

Single-particle aspects of mirror energy states across the sd-pf shells

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Mirror Energy Differences [MEDs]

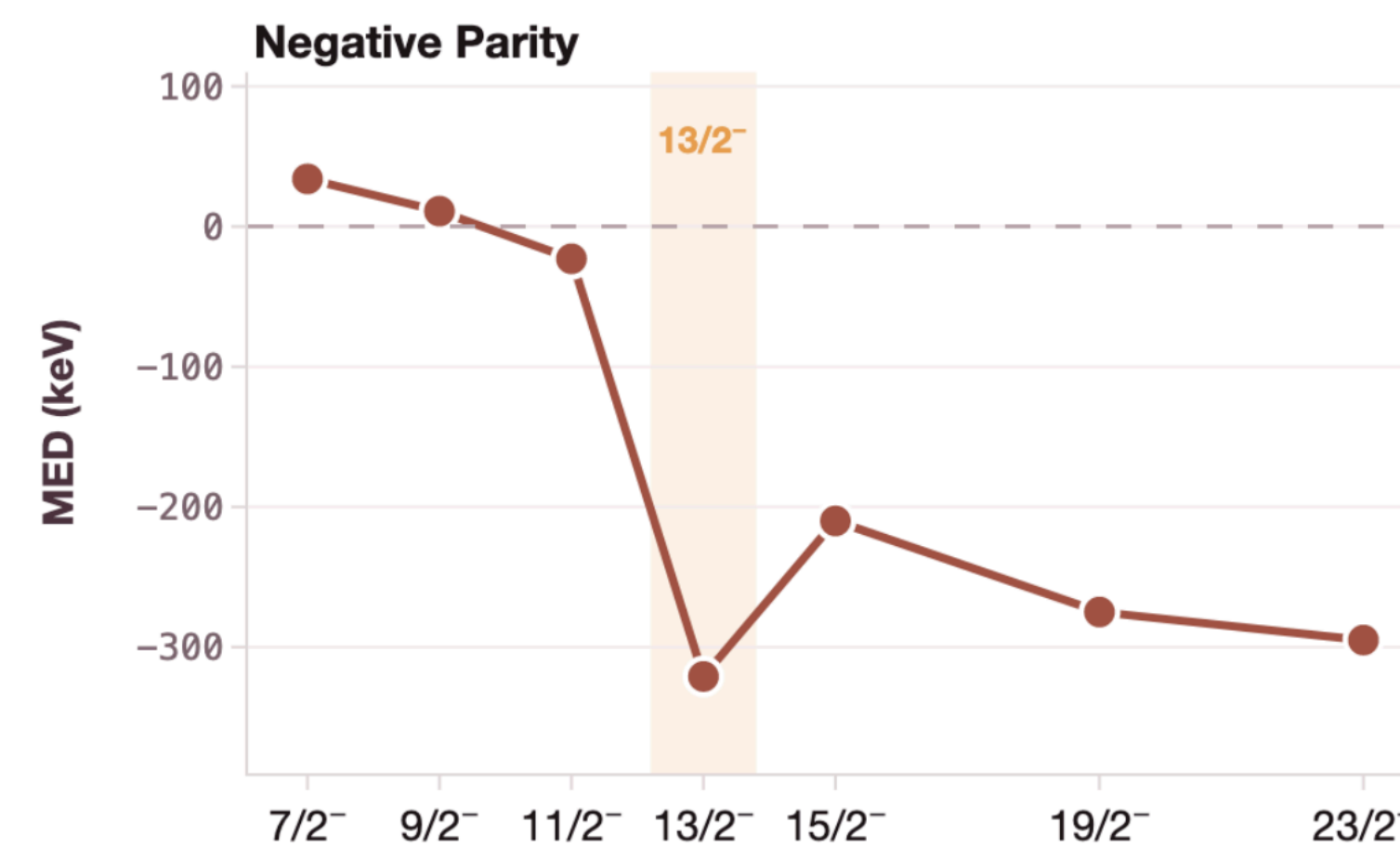
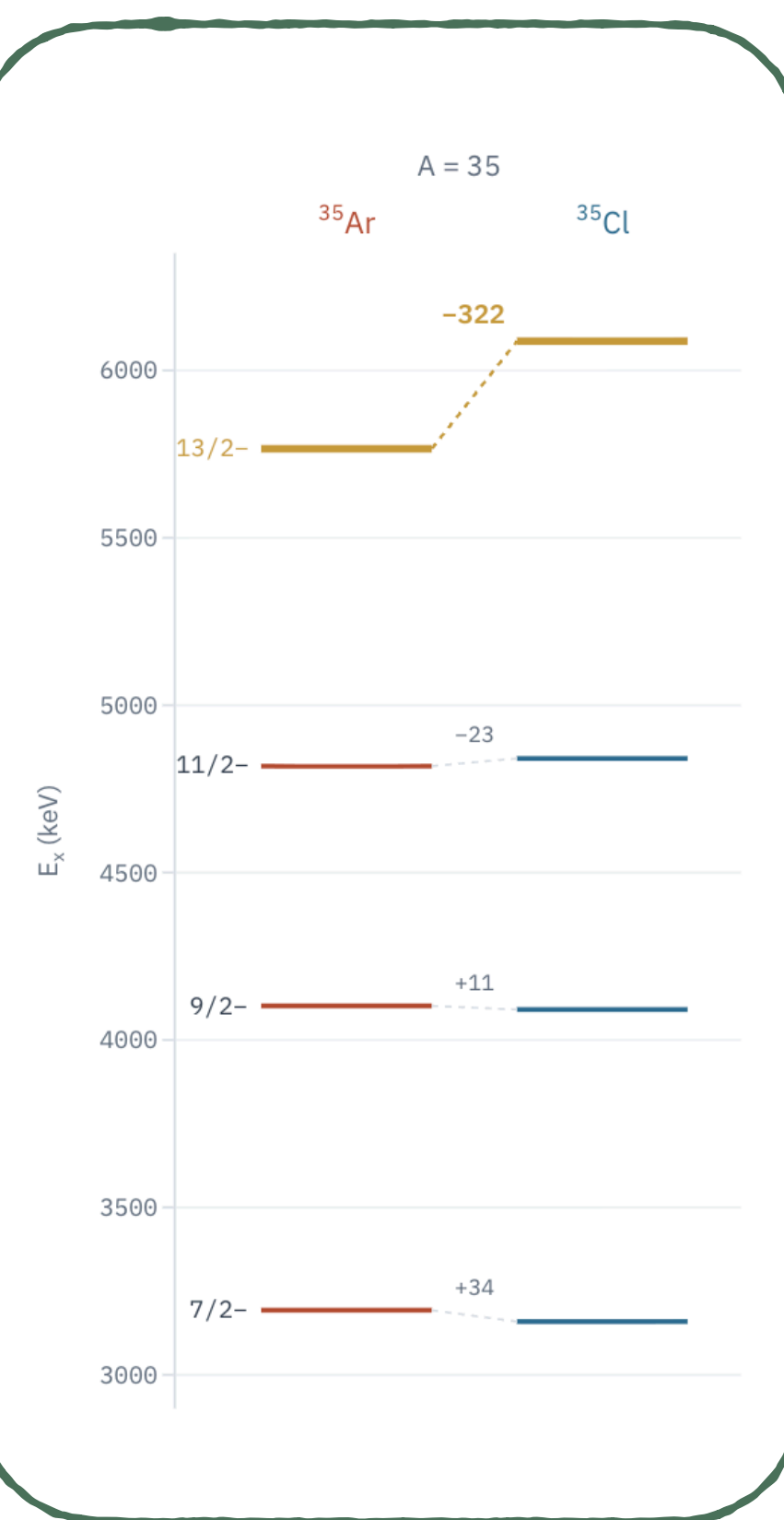
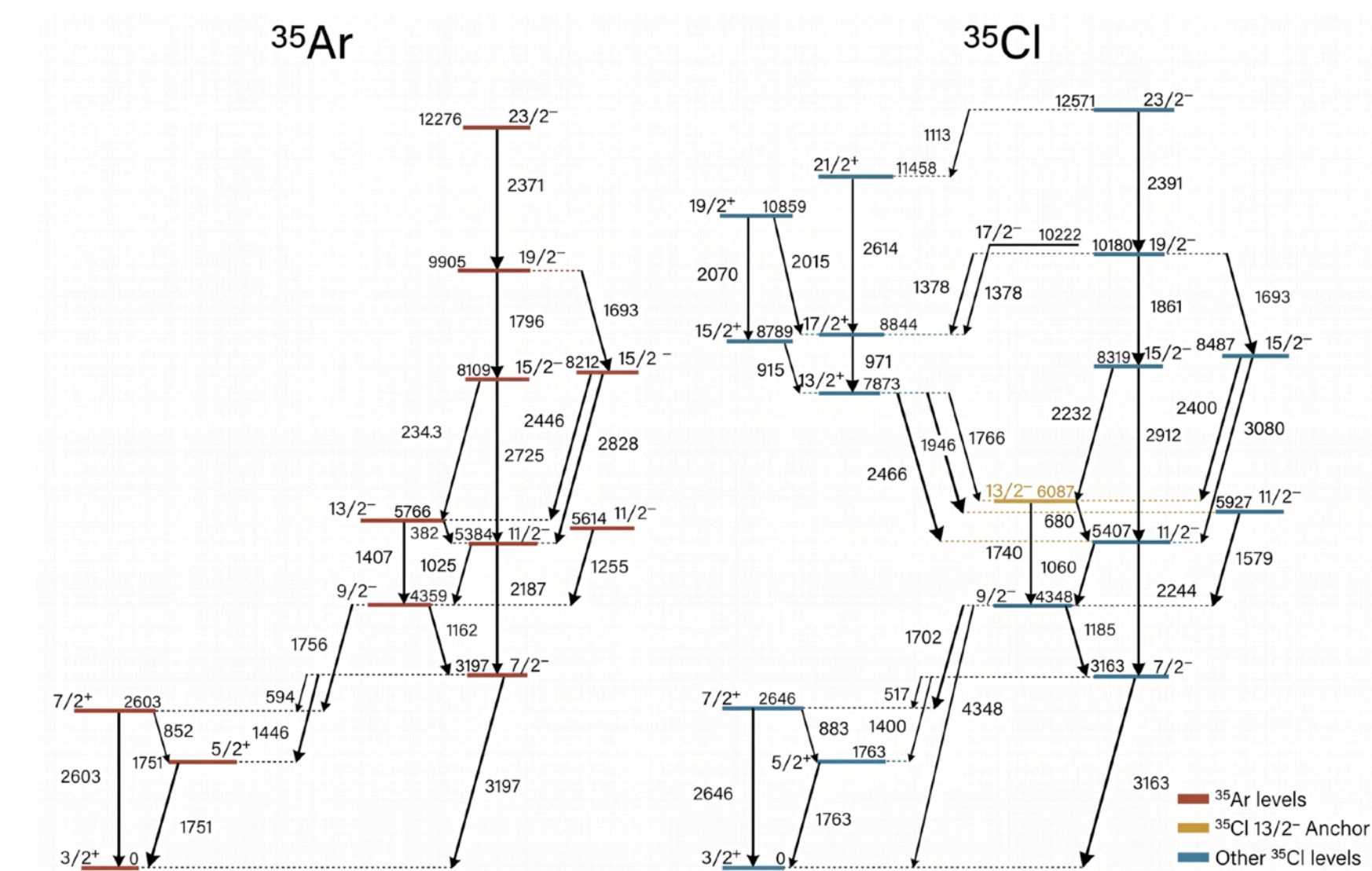
$$\text{MED}(J^\pi) = E_x(J^\pi; T_z = -1/2) - E_x(J^\pi; T_z = +1/2)$$

$$T_z = -1/2$$

Proton-rich. Valence nucleon is a **proton**.

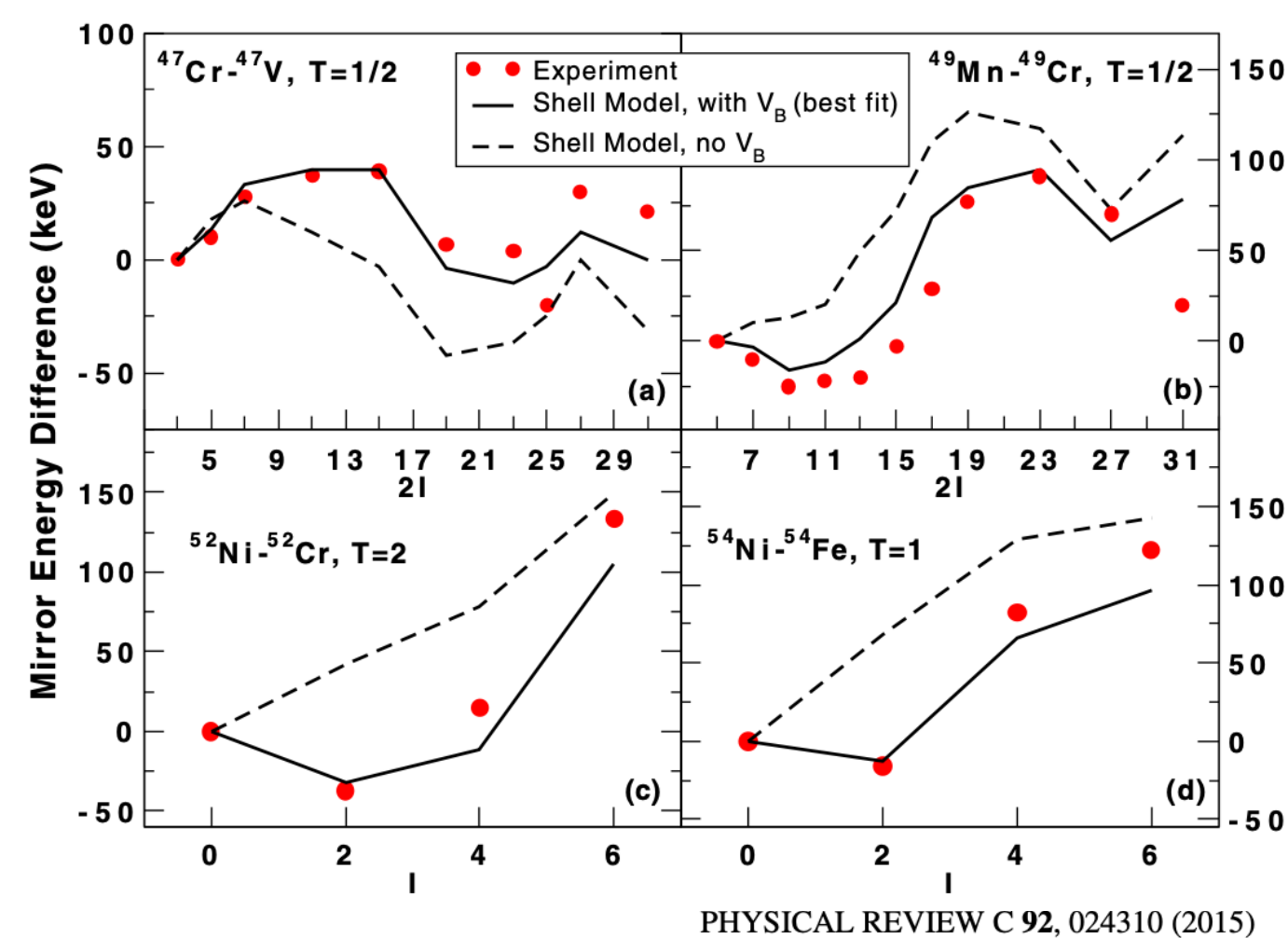
$$T_z = +1/2$$

Neutron-rich. Valence nucleon is a **neutron**.

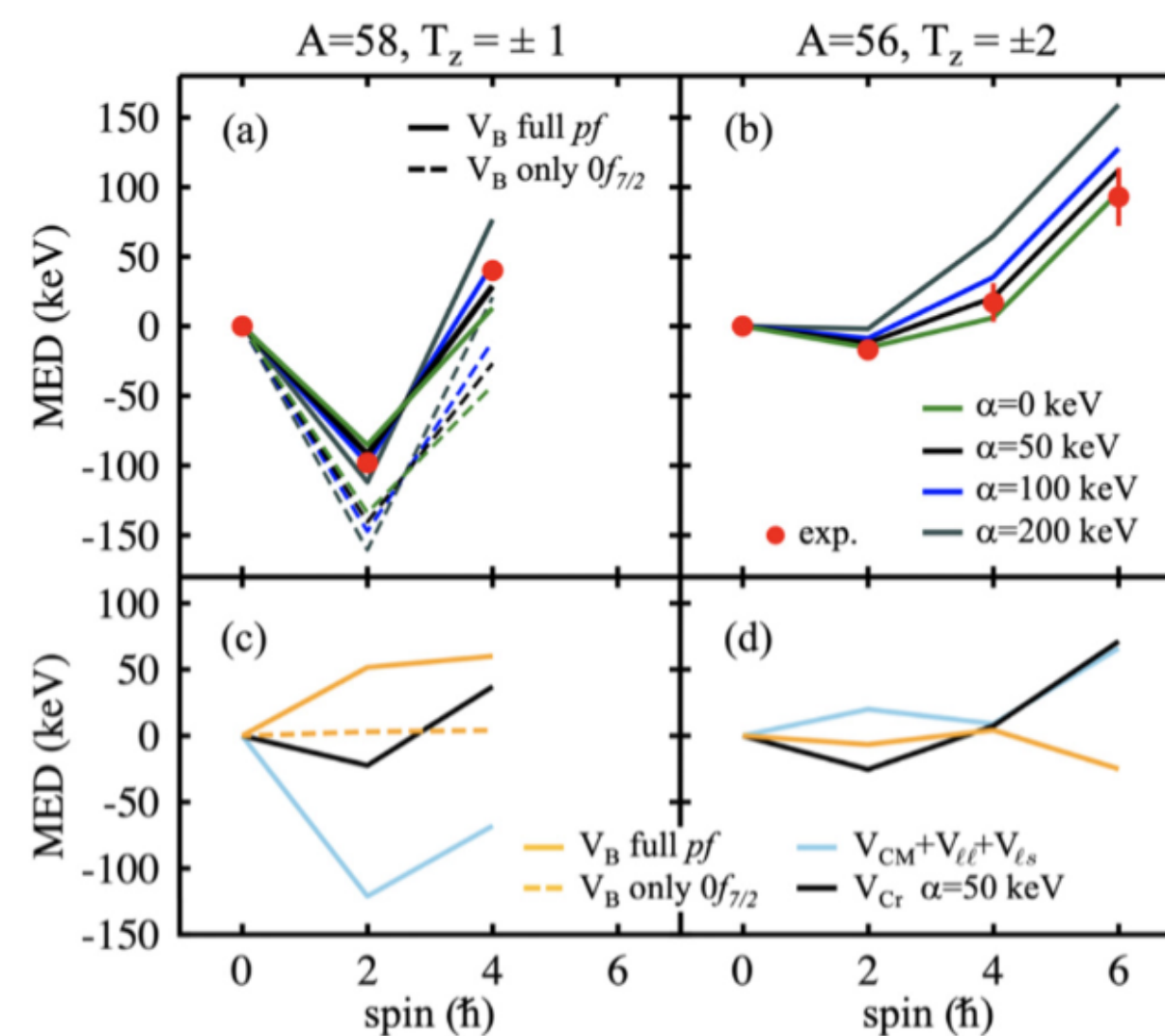


Mirror Energy Differences [MEDs]

