New vistas in quantum information aspects of gravity and quantum fields

Michal P. Heller

2408.15752 [PRL] and **2507.17847** [PRX in press] with Ori and Serantes

2412.17785 [PRL] with Papalini and Schuhmann

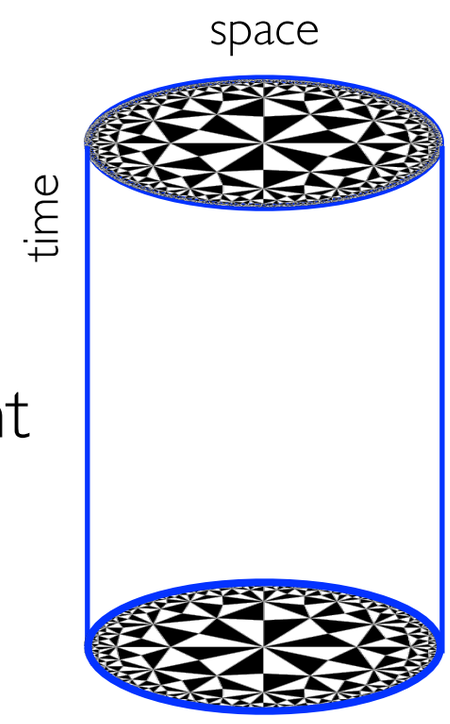
see also **2503.10753** for a broad review of complexity in holography

Introduction

Holography

hep-th/9711200 by Maldacena, hep-th/9802109 by Gubser, Klebanov, Polyakov, hep-th/9802150 by Witten, …

semi-classical gravity with
negative cosmological constant
and certain matter content



strongly-coupled quantum systems
with many degrees of freedom
certain cousins of QCD
certain critical systems (CFTs)
regimes of Sachdev-Ye-Kiteav model
and living on the boundary

The nature of the breakthrough: fully quantitative relation

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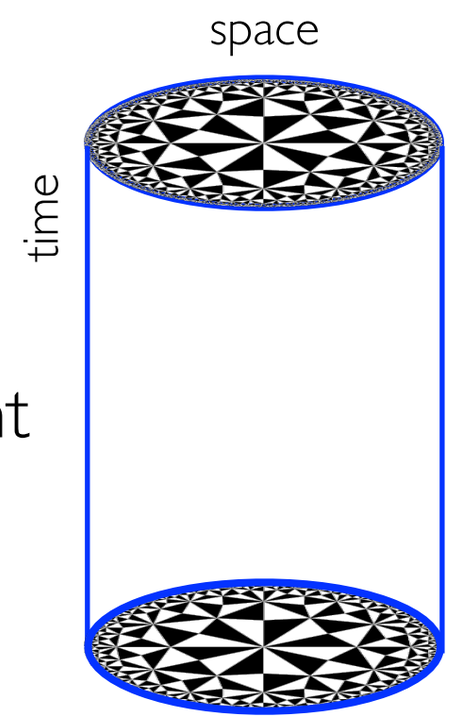

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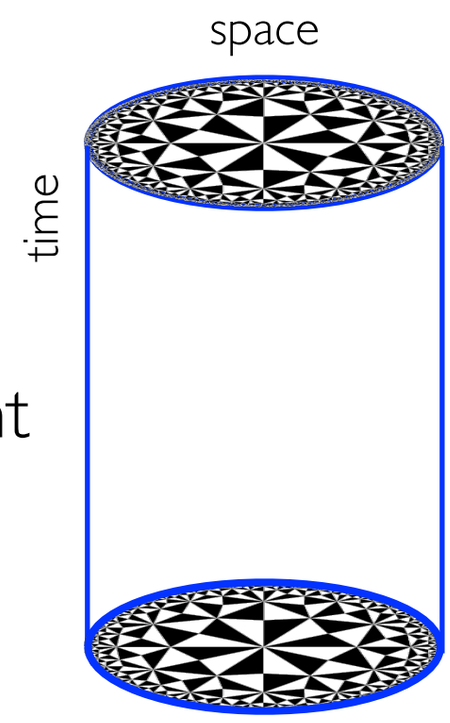

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Quantum information and spacetime: entanglement

hep-th/0603001 by Ryu, Takayanagi, …, 1304.4926 by Lewkowicz, Maldacena, …

Bekenstein-Hawking entropy
of the minimal area extremal surface
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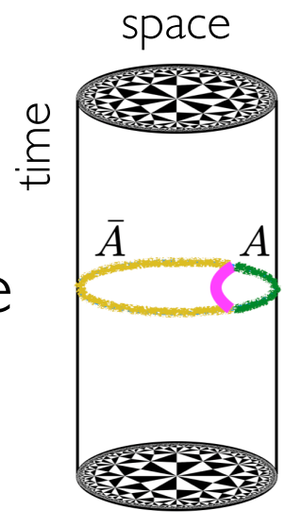

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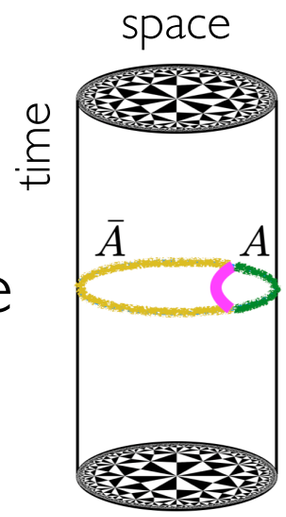

