PAS Committee for Mechanics

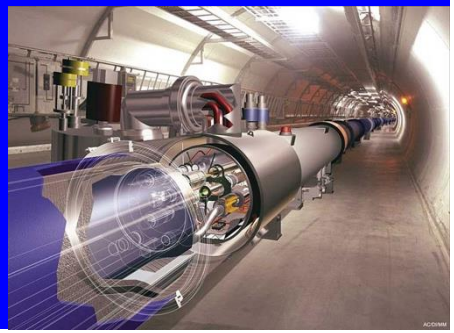
Radiation Hardening of Metastable Materials at Cryogenic Temperatures: From Irradiation-Induced Defects to Constitutive Modelling



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Division of Applied Mechanics and Biomechanics
Laboratory of Extremely Low Temperatures



Experiments in cryogenic conditions

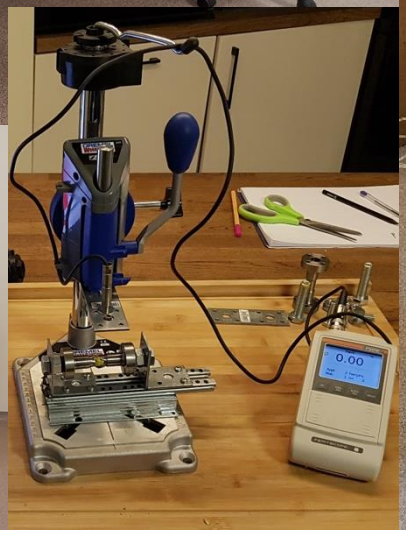


Laboratory of Extremely Low Temperatures

Experiments at cryogenic temperatures in LENT (77 K, 4.2 K)

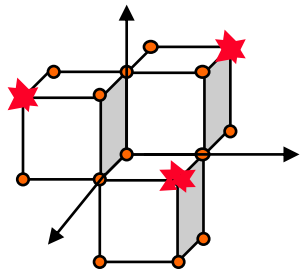








Ductile materials at extremely low temperatures



Materials used at extremely low temperatures: LHC, FCC, ITER

Modern materials for extremely low temperatures:

I Metals:

Al, Ag, Be, Cu, Sn, Ta, Ti, Zr, etc



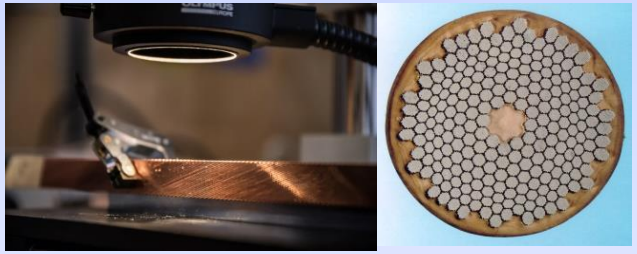
II Alloys:

304L, 316L, 316LN, 316Ti, P506, JK2LB, etc.

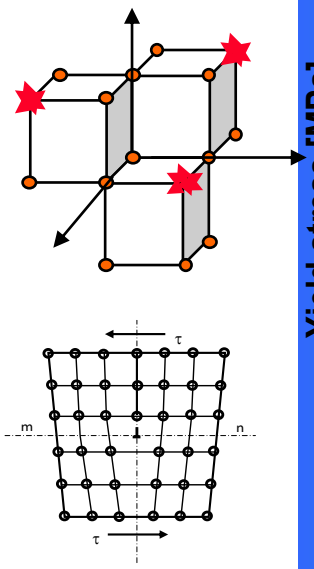


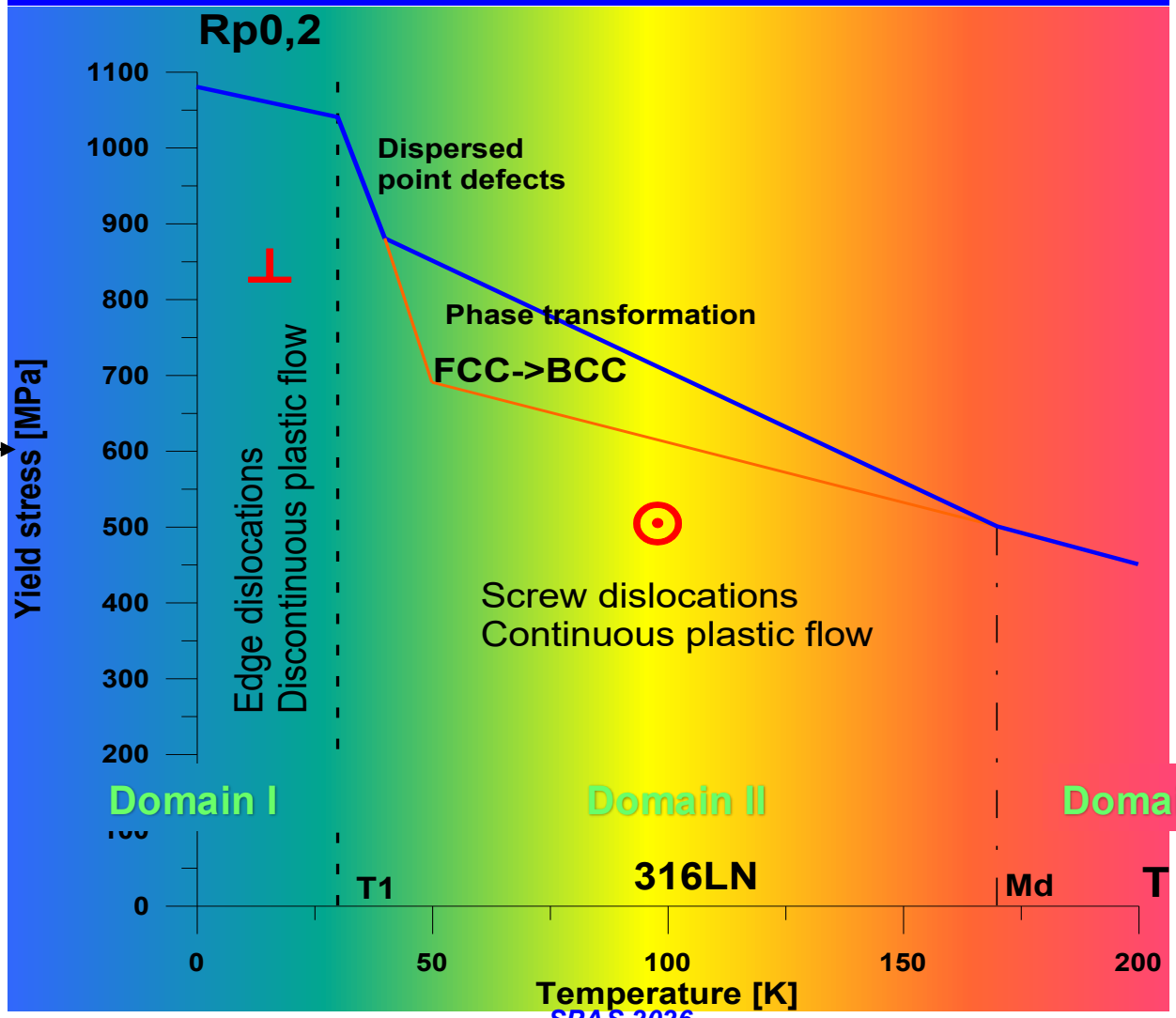
III Composites:

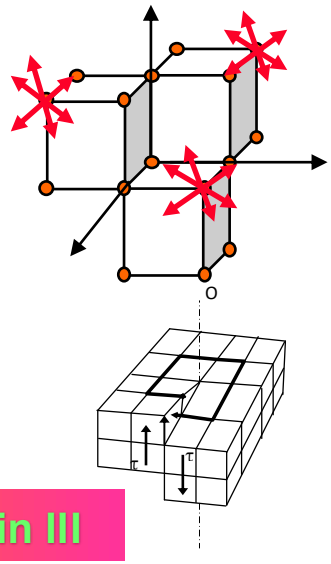
NbTi, NbAl, Nb_3Sn, MgB_2, Bi2212, etc.



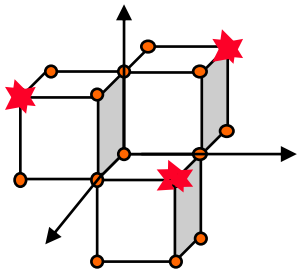
Dissipative phenomena observed in ductile metastable materials at cryogenic temperatures







Thermodynamics of weakly excited lattice



Thermodynamics of crystal lattice at extremely low temperatures

***Phonon as a quasiparticle**, consists in a collective excitation of lattice atoms. At extremely low temperatures only acoustic phonons are excited (wave-length is much higher than the lattice constant).*

Hamilton operator of lattice vibrations:

$$H = H_0 + \sum_{\alpha,\underline{k}} \hbar\omega_\alpha(\underline{k}) a_\alpha^+(\underline{k}) a_\alpha(\underline{k})$$

Number of phonons operator and its eigenvalues:

$$a_\alpha^+(\underline{k}) a_\alpha(\underline{k}) \longrightarrow N_\alpha(\underline{k})$$

Energy of lattice vibrations:

$$E = E_0 + \sum_{\alpha,\underline{k}} \hbar\omega_\alpha(\underline{k}) N_\alpha(\underline{k})$$

Phonon energy of lattice:

$$\Delta E = \sum_{\alpha,\underline{k}} \hbar\omega_\alpha(\underline{k}) f_0(\omega_\alpha) \Rightarrow \int_0^{\omega_{\max}} f_0(\omega)\nu(\omega)\hbar\omega d\omega$$

$$E_{ph} \sim N\left(\frac{T}{\Theta}\right)^3 T$$

A.M.Kosevich, Physical Mechanics of Real Crystals, Kyiv, 1981

Thermodynamics of crystal lattice at extremely low temperatures

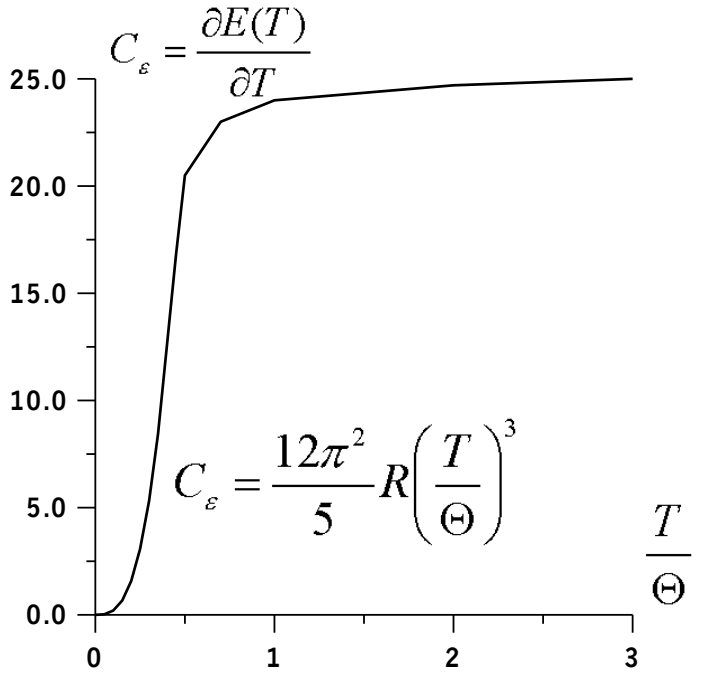
Specific heat at constant volume:

$$(C_{ph})_V = \left(\frac{\partial E_{ph}}{\partial T}\right)_V \sim N\left(\frac{T}{\Theta}\right)^3 \qquad (C_{el})_V \sim \frac{T}{T_F}$$

$$C_V = (C_{el})_V + (C_{ph})_V \sim aT + bT^3$$

$$\lim_{T\to 0} C_V = 0$$

$$dQ = mC_V dT \qquad \frac{dT}{dQ} = \frac{1}{mC_V}$$

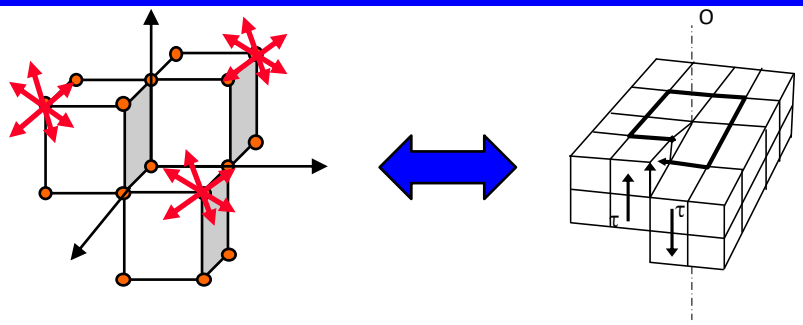


$$T \to 0 \Rightarrow \frac{dT}{dQ} \to \infty$$

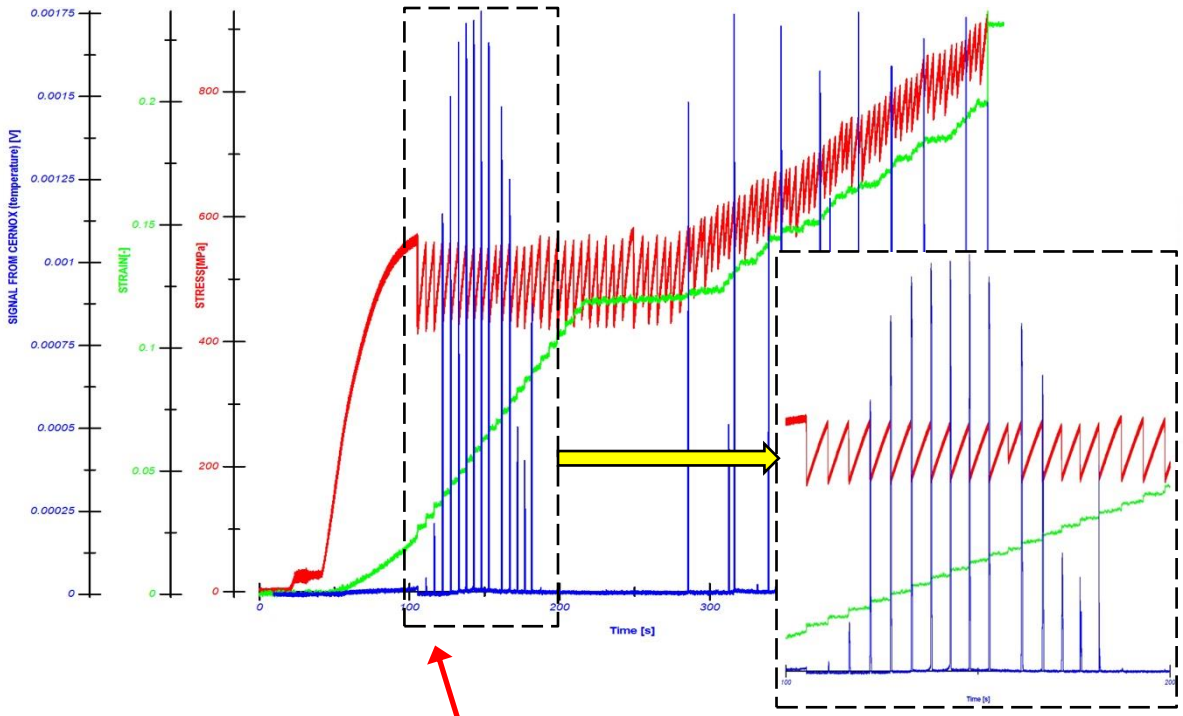
Small ΔQ causes large ΔT → "thermodynamic instability"

