Universal Mean Field in the Service of Nuclear Structure: From GDR and Super-Deformation to Exotic Symmetries

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59^{th} Zakopane Conference on Nuclear Physics,
"Extremes of the Nuclear Landscape"

Structuring the Presentation

Universal Mean Field $\leftrightarrow$ GEMINI $\leftrightarrow$ Choice of Subjects

INPUT: Consider ∼400 published texts & 12 000 citations

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$\rightarrow$ 400 texts within 40 minutes $\rightarrow$ 10 texts per minute

$\rightarrow$ Physics interests and citation numbers are not mutually proportional

$\rightarrow$ Four speakers share the big field of research: $\frac{1}{4} \times 1 \rightarrow$ Nonuniformities

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- Final choice: We choose JD to choose the Sub-Fields of Presentation

ACKNOWLEDGEMENTS

Various branches of this project were contributed
by collaborators from research institutions in and between:
Canada and USA on the one side and Japan on the other
Their names are collected in a dedicated Annexe.

Many thanks to the youngest (PhD) joining our research activities:

Irene Dedes, Jie Yang, David Rouvel and Kasia Mazurek

Very special thoughts are associated with:

• Numerous high-spin collaboration projects started at Daresbury Laboratory towards the early 1980'ies and continued over many years in the USA ... **Mark RILEY**

• Collaboration on Giant Dipole Resonances, nuclear evolution with spin and temperature, Jacobi shape transitions ... **Adam MAJ**

• Numerous fascinating projects involving nuclear exotic symmetries in relation with relativistic mean field theory ... **Nicolas SCHUCK**

Present Mean-Field Project Involves

the Following Mathematical-Physics Tools:

– Inverse Problem Theory (to control parameter adjustments)

– Group-, and Group Representation Theories (symmetries)

– Graph Theory (shape transition in deformation spaces)

Comprehensive List of Nuclear Phenomena Addressed in Collaboration with Experiment

1. Isomers

K-isomers,
Yrast trap isomers and yrast lines,
Shape isomers,
In particular fission isomers, Exotic symmetry isomers, etc.;

2. Nuclear Masses

Progress in measurements/reproduction of nuclear masses

3. Rotational band properties

Bands based on isomers,
Quasiparticle excitations vs. rotational bands,
Band crossings and interactions,
Shape evolution with spin,
So-called paring phase transitions, etc.;

4. Exotic symmetries and shapes

Tetrahedral and octahedral symmetries (freshly identified),
Super-deformation, Hyper-deformation
Toroidal shapes,
Shapes leading to tripartition, etc.;

5. Fission and exotic fission modes

Competing fission paths with Graph Theory treatment,
Local-minimum to local-minimum transitions, etc.;

6. Specific nuclear excitations modes

Modes involving high temperatures,
Modes involving high spins,
Giant Dipole Resonances,
Jacobi and Poincaré shape transitions, etc.;

Collaborations & Institutions Already Involved

- A few selected publications combining experiment and theory will be shown in the presentation, they involved diverse experimental laboratories

- **Among collaborating experimental institutions:**

CANADA: TRIUMF, University of Manitoba, University of Calgary, University of Victoria, Saint Mary's University, McGill University, University of British Columbia ...

GERMANY: Technische Universität Darmstadt, Johannes Gutenberg-Universität Mainz, FAIR (HFHF), Giesseen, GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Universität zu Köln ...

JAPAN Center for Nuclear Study, University of Tokyo, RIKEN Nishina Center ...

USA: Argonne National Laboratory, Lawrence Berkeley National Laboratory, University of Chicago, Colorado School of Mines, University of Notre Dame ...

Some bilateral contacts: IFJ, Cracow; IPHC, Strasbourg; Laboratoire National Henri Becquerel (LNE-LNHB); University of Jyväskylä; University of Gröningen.

Part I

Phenomenological Woods-Saxon Mean-Field Hamiltonian and its Universal Parameters

Mathematical forms of Woods-Saxon potentials
are defined today
using well known relativistic meson-exchange theory *)

However, it will be instructive to begin by presenting
often discussed phenomenological arguments
as an introduction

$\boxed{\text{We begin with the simplest, spherical realisations}}$

*) A. Bohr and B. Mottelson, Chapter 2 (and Appendix 2C/2D)
Nuclear Structure (Volume I)

Postulated Form of Spherical Woods-Saxon Potential

Simple geometrical features are conveniently described using Woods-Saxon Mean Field Hamiltonian:

$$\hat{H}^{WS} = \underbrace{\hat{t}}_{kinetic} + \underbrace{\hat{V}^{WS}_{cent}}_{central} + \underbrace{\hat{V}^{WS}_{so}}_{spin-orbit}$$

- **Central potential in the simplest, spherical-symmetry case:**

$$V^{WS}_{cent}(r; V_o, r_o, a_o) \stackrel{df}{=} \frac{V_o}{1 + \exp\left[(r - R_o)/a\right]}; \quad R_o = r_o A^{1/3}$$

- **Here we have 3 central-potential parameters in total:**

V_o - central potential depth parameter
r_o - central potential radius parameter
a_o - central potential diffuseness parameter

- **Addressing spin-orbit in the simplest spherical-symmetry case:**

$$V_{so}^{WS}(r;\lambda_{\ell\cdot s}, r_{\ell\cdot s}, a_{\ell\cdot s}) \stackrel{df}{=} \frac{1}{r}\frac{d\rho(r)}{dr}\;\hat{\vec{\ell}}\cdot\hat{\vec{s}} \to \frac{1}{r}\frac{d\mathcal{V}_{\ell\cdot s}^{WS}}{dr}\;\hat{\vec{\ell}}\cdot\hat{\vec{s}}$$

where by definition

$$\mathcal{V}_{\ell\cdot s}^{WS}(r;\lambda_{\ell\cdot s}, r_{\ell\cdot s}, a_{\ell\cdot s}) \stackrel{df}{=} \frac{\lambda_{\ell\cdot s}}{1+\exp\left[(r-R_{\ell\cdot s})/a_{\ell\cdot s}\right]};\;\; R_{\ell\cdot s} = r_{\ell\cdot s}A^{1/3}$$

[See A. Bohr and B. R. Mottelson, Nuclear Structure, Vol. 1, Eq. (2-144)]

– – – – – – – – – – –

- **Corresponding potential parameters:**

$\lambda_{\ell\cdot s}$ - **spin-orbit potential strength parameter**
$r_{\ell\cdot s}$ - **spin-orbit potential radius parameter**
$a_{\ell\cdot s}$ - **spin-orbit potential diffuseness parameter**

Further comments $\to\to\to$

Unique Features of the WS Parameterisation

As one attentive observer put it:

The Woods-Saxon potential has a very typical feature:

Each of its parameters relates to a specific, well defined, measurable mechanism

V_o - **Central potential depth** $\leftrightarrow$ **Nucleonic binding**
r_o - **Central potential radius** $\leftrightarrow$ **Nuclear sizes**
a_o - **Central potential diffuseness** $\leftrightarrow$ **Particle scattering**

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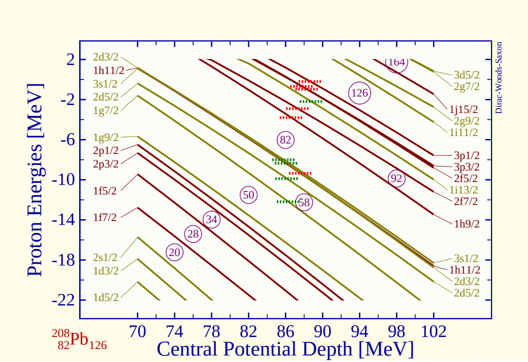
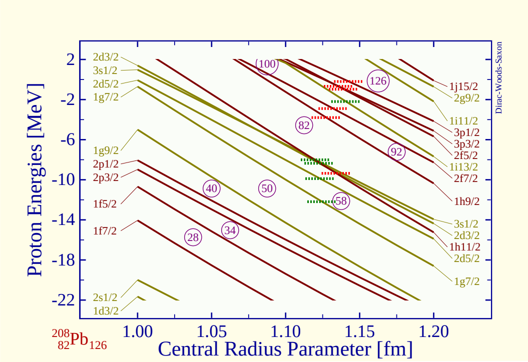
It is instructive to show parametric impact graphically:

- The nucleonic binding energies are inversely-proportional to the potential depth – (let us notice that this variation is nearly parallel)



Mechanism No. 1: The potential depth parameter is primarily responsible for the nucleonic binding energies. Observe nearly linear dependence and an ideal description of the experimental levels: ^{208}Pb.

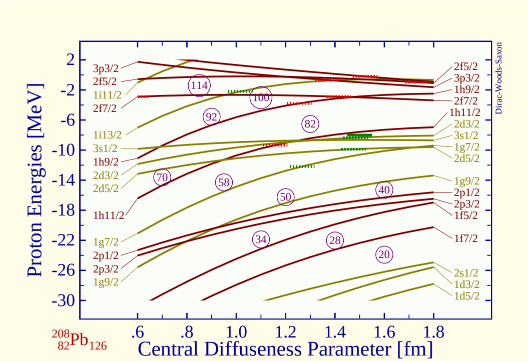
- The nucleonic binding energies vary nearly linearly also as functions of the central radius (even though some levels may cross "gently").



Mechanism No. 2: *The central-radius parameter is co-responsible for the nucleonic binding energies but impacts importantly the r.m.s. radii. Note the correspondence with experiment. Here: For ^{208}Pb.*

Nucleon Energies: Central Potential Impact - 3

- The central diffuseness parameter is the only one that clearly distinguishes among the eigen-energies with various quantum numbers.



Mechanism No. 3: Observe the existence of families of nearly parallel lines, which are characterised by the common ℓ quantum number.

An alert student noticed*):

Wait a moment: Here we are discovering a quite new, seemingly fundamental feature ...