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There is a sense in which holographic entanglement entropy acts as a tomography of dynamical spacetime geometry probing its emergence

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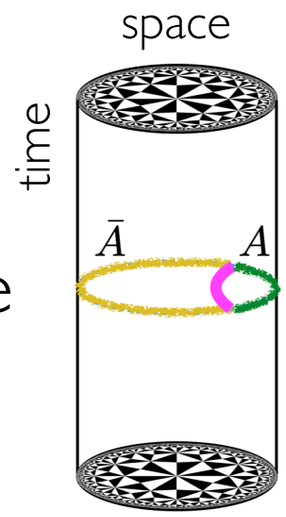

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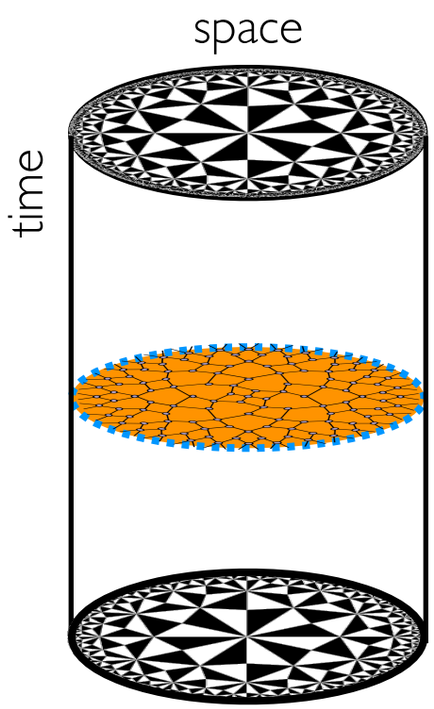
Broad relevance as it led to adoption of tools from quantum information science and quantum computing in the context of gravitational physics

Quantum information and spacetime: complexity

1402.5674 by Susskind, …, 2503.10753 with Baiguera, Balasubramanian, Caputa, Chapman, Haferkamp, Halpern



volume of maximal volume time slice

~

some notion of complexity
in the microscopic quantum system

…

Interesting, because the geometric object naturally characterizes black holes, which it universally grows linearly with time due to contributions from the inside.

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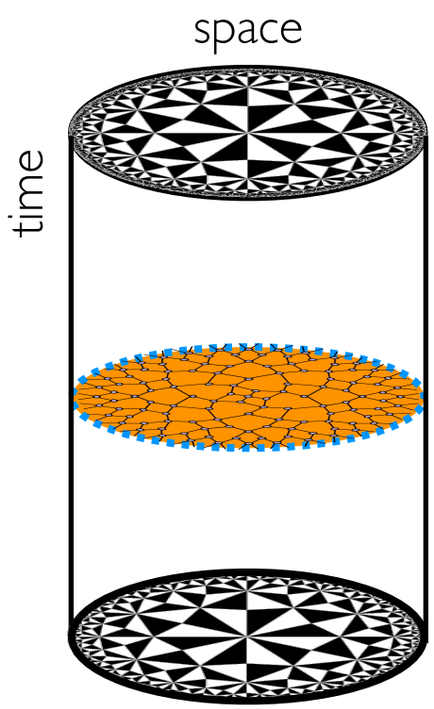

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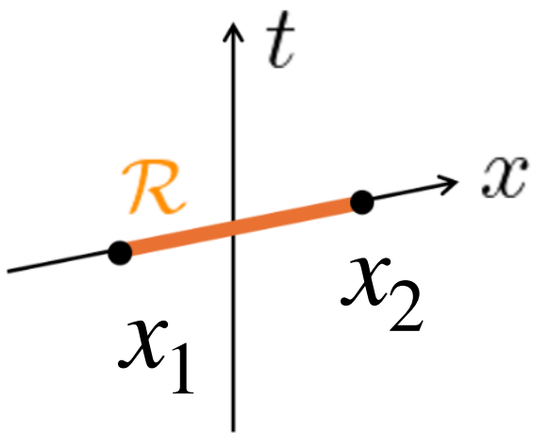
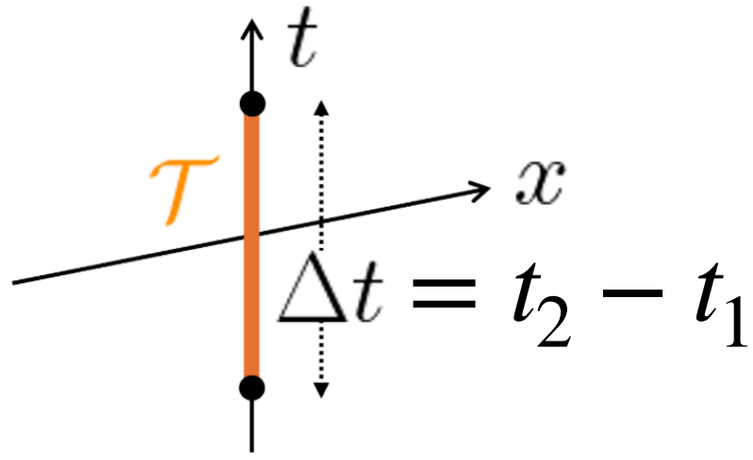
In contrast to holographic entanglement entropy,
microscopic interpretation of holographic complexity still elusive.