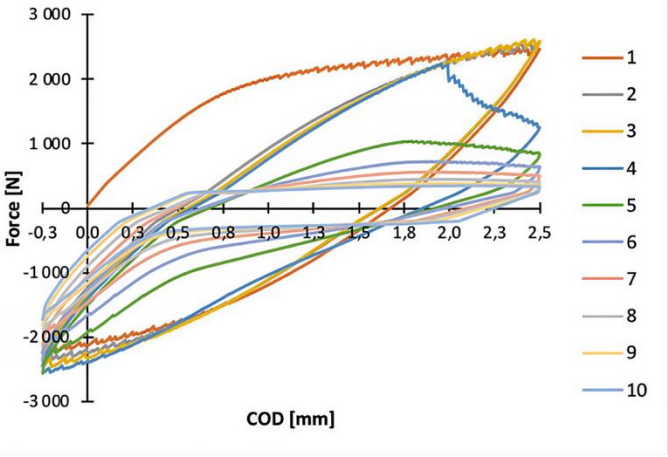
Skoczen, B., Springer, 2004

Phenomena occurring in metastable materials at extremely low temperatures

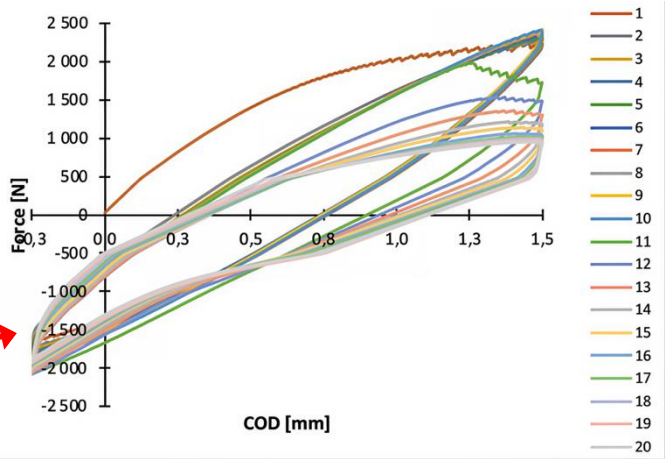


Intermittent Plastic Flow in grades 304 and 316L st. steel at 4.2K





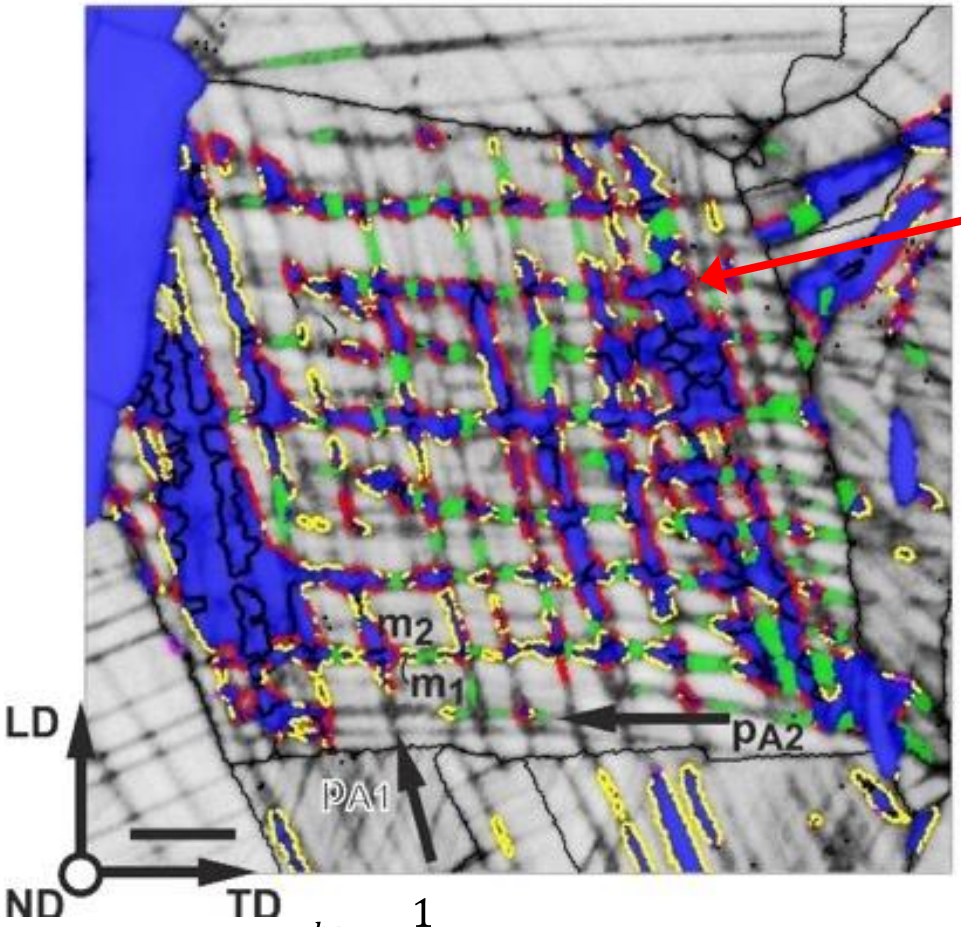
IPF during tension and compression

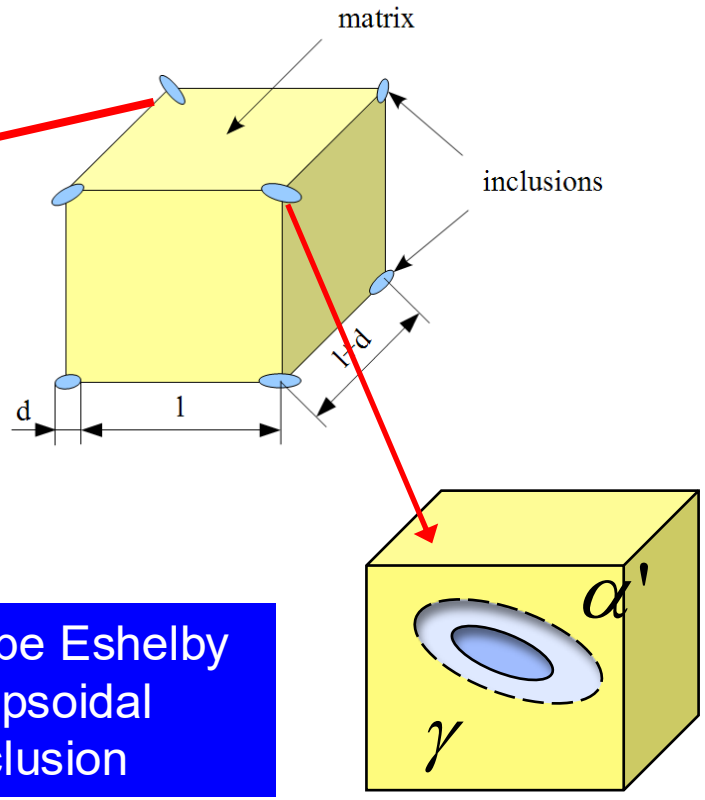


IPF in grade 304 stainless steel under monotonic loading at 4.2 K

IPF in grade 316L stainless steel under cyclic loading at 4.2 K

Mechanism of $\gamma \rightarrow \alpha'$ phase transformation: micro shear bands





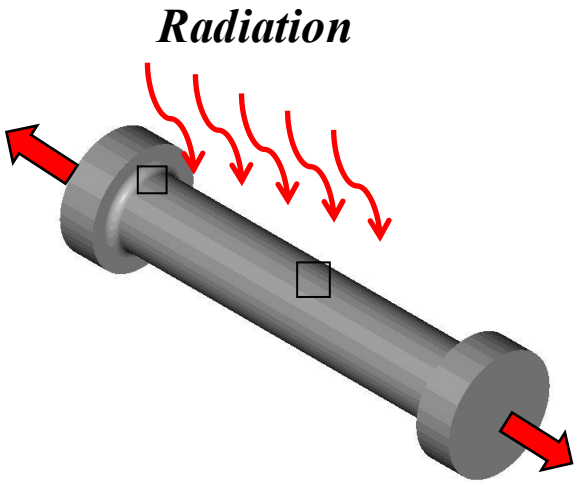
Type Eshelby ellipsoidal inclusion

$$\epsilon_{kl}^{bs} = \frac{1}{3}[\xi_{\varepsilon}\,\Delta v_{\varepsilon} + \xi_{\alpha\prime}\,\Delta v_{\alpha\prime}]\delta_{kl} + \xi_{\alpha\prime}\langle(\epsilon_{kl}^{bs})_{\mu\gamma}\rangle$$

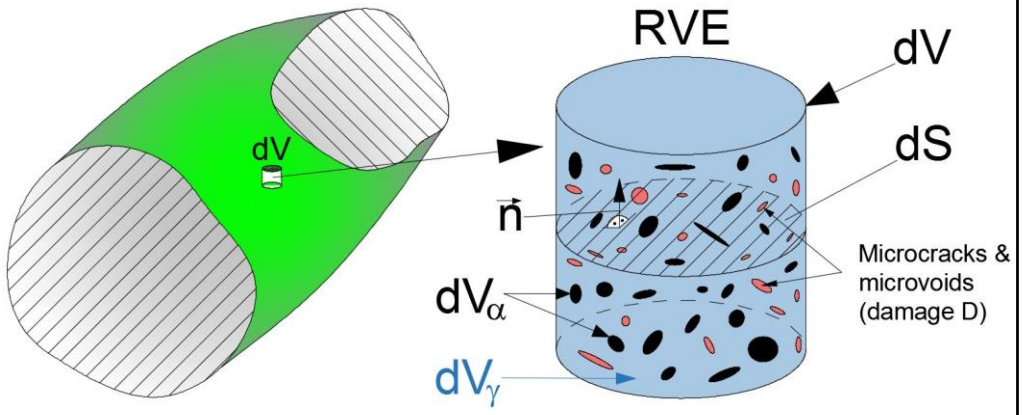
Eshelby, J.D., Proc. Roy. Soc. Lond Ser A, 1957

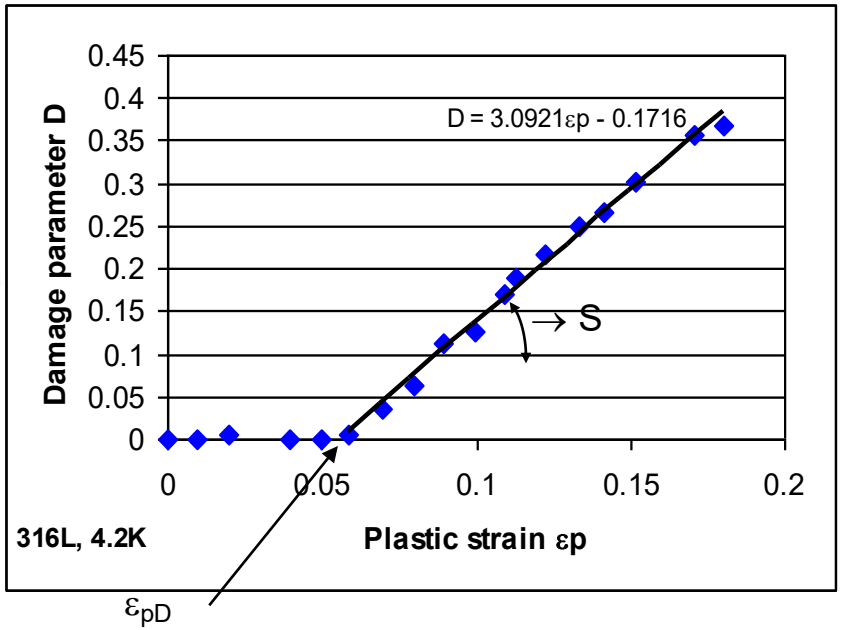
Martensite variants α', ε in blue and green. The interphase boundaries with disorientation Pitsch and Kurdjumov-Sachs are distinguished in red and yellow. Scale bar: 2 μm.

Evolution of microdamage



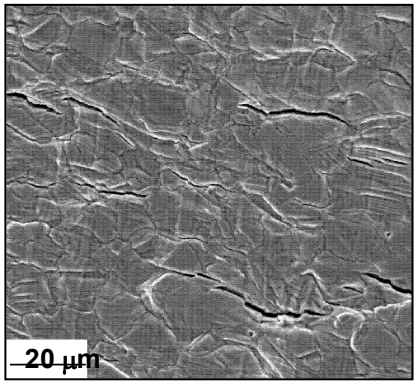
Damage of mechanical origin: isotropic versus anisotropic model





D – surface fraction of micro-cracks and micro-voids; D_{ij} – damage tensor (Murakami)

Murakami, S., IUTAM/ICM, 1990



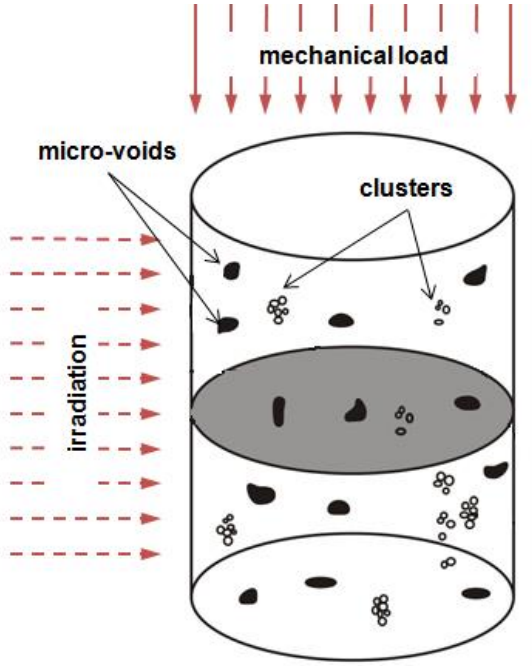
$$D = \frac{dS_D}{dS} \quad ; \quad 0 \le D \le 1 \quad \rightarrow \quad D_{ij} = \sum_{k=1}^{3} D_k \, n_i^{(k)} n_j^{(k)}$$