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Indeed:

- **If levels move together and in agreement with experiment when single parameter varies, then whole nucleonic spectrum is partially modelled by this (universally acting) parameter**

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Indeed:

- **If levels move together and in agreement with experiment when single parameter varies, then whole nucleonic spectrum is partially modelled by this (universally acting) parameter**

- **Summarising the idea of universality: we dispose of "only" 6 proton & 6 neutron parameters common for $\propto 3\,000$ nuclei in the whole Mass Table**

- **Next Task: Optimisation of Hamiltonian Parameters**

$^{*)}$**The person chooses to remain incognito**

Part II

Principles of Parameter Optimisation Employing Applied-Mathematics Tools:

Inverse Problem Theory

About the Importance of the Inverse Problem Theory

• After laborious theoretical constructions, we get terribly exhausted and forget that: *Professional parameter determination is a noble, mathematically sophisticated procedure based on statistical theories often more involved than the physical problems under our studies!*

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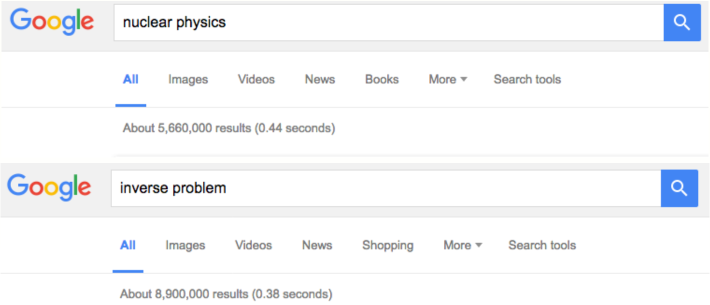

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Stochastic Interpretation of Predictive Power$^{*)}$

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- Observables will be characterised not only by related eigenvalues i.e. $\{f_j\}$

$$\big[\hat{\mathcal{F}}_1 \to \{f_1\}, \quad \hat{\mathcal{F}}_2 \to \{f_2\}, \quad \ldots \; \hat{\mathcal{F}}_p \to \{f_p\}\big]$$

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but also by distributions of probability of their validity - or applicability
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- These distributions are obtained using methods inverse problem theory

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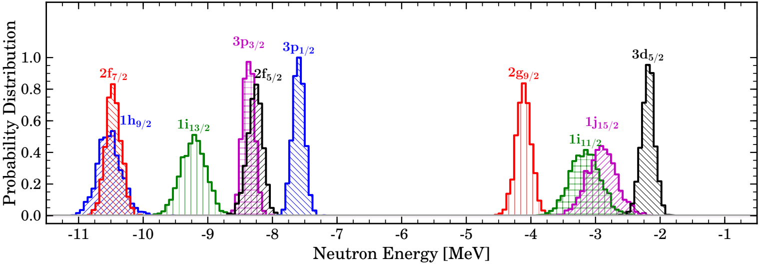
Single-neutron energy uncertainty probability distributions for ^{208}Pb with our 'universal' Woods-Saxon Hamiltonian



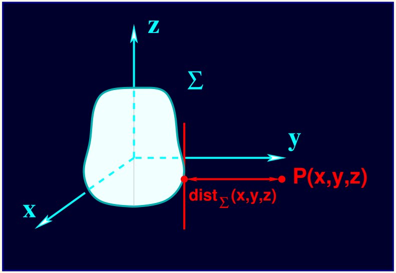
- **Observe that in the presented example uncertainties increase with angular momentum**

Part III

Main concepts of parameter optimisation and parameter adjustment stabilisation

W-S Mean-Field is a Functional of $\mathrm{dist}_\Sigma(\vec{r})$

Given surface Σ



Constructing distance function

Phenomenological Woods-Saxon potential respects automatically the symmetries of the underlying surface Σ:

$$\hat{V}^{\mathrm{WS}}_{\mathrm{Cent}}(\vec{r}) \equiv \frac{V_0}{1+\exp\{\mathrm{dist}_\Sigma(\vec{r})/a\}}$$

$$\hat{V}^{\mathrm{WS}}_{\mathrm{S-O}}(\vec{r}) \equiv \left[\nabla V^{\mathrm{WS}}_{\mathrm{Cent}}(\vec{r}) \wedge \hat{p}\right] \cdot \vec{s}$$

$$\boxed{\hat{V}_{\mathbf{mf}} = \hat{V}^{\mathbf{WS}}_{\mathbf{Cent}} + \hat{V}^{\mathbf{WS}}_{\mathbf{S-O}} + \hat{V}_{\mathbf{C}}}$$

$$\text{Surface}\,\Sigma: \quad R(\vartheta,\varphi) = R_o\, c(\{\alpha\})\left[1+\sum_\lambda \sum_{\mu=-\lambda}^{\lambda} \alpha_\lambda{}^\mu Y_{\lambda\mu}(\vartheta,\varphi)\right]$$

Hamiltonian: $\boxed{\hat{H}_{\mathbf{m-f}} = \hat{T} + \hat{V}_{\mathbf{m-f}}}$

Mean-Field Hamiltonian and Its Parameters

- More precisely, we have 3 parameters for the central potential

$$\hat{\mathcal{V}}^{WS}_{\rm Cent}(\vec{r},\alpha; V^c, r^c, a^c) = \frac{V^c}{1+\exp[{\rm dist}_\Sigma(\vec{r}, r^c;\alpha)/a^c]}, \leftrightarrow \frac{V^c}{1+\exp[(r-R^c)/a^c]},$$

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and 3 parameters for the spin-orbit potential

$$\hat{\mathcal{V}}^{so}_{WS}(\vec{r},\hat{p},\hat{s},\alpha;\lambda^{so}, r^{so}, a^{so}) = \frac{2\,\hbar c^2}{(2m^*)^2}[(\vec{\nabla}V^{so}_{WS})\wedge\hat{p}]\cdot\hat{s},$$

where

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= =

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= =

- **Our Hamiltonian formally depends on the two sets of 6 parameters each,**

$$\boxed{\{V^c, r^c, a^c, \lambda^{so}, r^{so}, a^{so}\}_{\pi,\nu}}$$

- **They are fitted to experimental values of single-nucleon level-energies in**

$$\boxed{{}^{16}\mathrm{O},\ {}^{40}\mathrm{Ca},\ {}^{48}\mathrm{Ca},\ {}^{56}\mathrm{Ni},\ {}^{90}\mathrm{Zr},\ {}^{132}\mathrm{Sn},\ {}^{146}\mathrm{Gd},\ {}^{208}\mathrm{Pb}}$$

Role of Parameter-Correlation Removal – Part I

- **One demonstrates within Inverse Problem Theory an importance of removal of parameter correlations**

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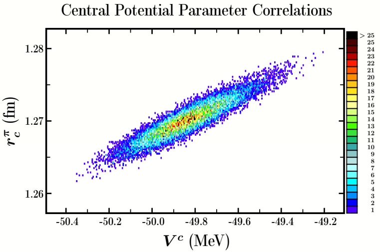
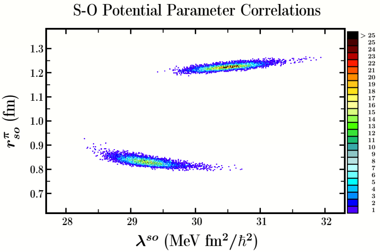
- **One demonstrates within Inverse Problem Theory an importance of removal of parameter correlations**

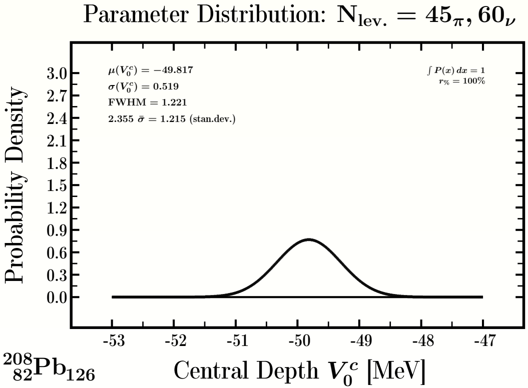
- **We determine the existence and remove parameter correlations employing Monte-Carlo simulations*)**





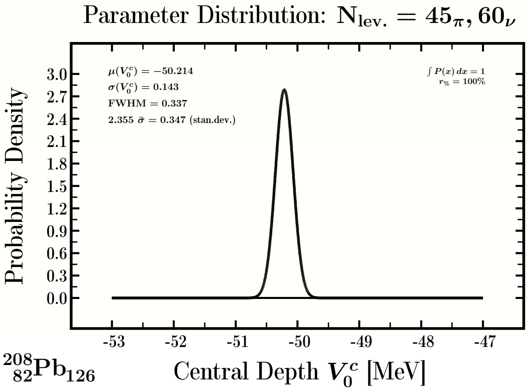
***) See PHYSICAL REVIEW C 103, 054311 (2021) and references therein**

- **Parametric correlations present**



Probability of Uncertainty. Here: Central potential depth, V_0^c, for Woods-Saxon Universal

- **Parametric correlations removed**



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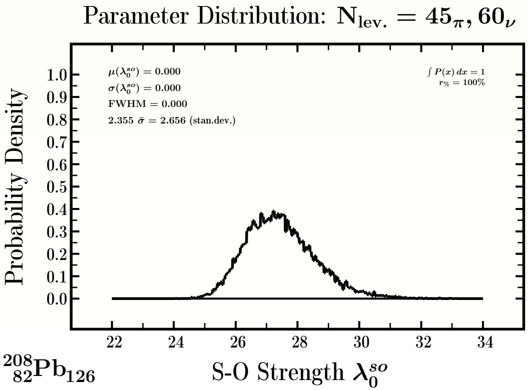

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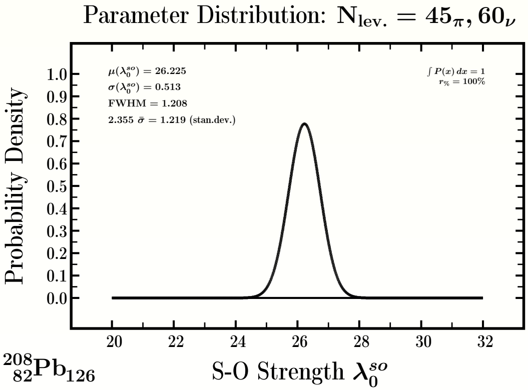

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Instructive Comparisons with Skyrme Hartree-Fock Hamiltonian

Sampling and Parametric Correlations

To follow the discussion it will be sufficient to know that the first order Skyrme Hamiltonian depends on the adjustable constants:

$$C_0^{\rho},\, C_1^{\rho},\, C_o^{\rho\alpha},\, C_0^{\tau},\, C_1^{\tau},\, C_0^{\nabla J}$$