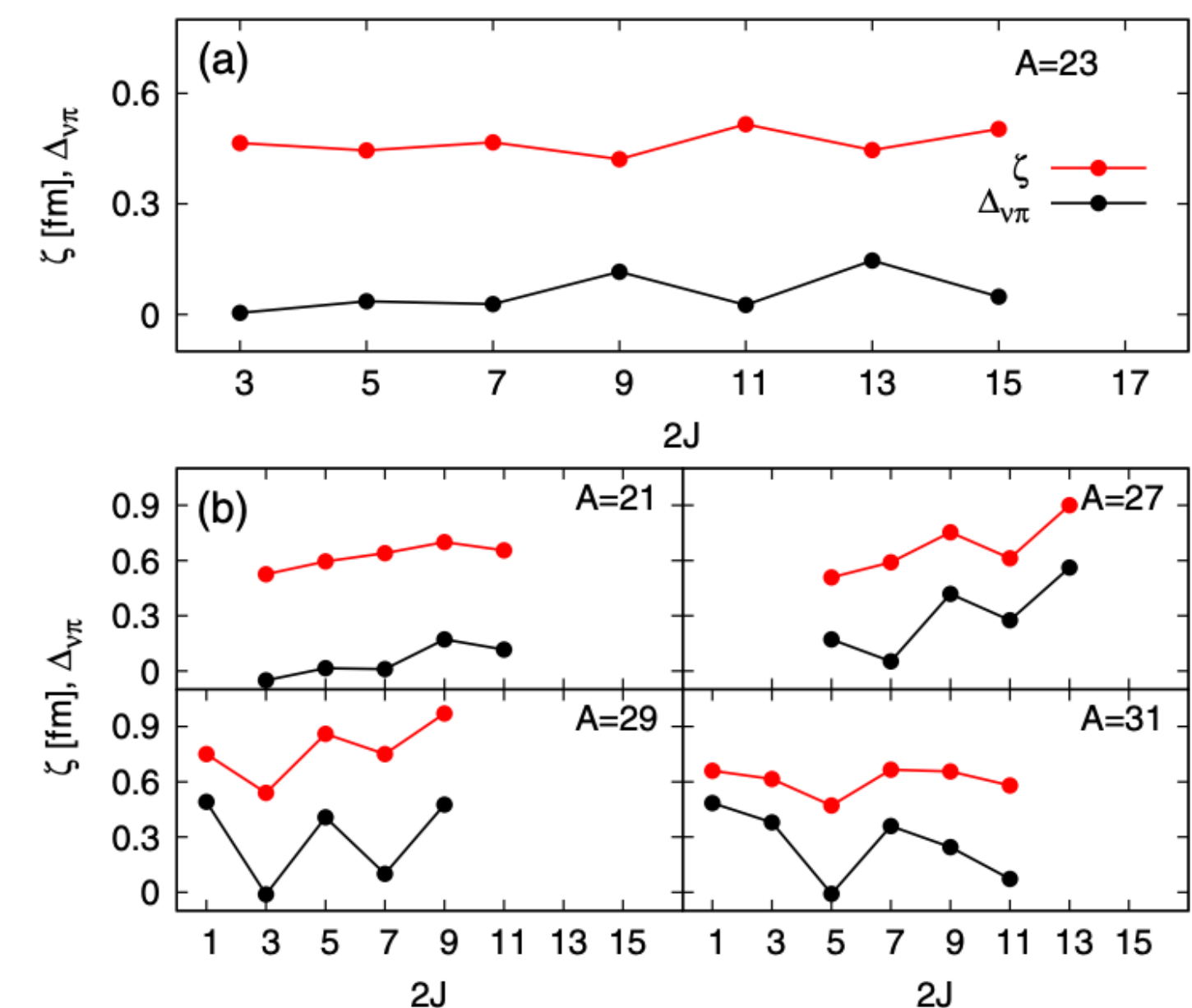
- Small (10's of keV) shifts in excitation energy between mirror nuclei
- Sensitive to nucleon alignments, both single-particle & coherent evolutions, & INC



Spin-Dependence of
isovector breaking term

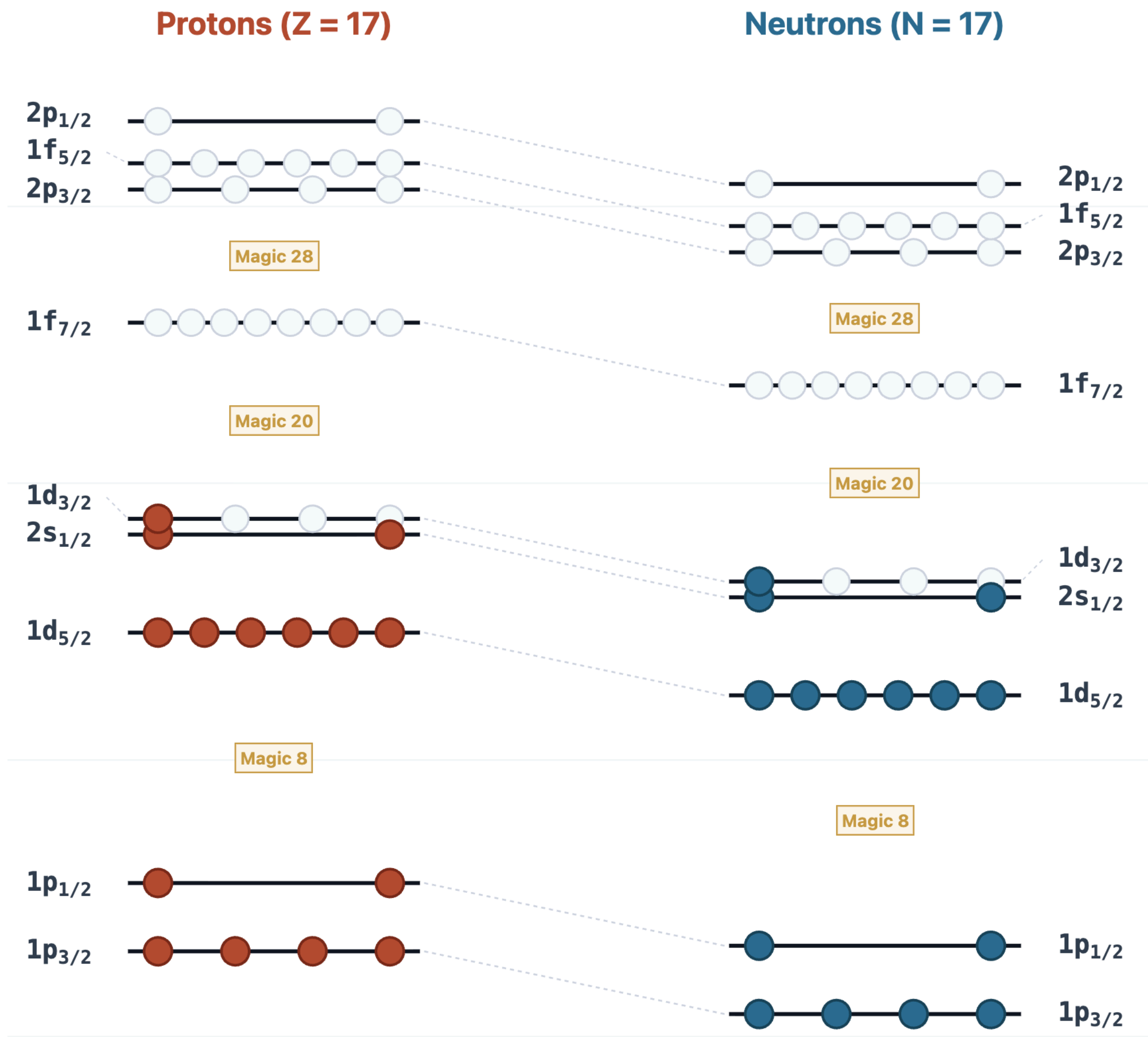


Spin-Isospin correlations
of breaking effects



Radii (skin) relation to MEDS

Working landscape 1s0d - 0f1p orbitals



A = 35 & 39 T = 1/2 Mirror Pairs

³⁵Cl-³⁵Ar, ³⁹K-³⁹Ca

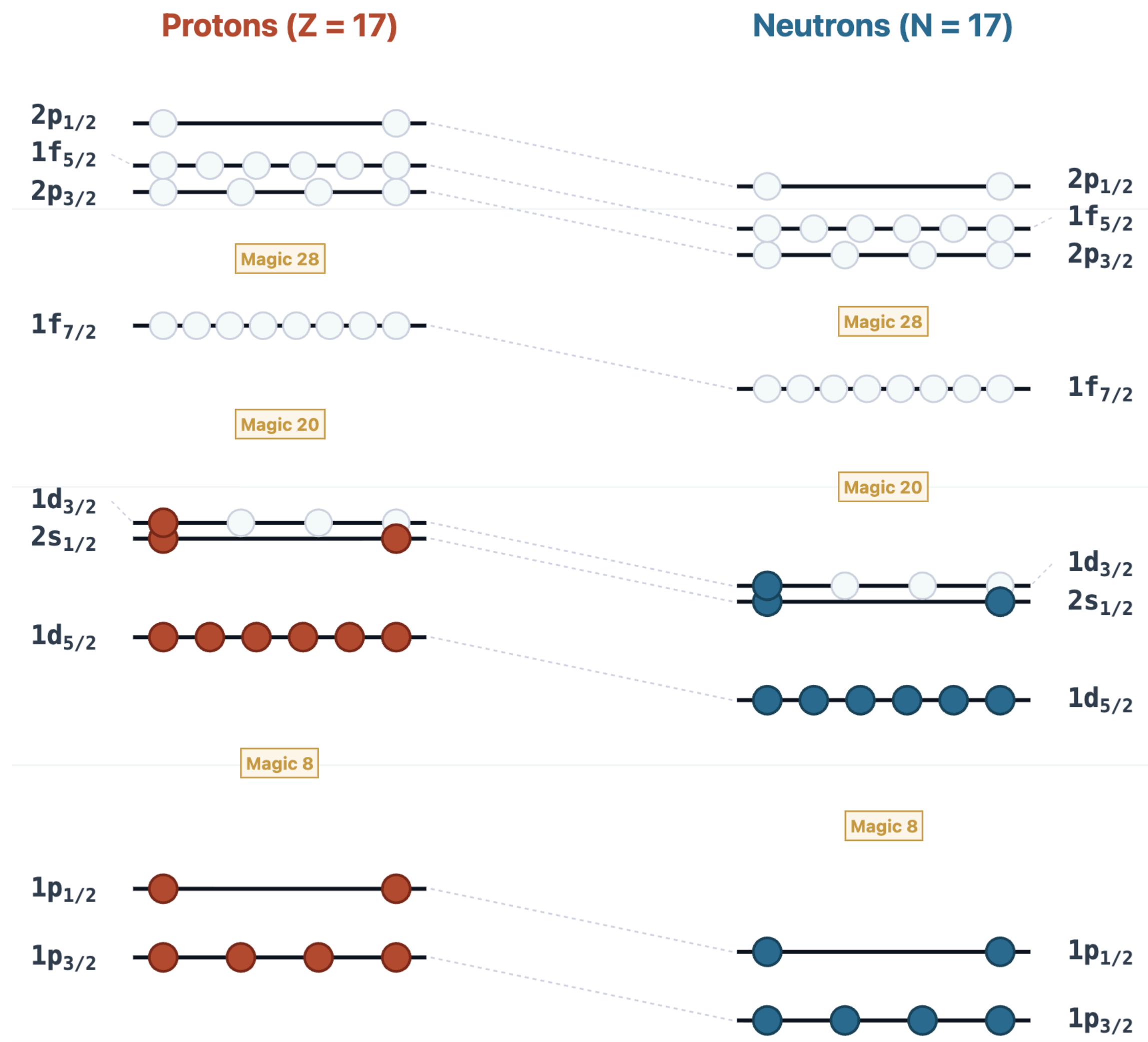
$$T = 0 \text{ Core: } \pi(0d_{3/2})^1 \nu(0d_{3/2})^1$$

$$J\pi = 0^+ - 3^+$$

Ground and isomeric state information for ³⁴ ₁₇ Cl				
E(level) (MeV)	J _π	Mass Excess (keV)	T _{1/2}	Decay Modes
0.0	0+	-24440.09 5	1.5265 s 4	ε+β+ = 100%
0.1464	3+	-24293.73 5	31.99 min 3	ε+β+ = 55.4% IT = 44.6%

Ground and isomeric state information for ³⁸ ₁₉ K				
E(level) (MeV)	J _π	Mass Excess (keV)	T _{1/2}	Decay Modes
0.0	3+	-28800.76 20	7.645 min 20	ε+β+ = 100%
0.1302	0+	-28670.54 20	924.40 ms 21	ε+β+ = 99.967% IT = 0.033%

Working landscape 1s0d - 0f1p orbitals



A = 35 & 39 T = 1/2 Mirror Pairs

³⁵Cl-³⁵Ar, ³⁹K-³⁹Ca

$\pi = +$ levels: $(1s0d)^{X-1}(0d_{3/2})^1$

$J_{\text{Max}} = (0d_{3/2})^3_{J=7/2} = 7/2^+$

↓
 $\pi \ \& \ \nu$

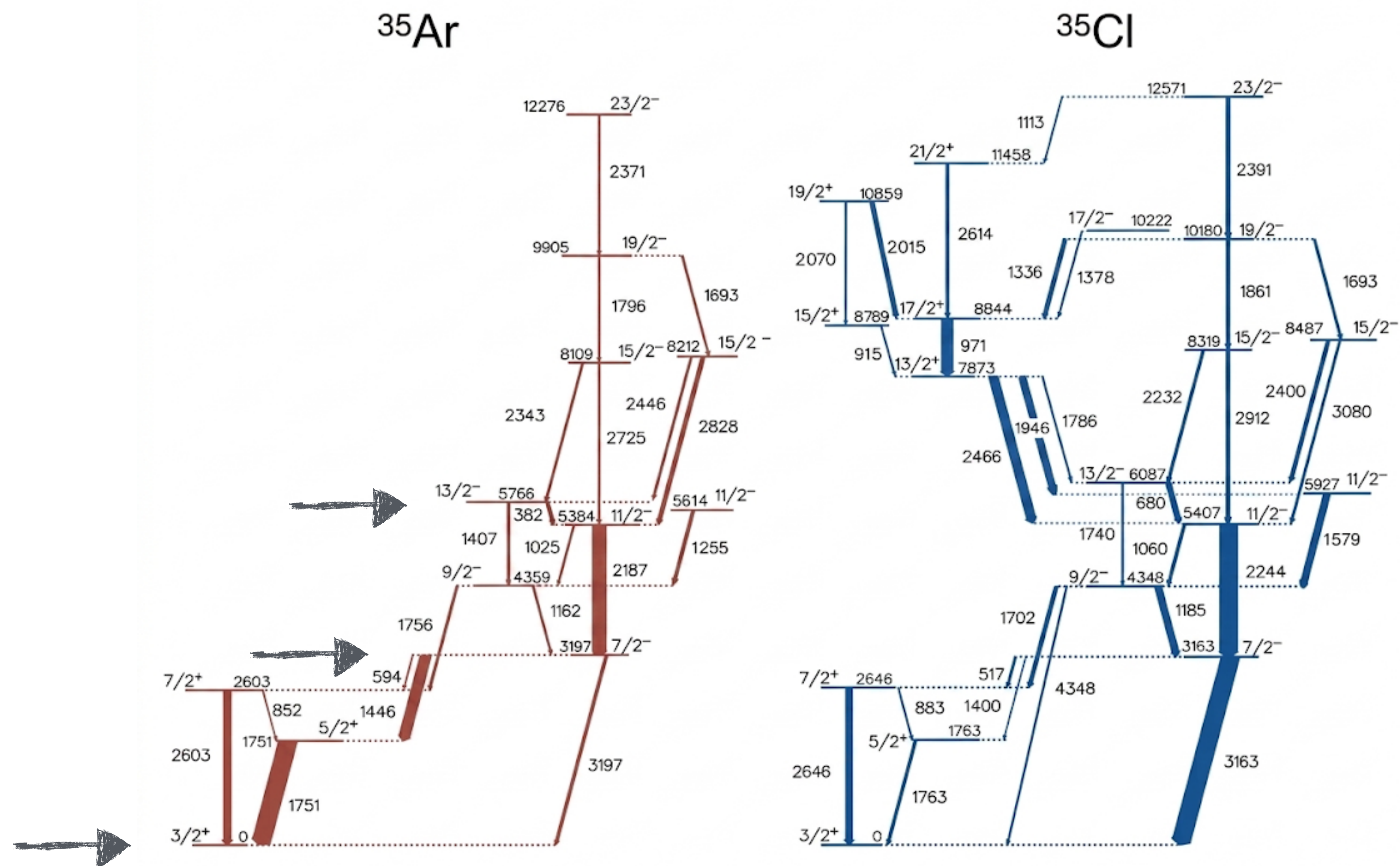
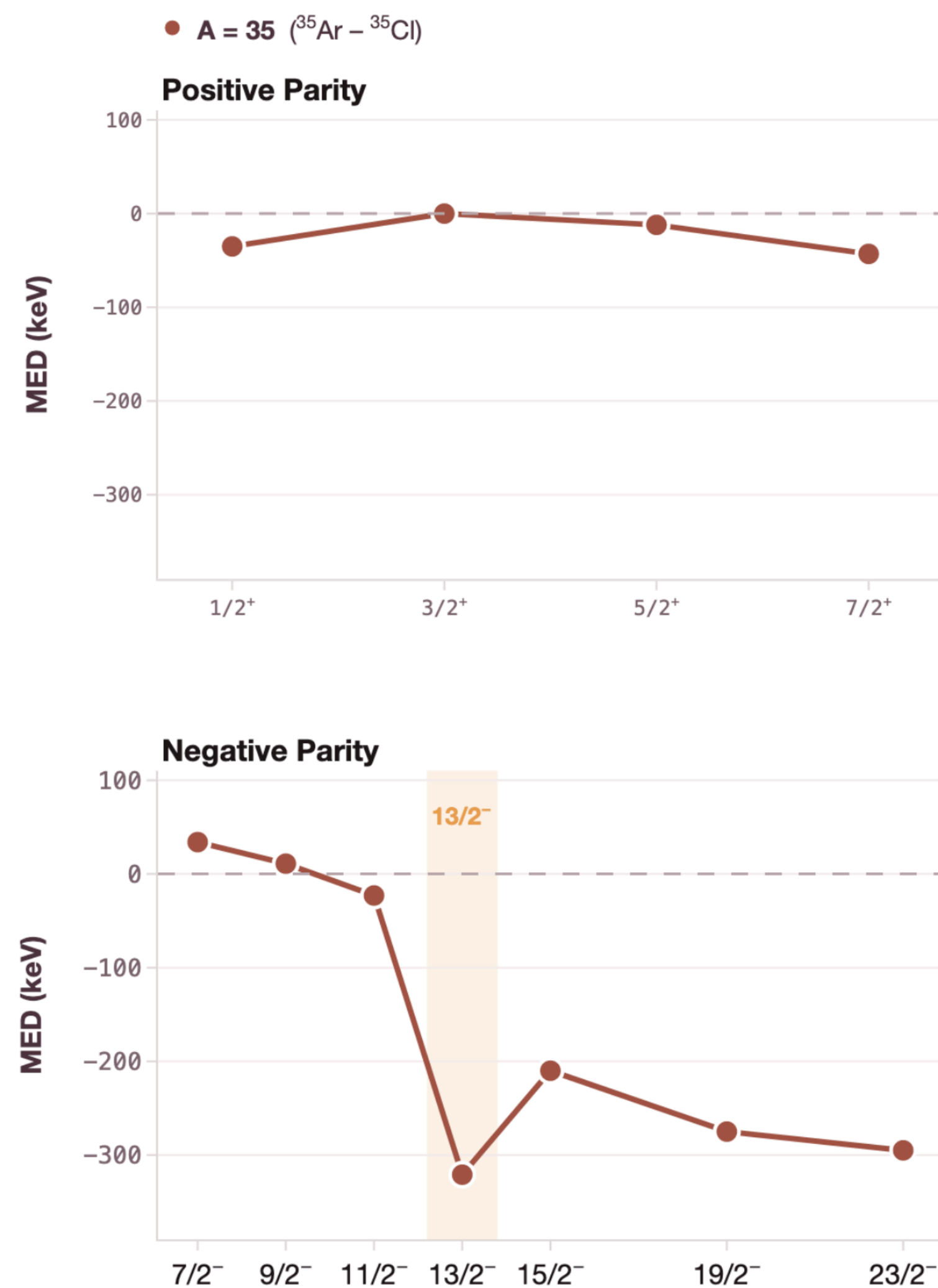
$\pi = -$ levels: $(1s0d)^{X-1}(0f1p)^1$

