNLO+NNLL calculation of open bottom at hadron colliders
- Straightforward extension of FONLL to higher order
- First NNLO+NNLL calculation of small-radius jets at LHC
- For everything shown: improved precision and agreement with data
- Possible to extend to other factorisable observables, e.g. EECs

Backup

Perturbative fragmentation functions

- So far FFs of non-perturbative nature (parton→hadron)
- Can also define perturbative FFs as follows: *Mele, Nason (1991)*

$$\sigma_b(m_b) = \sum_i \sigma_i(m_b = 0) \otimes D_{i\to b} + \mathcal{O}(m_b^n/E_b^n)$$

- Convolution with $D_{i\to b}$ restores correct behaviour of mass-independent and log-enhanced terms $(\ln^m p_{T,b}/m_b)$
- Works for any massive particle, multiple masses, multiple particles, etc.
- As long as $m_b \gg \Lambda_{\mathrm{QCD}}$: perturbatively calculable!
- For heavy quarks: known through NNLO in QCD

NLO: *Mele, Nason (1991)*, NNLO: *Melnikov, Mitov (2004); Mitov (2004)*

Perturbative fragmentation functions

$$\sigma_b(m_b) = \sum_i \sigma_i(m_b = 0) \otimes D_{i\to b} + \mathcal{O}(m_b^n/E_b^n)$$

- The $\sigma_i(m_b = 0)$ contain additional collinear singularities w.r.t. $\sigma_b(m_b)$
- $\Rightarrow$ The PFF must contain collinear poles which cancel these
- $\Rightarrow$ Predictable poles, which can be renormalised away
- Introduces new and arbitrary factorisation scale + corresponding RGEs (time-like DGLAP equations)
- Plan for resummation:
  1. Evaluate PFF at $\mu \sim m_b$ (no large log of m_b)
  2. Evolve to $\mu \sim Q$ (hard scale of process)
  3. Combine with massless cross sections evaluated at $\mu \sim Q$

The final resummation formula

- Factorising the mass dependence into FFs allows resummation
- But only accurate up to power corrections $\mathcal{O}(m_b/E_b)$ (leading-power, LP)
- At low p_T, resummation not needed
- $\Rightarrow$ Can use fixed-order result with full mass dependence
- 'Best prediction' across full spectrum: combine the two!
- Naïve matching:

$$\text{Best prediction} = (\text{FO massive}) + (\text{resummed LP}) - \underbrace{(\text{FO LP})}_{\text{double counting}}$$

The final resummation formula

- Resummation of PFF performed in previous work

  Czakon, TG, Mitov, Poncelet (2022)

- ‘Double counting’ is FO expansion of resummed calculation

- Combination of partonic cross sections with
  1. NNLO perturbative fragmentation functions

     Melnikov, Mitov (2004); Mitov 2005)
  2. NNLO PDF threshold matching conditions

     Buza, Matiounine, Smith, Migneron, van Neerven (1996); Bierenbaum, Blümlein, Klein (2009)
  3. NNLO α_s threshold matching condition

     Bernreuther, Wetzel (1982, 1983); Larin, van Ritbergen, Vermaseren (1994)

- These extra contributions now also implemented in STRIPPER

The final resummation formula

- Naïve matching:

  Best prediction = (FO massive) + (resummed LP) – (FO LP)

- But: LP cross sections blow up at low p_T

- Solution: change $E_T(m_b = 0) \longrightarrow E_T(m_b) \Leftrightarrow p_T^2 \longrightarrow p_T^2 - m_b^2$ for LP

- In practice still not enough: large resummation effects at low p_T

- Artificially suppress resummation effects at low p_T

- FONLL approach: Cacciari, Greco, Nason (1998)