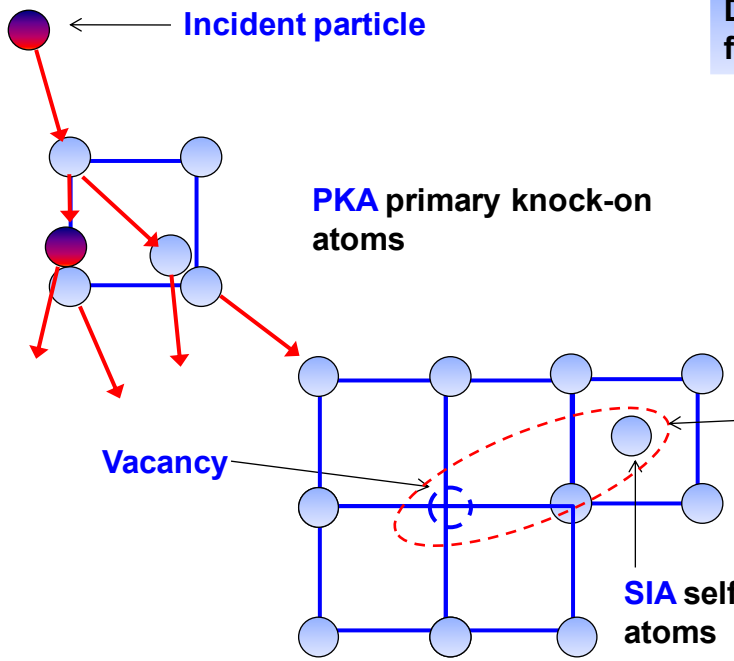
$$(D)_{,t} = \left(\frac{Y}{S}\right)^s p_{,t} H(p - p_D) \quad \rightarrow \quad (D_{ij})_{,t} = C_{ik} Y_{kl} C_{jl} p_{,t} H(p - p_D)$$

Chaboche, 1988; Lemaitre, 1992

Garion, C., Skoczeń, B., IJDM, 2003

Evolution of radiation induced damage in crystal lattice





Displacement cascade and formation of Frenkel pairs

↓

Kinchin & Pease, 1955

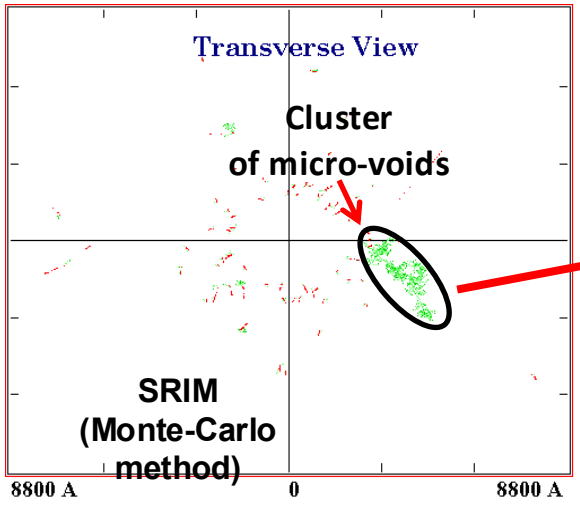
↓

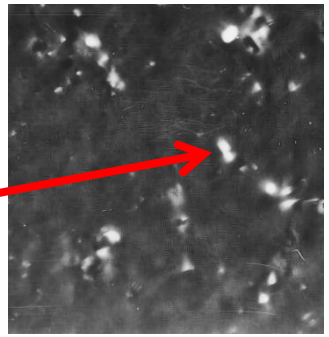
Norgett, Robinson, Torrens, 1975

↓

$$N_{NRT} = \frac{0.8E_{dam}}{2\overline{E}_d}$$

$$E_{dam} = NIEL$$

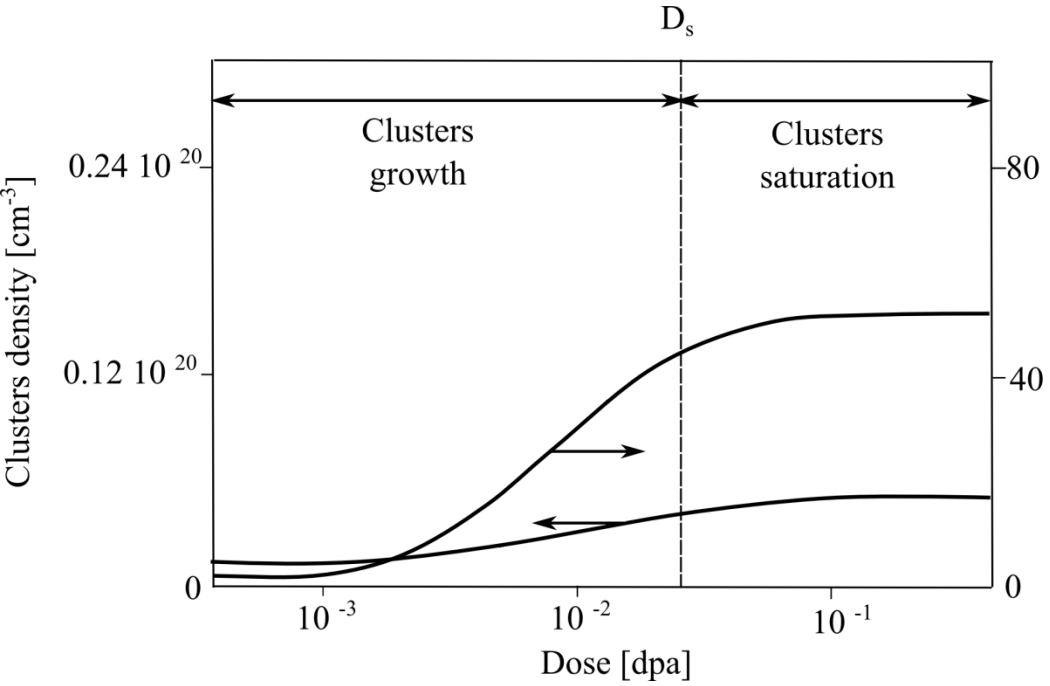




Defects due to irradiation:

1. SFT – stacking fault tetrahedron
2. Faulted or perfect dislocation loops
3. Voids – 3D vacancy clusters
4. Cavities – 3D vacancy clusters with impurities (He)

Lattice defects induced by irradiation: from physics to mechanics



dpa

Physics

$$q_c = \begin{cases} C_{qI}(dpa)^{n_{qI}} & for \quad dpa < D_S \\ C_{qII}(dpa)^{n_{qII}} & for \quad dpa \geq D_S \end{cases}$$

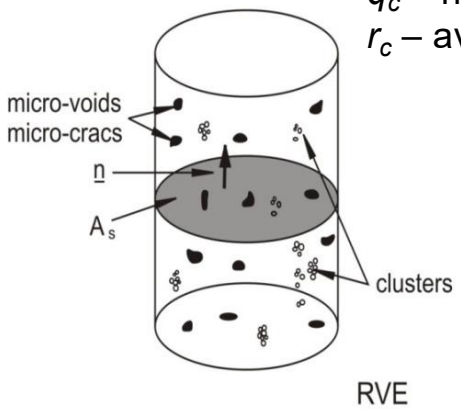
$$r_c = \begin{cases} C_r(dpa)^{n_r} & for \quad dpa < D_S \\ r_{cr} & for \quad dpa \geq D_S \end{cases}$$

$$q_A = \left(\sqrt[3]{q_V}\right)^2 = q_c^{\,2/3}$$

$$D_{r0} = q_A \pi r_{c0}^{\,2}$$

Mechanics

q_c – number of clusters per unit volume
r_c – average radius of clusters



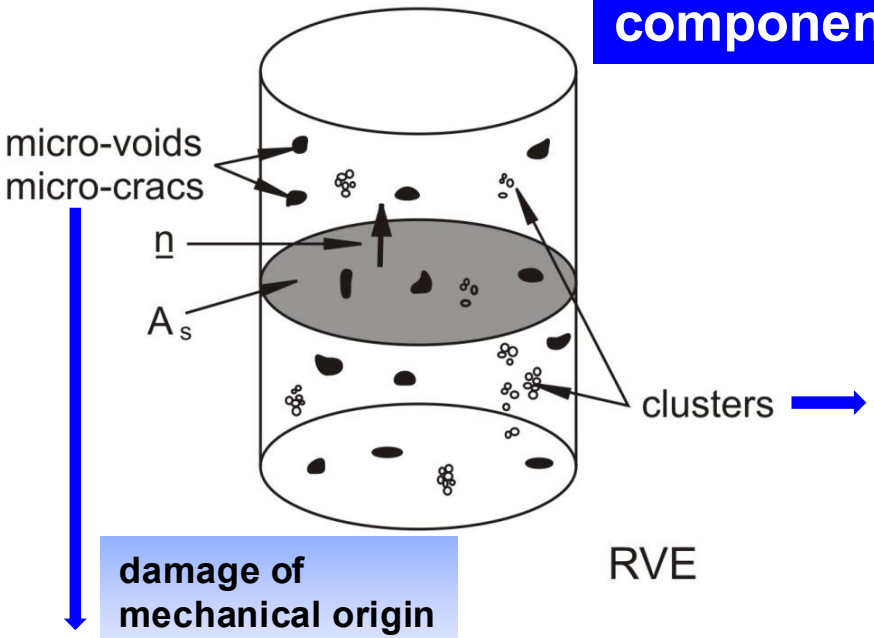
$$D = \frac{dS_D}{dS} \quad ; \quad 0 \leq D \leq 1$$

$$\xi = \frac{dV_D}{dV} \quad ; \quad 0 \leq \xi \leq 1$$

Skoczen, B, Ustrzycka A., IJP, 2016

Mechanical vs radiation damage: additive formulation

Postulate: both micro-damage components are treated in an additive way



damage of mechanical origin

$$\left(D_{ij}{}_m\right)_{,t} = C_{ik} Y_{kl} C_{jl} p_{,t}$$

anisotropic

$$D_r = D_{r0} + \int_0^{\hat{p}} dD_{rm}$$

radiation induced damage

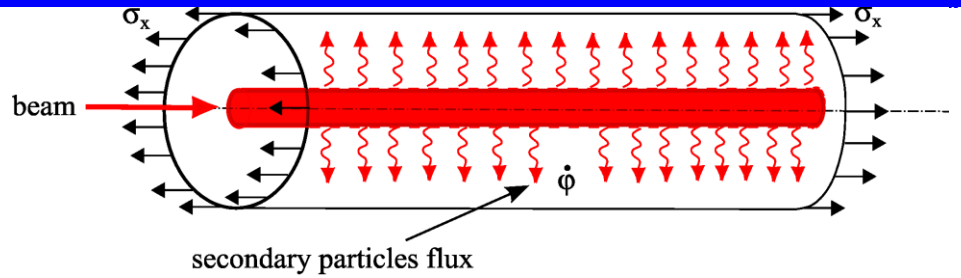
$$D_{ij}{}_r = \frac{D_r}{3}\delta_{ij}$$

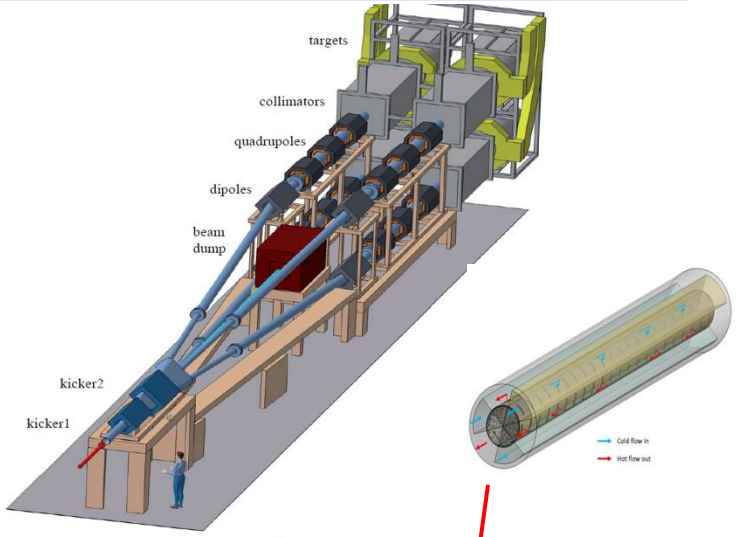
isotropic

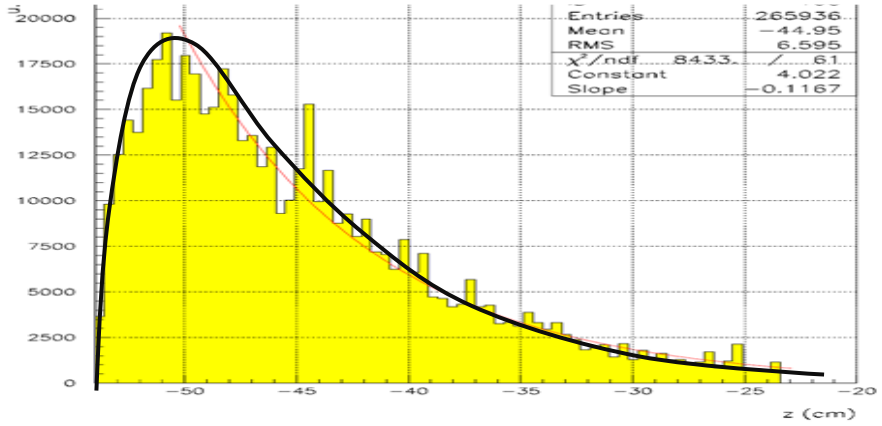
$$D_{ij} = D_{ij}{}_m + D_{ij}{}_r = D_{ij}{}_m + \frac{D_r}{3}\delta_{ij}$$

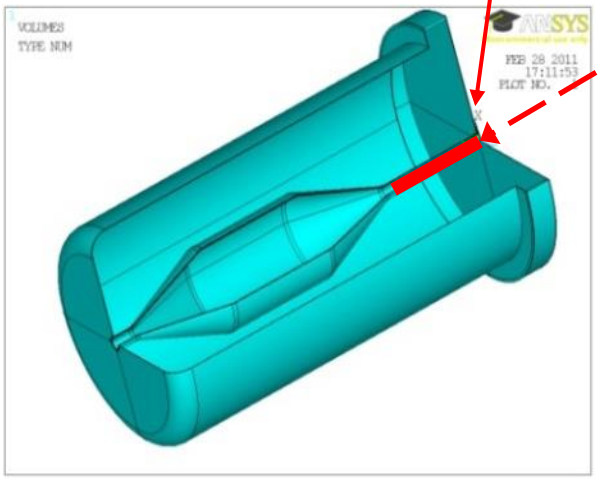
Lifetime estimation for irradiated components (*EUROv*)

Secondary particles flux: γ, n, p+, π± and e±

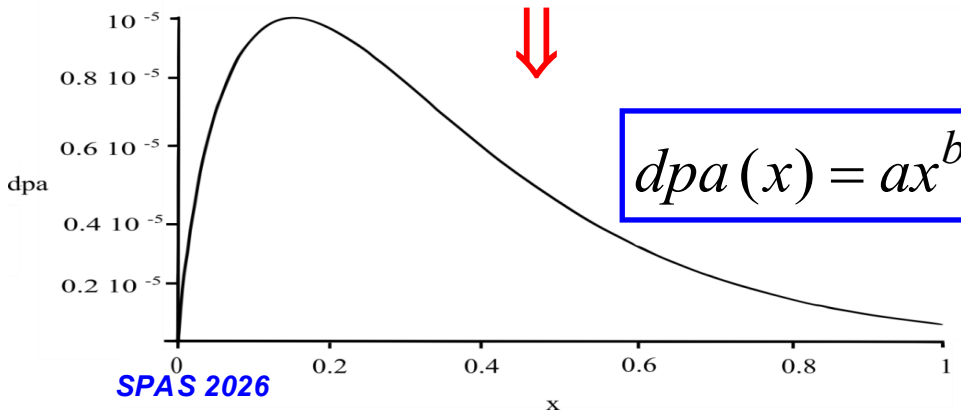








Typical distribution of particle flux along the target axis



$$dpa(x) = ax^b e^{cx}$$

Lifetime estimation for irradiated components: R-T versus GTN

Kinetics of evolution of radiation induced damage (clusters of voids) under mechanical loads