$J_{\text{Max}} = (0d_{3/2})^2_{J=3}(0f_{7/2})^1_{J=7/2} = 13/2^-$

↓ ↓
 $\pi \ \& \ \nu$ $\pi \ || \ \nu$

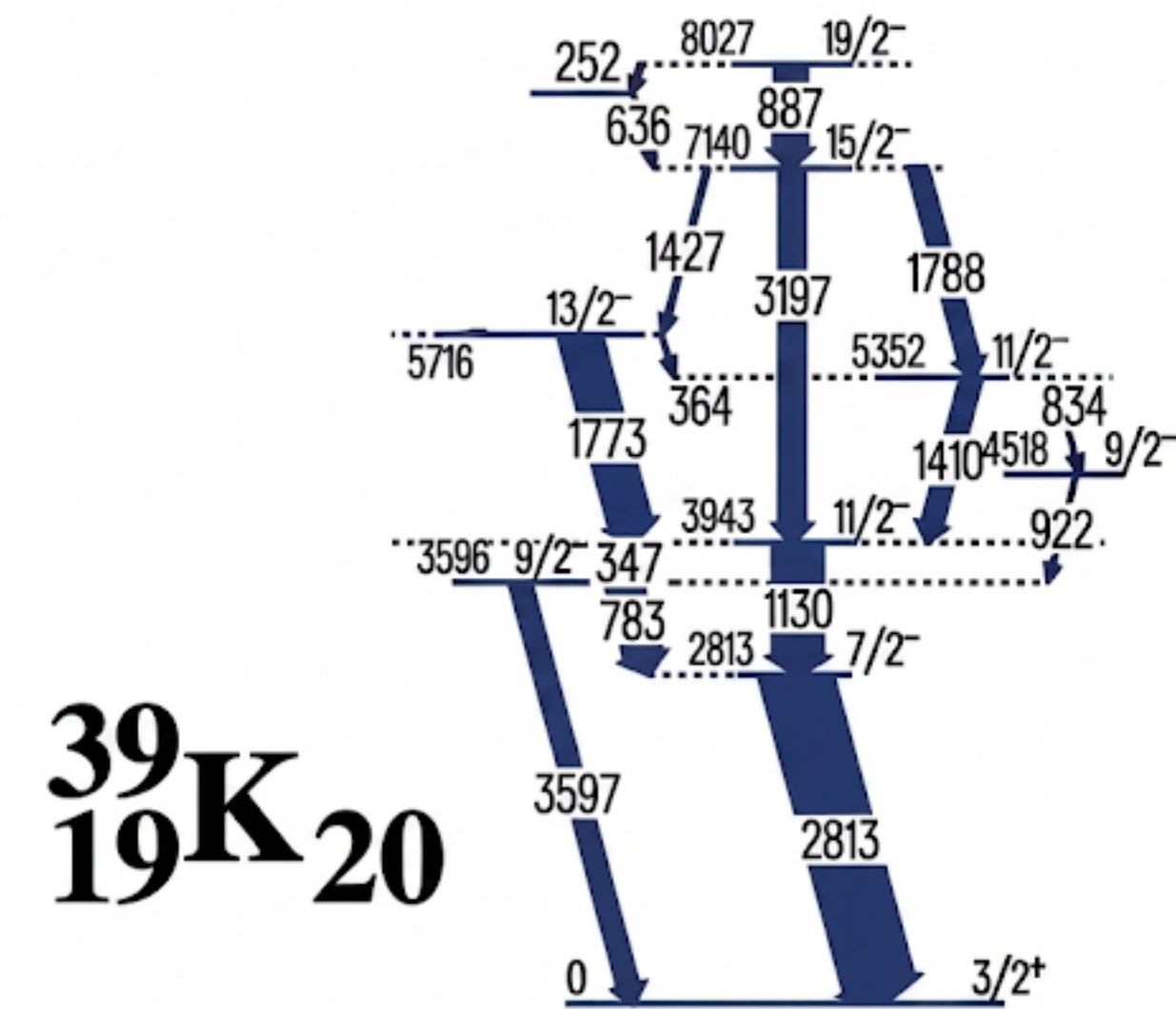
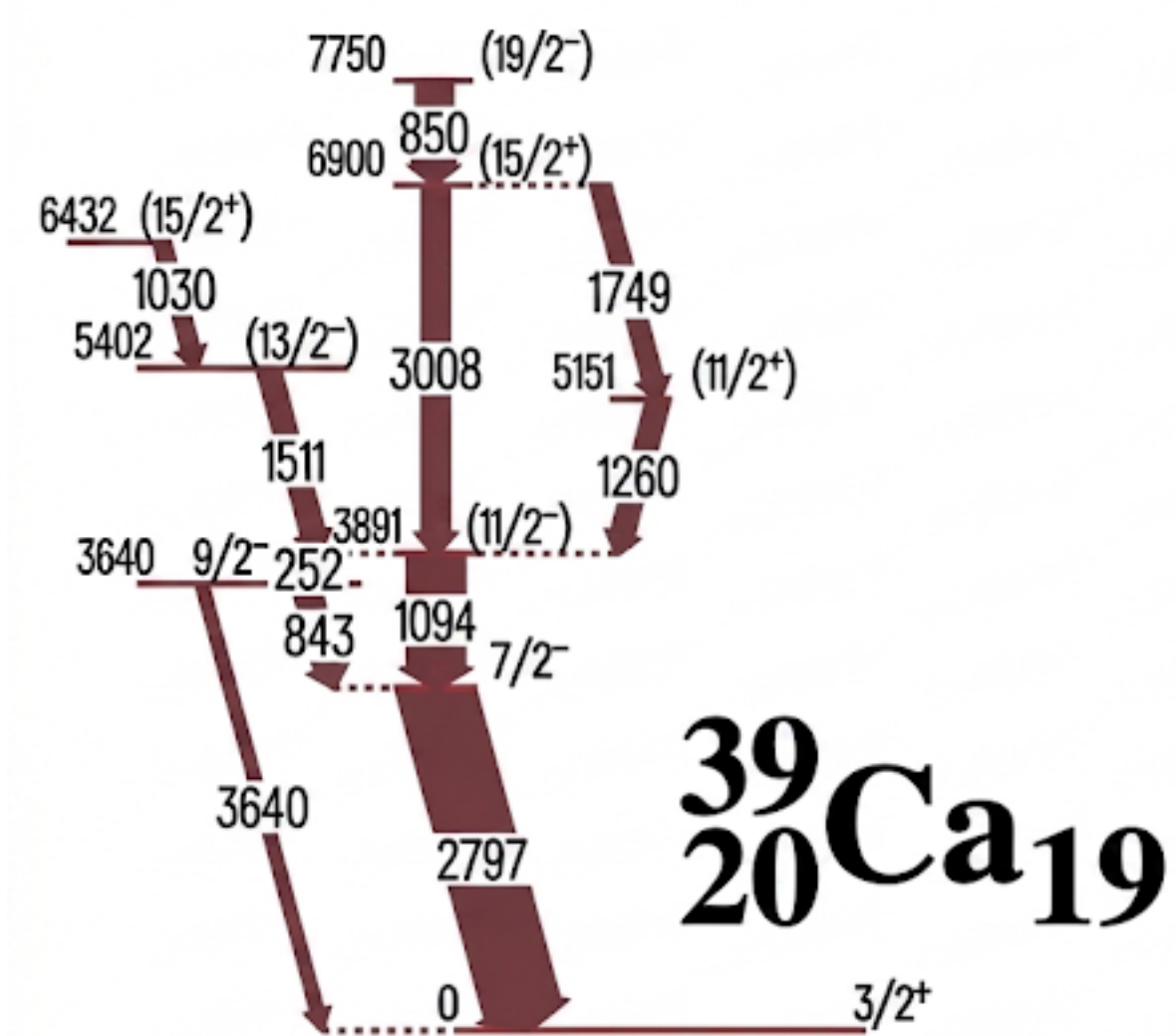
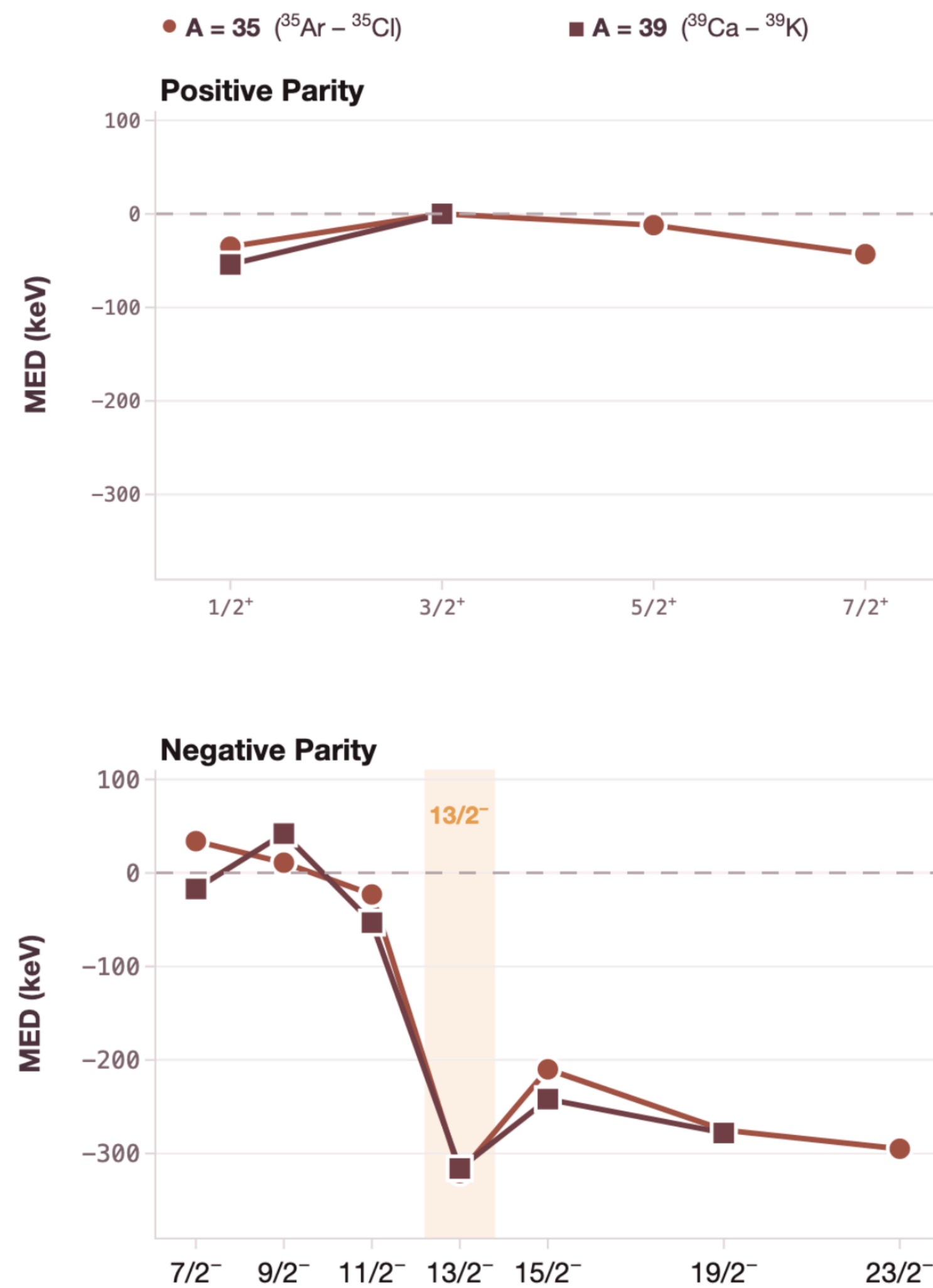
Experimental MEDs: $A = 31, 35 \text{ \& } 39$

- Extracted from detailed level schemes between $T = 1/2$ mirror pairs: $^{35}\text{Ar} - ^{35}\text{Cl}$