New ideas:
entanglement

New boundary observable

2210.09457, 2302.11695 by Doi, Harper, Mollabashi, Takayanagi, Taki





CFT$_{1+1}$: $S = \frac{c}{3} \log \frac{\sqrt{(x_1 - x_2)^2}}{\delta}$

standard entanglement

$$S = \frac{c}{3} \log \frac{\sqrt{-(t_1 - t_2)^2}}{\delta}$$
$$= \frac{c}{3} \left(\log \frac{\sqrt{(t_1 - t_2)^2}}{\delta} + i \frac{\pi}{2} \right)$$

temporal entanglement

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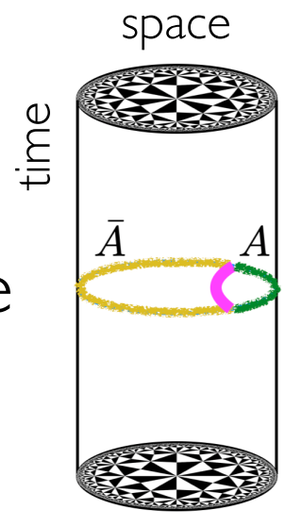

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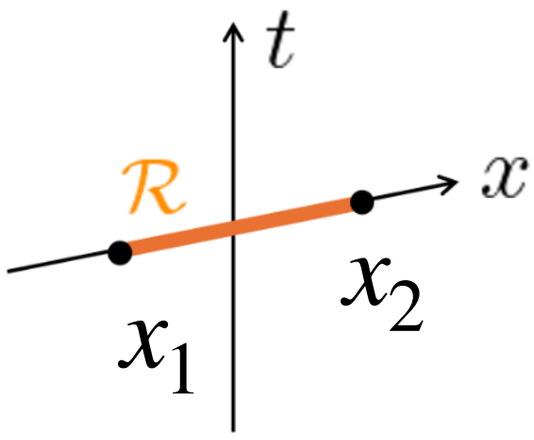

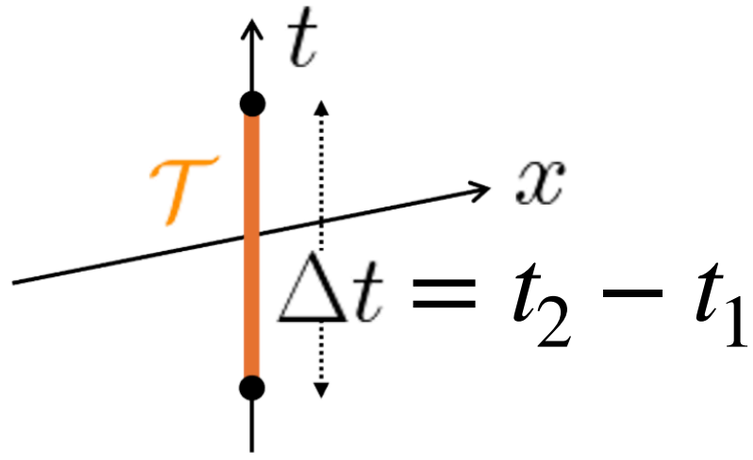

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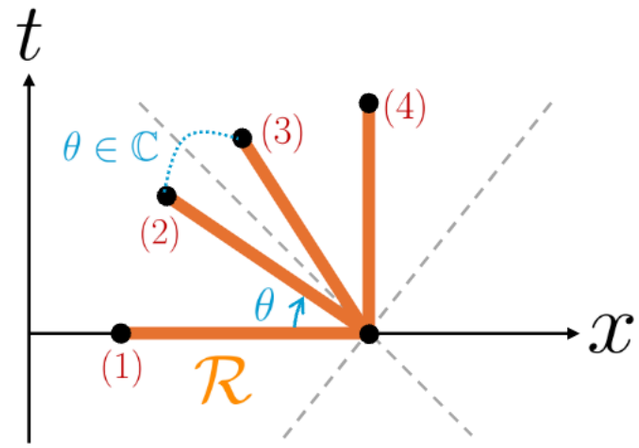
New result:
holographic temporal entanglement

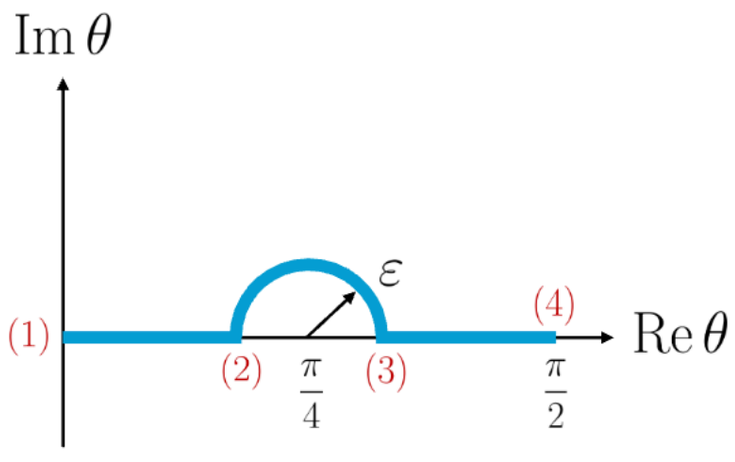
2408.15752 [PRL] and **2507.17847** with Fabio Ori and Alexandre Serantes