Parameter-Correlations in Skyrme-H-F

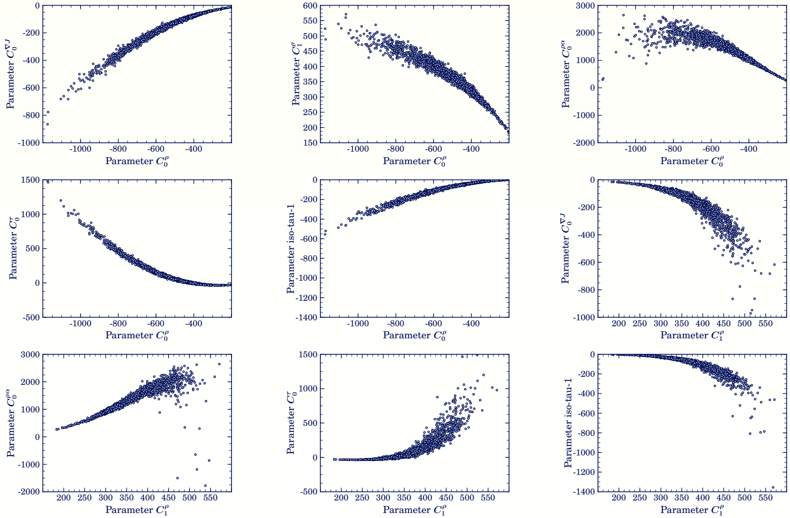

Illustration suggesting that majority of these parameters are strongly correlated excluding the prediction capacities of the model [B. Szpak, PhD thesis]

Skyrme-HF in the EDF Formulation up to N^3LO

• In their excellent article Dobaczewski et al. develop the HF formalism up to 6^{th} (next-to-next-to-next order): Number of Hamiltonian terms 2632. Taking into account symmetries → 1316; Compare with 18 - in first order.

Order	T-even	T-odd	Total	Galilean	Gauge
0	1	1	2	2	2
2	8	10	18	12	12
4	53	61	114	45	29
6	250	274	524	129	54
N^3LO	2x312	2x346	2x658	2x188	2x97
	624	692	$\boxed{1316}$	376	194

• Let us observe a very fast-growing number of terms. The results of B. Szpak suggest correlations everyone with everyone → 1 730 540 pairs!!!

[1] *For comments about Skyrme HF gauge invariance cf. e.g.*
J. Dobaczewski and J. Dudek, PRC 52 (1995) 1827

Inverse Problem in Quantum Theories vs. χ^2-Test

- Given parameters $\{p\} \rightarrow$ The Schrödinger equation produces data:

$$\hat{H}(p) \rightarrow \{E_i(p), \psi_i(p)\} \leftrightarrow \boxed{\hat{\mathcal{O}}_H(p) = d^{th} \leftarrow \textit{Direct Problem}}$$

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- **If $\hat{\mathcal{O}}_{\mathbf{H}}$ has no inverse we say that inverse problem is ill-posed**

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Ill-Posed: Correlation between parameters and the data is lost!

- In physics this issue remains unsolved: Instead of finding optimal parameters by solving the Inverse Problem $\to\to$ "one minimises χ^2"

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"Unfortunately, many practitioners of parameter estimation never proceed beyond determining the numerical values of the parameter fit. They deem a fit acceptable if a graph of data and model 'l o o k s g o o d'. This approach is known as chi-by-the-eye. Luckily, its practitioners get what they deserve" [what is meant is: "they" obtain a 'meaningless result']

• However χ^2-fitting is just the beginning; it must be accompanied by a number of operations assuring a minimum of stochastic sense

About the So-Called Chi-By-the-Eye "Method"

• In their introduction to the book chapter *'Modelling of Data'*, the authors of *'Numerical Recipes"* (p. 651), observe with sarcasm:

"Unfortunately, many practitioners of parameter estimation never proceed beyond determining the numerical values of the parameter fit. They deem a fit acceptable if a graph of data and model ' l o o k s g o o d '. This approach is known as chi-by-the-eye. Luckily, its practitioners get what they deserve" [what is meant is: "they" obtain a 'meaningless result']

Meaningless result ← less politely → Equivalent to random numbers

Phenomenological Woods Saxon Hamiltonian offers universal descriptions addressing 3000 nuclei using 6 uncorrelated parameters only.

Mean-Field from Predictive Power Perspective

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Microscopic Density Functional Theory with N^3LO H-F realisation proposes 2632 coupling constants – each of which is expected to be correlated with the others

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Phenomenological Woods Saxon Hamiltonian offers universal descriptions addressing 3000 nuclei using 6 uncorrelated parameters only.

Microscopic Density Functional Theory with N^3LO H-F realisation proposes 2632 coupling constants – each of which is expected to be correlated with the others

The priority should be given to solution offering better Predictive Power, with fewer parameters, moreover – totally uncorrelated

$\boxed{\textbf{We choose WS rather than HF}}$

Consequences for Our Project

• Our total energy calculations are performed using the Hamiltonian parametrisation with parametric correlations removed

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• Our total energy calculations are performed using the Hamiltonian parametrisation with parametric correlations removed

• According to Inverse Problem Theory this assures maximum stochastic stability of modelling predictions

(indeed verified: Narrowing stability peaks shown above)

Part IV

Testing Parametric Universality By Calculating:

High-K Isomers and Yrast Lines

Nuclear Spins Aligned With the Symmetry Axis

• One uses mean-field approach and the fact that in the case of axial symmetry, $\mathcal{O}_z$-axis, we have

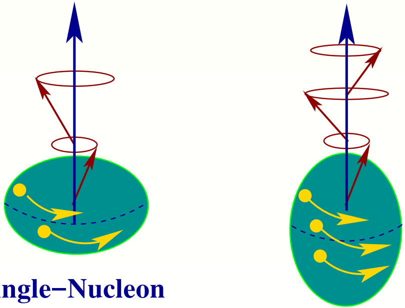
$$\boxed{[\hat{H}, \hat{\jmath}_z] = 0}$$

• Consequently

$$\boxed{\hat{H}\phi_{\nu,m_\nu} = e_{\nu,m_\nu}\phi_{\nu,m_\nu}}$$

$$\boxed{\hat{\jmath}_z\phi_{\nu,m_\nu} = m_\nu\phi_{\nu,m_\nu}}$$

Projections of Angular Momenta Are Conserved in the Presence of Axial Symmetry



Single−Nucleon Alignment

• The presence of axial symmetry (like any other symmetry) imposes certain hindrance mechanism: In the present case $\rightarrow\rightarrow\rightarrow$ K-isomers

Tilted Fermi Surface: Lagrange Multiplier Method

• We would like to find the lowest energy excited configurations. $\mathcal{C}$, denotes ensemble of indices of occupied states for protons and neutrons separately:

$$E^* = \sum_{\nu \in \{\mathcal{C}\}} e_\nu, \qquad (A)$$

• The number of particles $\mathcal{N}$ – is equal to N or Z

$$\sum_{\nu \in \{C\}} 1_\nu = \mathcal{N}, \qquad (B)$$

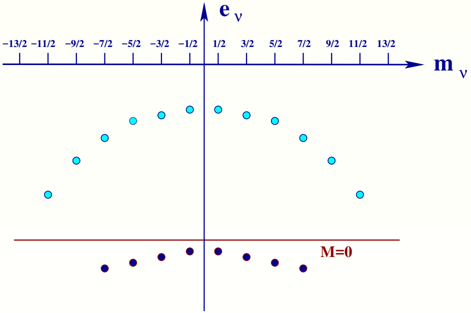
• The projected angular momentum, $\mathcal{M}$, corresponds either to proton or to the neutron contributions, M_Z or M_N, respectively

$$\sum_{\nu \in \{C\}} m_\nu = \mathcal{M} \quad (M_Z \text{ or } M_N). \qquad (C)$$

• According to the Lagrange theorem, minimisation of (A) under conditions (B) and (C) is equivalent to the minimisation of an auxiliary expression

$$\boldsymbol{\tilde{E}^*_M = \sum_{\nu \in \{C\}} (e_\nu - \lambda \cdot 1_\nu - \omega \cdot m_\nu),} \quad \boldsymbol{(D)}$$

where the so called Lagrange multipliers $\boldsymbol{\lambda}$ and $\boldsymbol{\omega}$ are so far unknown.



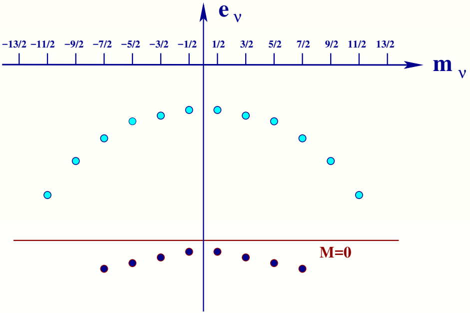
Minimisation of the expression in Eq. (D) under the conditions specified by Eqs. (A) - (B) is equivalent to searching for the lowest points (e_ν, m_ν)

$$\tilde{E}^*_M = \sum_{\nu\in\{C\}} (e_\nu - \lambda\cdot 1_\nu - \omega\cdot m_\nu),$$

what is equivalent to finding all points strictly below following straight line

$$\boxed{e = \lambda - \omega m \quad \leftrightarrow \quad \text{Tilted Fermi surface}}$$

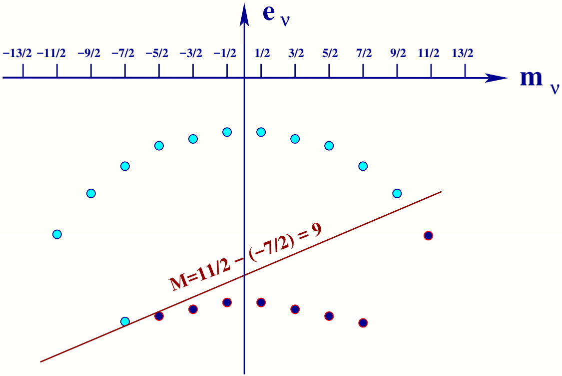
Tilted Fermi Surface: Energy Minimisation at Fixed Spin