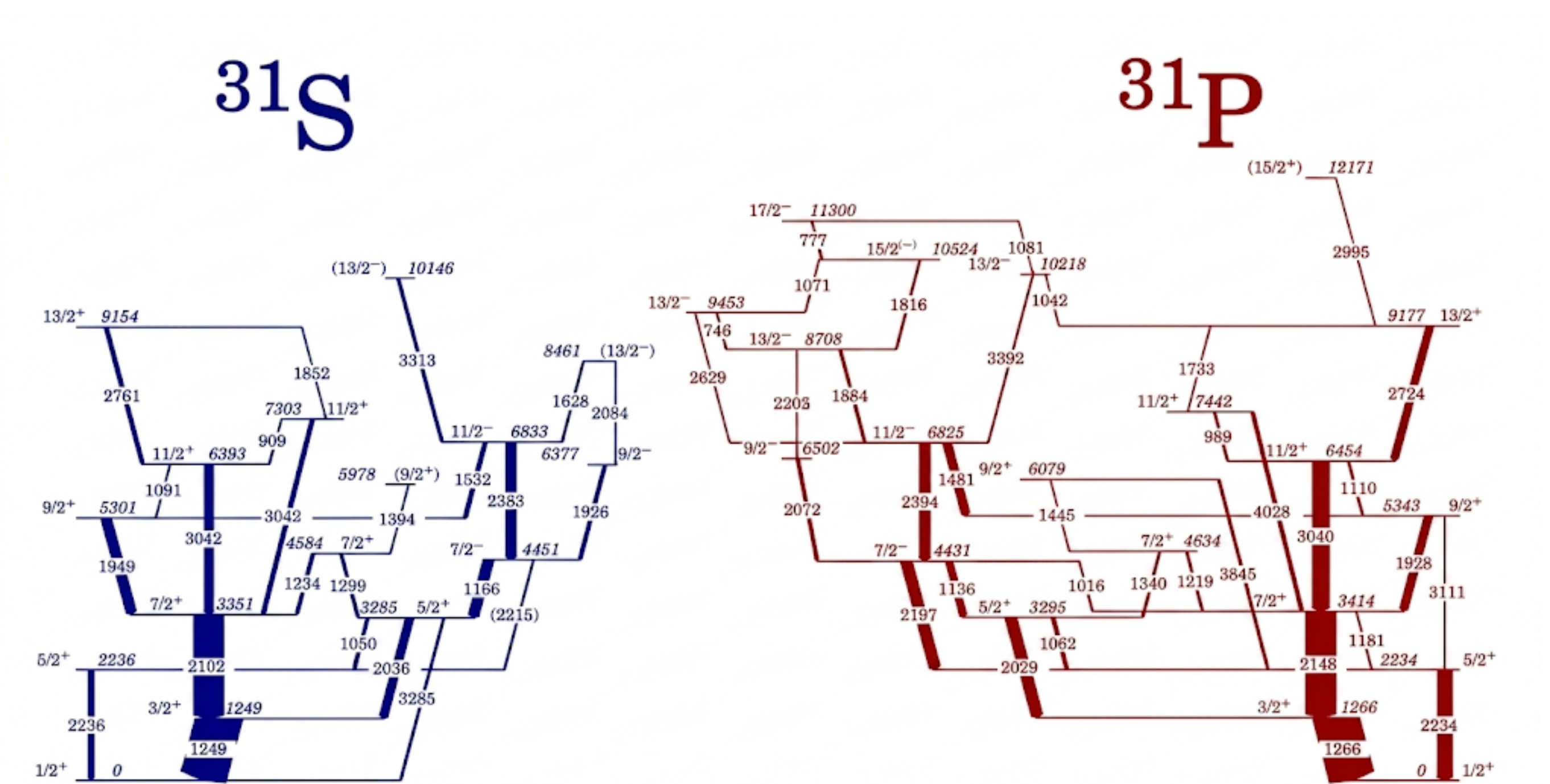
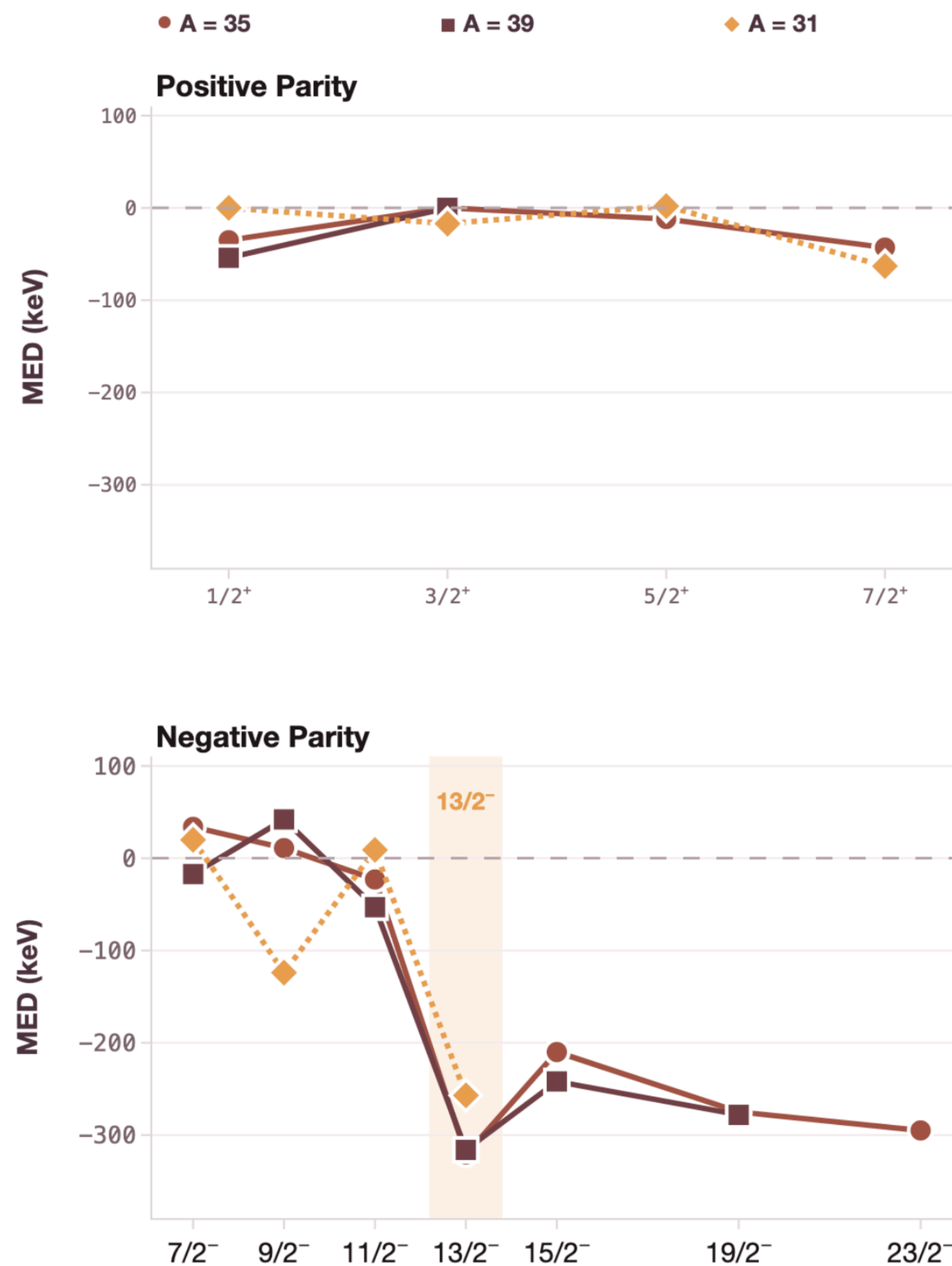
Experimental MEDs: $A = 31, 35$ & 39

- Extracted from detailed level schemes between $T = 1/2$ mirror pairs: $^{39}\text{Ca} - ^{39}\text{K}$



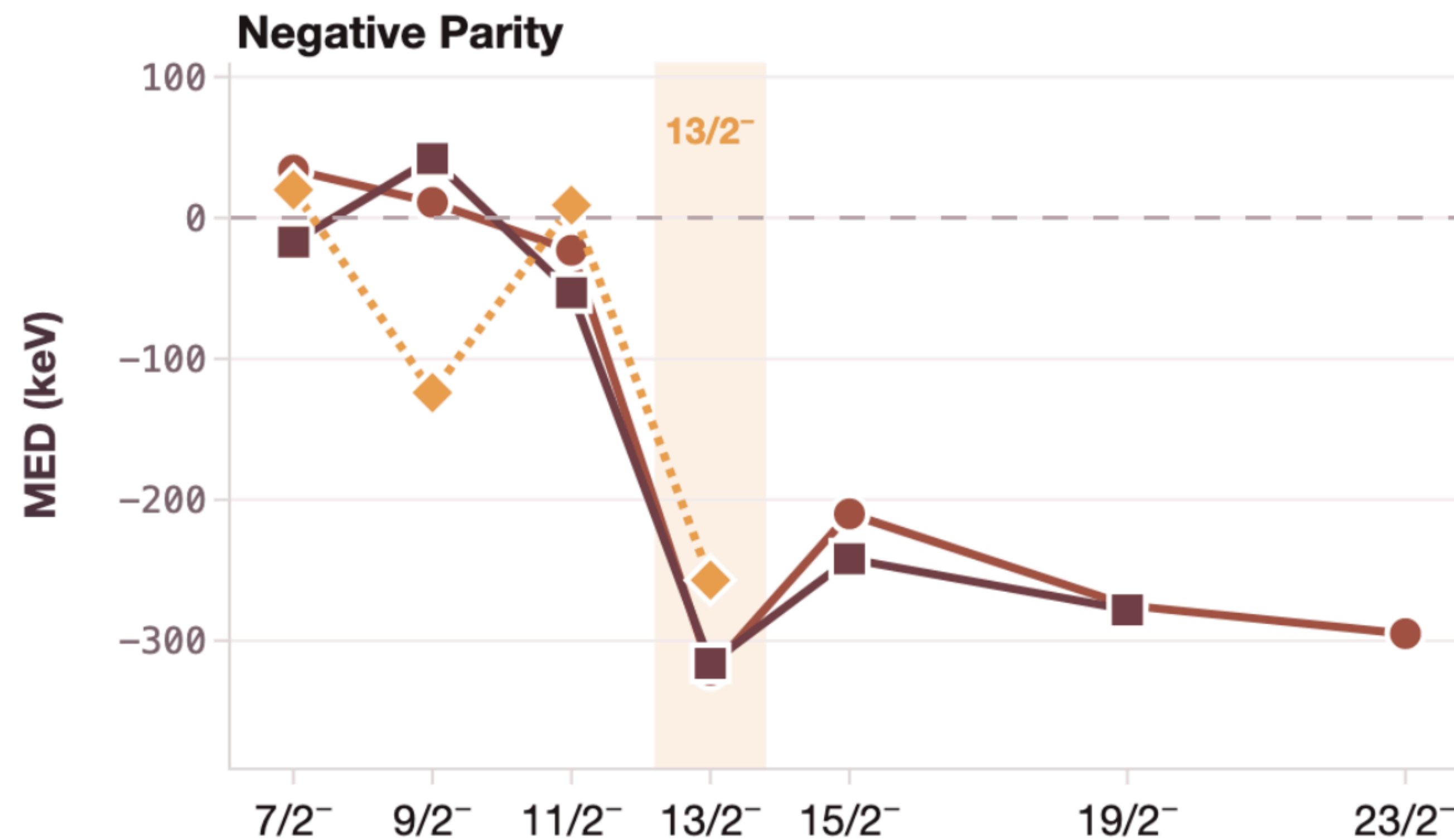
Experimental MEDs: $A = 31, 35$ & 39

- Extracted from detailed level schemes between $T = 1/2$ mirror pairs: $^{31}\text{S} - ^{31}\text{P}$



Alignment at the 13/2- level

- Signature: Search for a large single-nucleon component in the wave function



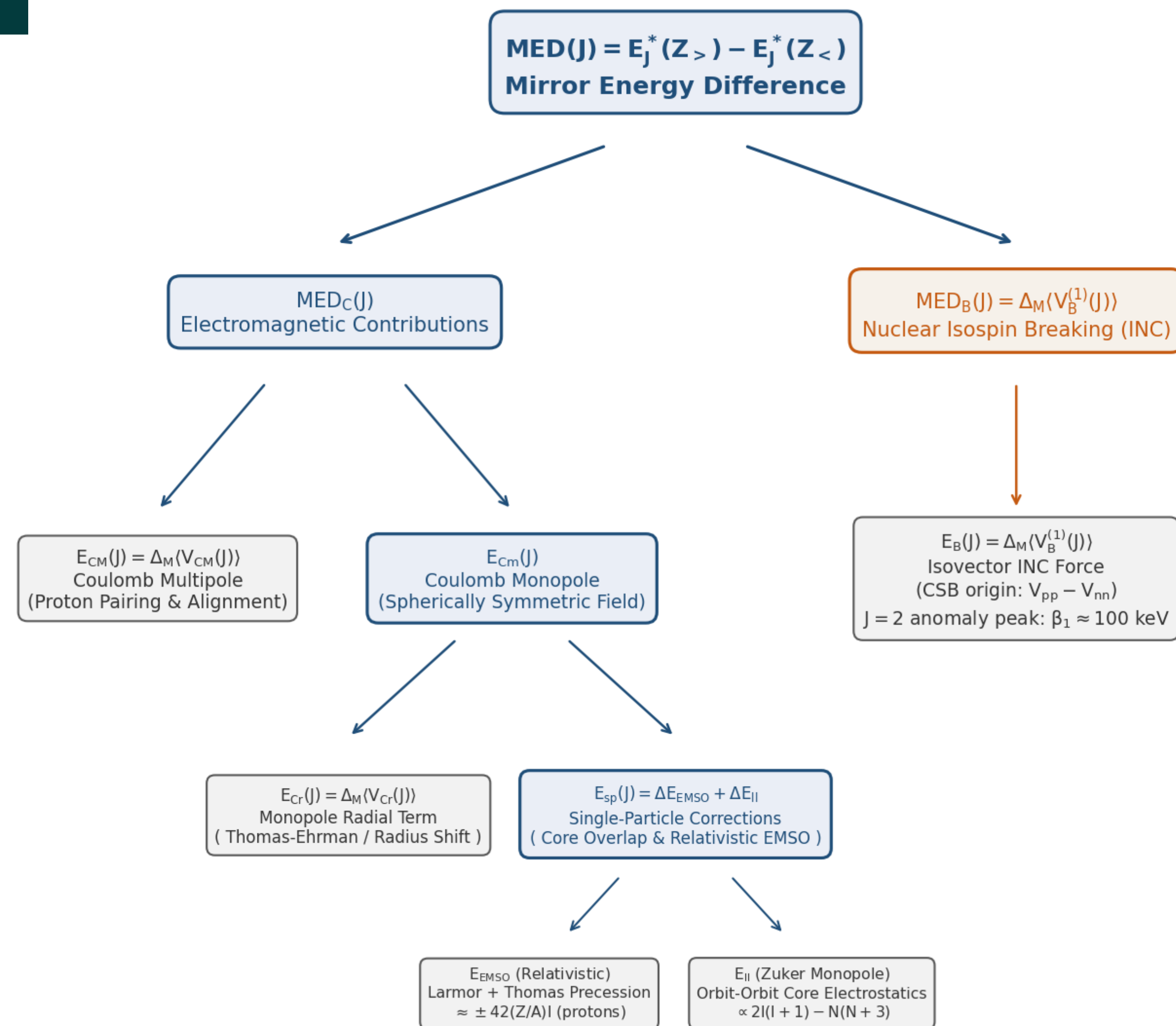
$$J_{\text{Max}} = (0d_{3/2})^2_{J=3} (0f_{7/2})^1_{J=7/2} = 13/2^-$$

$\pi \& \nu$ $\pi \parallel \nu$

Theoretical decomposition of the MED

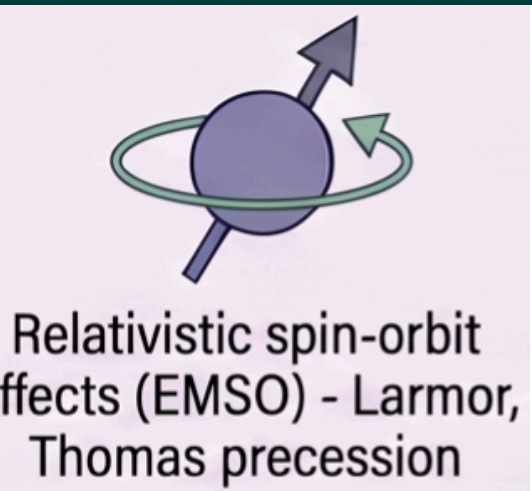
$$\begin{aligned} \text{MED} &= E_{\text{CM}} + E_{\text{Cm}} + E_{\text{INC}} \\ &= E_{\text{CM}} + \underline{E_{\text{EMSO}}} + \underline{E_{\ell\ell}} + E_{\text{Cr}} + E_{\text{INC}} \end{aligned}$$

- **E_{CM} (Coulomb Multipole):** Multipole Coulomb pair-breaking and angular momentum alignment; cancels for single valence nucleons outside identical cores.
- **E_{EMSO} (Electromagnetic Spin-Orbit):** Relativistic Foldy-Wouthuysen interaction; directly scales with single-particle strength C^2S .
- **E_{Cr} (Monopole Radius Effect):** Core dilation / radial expansion shift from occupying the larger $0f_{7/2}$ orbit.
- **$E_{\ell\ell}$ (Orbit-Orbit Coupling):** Relativistic kinetic shift across oscillator shells.
- **E_{INC} (Isospin Non-Conserving) & TES:** Nuclear CSB/CIB forces and near-threshold Thomas-Ehrman shifts.



$$\underline{E_{\text{sp}} \propto (E_{\text{EMSO}} + E_{\ell\ell}) \cdot C^2S}$$

Mirror Energy Differences [MEDs]



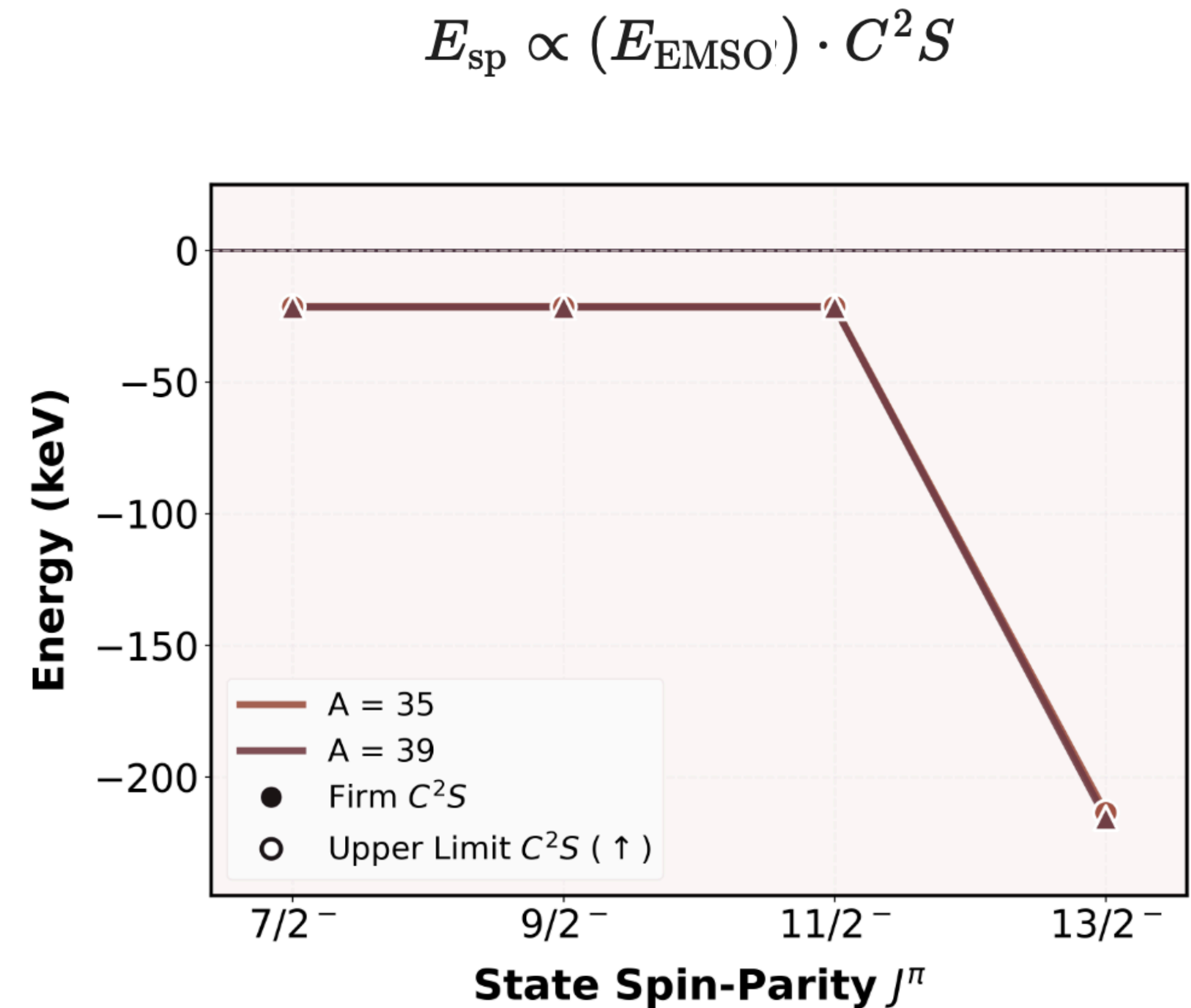
$$E_{\text{EMSO}} \approx (g_s - g_l) \frac{1}{2m_N^2 c^2} \left(-\frac{Ze^2}{R_C^3} \right) \langle \vec{l} \cdot \vec{s} \rangle$$

Promotion of nucleon: $0d_{3/2} \rightarrow 0f_{7/2}$

$$\begin{aligned} \Delta E_{\text{EMSO}} &= \Delta_{\text{pn}}(0f_{7/2}) - \Delta_{\text{pn}}(0d_{3/2}) \\ &= \sim -100 \text{ keV} - \sim 120 \text{ keV} = \sim 220 \text{ keV} \end{aligned}$$

'Bare' g-factor values, $R_C = 1.2 \text{ fm}$

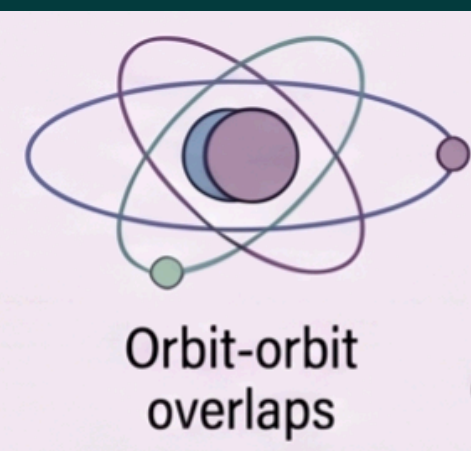
Demonstrative C^2S values: $[0.1, 1.0]$



$$(0d_{3/2})^2_{J=3} \otimes (0f_{7/2})^1_{j=7/2} \rightarrow J^\pi = 13/2^-$$

- Alignment Maximizes EMSO:** The maximally aligned $13/2^-$ state locks the valence $0f_{7/2}$ nucleon into maximum angular momentum, maximizing the spin-orbit interaction.