Prescription for holographic timelike entanglement entropy

2507.17847 with Fabio Ori and Alex Serantes



Analytically continue



all Ryu-Takayanagi candidates to the timelike regime

If more than one surface in the timelike part, minimize over the real part

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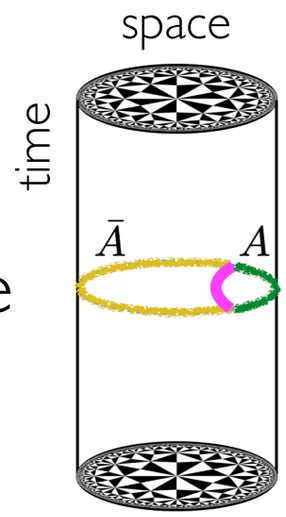

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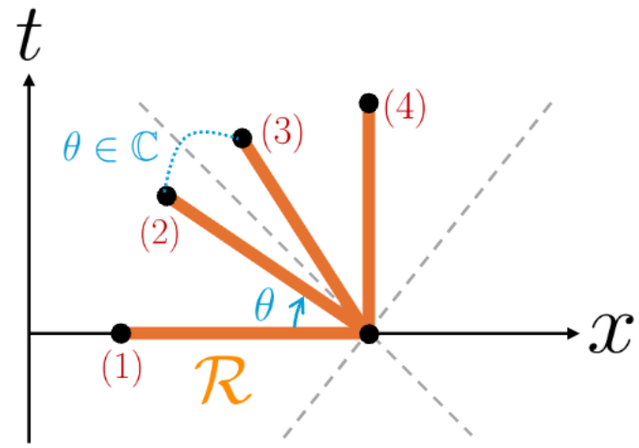

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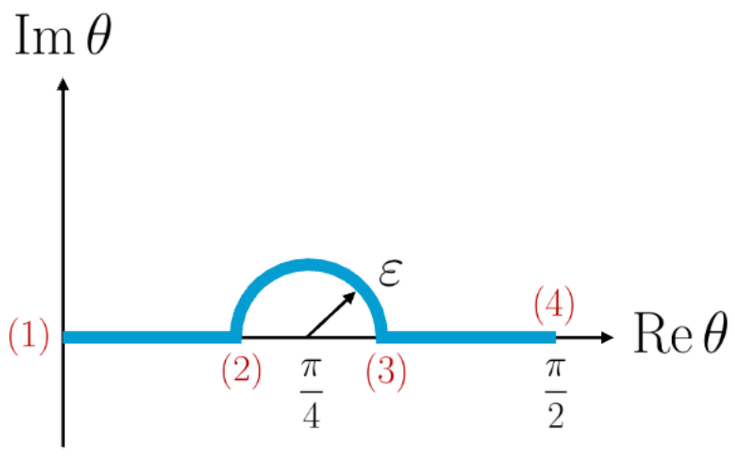

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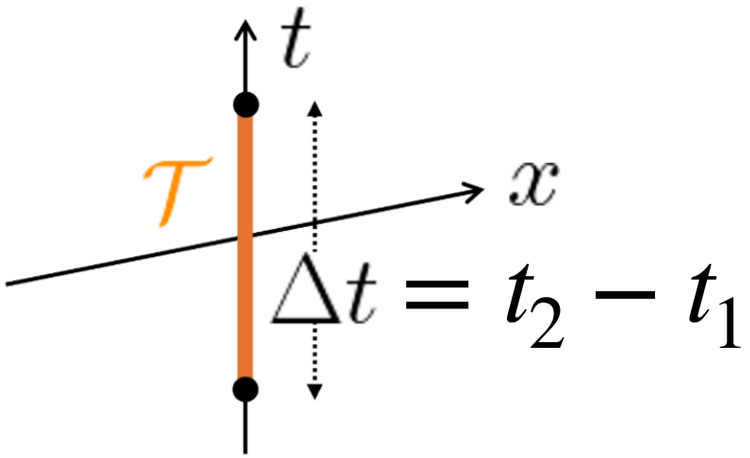
Outcome