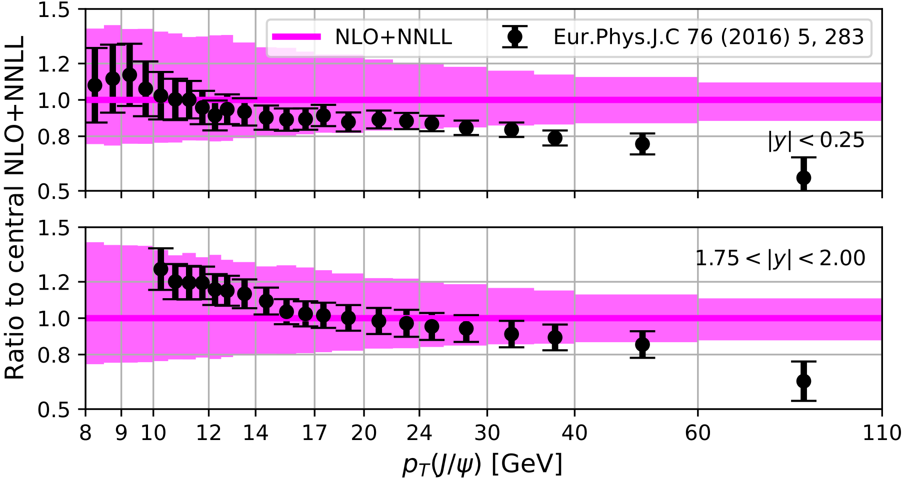
  FONLL = (FO massive) + $G(p_{T,b}) \cdot \big[$(resummed LP) – (FO LP)$\big]$

  $$G(p_T) = \frac{p_T^2}{p_T^2 + c^2 m_b^2}, \text{ with } c = 5$$

Soft-gluon resummation

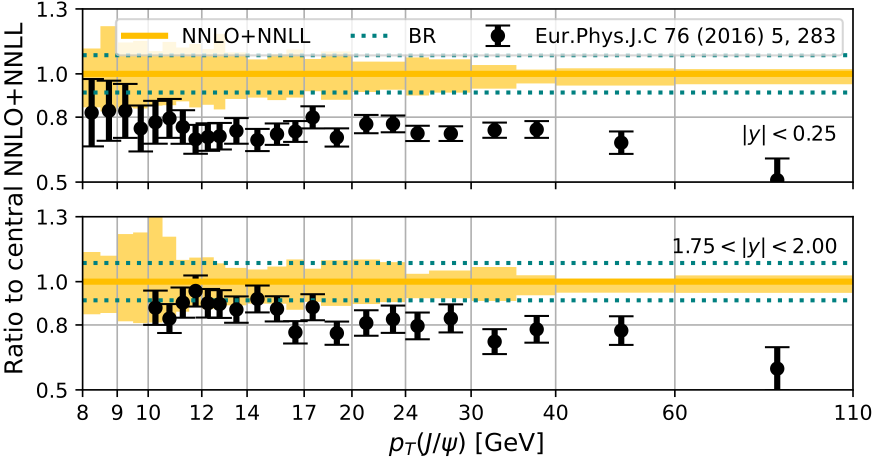
- PFF contains large threshold logs $\left(\frac{\ln^m(1-x)}{1-x}\right)_+$
- $\Rightarrow$ We perform soft-gluon resummation with NNLL accuracy
  Cacciari, Catani (2001); Cacciari, Gardi (2003); Gardi (2005); Aglietti, Corcella, Ferrera (2006); Fickinger, Fleming, Kim, Mereghetti (2016); Maltoni, Ridolfi, Ubiali, Zaro (2022); Czakon, TG, Mitov, Poncelet (2022)
- Well-known and tiny numerical impact here $\Rightarrow$ Skip details
- Recent studies: care must be taken at very large x
  Ghira, Marzani, Ridolfi (2023); Cacciari, Ghira, Marzani, Ridolfi (2024); Ghira, Mai, Marzani (2024)
- Fortunately not a problem here (large x not probed)
- Note: partonic cross section also contains threshold logarithms
- Those are not resummed here

Previous status: J/ψ from B-hadrons at 8 TeV (ATLAS)