Mirror Energy Differences [MEDs]



$$E_{ll} = \frac{-4.5Z_{cs}^{13/12}[2l(l+1)-N(N+3)]}{A^{1/3}(N+\frac{3}{2})} \text{ keV}$$

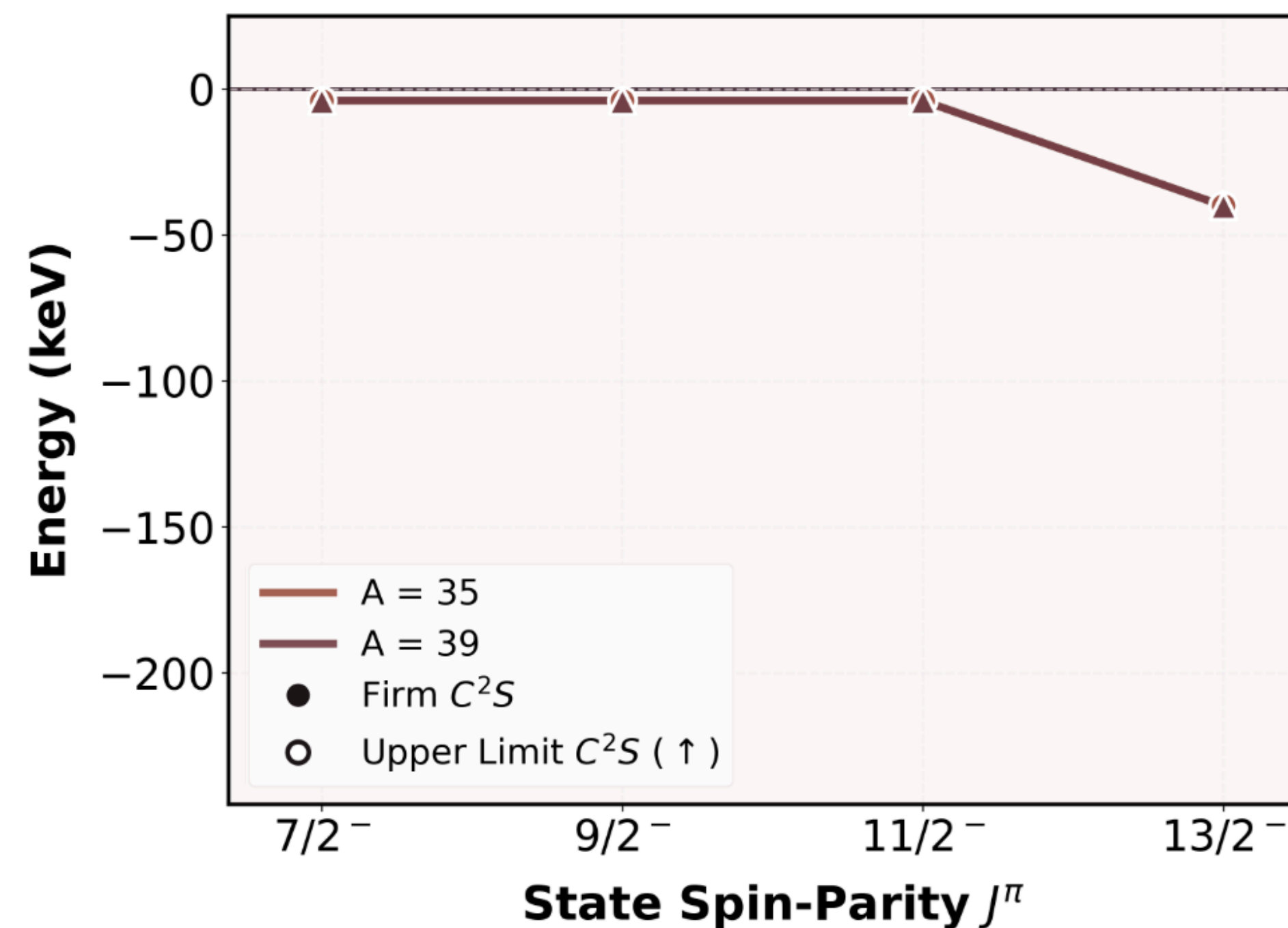
Impacts the proton orbital energies

$$\Delta E_{ll} = \Delta_p(\text{Of}_{7/2} - \text{Od}_{3/2}) = \sim 45 \text{ keV}$$

$$N = 2 \rightarrow N = 3$$

Demonstrative C^2S values: [0.1, 1.0]

$$E_{sp} \propto (E_{ll}) \cdot C^2S$$



Measure the single-neutron overlaps

Reaching $J^\pi = 13/2^-$ via single-step (d,p) transfer requires high-spin beam states.

BEAM IMPLEMENTATIONS

A = 35 at HELIOS / ATLAS: Produced an isomeric beam of ^{34m}Cl ($J^\pi = 3^+$, $T_{1/2} = 32.0$ min) via in-flight transfer.

A = 39 at ISS / ISOLDE: ^{38}K ground state is already $J^\pi = 3^+$ ($T_{1/2} = 7.6$ min), enabling clean direct transfer into ^{39}K .

Emphasis on C²S of the yrast 11/2⁻ to 13/2⁻

ANGULAR MOMENTUM SELECTION RULES

Single-neutron transfer carries $\Delta J \leq \ell + \frac{1}{2} = 3 + \frac{1}{2} = 3.5$ ($\ell = 3$ for $0f_{7/2}$).

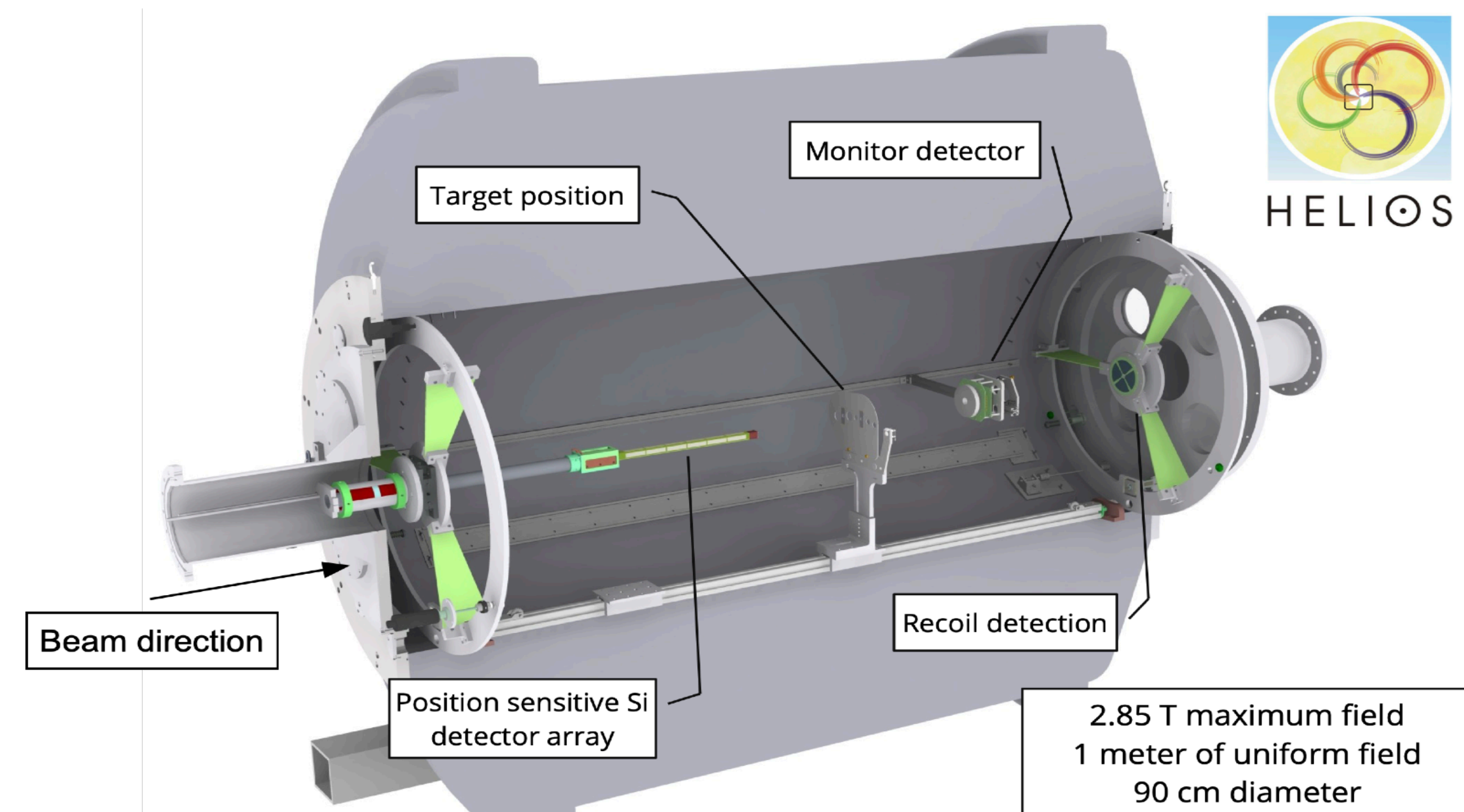
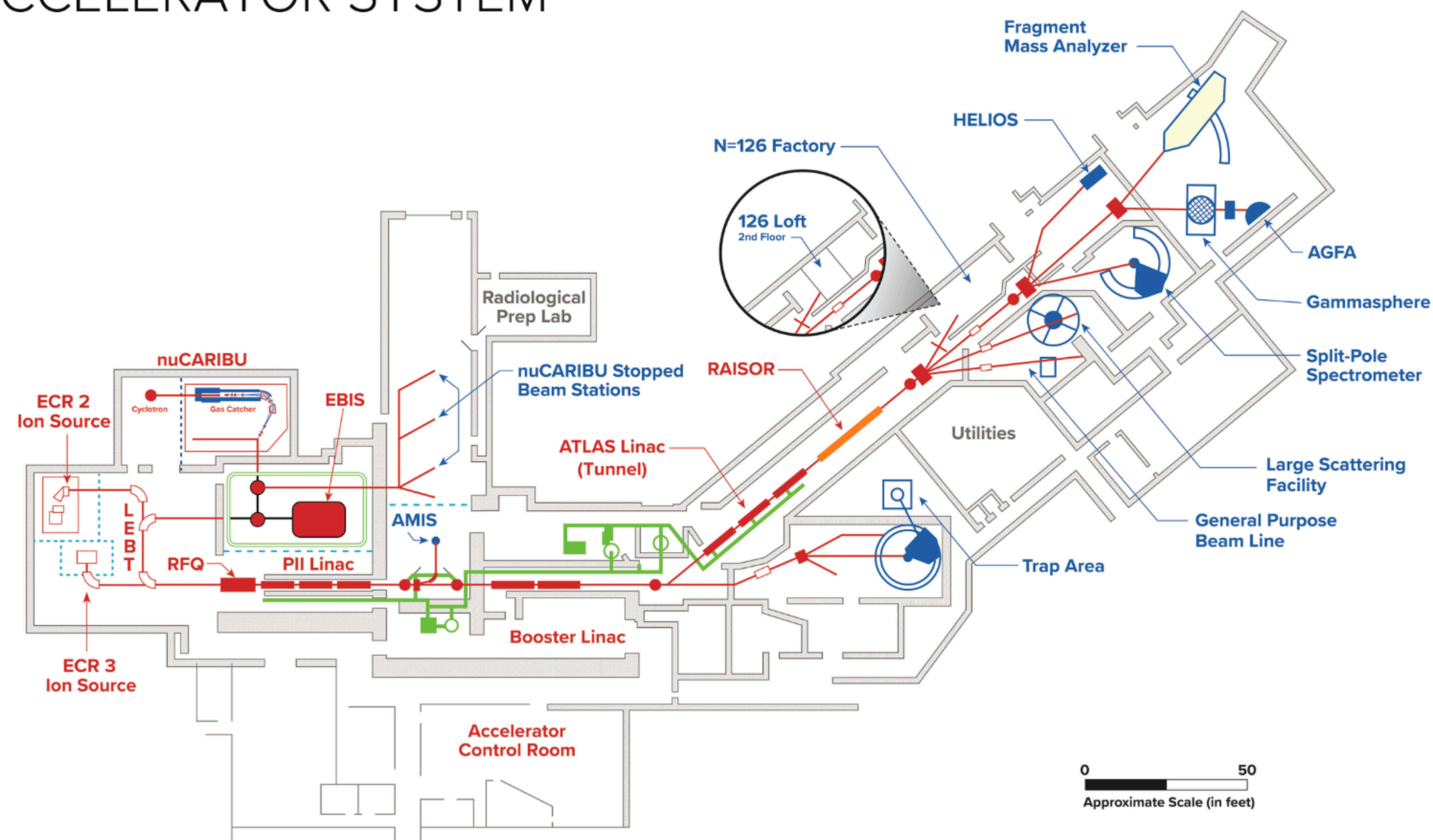
- From 0^+ ground states: $J_{\text{final}} \leq 0 + 3.5 = \mathbf{7/2^-}$ max. ($13/2^-$ is completely inaccessible!)
- From 3^+ initial states: $J_{\text{final}} \leq 3 + 3.5 = \mathbf{13/2^-}$ accessible in single-step transfer!

Ground and isomeric state information for $^{34}_{17}\text{Cl}$				
E(level) (MeV)	J π	Mass Excess (keV)	T $_{1/2}$	Decay Modes
0.0	0+	-24440.09 5	1.5265 s 4	$\epsilon+\beta^+ = 100\%$
0.1464	3+	-24293.73 5	31.99 min 3	$\epsilon+\beta^+ = 55.4\%$ IT = 44.6%

Ground and isomeric state information for $^{38}_{19}\text{K}$				
E(level) (MeV)	J π	Mass Excess (keV)	T $_{1/2}$	Decay Modes
0.0	3+	-28800.76 20	7.645 min 20	$\epsilon+\beta^+ = 100\%$
0.1302	0+	-28670.54 20	924.40 ms 21	$\epsilon+\beta^+ = 99.967\%$ IT = 0.033%

Leverage the Solenoid Spectrometer technique, isomeric in-flight rare-isotope beams & prolific ISOL beams