I Rice & Tracey (R-T) model:

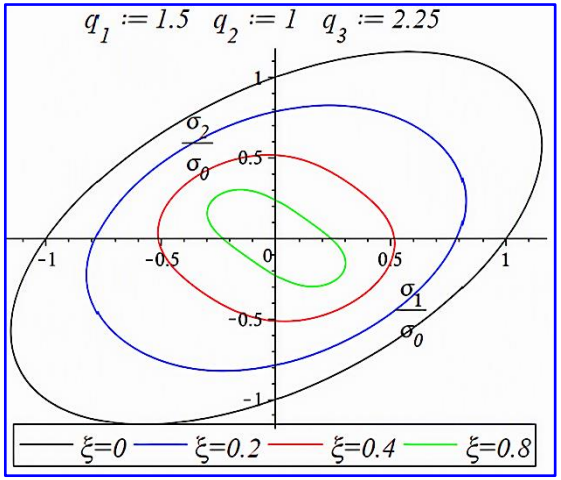
$$dr_c = r_c \alpha_r \exp\left(\frac{3\sigma_m}{2\sigma_{eq}}\right) dp \qquad \dot{p} = \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p}$$

II Gurson–Tvergaard–Needleman (GTN):

$$\xi_{,t} = (1-\xi) tr\left(\varepsilon_{ij\,,t}^p\right) + \xi_{nucl\,,t}$$

GTN yield surface:

$$\Phi(\sigma_{ij}, \sigma_M, \xi) = \frac{\sigma_e^2}{\sigma_M^2} + 2\,\xi q_1 cosh\left(q_2 \frac{3\sigma_h}{2\sigma_M}\right) - (1 + q_3\xi^2) = 0$$

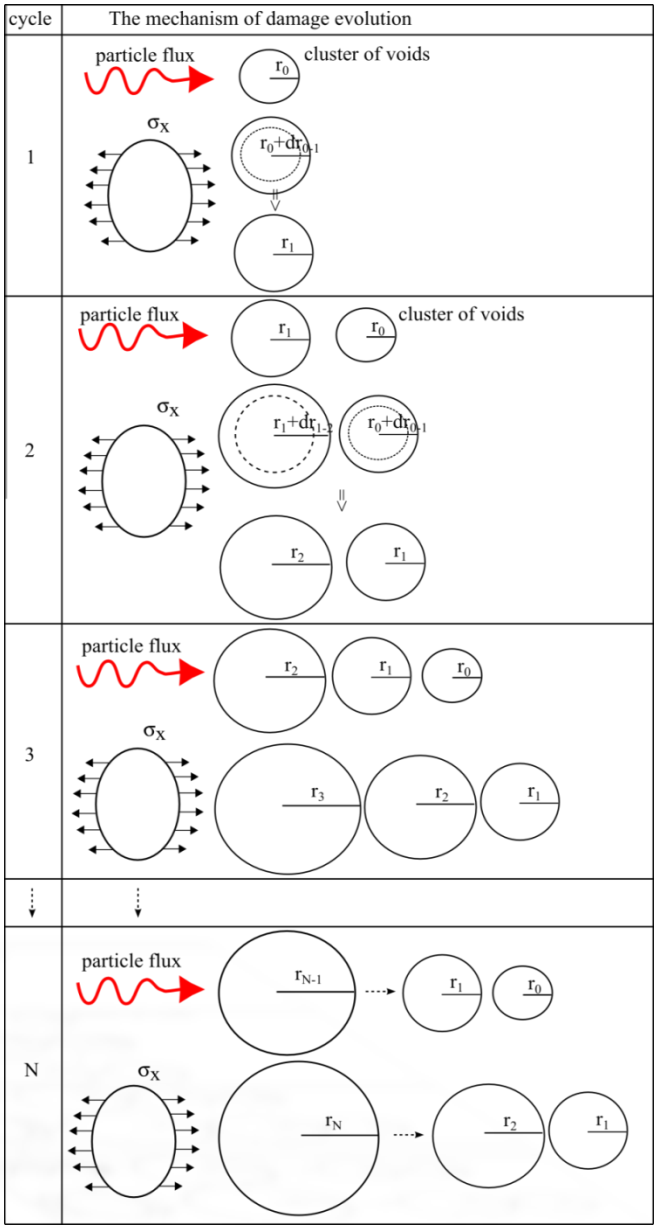


$$d\varepsilon_{ij}^p = d\lambda \frac{\partial \Phi}{\partial \sigma_{ij}} = \dot{\lambda}\left[\frac{3}{\sigma_0^2} s_{ij} + \frac{\xi}{\sigma_0} sinh\left(\frac{\sigma_{kk}}{2\sigma_0}\right)\delta_{ij}\right]$$

$$\dot{\xi} = \frac{V_m}{V}\frac{\dot{V}}{V} = (1-\xi)\frac{\dot{V}}{V} = (1-\xi)\mathrm{tr}\left(\dot{\varepsilon}_{ij}^p\right)$$

$$tr(\dot{\varepsilon}_{ij}^p) = \dot{\lambda}\frac{3\Phi}{\sigma_0} sinh\left(\frac{\sigma_{kk}}{2\sigma_0}\right)$$

Lifetime estimation for irradiated components



Rice & Tracey law

$$\int_{D_i}^{D_{i+1}} dD = q_A 2\pi \int_{r_i}^{r_{i+1}} r\, dr$$

$$\Delta D_{i\to i+1} = q_A \pi \left(r_{i+1}^2 - r_i^2 \right)$$

$$\int_{r_i}^{r_{i+1}} \frac{dr_c}{r_c} = \alpha_r \exp\left(\frac{3\sigma_m}{2\sigma_{eq}} \right) \int_0^{\tilde{p}} dp$$

$$r_{i+1} = r_i\, e^{A\tilde{p}} \qquad A := \alpha_r \exp\left(\frac{3\sigma_m}{2\sigma_{eq}} \right)$$

$$\boxed{\Delta D_{i\to i+1} = q_A \pi r_i^2 \left(e^{2A\tilde{p}} - 1 \right)}$$

Lifetime estimation for irradiated components

$$D_{r0} = q_A \pi r_{c0}^2$$

$$D_{rm1} = D_{r0} + \Delta D_{rm(0\to1)} = D_{r0} + q_A \pi r_{c0}^2 \left(e^{2A\tilde{p}} - 1\right)$$

$$D_{rm2} = D_{rm1} + \Delta D_{rm(1\to2)} + D_{r0} + \Delta D_{rm(0\to1)} = 2D_{r0} + q_A \pi r_{c0}^2 e^{2A\tilde{p}} + q_A \pi r_{c0}^2 e^{4A\tilde{p}} - 2q_A \pi r_{c0}^2$$

$$\vdots$$

$$D_{rm_{i+1}} = D_{rm_i} + D_{r0} + \Delta D_{rm(i\to i+1)} + \Delta D_{rm(i-1\to i)} + \ldots + \Delta D_{rm(0\to1)}$$

$$D_{rmN} = \underbrace{q_A \pi r_{c0}^2 e^{2A\tilde{p}} + q_A \pi r_{c0}^2 e^{4A\tilde{p}} + q_A \pi r_{c0}^2 e^{6A\tilde{p}} + \ldots + q_A \pi r_{c0}^2 e^{2NA\tilde{p}}}_{\text{Geometric series}}$$

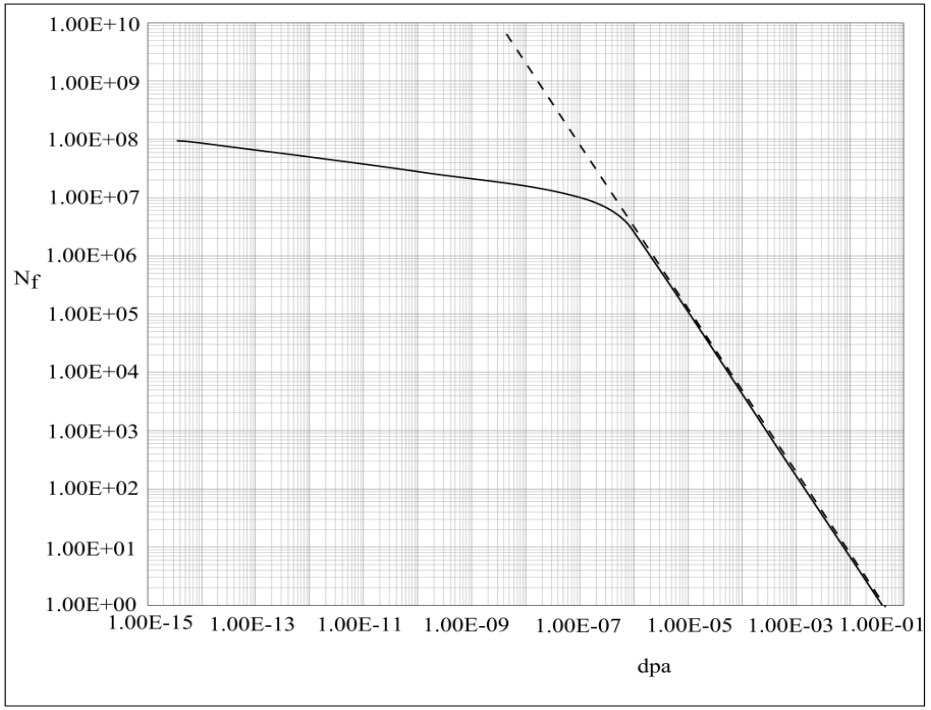
Number of cycles to failure N_f based on the critical damage criterion: $D_{rm}=D_{cr}$

$$D_{rmN} = q_A \pi r_{c0}^2 \sum_{n=1}^{N} e^{2nA\tilde{p}}$$

$$S_N = a_1 \frac{1-q^N}{1-q} \qquad S_N = q_A \pi r_{c0}^2 e^{2A\tilde{p}} \frac{1-e^{2ApN}}{1-e^{2Ap}}$$

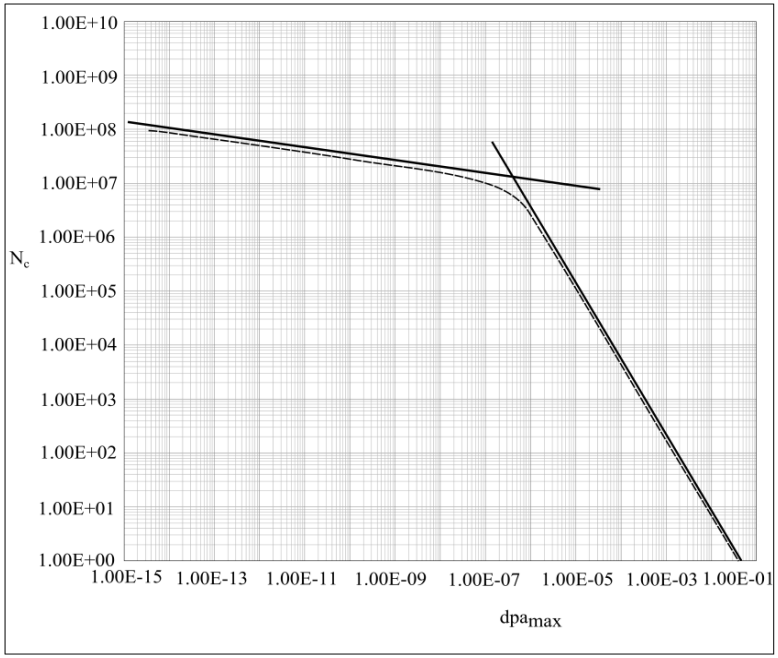
$$D_{rmN} = q_A \pi r_{c0}^2 e^{2A\tilde{p}} \frac{1-e^{2ApN}}{1-e^{2Ap}}$$

$$D_{rmN_f} = q_A \pi r_{c0}^2 e^{2A\tilde{p}} \frac{1-e^{2A\tilde{p}N_f}}{1-e^{2A\tilde{p}}} = D_{cr}$$

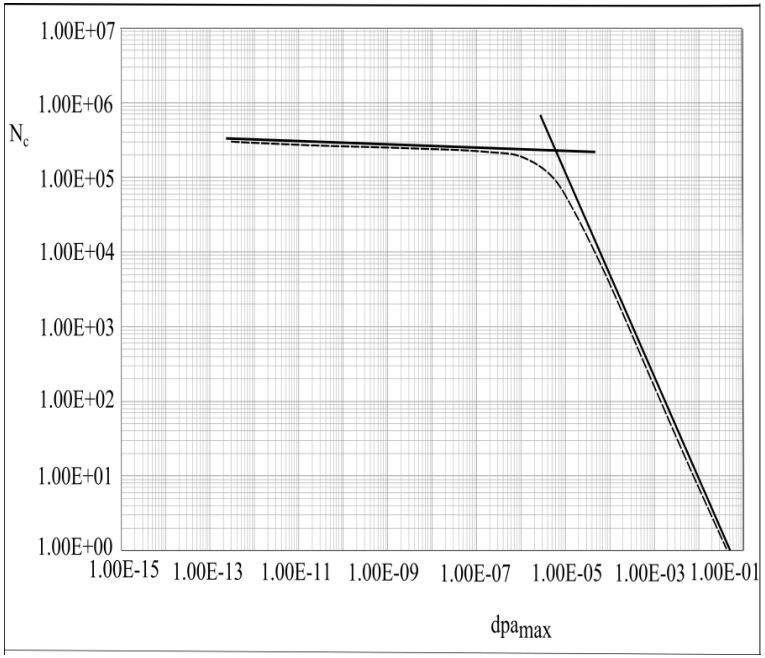


Bilinear approximations for R-T and Gurson models

Rice & Tracey model



Gurson model



$$\log(N_c) = a + b\log(dpa_{\max})$$

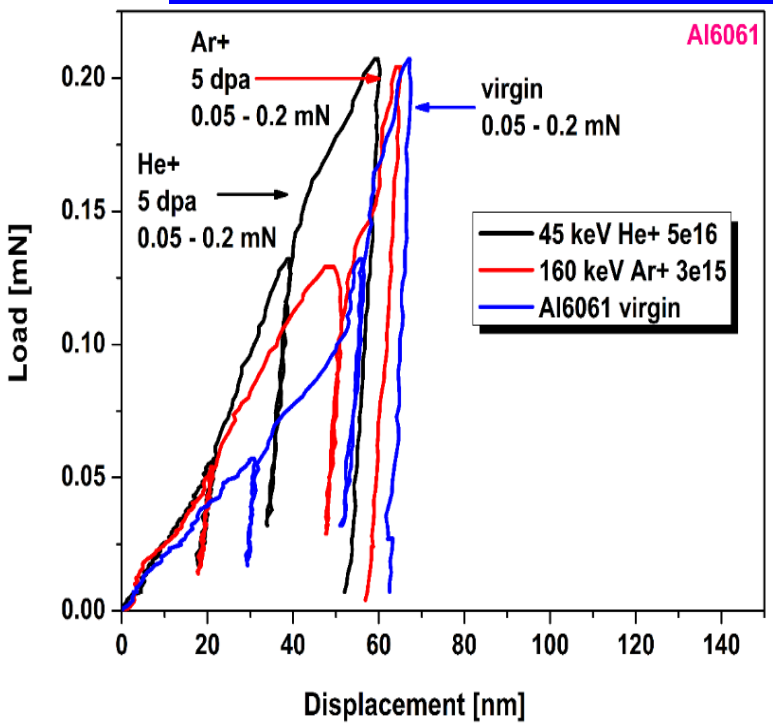
Analytical formula - useful tool for estimation of number of cycles to failure