For the **p**article-**h**ole excited-states we obtain at the same time the theoretical energy and theoretical spin →→→ A. Bohr hypothesis:

$$\boxed{I \approx M^*}$$

$$E^* = \sum_p e_{p,m_p} - \sum_h e_{h,m_h} \quad \text{and} \quad I \approx M^* = \sum_p m_p - \sum_h m_h$$



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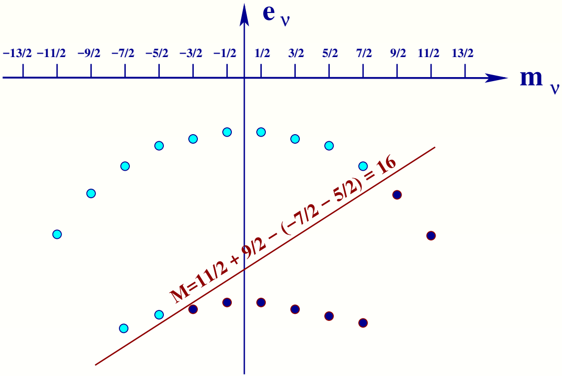

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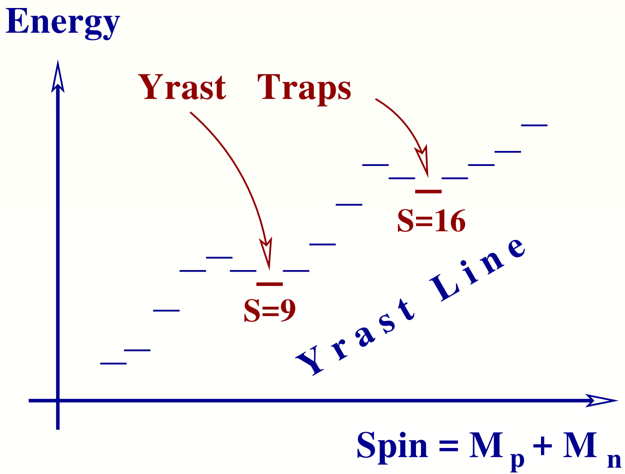
Some Particle-Hole Excitations Generate Yrast-Traps



$$E^* = \sum_p e_{p,m_p} - \sum_h e_{h,m_h} \quad \text{and} \quad I \approx M^* = \sum_p m_p - \sum_h m_h$$

K-Isomers

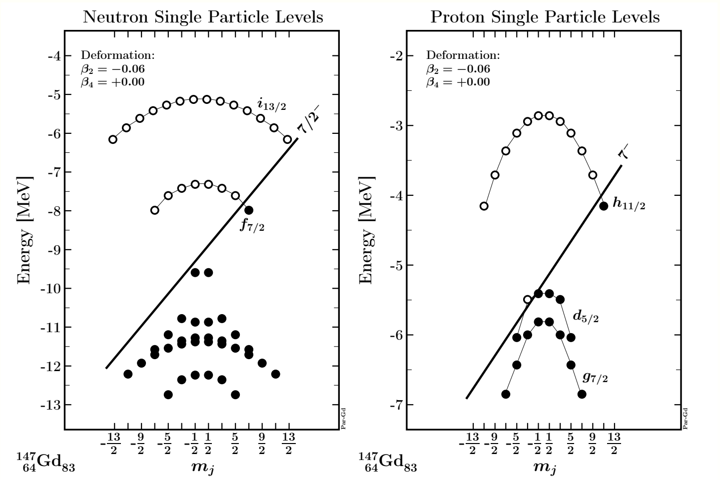
Example: $^{147}_{64}Gd_{83}$ – Realistic Calculations

K-Isomers

Example: ${}^{147}_{64}\mathrm{Gd}_{83}$ – Realistic Calculations

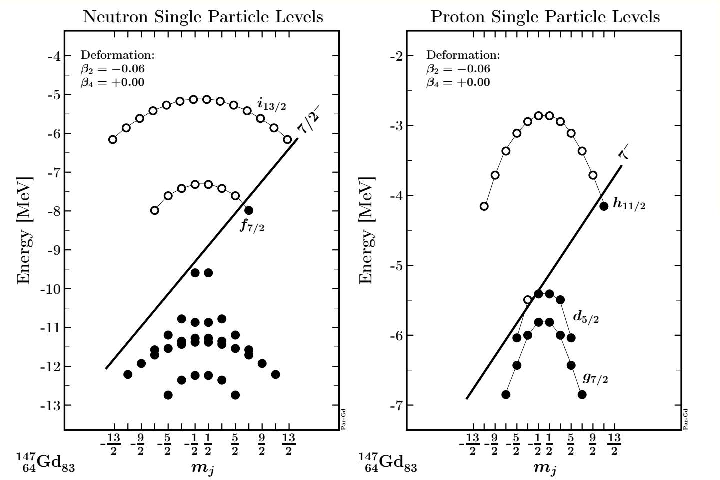
Realistic Phenomenological Mean Field
Universal Parametrisation: (Z,N)-Plane

A Powerful Tool: Tilted Fermi Surface Method (I)



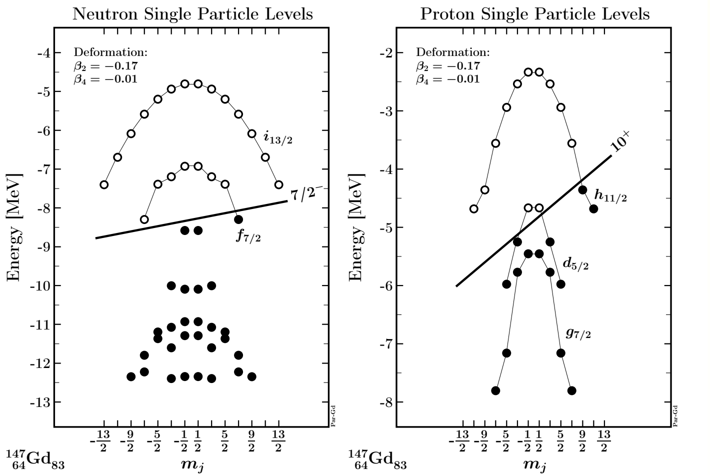
- Theoretically suggested spin I$=\frac{7}{2}+7=\frac{21}{2}$ $\rightarrow$ Prediction: $I^\pi = \frac{21}{2}^+$

A Powerful Tool: Tilted Fermi Surface Method (I)



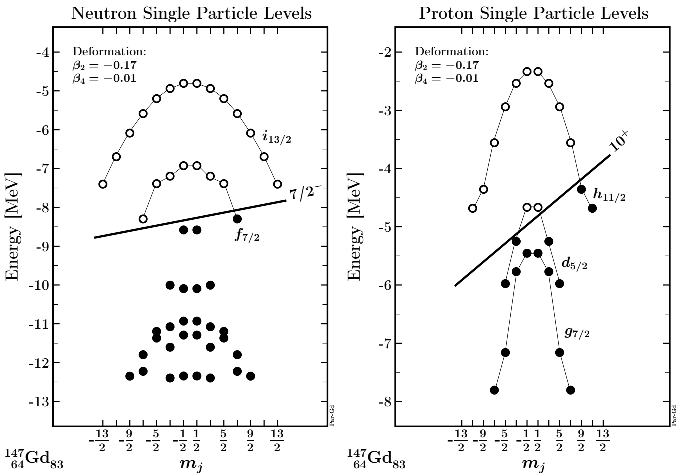
$$\text{Experiment}: \mathrm{I}^{\pi}_{\text{exp.}} = \frac{21}{2}^{+} (\mathbf{4.55\,ns}) \leftrightarrow \text{Mean Field theory } \mathrm{I}^{\pi}_{\text{mf}} = \frac{21}{2}^{+}$$

A Powerful Tool: Tilted Fermi Surfaces - (II)



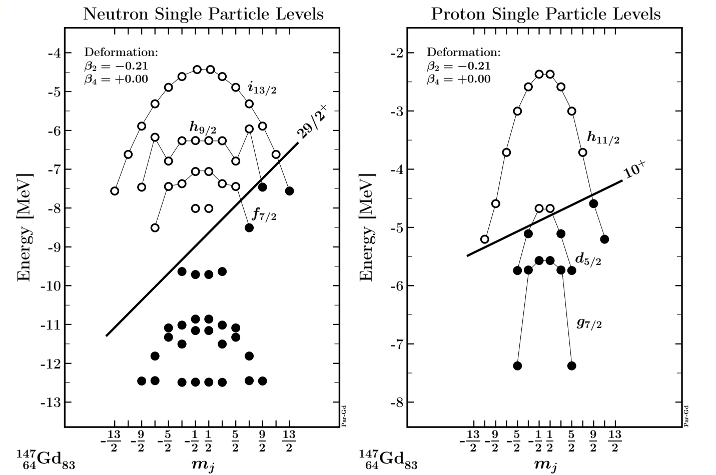
- Theoretically suggested spin I$= \frac{7}{2} + 10 = \frac{27}{2}$ $\rightarrow$ Prediction: $I^\pi = \frac{27}{2}^-$

A Powerful Tool: Tilted Fermi Surfaces - (II)



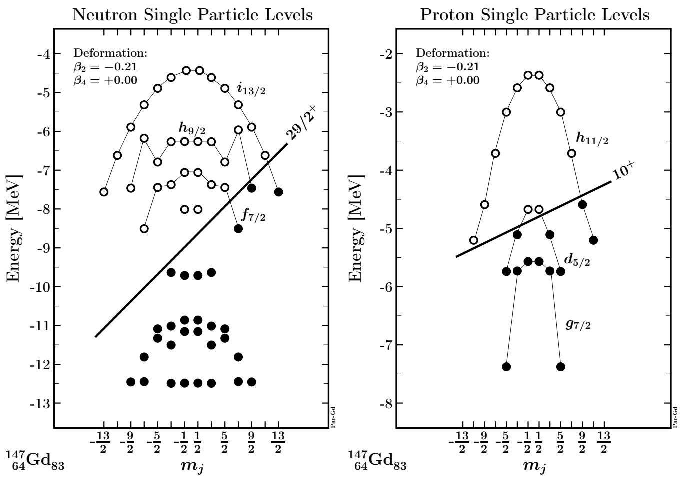
Experiment : $I^{\pi}_{exp.} = \frac{27}{2}^-$ **(26.8 ns)** $\leftrightarrow$ Mean Field theory $I^{\pi}_{mf} = \frac{27}{2}^-$