ATLAS ARGONNE TANDEM LINAC ACCELERATOR SYSTEM

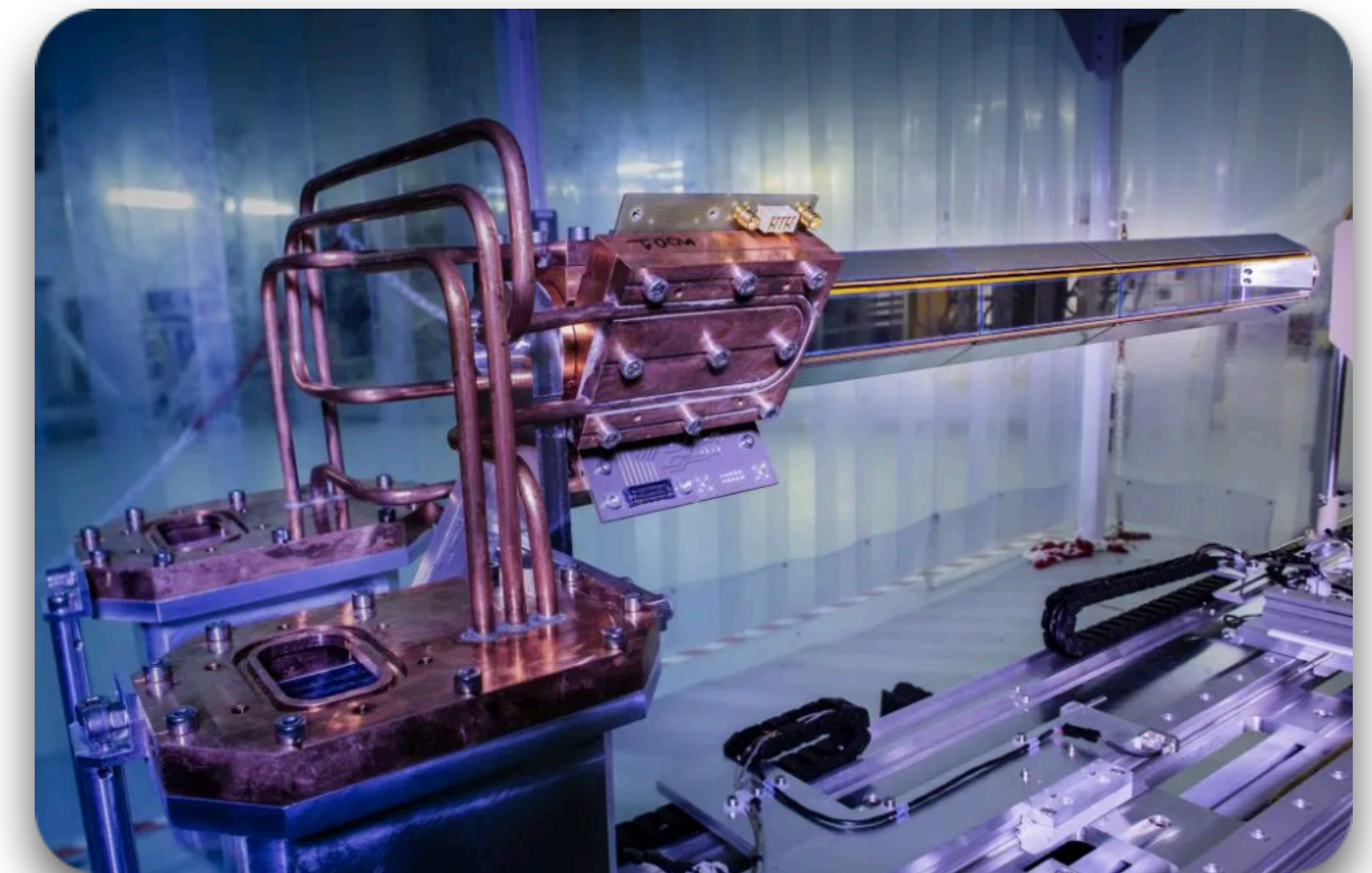
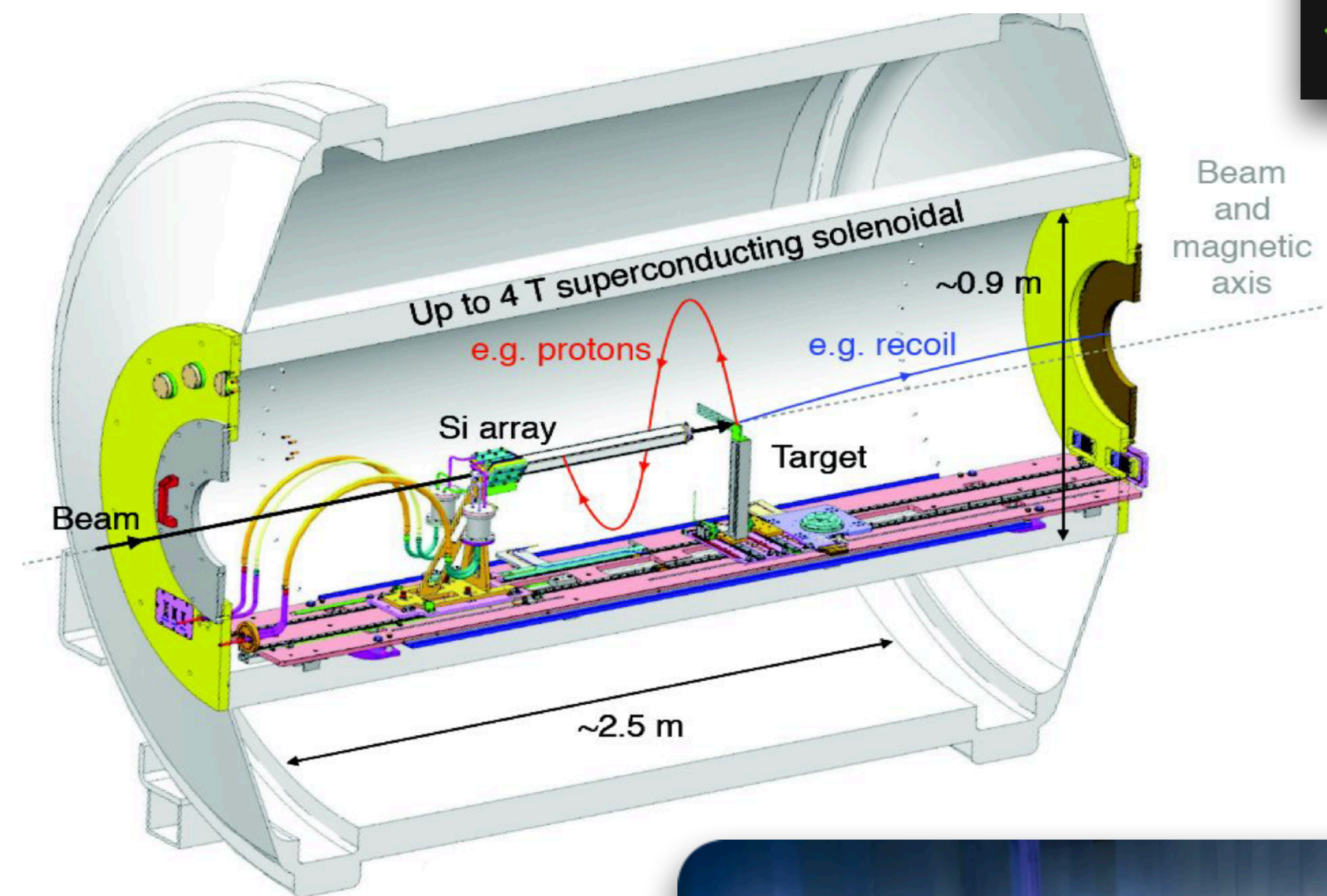


^{34}mCl in-flight beam production:
 $^{33}\text{S}(\text{d},\text{n})$ & $^{34}\text{S}(\text{p},\text{n})$ @ 10 MeV/u
50/50 ground state to isomer ratio

~150 $\mu\text{g}/\text{cm}^2$ deuterated polyethylene
 $0.4 - 2.0 \times 10^5$ pps/sec on target

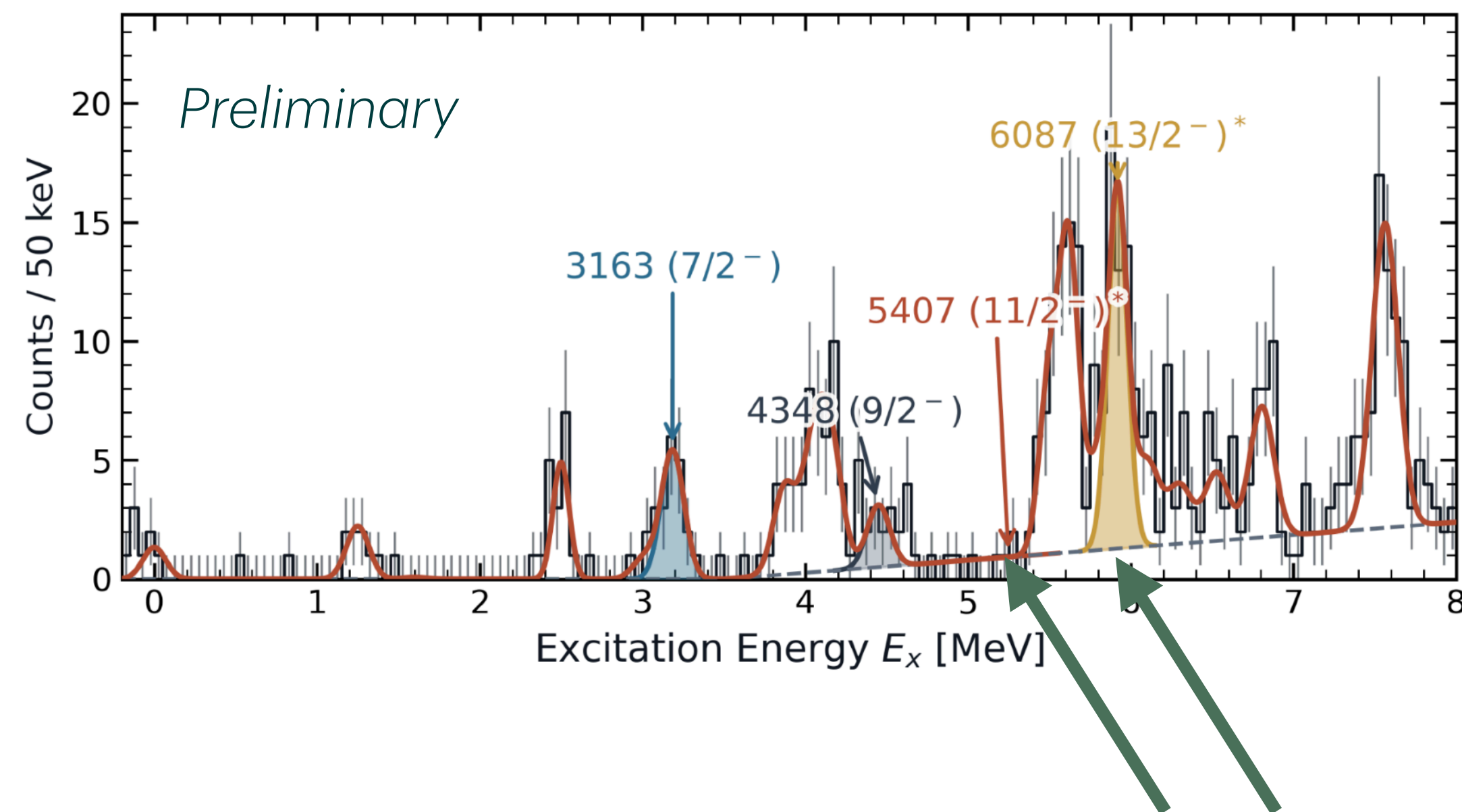


- $^{38}\text{K}^{13+}$ beam:
 - >99% purity [T=0, 3+ ground state]
 - 7.5 MeV/u & > 3×10^6 pps
- ISS Setup:
 - ISS Array & Luminosity detector [no recoil det.]
 - 2 CD_2 targets [$137, 93 \text{ ug/cm}^2$], 2.5 T field
 - ~4.5 Days, CD_2 + C data collection

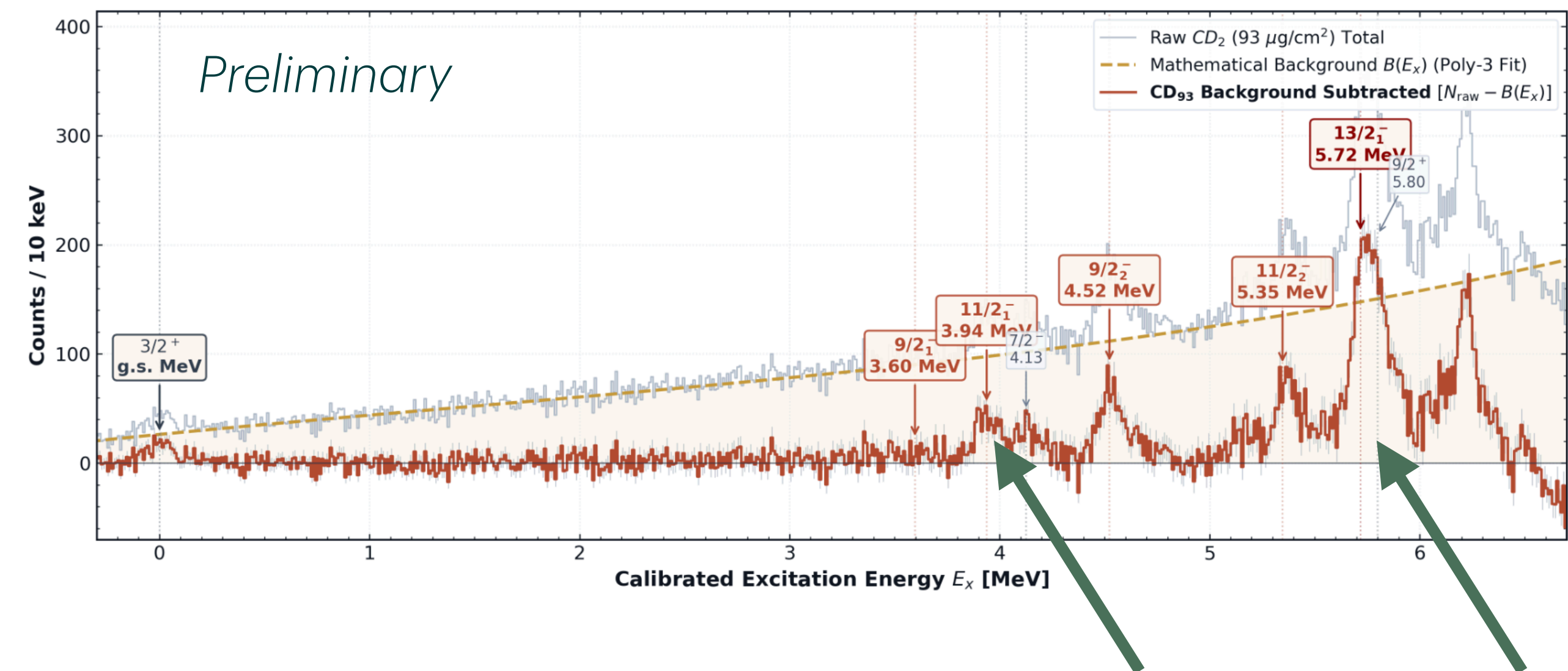


Experimental excitation energy spectra

$^{34}\text{mCl}(d,p)^{35}\text{Cl}$ @ 10 MeV/u



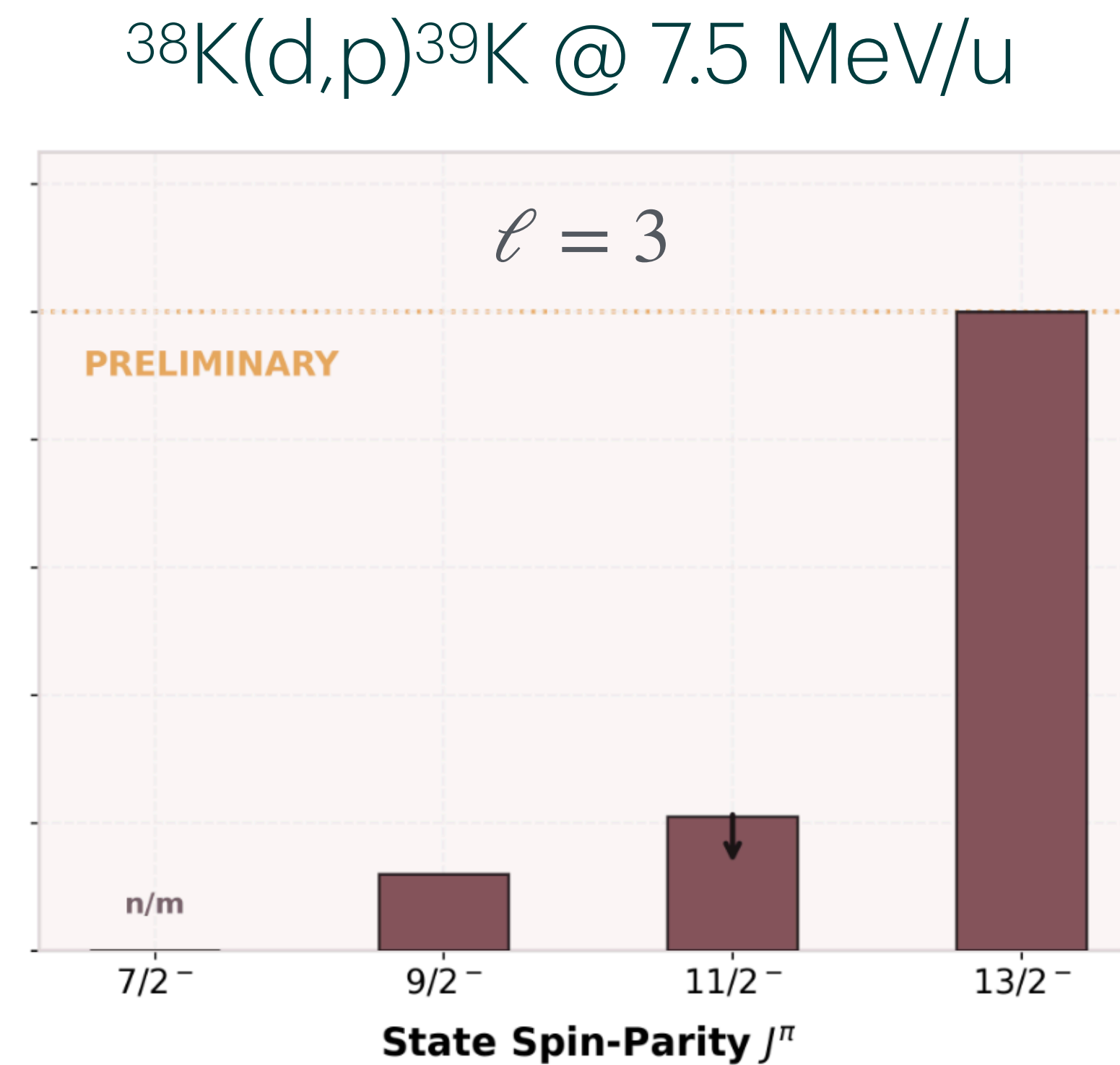
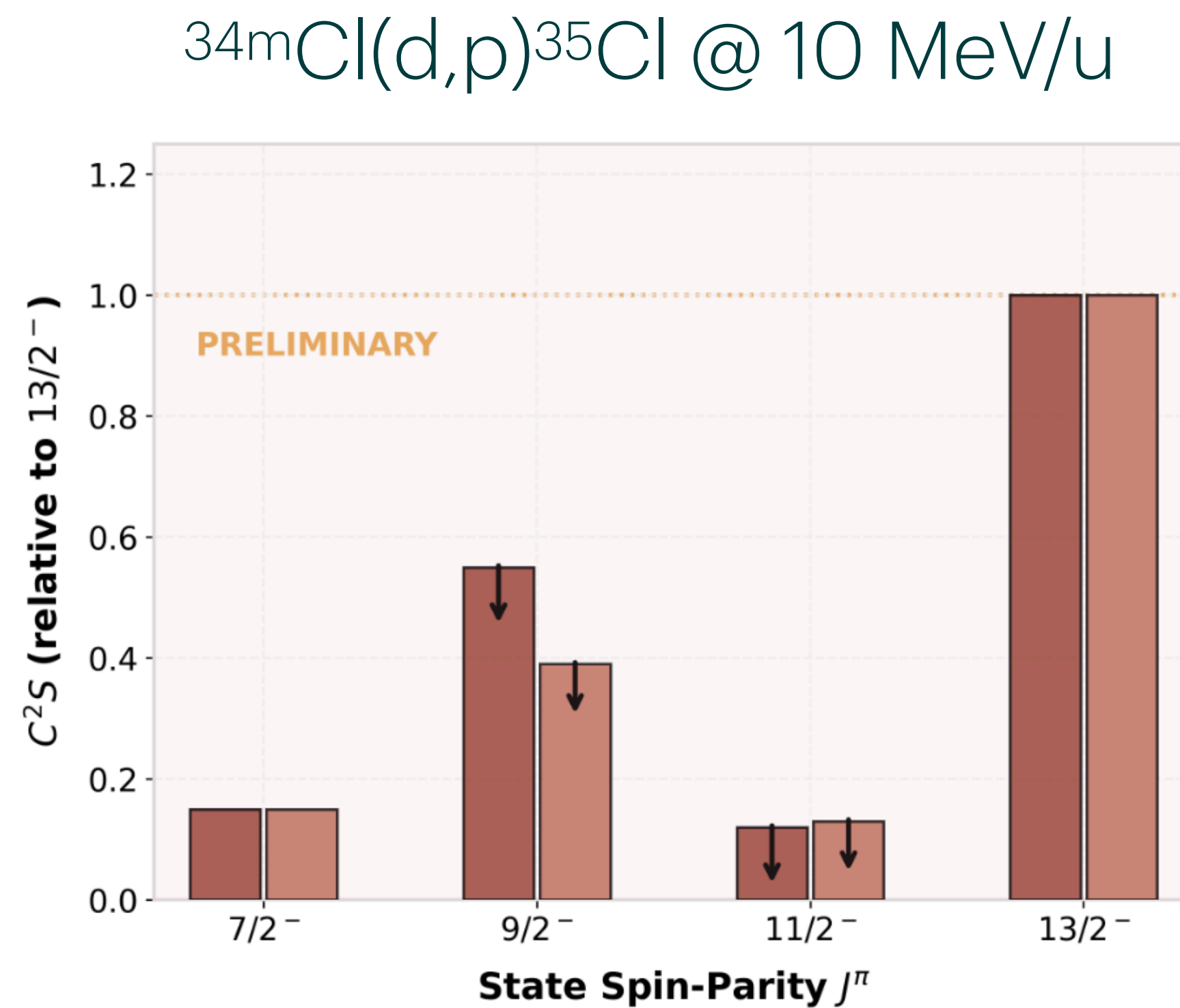
$^{38}\text{K}(d,p)^{39}\text{K}$ @ 7.5 MeV/u