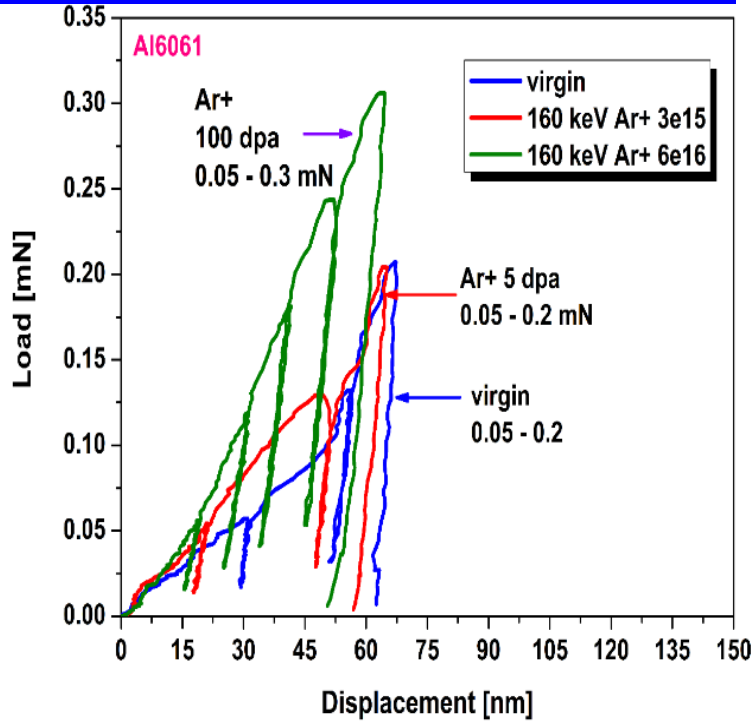
$$N_c = 10^a \, dpa_{\max}^b$$

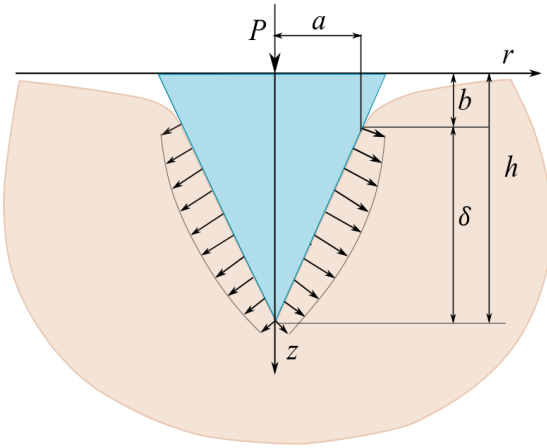
$$N_c = \begin{cases} 10^{-1.9}\, dpa_{\max}^{-1.4} & for \quad dpa_{\max} \geq 10^{-6} \\ 10^{6.1}\, dpa_{\max}^{-0.13} & for \quad dpa_{\max} < 10^{-6} \end{cases}$$

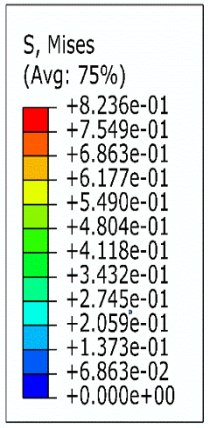
$$N_c = \begin{cases} 10^{-1.9}\, dpa_{\max}^{-1.4} & for \quad dpa_{\max} \geq 10^{-5} \\ 10^{5.43}\, dpa_{\max}^{-0.016} & for \quad dpa_{\max} < 10^{-5} \end{cases}$$

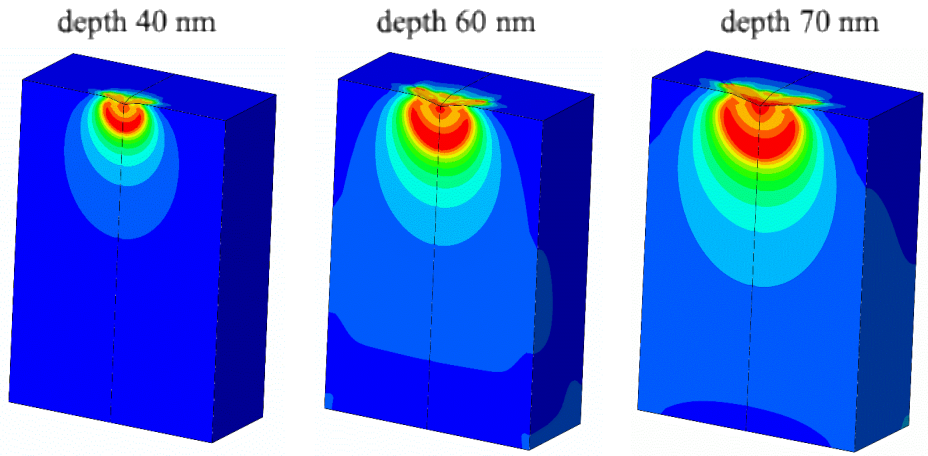
Nanoindentation of irradiated Al6061: experiment vs GTN



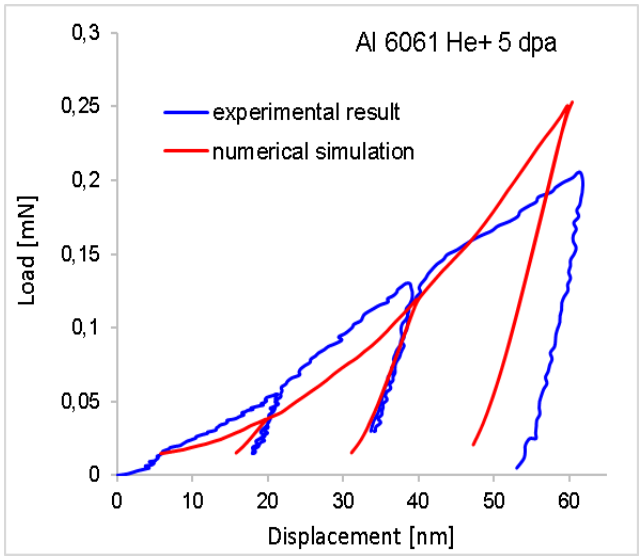


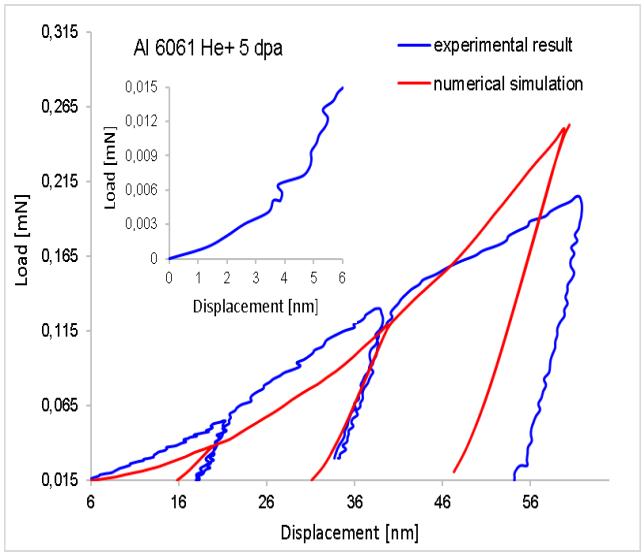


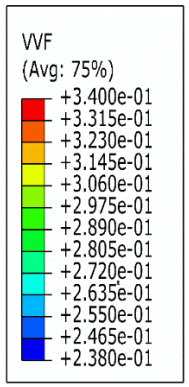


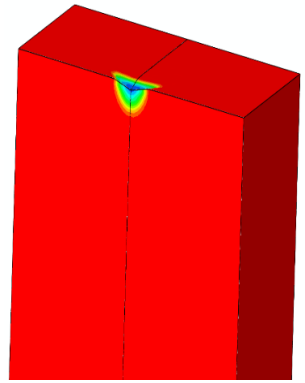


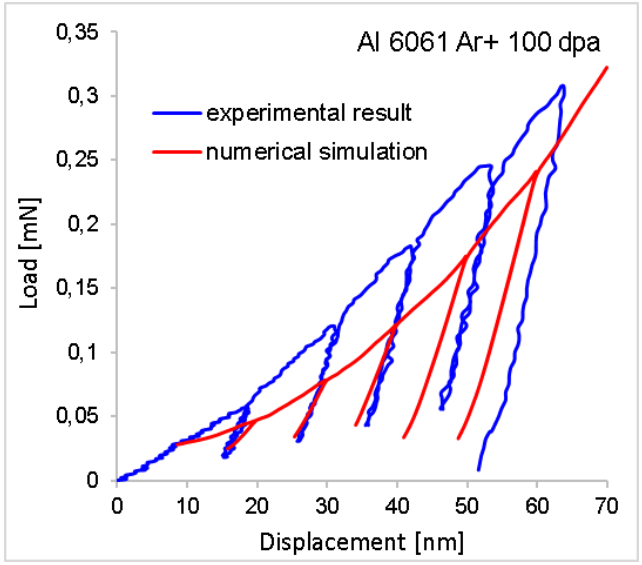
Nanoindentation of irr. Al6061: experiment vs. Gurson model

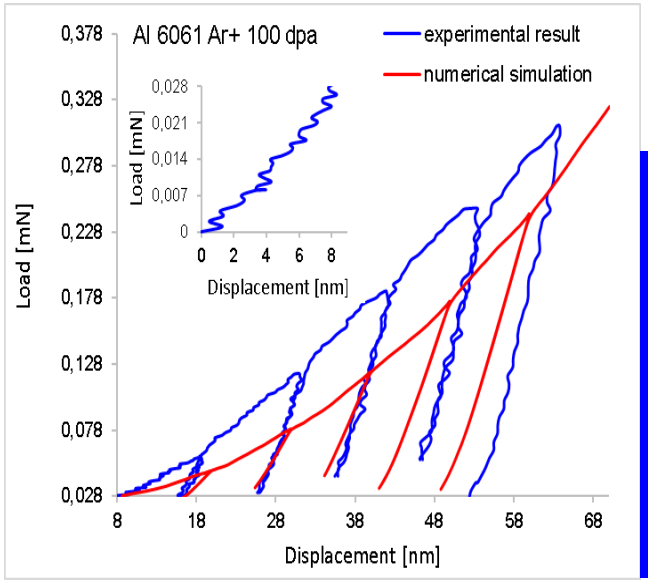


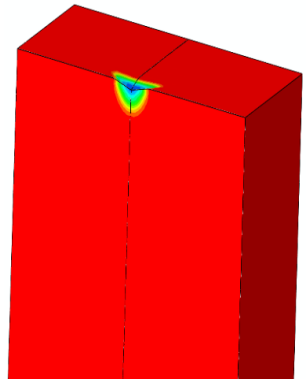


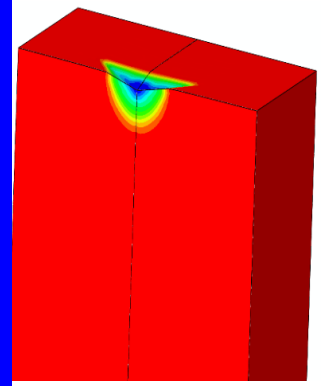










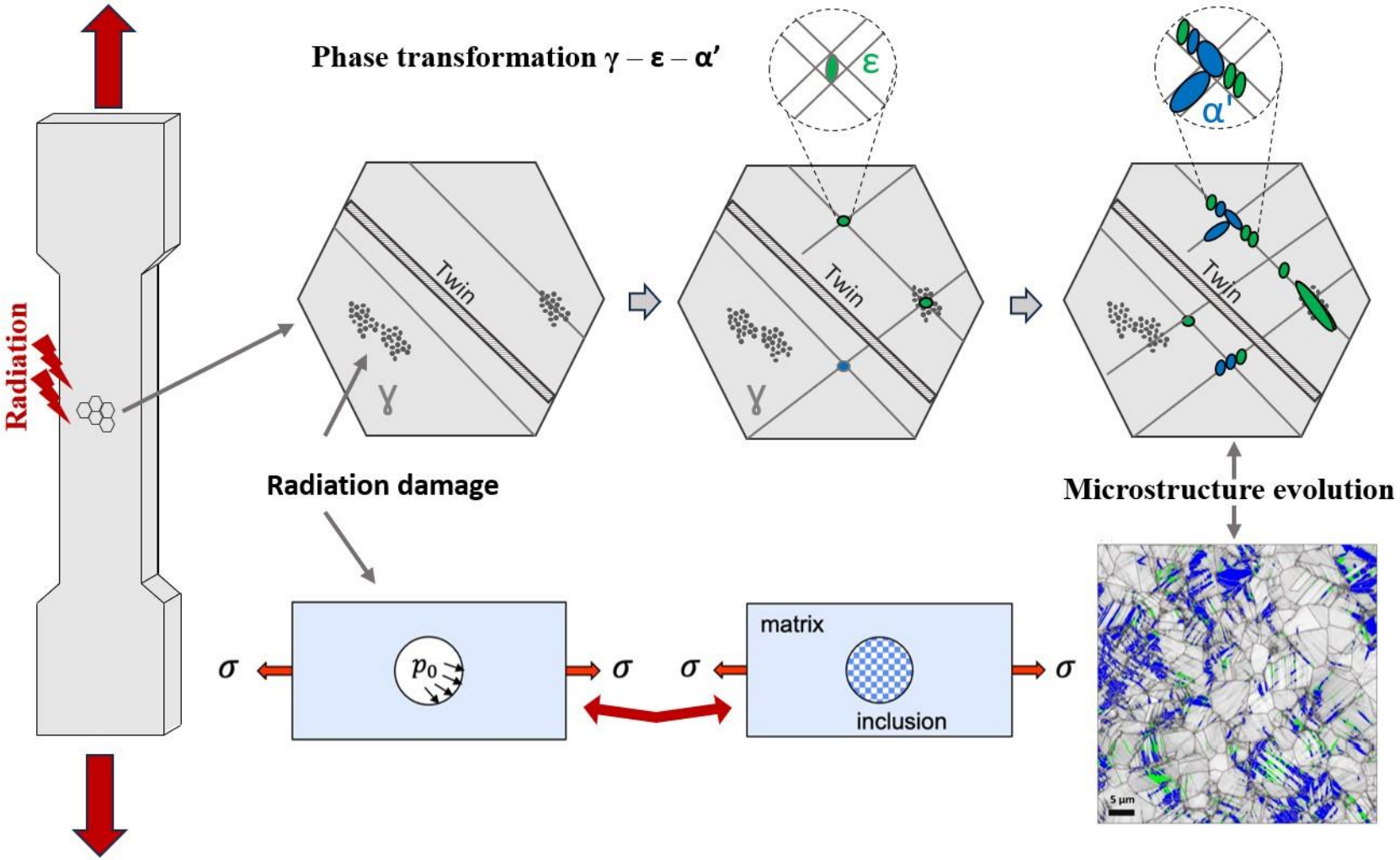


Evolution of porosity in the damaged material subjected to nano-indentation (100 dpa)