A Powerful Tool: Tilted Fermi Surfaces - (III)



- Theoretically suggested spin I$= \frac{29}{2} + 10 = \frac{49}{2}$ $\rightarrow$ Prediction: $I^\pi = \frac{49}{2}^+$

A Powerful Tool: Tilted Fermi Surfaces - (III)

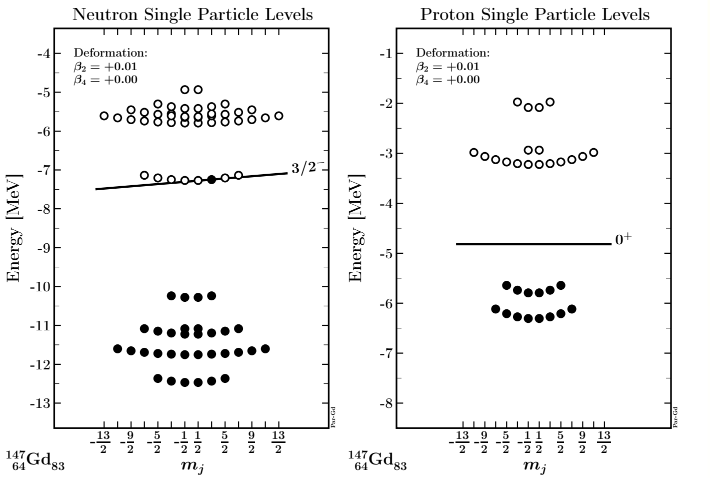


Experiment : $\mathrm{I}^{\pi}_{\mathrm{exp.}} = \frac{49}{2}^{+}$ **(510 ns)** $\leftrightarrow$ Mean Tield theory $\mathrm{I}^{\pi}_{\mathrm{mf}} = \frac{49}{2}^{+}$

These isomers were known for over 30 years or so...

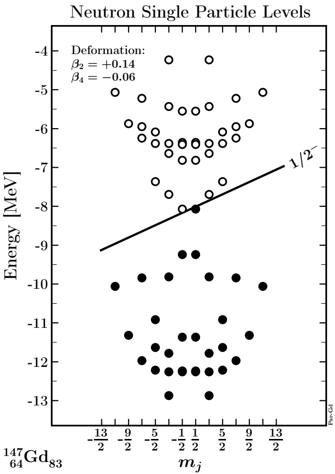
What About the "Newer" Isomers in ^{147}Gd?

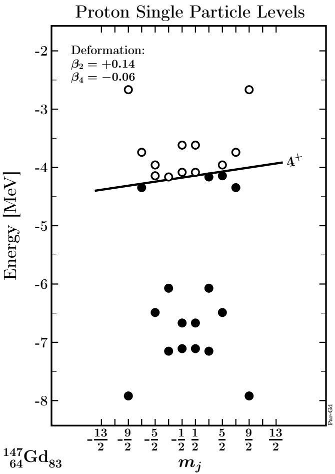
Negative Parity Isomers in ^{147}Gd - (IV)



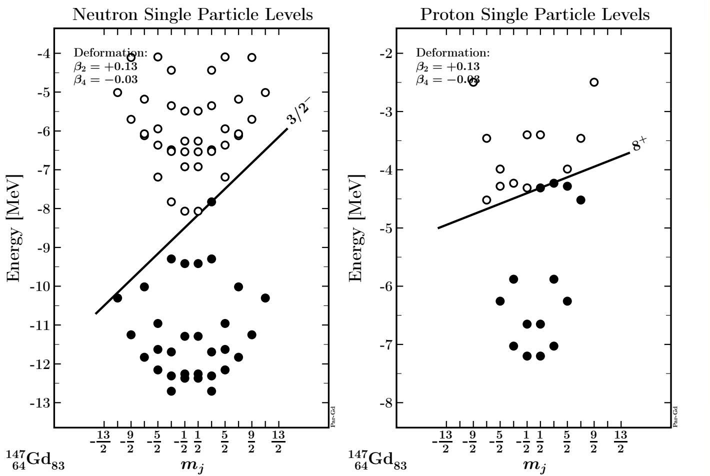
$$\text{Experiment : } I^{\pi}_{\text{exp.}} = \frac{3}{2}^{-}\,\mathbf{(0.2\,ns)} \;\leftrightarrow\; \text{Mean Field theory } I^{\pi}_{\text{mf}} = \frac{3}{2}^{-}$$

Negative Parity Isomers in ^{147}Gd - (V)



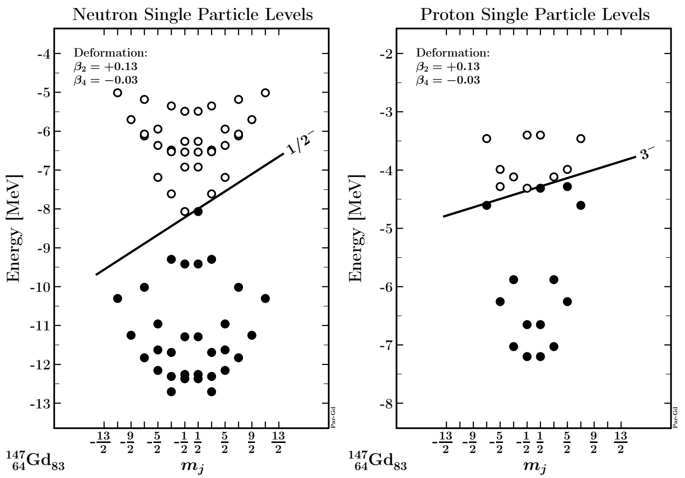


Experiment : $I^{\pi}_{\text{exp.}} = \frac{9}{2}^{-}$ **(0.35 ns)** $\leftrightarrow$ Mean Field theory $I^{\pi}_{\text{mf}} = \frac{9}{2}^{-}$



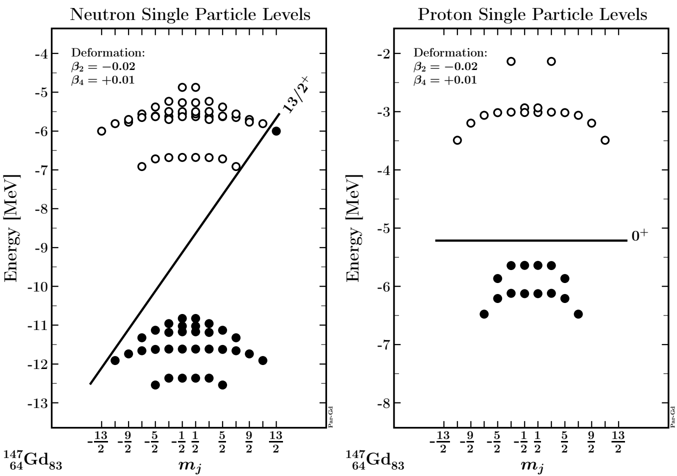
Experiment : $\mathrm{I}^{\pi}_{\mathrm{exp.}} = \frac{19}{2}^{-}$ **(0.37 ns)** $\leftrightarrow$ Mean Field theory $\mathrm{I}^{\pi}_{\mathrm{mf}} = \frac{19}{2}^{-}$

Positive Parity Isomers in ^{147}Gd - (VII)



Experiment : $\mathrm{I}^{\pi}_{\mathrm{exp.}} = \frac{7}{2}^{+}$ **(0.42 ns)** $\leftrightarrow$ Mean Field theory $\mathrm{I}^{\pi}_{\mathrm{mf}} = \frac{7}{2}^{+}$

Positive Parity Isomers in ^{147}Gd - (VIII)



$$\text{Experiment : } I^{\pi}_{\text{exp.}} = \frac{13}{2}^{+} (\mathbf{21.4\,ns}) \leftrightarrow \text{Mean} - \text{Field theory } I^{\pi}_{\text{mf}} = \frac{13}{2}^{+}$$

Full Experimental Confirmation $\leftrightarrow$ Universality

- **K-isomers: Attention! Highly non-trivial numerical effort involving $\underline{\underline{N \sim 10^6}}$ particle-hole configurations! Minimised over α_{20}, α_{40}, etc.**

- **Summary**: All 8 double titled Fermi surface solutions for ^{147}Gd (Tested lowest energy solutions according to Bohr tilted Fermi hypothesis) **correspond to the experimentally confirmed results**

No.	I^π	I_p^π	I_n^π	Isomer Lifetime
1	$3/2^-$	0^+	$3/2^-$	0.20 ns
2	$7/2^+$	3^-	$1/2^-$	0.42 ns
3	$9/2^-$	4^+	$1/2^-$	0.35 ns
4	$13/2^+$	0^+	$13/2^+$	21.4 ns
5	$19/2^-$	8^+	$3/2^-$	0.37 ns
6	$21/2^+$	7^-	$7/2^-$	4.55 ns
7	$27/2^-$	10^+	$7/2^-$	26.8 ns
8	$49/2^+$	10^+	$29/2^+$	510 ns
	...	...	...	...

Another Illustration of the Axial Symmetry and K-Conservation

Calculating Nuclear Yrast Lines

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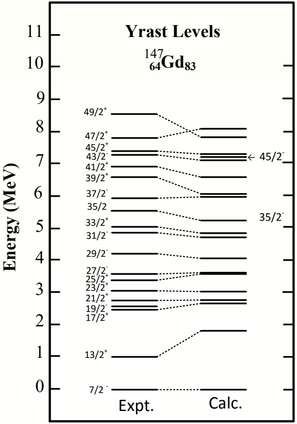

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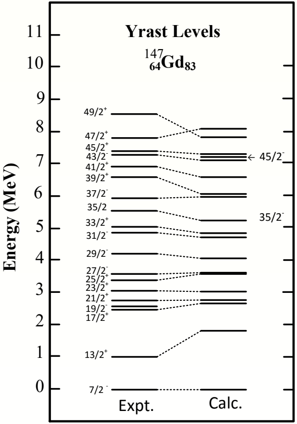

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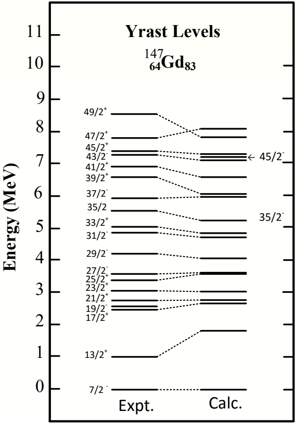
• We consider the number of mean-field configurations comparable to the sizes of the typical spherical shell-model Hamiltonian.