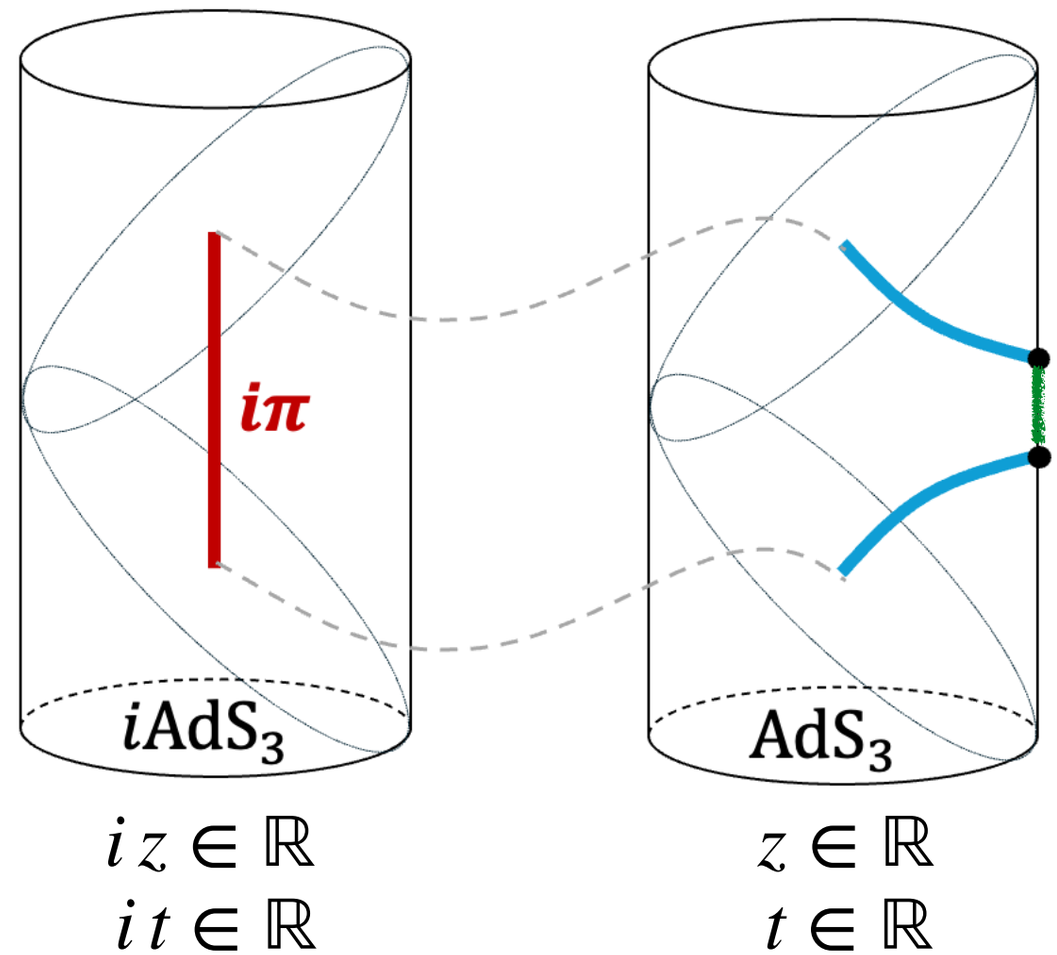
2408.15752 [PRL] and 2507.17847 with Fabio Ori and Alexandre Serantes

Carriers of holographic timelike entanglement entropy probe complex bulk, e.g.



$$S = \frac{c}{3}\log\frac{\sqrt{-(t_1-t_2)^2}}{\delta} = \frac{c}{3}\left(\log\frac{\sqrt{(t_1-t_2)^2}}{\delta} + i\frac{\pi}{2}\right)$$

$$ds^2 \sim \frac{dz^2 - dt^2 + dx^2}{z^2}$$



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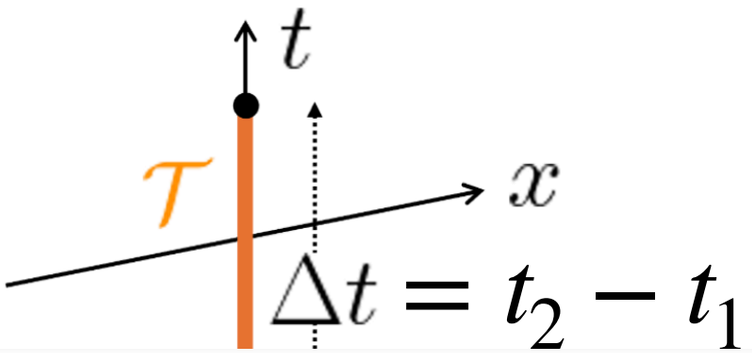