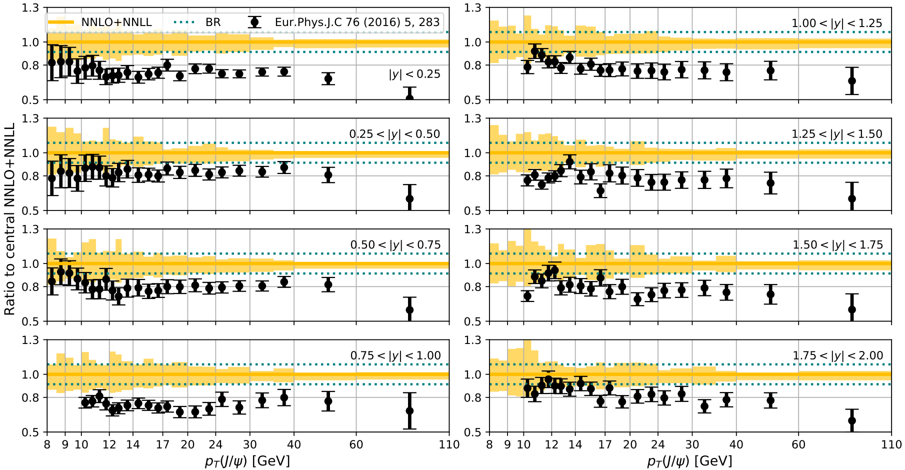
Clear deviation of some kind, but hard to interpret

Current status: J/ψ from B-hadrons at 8 TeV (ATLAS)



No more shape difference: deviation is just a constant factor!

Current status: J/ψ from B-hadrons at 8 TeV (ATLAS)



Most probably solution: BR($B \to J/\psi$) is off by 25% (3σ)

Could not have reached that conclusion using NLO!

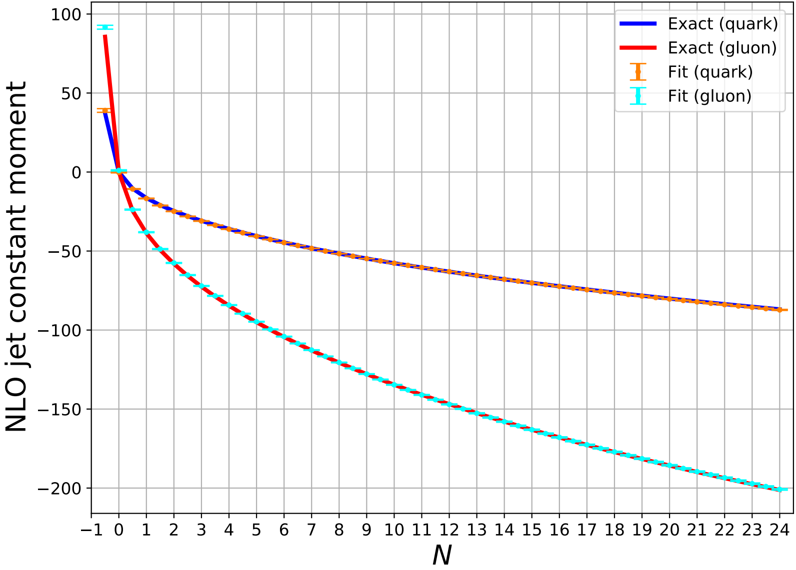
Approach

- DGLAP evolution performed by truncating at high order
- Converges well and gives precise control over included terms
- Matching trivial: σ = (exact NNLO) + (LP beyond NNLO)
- Requires convolutions with many different distributions
- In practice: α_s^5 for LL and NLL and α_s^4 for NNLL terms
- $\Rightarrow$ Need convolutions with $\left(\frac{\ln^5(1-x)}{1-x}\right)_+$
  $\Rightarrow$ Need very robust and stable code
- STRIPPER generalised to support arbitrary distributions

NNLO jet constant

- NNLL part of NNLO jet functions, i.e. $\mathcal{O}(\ln^0 R)$ not known
- But: can compute exact, fixed-order NNLO cross section
- Extract unknown terms by comparing exact and factorised result!
- In practice: cross section moments double-differential in y and $\hat{H}_T$
- Also split up cross section according to initial-state partons
- Allows to disentangle quark and gluon-initiated jet very well
- Computed at $R = 0.1$, power corrections found to be negligible
- Obtained the first 50 half-integer moments of both J_q and J_g

Cross-check: NLO jet constant



Result: NNLO jet constant