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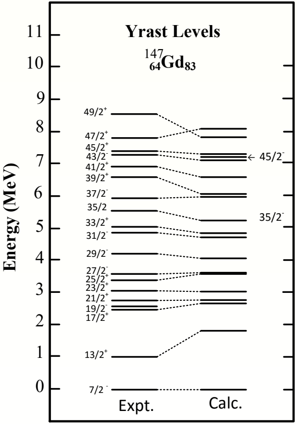
• We consider the number of mean-field configurations comparable to the sizes of the typical spherical shell-model Hamiltonian.

• It is natural to ask:

How many parameters have been fitted to obtain the result on the right?



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  - Or rather a manifestation of *predictive power?*

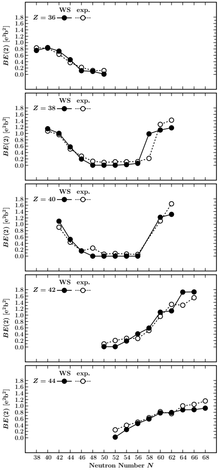
In other words: How many parameters are fitted to spectra?

NONE – no parameter adjusted to the presented data;
This is what is meant as Woods-Saxon Universal mean-field

Let us show another example →→

Quadrupole $\lambda = 2$ Transition Probabilities: $Z \in [36, 44]$

• We calculate total energy surfaces on the (Z,N)-square indicated, determine equilibrium deformations and proceed calculating BE2 $\rightarrow$



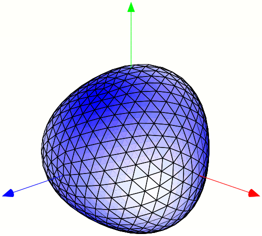
Reduced transition probabilities B(E2) for increasing N compared with experiment.

There are numerous other calculation-tests fully confirming examined parametric universality ...

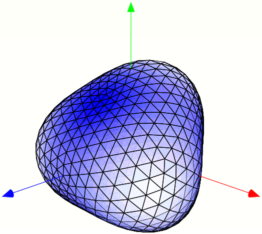
Part V

Properties and Identification of Molecular Shapes in Nuclei: Tetrahedral and Octahedral Symmetries

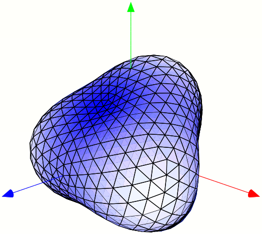
About Recently Identified Tetrahedral and Octahedral Symmetries



$\alpha_{32} \equiv t_3 = 0.1$



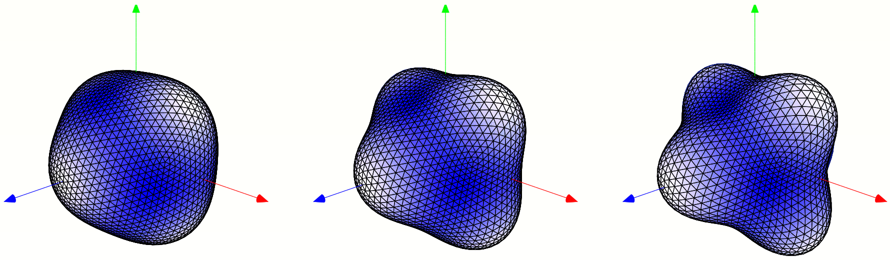
$\alpha_{32} \equiv t_3 = 0.2$



$\alpha_{32} \equiv t_3 = 0.3$

Examples: Nuclear TETRAHEDRAL symmetry shapes

About Recently Identified Tetrahedral and Octahedral Symmetries

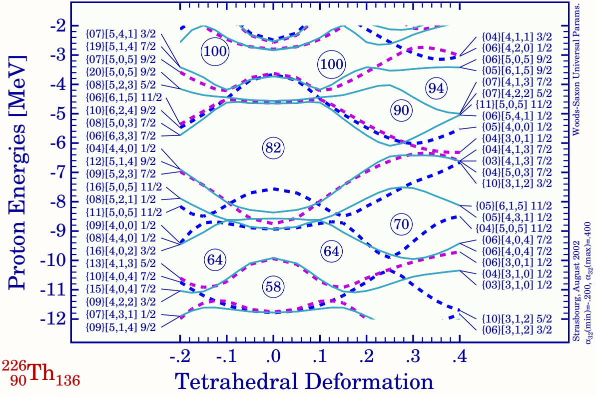


$o_4 = 0.1$ $\quad$ $o_4 = 0.2$ $\quad$ $o_4 = 0.3$

Examples: Nuclear OCTAHEDRAL symmetry shapes

Nuclear Tetrahedral Magic Gaps - Proton Spectra

Double group T_d^D has two 2-dimensional - and one 4-dimensional irreducible representations: Three distinct families of nucleon levels



Full lines $\leftrightarrow$ 4-dimensional irreducible representations - marked with double Nilsson labels. Observe huge gaps at N=64, 70, 90-94, 100.

Time for a Short Reminder:

How Did this Research Start?

VOLUME 88, No. 25 PHYSICAL REVIEW LETTERS 22 June 2002

Nuclear Tetrahedral Symmetry: Possibly Present throughout the Periodic Table

J. Dudek, A. Góźdź, N. Schunck, and M. Miśkiewicz

Abstract:

More than half a century after the fundamental, spherical shell structure in nuclei had been established, theoretical predictions indicate that the **shell gaps comparable or even stronger than those at spherical shapes may exist.** *Group-theoretical analysis supported by realistic mean-field calculations indicate that the corresponding nuclei are characterized by the* T_d^D *("double-tetrahedral") symmetry group.*

Strong shell-gap structure is enhanced by the existence of the four-fold degenerate levels; *it can be seen as a geometrical effect that does not depend on a particular realization of the mean field. Possibilities of discovering the* T_d^D *symmetry in experiment are discussed.*

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[This was a follow-up of an 8 years earlier pilot project:
X. Li and J. Dudek, Phys. Rev. C 94, R1250 **(1994)**.]

"Nuclear Pyramids"

Physics News Update http://www.aip.org/pnu/2002/split/593-1.html

Inside Science Research — Physics News Update

Number 593 #1, June 10, 2002 by Phil Schewe, James Riordon, and Ben Stein

Nuclear Pyramids

Physicists normally think of atomic nuclei as being something like a droplet with a roughly spherical shape. But if atoms can assemble into tiny pyramid structures (such as the ammonia molecule, NH_4), why not nuclei? It all depends on how the nuclear forces act in a nucleus.

A group of physicists from the Universite Louis Pasteur (Strasbourg, France) and the Marie Curie University (Lublin, Poland) have, for the first time, tried to imagine how stable nuclei could form with pyramid, or even cubic or octahedral shapes.

In chemistry many configurations are possible because the interactions (*e.g.*, Van der Waals, covalent, or hydrogen bonding) can extend over considerable distances.

The nuclear force, by contrast, is attenuated, and acts not much further than the size of nucleons (the protons and neutrons making up the nucleus). An excited pyramidal nucleus would turn in space, every now and then throwing out a high-energy photon (gamma ray). This would make for a characteristic spectrum, but one which would most likely require a gamma detection sensitivity only now being planned for experiments in the US and Europe.

Jerzy Dudek (jerzy.dudek@res.in2p3.fr, 33-388-10-6498) and his colleagues have worked out the "magic numbers" for those elements and isotopes most likely to be sustainable in tetrahedral form, nuclei with certain numbers of protons (e.g., 20, 32, 40, 55/58, 70) and neutrons.

For example, barium-126 (56 protons, 70 neutrons) and barium-146 (56 protons, 90 neutrons) have promise, whereas Ba-114 or Ba-168 do not. (**Dudek *et al.* (http://link.aps.org/abstract/PRL/v88/e252502)**, *Physical Review Letters*, 24 June 2002.)

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Monumental shapes

› 08 June 2002 by Charles Choi
› Magazine issue 2346. Subscribe and save

ANCIENT Egyptians may have been unwittingly emulating one of the most fundamental structures in existence. It turns out that the nuclei of some atoms may form tiny pyramids instead of spheres.

No shape is more compact than a sphere, which is why it's so common in nature from bubbles to water droplets to the Earth. Previous experiments have confirmed that although atomic nuclei can briefly form rugby ball or even egg-shaped structures, they are usually spherical or at least roughly round.

But now theoretical physicist Jerzy Dudek at the Louis Pasteur University in Strasbourg, France, and his colleagues say that it isn't always this way. In a paper to be published in Physical Review Letters, they suggest that nuclei could in theory form stable pyramidal shapes. Dudek explains that while spheres are usually the most stable shape for a nucleus, in some heavier elements there sometimes aren't the right number ...

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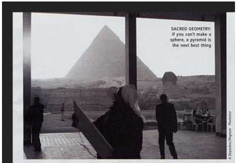


Monumental shapes

Their creators thought them unique, but as usual nature got there first

Science Popularisation "Gazeta Wyborcza" [2002]

These are just the first 2 paragraphs of a ∼3 pages article...

Gazeta.pl > Wyborcza > Nauka

Kwadratura jądra, czyli polska wizja atomu

Piotr Cieśliński (17-06-02 16:50)

Kant kuli
STUDIO GAZETA

Fizycy wyobrażali sobie dotychczas, że jądro atomowe przypomina kształtem kulę, czasami nieco wydłużoną. Tymczasem polscy naukowcy twierdzą, że jądra atomowe mogą być kanciaste! Odkrycie Polaków może zmienić kierunek badań nad superciężkimi pierwiastkami, bo - jak wiadomo - bryła kanciasta" toczy się w zupełnie inną stronę niż kulista...