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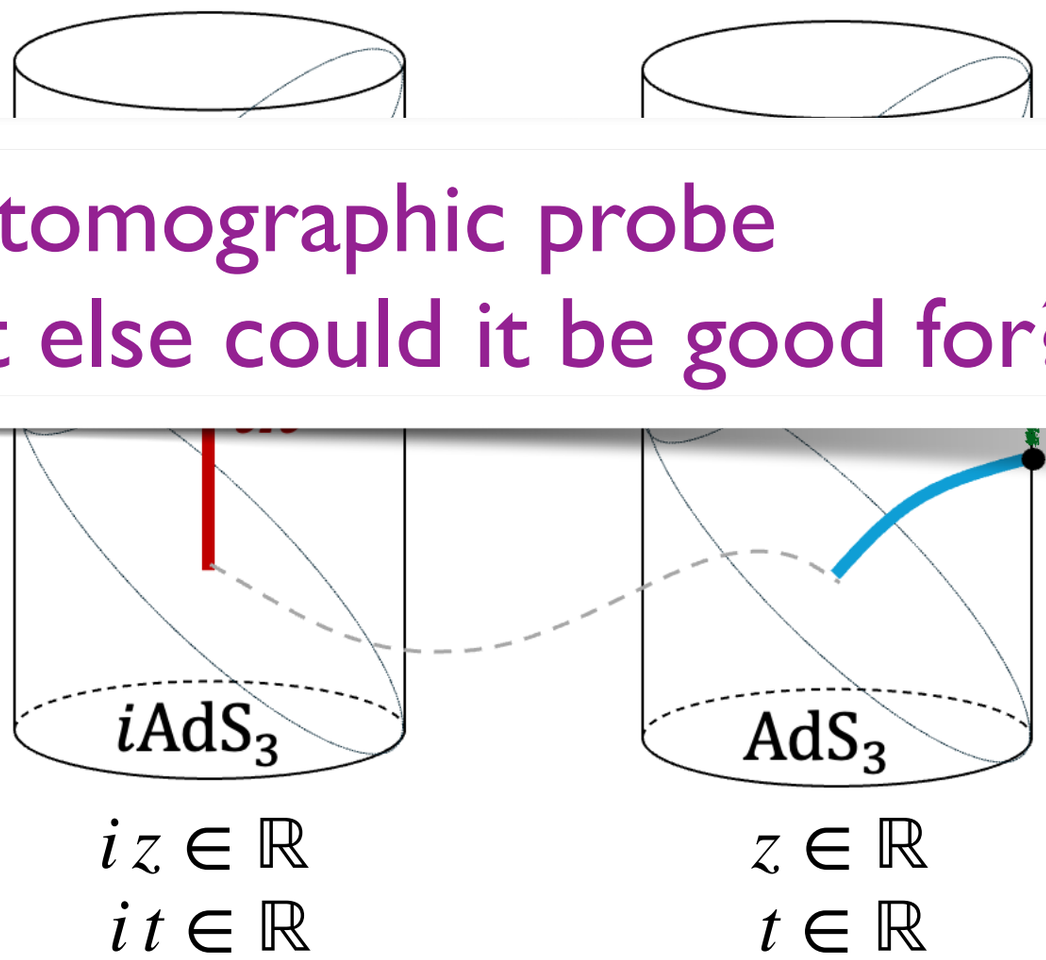

This shows that as a bulk tomographic probe temporal entangle is subtle. What else could it be good for?

Vision 2507.17847 with Fabio Ori and Alexandre Serantes

If one has a closed form expression for a function, then one also has its analytic continuation implicitly and often also explicitly, c.f. $\frac{c}{3}\log\frac{x_2-x_1}{\delta}$

However, entanglement entropy is notoriously difficult to compute in quantum systems and even in holography one most often gets it numerically

As a result, our prescription for computing holographic timelike entanglement entropy allows to probe novel facets of entanglement entropy akin to how e.g. shear viscosity is encoded in Euclidean thermal correlators

New ideas:
complexity

Krylov (spread) complexity: definition

2202.06957 by Balasubramanian, Caputa, Magan and Wu

Consider the time evolution operator acting on some initial state

$$|\psi(t)\rangle = e^{-iHt}|R\rangle = |R\rangle + (-i\,H)\,t\,|R\rangle + \frac{1}{2}(-i\,H)^2 t^2\,|R\rangle + \ldots$$

It gives a sequences of orthogonal directions (Krylov basis) in the Hilbert space:

$$|0\rangle = |R\rangle \qquad |1\rangle \sim H|R\rangle - \langle R|H|R\rangle|R\rangle \qquad |2\rangle \sim H^2|R\rangle + \ldots$$

that subsequent powers of t (time progression) trigger

Krylov (spread) complexity:

$$C_K \equiv \langle\psi(t)|\left(\sum_n n\,|n\rangle\langle n|\right)|\psi(t)\rangle$$

average order number of the Krylov basis element occupied via time evolution

Krylov (spread) complexity: properties

2503.10753 with Baiguera, Balasubramanian, Caputa, Chapman, Haferkamp, Halpern

Early times: $C_K \sim t^2$

$$C_K \equiv \langle \psi(t) | \left(\sum_n n \, |n\rangle\langle n| \right) | \psi(t) \rangle$$

Later times: possibility of growth in time via occupying higher and higher n

Sine dilaton gravity / (DS)SYK correspondence

2404.03535 by Blommaert, Mertens and Papalini

$$H_{\mathrm{SYK}} = i^{p/2} \sum_{1 \leq i_1 < \cdots < i_p \leq N} J_{i_1 \cdots i_p} \psi_{i_1} \cdots \psi_{i_p}$$

large N keeping $\log(q) \equiv \frac{p^2}{N}$ fixed ↓ analogous to the `t Hooft limit in QCD

$$\int \mathcal{D}g \mathcal{D}\Phi \, \exp\left(\frac{1}{2}\int \mathrm{d}^2x\sqrt{g}\left(\Phi R + \frac{\sin(2\,|\log q|\,\Phi)}{|\log q|}\right)\right)$$

It has black holes solutions that give rise to growth of holographic complexity

New results:
complexity

Our result

2412.17785 with Papalini and Schuhmann

$$C_V \equiv \langle l \rangle = -\,\partial_\Delta \langle e^{-\Delta\, l} \rangle \Big|_{\Delta \to 0} = C_K$$

for all β and arbitrary q

Positioning

2412.17785 with Papalini and Schuhmann

$$C_V \equiv \langle l \rangle = -\,\partial_\Delta \langle e^{-\Delta\, l} \rangle \Big|_{\Delta \to 0} = C_K$$