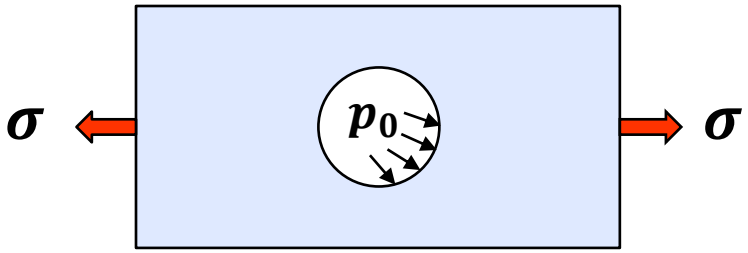
Hardening of irradiated materials

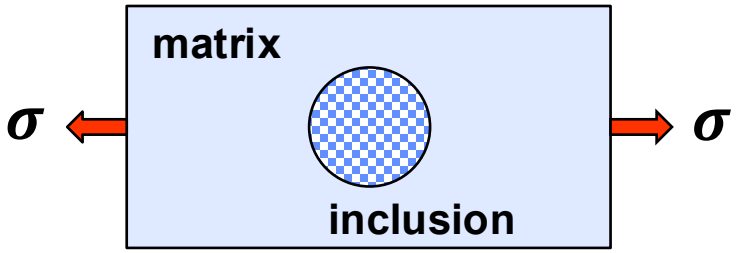
Metastable materials subjected to irradiation in cryogenic conditions



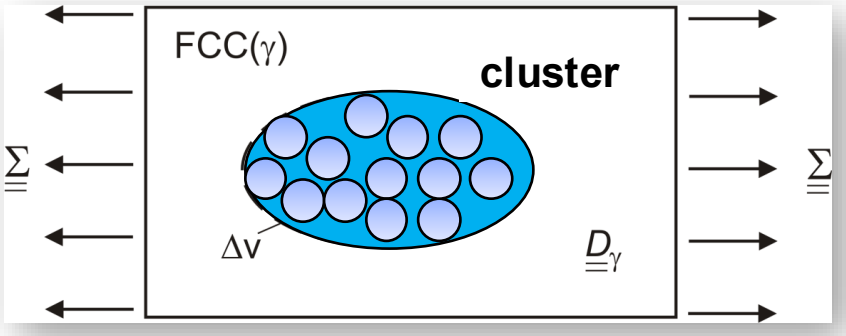
Type Eshelby entities: the equivalent inclusion principle

Assumptions:
- small strains approach
- perfect gas inside the void at a constant temperature T
- pressurized void is equivalent to inclusion subjected to hydrostatic stress





$$\Delta p = -3p_0\Delta\varepsilon$$



$$E_{p_{ijkl}} = 3k_pJ_{ijkl} + 2\mu_pK_{ijkl}$$

$$\mu_p = 0 \;\; ; \;\; \nu_p = 0 \;\; ; \;\; k_p \neq 0$$

$$\Delta\sigma = E_p\Delta\varepsilon \;\; ; \;\; E_p = -3p_0$$

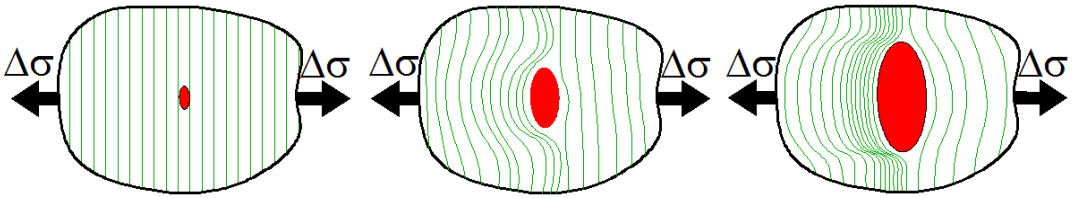
$$\Delta\sigma = -3p_0\Delta\varepsilon$$

Interaction of dislocations with pressurized voids (helium)

The Orowan mechanism:

$$\tau_p = \frac{\mu b}{d}\sqrt[3]{\frac{6\xi_0}{\pi}}\left(1+\frac{1}{3}\frac{\Delta\xi}{\xi_0}\right) \qquad \Delta\xi = 3C_1\xi_0\Delta p$$

$$\tau_p = \frac{\mu b}{d}\sqrt[3]{\frac{6\xi_0}{\pi}}(1+C_1\Delta p)$$



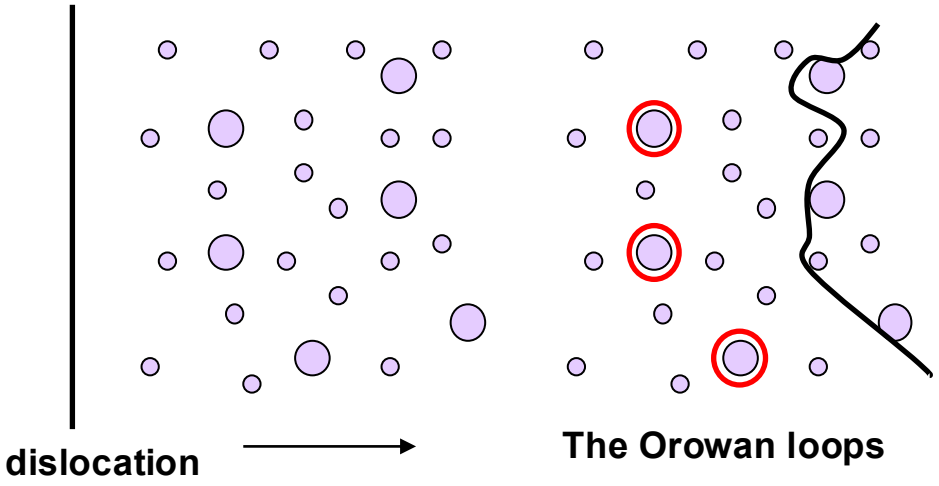
Using the Taylor factor:

$$\sigma_p = M\tau_p = MA_0\sqrt[3]{\xi_0}\left(1+\frac{1}{3}\frac{\Delta\xi}{\xi_0}\right)$$

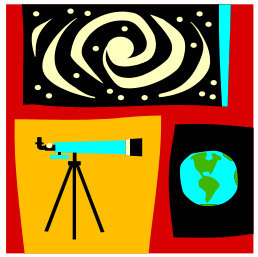
Hardening modulus:

$$C = C_0\left(1+\frac{1}{3}\frac{\Delta\xi}{\xi_0}\right) \rightarrow$$

$$C = C_0\,(1+h\Delta\xi)$$



Uniaxial case – closed form analytical solution for hardening



$$d\sigma = d\sigma_i + d\sigma_{MT}$$

General law

$$d\sigma_i = C_0(1 + h\Delta\xi)d\varepsilon^p \quad ; \quad d\sigma_{MT} = E_H\, d\varepsilon^p = C_{MT}\, d\varepsilon^p$$

$$\Delta\sigma_i = C_0(\xi_0)\left(\varepsilon^p + \frac{1}{2}C_1(\varepsilon^p)^2\right)$$

Interaction of dislocations with voids

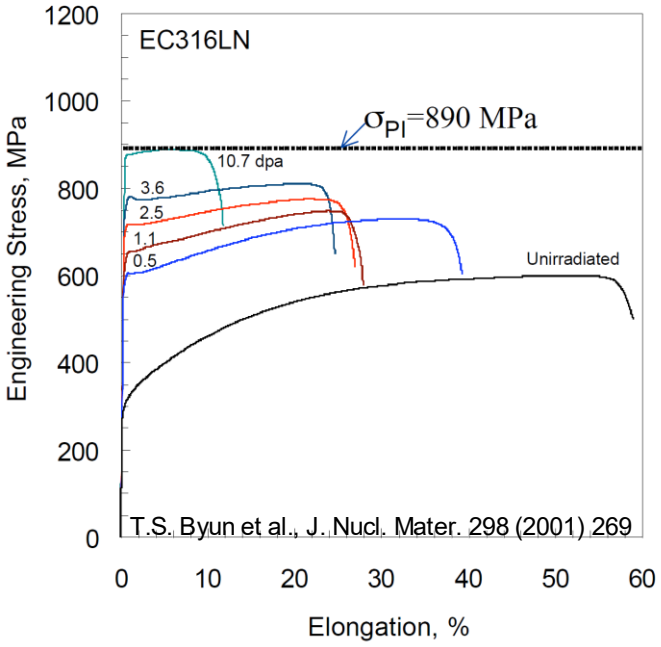
$$\Delta\sigma_{MT} = -\frac{5}{2}\mu\eta_0\frac{\xi_0}{C_1}\left[\frac{1}{4}(\chi^2 - 1) - \frac{4}{81}\xi_0(\chi^3 - 1) + -\frac{2}{81}{\xi_0}^2(\chi^4 - 1)\right]$$

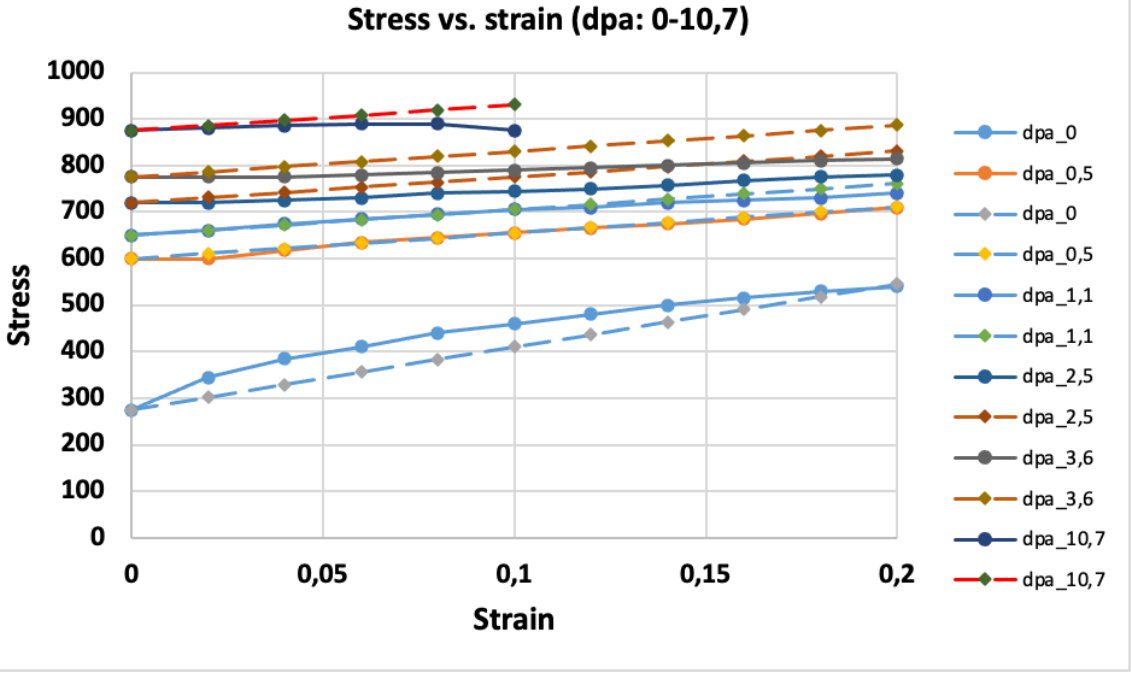
$$\chi = 1 + 3C_1\varepsilon^p \qquad \eta_0 = \frac{C_i}{E} = \frac{M\frac{\mu b}{d}\sqrt[3]{\frac{6}{\pi}}\sqrt[3]{\xi_0}}{E}$$

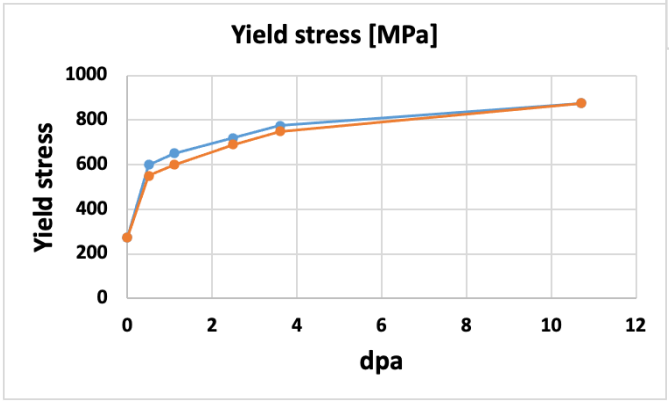
MT homogenization

$$\sigma = \sigma_0 + C_0(\xi_0)\left(\varepsilon^p + \frac{1}{2}C_1(\varepsilon^p)^2\right)$$
$$-\frac{5}{2}\mu\frac{C_i(\xi_0)}{E}\frac{\xi_0}{C_1}\left[\frac{1}{4}(\chi^2 - 1) - \frac{4}{81}\xi_0(\chi^3 - 1) + -\frac{2}{81}{\xi_0}^2(\chi^4 - 1)\right]$$

Uniaxial case: 316LN stainless steel irradiated to high dpa level





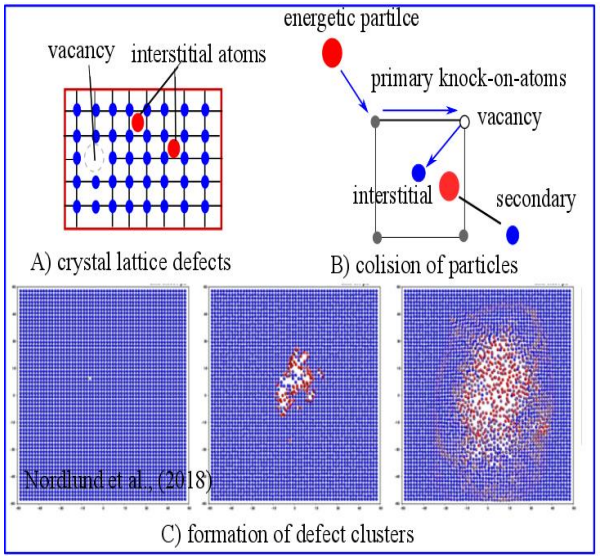


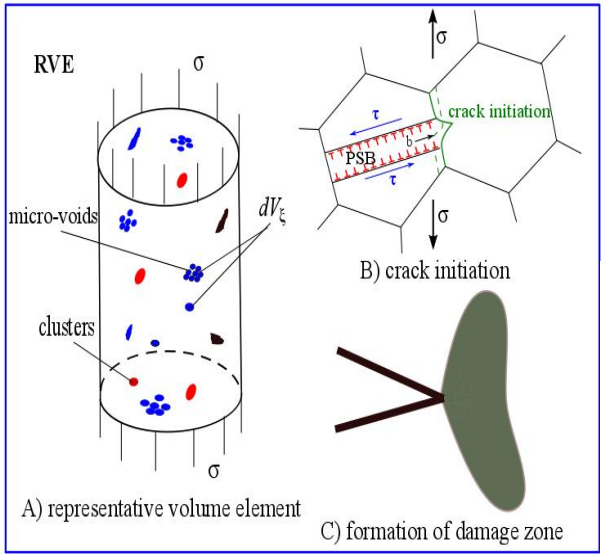
Radiation induced hardening comprising:

- **massive interaction of dislocations with the pressurized voids,**
- **evolution of tangent stiffness expressed by the Mori-Tanaka homogenization.**

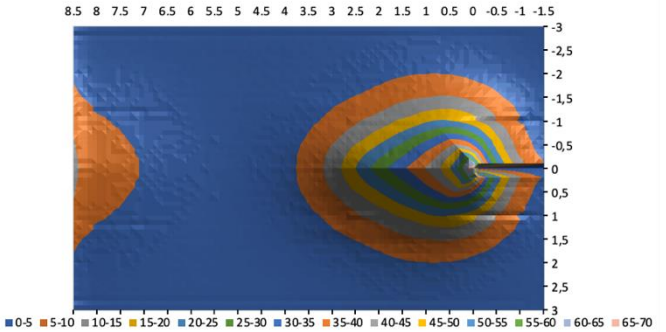
Fracture at extremly low temperatures

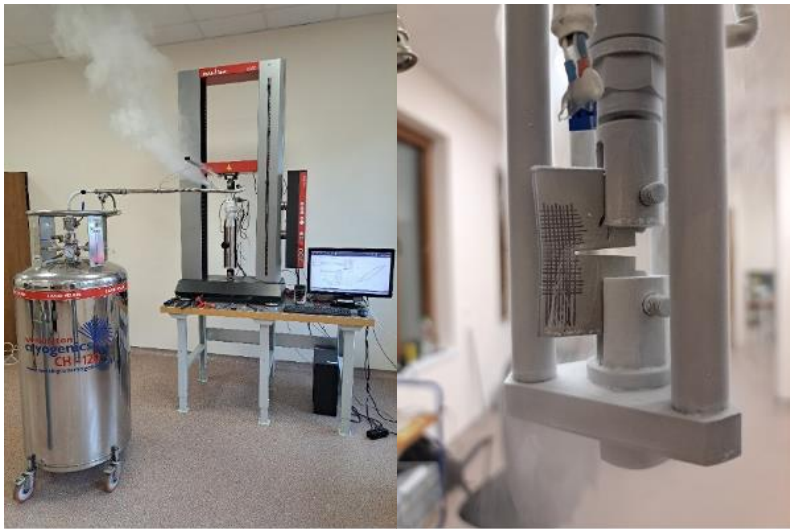
Fracture induced fcc-bcc phase transformation in 316L at 4.2 K

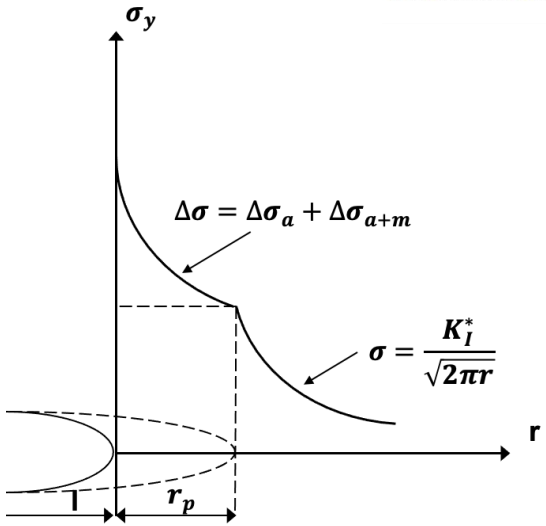


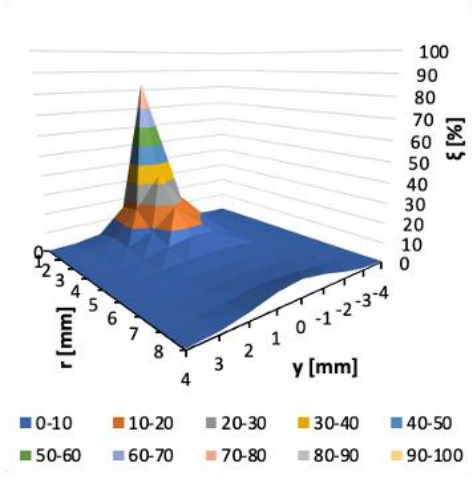


Initiation of fracture in the irradiated metastable materials loaded in liquid helium (4.2 K).









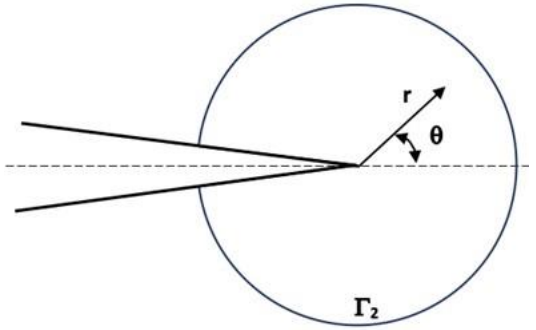
Fracture at extremly low temperatures – the Hutchinson solution

Type Airy stress function

$$\sigma_r = r^{-1}\psi_{,r} + r^{-2}\psi_{,\theta\theta}$$

$$\sigma_\theta = \psi_{,rr}$$

$$\sigma_{r\theta} = r^{-2}\psi_{,\theta} - r^{-1}\psi_{,r\theta}$$



Hencky-Ilyushin deformation theory

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p = (1+\nu)\sigma_{ij} - \nu\sigma_{kk}\delta_{ij} + \varphi s_{ij} \qquad \varphi = \chi\left(1 - \sigma_i^{-1}\right)$$

Linear hardening

$$\sigma_i = \left(\frac{3}{2}s_{ij}s_{ij}\right)^{\frac{1}{2}} \qquad \sigma_i = 1 + \frac{\eta}{1-\eta}\varepsilon_i^p \qquad \varepsilon_i^p = \left(\frac{2}{3}\varepsilon_{ij}^p\varepsilon_{ij}^p\right)^{\frac{1}{2}}$$

Hutchinson J.W., JMPS, 1967

Fracture at extremly low temperatures – the Hutchinson solution

Compatibility equation

$$r^{-1}(r\varepsilon_\theta)_{,rr} + r^{-2}(\varepsilon_r)_{,\theta\theta} - r^{-1}(\varepsilon_r)_{,r} - 2r^{-2}\left(r(\varepsilon_{r\theta})_{,\theta}\right)_{,r} = 0$$

First variation of the complementary energy

$$\eta^{-1}\Delta\Delta\psi$$
$$-\frac{\lambda}{3}\{[r^{-2}\partial^2_{,\theta\theta} - r^{-1}\partial_{,r}][\sigma_i^{-1}(2r^{-2}\psi_{,\theta\theta} + 2r^{-1}\psi_{,r} - \psi_{,rr})]$$
$$+ r^{-1}\partial_{,rr}[\sigma_i^{-1}(2r\psi_{,rr} - \psi_{,r} - r^{-1}\psi_{,\theta\theta})] + 6r^{-2}\partial^2_{,r\theta}[r\sigma_i^{-1}\partial_{,r}(r^{-1}\psi_{,\theta})]\} = 0$$

Boundary conditions

$$\psi = 0 \;;\quad \psi_{,\theta} = 0$$

The solution

$$\psi = Kr^s\tilde{\psi}(\theta)$$

$$\sigma_r = Kr^{s-2}\tilde{\sigma}_r(\theta) = Kr^{s-2}(s\tilde{\psi} + \tilde{\psi}_{,rr})$$

$$\sigma_\theta = Kr^{s-2}\tilde{\sigma}_\theta(\theta) = Kr^{s-2}s\tilde{\psi}(s-1)$$

$$\sigma_{r\theta} = Kr^{s-2}\tilde{\sigma}_{r\theta}(\theta) = Kr^{s-2}(1-s)\tilde{\psi}_{,\theta}$$

Fracture at extremly low temperatures – the Hutchinson solution

The plastic zone is assumed small compared to the length of the macro-crack

$$\psi = Kr^s\tilde{\psi} \quad ; \quad \sigma_i = Kr^{s-2}\tilde{\sigma}_i$$

The stress function fulfils the biharmonic equation

$$\Delta\Delta\psi = 0 \qquad \psi = K\sqrt{r}\left(cos\frac{\theta}{2} + \frac{1}{3}cos\frac{3}{2}\theta\right)$$

$$\sigma_i = \frac{K}{\sqrt{r}}\left(cos^2\frac{\theta}{2} + \frac{3}{4}sin^2\theta\right)^{\frac{1}{2}}$$

The Rice integral takes the form

$$\int_{\Gamma_2} Wdy - \sigma_{ij}n_j u_{i,x}ds = 2\pi K^2\eta^{-1} = \pi\sigma^{\infty 2} \qquad K = \sigma^{\infty}\sqrt{\frac{\eta}{2}}$$

The plastic strains $\varepsilon_{ij}^p = \varphi s_{ij}$ $\qquad \varphi = \chi\left(1 - \sigma_i^{-1}\right)$

The phase transformation in the vicinity of the macro-crack: NEW

Plastic strain intensity

$$\varepsilon_i^p = \sqrt{\frac{2}{3}\varepsilon_{ij}^p \varepsilon_{ij}^p} = (\eta^{-1} - 1)(\sigma_i - 1)$$

$$\bar{\varepsilon}_i^p = (\eta^{-1} - 1)\frac{\bar{\sigma}_0}{E}\left(\frac{\sigma^\infty}{\sqrt{2r}}\sqrt{\eta\left(cos^2\frac{\theta}{2} + \frac{3}{4}sin^2\theta\right)} - 1\right)$$

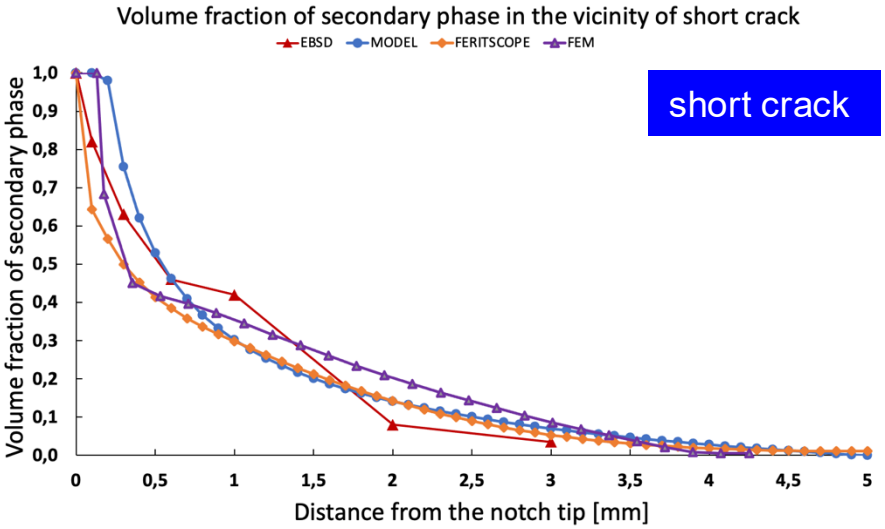
Kinetics of the phase transformation

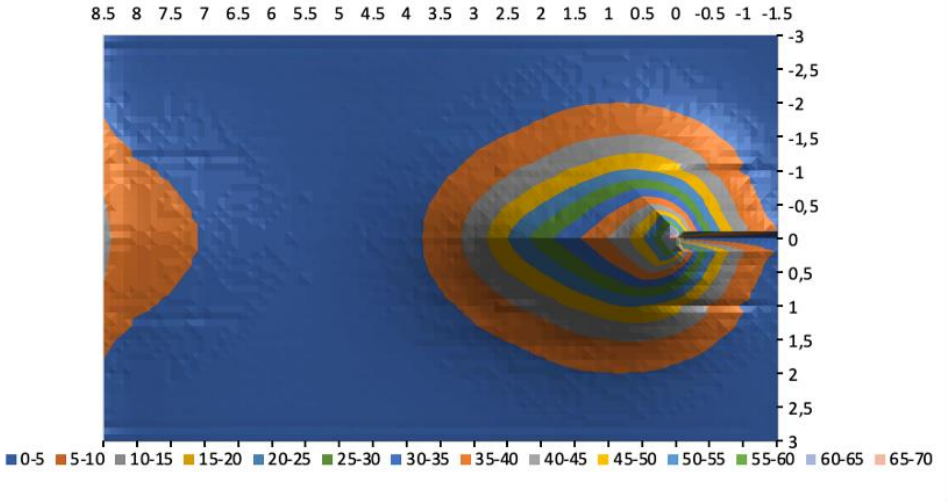
$$\xi_{,t} = A\left(\bar{\varepsilon}_i^p\right)_{,t} \qquad \bar{\varepsilon}_i^p \geq \bar{\varepsilon}_{i\,\xi}^p \qquad \int_0^{\hat{\xi}} d\xi = A\int_{\bar{\varepsilon}_{i\,\xi}^p}^{\widehat{\bar{\varepsilon}_i^p}} d\bar{\varepsilon}_i^p \qquad \xi = A\left(\bar{\varepsilon}_i^p - \bar{\varepsilon}_{i\,\xi}^p\right)$$