ZOBACZ TAKŻE

- Pierwiastki chemiczne (11-12-00, 14:07)

Zdaniem polskich fizyków niektóre jądra mają kształt zbliżony do czworościanu foremnego - równobocznej piramidy o trójkątnej podstawie. "Wszyscy jesteśmy zbudowani z piramid" - żartobliwie skomentował polskie odkrycie tygodnik "New Scientist", sugerując przy tym, że starożytni Egipcjanie nieświadomie wzorowali się na kształcie jąder atomowych. A co powiedzieć o intuicji starożytnych Greków? Wprawdzie Pitagorejczycy wierzyli, że wszystkie ciała niebieskie muszą być bryłami kulistymi, a ich drogi okręgami, gdyż kula jest najdoskonalszą bryłą, tak jak okrąg najdoskonalszą figurą. Niemniej jednak cztery podstawowe żywioły, z których miała składać się materia - ogień, woda, ziemia, powietrze - wielki Platon utożsamił z kanciastymi bryłami foremnymi: czworościanem, sześcianem, ośmiościanem i dziesięciościanem.

In these lines: *"... – because as it is well known – a round body is rolling in totally different direction as compared to the one with corners..."*

VOLUME 97, 072501 (2006) PHYSICAL REVIEW LETTERS 18 August 2006

Island of Rare Earth Nuclei with Tetrahedral and Octahedral Symmetries: Possible Experimental Evidence

J. Dudek, D. Curien, N. Dubray, J. Dobaczewski, V. Pangon, P. Olbratowski, and N. Schunck

Abstract:

Calculations using realistic mean-field methods suggest the existence of nuclear shapes with tetrahedral T_d and/or octahedral O_h symmetries sometimes at only a few hundreds of keV above the ground states in some rare earth nuclei around ^{156}Gd and ^{160}Yb. The underlying single-particle spectra manifest exotic fourfold rather than Kramers's twofold degeneracies. The associated shell gaps are very strong, leading to a new form of shape coexistence in many rare earth nuclei. We present possible experimental evidence of the new symmetries based on the published experimental results –although an unambiguous confirmation will require dedicated experiments.

"Smallest Pyramids in the Universe"

AIP American Institute of Physics SEARCH AIP go home | contact us | site map

Inside Science Research — Physics News Update

Number 789 #1, August 22, 2006 by Phil Schewe, Ben Stein, and Davide Castelvecchi

Smallest Pyramids in the Universe

French physicists believe they can solve the mystery behind dozens of nuclear experiments conducted years ago. The experiments, conducted with a variety of detectors, energies, and colliding nuclear species, left puzzling results, so puzzling and hard to interpret that many of the experimenters turned their attention to the study of highly spinning nuclei, a quite fashionable subject at the time.

Now, Jerzy Dudek of the Université Louis Pasteur in Strasbourg, France, and his colleagues at Warsaw University and the Universidad Autonoma de Madrid claim that the old results can be explained by arguing that some nuclei, made in the tempestuous conditions of a sufficiently high-energy collision, can exist in the form of a tetrahedron or a octahedron.

Like a pyramid-shaped methane (CH_4) molecule held together by the electromagnetic force, a pyramidal nucleus would consist of protons and neutrons held together by the strong nuclear force. Such a nuclear molecule -- in effect the smallest pyramid in the universe -- would be only a few femtometers (10^{-15} meter) on a side and millions of times smaller in volume than methane molecules.

Just as there are so-called "magic" nuclei with just the right number of neutrons and protons that readily form stable spherical nuclei, so there are expected to be such magic numbers for forming pyramid nuclei too. Stable, in this case, means that the state persists for 10^{12} to 10^{14} times longer than the typical timescale for nuclear reactions, namely 10^{-21} seconds.

Dudek (contact jerzy.dudek@ires.in2p3.fr, +33-3-88-10-64-98) says that gadolinium-156 and ytterbium-160 are nuclei very conducive to residing in a stable pyramid configuration. Nuclei might exist also in stable octahedral (diamond) forms also. These nuclei would all possess a quantum property not seen before in nuclei: in the process of filling out an energy-level diagram for the nucleus, four nucleons of the same kind (neutrons or protons) could share a single energy level instead of the customary one or two permitted nucleons.

This rule-of-four would inhibit the normally observed decay patterns by which non-spherical nuclei throw off energy, usually by emitting gamma rays. In fact, in the case of nuclear pyramids it is expected to result in new and unprecedented decay rules. This inhibition would explain the puzzling results of earlier experiments.

Dudek and his colleagues plan to test these ideas in upcoming experiments.

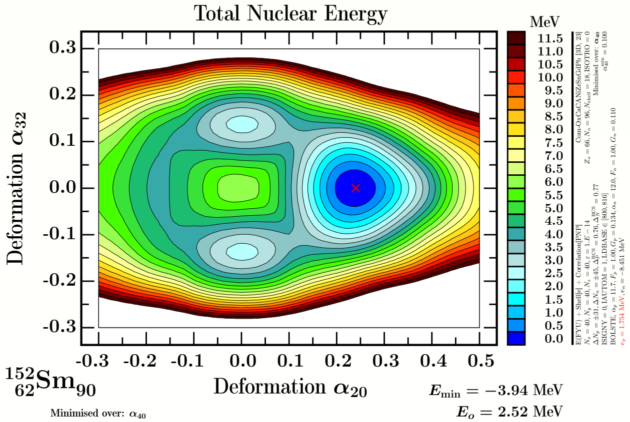
Dudek *et al.* (http://link.aps.org/abstract/PRL/v97/e072501), Physical Review Letters, 18 August 2006

Contact Jerzy Dudek
Université Louis Pasteur
jerzy.dudek@ires.in2p3.fr
Tel: +33-3-88-10-64-98

Back to Physics News Update (/pnu)

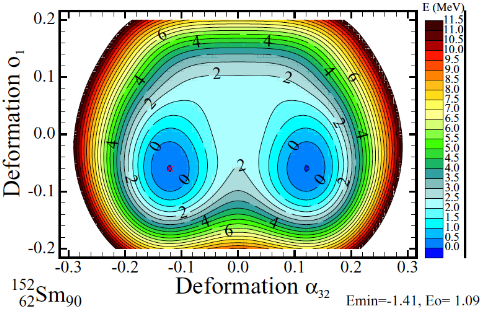
This Discussion Is Focussed on ^{152}Sm – Part I

- **Based on: JD et al., PHYSICAL REVIEW C 97, 021302(R) (2018)**



- **Symmetric minima at $\alpha_{20} = 0$ represent tetrahedral symmetry**

- **Based on: JD et al., PHYSICAL REVIEW C 97, 021302(R) (2018)**



- **Tetrahedral minima at $\alpha_{32} = \pm 0.13$ are accompanied by non-vanishing octahedral deformation $o_1 \approx -0.06$**

- **In this way our theory predicts a coexistence of T_d and O_h**

From Predictions to Experimental Identification

- **We had to address the experimental identification methods which allow to confirm the presence of T_d and O_h symmetries**
- **The formal quantum mechanical and mathematical solution of this problem is very well known, see for instance Refs. [1,2]**
- **It defines the unique structure of molecular rotational bands through derivation of the quantum T_d and O_h rotor properties**

We will present the T_d and O_h rotational band properties avoiding mathematical details and formalism (see what follows)

[1] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, First Edition 1948;
[2] M. Hamermesh, *Group Theory*, 1962

And This Looks Like an Original Copy from Kraków

- What?

How to identify tetrahedral symmetry!

And This Looks Like an Original Copy from Kraków

- What?
How to identify tetrahedral symmetry!

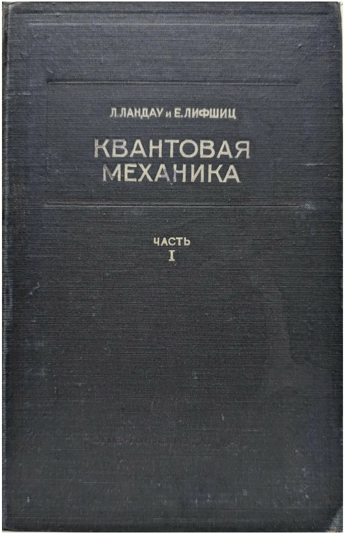
- Where to find:

In: L. D. Landau and E. M. Lifshitz, Quantum Mechanics, First Edition 1948

- Which bands to measure?

$$\underbrace{A_{1g}\ :\ 0^+, 4^+, 6^+, 8^+, 9^+, 10^+,\ \ldots,\ \ I^\pi = I^+}_{\text{with corresponding energies forming a common parabola}}$$

$$\underbrace{A_{2u}\ :\ 3^-, 6^-, 7^-, 9^-, 10^-, 11^-, \ldots,\ I^\pi = I^-}_{\text{Forming another (common) parabola}}$$



Quantum Rotors: Tetrahedral vs. Octahedral

- The tetrahedral symmetry group has 5 irreducible representations
- The $\mathrm{T_d}$ ground-state $I^\pi = 0^+$ belongs to A_1 representation given by

$$\underbrace{\mathrm{A}_1: \quad 0^+, 3^-, 4^+, \underbrace{(6^+, 6^-)}_{\text{doublet}}, 7^-, 8^+, \underbrace{(9^+, 9^-)}_{\text{doublet}}, \underbrace{(10^+, 10^-)}_{\text{doublet}}, 11^-, \underbrace{2\times 12^+, 12^-}_{\text{triplet}}, \cdots}_{\textbf{with corresponding energies forming a common parabola}}$$

- There are no states with spins $I = 1, 2$ and 5. We have parity doublets: $I = 6, 9, 10 \ldots$, at energies: $E_{6^-} = E_{6^+}$, $E_{9^-} = E_{9^+}$, etc.

$$\boxed{\textit{Compare with } I^\pi = 0^+, 2^+, 4^+, \ \ldots}$$

Quantum Rotors: Tetrahedral vs. Octahedral

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$$\boxed{\textit{Compare with } I^\pi = 0^+, 2^+, 4^+,\ \ldots}$$

- One shows the analogous structures in the octahedral symmetry

$$\underbrace{A_{1g} :\ 0^+, 4^+, 6^+, 8^+, 9^+, 10^+,\ \ldots,\quad I^\pi = I^+}_{\text{with corresponding energies forming a common parabola}}$$

$$\underbrace{A_{2u} :\ 3^-, 6^-, 7^-, 9^-, 10^-, 11^-, \ldots,\quad I^\pi = I^-}_{\text{Forming another (common) parabola}}$$