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It generalizes pioneering results of 2305.04355 by Rabinovici, Sánchez-Garrido, Shir and Sonner from a single point $\beta \to 0, q \to 1$ to the full $\beta - q$ parameters plane

Our key insight I

2412.17785 with Papalini and Schuhmann

reference (starting) state used by

2305.04355 by Rabinovici, Sánchez-Garrido, Shir and Sonner

Two options: C_K for $e^{-iT(t-i\beta/2)}|0\rangle \neq C_K$ for $e^{-iTt}\, e^{-\beta T/2}|0\rangle$

$|R\rangle$ $\qquad$ $|R\rangle \equiv |TFD(t=0)\rangle$

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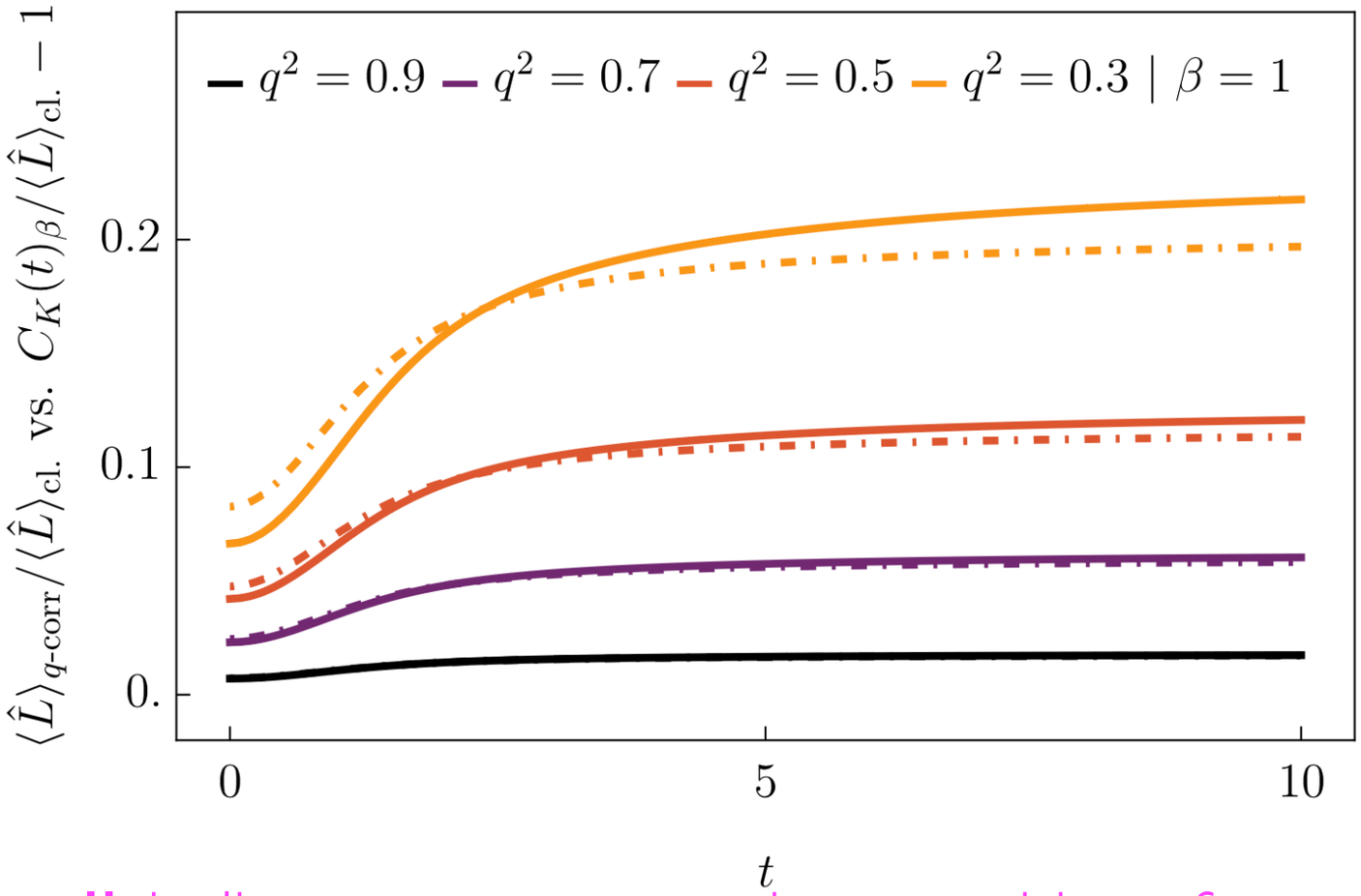
C_V and, hence, gravity gives **this**

Consequence I: gravity assigns Krylov cost also to Euclidean state preparation!