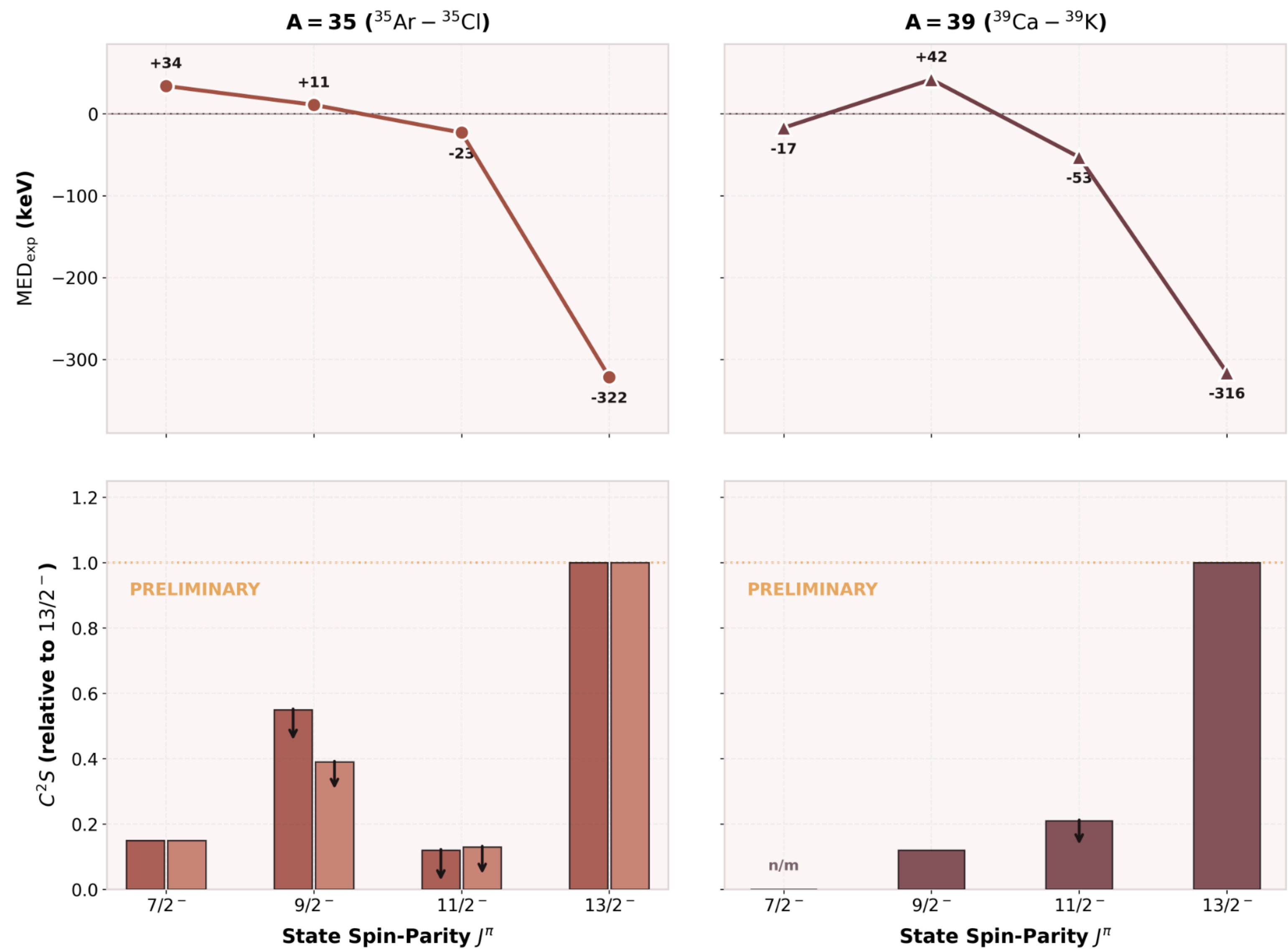
$\text{Of}_{7/2}$ orbital angular momentum transfer confirmed for $13/2^-$ through angular distributions

Single-neutron overlap results

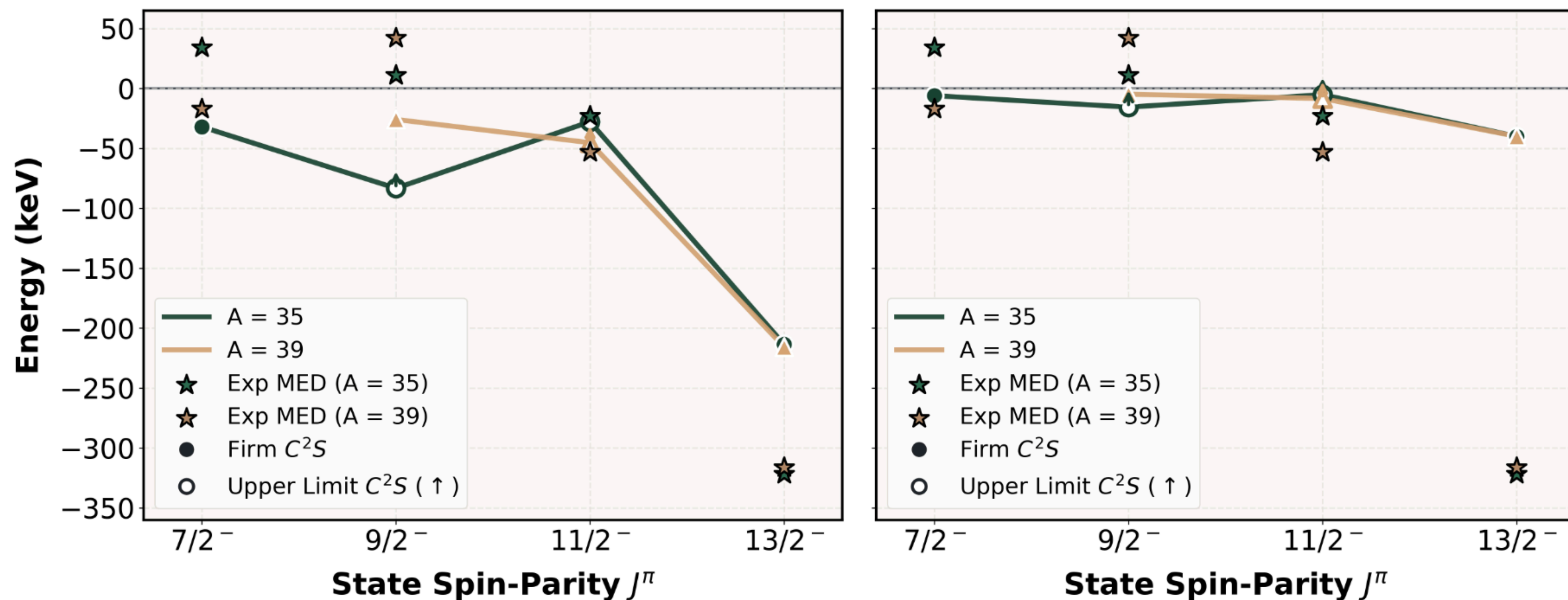
C^2S values extracted from observed (d,p) cross sections via a DWBA analysis



Relationship to the MEDs



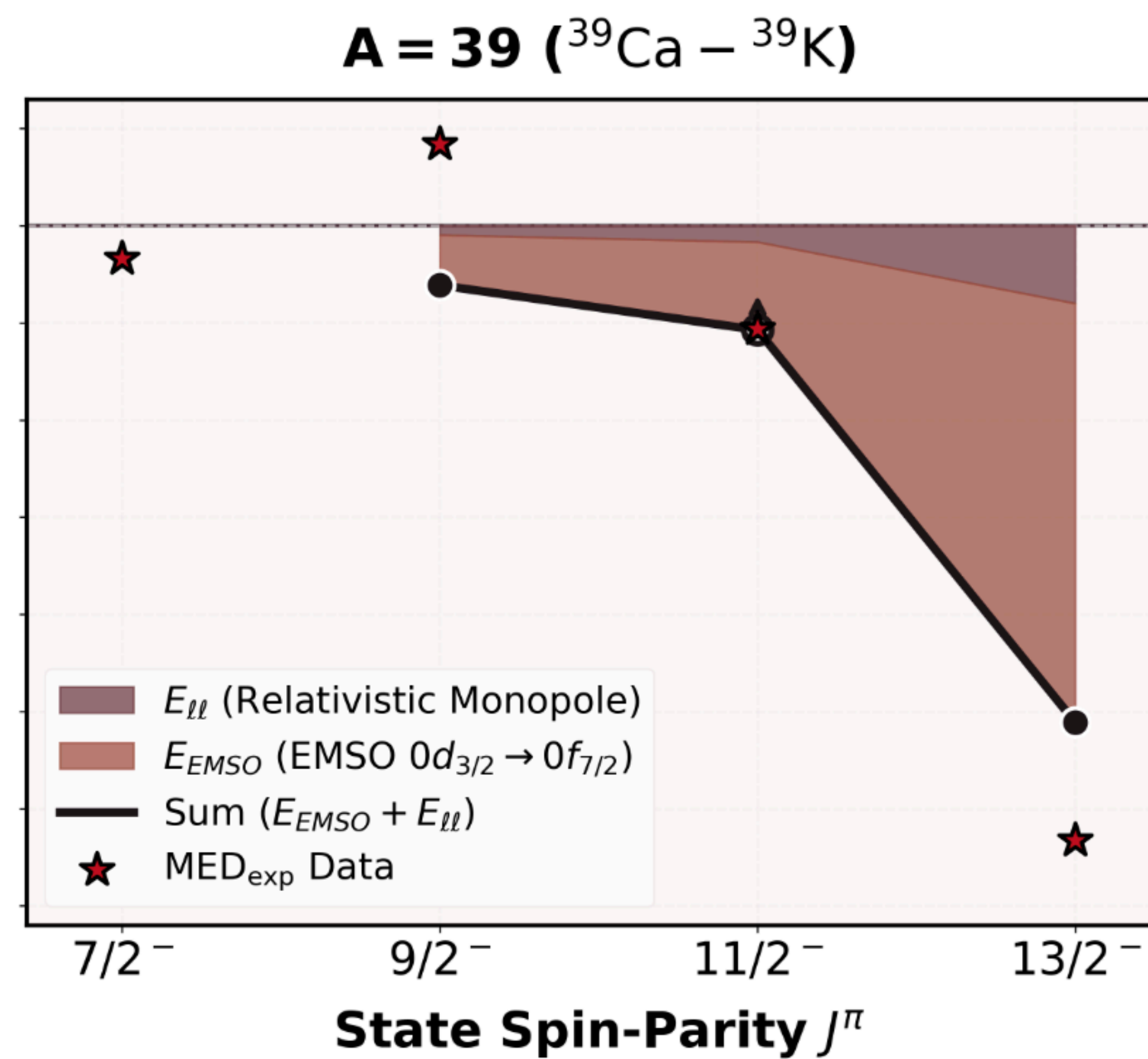
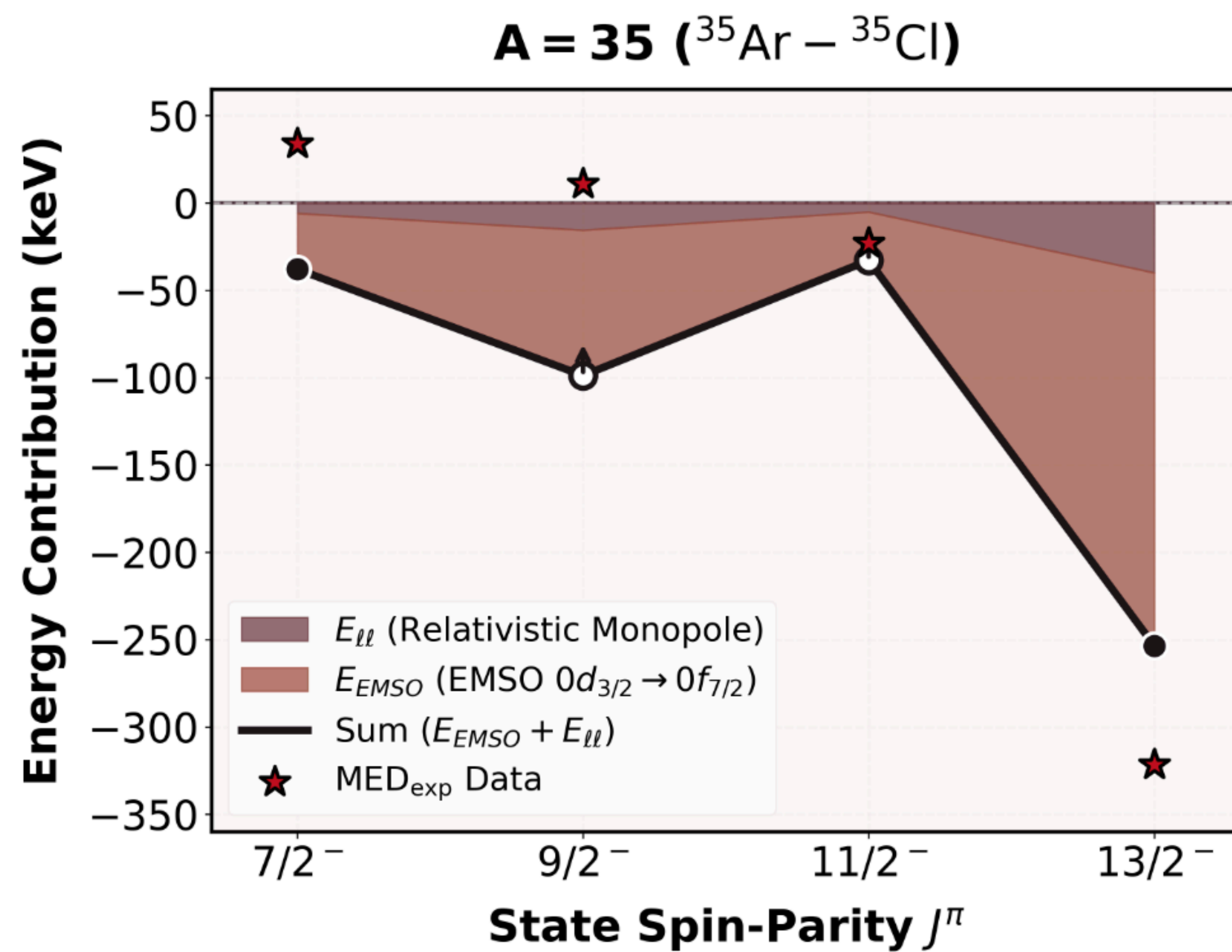
Substitution of the empirical C²S