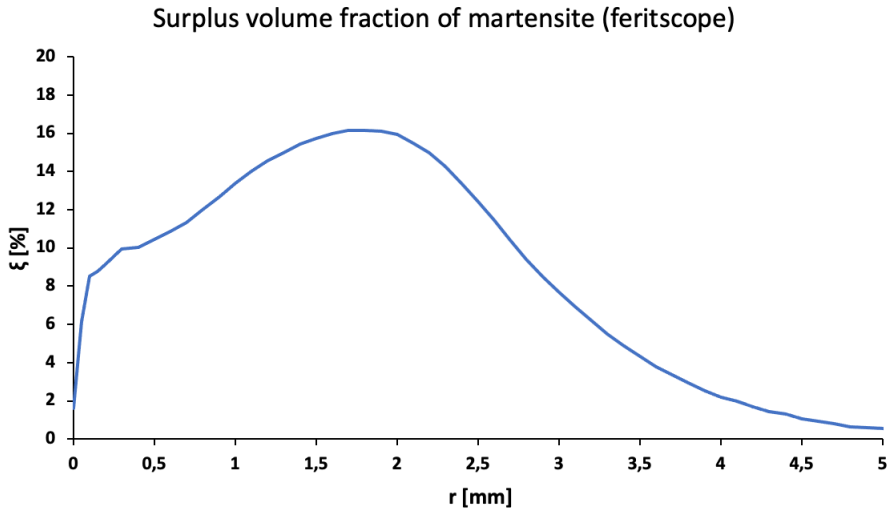
Distribution of the secondary phase

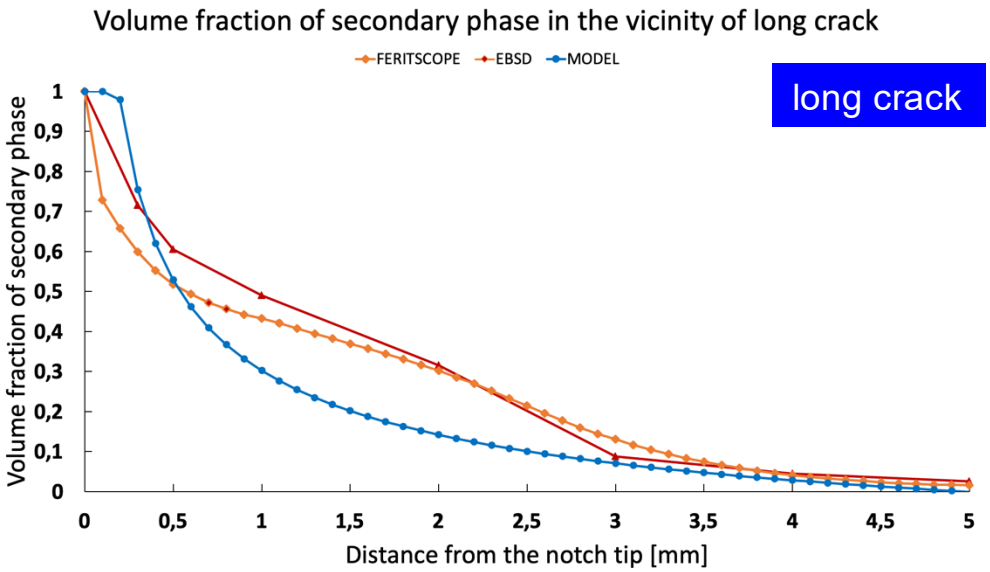
$$\xi = A\left[(\eta^{-1} - 1)\frac{\bar{\sigma}_0}{E}\left(\frac{\sigma^\infty}{\sqrt{2r}}\sqrt{\eta\left(cos^2\frac{\theta}{2} + \frac{3}{4}sin^2\theta\right)} - 1\right) - \bar{\varepsilon}_{i\,\xi}^p\right]$$

Constitutive model versus in situ magnetometric measurements









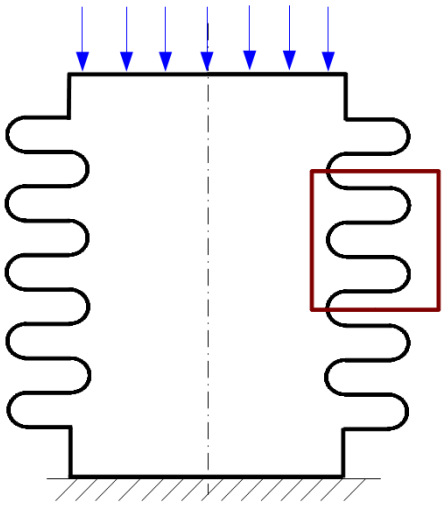
Schmidt R., Schmidt E., Skoczen B., IJP, 2025

SPAS 2026

Applications of the constitutive models

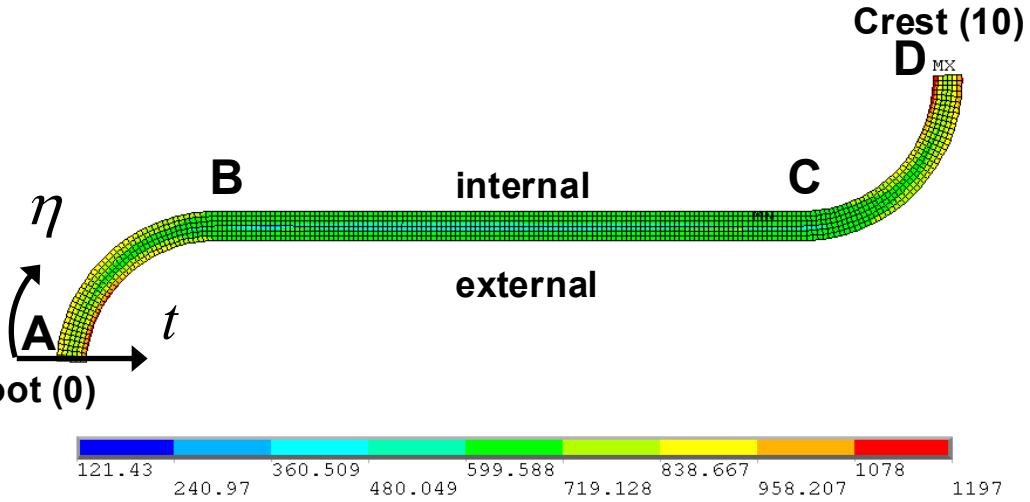


Compensation system of CERN Large Hadron Collider (LHC)

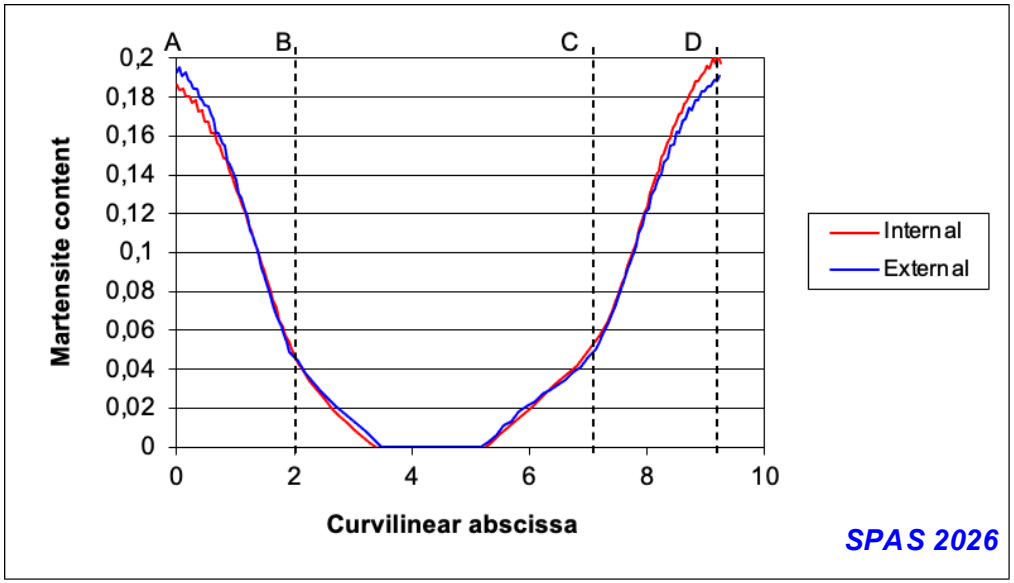


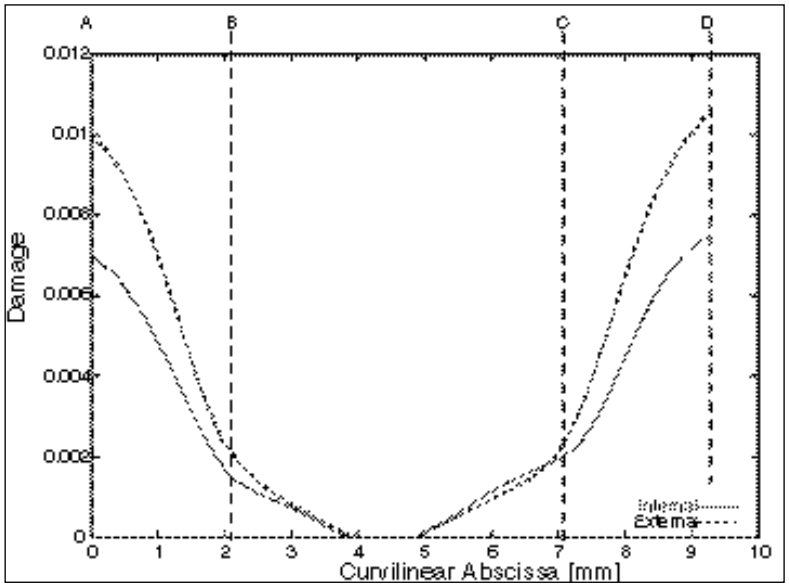




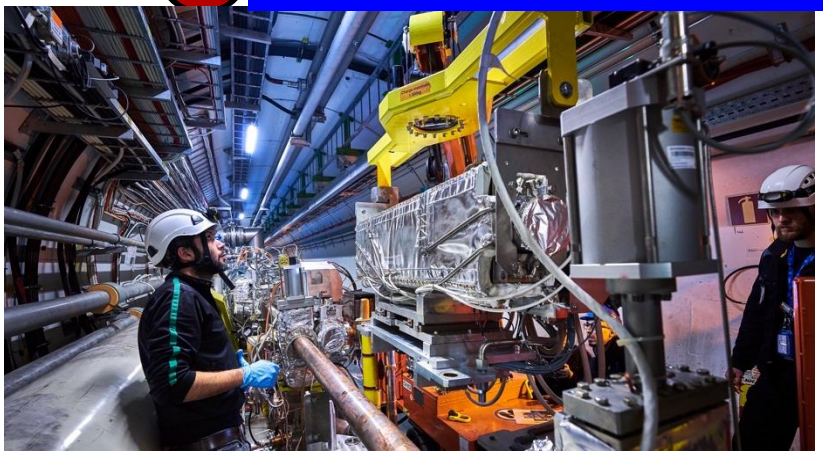


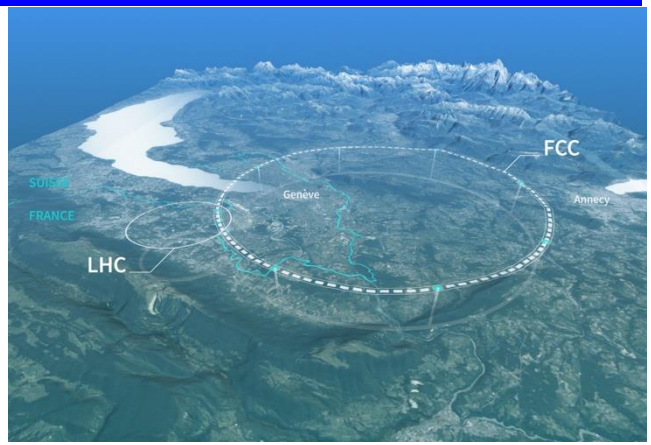
Profile of α' phase at 77 K



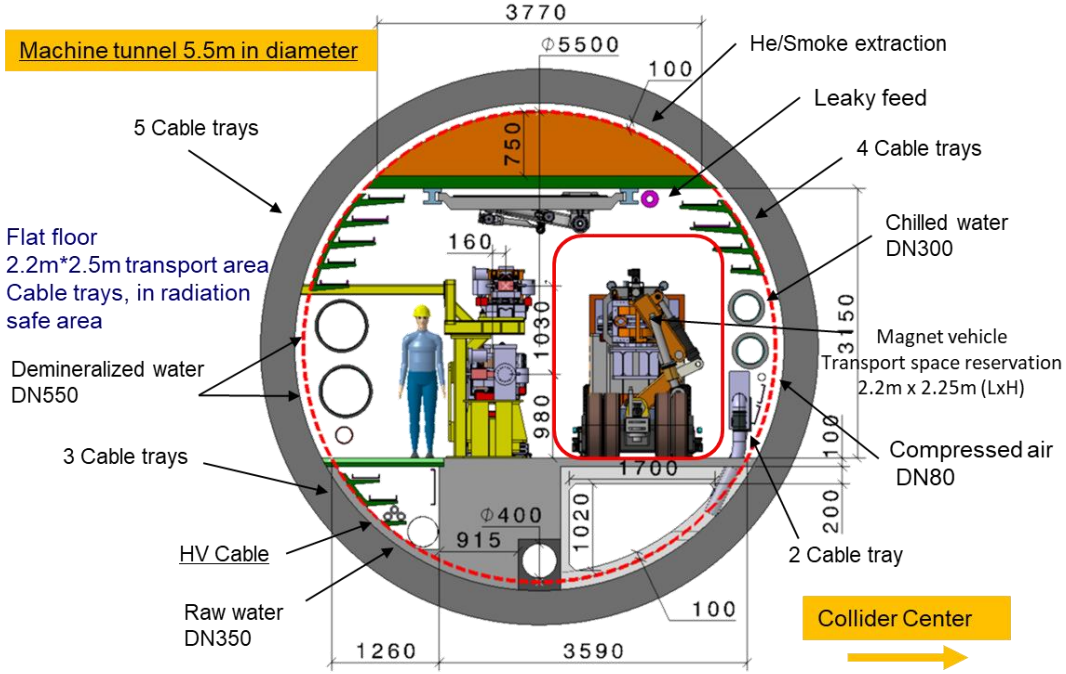


CERN Future CircularCollider (FCC) – high intensity radiation

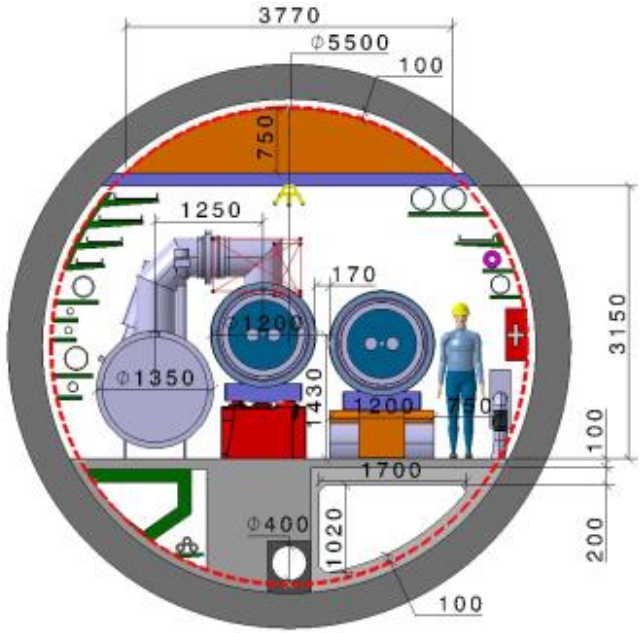




Electron-Positron Collider



Proton-Proton Collider





Conclusions:

1. As the temperature approaches absolute zero, the **thermodynamic instability** takes place leading to thermo-mechanical coupling.
2. Physically based **constitutive models** describing the phenomena occurring in ductile materials at extremely low temperatures were developed.
3. The evolution of **ductile damage** and **fracture** in the vicinity of absolute zero was investigated.
4. Closed-form **analytical solutions** were obtained for the cases of uniaxial loading.
5. In particular, a new model of the **zone adjacent to the crack tip** was developed, incorporating the field of fcc-bcc phase transformation.

Thank you for your attention!









CUT Laboratory of Extremely Low Temperatures

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