More Surprises from Hartree-Fock Approach
by our Fukuoka-Strasbourg Collaboration

Our Realisation of the Gogny Hartree Fock Bogolyubov Approach with Particle Number, Parity and Angular Momentum Projections

$\rightarrow\rightarrow\rightarrow$

Degeneracies Increase when α_{32} Deformation Grows

SHINGO TAGAMI, YOSHIFUMI R. SHIMIZU, AND JERZY DUDEK PHYSICAL REVIEW C **87**, 054306 (2013)

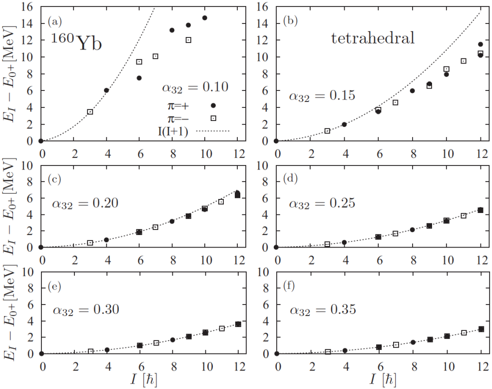
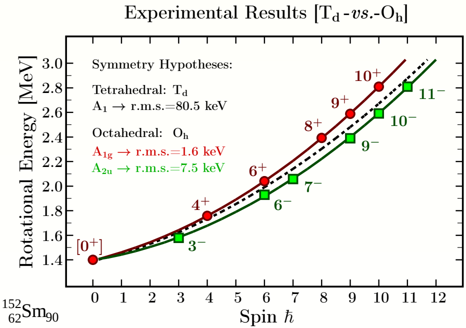


FIG. 4. Calculated spectra of tetrahedral states in ^{160}Yb with $\alpha_{32} = 0.10, 0.15, 0.20, 0.25, 0.30$, and 0.35, respectively, for (a), (b), (c), (d), (e), and (f). The dotted line in each panel denotes an ideal $I(I+1)$ sequence going through the first excited 3^- state. Note that almost exact degeneracies for $I = (6^+, 6^-), (9^+, 9^-), (10^+, 10^-), (2 \times 12^+, 12^-)$ states are obtained for $\alpha_{32} \geqslant 0.25$ demonstrating the nearly perfect rotor character of the rotational excitation of the system.

Emphasise that computer codes "know nothing about group theory", the only notion about tetrahedral symmetry enters via expected values Q_{32}

MAGIC???

- **These two sequences represent the coexistence between tetrahedral and octahedral symmetries. Curves represent the parabolic fit and are not meant to guide the eye. This is the first evidence based on the experimental data**



FROM: **Spectroscopic criteria for identification of nuclear tetrahedral and octahedral symmetries: Illustration on a rare earth nucleus**

J. Dudek et al., PHYSICAL REVIEW C 97, 021302(R) (2018)


The first observation


The first observation
of new shape symmetry

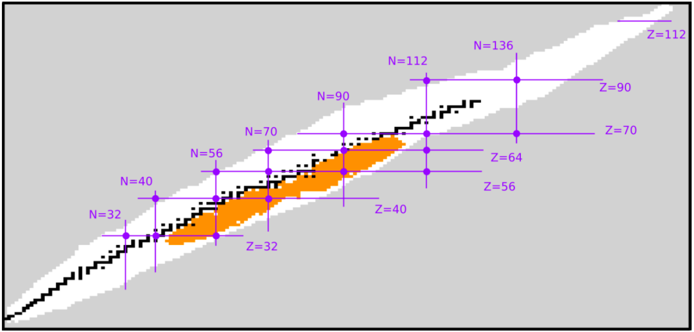

The first observation
of new shape symmetry
may not contain


The first observation
of new shape symmetry
may not contain
the full truth

We Will Need to Strengthen the Evidence

Possible Collaboration
with
Multiple-Reflection Time-of-Flight Mass Spectrometer
Team
[MR-TOF-MS]

- Chart of nuclides with the tetrahedral magic numbers in purple and the isotopes accessible via spontaneous fission of ^{252}Cf in orange



- Measurements programmed with application among others of the MR-TOF-MS [Multiple-Reflection Time-of-Flight Mass Spectrometer]

Good News from European Funding Agencies

- **While calculating potential energy surfaces and running other tests for the identification of tetrahedral symmetry we developed many advanced computer codes**

- **We constructed $\sim 5 \times 10^5$ various potential energy 2D projections with the idea of creating an internet site data base**

- **... and there seem to be some good news to shear ...**

- **EURO-LABS has financed our MeanField4Exp proposal of the service based on our computer codes and addressing experimentalists working in nuclear accelerator centres**

- **After 4 years of installation efforts in Kraków the project has just been completed $\rightarrow$**

EUROPEAN LABORATORIES FOR ACCELERATOR BASED SCIENCE

SHORT NAME: **EURO-LABS**

- **About MeanField4Exp Proposal**

– **Providing Research Infrastructure users a mean-field theory based universal-use software allowing a non-expert to produce state-of-the-art and standard today theory results comparable with experiment**

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– Bypassing 'the standard'; Proponents offer a new dimension of high actuality today:

Theory Results with Theoretical Uncertainties

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– Large spectrum of nuclear structure subfields covers many mechanisms of potential interest at GSI/FAIR, GANIL/SPIRAL2, LNL/SPES, ALTO, ISOLDE, JYFL, HIL Warsaw, CCB Cracow, and, and, and ...

Good News from the European Funding Agencies

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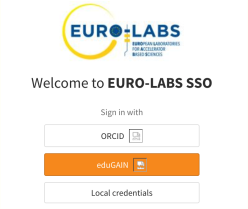
– Subjects covered in the European Project cover much more than presented today

Recent Email Transfer to JD from Professor Maj



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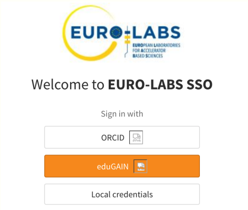

From: Sullivan, Conor < C.M.Sullivan@liverpool.ac.uk >
To: adam.maj@ifj.edu.pl
Subject: MeanField4Exp - Fantastic!

Hi Adam,

Thank you so much for the recommendation on this website.
It's absolutely fantastic and so easy to use!
What a triumph for the collective efforts of Jerzy
and all his collaborators over the years.

Conor

Summary and Conclusions (1)

Mean Field and Physics Issues in Nuclear Structure

- **Physics of nuclear isomers,**
- **Collective rotation and pairing,**
- **Nuclear masses, shapes and symmetries,**
- **Fission, tri-partition and exotic fission modes,**
- **High-rank symmetries, properties and identification,**
- **Spin & temperature, GDR, Jacobi and Poincaré transitions.**

Summary and Conclusions (1)

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Mean-Field and Advanced Applied Mathematics Methods

- Graph Theory
- Inverse Problem Theory
- Group-, and Group-Representation Theories

Physics Issues from the Gordon-Conference Series

Main Lines in J. Dudek Gordon presentations (after GEMINI)

Gordon

- **1987: Theoretical Framework of Super-deformation, ^{152}Dy**

Physics Issues from the Gordon-Conference Series
Main Lines in J. Dudek Gordon presentations (after GEMINI)

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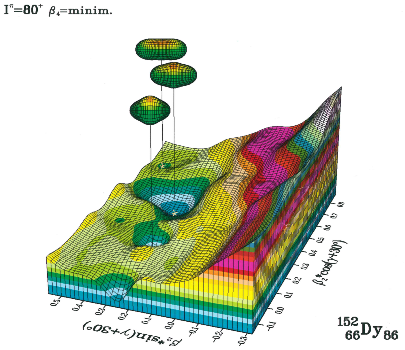

GEMINI ↔ The so-called (hmmm) famous Dudek plot

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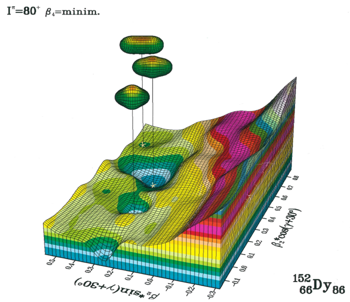

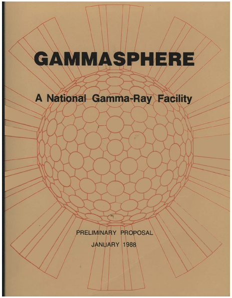

Dudek plot on the front page of the original proposal

Summary and Conclusions (3)

Physics Issues from the Gordon-Conference Series
J. Dudek presentation series (After GEMINI)

- **1987: Theoretical Framework of Super-deformation – ^{152}Dy**
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- **1993: Symmetries at Large Deformations: C_4-Symmetry,**

Summary and Conclusions (3)

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=”= Dudek plots,
=”= The so called identical (twin) SD bands,
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- **2001: Fresh Insight into Relativistic Mean Field Theory**
- **2007: Prediction of Tetrahedral and Octahedral Symmetries**
  Rotational bands with neither E1 nor E2 transitions

GEMINI: This led to a (hmmm) famous Dudek-ism:
Tetrahedral nuclei are “experimentally invisible”
to standard quadrupole transitions

Physics Issues Emphasized in this Presentation (Universality, Predictive Power, Parametric Correlation Removal, Mean-Field as a Reliable Collaboration-Tool)

- Identification of the spherical experimental single nucleon levels*)
- Include experimental uncertainties to find modelling uncertainties
- Inverse Problem Theory Combined with Monte-Carlo Simulations

*) Note: Experimental single nucleon levels are strongly model dependent objects

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- WS Hamiltonian: Identified and removed parametric correlations
- Predicting & Identifying Tetrahedral and Octahedral Symmetries

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Summary and Conclusions (3)

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Projects for International Community

- 2016: Guest Editor: Focus issue for the 40 year anniversary of the 1975 Nobel Prize to Aage Bohr, Ben Mottelson and Leo Rainwater
- Proposal: EuroLabs MeanField4Exp theory project for experiment
- Acting as co-chairman of 5 Editions of the SSNET Conferences

*) Note: Experimental single nucleon levels are strongly model dependent objects

Thank You