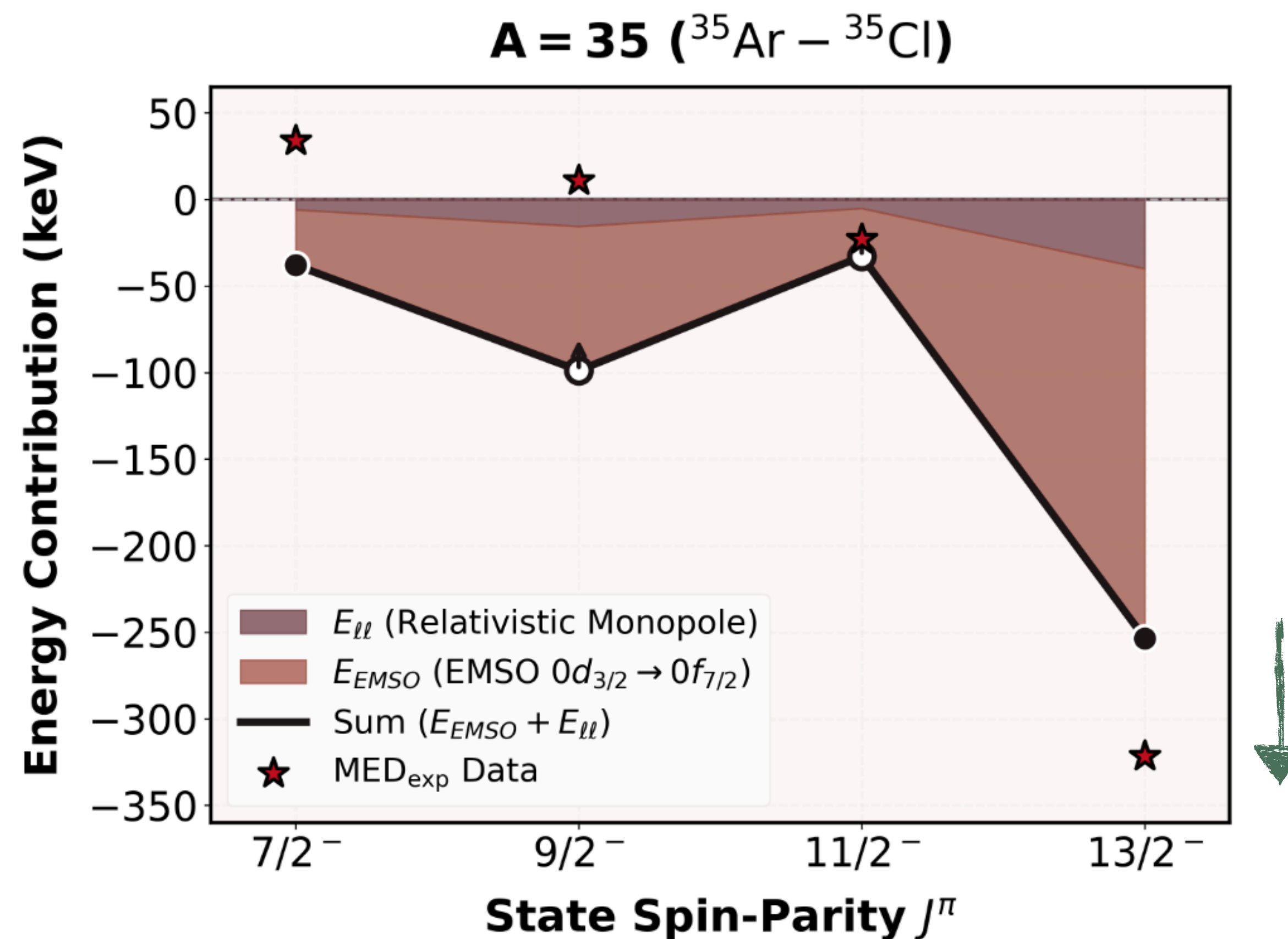
$$E_{\text{EMSO}} \approx (g_s - g_l) \frac{1}{2m_N^2 c^2} \left(-\frac{Ze^2}{R_C^3} \right) \left\langle \vec{l} \cdot \vec{s} \right\rangle$$

$$E_{ll} = \frac{-4.5 Z_{\text{cs}}^{13/12} [2l(l+1) - N(N+3)]}{A^{1/3} (N + \frac{3}{2})} \text{ keV}$$

Substitution of the empirical C²S



Remaining 13/2- MED energy



$$\text{MED} = E_{\text{CM}} + E_{\text{Cm}} + E_{\text{INC}}$$

$$= E_{\text{CM}} + E_{\text{EMSO}} + E_{\ell\ell} + E_{\text{Cr}} + E_{\text{INC}}$$

Of order 100 keV still remains:

Coulomb Multipole: E_{CM}

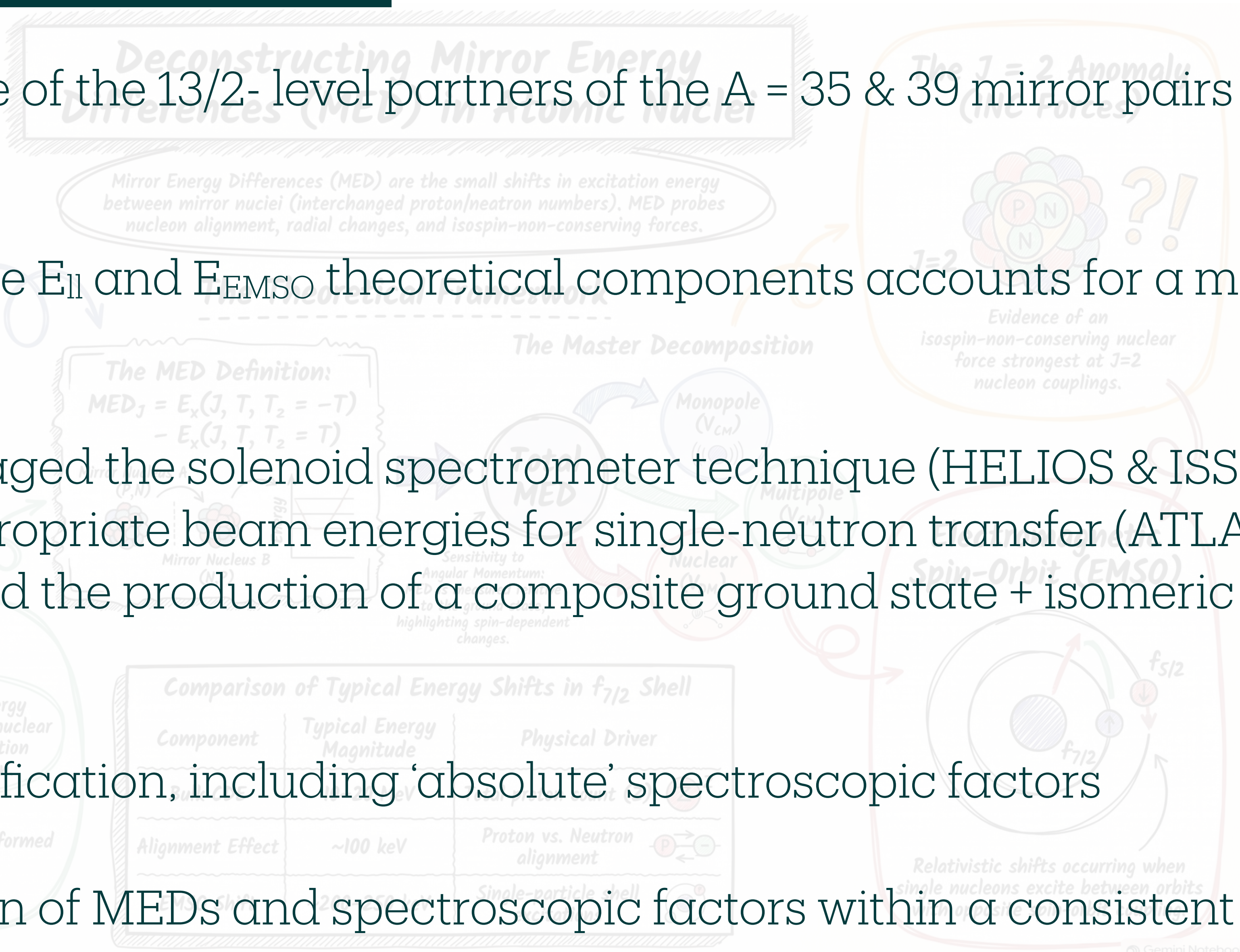
Probable source of bulk of remaining energy
Calculable with many-body wave function
[100 - 200 keV from previous calculations]

Radial monopole: E_{Cr}

May play a role on the 10's of keV level

Summary & Outlook

- The single-neutron nature of the 13/2- level partners of the $A = 35$ & 39 mirror pairs was empirically established
- A simple application of the E_{II} and E_{EMSO} theoretical components accounts for a majority of the MED for the 13/2-
- The measurements leveraged the solenoid spectrometer technique (HELIOS & ISS), rare-isotope beams at the appropriate beam energies for single-neutron transfer (ATLAS & HIE-ISOLDE), and included the production of a composite ground state + isomeric state in-flight beam
- Future focus: Error quantification, including 'absolute' spectroscopic factors
- Future focus: Computation of MEDs and spectroscopic factors within a consistent interaction



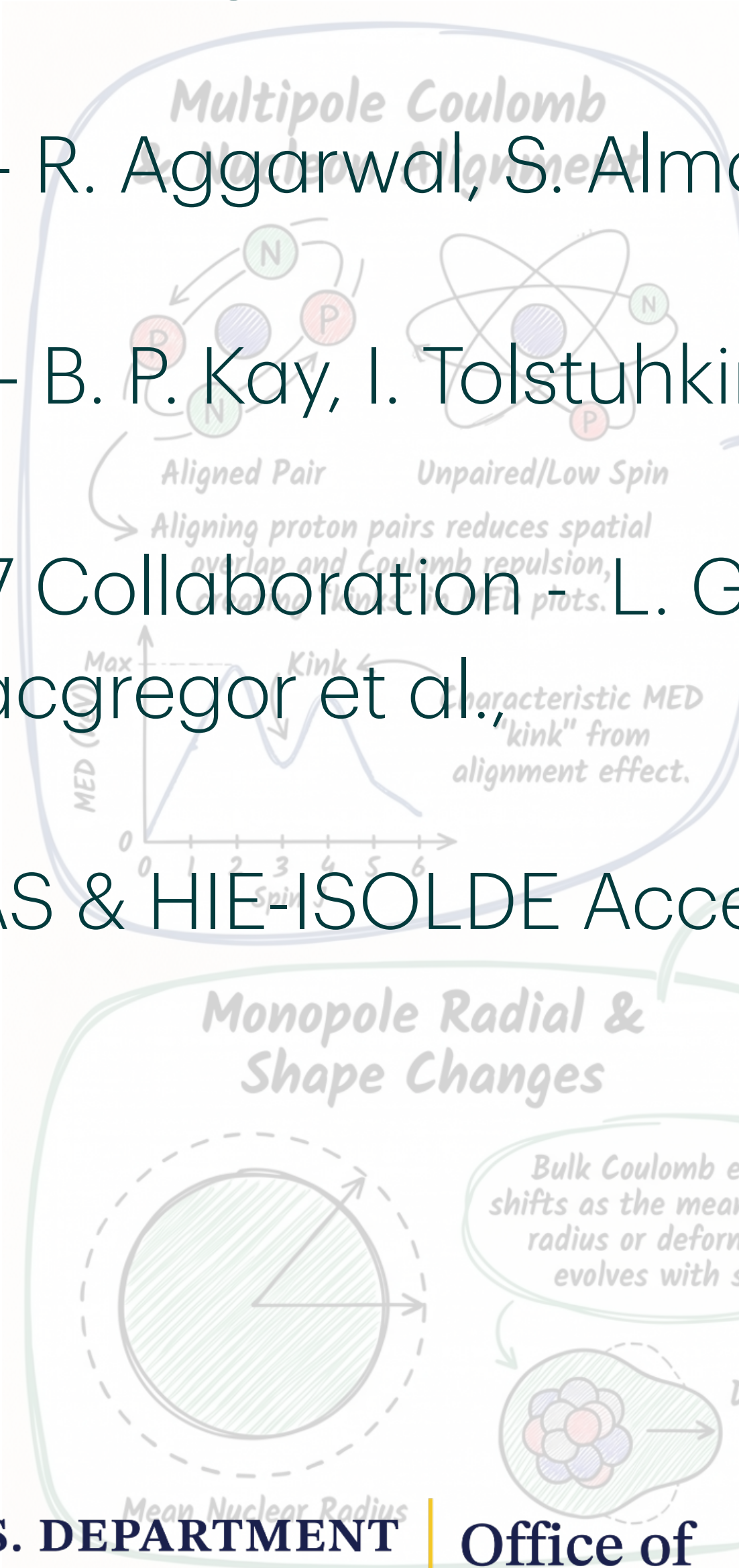
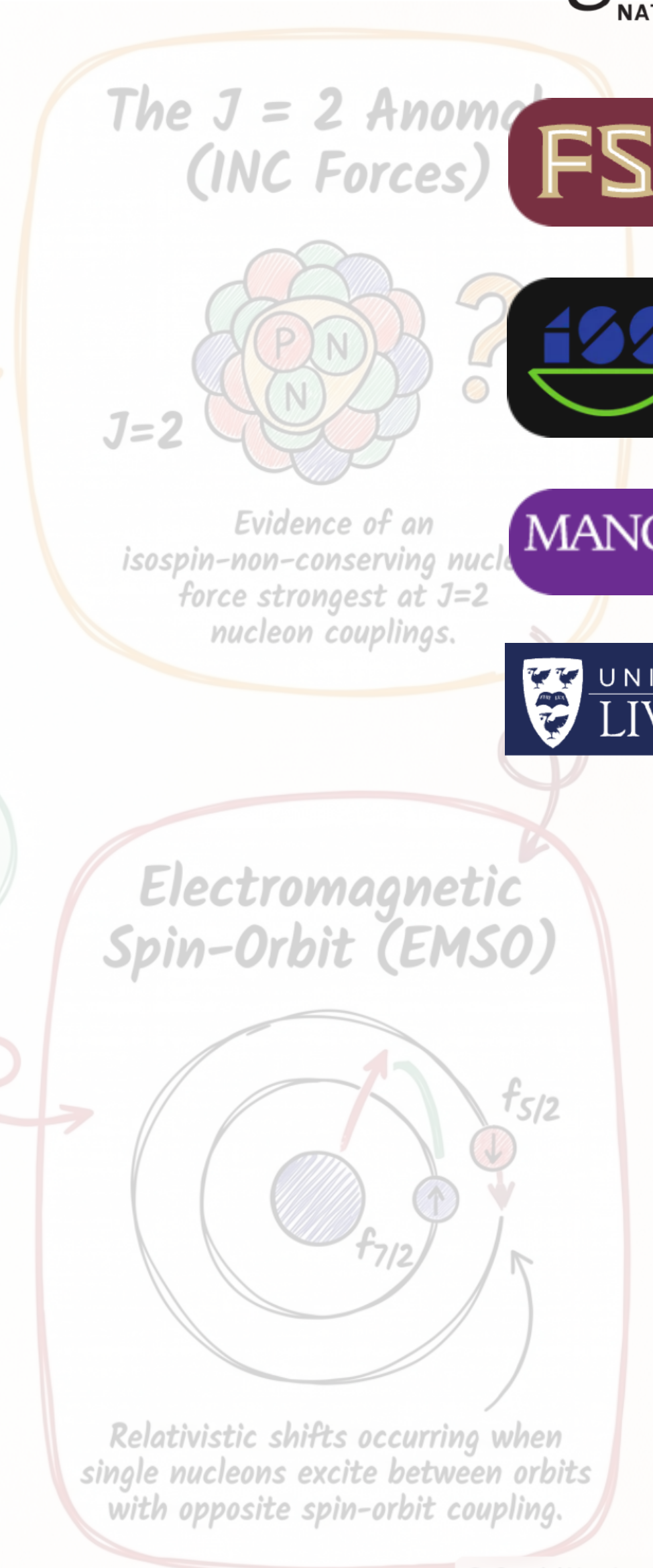
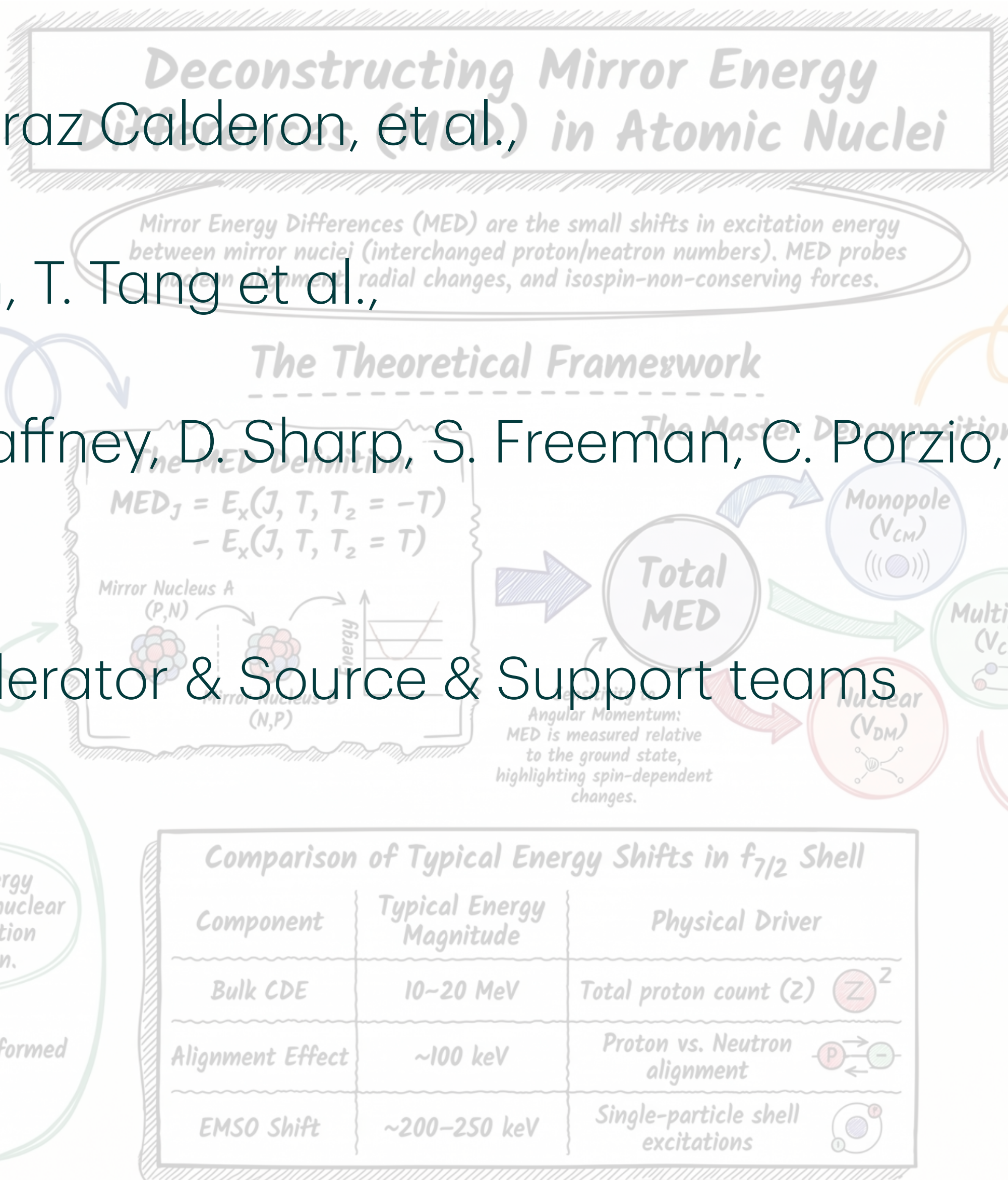
Acknowledgments

- FSU - R. Aggarwal, S. Almaraz Calderon, et al.,
- ANL - B. P. Kay, I. Tolstuhkin, T. Tang et al.,
- IS757 Collaboration - L. Gaffney, D. Sharp, S. Freeman, C. Porzio, P. Macgregor et al.,
- ATLAS & HIE-ISOLDE Accelerator & Source & Support teams

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