Our key insight II 2412.17785 with Papalini and Schuhmann

Our result is valid for any q and series expanding around $q = 1$ gives a first bulk quantum correction on top of the classical gravity result



Consequence II: bulk quantum correction transitions from t^2 to t behaviour

Speculation: it might a Krylov spread complexity of the bulk quantum field

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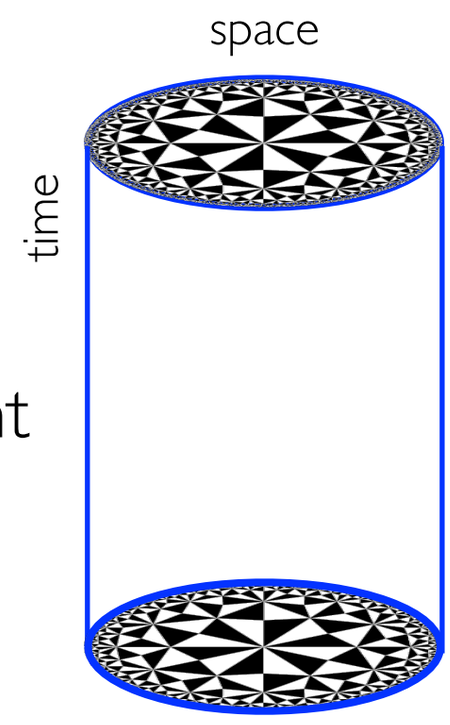

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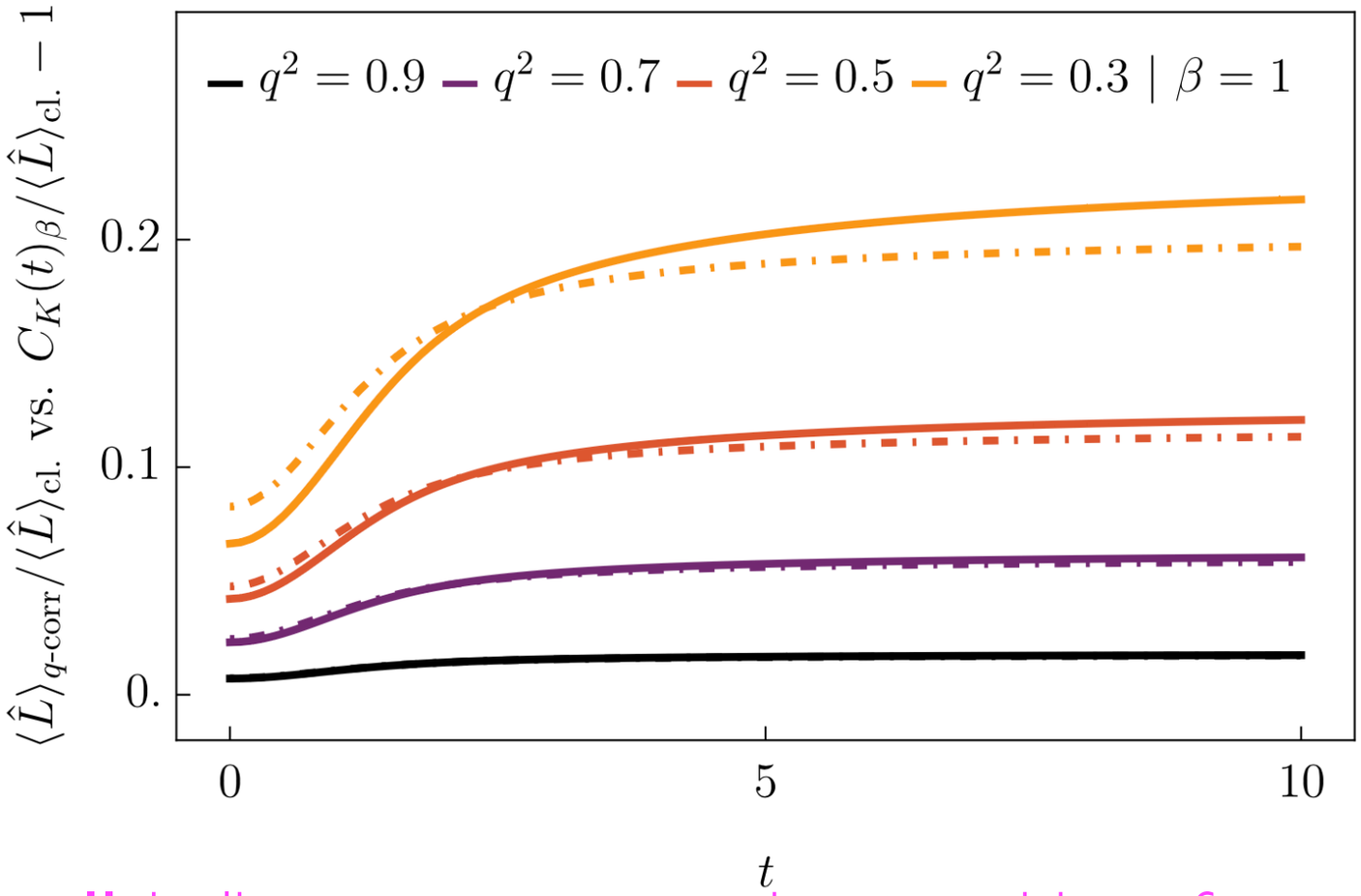

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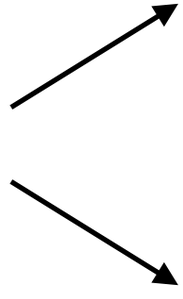
Outlook

Summary and outlook

The past 18 years have witnessed an explosion of understanding about gravity coming from quantum information science and quantum many body physics

This talk:



defining holographic temporal entanglement

providing 2-parameter family of exact bulk complexity notions

They are novel facets in the quest for geometrization of entanglement and complexity within the holographic paradigm